Reference: Simply-typed λ-calculus

Alex Kavvos

Figure 1: Statics of the simply-typed λ-calculus (with numbers)

$$\textsc{Var}\ \frac{}{\Gamma, x : \sigma \vdash x : \sigma} \qquad \textsc{Num}\ \frac{n \in \mathbb{N}}{\Gamma \vdash \mathsf{num}[n] : \mathsf{Num}} \qquad \textsc{Plus}\ \frac{\Gamma \vdash e_1 : \mathsf{Num} \qquad \Gamma \vdash e_2 : \mathsf{Num}}{\Gamma \vdash \mathsf{plus}(e_1; e_2) : \mathsf{Num}}$$

$$\textsc{Times}\ \frac{\Gamma \vdash e_1 : \mathsf{Num} \qquad \Gamma \vdash e_2 : \mathsf{Num}}{\Gamma \vdash \mathsf{times}(e_1; e_2) : \mathsf{Num}} \qquad \textsc{Let}\ \frac{\Gamma \vdash e_1 : \sigma_1 \qquad \Gamma, x : \sigma_1 \vdash e_2 : \sigma_2}{\Gamma \vdash \mathsf{let}(e_1; x.\, e_2) : \sigma_2} \qquad \textsc{Unit}\ \frac{}{\Gamma \vdash \langle\rangle : \mathbf{1}}$$

$$\textsc{Prod}\ \frac{\Gamma \vdash e_1 : \tau_1 \qquad \Gamma \vdash e_2 : \tau_2}{\Gamma \vdash \langle e_1, e_2 \rangle : \tau_1 \times \tau_2} \qquad \textsc{Proj-1}\ \frac{\Gamma \vdash e : \tau_1 \times \tau_2}{\Gamma \vdash \pi_1(e) : \tau_1} \qquad \textsc{Proj-2}\ \frac{\Gamma \vdash e : \tau_1 \times \tau_2}{\Gamma \vdash \pi_2(e) : \tau_2} \qquad \textsc{Abort}\ \frac{\Gamma \vdash e : \mathbf{0}}{\Gamma \vdash \mathsf{abort}(e) : \tau}$$

$$\textsc{Inl}\ \frac{\Gamma \vdash e : \tau_1}{\Gamma \vdash \mathsf{inl}(e) : \tau_1 + \tau_2} \qquad \textsc{Inr}\ \frac{\Gamma \vdash e : \tau_2}{\Gamma \vdash \mathsf{inr}(e) : \tau_1 + \tau_2}$$

$$\textsc{Case}\ \frac{\Gamma \vdash e : \tau_1 + \tau_2 \qquad \Gamma, x : \tau_1 \vdash e_1 : \tau \qquad \Gamma, y : \tau_2 \vdash e_2 : \tau}{\Gamma \vdash \mathsf{case}(e; x.\, e_1; y.\, e_2) : \tau} \qquad \textsc{Lam}\ \frac{\Gamma, x : \sigma \vdash e : \tau}{\Gamma \vdash \lambda x : \sigma.\, e : \sigma \to \tau}$$

$$\textsc{App}\ \frac{\Gamma \vdash e_1 : \sigma \to \tau \qquad \Gamma \vdash e_2 : \sigma}{\Gamma \vdash e_1(e_2) : \tau}$$

Figure 2: Dynamics of the simply-typed λ-calculus