

Do Courts Apply a Private Company Discount or a Marketability Discount?

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ABSTRACT

The value of an unlisted entity remains a contentious issue. This is notably because there is no consensus on the nature, size, and drivers of the so-called discount for lack of marketability (“DLOM”). The DLOM is the estimated percentage difference in value between an unlisted and an all-else-equal listed company.

The traditional methods for estimating the DLOM are essentially based on financial and transactional data. It is notoriously difficult to obtain information about the internal organization of private companies and this explains why the knowledge of the DLOM determinants remains limited.

These limitations have led us to consider an alternative data source. Specifically, we have based our research on a unique dataset of U.S. court decisions that apply a DLOM to a private company and justify the percentage discount by reference to the specifics of the valuation subject.

This article shows that courts apply different discount percentages depending on whether they value operating or non-operating companies. The difference between the DLOM applied on operating companies and the DLOM applied on non-operating companies is 7 percent. This paper explains the difference by

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distinguishing between a private company discount and a marketability discount.

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I. INTRODUCTION

The understanding of the discount for lack of marketability (“DLOM”) has evolved over time. The DLOM is, counterintuitively and despite its name, not necessarily or exclusively explained by the difference in marketability between public and private companies. We posit in this paper that the DLOM has two different appearances: it can be either a private company discount or a marketability discount. The difference has a direct and important impact on the valuation of private companies. We test our central hypothesis on the basis of a unique dataset of court cases that provide rich contextual information and offer novel insights into the DLOM’s determinants. We outline our reasoning and approach below.

A. The DLOM is a Polysemantic Concept

An unlisted company is valued less than an otherwise comparable listed company. The discount goes by several names: private company discount, marketability discount, liquidity discount, etc. The discount is an economic and legal reality—it is applied by practitioners and courts. In this article, we attempt to shed light on the discount’s nature, size, and components.

First, it is important to clarify that the discount is a deduction from a marketable baseline valuation. The accepted business valuation approaches (income approach, market approach, and asset-based approach) typically provide value indications for a marketable interest.¹ Depending on the reference value (controlling interest or minority value), two successive deductions may be required: a discount for lack of control (“DLOC”) and a DLOM.² This two-step approach is followed by valuation experts and the courts:

In valuing stock in a closely held corporation, the courts generally engage in a multistep process. First, the value of a corporation as a whole is determined. If there is only one class of stock, the value of each share of stock is calculated by dividing the number of shares into that

¹ Robert Reilly & Aaron Rotkowsky, *The Discount for Lack of Marketability: Update on Current Studies and Analysis of Current Controversies*, 61 TAX LAW. 241, 243 (2007).

² We use the terms DLOC and DLOM for consistency. The courts tend to use the terms “lack of control discount” and “minority discount” as synonyms and the same goes for the terms “lack of marketability discount” and “liquidity discount.” See Stephen J. Leacock, *Lack of Marketability and Minority Discounts in Valuing Close Corporation Stock: Elusiveness and Judicial Synchrony in Pursuit of Equitable Consensus*, WM. & MARY BUS. L. REV. 683, 697 (2016).

value. An appropriate discount to reflect lack of control and liquidity [...] then is applied to the block of stock [...]. At the outset, application of each factor depends upon whether the court is valuing the corporation as a whole from the perspective of a controlling or minority shareholder.³

The traditional levels-of-value chart (and the court's multistep valuation process) appears as follows:⁴

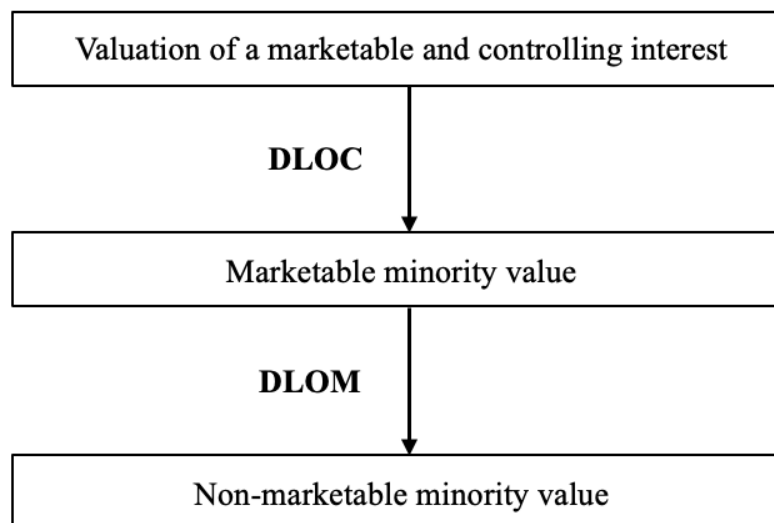


Illustration 1: Levels of Value Chart

This two-stage deduction model has taken time to develop. Early court decisions did not systematically distinguish between the two discounts. For example, in *Barnes v. Commissioner*, the U.S. Tax Court justified important DLOMs of 40 percent and 45 percent reasoning that “[stocks] represent very small minority interests that have no ability to direct the affairs of either company or cause the sale of its assets.”⁵

³ James R. Repetti, *Minority Discounts: The Alchemy in Estate and Gift Taxation*, 50 TAX L. REV. 415, 424 (1995).

⁴ Z. CHRISTOPHER MERCER & TRAVIS W. HARMS, BUSINESS VALUATION: AN INTEGRATED THEORY 482 (3d ed. 2021); *see id.* at 25 (for the traditional model); *see also id.* at 19–69 (the authors’ view on the levels of value).

⁵ 76 T.C.M. 881, 888–89 (1998).

Later courts made a clear conceptual distinction between the DLOC and the DLOM but applied a conflated discount. This is shown in *Furman v. Commissioner*, where the court stated: “[w]e also recognize that while the minority and marketability discounts may be conceptually distinct [...], the boundaries are often less clear in practice, [...] and conclude that a 40–percent combined minority and marketability discount is appropriate in this case.”⁶

The use of conflated discounts illustrates a lack of understanding and consensus on whether the combination of discounts should be additive or multiplicative. A clearer approach was developed by the court in *Estate of Magnin v. Commissioner*.⁷ In *Magnin*, the Tax Court noted that one of the parties added together the 35 percent marketability and the 25 percent minority discount to get a combined discount of 60 percent.⁸ The court held that “[i]n order to ensure accuracy, the minority interest discount should be applied first and then the marketability and liquidity discount should be applied to this figure.”⁹

As correctly observed by the *Magnin* Court, a 25 percent minority discount applied to a baseline value of \$100 is \$75, and a 35 percent marketability discount applied to the new value of \$75 results in a value of \$48.75. Thus, the combined discount rate is 51.25 percent, not 60 percent.¹⁰ *Magnin* established a more disciplined approach to valuation than previously used—now the courts generally determine separate DLOCs and DLOMs and apply a multiplicative logic instead of an additive one.¹¹ Although every case is different, certain scholars have argued that the Tax Court decisions converge on average DLOCs and DLOMs of around 25 percent and 15 percent, respectively.¹²

⁶ 75 T.C.M. (CCH) 2206, 2219 (1998).

⁷ 81 T.C.M. (CCH) 1126 (2001).

⁸ *Id.* at 1141.

⁹ *Id.*

¹⁰ *Id.* at 1141 n.36.

¹¹ Stephen J. Leacock noted in 2001 (the year in which the *Magnin* decision was rendered) that “a historical evaluation of cases up to the present indicated that both judges and lawyers are becoming increasingly more respectful of substantive financial theory [...]” Stephen J. Leacock, *The Anatomy of Valuing Stock in Closely Held Corporations: Pursuing the Phantom of Objectivity into the New Millennium*, 2001 COLUM. BUS. L. REV. 161, 200 (2001).

¹² Mary M. Anderson et. al, *An Empirical Investigation of the Minority Interest and Marketability Discounts in Valuation of Closely Held Stock for Estate and Gift Tax Purposes*, 22 J. APPLIED BUS. RES. 89, 99–102 (2006).

B. *The DLOM is Not Only Explained by the Lack of Marketability*

The DLOM was traditionally explained as equilibrium compensation (in the form of a discount) to investors for bearing additional risks in the form of opportunity costs.¹³ However, a more recent view is that private companies are different from their public counterparts and that the valuation discount can be explained by a variety of factors of which marketability is only one.¹⁴

Some authors go so far to say that the lack of marketability has no impact on the DLOM.¹⁵ Rather, because the term stands for “discount of marketability,” the very notion of a DLOM may be wrong or at least misleading. Gilbert E. Matthews argues that “to the extent that there is a [private-company] discount, it derives *not* from the fact that its shares are not traded in a public market but, in fact, *from the characteristics of the company*.”¹⁶ Recent studies have sometimes found a private company *premium*, adding evidence that the valuation differences between private and public companies are not only the result of a difference in marketability.¹⁷

The DLOM can thus be explained by factors other than marketability. Indeed, several studies advance the size of the company as an explanatory factor (public companies are on average much bigger than private companies).¹⁸ Another factor that may contribute to the discount is the

¹³ The inability to convert an asset to cash can cause investors to miss out on opportunities to allocate capital to assets with higher returns. See Mukesh Bajaj et al., *Firm Value and Marketability Discounts*, 27 J. CORP. L. 89, 92–93 (2001).

¹⁴ Jay E. Fishman, *Advanced Concepts of Discounts for Marketability: New Studies*, FAM. L. TRIAL INST. 1, 7 (May 2019), <https://www.familylawtrialinstitute.org/wp-content/uploads/2019/05/5.-Advanced-Concepts-of-DLOM.New-Studies.pdf> (“A more recent view indicates that marketability is one of several factors affecting the observed discount.”).

¹⁵ See Gilbert E. Matthews, *Private Company Discounts Are Not Caused by Lack of Marketability*, 22 BUS. VALUATION UPDATE 1, 6 (2016).

¹⁶ *Id.* at 3 (emphasis added).

¹⁷ See Anthony J. Anderson & Michael S. Long, *Evidence on Lack of Liquidity for Small Public Firms*, 7 J. OF BUS. VALUATION & ECON. LOSS ANALYSIS 1, 24 (2012); Louis T. W. Cheng & Simon Mak, *Examining the Discount-Premium Puzzle of Private Company Valuation: The Case of China*, 7 INT’L REV. ACCT., BANKING & FIN. 1, 26 (2015); Jeffrey F. Jaffe et. al, *Do Unlisted Targets Sell at Discounts*, 54 J. FIN. & QUANTITATIVE ANALYSIS 1371, 1395 (2019).

¹⁸ See Robert Comment, *Business Valuation, DLOM and Daubert: The Issue of Redundancy*, 29 BUS. VALUATION REV. 83, 83–96 (2010). Comment found that the application of a large DLOM is generally redundant to the discount for lack of size (DLOS). *Id.* He identified the size premium that is included in the discount rate on the basis of fairness opinions and

absence of information. Private firms provide less information to the market than a comparable public firm and is allegedly one of the reasons why they face significantly higher costs of capital than public firms.¹⁹ Other authors have pointed to the limited diversification of most private firms and their dependence on suppliers, customers, or management.²⁰

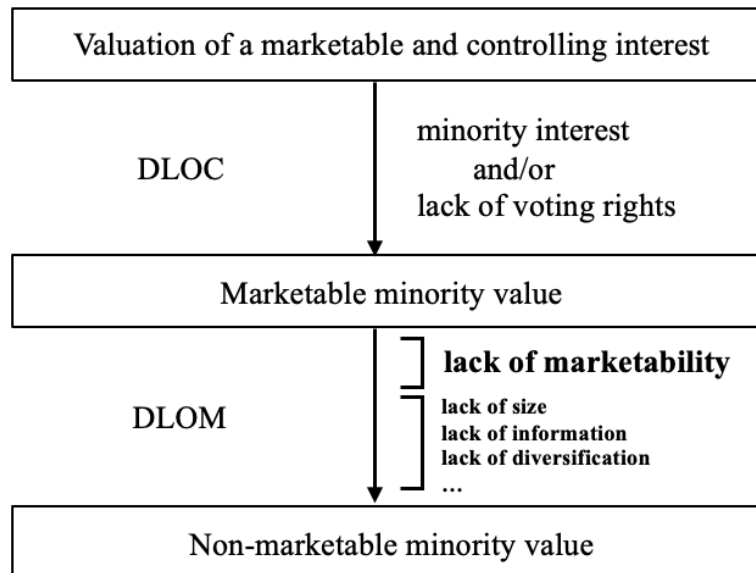


Illustration 2: DLOM Determinants

C. Research Question and Hypothesis

Scholars and valuation specialists most often refer to a DLOM when discussing the valuation discount for private companies. In their wake, courts do not seem to distinguish between discounts for lack of marketability and discounts that also adjust for other differences between public and private companies. Or do they? In a seminal article published in 2001, Bajaj et al. set out what the courts should do:

shows WACC differences of 8.8 percent between the smallest and the largest companies in his sample. *Id.* at 10. Comment's study suggests that the real DLOM is likely small ("just several percent"). *Id.* at 22.

¹⁹ Elisabeth de Fontenay, *The Deregulation of Private Capital and the Decline of the Public Company*, 68 HASTINGS L. J. 445, 491–92 (2017). This effect is somewhat mitigated because public company information allows private companies to freeride for information purposes. *Id.*

²⁰ See Matthews, *supra* note 16 at 3.

In our opinion, when valuing an operating company that is privately held (or its securities), the appropriate benchmark for discounts is provided by the total private placement discount or the discount observed in the acquisition approach. This is because, whether it is the marketability restriction per se or other factors, the relevant analysis aims to determine the total valuation discount. However, when it is appropriate to only consider the effect of marketability restrictions, as is the case in valuation of non-controlling interest in a non-operating partnership which holds assets of known value (e.g., a family limited partnership), the distinction between the total valuation discount and liquidity discount is key. In such cases, the applicable discount is only for the lack of liquidity.²¹

Thus, Bajaj et al. suggested a way to test the difference between DLOMs that reflect a marketability discount and DLOMs that reflect a total private company discount.²² The hypothesized valuation difference between operating and non-operating companies is illustrated below (and will hereinafter be referred to as the “Bajaj Hypothesis”).

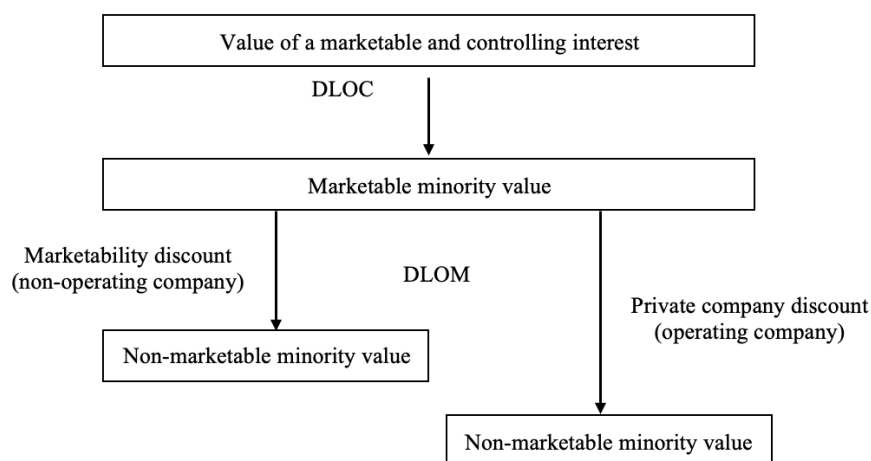


Illustration 3: Bajaj Hypothesis

This article tests the Bajaj Hypothesis using data derived from court decisions. The remainder of this article is organized as follows. In Section

²¹ Bajaj et al., *supra* note 14 at 114-15.

²² *Id.* at 114.

II, this paper demonstrates how the DLOM has been estimated and explained in the finance literature. Afterwards, this article presents the dataset used and analysis (Section III) and the results of various testing strategies for the Bajaj hypothesis (Section IV). Finally, Section V discusses and summarizes the conclusions of the study.

II. EMPIRICAL ESTIMATES OF THE DLOM

It is generally accepted that investors pay lower prices for assets that are not readily marketable in comparison to otherwise equal but liquid assets.²³ A company's valuation does not usually include this discount. As Bajaj et. al. pointed out:

The usual valuation methodologies, which utilize cash flows or market transactions, do not explicitly account for the marketability of an asset. Hence, in order to value an asset that is not marketable, the usual approach is to value the asset as if it were marketable, then apply a marketability discount to this estimated value.²⁴

The nature, size, dynamics, and determinants of the discount are still not entirely understood. Generally, in practice, DLOMs of 20 percent to 40 percent are used for valuing private businesses. This rule-of-thumb is a generalization of conclusions from three different types of empirical studies.

A. IPO Studies

One empirical technique to estimate the marketability discount compares the price of an asset during a period in which it is relatively non-marketable to a period in which it is marketable. The so-called IPO studies estimate the DLOM by comparing the “non-marketable” stock price in private transactions before the IPO to the “marketable” IPO price.

The primary research in this field was undertaken by William L. Silber.²⁵ Pre-IPO studies have been popularized by Willamette Management

²³ Linda H. Chen et al., *Risk, Illiquidity or Marketability: What Matters for the Discounts on Private Equity Placements?*, 57 J. BANKING & FIN. 41, 4–50 (2015).

²⁴ Bajaj et al., *supra* note 14 at 90.

²⁵ William L. Silber, *Discounts on Restricted Stock: The Impact of Illiquidity on Stock Prices*, 47 FIN. ANALYSTS J. 60, 60–64 (1991).

Associates (“WMA”) and by John Emory under whose direction a number of studies have been conducted for periods between 1980 and 2002.²⁶

Emory study	# of obs.	Mean discount	Median discount	WMA study	# of obs.	Mean discount	Median discount
1980- 1981	12	59%	68%	1975- 1978	31	34.0%	52.5%
1985- 1986	19	43%	43%	1979	17	55.6%	62.7%
1987- 1989	21	38%	43%	1980- 1982	113	48.0%	56.5%
1989- 1990	17	46%	40%	1983	214	50.1%	60.7%
1990- 1991	30	34%	33%	1984	33	43.2%	73.1%
1992- 1993	49	45%	43%	1985	25	41.3%	42.6%
1994- 1995	45	45%	47%	1986	74	38.5%	47.4%
1995- 1997	84	43%	41%	1987	40	36.9%	43.8%
1997- 2000	266	50%	52%	1988	19	41.5%	51.8%
				1989	19	47.3%	50.3%
				1990	23	30.5%	48.5%
				1991	34	24.2%	31.8%
				1992	75	41.9%	51.7%
				1993	110	46.9%	53.3%
				1994	48	31.9%	42.0%
				1995	66	32.2%	58.7%
				1996	22	31.5%	44.3%
				1997	44	28.4%	35.2%
				1998	21	35.0%	49.4%
				1999	28	26.4%	27.7%
				2000	15	18.0%	31.9%

²⁶ See Reilly, *supra* note 2 at 251, 259.

2001	2	-195.8%	-195.8%
2002	7	55.8%	76.2%

Illustration 4: IPO Studies

The WMA results for 2001 and 2002 have been explained by the fact that there were too few private market stock sale transactions in these years to be statistically meaningful.²⁷ If we disregard these anomalous WMA results, IPO studies show a mean and a median discount in the mid-40 percent range which seems to be implausibly large.²⁸ IPO studies present several difficulties of which the selection bias is the most important. Indeed, by design, these studies can only include companies that had a successful IPO. Another caveat is that pre-IPO studies do not ensure that a private transaction was concluded on an arm's length basis. Buyers of shares prior to the IPO may be officers, affiliates, or related parties with commercial ties to the issuer.²⁹

B. Restricted Stock Studies

The second empirical technique used to estimate the marketability discount is a comparison of share prices of two claims on the same underlying assets where one claim is marketable and the other one not.³⁰ This approach is illustrated in private placement studies.³¹

Private placement is the private sale of registered or unregistered securities by a company to a select group of sophisticated investors and accredited institutions.³² In the case of unregistered securities, they are commonly referred to as restricted stock (or letter stock) because they are restricted from trading under Rule 144 of the U.S. Securities Act of 1933.³³ Resale in the public marketplace is not possible unless and until the stock is

²⁷ SHANNON P. PRATT, *VALUING A BUSINESS: THE ANALYSIS AND APPRAISAL OF CLOSELY HELD COMPANIES* 437 (5th ed. 2008).

²⁸ In *McCord v. Comm'r*, the pre-IPO studies came under criticism by the Tax Court because of implausibly large discounts. 120 T.C. 358, 369 (2003).

²⁹ See generally Stanley Block, *The Liquidity Discount in Valuing Privately Owned Companies*, 17 J. APPLIED FIN. 33 (2007); Comment, *Business Valuation, DLOM and Daubert*, *supra* note 19; John Koeplin et al., *The Private Company Discount*, 12.4 J. APPLIED CORP. FIN. 94 (2000).

³⁰ See Koeplin et al., *supra* note 30 at 95.

³¹ *Id.*

³² *Private Placements, Explained*, FINRA (Dec. 7, 2020), <https://www.finra.org/investors/insights/private-placements-explained>.

³³ See 17 C.F.R. § 230.144 (2011).

registered or after a legal holding period (two years in the original version of the Securities Act).³⁴

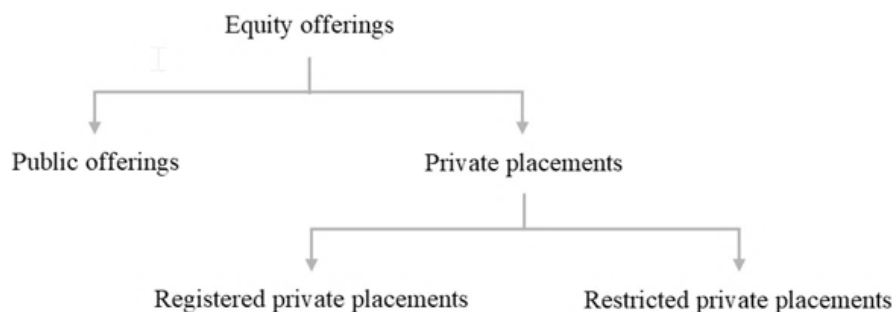


Illustration 5: Equity Offerings

Early restricted stock studies conducted in the 1970s found discounts between 30–40 percent, but in later studies, the value declined to 25 percent and below.³⁵ The declining size of the liquidity discount found in successive studies can be explained primarily by changes in Rule 144.³⁶ In 1990, qualified institutional investors were allowed to trade unregistered securities among themselves without filing registration statements.³⁷ This made the restricted stock more liquid and reduced the liquidity discount.³⁸ The holding period for restricted stock was reduced to one year in 1997 and to six months in 2008.³⁹

We provide an overview of a selected number of private placement studies below (with a difference between registered and unregistered stock when available).

³⁴ *Id.*

³⁵ See, e.g., Michael J. Maher, *Discounts for Lack of Marketability for Closely Held Business Interests*, 54 TAXES 562, 571 (1976); Robert E. Moroney, *Most Courts Overvalue Closely Held Stocks*, 51 TAXES 144, 151, 153, 156 (1973); Reilly, *supra* note 2 at 250; Robert R. Trout, *Estimation of the Discount Associated with the Transfer of Restricted Securities*, 55 TAXES 381, 384 (1977).

³⁶ 17 C.F.R. § 230.144.

³⁷ Reilly, *supra* note 2 at 257–58.

³⁸ *Id.*

³⁹ *Id.* at 258.

Author(s)	Year	# of Obs.	Years	Mean Discount	Median Discount
Silber	1991	69	1981-1988	33.75%	
Hertzel and Smith	1993	106	1980-1987	20.14%	13.25%
Bajaj et. al.	2001	88	1990-	22.21%	20.67%
Registered		37	1995	14.04%	9.85%
Unregistered		51		28.13%	26.47%
Wu	2004	338	1986-	8.7%	19.4%
Registered		37	1997	9.3%	15.3%
Unregistered		301		8.7%	19.8%
Chu et. al.	2004	142	1985-	19.22%	16.42%
Registered		21	1995	17.78%	12.00%
Registration rights		121		19.47%	16.67%
Krishnamurthy et. al.	2005	391	1983-	19.44%	
Registered		293	1992	15.41%	
Registration pending		23		23.31%	
Unregistered		75		34.02%	
Barclay et. al.	2007	594	1979-1997	18.7%	17.4%
Inget	2009	1125	2000-2007	9.38%	11.48%
Wruck and Wu	2009	1854	1980-1999	11.33%	10.96%
Comment	2011	153	2007-	10.5%	
Registered		68	2009	9.4%	
Registration rights		39		8.5%	
Unregistered		46		13.8%	
Finnerty	2013	52	1991-	20.82%	19.78%
		223	1997	14.62%	11.89%
			1997-2007		
Chipalkatti et. al.	2013	338	1991-2000	22.77%	18.71%
Garg and Kumar	2014	60	2005-2011	10.81%	
Chen et. al.	2015	496	1999-2012	10.33%	10.45%

Illustration 6: Private Placement Studies

In early restricted-stock studies, it was standard practice to limit the sample to restricted stock and sustain the assumption that the regulatory restriction is the only cause of the discount.⁴⁰ This interpretation was abandoned after the publication of a landmark study by William L. Silber in 1991.⁴¹ Silber concluded that the high valuation discount (33.75 percent in his sample) was not necessarily exclusively caused by the reduced liquidity.⁴² Specifically, he found that the DLOM is lower for firms that have higher revenues, positive earnings, and a customer relationship with investor(s), and that the discount increases when the fraction of shares placed becomes more important.⁴³

A 1993 study by Michael Hertz and Richard L. Smith compared private placements of restricted (i.e., unregistered) stock and registered shares.⁴⁴ They found that the discount for restricted shares was only 13.5 percent greater than that for registered shares.⁴⁵ The difference can be seen as a more precise estimate of the marketability discount because it controls for the specifics of the firms that resort to private placements for their financing.⁴⁶ However, Hertz and Smith are careful in their conclusions and state that “it [is] unlikely that a pure illiquidity effect can explain the magnitude of the discounts we find.”⁴⁷

Bajaj et. al. found a differential of 14.09 percent between the discount for unregistered and registered private placements.⁴⁸ They examined several other factors in a multivariate regression (including the percentage of shares issued, an indicator of the firm’s financial health, and the standard deviation of the firm’s returns) and concluded that the discount associated with marketability was only 7.23 percent.⁴⁹ A study by Robert Comment utilized the same method and examined 15 explanatory variables that reflected solvency, buyer skepticism, potential for dilution, arm’s length character,

⁴⁰ Robert Comment, *Revisiting the Illiquidity Discount for Private Companies: A New (and “Skeptical”) Restricted-Stock Study*, 24 J. APPLIED CORP. FIN. 80, 84 (2012); Bajaj et. al., *supra* note 14 at 98.

⁴¹ See Silber, *supra* note 26 at 60–64.

⁴² *Id.*

⁴³ *Id.*

⁴⁴ Michael Hertz & Richard L. Smith, *Market Discounts and Shareholder Gains for Placing Equity Privately*, 48 J. FIN. 459, 480 (1993).

⁴⁵ *Id.*

⁴⁶ Z. Christopher Mercer, *A Current View of the Restricted Stock Studies and Restricted Stock Discounts*, 40 BUS. VALUATION REV. 43, 59 (2021) (noting that the issuing firms are generally small, risky, and concentrated in a small group of industries).

⁴⁷ Hertz, *supra* note 45 at 480.

⁴⁸ See Bajaj et al., *supra* note 14 at 107.

⁴⁹ *Id.* at 114.

and marketability.⁵⁰ He estimated the discount that can be tied directly to the regulatory restriction at 5.2 percent.⁵¹

The current view is that the overall discount in private placements of restricted stock is impacted by a variety of factors, of which marketability is only one.⁵² The other explanations advanced for the private placement discount are not mutually exclusive and include (i) monitoring and ownership concentration;⁵³ (ii) information asymmetry;⁵⁴ (iii) management entrenchment;⁵⁵ and (iv) financial distress.⁵⁶

In short, private placement studies have established that the valuation differences between restricted and publicly traded stock have to do with more than marketability. The few studies that single out the effect of marketability estimate the effect in a range between 5 percent and 14 percent. Note also that these studies only look, by design, into private placements by

⁵⁰ See Comment, *Revisiting the Illiquidity Discount for Private Companies*, *supra* note 41 at 87.

⁵¹ See *id.* at 89.

⁵² See *id.* at 84-85.

⁵³ This hypothesis holds that the private placement discount reflects the compensation to purchasers of stock for monitoring and expert advice. The placement can have an impact on the resulting ownership of the issuer (concentration of ownership is a common by-product of private placements), and a concentration of ownership is deemed to contribute to enhanced monitoring. See, e.g., Karen H. Wruck, *Equity Ownership Concentration and Firm Value: Evidence from Private Equity Financings*, 23 J. FIN. ECON. 3, 4 (1989); Bajaj et al., *supra* note 14 at 93; Karen H. Wruck & YiLin Wu, *Relationships, Corporate Governance, and Performance: Evidence from Private Placements of Common Stock*, 15 J. CORP. FIN. 1, 21 (2009).

⁵⁴ This view holds that private placements are used by firms that have higher information asymmetries and discounts reflect the compensation for information acquisition costs. See, e.g., Hertz, *supra* note 45 at 465-66; Bajaj et al., *supra* note 14 at 98-99; Srinivasan Krishnamurthy et al., *Does Investor Identity Matter in Equity Issues?: Evidence from Private Placements*, 14 J. FIN. INTERMEDIATION 210, 211 (2005); Rahsan B. Inget, *Private Equity Placements and Illiquidity Discount*, PURDUE UNIV. <https://ssrn.com/abstract=1583562> (last modified Jul. 30, 2010).

⁵⁵ This hypothesis suggests that "private placements are often made to passive investors, thereby helping management solidify their control of the firm." See Michael Barclay et al., *Private Placements and Managerial Entrenchment*, 13 J. CORP. FIN. 461, 461 (2007). A private placement (contrary to a block trade which is set up by an existing block holder) is organized by management. The discount compensates the block purchasers for the consequences of their passivity. See YiLin Wu, *The Choice of Equity-Selling Mechanisms*, 74 J. FIN. ECON. 93, 96 (2004); see also Wruck, *supra* note 54 at 4; and Krishnamurthy et al., *supra* note 55 at 211.

⁵⁶ The discount can sometimes be explained by reference to the financial distress experienced prior to and around the time of the issue. The discount is compensation to private investors for their willingness to provide capital when public funding would be difficult to secure at short notice or subject to severe underpricing. Shin-Herng Chu et al., *Comparing the Characteristics and Performance of Private Equity Offering Firms with Seasoned Equity Offering Firms*, 1 J. ECON. & MGMT. 57, 58-59 (2005).

public firms (i.e., there is an established market on which the restricted shares can eventually be traded).⁵⁷

C. Acquisition Studies

The multiples approach compares acquisitions of private and public companies on the basis of the price multiples of various relevant financial parameters.⁵⁸ The valuation discount is thus calculated as $1 - (\text{private company multiple} / \text{public company multiple})$.⁵⁹

The valuation multiplier approach was first proposed by Koeplin et al.⁶⁰ These authors examined 192 acquisitions of private companies between 1984 and 1998.⁶¹ For each of the private acquisitions, they identified the acquisition of a public company in the same country, year, and industry (the “matched pairs” approach).⁶² They calculated the ratio enterprise value to EBIT, EBITDA, sales, and book value, and concluded that private transactions take place at a discount of about 20 percent.⁶³ The Koeplin et al. study has been repeated by both Block and Officer wherein Block found a significant relationship between the industry and the size of the discount.⁶⁴

De Franco et al. uses a multivariate regression procedure to show that the discount paid for private firms relative to public firms goes beyond simple differences in liquidity. They point specifically to the information quality facing the buyer as an explanatory factor.⁶⁵ Paglia and Harjoto introduce industry, size, and profitability as explanatory factors.⁶⁶

More recent articles find no conclusive evidence that unlisted targets sell at discounts.⁶⁷ These results thus run counter to prior academic evidence and

⁵⁷ See PRATT, *supra* note 28 at 419.

⁵⁸ See, e.g., Koeplin et al., *supra* note 30 at 99.

⁵⁹ *Id.*

⁶⁰ *Id.* at 96-97.

⁶¹ *Id.* at 97.

⁶² *Id.*

⁶³ *Id.*

⁶⁴ See Block, *supra* note 30 at 40; Micah S. Officer, *The Price of Corporate Liquidity: Acquisition Discounts for Unlisted Targets*, 83 J. FIN. ECON. 571, 572-73. (2007).

⁶⁵ See Gus De Franco et al., *Do Private Company Targets that Hire Big 4 Auditors Receive Higher Proceeds?*, 28 CONTEMP. ACCT. RES. 215, 216 (2011).

⁶⁶ See John Paglia & Maretno Harjoto, *The Discount for Lack of Marketability in Privately Owned Companies: A Multiples Approach*, 5 J. OF BUS. VALUATION & ECON. LOSS ANALYSIS 1 (2010).

⁶⁷ See Anthony J. Anderson, *supra* note 18; Cheng, *supra* note 18; Jaffe et. al, *supra* note 18.

the views of practitioners, courts, and regulators.⁶⁸ The premium transactions reported in the studies indicate that illiquidity is not the only factor that explains the differences in multiples.

The acquisitions approach can at best provide upper bound estimates for the DLOM. The discount resulting from this method can best be described as a private company discount as carefully admitted by Koeplin et al.:

“[W]hat we are capturing is more than a marketability discount. The companies that choose to be acquired before they go public may be fundamentally different than the companies that first get listed and are subsequently sold.”⁶⁹

Block also notes that “it is possible that there are systematic differences in the characteristics of the private and public firms that are acquired.”⁷⁰

The acquisition studies are interesting because they consider and compare private companies (IPO studies and restricted stock studies are based on public company information). On the other hand, the acquisition values for both private and public targets are possibly contaminated with strategic value components. Other critiques on the method include the observation that it is difficult to find a good matching pair for acquisitions,⁷¹ and that the results vary widely depending on the multiple that is considered.⁷²

Please refer to the table below for an overview of the most authoritative acquisition studies (with an indication of the average discount for the different multiples). The valuation discounts are near the 20 percent mark reported in the Koeplin et al. study with notable exceptions for the price to book ratio reported by Officer (a result he explains by the fact that unlisted

⁶⁸ Anderson and Long found that small private firms (selling price USD 20-100 million) sell at a premium compared to similar public firms. Anthony J. Anderson, *supra* note 18. They explained this phenomenon by the high agency costs that public owners of small firms face. *Id.* Cheng and Mak examined Chinese private transactions and concluded that premium deals are a significant phenomenon. Cheng, *supra* note 18. They also identified firm size as an important factor. *Id.* They explained the premium deals by referring to the less developed stock market and the “strong growth element” in these transactions. *Id.*

⁶⁹ Koeplin et al., *supra* note 30 at 96.

⁷⁰ Block, *supra* note 30 at 38.

⁷¹ Paglia, *supra* note 67.

⁷² See Comment, *Revisiting the Illiquidity Discount for Private Companies*, *supra* note 41. Note also that this method is based on accounting data. *Id.* These are historical (rather than forecasted) data, and the results can be different across countries because of different accounting standards; Koeplin et al., *supra* note 30 at 94.

targets are young, high growth companies)⁷³ and the high ratios reported by Paglia and Harjoto (which they simply acknowledge to be “significantly larger than findings from existing studies”).⁷⁴

Study	Year	# of Obs.	EBITDA	Income	EBIT	Sales	Book
Koeplin et. al.	2000	192	21.1%		17.1%	9.8%	7.6%
Block	2007	91	24.56%		27.10%	26.35%	16.25%
Officer	2007	364	17.18%	22.85%		18.15%	-15.61%
Paglia & Harjoto	2010	431	50%			75%	
DeFranco et. al.	2011	664	39.7%			20.2%	

Illustration 7: Valuation Multiplier Studies

D. Limitations of the Empirical Methods

The overview of the existing estimation methods raises some important issues. First, all of these methods are somewhat flawed. This is especially the case for the IPO method which suffers from a selection bias (only successful IPO companies are included in the sample). The two other methods lead to more reasonable results. The restricted stock studies do not sufficiently control for determinants of the discount. The best available information shows that the difference in discounts between registered and unregistered private placements is about 14 percent and that this can be considered an upper bound for the marketability discount. The acquisition approach is interesting in that it looks at private company information, which is a clear improvement compared to the private placement studies. However, these acquisition studies, too, offer little insights into the determinants of the DLOM and their magnitude.

⁷³ See Officer, *supra* note 65 at 581.

⁷⁴ Paglia, *supra* note 67 at 12-13, 17.

III. SELECTION AND ANALYSIS OF COURT CASES

A. Dataset

The limitations of traditional empirical studies have compelled us to look to court cases as an alternative source of information for factors affecting the DLOM; in particular U.S. tax cases, in which the DLOM is in controversy represent an illuminating data source.⁷⁵ These cases, which pit taxpayers against the IRS, typically raise points in two different contexts: court cases that settle estate tax disputes and court cases that settle gift tax disputes.

Estate tax cases, which make up the majority of our data set, arise when closely held stock is transferred from a deceased shareholder to the estate. In this instance, the application of a DLOC and DLOM lowers the death-date value of the deceased shareholder's stock, thereby reducing the estate tax liability of the deceased shareholder's estate.⁷⁶ As a result, the recipient shareholder (i.e., the estate) argues for a *higher* DLOM so as to minimize the share value. Conversely, the IRS takes the opposite position, arguing that the DLOM proposed by the estate is inflated and that the shares at issue are actually worth far more.

Gift tax cases, although also stemming from the desire to minimize tax consequences, operate slightly differently than estate tax cases. In general, gift tax cases fall into two categories. The first category occurs when a shareholder of closely-held stock gifts a portion of their stock to another

⁷⁵ We have excluded appraisal cases (cases brought by minority shareholders who dissent from certain corporate actions and want to receive the appraised fair value of their shares). The reason is that there is no unanimity about the reference value to be used in these type of cases, and legislatures and courts refer most often to a "fair value." This is an ambiguous term that can be interpreted at will. In determining fair value in the context of appraisal rights, most (but not all) jurisdictions regard shareholder level minority and liquidity discounts as prohibited as a matter of law, notably on the premise that the majority shareholder would otherwise reap a windfall at the expense of the minority. The majority view is that a minority shareholder is entitled to a proportionate interest in the company's going concern value and that this excludes shareholder level minority and marketability discounts. See Leacock, *Lack of Marketability and Minority Discounts in Valuing Close Corporation Stock*, *supra* note 3 at 723; see also Gilbert E. Matthews, *Statutory Fair Value in Dissenting Shareholder Cases: Part I*, 36 BUS. VALUATION REV. 15, 17 (2017).

⁷⁶ Federal tax law allows for a relatively large amount of a decedent's estate to be excluded from tax. 26 U.S.C. § 2010(c)(3). The amount excluded from estate tax for 2022 is \$12,060,000 per individual. See *Estate Tax*, INTERNAL REVENUE SERVICE (Nov. 15, 2021), <https://www.irs.gov/businesses/small-businesses-self-employed/estate-tax>. However, any amount over this "estate tax exclusion" is taxed at a relatively high rate. See also 26 U.S.C. § 2001(c) (setting the estate tax rate at 40 percent).

person.⁷⁷ In these instances, the transferor of the stock will argue for a large discount so that the gift is as small as possible in the eyes of the IRS.⁷⁸ The second category of gift tax cases occur when a closely-held corporate shareholder transfers stock for the purposes of a charitable donation.⁷⁹ There, the transferor argues for a smaller stock discount so that their gift is as large as possible. Aside from making the transferor look more generous, arguing for a larger gift also reduces the transferor's tax liability by shielding a higher portion of taxable income.

B. Data Analysis

We found the cases for our analysis via a keyword search in Lexis Advance ("discount" AND "lack of marketability") narrowed by practice area (tax law), courts (federal), and timeline.⁸⁰

The resulting decisions were then screened to verify whether they contained a positive legal decision on a percentage DLOM. We rejected cases that did not contain an unequivocal decision on the DLOM and cases where the court decided on the value of fractional interests in real estate or of shares in a listed company. We also excluded cases that had been overruled on grounds that invalidate the DLOM decision.

The remaining cases were examined by at least two qualified reviewers who consolidated their findings in a control sheet that identified a number of explanatory variables for the DLOM. In this detailed process, a limited number of additional cases were rejected (on the basis of the criteria set out above) and eventually, a total of eighty cases were retained in the dataset. As some of these cases dealt with more than one company or more than one gift,

⁷⁷ For an example of a gift tax case where the transferor argues for a higher DLOM, see *Nelson v. Comm'r*, 119 T.C.M. (CCH) 1554 (T.C. 2020); see also *Grieve v. Comm'r*, 119 T.C.M. (CCH) 1174 (T.C. 2020).

⁷⁸ This is because smaller gifts leave more room for the transferor to bequest future tax-free gifts under the lifetime Federal Gift Tax Exclusion. See Repetti, *supra* note 4 at 471.

⁷⁹ For an example of a gift tax case where the transferor argues for a lower DLOM, see *Bergquist v. Comm'r*, 131 T.C. 8 (T.C. 2008); see also *Koblick v. Comm'r*, 91 T.C.M. (CCH) 959 (T.C. 2006).

⁸⁰ We analyzed cases that were decided between January 1990 and beyond (through June 2021). This cut-off was motivated not only by the desire to obtain a high quantity of data points, but also by the fact that restricted stock studies—an important reference point for deciding courts—generally show a sharp decline in the average DLOM during this period (from about 35 percent pre-1990 to a range of 20–25 percent post-1990). Reilly, *supra* note 2 at 257. Therefore, post-1990 cases represent a more stable pool of data in comparison to if we had mixed these cases in with their pre-1990 predecessors.

the total number of observations is higher than the number of court cases (137 observations for 80 court cases).⁸¹

The rigid selection criteria for the cases combined with the thorough review by two independent coders as well as the consolidation of findings based on the principle of interrater reliability, have resulted in a robust database on which we have run our analysis.

C. *The Value of Case Analysis*

The above estate and gift tax cases represent a rich source of data because the court is tasked with determining the fair market value of the shares at issue.⁸² This is a decision that necessarily requires the court to determine the correct DLOM that should apply.⁸³ Therefore, although the determination of the fair market value is a factual matter, courts look to the parties' "dueling" valuation experts for guidance.⁸⁴ These experts, who argue for a DLOM which allies with their party's interests, typically support their assessment of the DLOM with factual information that private companies would not otherwise disclose. In particular, many of these facts involve variables that have been theorized to impact the DLOM, such as information concerning the company's size, profitability, industry etc. The court may also consider other elements as it is provided with rich contextual information about the company and its shareholders. As a result, the court's ultimate decision as to the correct fair market value reflects a rich palette of data and variables which are determinative of the DLOM.

⁸¹ See *infra* Exhibit A for a listing of the court cases used in the analysis.

⁸² Ronald H. Jensen, *The Magic of Disappearing Wealth Revisited: Using Family Limited Partnerships to Reduce Estate and Gift Tax*, 1 PITT. TAX. REV. 155, 163-64, 207 (2004).

⁸³ *Id.* at 163-64. The fair market value test does not preclude the application of lack of control and/or lack of marketability discounts (a matter that is debated when it comes to appraisal cases). Leacock, *Lack of Marketability and Minority Discounts in Valuing Close Corporation Stock*, *supra* note 3 at 730.

⁸⁴ The determination of the fair market value is a factual matter. The courts look to valuation experts for guidance but remain free to accept, reject or modify the valuation conclusions of the adversarial parties or their respective experts. Leacock, *Lack of Marketability and Minority Discounts in Valuing Close Corporation Stock*, *supra* note 3 at 730-31; Estate of Adell v. Comm'r, 108 T.C.M (CCH) 107 (T.C. 2014) ("As the finder of fact, the Court considers the underlying facts agreed upon by the parties and presented at trial and looks to the experts' reports for guidance on deciding the valuation issue.").

IV. METHODOLOGY

A. Tests for the Bajaj Hypothesis

We tested the Bajaj Hypothesis in multiple ways. Specifically, our dataset allows us to determine the operating character of a company by testing the impact of three different variables on the DLOM:

(a) Asset-Based Valuation Variable. The asset-based valuation variable indicates whether the valuation method withheld by the court was based on the company's assets. An asset-based approach is deemed appropriate for holding (non-operating) companies. As such, we expect a lower DLOM for such companies (as compared to companies that are valued based on an income, a market, or a mixed approach).⁸⁵ In our dataset, 76 out of the 137 DLOM decisions applied an asset-based approach.

(b) The Standard Industrial Classification (SIC) Codes. We expect companies with two-digit SIC codes designated within the sixty to sixty-seven code range (Industry H) to be predominantly passive (non-operating) vehicles, resulting in a lower DLOM.⁸⁶ We found that 76 out of the 137 observations in our dataset belong within this range. As for observations outside this range, we obtained detailed information on their attributions. However, these observations are scattered across the other SIC division codes, thus we could not determine a statistical representative number. Therefore, we grouped them as "non-60-67."⁸⁷

(c) Operating Company. The indicator variable "operating company" denotes companies with operating revenues. We expect operating companies to have comparatively higher DLOMs. We have identified 80 out of the 137 observations as "operating."⁸⁸

⁸⁵ Non-operating companies are generally valued on the basis of the assets method. *See, e.g., Estate of Smith v. Comm'r*, 78 T.C.M. (CCH) 745 (T.C. 1999) ("It is well established that, in general an asset-based method of valuation applies in the case of corporations that are essentially holding corporations, while an earnings-based method applies for corporations that are going concerns.").

⁸⁶ The two-digit SIC codes are catalogued within Division H and includes establishments operating primarily in the fields of finance, insurance, and real estate. *See 2-Digit SIC (Standard Industrial Classification) Codes*, N.C. STATE UNIV. MCKIMMON CTR. FOR EXTENSION & CONTINUING EDUC. (Jun. 16, 2021, 10:49 AM), <https://mckimmoncenter.ncsu.edu/2digitsiccodes/>.

⁸⁷ The split of the 137 observations in our dataset over the various industry classes can be found in Exhibit B.

⁸⁸ The coding is based on an identical analysis performed by at least two independent reviewers, *see supra* Section III. B.

We first present the univariate linear regression results for each of the above explanatory variables. A regression analysis is a recognized statistical method to estimate the relationship between a dependent (output) variable and one or more independent variables.⁸⁹ In principle, the nature of the DLOM variable (an output variable that can take the value of zero or one, or any value between zero and one) requires us to apply the fractional regression method.⁹⁰ The fractional regression method results are more difficult to interpret, and they remain close to those of a linear regression (since the observed values of the dependent variable stay away from its boundaries). On this basis we chose to present only the linear regression results and to report the fractional regression results in an appendix (Exhibit C).

Our dataset contains several “conflated discounts” (i.e., discounts that reflect a combined DLOM and DLOC). In these cases (a total of seventeen observations), we have replaced the missing DLOC and DLOM based on a proportional allocation.⁹¹

⁸⁹ Catherine Cote, *What is Regression Analysis in Business Analytics?*, HARV. BUS. SCH. ONLINE (Dec 14, 2021), <https://online.hbs.edu/blog/post/what-is-regression-analysis>. The output results indicate not only the estimated size effect of the dependent variables on the DLOM but give us two important additional indicators: the p-value indicates the probability that the observed effect is due to random sampling error. The other relevant indicator is the R-Squared (R^2 or the coefficient of determination). This is a statistical measure in a regression model that determines the proportion of variance in the dependent variable that can be explained by the independent variable. In other words, R-squared is the predictive value: a measure that shows how well the data fit the regression model. The adjusted R-squared is a modified version of R-squared that has been adjusted for the number of independent variables in the model.

⁹⁰ Michael Clark, *Fractional Regression*, MICHAEL CLARK (Aug. 20, 2019), <https://m-clark.github.io/posts/2019-08-20-fractional-regression/>.

⁹¹ In cases that decide on a conflated discount (DTOT), we have applied a proportional allocation based on the nonlinear constraint that $DTOT = DLOM + DLOC - (DLOC * DLOM)$. Based on seventy-seven observations in our dataset, we concluded that the average \overline{DLOC} was 16.49 percent. We excluded the observations that had conflated discounts, missing observations, as well as observations that dealt with a controlling interest (for which the DLOC is zero). (Similarly, based on the same seventy-seven observations, we found an average \overline{DLOM} of 24.06 percent; an average \overline{DTOT} of 36.35 percent; and an average $\overline{DLOC * DLOM}$ value of 4.20 percent. The conflated discounts (DTOT) in our sample have been allocated over a DLOM and a DLOC in a manner that deviates as less possible from the relative sample estimates μ_{DLOC} , μ_{DLOM} and $\mu_{DLOC * DLOM}$ based on the following constrained quadratic optimization:

- Minimize:

$$\left(\frac{DLOC}{DTOT} - \mu_{DLOC}\right)^2 + \left(\frac{DLOM}{DTOT} - \mu_{DLOM}\right)^2 + \left(\frac{DLOC * DLOM}{DTOT} - \mu_{DLOC * DLOM}\right)^2$$

- Subject to:

$$DLOC + DLOM - (DLOC * DLOM) = DTOT$$

- DLOC and DLOM both between zero and one.

Variable	Estimate	Std. Error	t-value	p-value
Intercept	0.26192	0.01104	23.72912	0.00000
V_ASSET_VAL	-0.04403	0.01482	-2.97066	0.00352
Multiple R-squared: 0.0614				
Adjusted R-squared: 0.0544				

Table 1: Asset-based Valuation

Variable	Estimate	Std. Error	t-value	p-value
Intercept	0.27632	0.01047	26.38272	0.00000
V_INDUSTRY_H	-0.06997	0.01406	-4.97620	0.00000
Multiple R-squared: 0.1550				
Adjusted R-squared: 0.1487				

Table 2: SIC Code 60-67

Variable	Estimate	Std. Error	t-value	p-value
Intercept	0.19718	0.01088	18.12963	0.00000
V_OPERATING	0.06906	0.01423	4.85218	0.00000
Multiple R-squared: 0.1485				
Adjusted R-squared: 0.1422				

Table 3: Operating Company

The three tests produce statistically significant results in line with our predictions. When we introduce the three variables in a multivariate regression, only the indicator variables for “Operating Company” and “Industry H” remain relevant at the 10 percent level.

Variable	Estimate	Std. Error	t-value	p-value
Intercept	0.23618	0.02638	8.95267	0.00000
V_OPERATING	0.04055	0.02219	1.82767	0.06984
V_INDUSTRY_H	-0.04359	0.01955	-2.22920	0.02748
V_ASSET_VAL	0.00329	0.01820	0.18059	0.85696
Multiple R-squared: 0.1716				
Adjusted R-squared: 0.1529				

Table 4: Multiple Regression

Because we have two variables that are relevant at the 10 percent level, we check the impact of control variables for each of them. The results are reported in Section B below and on the basis of the R-squared of the models, we conclude that the variable “Operating Company” is the most reliable indicator. This should come as no surprise since it is based on an in-depth qualitative analysis by at least two reviewers.

B. Impact of Control Variables

Earlier studies have demonstrated the impact of other variables on the DLOM, enabling us to verify whether our results hold after controlling for these known determinants. Specifically, we introduced:

1. The size of the company being valued (undiscounted equity value in million USD and actualized on the basis of the CPI published by the World Bank—five missing values have been replaced by the average 120.33 million USD over the 132 observations). We use the natural logarithm of the size because this value is normally distributed. Larger companies are associated with a reduced risk (and thus a lower DLOM);
2. The availability of information (22 out of the 137 observations concern companies that make audited accounts available to their shareholders). The availability of audited information should reduce information asymmetry and thus the DLOM;
3. An indicator variable for profitability (six missing values, 117 out of the 131 available observations are for companies that are profitable in the accounting year preceding the valuation date). Profitable companies (and even more companies distributing dividends) are thought to have lower DLOMs; and
4. An indicator variable for diversification (two missing values, 42 out of the 135 available observations have diversified operations, i.e., they are active in more than one line of business on the valuation date). Diversified companies have a lower risk (and thus a reduced DLOM).

We also introduced the open character of a company (i.e., its ability and willingness to accept third party shareholders) as a control variable.⁹² 79 observations out of the 137 available observations possess an open character. Finally, we explored whether control has an impact on the DLOM. Specifically, we examined whether controlling stakes (with a DLOC equal to zero) have lower DLOMs (i.e., we hypothesize that a controlling stake is

⁹² The open character of a company was determined on the basis of a judicial finding or evidence of arm’s length third party transactions. In the absence of conclusive indications, we used a scoring table that considered (i) the legal form of the company, (ii) the presence of material transfer restrictions on the shares, (iii) the number of and relationships between the shareholders and (iv) the stated or implicit intent of the owners of the company (as reflected in the published case).

more marketable). Thus, we add a dichotomous, “synthetic,” or derived variable indicating whether the DLOC is zero. This is the case for 21 observations out of the 137.

We present our linear regression results below. It is important to reiterate that the response variable can take any value between zero and one and the correct methodology to be applied is a fractional regression for which the results are reported in Exhibit C.

Variable	Estimate	Std. Error	t-value	p-value
Intercept	0.30458	0.02527	12.05476	0.00000
Log(V_SIZE)	-0.00055	0.00407	-0.13451	0.89322
V_OPEN	-0.06117	0.01410	-4.33936	0.00003
V_INDUSTRY_H	-0.05005	0.01497	-3.34311	0.00110
V_PROFITABILITY	0.00447	0.02310	0.19330	0.84705
V_AUDIT	0.02868	0.01811	1.58359	0.11585
V_DIVERSIFIED	-0.00715	0.01552	-0.46103	0.64559
V_DLOC_ZERO	-0.04307	0.02010	-2.14314	0.03407

Multiple R-squared: 0.2974

Adjusted R-squared: 0.2574

Table 5a: Industry H with Control Variables

Variable	Estimate	Std. Error	t-value	p-value
Intercept	0.24798	0.02382	10.41052	0.00000
Log(V_SIZE)	-0.00032	0.00384	-0.08244	0.93443
V_OPEN	-0.07096	0.01359	-5.21998	0.00000
V_OPERATING	0.06407	0.01477	4.33913	0.00003
V_PROFITABILITY	0.00363	0.02242	0.16211	0.87149
V_AUDIT	0.00772	0.01854	0.41672	0.67761
V_DIVERSIFIED	-0.00503	0.01506	-0.33403	0.73893
V_DLOC_ZERO	-0.04729	0.01958	-2.41501	0.01721

Multiple R-squared: 0.3353

Adjusted R-squared: 0.2975

Table 5b: Operating with Control Variables

The introduction of the control variables does improve the predictive value of the model (adjusted R-squared) in an important manner. We note

that the Model 5b (including V_OPERATING) has the highest predictive value (R-squared) and that the impact of this variable remains stable compared to its estimated coefficient in a univariate regression.

The significance of DLOC_ZERO attracts the attention as it seems to indicate that in cases where a controlling stake is valued (i.e., for which the DLOC is zero), the DLOM is about 4.7 percent lower. This implies that a controlling stake is more marketable (hence a lower DLOM). As far as we are aware, this is the first empirical evidence that DLOC and DLOM are not two unrelated discounts as commonly suggested in court decisions.⁹³ We also observe that companies that are open to external shareholders (V_OPEN) are rewarded by a lower DLOM. The other control variables are not significant.

C. Summary Results

Based on the above findings, we only retained the variables which are statistically significant at the five percent level to determine the estimated effect of the operating character of a company on the DLOM.

Variable	Estimate	Std. Error	t-value	p-value
Intercept	0.24028	0.01234	19.47522	0.00000
V_OPEN	-0.06441	0.01282	-5.02359	0.00000
V_OPERATING	0.07039	0.01281	5.49300	0.00000
V_DLOC_ZERO	-0.04399	0.01758	-2.50255	0.01354

Multiple R-squared: 0.3205

Adjusted R-squared: 0.3051

Table 6: Final Model

Our analysis supports the Bajaj Hypothesis. There is an important and significant difference between the DLOM applied on operating and on non-operating companies. The difference we find is about 7.0 percent which is statistically significant at the 0.0 percent level (p-value).

⁹³ See Leacock, *The Anatomy of Valuing Stock in Closely Held Corporations*, *supra* note 12 at 199 (“[The] minority interest discount is distinguishable from the lack of marketability discount, because a lack of marketability can exist whether or not a minority or controlling interest is involved.”).

V. CONCLUSION

In conclusion, our study supports the Bajaj Hypothesis: the courts apply a different discount to operating companies versus non-operating companies. We found a difference of 7.0 percent, which is statistically significant.

To the extent that the lower discount applied on non-operating companies can be assimilated to a pure marketability discount, the percentage we found is high compared to the percentages derived by Bajaj et. al. and by Comment in multivariate regressions based on their respective restricted stock studies.

We believe that the court's DLOM decisions for non-operating companies might reflect an anchoring bias. The courts are provided with expert analysis that is often based on private placement and acquisition studies. These results reflect total private company discounts, and the courts only dare to venture into the lower bounds of these estimates. These discounts are thus overly generous when one wants to capture only the effect of a lack of marketability. The few studies that attempted to isolate the effect of marketability point to percentages between 5 percent and 14 percent. Admittedly, there are only a limited number of studies, and they too have been criticized.⁹⁴

This means, however, that the courts are double counting. The valuation of a non-operating company's assets reflects their fair market value (after appropriate discounts). Any adjustment (other than for marketability) at the company level would thus represent an excessive discount. As long as the courts hold onto their anchor (the total private company discount as reflected in certain studies), any DLOM on a non-operating company is probably higher than warranted. This might be an opportunity for tax planning and legal strategy.

⁹⁴ Reilly, *supra* note 2 at 257-58. Notably, the Bajaj et al. study was criticized because the restricted placements in their study concern important blocks that might include elements of control (which can have a positive influence on the private placement price). *See generally* Ashok B. Abbott, *Estimating the Discount for a Lack of Marketability: A Best Fit Model*, 15 VALUATION STRATEGIES 20 (2012). We also note that restricted stock studies involve companies that already have an established public trading market and private companies could be perceived as considerably riskier.

Exhibit A: Cases and Data Points

#	Date	Case	Obs.	Reporter
1	3-May-2021	Estate of Jackson v. Comm'r	1	T.C. Memo 2021-48 *; 2021 Tax Ct. Memo LEXIS 74 **; 121 T.C.M. (CCH) 1320
2	27-Oct-2020	Lucero v. United States	1	2020 U.S. Dist. LEXIS 199605 *; 2020 WL 6281591
3	10-Jun-2020	Nelson v. Comm'r	2	T.C. Memo 2020-81 *; 2020 Tax Ct. Memo LEXIS 79 **
4	2-Mar-2020	Grieve v. Comm'r	2	T.C. Memo 2020-28 *; 2020 Tax Ct. Memo LEXIS 28 **
5	19-Aug-2019	Estate of Jones v. Comm'r	2	T.C. Memo 2019-101 *; 2019 Tax Ct. Memo LEXIS 108 **
6	26-Mar-2019	Kress v. United States	3	372 F. Supp. 3d 731 *; 2019 U.S. Dist. LEXIS 49850 **; 2019-1 U.S. Tax Cas. (CCH) P60,711
7	24-Oct-2018	Estate of Streightoff v. Comm'r	1	T.C. Memo 2018-178 *; 2018 Tax Ct. Memo LEXIS 179 **; 116 T.C.M. (CCH) 437
8	9-Dec-2015	Redstone v. Comm'r	1	T.C. Memo 2015-237 *; 2015 Tax Ct. Memo LEXIS 242 **; 110 T.C.M. (CCH) 564
9	11-Feb-2014	Estate of Richmond v. Comm'r	1	T.C. Memo 2014-26 *; 2014 Tax Ct. Memo LEXIS 26 **; 107 T.C.M. (CCH) 1135
10	18-Oct-2013	Estate of Tanenblatt v. Comm'r	1	T.C. Memo 2013-263 *; 2013 Tax Ct. Memo LEXIS 273 **
11	8-Apr-2013	Estate of Koons v. Comm'r	1	T.C. Memo 2013-94 *; 2013 Tax Ct. Memo LEXIS 98 **; 105 T.C.M. (CCH) 1567
12	7-Feb-2013	Estate of Kite v. Comm'r	1	T.C. Memo 2013-43 *; 2013 Tax Ct. Memo LEXIS 43 **; 105 T.C.M. (CCH) 1277
13	28-Jun-2011	Estate of Gallagher v. Comm'r	1	2011 Tax Ct. Memo LEXIS 150 *; T.C. Memo 2011-148; 101 T.C.M. (CCH) 1702
14	22-Jun-2011	Estate of Giustina v. Comm'r	1	2011 Tax Ct. Memo LEXIS 141 *; T.C. Memo 2011-141; 101 T.C.M. (CCH) 1676
15	13-May-2010	Pierre v. Comm'r	1	2010 Tax Ct. Memo LEXIS 143 *; T.C. Memo 2010-106; 99 T.C.M. (CCH) 1436
16	2-Oct-2009	Estate of Murphy v. United States	3	2009 U.S. Dist. LEXIS 94923 *; 2009-2 U.S. Tax Cas. (CCH) P60,583; 104 A.F.T.R.2d (RIA) 2009-7703

#	Date	Case	Obs.	Reporter
17	29-Jan-2009	Estate of Marjorie deGreeff Litchfield v. Comm'r	2	2009 Tax Ct. Memo LEXIS 21 *; T.C. Memo 2009-21; 97 T.C.M. (CCH) 1079
18	22-Jul-2008	Bergquist v. Comm'r	2	131 T.C. 8 *; 2008 U.S. Tax Ct. LEXIS 20 **; 131 T.C. No. 2
19	27-May-2008	Holman v. Comm'r	3	130 T.C. 170 *; 2008 U.S. Tax Ct. LEXIS 12 **; 130 T.C. No. 12
20	5-May-2008	Astleford v. Comm'r	3	2008 Tax Ct. Memo LEXIS 129 *; T.C. Memo 2008-128; 95 T.C.M. (CCH) 1497
21	28-Sep-2006	Dallas v. Comm'r	2	2006 Tax Ct. Memo LEXIS 216 *; T.C. Memo 2006-212; 92 T.C.M. (CCH) 313
22	9-May-2006	Huber v. Comm'r	1	2006 Tax Ct. Memo LEXIS 97 *; T.C. Memo 2006-96; 91 T.C.M. (CCH) 1132; RIA TM 56510
23	10-Mar-2006	Temple v. United States	4	423 F. Supp. 2d 605 *; 2006 U.S. Dist. LEXIS 16171 **; 2006-1 U.S. Tax Cas. (CCH) P60,523; 97 A.F.T.R.2d (RIA) 2006-1649
24	11-Oct-2005	Estate of Kelley v. Comm'r	1	2005 Tax Ct. Memo LEXIS 236 *; T.C. Memo 2005-235; 90 T.C.M. (CCH) 369
25	31-May-2005	Estate of Jelke v. Comm'r	1	2005 Tax Ct. Memo LEXIS 128 *; T.C. Memo 2005-131; 89 T.C.M. (CCH) 1397
26	15-Mar-2005	Estate of Bongard v. Comm'r	2	124 T.C. 95 *; 2005 U.S. Tax Ct. LEXIS 8 **; 124 T.C. No. 8
27	26-Jul-2004	Estate of Thompson v. Comm'r	1	499 F.3d 129 *; 2007 U.S. App. LEXIS 20066 **; 2007-2 U.S. Tax Cas. (CCH) P60,546; 100 A.F.T.R.2d (RIA) 2007-5792
28	29-Dec-2003	Estate of Green v. Comm'r	1	2003 Tax Ct. Memo LEXIS 348 *; T.C. Memo 2003-348; 86 T.C.M. (CCH) 758; RIA TM 55384
29	25-Sep-2003	Peracchio v. Comm'r	1	2003 Tax Ct. Memo LEXIS 279 *; T.C. Memo 2003-280; 86 T.C.M. (CCH) 412
30	3-Sep-2003	Lappo v. Comm'r	2	2003 Tax Ct. Memo LEXIS 257 *; T.C. Memo 2003-258; 86 T.C.M. (CCH) 333
31	20-Aug-2003	Hess v. Comm'r	1	2003 Tax Ct. Memo LEXIS 250 *; T.C. Memo 2003-251; 86 T.C.M. (CCH) 303

#	Date	Case	Obs.	Reporter
32	13-Jun-2003	Estate of Deputy v. Comm'r	1	2003 Tax Ct. Memo LEXIS 174 *; T.C. Memo 2003-176; 85 T.C.M. (CCH) 1497
33	14-May-2003	McCord v. Comm'r	1	120 T.C. 358 *; 2003 U.S. Tax Ct. LEXIS 16 **; 120 T.C. No. 13
34	23-Aug-2002	Okerlund v. United States	2	53 Fed. Cl. 341 *; 2002 U.S. Claims LEXIS 221 **; 2002-2 U.S. Tax Cas. (CCH) P60,447; 90 A.F.T.R.2d (RIA) 2002-6124
35	1-Aug-2002	Dunn v. Comm'r	1	301 F.3d 339 *; 2002 U.S. App. LEXIS 15453 **; 59 Fed. R. Serv. 3d (Callaghan) 529
36	17-Jun-2002	Estate of Bailey v. Comm'r	1	2002 Tax Ct. Memo LEXIS 159 *; T.C. Memo 2002-152; 83 T.C.M. (CCH) 1862; T.C.M. (RIA) 54788
37	9-Apr-2002	Estate of Mitchell v. Comm'r	1	2002 Tax Ct. Memo LEXIS 100 *; T.C. Memo 2002-98; 83 T.C.M. (CCH) 1524; T.C.M. (RIA) 54715
38	5-Feb-2002	Estate of Heck v. Comm'r	1	2002 Tax Ct. Memo LEXIS 38 *; T.C. Memo 2002-34; 83 T.C.M. (CCH) 1181; T.C.M. (RIA) 54639
39	3-Oct-2001	Estate of Elma Middleton Dailey v. Comm'r	2	2001 Tax Ct. Memo LEXIS 299 *; T.C. Memo 2001-263; 82 T.C.M. (CCH) 710
40	24-Aug-2001	Adams v. United States	1	2001 U.S. Dist. LEXIS 13092 *; 2001-2 U.S. Tax Cas. (CCH) P60,418; 88 A.F.T.R.2d (RIA) 2001-6057
41	6-Jul-2001	Estate of H.A. True v. Comm'r	9	2001 Tax Ct. Memo LEXIS 199 *; T.C. Memo 2001-167; 82 T.C.M. (CCH) 27
42	9-May-2001	Estate of Marcia P. Hoffman v. Comm'r	1	2001 Tax Ct. Memo LEXIS 136 *; T.C. Memo 2001-109; 81 T.C.M. (CCH) 1588
43	27-Mar-2001	Wall v. Comm'r	1	2001 Tax Ct. Memo LEXIS 97 *; T.C. Memo 2001-75; 81 T.C.M. (CCH) 1425
44	6-Mar-2001	Estate of Jones v. Comm'r	2	116 T.C. 121 *; 2001 U.S. Tax Ct. LEXIS 11 **; 116 T.C. No. 10; 116 T.C. No. 11
45	2-Feb-2001	Janda v. Comm'r	1	2001 Tax Ct. Memo LEXIS 34 *; T.C. Memo 2001-24; 81 T.C.M. (CCH) 1100; T.C.M. (RIA) 54231
46	30-Nov-2000	Knight v. Comm'r	1	115 T.C. 506 *; 2000 U.S. Tax Ct. LEXIS 88 **; 115 T.C. No. 36

#	Date	Case	Obs.	Reporter
47	18-Aug-2000	Estate of Borgatello v. Comm'r	1	2000 Tax Ct. Memo LEXIS 309 *; T.C. Memo 2000-264; 80 T.C.M. (CCH) 260; T.C.M. (RIA) 54013
48	4-Aug-2000	Godley v. Comm'r	4	2000 Tax Ct. Memo LEXIS 284 *; T.C. Memo 2000-242; 80 T.C.M. (CCH) 158; T.C.M. (RIA) 53984
49	27-Jun-2000	Estate of Klauss v. Comm'r	1	2000 Tax Ct. Memo LEXIS 228 *; T.C. Memo 2000-191; 79 T.C.M. (CCH) 2177; T.C.M. (RIA) 53923
50	11-Apr-2000	Estate of Maggos v. Comm'r	1	2000 Tax Ct. Memo LEXIS 154 *; T.C. Memo 2000-129; 79 T.C.M. (CCH) 1861
51	20-Mar-2000	Gow v. Comm'r	4	2000 Tax Ct. Memo LEXIS 108 *; T.C. Memo 2000-93; 79 T.C.M. (CCH) 1680
52	15-Feb-2000	Estate of Weinberg v. Comm'r	1	2000 Tax Ct. Memo LEXIS 58 *; T.C. Memo 2000-51; 79 T.C.M. (CCH) 1507
53	5-Nov-1999	Estate of Smith v. Comm'r	2	1999 Tax Ct. Memo LEXIS 425 *; T.C. Memo 1999-368; 78 T.C.M. (CCH) 745
54	14-Oct-1999	Estate of Marmaduke v. Comm'r	2	1999 Tax Ct. Memo LEXIS 397 *; T.C. Memo 1999-342; 78 T.C.M. (CCH) 590
55	23-Aug-1999	Estate of Hendrickson v. Comm'r	1	1999 Tax Ct. Memo LEXIS 318 *; T.C. Memo 1999-278; 78 T.C.M. (CCH) 322; T.C.M. (RIA) 99278
56	29-Jul-1999	Gross v. Comm'r	1	1999 Tax Ct. Memo LEXIS 290 *; T.C. Memo 1999-254; 78 T.C.M. (CCH) 201; T.C.M. (RIA) 99254
57	10-Mar-1999	Desmond v. Comm'r	1	1999 Tax Ct. Memo LEXIS 84 *; T.C. Memo 1999-76; 77 T.C.M. (CCH) 1529; T.C.M. (RIA) 99076
58	17-Nov-1998	Barnes v. Comm'r	2	1998 Tax Ct. Memo LEXIS 410 *; T.C. Memo 1998-413; 76 T.C.M. (CCH) 881; T.C.M. (RIA) 98413
59	8-Aug-1998	King v. Comm'r (Estate of Brookshire)	1	1998 Tax Ct. Memo LEXIS 370 *; T.C. Memo 1998-365; 76 T.C.M. (CCH) 659
60	30-Jun-1998	Estate of Davis v. Comm'r	1	110 T.C. 530 *; 1998 U.S. Tax Ct. LEXIS 35 **; 110 T.C. No. 35
61	30-Apr-1998	Furman v. Comm'r	2	1998 Tax Ct. Memo LEXIS 158 *; T.C. Memo 1998-157; 75 T.C.M. (CCH) 2206
62	19-Mar-1998	Dockery v. Comm'r	2	1998 Tax Ct. Memo LEXIS 114 *; T.C. Memo 1998-114; 75 T.C.M. (CCH) 2032

#	Date	Case	Obs.	Reporter
63	27-Oct-1997	Estate of Fleming v. Comm'r	1	1997 Tax Ct. Memo LEXIS 566 *; T.C. Memo 1997-484; 74 T.C.M. (CCH) 1049; 3 U.S. Tax Cas. (CCH) P45,035
64	5-Feb-1997	Gray v. Comm'r	1	1997 Tax Ct. Memo LEXIS 66 *; T.C. Memo 1997-67; 73 T.C.M. (CCH) 1940
65	26-Aug-1996	Estate of Barudin v. Comm'r	1	1996 Tax Ct. Memo LEXIS 403 *; T.C. Memo 1996-395; 72 T.C.M. (CCH) 488
66	11-Mar-1996	Kosman v. Comm'r	3	1996 Tax Ct. Memo LEXIS 107 *; T.C. Memo 1996-112; 71 T.C.M. (CCH) 2356
67	4-Dec-1995	Wheeler v. United States	1	1995 U.S. Dist. LEXIS 21432 *; 77 A.F.T.R.2d (RIA) 96-1405
68	7-Aug-1995	Estate of McCormick v. Comm'r	4	1995 Tax Ct. Memo LEXIS 367 *; T.C. Memo 1995-371; 70 T.C.M. (CCH) 318
69	12-Jun-1995	Mandelbaum v. Comm'r	1	1995 Tax Ct. Memo LEXIS 256 *; T.C. Memo 1995-255; 69 T.C.M. (CCH) 2852
70	28-Mar-1995	Estate of Frank v. Comm'r	1	1995 Tax Ct. Memo LEXIS 178 *; T.C. Memo 1995-132; 69 T.C.M. (CCH) 2255
71	27-Oct-1994	Estate of Luton v. Comm'r	2	1994 Tax Ct. Memo LEXIS 550 *; T.C. Memo 1994-539; 68 T.C.M. (CCH) 1044
72	19-Oct-1994	Estate of Lauder v. Comm'r	1	1994 Tax Ct. Memo LEXIS 535 *; T.C. Memo 1994-527; 68 T.C.M. (CCH) 985
73	11-May-1994	Estate of Simpson v. Comm'r	2	1994 Tax Ct. Memo LEXIS 217 *; T.C. Memo 1994-207; 67 T.C.M. (CCH) 2938
74	8-Dec-1993	Estate of Ford v. Comm'r	6	1993 Tax Ct. Memo LEXIS 595 *; T.C. Memo 1993-580; 66 T.C.M. (CCH) 1507
75	10-Nov-1993	Estate of Jung v. Comm'r	1	101 T.C. 412 *; 1993 U.S. Tax Ct. LEXIS 69 **; 101 T.C. No. 28
76	1-Feb-1993	Estate of Bennett v. Comm'r	1	1993 Tax Ct. Memo LEXIS 47 *; T.C. Memo 1993-34; 65 T.C.M. (CCH) 1816
77	30-Aug-1990	Estate of Murphy v. Comm'r	1	1990 Tax Ct. Memo LEXIS 520 *; T.C. Memo 1990-472; 60 T.C.M. (CCH) 645; T.C.M. (RIA) 90472
78	1-Aug-1990	Estate of Lenheim v. Comm'r	5	1990 Tax Ct. Memo LEXIS 420 *; T.C. Memo 1990-403; 60 T.C.M. (CCH) 356; T.C.M. (RIA) 90403

#	Date	Case	Obs.	Reporter
79	31-May-1990	Estate of Dougherty v. Comm'r	1	1990 Tax Ct. Memo LEXIS 292 *; T.C. Memo 1990-274; 59 T.C.M. (CCH) 772; T.C.M. (RIA) 90274
80	28-Feb-1990	Estate of Newhouse v. Comm'r	1	94 T.C. 193 *; 1990 U.S. Tax Ct. LEXIS 9 **; 94 T.C. No. 14
TOTAL NUMBER OF OBSERVATIONS			137	

Exhibit B: Descriptive Statistics

A. Dependent variable (DLOM)

N	Range	Minimum	Maximum	Mean	Std. Deviation
137	50.00	0.00	50.00	23.75	8.8656

These numbers include cases for which a conflated discount (i.e., a combined DLOM and DLOC) was decided and for which a proportional allocation has been made based on our sample averages as explained in footnote 92. There are no missing values.

A QQ-plot and a Shapiro-Wilks test confirm that the DLOM is normally distributed.

B. Independent variables

We report descriptive statistics for the main independent variables below. The independent variables are categorical variables for which the independent reviewers came to concurring conclusions (no missing information).

Categorical Variables	1	0	Missing	Total (N)
V_ASSET_VAL	76	61	0	137
V_OPERATING	80	57	0	137

SIC Codes & Industry Classification

Division	Industry	SIC Range	# of Obs.
A	Agriculture, Forestry, & Fishing	01-09	7
B	Mining	10-14	2
C	Construction	15-17	4
D	Manufacturing	20-39	14
E	Transportation & Public Utilities	40-49	9
F	Wholesale Trade	50-51	5
G	Retail Trade	52-59	8
H	Finance, Insurance, & Real Estate	60-67	76
I	Services	70-89	12
J	Public Administration	91-98	0
K	Non classifiable Establishments	99	0
TOTAL			137

A variance inflation factor (VIF) analysis shows that Asset_Val, Operating and Industry H are moderately correlated.

C. Control variables

We report key statistics for the continuous and categorical control variables below. The missing information corresponds to the cases and observations for which there is no information available or for which the independent reviewers came to conflicting conclusions.

Continuous Variables	N	Min.	Max.	Mean	Std. Deviation
V_SIZE (\$M)	137	0.198	3149.63	120.33	372.16

The numbers for size include 5 missing observations that have been replaced by the average of the available observations. In the regression we use the natural logarithm of the size as the control variable. A QQ plot and Shapiro-Wilks test confirm the normal distribution of this variable.

Categorical Variables	1	0	Missing	Total (N)
V_AUDIT	22	115	0	137
V_DIVERSIFIED	42	93	2	137
V_PROFITABLE	117	14	6	137
V_DLOC_ZERO	21	116	0	137
V_OPEN	79	58	0	137

Exhibit C: Fractional Regression Results**A. Tests for the Bajaj Hypothesis**

(a) Asset-based Valuation

Variable	Estimate	Std. Error	t-value	p-value
Intercept	-1.03599	0.05918	-17.50464	0.00000
V_ASSET_VAL	-0.24196	0.08180	-2.95781	0.00366

R-squared: 0.0614

(b) SIC code 60-67

Variable	Estimate	Std. Error	t-value	p-value
Intercept	-0.96280	0.05505	-17.48966	0.00000
V_INDUSTRY_H	-0.38429	0.07746	-4.96104	0.00000

R-squared: 0.1550

(c) Operating Company

Variable	Estimate	Std. Error	t-value	p-value
Intercept	-1.40404	0.06465	-21.71650	0.00000
V_OPERATING	0.39023	0.08120	4.80578	0.00000

R-squared: 0.1485

(d) Multiple Regression

Variable	Estimate	Std. Error	t-value	p-value
Intercept	-1.19470	0.14579	-8.19486	0.00000
V_ASSET_VAL	0.23497	0.12412	1.89317	0.06051
V_INDUSTRY_H	-0.23202	0.10749	-2.15851	0.03268
V_OPERATING	0.01511	0.09827	0.15376	0.06984

R-squared: 0.1768

B. Impact of Control Variables

a) Industry H with Control Variables

Variable	Estimate	Std. Error	t-value	p-value
Intercept	-0.81591	0.13533	-6.02899	0.00000
Log(V_SIZE)	-0.00321	0.02247	-0.14294	0.88657
V_OPEN	-0.33151	0.07681	-4.31617	0.00003
V_INDUSTRY_H	-0.27434	0.08269	-3.31755	0.00119
V_PROFITABILITY	0.02824	0.12431	0.22714	0.82069
V_AUDIT	0.14451	0.09830	1.47021	0.14406
V_DIVERSIFIED	-0.04167	0.08577	-0.48585	0.62794
V_DLOC_ZERO	-0.25293	0.11613	-2.17796	0.03132

R-squared: 0.3019

b) Operating with Control Variables

Variable	Estimate	Std. Error	t-value	p-value
Intercept	-1.13832	0.13020	-8.74279	0.00000
Log(V_SIZE)	-0.00293	0.02138	-0.13694	0.89130
V_OPEN	-0.38933	0.07470	-5.21199	0.00000
V_OPERATING	0.36511	0.08342	4.37677	0.00003
V_PROFITABILITY	0.03047	0.12155	0.25067	0.80248
V_AUDIT	0.04172	0.09962	0.41873	0.67614
V_DIVERSIFIED	-0.03390	0.08342	-0.40637	0.68517
V_DLOC_ZERO	-0.28225	0.11414	-2.47278	0.01477

R-squared: 0.3377

C. Final Model

Variable	Estimate	Std. Error	t-value	p-value
Intercept	-1.17610	0.06945	-16.93346	0.00000
V_OPEN	-0.35520	0.07102	-5.00146	0.00000
V_OPERATING	0.40080	0.07338	5.46176	0.00000
V_DLOC_ZERO	-0.26363	0.10442	-2.52469	0.01276

R-squared: 0.3252