

## REFERENCE: PCF

Alex Kavvos

Figure 1: Statics of PCF

$\frac{\text{VAR}}{\Gamma, x : \sigma \vdash x : \sigma}$	$\frac{\text{ZERO}}{\Gamma \vdash \text{zero} : \text{Nat}}$	$\frac{\text{Succ} \quad \Gamma \vdash e : \text{Nat}}{\Gamma \vdash \text{succ}(e) : \text{Nat}}$
$\frac{\text{LAM} \quad \Gamma, x : \sigma \vdash e : \tau}{\Gamma \vdash \lambda x : \sigma. e : \sigma \rightarrow \tau}$	$\frac{\text{APP} \quad \Gamma \vdash e_1 : \sigma \rightarrow \tau \quad \Gamma \vdash e_2 : \sigma}{\Gamma \vdash e_1(e_2) : \tau}$	
$\frac{\text{IFZERO} \quad \Gamma \vdash e : \text{Nat} \quad \Gamma \vdash e_0 : \tau \quad \Gamma, x : \text{Nat} \vdash e_1 : \tau}{\Gamma \vdash \text{ifz}(e; e_0; x. e_1) : \tau}$		$\frac{\text{FIX} \quad \Gamma, x : \tau \vdash e : \tau}{\Gamma \vdash \text{fix}(x : \tau. e) : \tau}$

Figure 2: Dynamics of PCF

$\frac{\text{VAL-ZERO}}{\text{zero val}}$	$\frac{\text{VAL-SUCC} \quad e \text{ val}}{\text{succ}(e) \text{ val}}$	$\frac{\text{VAL-LAM}}{\lambda x : \tau. e \text{ val}}$	$\frac{\text{D-SUCC} \quad e \mapsto e'}{\text{succ}(e) \mapsto \text{succ}(e')}$
$\frac{\text{D-APP-1} \quad e_1 \mapsto e'_1}{e_1(e_2) \mapsto e'_1(e_2)}$	$\frac{\text{D-BETA}}{(\lambda x : \tau. e_1)(e_2) \mapsto e_1[e_2/x]}$		
$\frac{\text{D-FIX}}{\text{fix}(x : \tau. e) \mapsto e[\text{fix}(x : \tau. e)/x]}$	$\frac{\text{D-IFZ-1} \quad e \mapsto e'}{\text{ifz}(e; e_0; x. e_1) \mapsto \text{ifz}(e'; e_0; x. e_1)}$		
$\frac{\text{D-IFZ-ZERO}}{\text{ifz}(\text{zero}; e_0; x. e_1) \mapsto e_0}$	$\frac{\text{D-IFZ-SUCC} \quad \text{succ}(e) \text{ val}}{\text{ifz}(\text{succ}(e); e_0; x. e_1) \mapsto e_1[e/x]}$		