Reference: PCF

Alex Kavvos

Figure 1: Statics of PCF

$$\textsc{Var}\ \frac{}{\Gamma, x : \sigma \vdash x : \sigma} \qquad \textsc{Zero}\ \frac{}{\Gamma \vdash \mathsf{zero} : \mathsf{Nat}} \qquad \textsc{Succ}\ \frac{\Gamma \vdash e : \mathsf{Nat}}{\Gamma \vdash \mathsf{succ}(e) : \mathsf{Nat}}$$

$$\textsc{Lam}\ \frac{\Gamma, x : \sigma \vdash e : \tau}{\Gamma \vdash \lambda x : \sigma.\, e : \sigma \rightharpoonup \tau} \qquad \textsc{App}\ \frac{\Gamma \vdash e_1 : \sigma \rightharpoonup \tau \qquad \Gamma \vdash e_2 : \sigma}{\Gamma \vdash e_1(e_2) : \tau}$$

$$\textsc{IfZero}\ \frac{\Gamma \vdash e : \mathsf{Nat} \qquad \Gamma \vdash e_0 : \tau \qquad \Gamma, x : \mathsf{Nat} \vdash e_1 : \tau}{\Gamma \vdash \mathsf{ifz}(e; e_0; x.\, e_1) : \tau} \qquad \textsc{Fix}\ \frac{\Gamma, x : \tau \vdash e : \tau}{\Gamma \vdash \mathsf{fix}(x : \tau.\, e) : \tau}$$

Figure 2: Dynamics of PCF

$$\textsc{Val-Zero}\ \frac{}{\mathsf{zero}\ \mathsf{val}} \qquad \textsc{Val-Succ}\ \frac{e\ \mathsf{val}}{\mathsf{succ}(e)\ \mathsf{val}} \qquad \textsc{Val-Lam}\ \frac{}{\lambda x : \tau.\, e\ \mathsf{val}} \qquad \textsc{D-Succ}\ \frac{e \longmapsto e'}{\mathsf{succ}(e) \longmapsto \mathsf{succ}(e')}$$

$$\textsc{D-App-1}\ \frac{e_1 \longmapsto e_1'}{e_1(e_2) \longmapsto e_1'(e_2)} \qquad \textsc{D-Beta}\ \frac{}{(\lambda x : \tau.\, e_1)(e_2) \longmapsto e_1[e_2/x]}$$

$$\textsc{D-Fix}\ \frac{}{\mathsf{fix}(x : \tau.\, e) \longmapsto e[\mathsf{fix}(x : \tau.\, e)/x]} \qquad \textsc{D-Ifz-1}\ \frac{e \longmapsto e'}{\mathsf{ifz}(e; e_0; x.\, e_1) \longmapsto \mathsf{ifz}(e'; e_0; x.\, e_1)}$$

$$\textsc{D-Ifz-Zero}\ \frac{}{\mathsf{ifz}(\mathsf{zero}; e_0; x.\, e_1) \longmapsto e_0} \qquad \textsc{D-Ifz-Succ}\ \frac{\mathsf{succ}(e)\ \mathsf{val}}{\mathsf{ifz}(\mathsf{succ}(e); e_0; x.\, e_1) \longmapsto e_1[e/x]}$$