Problem Sheet 6

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The following questions are about call-by-name (CBN) and call-by-value (CBV).

1. Show that the STLC+print term $e \stackrel{\text{def}}{=} e_1(e_2)$ is well-typed. Then, reduce it to a value twice: once using the CBN, and once using the CBV dynamics. Feel free to skip ε in transitions that do not print anything.

$$\begin{aligned}
u &\stackrel{\text{def}}{=} \lambda x : \mathsf{Num}.\, \mathsf{print}(\text{`batman'}; \mathsf{plus}(x; \mathsf{num}[1])) \\
e_1 &\stackrel{\text{def}}{=} \lambda f : \mathsf{Num} \to \mathsf{Num}.\, f(f(\mathsf{num}[2])) \\
e_2 &\stackrel{\text{def}}{=} \mathsf{print}(\text{`na'}; u) \\
e &\stackrel{\text{def}}{=} e_1(e_2)
\end{aligned}$$

2. Do you need to prove progress and preservation for the CBV STLC, or can you say that they have already been established? Do not do any proofs in the answer to this question (unless you want to).

The following question is about Modernised Algol.

3. (∗) Assuming that weakening is a rule of the system for both terms and commands, show that the following typing rules concerning various definable constructs are derivable.

   [Hint: Do them in the order presented below, so that you may assume that some rules are derivable while showing the next one.]

$$\frac{\Gamma \vdash_\Sigma m_1\ \mathsf{ok} \qquad \Gamma, x : \mathsf{Nat} \vdash_\Sigma m_2\ \mathsf{ok}}{\Gamma \vdash_\Sigma \{x \leftarrow m_1; m_2\}\ \mathsf{ok}} \qquad \frac{\Gamma \vdash_\Sigma m_1\ \mathsf{ok} \qquad \Gamma \vdash_\Sigma m_2\ \mathsf{ok}}{\Gamma \vdash_\Sigma \{m_1; m_2\}\ \mathsf{ok}}$$

$$\frac{\Gamma \vdash_\Sigma e : \mathsf{Cmd}}{\Gamma \vdash_\Sigma \mathsf{do}\, e\ \mathsf{ok}} \qquad \frac{\Gamma \vdash_\Sigma m\ \mathsf{ok} \qquad \Gamma \vdash_\Sigma m_1\ \mathsf{ok} \qquad \Gamma \vdash_\Sigma m_2\ \mathsf{ok}}{\Gamma \vdash_\Sigma \mathsf{if}\, m\, \mathsf{then}\, m_1\, \mathsf{else}\, m_2\ \mathsf{ok}}$$

$$\frac{\Gamma \vdash_\Sigma m\ \mathsf{ok} \qquad \Gamma \vdash_\Sigma m^\star\ \mathsf{ok}}{\Gamma \vdash_\Sigma \mathsf{while}\,(m)\{m^\star\}\ \mathsf{ok}} \qquad \frac{\Gamma, x : \tau \vdash_\Sigma m\ \mathsf{ok}}{\Gamma \vdash_\Sigma \mathsf{proc}\,(x : \tau)\,\{m\} : \tau \rightharpoonup \mathsf{Cmd}}$$

$$\frac{\Gamma \vdash_\Sigma e_1 : \tau \rightharpoonup \mathsf{Cmd} \qquad \Gamma \vdash_\Sigma e_2 : \tau}{\Gamma \vdash_\Sigma \mathsf{call}\, e_1(e_2)\ \mathsf{ok}}$$