4069 State Variable Filter

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1 Introduction

The design for the Yorick filter was found on a thread on the the electro music forum. A user called Hammer had created a filter based around the CD4069 CMOS inverter IC which could run off 5 volts, and give CV control of the cutoff. This document tries to run through the maths on how it works, and is mainly to try and help with figuring out the correct values for the design.

In the first version that Hammer shared they used JFETs as a variable resistance element to handle the voltage control, but specified the 2N5457 which is getting a bit pricy to get hold of, and isn't ideal if we want to keep things cheap. The J113 JFET should work reasonably well in it's place, but it's worth running through the maths to make sure, and see if and how we can improve things.

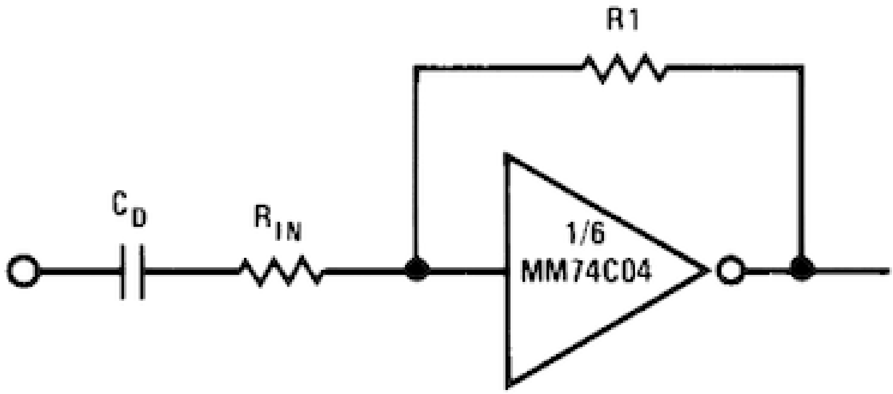
1.1 CMOS Inverter Linear Applications

1.1.1 Inverters as OpAmps

The use of 4069 inverters as amplifiers isn't that uncommon, and it turns out that they can work pretty well. This thread on ModWiggler has some really useful info on the subject, and famously the Wasp filter uses inverters as the integrators that are paired with the 3080 OTAs.

In terms of the analysis, we can assume that it will follow similar behaviour to an inverting op-amp circuit.

Figure 1: A CMOS inverter biased for linear mode operation, taken from the National Semiconductor Application Note 88 July 1973



Here the equation for the output voltage is $V_{out} = -V_{in} * \frac{R1}{R_{in}}$.

1.1.2 Inverters as Integrators

Knowing that the inverter works roughly the same as an inverting op-amp, we can also make the same assumption about how it works as an integrator.

Assuming that the input of the inverter self-biases to half the supply voltage, we can treat it as a virtual ground, meaning that the sum of all currents at that point should equal zero. This means that the current through the capacitor C will be equal to the current flowing through R_i.

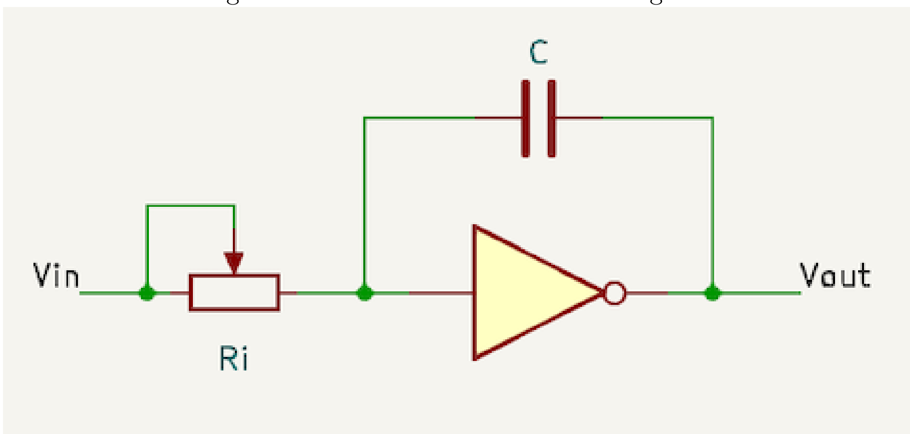
Working in the Laplace domain to make the analysis easier, we can state the following:

s is a complex frequency variable that can represent the phase and amplitude of a signal.

$v_{in}(s)$ is the input voltage in terms of s.

R_i is the value of the resistor through which the input signal is fed to the integrator.

Figure 2: A CMOS inverter based integrator



C is the value of the integrators' capacitors.

$v_c(s)$ is the voltage across the capacitor in terms of s.

$i_c(s)$ is the current flowing into the capacitor in terms of s.

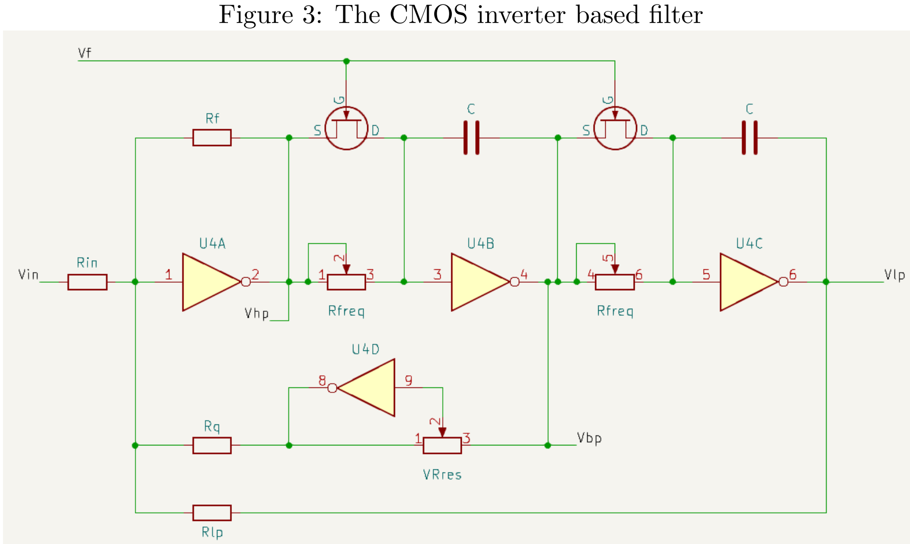
$$
\begin{aligned}
v_c(s) &= \frac{i_c(s)}{Cs} \\
i_c(s) &= \frac{v_{in}(s)}{R_i} \\
v_{out}(s) &= -v_c(s) \\
&= -\frac{i_c(s)}{Cs} \\
&= -\frac{v_{in}(s)}{R_i}\frac{1}{Cs} \\
&= -\frac{v_{in}(s)}{CsR_i}
\end{aligned}
$$

In this case, the resistors R_i are variable, which we can represent as x, being a value from 0 to 1. Using this, we can take the above equation and turn it into a transfer function $\alpha(s)$, representing $\frac{v_{out}(s)}{v_{in}(s)}$.

$$
\begin{aligned}
v_{out}(s) &= -\frac{v_{in}(s)}{CsxR_i} \\
\frac{v_{out}(s)}{v_{in}(s)} &= -\frac{1}{CsxR_i} \\
\alpha(s) &= -\frac{1}{CsxR_i} \\
\alpha(s) &= -x\frac{1}{CsR_i}
\end{aligned}
$$

1.2 Filter Analysis

Figure 3: The CMOS inverter based filter



For the time being we're going to ignore the JFETs and any affect that they might have on the circuit. Having them in parallel with the variable resistors means they can affect the overall resistance of the input to the two integrators and change the frequency response, but we can investigate their affect later.

The filter circuit actually gives us the high-pass, band-pass and low-pass signals as outputs, but we'll be primarily focussed on the low-pass signal here. It's still helpful to refer to these signals as v_{hp}, v_{bp} and v_{lp} though.

1.2.1 Resonance Feedback Amplifier

Inverter U4D along with variable resistor VR_q is responsible for the amount of band-pass signal getting fed back to the input, which controls the overall resonance. Treating it like the inverting amplifier, we can see that as the potentiometer is turned to bring the wiper to pin 1, the effecting feedback resistance will drop and the input resistence will rise, giving a lower gain. Turning the potentiometer to bring the wiper to pin 3 will instead increase the gain.

If y is a value from 0 to 1 for the rotation of the potentiometer, then the gain will be $(-y)VR_{res}$. In the final circuit design it would probably be worth having another resistor in series before the potentiometer to limit the gain.

To simplify the equations for the time being, we'll just say this is a gain factor β which is controlled by the potentiometer rotation.

1.2.2 Input Mixer

The first stage of the filter is a summing mixer built around U4A.

$$\frac{v_{in}(s)}{R_{in}} + \frac{v_{hp}(s)}{R_f} + \frac{v_{lp}(s)}{R_{lp}} + \frac{\beta v_{bp}(s)}{R_q} = 0 \tag{1}$$

It is useful to note that $v_{bp}(s) = v_{hp}(s)\alpha(s)$, and $v_{lp}(s) = v_{hp}(s)\alpha(s^2)$. Also to simplify here we'll set $R_{lp} == R_f$.

1.2.3 High-pass

$$\begin{aligned}
\frac{v_{hp}(s)}{R_f} &= -\frac{v_{in}(s)}{R_{in}} - \frac{v_{lp}(s)}{R_f} - \frac{v_{bp}(s)\beta}{R_q} \\
\frac{v_{hp}(s)}{R_f} &= -\frac{v_{in}(s)}{R_{in}} - \frac{v_{hp}(s)\alpha(s^2)}{R_f} - \frac{\beta v_{hp}(s)\alpha(s)}{R_q} \\
\frac{v_{in}(s)}{R_{in}} &= -\frac{v_{hp}(s)}{R_f} - \frac{v_{hp}(s)\alpha(s^2)}{R_f} - \frac{\beta v_{hp}(s)\alpha(s)}{R_q} \\
H_{hp}(s) &= \frac{v_{hp}(s)}{v_{in}(s)} \\
\frac{v_{in}(s)}{R_{in}} &= -v_{hp}(s)(\frac{1}{R_f} + \frac{\alpha(s^2)}{R_f} + \frac{\beta\alpha(s)}{R_q}) \\
\frac{1}{R_{in}} &= -H_{hp}(s)(\frac{1}{R_f} + \frac{\alpha(s^2)}{R_f} + \frac{\beta\alpha(s)}{R_q}) \\
\frac{-1}{R_{in}} &= H_{hp}(s)(\frac{1}{R_f} + \frac{\alpha(s^2)}{R_f} + \frac{\beta\alpha(s)}{R_q}) \\
H_{hp}(s) &= \frac{-1/R_{in}}{\frac{1}{R_f} + \frac{\alpha(s^2)}{R_f} + \frac{\beta\alpha(s)}{R_q}} \\
&= \frac{-R_f/R_{in}}{1 + \frac{\beta R_f\alpha(s)}{R_q} + \alpha(s^2)}
\end{aligned}$$

1.2.4 Low-Pass

$$
\begin{aligned}
H_{lp}(s) &= \frac{v_{lp}(s)}{v_{in}(s)} \\
&= \frac{v_{hp}(s)}{v_{in}(s)}\alpha^2(s) \\
&= \frac{-R_f/R_{in}}{\frac{1}{\alpha^2(s)} + \frac{R_f\beta}{R_q\alpha(s)} + 1} \\
&= \frac{-R_f/R_{in}}{\frac{1}{\alpha^2(s)} + \beta\frac{R_f}{R_q}\frac{1}{\alpha(s)} + 1} \\
&= \frac{-R_f/R_{in}}{\frac{1}{(-x\frac{1}{CsR_i})^2} + (-y)VR_{res}\frac{R_f}{R_q}\frac{1}{-x\frac{1}{CsR_i}} + 1} \\
&= \frac{-R_f/R_{in}}{\frac{1}{\frac{1}{s^2}(x\frac{1}{CR_i})^2} + yVR_{res}\frac{R_f}{R_q}\frac{1}{x\frac{1}{s}\frac{1}{CR_i}} + 1} \\
&= \frac{-R_f/R_{in}}{\frac{s^2}{(x\frac{1}{CR_i})^2} + yVR_{res}\frac{R_f}{R_q}\frac{s}{x\frac{1}{CR_i}} + 1}
\end{aligned}
$$

At this point we'll also make the simplification that $R_q = R_f$, giving the final transfer function for the low-pass output.

$$
H_{lp}(s) = \frac{-R_f/R_{in}}{\frac{s^2}{(x\frac{1}{CR_i})^2} + yVR_{res}\frac{s}{x\frac{1}{CR_i}} + 1} \tag{2}
$$

1.2.5 Calculating Gain, Cutoff and Resonance

For reference, the standard second-order filter transfer functions are:

$$
\begin{aligned}
H_{lp}(s) &= \frac{1}{s^2T^2 + \frac{1}{q}sT + 1} \\
H_{bp}(s) &= \frac{1}{\frac{q}{sT} + qsT + 1} \\
H_{hp}(s) &= \frac{1}{\frac{1}{s^2T^2} + \frac{1}{qsT} + 1}
\end{aligned}
$$

By comparing our transfer function with the standard one for a low-pass, we can work out the following.

Pass-band gain, $-\frac{R_f}{R_{in}}$

Cutoff frequency, $x\frac{1}{CR_i}$

Resonance, $\frac{1}{yVR_{res}}$

1.3 Calculating Cutoff Frequencies

Given the calculation for frequency, and picking some standard values we can calculate cutoff for different i_{cv} values.

R_i is between 100r and 100k

C is 100nF

$$f = \frac{1}{CR_i} \tag{3}$$

for R_i of 100r $f = 100000Hz$

for R_i of 1k $f = 10000Hz$

for R_i of 10k $f = 1000Hz$

for R_i of 50k $f = 200Hz$

for R_i of 100k $f = 100Hz$

1.4 Calculating Resonance

The resonance here is directly proportional to the gain of the resonance feedback stage. A value of around 0.5 is the best minimum, with higher values being more resonant. This means that a maximum gain of 2 is best for the stage, and the mimimum being close to 0.