Reference: Simply-typed λ -calculus

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Figure 1: Statics of the simply-typed λ -calculus (with numbers)

Figure 2: Dynamics of the simply-typed λ -calculus

Val-Unit	Val-Pair	Val-Inl	Val-Inr	Val-Lam	
$\overline{\langle \rangle}$ val	$\overline{\langle e_1, e_2 angle}$ val	$\overline{inl(e) val}$	$\overline{inr(e) val}$	$\overline{\lambda x}: au.\ e$ val	
$rac{n \in \mathbb{N}}{num[n] \ val}$		$\frac{n_1+n_2=n}{plus(num[n_1];num[n_2]) \longmapsto num[n]}$		$\frac{e_1 \longmapsto e_1'}{plus(e_1; e_2) \longmapsto plus(e_1'; e_2)}$	
$\frac{\text{D-Plus-2}}{e_1 \text{ val}} e_2 \longmapsto \text{plus}(e_1; e_2) \longmapsto \text{plus}(e_2; e_3) $				$\frac{\text{D-Proj-Tuple-1}}{\pi_1(\langle e_1, e_2 \rangle) \longmapsto e_1}$	
$\frac{\text{D-Proj-Tuple-2}}{\pi_1(\langle e_1, e_2 \rangle) \longmapsto e_2}$		D-Proj-1 $\frac{e \longmapsto e'}{\pi_1(e) \longmapsto \pi_1(e')}$	$ \frac{e \longmapsto e'}{\pi_2(e) \longmapsto \pi_2} $		
$\frac{e \longmapsto e'}{abort(e) \longmapsto abo}$	$\overline{\operatorname{ort}(e')}$	$\frac{\text{D-Case-Inl}}{case(inl(e); x. e_1;}$	$y. e_2) \longmapsto e_1[e/x]$		
D-Case-Inr $\overline{case(inr(e); x. e_1; y. e_1)}$	$e_2) \longmapsto e_2[e/y]$				
$e_1 \vdash e_1 \vdash e_1 \vdash e_1 \vdash e_2) \vdash e_1 \vdash e_2 \vdash e_3 \vdash e$	$\xrightarrow{\rightarrow e_1'} e_1'(e_2)$	D-Beta $(\lambda x : \tau. e_1)(e_2) \longmapsto e_1[e_2/x]$			