

Dynamic Programming - Pole Cutting

COMS10018 - Algorithms

Dr Christian Konrad

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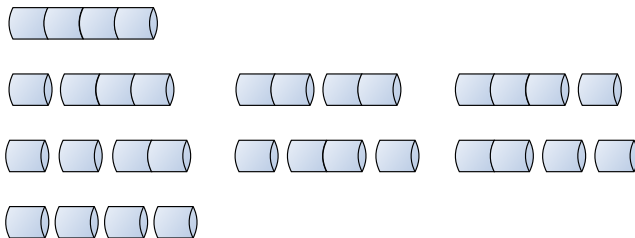
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Problem:

- Find best out of 2^{n-1} possibilities
- We could disregard similar cuts, but we would still have an exponential number of possibilities
- A fast algorithm cannot try out all possibilities

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- Optimal revenue r_n :

$$r_n = p(i_1) + p(i_2) + \cdots + p(i_k)$$

Pole Cutting (5)

What are the optimal revenues r_i ?

length i	1	2	3	4	5	6	7	8	9	10
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r_8	=	22		$8 = 2 + 6$
r_9	=	25		$9 = 3 + 6$
r_{10}	=	30		$10 = 10$

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- p_n corresponds to the situation of no cut at all
- $r_i + r_{n-i}$: initial cut into two pieces of sizes i and $n - i$

Pole Cutting: Dynamic Programming Formulation

Simpler Recursive Formulation: Let $r_0 = 0$

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Recursive Top-down Implementation

Recall:

$$r_n = \max_{1 \leq i \leq n} (p_i + r_{n-i}) \text{ and } r_0 = 0 .$$

Direct Implementation:

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Require: Integer  $n$ , Array  $p$  of length  $n$  with prices  
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 $q \leftarrow -\infty$   
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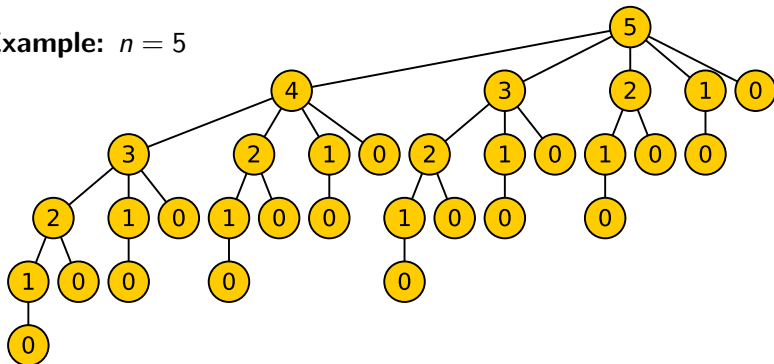
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How efficient is this algorithm?

Recursion Tree for CUT-POLE

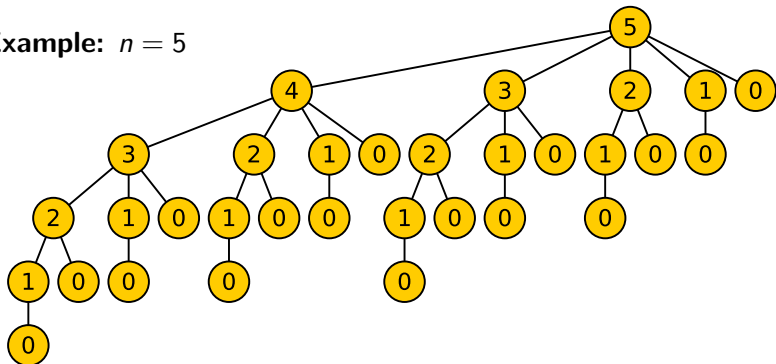
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Recursion Tree for CUT-POLE

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This implies $T(i) = 2^i$.

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- Observe: We compute solutions to subproblems many times
- Avoid this by storing solutions to subproblems in a table!
- This is a key feature of dynamic programming

Implementing the Dynamic Programming Approach

Top-down with memoization

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- Fill table T from smallest to largest index

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- When computing r_i , store r_i in a table T (of size n)
- Before computing r_i again, check in T whether r_i has previously been computed

Bottom-up

- Fill table T from smallest to largest index
- No recursive calls are needed for this

Top-down Approach

Require: Integer n , Array p of length n with prices
Let $r[0 \dots n]$ be a new array
for $i = 0 \dots n$ **do**
 $r[i] \leftarrow -\infty$
return MEMOIZED-CUT-POLE-AUX(p, n, r)

Algorithm MEMOIZED-CUT-POLE(p, n)

- Prepare a table r of size n
- Initialize all elements of r with $-\infty$
- Actual work is done in MEMOIZED-CUT-POLE-AUX, table r is passed on to MEMOIZED-CUT-POLE-AUX

Top-down Approach (2)

```
Require: Integer  $n$ , array  $p$  of length  $n$  with prices, array  $r$  of  
revenues  
if  $r[n] \geq 0$  then  
    return  $r[n]$   
if  $n = 0$  then  
     $q \leftarrow 0$   
else  
     $q \leftarrow -\infty$   
    for  $i = 1 \dots n$  do  
         $q \leftarrow \max\{q, p[i] + \text{MEMOIZED-CUT-POLE-AUX}(p, n -$   
             $i, r)\}$   
     $r[n] \leftarrow q$   
    return  $q$ 
```

Algorithm MEMOIZED-CUT-POLE-AUX(p, n, r)

Observe: If $r[n] \geq 0$ then $r[n]$ has been computed previously

Bottom-up Approach

Require: Integer n , array p of length n with prices

Let $r[0 \dots n]$ be a new array

$r[0] \leftarrow 0$

for $j = 1 \dots n$ **do**

$q \leftarrow -\infty$

for $i = 1 \dots j$ **do**

$q \leftarrow \max\{q, p[i] + r[j - i]\}$

$r[j] \leftarrow q$

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Algorithm BOTTOM-UP-CUT-POLE(p, n)

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Runtime: Two nested for-loops

$$\sum_{j=1}^n \sum_{i=1}^j O(1) = O(1) \sum_{j=1}^n \sum_{i=1}^j 1 = O(1) \sum_{j=1}^n j = O(1) \frac{n(n+1)}{2} = O(n^2) .$$

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Runtime of Top-down Approach $O(n^2)$

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Dynamic Programming

- Solves a problem by combining subproblems
- Subproblems are solved at most once, store solutions in table
- If a problem exhibits *optimal substructure* then dynamic programming is often the right choice

Runtime of Top-down Approach $O(n^2)$

(please think about this!)

Dynamic Programming

- Solves a problem by combining subproblems
- Subproblems are solved at most once, store solutions in table
- If a problem exhibits *optimal substructure* then dynamic programming is often the right choice
- Top-down and bottom-up approaches have the same runtime