

PROBLEM SHEET 3

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The following questions are about the dynamics of numbers and strings.

1. Draw derivations that justify the following transitions.

(i) $\text{plus}(\text{num}[1]; \text{num}[1]) \mapsto \text{num}[2]$

(ii) $\text{times}(\text{plus}(\text{num}[1]; \text{num}[1]); \text{num}[2]) \mapsto \text{times}(\text{num}[2]; \text{num}[2])$

(iii) $\text{len}(\text{let}(\text{str}['a']; v. \text{cat}(v; \text{str}['b']))) \mapsto \text{len}(\text{cat}(\text{str}['a']; \text{str}['b']))$

2. Write down transition sequences that justify the following multi-step transitions.

(i) $\text{times}(\text{plus}(\text{num}[1]; \text{num}[1]); \text{num}[2]) \mapsto^* \text{num}[4]$

(ii) $\text{times}(\text{len}(\text{let}(\text{str}['a']; v. \text{cat}(v; \text{str}['b']))); \text{num}[2]) \mapsto^* \text{num}[4]$

3. Are the following terms well-typed? Write down transition sequences that reduce them to values.

(i) $\text{let}(\text{str}['a']; z. \text{plus}(\text{len}(z); \text{len}(z)))$

(ii) $\text{let}(\text{len}(\text{str}['a'])); z. \text{plus}(z; z)$

(iii) $\text{plus}(\text{let}(\text{len}(\text{str}['a'])); z. \text{plus}(z; z)); \text{num}[1])$

4. The rules D-PLUS-1 and D-PLUS-2 of the dynamics enforce that e_1 is evaluated *before* e_2 when computing the value of $\text{plus}(e_1; e_2)$. Propose alternative versions of these rules that evaluate e_2 before e_1 . Would you expect your rules to affect the final value that is returned?
5. Prove that if e val then either $\vdash e : \text{Num}$ or $\vdash e : \text{Str}$.
6. Prove that multi-step transitions are transitive, i.e. that the following rule is admissible:

$$\frac{e_1 \mapsto^* e_2 \quad e_2 \mapsto^* e_3}{e_1 \mapsto^* e_3}$$

[Hint: perform an induction on the premise $e_1 \mapsto^* e_2$.]

7. (*) Complete the proof of preservation.
8. Complete the proof of progress.
9. (Hard, trick, highly optional.) We proved preservation by induction on $e \mapsto e'$, while we proved progress by induction on $\vdash e : \sigma$. Why did we make that choice? Could we have performed an induction on $\vdash e : \sigma$ for both? Discuss.