Firm Value and Marketability Discounts

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I. Introduction

The issue of valuation is a continuing one in legal literature since it resides at the heart of many controversies.[1] Valuation of securities of a closely held company, for which there is no ready market, is particularly challenging. For example, such valuation is critical in cases in which minority dissenting shareholders to a merger demand an appraisal remedy, or when a gift ownership interest in a partnership, a limited liability company, or a closely held company must be valued for tax purposes.

In general, investors value marketability. Therefore, other things being equal, investors will pay more for an asset that is readily marketable than for an otherwise identical asset that is not readily marketable. The usual valuation methodologies, which utilize cash flows or market transactions, do not explicitly account for the marketability of an asset. Hence, in order to value an asset that is not marketable, the usual approach is to value the asset as if it were marketable, then apply a marketability discount to this estimated value.[2] The challenge in valuing non-marketable assets lies in determining the appropriate discount to be applied in a given situation.

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   1. *See generally* Jay W. Eisenhofer & John L. Reed, *Valuation Litigation*, 22 DEL. J. CORP. L. 37 (1997) (discussing the various valuation issues arising during litigation for such causes of action as shareholder appraisal actions, breach of fiduciary duty, fraud, breach of contract, bankruptcy reorganization, and tax proceedings).

   2. One note of caution is in order: when employing a valuation approach based on market transactions, if the asset in question is being valued with reference to a transaction price observed for a similarly non-marketable asset, no marketability discount need be taken. For example, if a house is appraised on the basis of transactions in similar houses, one need not apply an additional discount to the appraised value.

The concept of marketability refers to how quickly an asset can be converted to cash without the owner incurring substantial transaction costs or price concessions.[3] Certain financial assets, such as actively traded common stocks, are considered to be among the most marketable of assets. If an individual wishes to convert their common stock holdings into cash during regular trading hours, this can be accomplished nearly instantaneously with modest transaction costs and a minimal impact on the market price of the stocks.[4] On the other hand, real estate is considered to be far less marketable. If an individual wishes to sell real estate holdings, it is typically a lengthy process, often involving substantial transaction costs and price concessions.

In legal settings, marketability discounts elicit two primary types of controversy: (i) whether a marketability discount should be applied, and (ii) the appropriate level of discount to be applied. For example, in dissenters' rights cases, which generally involve valuation of non-marketable assets, there is considerable disagreement about whether any marketability discount should be applied. Courts in New York, Illinois, Oregon, Washington, Georgia, Kentucky, Indiana, and Colorado have decided that it is appropriate to apply lack-of-marketability discounts. Courts in Delaware, New Jersey, Nebraska, and Missouri, however, have determined that such discounts are unfair.[5] Second, there is disagreement about the appropriate level of the discount. A variety of methodologies have been employed to estimate these discounts in practice. These alternative methodologies yield a wide range of proposed discounts.[6]

Existing methodologies estimate the marketability discount in one of three ways. One group of studies estimates the discount by comparing the price of an asset during a period in which it is marketable to the price of the same asset during a period in which it is non-marketable. A second group of studies contemporaneously compares the prices of two claims on the same underlying asset, where one of the claims is marketable and the other is not. The third group of studies compares transaction prices of assets that are readily marketable with prices of otherwise similar assets having restricted marketability.

In the following section we review economic and financial theory regarding the value of marketability and what might determine the marketability discount. Section III of this study summarizes and critiques prior empirical studies of the marketability discount. As part of this summary, we identify the limitations associated with using each of the three estimation techniques described above. In Section IV we provide new empirical evidence concerning the magnitude of the marketability discount and its cross-sectional determinants. This new evidence comes from an update of prior empirical studies using restricted stock. The results of our analysis allow us to provide some useful

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3. "Price concessions" refer to the phenomenon whereby the sale of an asset takes place at a discount relative to the asset's current market price.

4. An exception to this statement is the trading of large blocks of common stock. For example, Mikkelson and Partch show that blocks of shares sold in secondary sales are sold at prices below the prevailing market price of the shares. *See* Wayne H. Mikkelson & M. Megan Partch, *Stock Price Effects and Costs of Secondary Distributions*, 14 J. FIN. ECON. 165, 192 (1985).

5. *See generally* John D. Emory, Jr., Comment, *The Role of Discounts in Determining "Fair Value" Under Wisconsin's Dissenters' Rights Statutes: The Case for Discounts*, 4 WIS. L. REV. 1155 (1995) (reviewing state case law concerning the applicability of marketability discounts for establishing fair value in dissenter appraisal litigation).

6. *See generally* Robert F. Reilly, *The Identification and Quantification of Discounts and Premiums*, AM. BANKR. INST. J., May 1996, at 30.

prescriptions for the valuation of non-readily-marketable assets such as shares of closely held firms.

II. Marketability and the Pricing of Assets

In theory, the price of any asset is given by the present value of the cash flows to be received from owning the asset.[7] In the absence of taxes, transaction costs, and other restrictions on trade, an asset can be instantaneously converted to cash at this equilibrium price. In the presence of transaction costs, however, buyers will demand a discount on the price of the asset equal to their cost of converting the asset to cash. Similarly, investors will demand a discount if they are unable to sell the asset for a period of time. This discount represents compensation to the investor for the inability to convert the asset to cash. This inability potentially causes the investor to miss opportunities to allocate capital to assets with higher returns.

The marketability of an asset refers to the degree to which an asset can be converted to cash quickly, without incurring large transaction costs or price concessions.[8] All else equal, the more marketable an asset, the higher the price an investor will be willing to pay for the asset.[9]

Longstaff (1995) derives an upper bound on the value of marketability using option pricing theory.[10] Specifically, he considers a hypothetical investor with perfect market timing ability; *i.e.*, the investor has the ability to identify the point in time in which the price of a security reaches its maximum.[11] If this investor is restricted from selling the security for a fixed period of time, the investor forgoes the ability to sell the security at its maximum price. The difference between the present value of the price received from selling the security after marketability restrictions are relaxed and the present value from selling the security at its maximum price represents the value of marketability.[12] Using standard techniques for option pricing literature and certain assumed parameter values, Longstaff estimates discounts for lack of marketability as a function of marketability restriction period and the standard deviation of the security's returns.[13] He concludes that marketability discounts can be economically relevant even when the period of illiquidity is relatively short.[14]

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7. This view of asset valuation is standard in the finance literature. *See* Tom Copeland et al., Valuation: Measuring and Managing the Values of Companies (2d ed. 1994) (stating an argument that, in an attempt to maximize a firm's equity value, the discounted cashflow approach will reward corporate managers with higher share prices than attention to accounting earnings).

8. *See* Zvi Bodie et al., Investments 893 (2d ed. 1993) (arguing that an asset's liquidity (*i.e.*, marketability) represents the speed or ease with which an asset can be sold and still command a price close to its inherent or fair value).

9. *See generally* Yakov Amihud & Haim Mendelson, *Liquidity and Asset Prices: Financial Management Implications*, 17 Fin. Mgmt. 5 (1988) (discussing the relation between marketability and asset prices in a corporate framework); Yakov Amihud & Haim Mendelson, *Liquidity, Asset Prices and Financial Policy*, 47 Fin. Analyst J. 56 (1991) (discussing financial policy in relation to liquidity).

10. *See* Francis A. Longstaff, *How Much Can Marketability Affect Security Values?*, 50 J. Fin. 1767, 1767 (1995).

11. *Id.* at 1769.

12. *Id.* at 1768.

13. *Id.* at 1770.

14. *Id.* at 1773-74. Longstaff's analysis ignores the possibility that investors could synthetically create a

Several factors are likely to influence an asset's marketability. First, the more uncertain the asset's value, the higher will be the potential opportunity cost associated with a lack of marketability. Hence, the discount demanded by investors will be higher.[15] Similarly, the more difficult it is for an outsider to appraise the value of an asset, the less marketable that asset will be. Hence, potential investors will demand a discount in price. Third, the extent to which there are close substitutes for an asset will influence its marketability. For example, the fact that many common stocks are close substitutes for one another contributes to the high degree of liquidity present in the stock market.[16] Conversely, the lack of substitutes in the market for fine art contributes to the lack of liquidity in that market.[17] Fourth, the duration of restrictions on trade is likely to influence an asset's marketability discount. All else equal, the longer the period that an asset is non-marketable, the lower its value will drop.[18] Fifth, the larger the amount of the asset being sold, the less marketable the asset will be.[19] This effect potentially comes from two different sources. The more of an asset that is sold, relative to the total outstanding, the less liquid the remaining market for the asset.[20] Furthermore, the larger the size of the transaction, the smaller the pool of potential buyers due to capital constraints and desires for diversification of personal portfolios.[21]

In sum, marketability has inherent value to investors because the lack of marketability increases potential opportunity costs for holders of assets. Consequently, all else equal, asset values will be positively correlated with the marketability of the asset.[22] Stated differently, expected returns from holding assets will be negatively correlated with the asset's marketability. The higher expected returns for less marketable assets reflect equilibrium compensation to investors for bearing the additional risks (in the form of

liquid position by trading in derivative securities. We later discuss how this possibility affects estimates of the marketability discount. He also ignores the availability of additional capital for the holder of non-marketable security to profitably exploit trading opportunities emanating from the assumed timing ability. If such timing ability was less than perfect, his estimates of marketability discount would be lower. Thus, in the view of the authors, the contribution of Longstaff's work is to show the relationship between riskiness of a non-marketable asset and its marketability discounts and not necessarily a quantification of such discounts in the real world.

15. MARK HIRSCHEY, INVESTMENTS: THEORY AND APPLICATIONS (2001) (discussing the proposition that uncertainty in asset values is viewed by investors as risk and that they will demand higher rates of return to hold riskier assets).

16. To the extent that two stocks are substitutes for each other, the depth of the market for those kinds of equities will increase. With greater depth, more of a stock can be traded without a corresponding change in the price. For a fuller discussion of the concept of market depth and how it is similar to a price elasticity, see Stephen P. Ferris et al., *A Microstructure Examination of Trading Activity Following Stock Splits*, 5 REV. QUANTITATIVE FIN. & ACCT. 27, 31 (1995).

17. LAWRENCE J. GITMAN & MICHAEL D. JOEHNK, FUNDAMENTALS OF INVESTING 157 (8th ed. 2001).

18. *See* HIRSCHEY, *supra* note 15, at 682-84 (discussing the impact of liquidity on share prices and their value).

19. THOMAS E. COPLAND & J. FRED WESTON, FINANCIAL THEORY AND CORPORATE POLICY 370 (3d ed. 1988).

20. The impact of large asset sales on the value and liquidity of the remaining assets is examined with the price pressure hypothesis. *See* Myron Scholes, *The Market for Securities: Substitution Versus Price Pressure and the Effects of Information on Share Prices*, 45 J. BUS. 179, 179-211 (1972).

21. This phenomenon is consistent with the trading range argument of stock splits that contends that stocks split in an attempt to reduce their cost and thereby increase the overall liquidity of the issue. *See* JAMES C. VAN HORNE, FINANCIAL MANAGEMENT AND POLICY 326-27 (12th ed. 2001).

22. GITMAN & JOEHNK, *supra* note 17, at 157.

opportunity costs). This is caused by the inability to convert the asset to cash quickly, with minimal price impact.[23]

III. Prior Empirical Estimates of the Marketability Discount

Existing empirical methodologies to estimate the lack of marketability discount can be assigned to one of three categories of technique. The first estimates the marketability discount by comparing the price of an asset during a period in which it is relatively non-marketable to a period in which it is more marketable. This approach is typically implemented by comparing share prices of firms following initial public offerings (IPO) to transaction prices in those same shares prior to the IPO.[24] This Article refers to this approach as the IPO approach. The second approach compares share prices of two claims on the same underlying asset, where one claim is marketable and the other is not. This approach is typically implemented by examining issues of restricted stock. This Article refers to this as the restricted stock approach. The third approach uses a comparable set of marketable companies to estimate what the market value of a non-marketable asset would be if it were marketable. This approach compares acquisition prices of private (*i.e.*, non-marketable) companies with those of comparable public companies, and is referred to as the acquisitions approach.

This section reviews and critiques each of these approaches. The average discounts are estimated under each approach, including the cross-sectional determinants of the discounts (if any). Further, the limitations inherent in each approach are discussed.

A. The IPO Approach

The IPO approach attempts to quantify the discount associated with the lack of marketability by comparing post-IPO share prices with transaction prices in the same shares prior to the IPO. John Emory and Willamette Management Associates (WMA) each conducted a series of studies using the IPO approach.[25] Emory's studies examine prices for private stock transactions completed within five months of a subsequent IPO. These prices are then compared with the price at which this firm's shares were offered in its IPO.

As reported in Panel A of Table 1, Emory's studies suggest that private transactions take place at an average discount of approximately 45% relative to the price in the same share in the IPO aftermarket. Moreover, the discount is remarkably consistent over the

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23. Note that this does not imply that the expected premium per unit of illiquidity is constant. Investors that place a low value on liquidity will tend to own longer-term, less liquid securities; while investors that place a high value on liquidity will tend to own shorter-term, more liquid securities. This clientele effect suggests that asset returns will be an increasing, but concave, function of illiquidity. For a formal derivation of this idea and empirical evidence, see Yakov Amihud & Haim Mendelson, *Asset Pricing and the Bid-Ask Spread*, 17 J. Fin. Econ. 223 (1986).

24. Companies can issue equity to the public at multiple times. An initial public offering is the first time that a company issues shares to the public, and thus, a portion of its equity is owned by public investors. Subsequent issues of shares by the company are referred to as seasoned offerings. *See* Douglas R. Emery et al., Principles of Financial Management 46 (1998).

25. *See* Emory, *supra* note 5; Shannon P. Pratt et al., Valuing A Business: The Analysis and Appraisal of Closely Held Companies 344-49 (1996).

1985 to 1997 period, ranging between 42% and 45% (1980-1981 is an exception with an average discount of 60%).

WMA conducted a similar series of studies that are summarized in *Valuing a Business*.[26] WMA examined transactions occurring in the three years prior to each firm's IPO. Then the transaction prices were adjusted for changes in industry price/earnings multiples over the period between the pre-IPO transactions dates and the IPO date. As reported in Panel B of Table 1, WMA found marketability discounts similar to those found in the Emory studies.

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26. *See* PRATT ET AL., *supra* note 25, at 345.

Table 1: Summary of Marketability Discounts from IPO Studies[27]

Panel A reports marketability discounts from the series of studies conducted by John Emory over the period 1980 to 1997. Panel B reports marketability discounts from the series of studies conducted by Willamette Management Associates over the period 1975 to 1992.

Panel A: Emory Studies

Sample Period	Number of Transactions	Average Discount
1980-1981	13	60%
1985-1986	21	43%
1987-1989	27	45%
1989-1990	23	45%
1990-1992	35	42%
1991-1993	54	45%
1994-1995	46	45%
1995-1997	91	43%

Panel B: Willamette Management Associates Studies

Sample Period	Number of Transactions	Average Discount
1975-1978	31	55%
1979	17	63%
1980-1982	113	56%
1984	33	74%
1985	25	43%
1986	74	48%
1987	40	44%
1988	19	52%
1989	19	50%
1990	23	49%
1991	34	32%
1992	75	52%

There are several difficulties with interpreting the discounts from the IPO studies as discounts for the lack of marketability. First, the discounts appear to be implausibly large. Suppose that an IPO takes place at a price of $10 per share. A discount of 45% for pre-IPO private transactions suggests that an investor could purchase a less marketable share for $5.50, then sell it for $10 following the IPO, earning an 82% rate of return ($4.50 profit on an investment of $5.50 represents a return of $4.50/$5.50 = 82%). Assuming

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27. *Id.* at 408, 410.

that the private transaction takes place six months prior to the IPO (approximately the period of time in the Emory studies), this implies an annualized rate of return of 231%.[28]

Second, transactions prior to the IPO are likely to be very different types of transactions from those that take place at the time of the IPO. Buyers of shares prior to the IPO are likely to be insiders who provide some sort of service to the firm. For example, venture capitalists may provide their expertise to the management of the firm. Thus, part of the discount may reflect equilibrium compensation for these services rather than compensation for the lack of marketability. Buyers of the stock at the time of the IPO are less likely to be insiders, therefore, the sale price is less likely to be discounted to provide compensation for services performed.

Third, the IPO approach is subject to a serious sample selection problem. Firms will choose to issue shares through an initial public offering when their prospects improve. By contrast, those firms that do poorly will elect to bypass the IPO or may fail altogether. At the time of the private transaction, the market can only price the shares based on its probability assessment of the different possible outcomes. Once an IPO takes place, this uncertainty is resolved and only the successful (and hence higher valued) firms issue shares.

An example clarifies the intuitiveness of the underlying sample selection problem. Suppose that investors know that the value of a share tomorrow will be either $20 or $0 with equal probability. If the value turns out to be $20, the firm will issue shares via an IPO. Otherwise, it will not. Assuming no marketability discount, rational investors will price the shares today at their expected value, $10 (expected value = 50% * $20 + 50% * $0 = $10). An analyst using the IPO approach to estimate the marketability discount would observe an IPO only if the value of the firm turned out to be $20. Hence, the analyst would conclude that the marketability discount was 50% even though the true discount was zero. Consequently, the IPO approach likely generates inflated estimates of the marketability discount.

B. The Restricted Stock Approach

The restricted stock approach is another approach to quantify marketability discount. Securities that are privately placed with accredited investors (rather than offered to the public at large) do not have to be registered with the Securities and Exchange Commission (SEC). However, shares issued in unregistered private placements are considered restricted securities under SEC Rule 144.[29] Consequently, buyers of unregistered shares must wait for a specified period of time before the shares become marketable.[30] Unregistered shares placed privately are typically sold at a discount. There are several economic factors that contribute to this discount. One of these factors is the lack of marketability that characterizes these instruments. Therefore, a portion of the

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28. One hundred dollars invested at the beginning of the year would earn 82% in the first six months, giving the investor $182, which could then be reinvested. After another six months of earning 82%, the interest on the $182 would be 82% * $182, or $149. Adding $149 to the previously earned $182 yields $331, meaning a profit of $231 was obtained on a one year investment of $100. This represents a 231% annual return.

29. 17 C.F.R. § 230.144(a)(3) (2001).

30. Prior to February 1997, investors had to wait for two years for their shares to become marketable. As of February 1997, this waiting period has been reduced to one year. *See* Revision of Holding Period Requirements in Rules 144 and 145, Securities Act Release No. 33,7390, 63 SEC Docket 2077 (Feb. 20, 1997).

overall discount observed in private placements of restricted securities represents a marketability discount.

Table 2 summarizes the results of several empirical studies that have estimated the overall discounts on restricted stock. Average discounts on unregistered shares are sizable, ranging from 20% to 35%.[31] However, even the overall discounts on registered stocks—of which the marketability discount is only a part—are much smaller than the average estimates of marketability discounts generated by the IPO approach. This is undoubtedly a reflection of the selection bias inherent in the IPO approach that was described earlier.

Table 2: Summary of Marketability Discounts from Restricted Stock Studies[32]

A list of the studies estimating the marketability discount, the years covered in the studies and the average or median percentage difference between the restricted stock offer price and the price of the same firm's publicly traded shares.

Study	Sample Period	Average Discount
SEC	1966-1969	23%
Gelman	1968-1970	33%
Trout	1968-1972	34%
Maher	1969-1973	35%
Willamette Management	1981-1984	31%
Silber	1981-1988	34%
Johnson	1991-1995	20%

Although the studies listed in Table 2 are informative with respect to average discounts, they mask the fact that there is substantial cross-sectional variation in the discounts of restricted shares. For example, the SEC study cited in Table 2 finds that discounts range from -15% to 80%. Silber investigated this variation by comparing the characteristics of those firms having discounts below the median in his sample (35%) and those above the median.[33] He found that those firms in the high discount group are characterized by (i) a higher number of shares issued as a fraction of total shares outstanding, (ii) smaller dollar size of the restricted stock issue, (iii) lower earnings, (iv) lower revenues, and (v) smaller market value of equity. Moreover, in a cross-sectional regression model, Silber found that discounts are positively related to revenues, to a dummy variable denoting positive earnings, and to a dummy variable denoting whether

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31. For cases illustrating a discount for restricted stock in a publicly held firm, see *Trust Serv. of Am., Inc. v. United States*, 885 F.2d 561, 568-69 (9th Cir. 1989), and *Estate of Stratton v. Comm'r*, 45 T.C.M. (CCH) 432 (1982).

32. 5 INSTITUTIONAL INVESTOR STUDY REPORT OF THE SECURITIES AND EXCHANGE COMMISSION 2444-56 (1971); M. Gelman, *An Economist-Financial Analysts Approach to Valuing Stock of a Closely Held Company*, 36 J. TAX'N 353, 353-54 (1972); R. R. Trout, *Estimation of the Discount Associated with the Transfer of Restricted Securities*, 55 TAXES 381, 381-85 (1977); J. M. Maher, *Discounts for Lack of Marketability for Closely-Held Business Interests*, 54 TAXES 562, 562-71 (1976); PRATT, *supra* note 25, at 342-44; W. L. Silber, *Discounts on Restricted Stock: The Impact of Illiquidity on Stock Prices*, 47 FIN. ANALYSTS J. 60, 60-64 (1991); B. A. Johnson, *Quantitative Support for Discounts for Lack of Marketability*, 16 BUS. VALUATION REV. 152, 152-55 (1999).

33. Silber, *supra* note 32, at 60-64.

the buyer of the restricted shares had a special relationship with the issuing firm (*i.e.*, either a board representation or customer relationship).[34] Discounts are negatively related to the size of the restricted stock issue relative to total shares outstanding.

These findings suggest that it is incorrect to simply apply a constant discount for all issues of restricted stock. Rather, it appears that several characteristics of the firm and the restricted stock issue are associated with the discount in price relative to the firm's publicly traded shares.

Even if an analyst properly incorporates issue and issuer characteristics, it is premature to label the resulting discount a "marketability discount." For example, it is often the case that private equity investors commit to provide the issuing firm with advice and oversight following the private placement of equity. Moreover, these investors often commit to providing capital in the future, provided that the issuing firm meets a set of predetermined goals for financial performance. Consequently, at least a portion of the price discount observed in private equity placements might reflect compensation to these investors for future services rendered, rather than compensation for the lack of marketability.

Early restricted stock studies only examined restricted stock issues, not other private placements of equity.[35] They were, therefore, incapable of distinguishing between the two possible causes of the discount. However, more recent academic studies by Wruck (1989)[36] and Hertzel and Smith (1993)[37] do compare restricted stock to similar issues of registered shares. Wruck found that the difference in average discounts between the restricted shares in her study and registered shares was 17.6%, while the difference in median discounts was 10.4%. Since each type of shares may have been given as compensation, but only restricted shares have marketability impairments, this suggests that the difference in discounts is a gauge for the marketability discount. Consequently, the marketability discount is considerably smaller than studies based on the entire restricted stock discount would indicate.

The Wruck study did not, however, control for systematic differences in the characteristics of firms issuing registered and restricted shares. Hence, the difference in discounts could have been caused by factors other than differences in marketability. Hertzel and Smith provided a more comprehensive examination of the private placement market in 1993. Their study indicates that private placements are often undertaken by firms with limited tangible assets, those engaged in speculative development of new products and those in financial distress, and that discounts tend to be higher for these types of firms.[38] They speculate that discounts are required on the stock of these firms to serve as compensation for the higher information and monitoring costs associated with

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34. *Id.*

35. *See* Thomas Frieblob, *What are the Effects of Differing Types of Restrictions on Closely-Held Stocks?*, 58 J. TAX'N 240 (1983); Richard D. Johnson & George A. Racette, *Discounts on Letter Stocks Do Not Appear to be a Good Base on Which to Estimate Discounts for Lack of Marketability on Closely Held Stocks*, 59 TAXES 574 (1981).

36. Karen H. Wruck, *Equity Ownership Concentration and Firm Value: Evidence from Private Equity Financings*, 23 J. FIN. ECON. 3, 17 (1989).

37. Michael Hertzel & Richard L. Smith, *Market Discounts and Shareholder Gains for Placing Equity Privately*, 48 J. FIN. 459, 459-469 (1993).

38. *Id.* at 480.

the investments.[39] Therefore, discounts are not solely by compensation for future services rendered or due to the lack of marketability, but are also caused by these factors. After controlling for non-marketability determinants of private placement discounts in a multivariate regression framework, Hertzel and Smith found that the discount for restricted shares was only 13.5% greater than that of registered shares.[40] This is further evidence that the marketability discounts suggested by early restricted stock studies were too high.

There is yet another reason to be skeptical of marketability discounts estimated using the average discounts in a restricted stock approach. Such an approach assumes that investors cannot transform their restricted shares into a liquid security. If investors could do so, the costs of executing the transaction would represent the upper bound on the discount that investors would demand for non-marketability.

Because of recent innovations in financial markets, transactions of this type are now available to investors. Examples of such transactions include equity swaps and zero-cost collars.[41] An equity swap (sometimes referred to as a diversification swap) is an agreement with a commercial or investment bank to exchange the future cash flows on a notional amount invested in restricted shares for cash flows that the same notional amount will generate in an index (such as the S&P 500 Index). A zero-cost collar uses the notion of put-call parity to allow the holder of a restricted share to effectively liquidate those holdings.[42] Through the appropriate combination of borrowing, selling a call option, and buying a put option, the investor can create a position that generates the same cash flows as would an immediate sale of the restricted security.[43] Moreover, zero-cost collars do not necessarily have to be constructed using exchange-traded options. Private agreements with commercial and investment banks allow the investor to customize the strike price of the options, the term to maturity, and the transactional amount.

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39. *Id.* at 484.

40. In fact, Hertzel and Smith argue that even the 13.5% difference in discount between registered and restricted stock probably overstates the costs of illiquidity. In addition, they argue that discounts of this magnitude would provide strong incentives for firms to register shares prior to issue or to commit to quickly register shares following the private sale. Moreover, because buyers of the restricted shares tend to be institutional investors that do not value liquidity highly (e.g., life insurance companies and pension funds), it seems unlikely that such investors would require substantial marketability discounts for the commitment not to resell quickly. *Id.* at 480.

41. *See* J. Bettis et al., Insider Trading in Derivative Securities: An Empirical Examination of the Use of Zero-Cost Collars and Equity Swaps by Corporate Insiders (unpublished manuscript, on file with Author); *see also* P. Bolster et al., *Executive Equity Swaps and Corporate Insider Holdings*, 25 FIN. MGMT. 14, 14-24 (1996).

42. *See* HIRSCHEY, *supra* note 15, at 753 (discussing put-call parity that describes the nature of the relationship that must hold between the price of a put option and a call option on the same underlying equity).

43. Suppose that an investor buys a European put option and writes (sells) a European call option, both with an exercise price equal to the market value of the stock position assuming it was immediately marketable inflated at risk-free rate and with a term equal to the period of the marketability restriction. The Put-Call Parity Theorem implies that the investor will pay zero net cost for this position. (The sale proceeds from the writing of the call option will cover the cost of the put option.) Yet, such a "collar" will ensure that the investor will get an amount with the present value equal to the market value of the shares as if these shares were immediately marketable. Of course, since the payoffs from such a collar and from the stock position is guaranteed, the investor could borrow an amount equal to the value of the stock as if it were fully marketable against the collar and the restricted stock. Therefore, absent transaction costs, the cost of circumventing the marketability restriction by using the zero-cost collar will be zero.

The existence of these synthetic means of creating liquidity has important implications for estimating marketability discounts. Specifically, these imply an upwards bias in the estimates obtained from using restricted stock. If liquidity restrictions can be circumvented using either equity swaps or zero-cost collars at a modest cost, then true marketability discounts are likely to be much smaller than those estimated from the discount on restricted stock.

In sum, the restricted stock approach represents an improvement over the IPO approach in estimating the discount associated with a lack of marketability. Although early studies of restricted stock issues place the overall restricted stock discount in the range of 35%, this figure ignores other possible determinants of the discount other than lack of marketability. This figure also ignores cross-sectional differences in the discount and ignores the possibility of synthetically creating a liquid position. Once these other factors are taken into account, estimates of the marketability discount are less than one-half of the previous estimates.

C. The Acquisition Approach

The acquisitions approach estimates the marketability discount by comparing acquisition prices for public and private companies.[44] Koeplin, Sarin, and Shapiro (hereinafter Koeplin) examined 192 acquisitions of private companies between 1984 and 1998, excluding financial and regulated firms.[45] For each of the private company acquisitions, Koeplin identified the acquisition of a public company in the same country, the same year, and the same industry. In cases where several public company acquisitions met these criteria, the company with sales that were closest to those of the private company was chosen. Thus, each private company was matched with a similar public company that was also acquired around the same time.

The final sample included 84 acquisitions of private companies in the United States (domestic transactions) and 108 acquisitions of private companies outside the United States (foreign transactions).[46] On average, the private companies were smaller than their public counterparts (as measured by total assets and revenues).[47] In addition, among domestic transactions, earnings growth rates were higher for the private companies than for their matched firms. For foreign transactions, however, this pattern was reversed.[48]

To estimate the discount associated with private companies, Koeplin calculated the ratio of the firm's enterprise value to its earnings, sales, and book value. Enterprise value is defined as the purchase price of the acquisition multiplied by the number of outstanding shares, plus the book value of the firm's liabilities.[49] These ratios are then compared for the acquisitions of the private and comparable public companies.

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44. *See* Jay Eisenhofer & John Reed, *Valuation Litigation*, 22 DEL. J. CORP. L. 37, 116 (1997) (reviewing the issues surrounding the methodology of valuation in the context of litigation, including the use of comparable companies and comparable acquisitions approaches).

45. For further details, see John Koeplin et al., *The Private Company Discount*, 12 J. APPLIED CORP. FIN. 94 (2000).

46. *Id.* at 97.

47. *Id.*

48. *Id.*

49. *Id.* at 98 tbl.2.

Table 3: Marketability Discounts from Acquisitions of Private Companies

Means and medians of valuation ratios and private company discount of 84 (108) private and public transactions in the United States (outside the United States) between 1984 and 1998. The private company discount is measured as 1–(private company multiple/public company multiple). The sample was obtained from Securities Data Corporation (SDC) by first identifying acquisitions of private companies. Each of these transactions was matched with an acquisition of a public company that was closest in size and occurred in the same year, country, and similar industry.

Panel A: Domestic Transactions

	Private Targets		Public Targets		Discount	
	Mean	Median	Mean	Median	Mean	Median
Enterprise Value/ EBIT	11.76	8.58	16.39	12.47	28.26***	30.62***
Enterprise Value/ EBITDA	8.08	6.98	10.15	8.53	20.39***	18.14***
Enterprise Value/ Book Value	2.35	1.85	2.86	1.73	17.81***	-7.00
Enterprise Value/ Sales	1.35	1.13	1.32	1.14	-2.28	0.79

Panel B: Foreign Transactions

	Private Targets		Public Targets		Discount	
	Mean	Median	Mean	Median	Mean	Median
Enterprise Value/ EBIT	16.26	11.37	28.97	12.09	43.87***	5.96**
Enterprise Value/ EBITDA	11.96	7.10	25.91	9.28	53.85**	23.49*
Enterprise Value/ Book Value	2.41	1.35	3.70	1.68	34.86	19.64
Enterprise Value/ Sales	2.63	1.35	4.59	1.63	42.70	17.18

***, **, and * denote significance at the 0.01, 0.05, and 0.10 levels, respectively.

Table 3 summarizes the results of the Koeplin study. In general, private companies are purchased at a substantial discount from comparable public companies. For example, among domestic private transactions, the average ratio of enterprise value to earnings before interest, taxes, depreciation and amortization (EBITDA) is 8.08.[50] By contrast, the ratio of enterprise value to EBITDA is 10.15 for domestic public transactions.[51] This implies that the purchase price of a private transaction takes place at a discount of 20.4% relative to an acquisition of a matched public firm with the same EBITDA.[52] Similar

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50. *Id.*

51. *Id.*

52. *Id.* The private firm is valued at 8.08 times EBITDA, while the public firm is valued at 10.15 times its

discounts are obtained using the ratio of enterprise value to earnings before interest and taxes (EBIT).[53] By contrast, smaller discounts are obtained using the ratios of enterprise value to sales and enterprise value to book value.[54]

It is also evident from Table 3 that average discounts for private transactions are greater in foreign transactions than in domestic transactions.[55] For example, average discounts are 44% for the EBIT multiple and 54% for the EBITDA multiple.[56] Median discounts, however, are only 6% and 23% for the EBIT and EBITDA multiples, respectively.[57] The greater variation in discounts among foreign transactions is likely due to the fact that different countries have different accounting standards.[58] As a result, multiples could vary across countries simply because of different accounting standards.[59]

There are limitations involved in comparing the discounts estimated using the acquisitions approach to marketability discounts. First, it is possible that there are systematic differences in the characteristics of the private and public firms that are acquired. As noted earlier, private companies tend to be smaller and exhibit higher earnings growth rates prior to the acquisition.[60] The observed differences in discounts might be attributable to these differences in characteristics rather than a difference in marketability.[61] To explore this possibility, Koeplin estimates cross-sectional regressions of the different multiples on "size, growth, industry, and a dummy variable" denoting whether the transaction was an acquisition of a private company.[62] Even after including these control variables, the discount associated with domestic private acquisitions remains similar in magnitude to that reported in Table 3.[63] For foreign transactions, however, the estimated discount is statistically significant only for the EBIT multiple.[64]

Second, private companies are typically closely held. Therefore, it is possible that the owners of the private company, who are also likely to be the company's senior managers, are compensated in the acquisition by other means.[65] For example, senior managers of the private firm might prefer to receive part of their consideration in the acquisition "in the form of an employment contract."[66] If these employment contracts represent above-market compensation, this will reduce the value of the private companionship relative to that of the otherwise comparable public company.[67]

EBITDA. The ratio of the private firm's valuation multiple to that of public firms is 8.08 / 10.15 or 79.6%. This implies that private firms are valued at a 20.4% discount relative to public firms.

53. Koeplin et al., *supra* note 45, at 99.

54. *Id.* at 99-100 (2000) (stating that the lower statistical significance of discounts computed using ratios of sales and book value is due to the greater variability in these multiples across firms).

55. *Id.* at 99.

56. *Id.*

57. *Id.*

58. Koeplin et al., *supra* note 45, at 100

59. *Id.*

60. *Id.*

61. *Id.*

62. *Id.*

63. Koeplin et al., *supra* note 45, at 100.

64. *Id.*

65. *Id.* at 101.

66. *Id.*

67. *Id.*

Overall, Koeplin was careful in describing the observed differences as a "private company discount" rather than a discount due to lack of liquidity per se.[68] The acquisitions approach represents an independent assessment of the private company valuation discount. However, because other factors might be driving these differences in valuation ratios of public and private acquisitions, estimates using the acquisitions approach should be viewed only as "an upper bound" for the marketability discount.[69] The "true" marketability discount may be considerably smaller.[70]

D. What Do We Conclude About the Existing Estimation Methods?

Our review of the existing methods for estimating the marketability discount yields several conclusions. First, the IPO approach is seriously flawed in at least two respects. First, transactions prior to the IPO are likely to include compensation to investors for providing other services to the firm. Second, and more important, only firms that are ex post successful will conduct an IPO.[71] Hence, the IPO price will appear high relative to the price of pre-IPO transactions. As a result, the IPO approach produces estimates that seriously overstate the marketability discount.

The restricted stock approach and the acquisitions approach yield more reasonable estimates of the marketability discount. Nonetheless, each approach has its limitations. Most studies using the restricted stock approach produce fairly large discounts. However, these studies do not properly control for other determinants of the discount on privately placed shares. The one study that does so, Hertzel and Smith, estimates the marketability discount at 13.5%.[72] However, our knowledge of the cross-sectional determinants of this discount is limited and most of the studies using the restricted stock approach (including Hertzel and Smith) use data prior to the 1990s.[73] Similarly, although the acquisitions approach produces estimates of the marketability discount that are similar in magnitude to those of the early restricted stock studies, the approach has not yielded cross-sectional determinants of the discount. Until these other factors are determined, uncertainty remains that observed differences in value between public and private held acquisitions might be due to considerations other than the lack of marketability.

Given these limitations in the prior studies, this Article extends the empirical literature on marketability discounts. Specifically, it provides new evidence concerning the use of the restricted stock approach. It analyzes a dataset that includes transactions from the 1990s and undertakes a comprehensive cross-sectional analysis of the subsequent discounts.

IV. AN ANALYSIS OF DISCOUNTS IN PRIVATE EQUITY PLACEMENTS

In this section, we report the results from our empirical analysis of a comprehensive sample of private placement transactions in the 1990s. This data allows us to accomplish

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68. *See* Koeplin et al., *supra* note 45, at 100 (using four valuation multiples for both public and private firms and comparing multiples for domestic and foreign firms to measure "private company discount").

69. *Id.* at 101.

70. *Id.*

71. *Id.* at 100.

72. Hertzel & Smith, *supra* note 37, at 480.

73. *Id.* (using data from 106 private placements between Jan. 1, 1980 and May 3, 1987).

two objectives. First, it updates the estimates obtained from using the restricted stock approach by including transactions from more recent time periods. Second, we examine how discounts based on this approach cross-sectionally vary based on various firm and issue characteristics.

A. Sample Selection and Data Sources

Our sample of private equity placements comes from several sources. We first identified all private placements completed between January 1, 1990 and December 31, 1995 that were reported in the electronic database of Securities Data Corporation (SDC), which provides information on all private placement transactions and related details. We also verified SDC's information by searching for original news announcements on these placements in the Dow Jones News Retrieval Service (DJNR), using various keywords linked to private placement of equity.[74]

Verification through the DJNR led us to discard the majority of announcements in the initial list generated by the SDC database. Announcements were discarded at this stage because they either represented multiple news articles describing the same transaction or did not pertain to a pure equity transaction. We also eliminated stories pertaining to foreign firms because accounting and price data are not easily available for such firms. In cases of multiple announcements, we chose the earliest announcement date as the event date.

The final sample consists of eighty-eight observations. To the best of the authors' knowledge, no published study has done a detailed analysis of post-1990 data on a sample as broad as this sample.[75] For the firms represented in this sample, we obtained data on stock prices and trading volume from the Center for Research in Security Prices (CRSP) database of NYSE, AMEX, and NASDAQ firms. To compute discounts, we followed Hertzel and Smith and used the stock price ten trading days after the announcement date so the price reflects any information contained in the announcement.[76]

Accounting data was obtained from Compustat. Compustat is a database that provides up to twenty years of detailed financial data on over 20,600 public companies, including 10,096 active companies and 10,524 that are not currently active. This information is obtained from documents filed with the SEC. Where possible, Compustat modifies company data to conform to a standardized format for the treatment of depreciation and other items.

B. Data Description

Table 4 presents summary information on the private placements included in our sample and on the firms making these placements.

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74. Specifically, we used synonyms for private transactions, such as private sales and offerings, stock purchase agreements, or purchase of a stake in the firm. We searched for these phrases in conjunction with synonyms for equity, such as common and treasury stock or shares.

75. The studies conducted by the SEC and by Hertzel and Smith contain a larger number of observations, but they analyze periods prior to 1990. *See supra* note 73 and accompanying text.

76. Hertzel & Smith, *supra* note 37, at 484.

Table 4: Sample Description for Private Equity Placements

Time profile, industry profile, and exchange listing profile for the sample of 88 private placements of equity announced between January 1, 1990 and December 31, 1995.

Panel A: Time Profile

Year	Issues	Cumulative
1990	4	4
1991	15	19
1992	22	41
1993	24	65
1994	11	76
1995	12	88

Panel B: Industry Profile

Industry	SIC Code[77]	Number of Issues
Chemicals and Allied Products	28	21
Computers and Commercial Machinery	35	6
Electrical Equipment	36	7
Instruments for Analysis and Control	38	7
Business Services	73	8
Other Industries	N/A	39

Panel C: Exchange Listing Profile

Listing Exchange	Number of Issues
Over the Counter	72
American Stock Exchange	9
New York Stock Exchange	7

Panel A of Table 4 provides a time profile of the sample private placements. As indicated in the table, forty-six of the eighty-eight placements in the sample (52%) take place in just two of the sample years, 1992 and 1993. The number of issues in each of these years is about twice the annual number in the years 1990, 1994 and 1995. This variation in frequency over time is consistent with previous findings which suggested that equity placements tend to take place at different rates in different years, depending on firm-specific and macro-economic factors.[78]

Panel B of Table 4 shows that 56% of the private placements in our sample period are made by firms in just five industry groups (as identified by 2-digit SIC codes).

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77. Standard Industrial Classification (SIC) code. This classification is generally based on the industry from which the company derives most of its revenues.

78. *See, e.g.*, H. Choe et al., *Common Stock Offerings Across The Business Cycle: Theory and Evidence*, 1 J. EMPIRICAL FIN. 1, 3-31 (1993).

Interestingly, these industries tend to be technology driven and hence are plausibly characterized by larger information asymmetries between insiders and potential outside investors.[79] This finding is consistent with the idea that part of the discount represents compensation to buyers for information costs and for providing monitoring services following the private placement.

Not surprisingly, Panel C of Table 4 shows that most of the firms making private placement announcements tend to be traded in the over-the-counter (OTC) market. OTC firms have a propensity to be smaller and are typically not followed by many financial analysts. This result lends further support to the claim that firms making private placement of equity tend to be characterized by greater information asymmetry among insiders and potential outside investors.

C. Discounts on Private Placements: An Initial Assessment

Table 5 presents summary information on certain salient characteristics of the private placements in our sample. On average, these placements raised $13.04 million (median = $7.84 million). The number of shares offered in the private placement was, on average, 15.87% of the total number of shares outstanding after the issue (median = 14.50%). Reflecting the fact that private equity issuers tend to be smaller firms, the issuing firms have an average market capitalization of equity equal to $117.74 million (median = $61.50 million).

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79. Because of the specialized knowledge associated with the operations and product development of high technology firms, it is reasonable that corporate insiders such as managers and officers are likely to possess greater information about the firm than outside investors. Further, the costs to monitor such firms will be higher than that for low technology firms because of the specialized knowledge that analysts will need to possess to assess the firm's prospects. *See* Stephen Ferris & Atulya Sarin, *Security Analysis and Corporate Diversification*, 5 ADVANCES IN FIN. ECON. 105, 105-38 (2000) (discussing corporate attributes and the level of analyst following).

Table 5: Private Placement Summary Statistics

Summary statistics for offer size and price discounts. The sample includes 88 private placements of equity announced between January 1, 1990 and December 31, 1995.

Panel A: Issue Characteristics

Characteristic	Mean	Median	Minimum	Maximum
Amount Offered ($ million)	13.04	7.84	0.82	126
Market Capitalization of Equity	117.74	61.50	1.78	1,190.20
Shares Offered/Total Shares After Issue (%)	15.87	14.50	0.92	44.52

Panel B: Discounts on Private Placements

	Number	Mean	Median	Minimum	Maximum
All Issues	88	22.21	20.67	-14.28	68.00
Registered Issues	37	14.04	9.85	-14.28	62.13
Unregistered Issues	51	28.13	26.47	-7.14	68.00

Panel B of Table 5 shows that, on average, privately placed shares were issued at a discount of 22.21% relative to the market price of the firm's publicly traded shares. However, a closer inspection of these placements reveals that the entire discount cannot be attributed to a lack of marketability. In thirty-eight cases out of eighty-eight, the private placement involved registered shares. Registered shares can be transacted freely, and the fact that the firm was publicly traded meant there was a ready market for these shares. Despite their marketability, registered shares were also placed at discounts averaging 14.04% (median = 9.85%). This result is corroborated by the analysis of Hertzel and Smith, who found discounts averaging 15.6% in a similar sample of registered private placements spanning the years 1980 to 1987.[80] Clearly, the discounts on private placements are being generated, at least in part, by factors that are distinct from the marketability of these issues.

If discounts on registered shares do not indicate lack of marketability, it is natural to look to placements of unregistered shares for an estimate of the marketability discount. Panel B of Table 5 shows that private placements of unregistered shares involve an average discount of 28.13%, which is about 14.09% higher than the average discount on registered placements. Since the most visible difference between the two types of issues is in their marketability, it is tempting to infer that at least this *differential discount* of 14.09% is a reflection of the marketability discount.

However, even this inference is premature. In drawing this conclusion, it is implicitly assumed that all the other economic factors that contribute to private placement discounts are identical across the two samples of issues, registered and unregistered. The next section presents evidence that this is not the case. The registered issues in our sample differ from unregistered issues not just in marketability but also in other factors that contribute to discounts. Therefore, in order to assess the discount that is necessitated solely by the lack of marketability, the influence of these additional factors will first have to be controlled for.

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80. Hertzel & Smith, *supra* note 37, at 479 tbl.VI.

A medical analogy may help to clarify this point. A casual comparison of smokers and non-smokers may suggest that, on average, smokers live five years less than non-smokers. However, it would be a mistake to infer that smoking alone reduces longevity by five years because it may turn out that smokers also face greater personal or professional stress than non-smokers, and that this additional stress alone would reduce their longevity. Therefore, in order to capture the incremental damage caused by smoking, it is necessary to first control for the influence of other medical factors that also reduce life span.

What factors, other than marketability, necessitate discounts on privately placed shares? As Hertzel and Smith point out, investors who purchase privately placed shares have to expend considerable resources in assessing the quality of the issuing firm.[81] Moreover, investors who purchase private placements may become significant blockholders in the company, with the ability and the incentive to monitor managers and discipline their behavior.[82] As rational economic agents, they will bear these costs only if they are adequately compensated for doing so. The discount offered to buyers is compensation for the cost of assessing the quality of the firm and for the anticipated costs of monitoring the future decisions of its managers. In general, the harder it is to assess the current quality of the firm, and the more important it is to actively monitor the actions of managers in the future, the greater the discount investors will have to be offered in order to accept a private placement. This perspective suggests that a number of observable characteristics of the firm and its private placement—distinct from considerations of marketability—should be related to the discount observed on the transaction.

1. The Fraction of Total Shares Offered in the Placement

The value of the firm is harder to assess if the firm relies heavily on future growth opportunities for its value rather than on existing assets. Growth-oriented firms have a greater need for external finance than mature firms, and are likely to issue a greater fraction of their existing equity in new private placements.[83] Therefore, the discount on a private placement should increase with the fraction of total shares being placed in the issue.

2. Business Risk

Firms whose business operations are inherently risky are also more difficult to assess than those in relatively 'safe' or predictable lines of business. Therefore, the greater the degree of business risk, the higher the discount demanded by investors to accept a private placement of equity. Greater business risk also increases the importance of monitoring managerial decisions in the future, adding further to the discount required.

3. Financial Distress

A similar argument suggests that firms facing financial distress require much more intensive scrutiny when assessing value. They also will benefit from greater levels of

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81. *Id.* at 460.
82. *Id.* at 467-68.
83. *Id.* at 484.

monitoring by their blockholders. Therefore, firms in financial distress will have to offer a greater discount to potential buyers of their private placements than would firms that are financially secure.

4. Total Proceeds from the Placement

Finally, the acquisition of information about firms is likely to be affected by economies of scale. Potential buyers of private placements have to incur certain lump-sum expenses in obtaining data and acquiring the internal resources necessary to analyze this data. The larger the scale of the new private placement, the easier it is to absorb these fixed costs. As a result, the larger the proceeds of the private placement, the lower the discount on the issue ought to be.

Table 6 provides initial evidence that these factors do, indeed, influence the level of discounts on private placements. To show how these factors vary with the level of discounts, the eighty-eight private placements in our sample were first separated into three categories, according to the level of discount observed. The private placements were then analyzed to determine how these factors differed across the three groups. The first group contains the twenty-nine placements with the lowest discounts, the second group the twenty-nine placements with intermediate levels of discounts and the third category the thirty placements with the highest levels of discounts. The Low Discount Group has an average discount of just 2.21%, the average discount of the Medium Discount Group is 20.36%, and the average discount of the High Discount Group is 43.33%.

Table 6: A Comparison of Firm Characteristics by Private Placement Discount
Mean (median) values for characteristics of firms in each of three groups. The groups are obtained by ranking the sample transactions in ascending order according to the percentage discount. The sample includes 88 private placements of equity between January 1, 1990 and December 31, 1995.[84]

Characteristics	Low Discount		Medium Discount		High Discount	
	Mean	Median	Mean	Median	Mean	Median
Discount	2.21%	3.49%	20.36%	20.59%	43.33%	40.32%
Proceeds ($ mil.)	12.26	10.00	17.06	9.85	9.90	4.96
Shares Issued	11.53%	10.05%	15.16%	14.59%	20.76%	17.49%
Volatility	3.74%	4.15%	4.47%	4.44%	5.90%	5.25%
Z-Score	21.93	3.56	10.14	5.42	3.24	1.85

Table 6 shows that most of the variables hypothesized as potential determinants of the discount do, indeed, vary systematically with the level of discounts. As predicted, the fraction of total shares issued is systematically higher in high-discount placements than in low-discount placements. This is consistent with the speculation that firms with ambitious growth plans require closer scrutiny by investors prior to the consummation of the placement and greater monitoring subsequent to it. Similarly, the degree of business risks confronting the firm, as measured by the volatility of the firm's returns, is also greater in high-discount placements than in low-discount placements.

Finally, companies making high-discount placements are financially weaker than those making low-discount placements, as indicated by their Z-Scores. The Z-Score, originally devised by Altman to predict corporate bankruptcy,[85] has subsequently come to be recognized as a convenient measure of a firm's financial performance.[86] The Z-Score is calculated as a linear function of financial ratios that indicate the company's solvency, liquidity, return on assets, debt-servicing capacity, etc. Altman's research indicates that a Z-Score below 2.6 suggests financial distress.[87] In general, the higher the Z-Score of a company, the stronger its financial position. As Table 6 shows, issues that involve low discounts are made by firms with systematically higher Z-Scores, indicating their superior financial strength**.**

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84. Notes for Table 6:
1. "Shares Issued" indicates the new shares issued in the private placement as a percentage of the total shares outstanding after the issue.
2. "Volatility" indicates the standard deviation of the firm's daily returns measured over the 250 trading days prior to the issue.
3. The "Z-Score," devised by Altman, is a widely used measure of the financial health of the company. The Z-Score is computed as a function of financial ratios that indicate the company's solvency, liquidity, return on assets, debt-servicing capacity, etc. The higher the Z-Score of a company, the stronger its financial position. *See* Edward I. Altman, *Financial Ratios, Discriminant Analysis and the Prediction of Corporate Bankruptcy*, 23 J. FIN. 589, 589-605 (1968).

85. *Id.*
86. ALAN C. SHAPIRO, MODERN CORPORATE FINANCE 756 (1990).
87. *Id.* at 757.

The only variable that does not appear to be systematically related to discounts is the size of the issue in dollar terms ("proceeds"). This suggests that economies of scale in processing information may not be sufficiently significant in determining the discounts demanded by investors for private placements.

If the factors discussed above were present in equal measure among both registered and unregistered placements, the additional discount observed in unregistered issues could have been attributed solely to their lack of marketability. However, Table 7 shows that this is not the case. Registered issues differ from unregistered issues not just in terms of marketability, but also in terms of factors which are unrelated to marketability but which nonetheless influence discounts.

Table 7: Comparison of Firm Characteristics Between Registered and Unregistered Issues

This table compares the median values, across registered and unregistered issues, of characteristics that influence the size of the private placement discount for reasons unrelated to marketability. The sample includes 88 private placements of equity between January 1, 1990 and December 31, 1995.

Characteristic	Unregistered Issues	Registered Issues
Proceeds of the Issue ($ mil.) [**]	5.00	11.00
Fraction of Shares Issued (%)	15.07	13.10
Returns Volatility	4.73	4.22
Z-Score [**]	2.19	5.34

The presence of two asterisks (**) with a variable in Table 7 indicates that differences in the median values of this variable between registered and unregistered issues is too great to have occurred merely by chance.[88] Table 7 reveals that firms making unregistered issues are significantly weaker financially than those making registered issues, as revealed by their lower Z-Scores. As shown in Table 8, this difference in financial strength alone would have justified a higher discount on unregistered issues, even if they suffered no lack of marketability. Unregistered issues also tend to be smaller in terms of issue proceeds than registered issues, a factor which may also contribute to higher discounts, although this affect appears to be weak, as shown in Table 7.

Therefore, if unregistered issues have an average additional discount of 14.09% relative to registered issues, part of the additional discount simply reflects the fact that unregistered issues tend to be made by financially weaker companies, and that they tend to involve smaller issue proceeds. Both of these factors lead to higher discounts. Only the remaining portion of the additional discount associated with unregistered issues can be attributed to their lack of marketability.

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88. More precisely, these asterisks indicate that the difference in medians is significant at a confidence level of at least 95%. This determination was made on the basis of two well-known statistical tests for differences in medians: the Wilcoxon Signed-Rank Test and the Kruskal-Wallis Test. JOSEPH G. VAN MATRE & GLENN H. GILBREATH, STATISTICS FOR BUSINESS AND ECONOMICS 623-30 (3d ed. 1987).

D. Determinants of Private Placement Discounts: A Regression Analysis

The central challenge in identifying the impact of non-marketability on private placement discounts is to control the simultaneous influence exerted on the discount by other extraneous factors. The standard procedure for analyzing the simultaneous influence of several factors on a variable of interest, the subject variable, is the technique of multivariate regression.[89] Multivariate regression provides estimates of how a given explanatory factor contributes to the subject variable, *given* the contributions of all other specified factors. The objective of multivariate regression is to find the specification that best describes the variations observed in the subject variable across the sample.[90]

In order to explain private placement discounts, we considered four factors within a multivariate regression framework. The choice of these variables was dictated both by theoretical considerations and an initial assessment of the data.

- The Percentage of Total Shares Issued: Theoretical arguments, articulated earlier in this paper, suggest that private placement discounts should be an *increasing* function of the fraction of total shares issued, all other factors held constant. This prediction is borne out by an initial assessment of the placements in our sample, as reported in Table 7.

- The Z-Score of the Issuing Company: As explained earlier, the Z-Score is a numerical indicator of the firm's financial health. The stronger the firm, the higher its Z-Score. Theory suggests that private placement discounts should be a *decreasing* function of the issuer's Z-Score, other factors held constant.

- The Standard Deviation of the Issuing Firm's Returns: This variable measures the volatility in the issuer's publicly traded shares, which indicates the degree of business risks to which the firm is exposed. Theory suggests that private placement discounts should be an *increasing* function of the firm's riskiness, other factors held constant.

- Whether or Not the Issue is Registered: Finally, we consider the impact of marketability. We specify an "indicator" variable that is assigned a value of 1 if the issue is registered and 0 otherwise. Since registered shares are more readily marketable, issues denoted as "1" (i.e. registered issues) should have lower discounts than issues denoted "0"(unregistered issues), holding other factors constant.

We decided to exclude the level of proceeds as an explanatory variable because our initial assessment of the data, summarized in Table 6, suggested that this factor did not exhibit a systematic relationship with private placement discounts.[91]

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89. J. JOHNSTON, ECONOMETRIC METHODS 12-16 (3d ed. 1984).

90. *Id.*

91. This assessment was subsequently verified by studying an alternative specification in which proceeds were included as an explanatory variable. The results revealed that proceeds did not have a statistically significant relationship with private placement discounts even in a multivariate regression. Since proceeds failed to add any insight into the behavior of discounts, this specification was not pursued further.

Regression analysis estimates a model of the following form:[92]

Discount = $a + b_1 \times$ Fraction of Shares Issued $+ b_2 \times$ Z-Score
$+ b_3 \times$ Standard Deviation of Returns $+ b_4 \times$ Registration Indicator.

The data provided by the sample is used to estimate the coefficients a, b_1, b_2, b_3 and b_4. The "intercept" term a identifies that component of the discount, if any, that is unrelated to any of the explanatory factors included in the model. The coefficients b_1 to b_4 reveals the percentage percent the private placement discount will change if the associated factor increases by one unit, all else remaining constant. Of particular interest to us is the coefficient b_4. For example, if the value of b_4 turns out to be -5.0, we can infer that a registered issue, which is assigned a Registration Indicator of 1, would have a discount that is 5% *lower* than an otherwise identical issue that is unregistered (Indicator = 0). Therefore, the value of the coefficient b_4 provides an estimate of the discount that is attributable *purely* to the lack of marketability, when all other factors are held constant.

We estimated the above regression model using the sample of 88 private placements occurring between January 1, 1990 and December 31, 1995. The results of this regression are reported in Table 8. This table reports the coefficients associated with each explanatory factor, as well as diagnostic measures that indicate the statistical significance of these factors in explaining discounts.

Table 8: Cross-Sectional Regression Results Explaining Price Discounts for the Sample of Private Placement Transactions

Estimates from ordinary least square regressions of price discounts on explanatory factors. The sample includes 88 private placement transactions spanning the period January 1, 1990 to December 31, 1995. 't-Statistics' are computed using White's adjustment for heteroskedasticity.[93]

Variable	Coefficient	Std. Error	t-Statistic	Prob.
Intercept	4.91	5.526399	0.889140	37.65%
Percentage of Shares Issued	0.40	0.204315	1.999084	4.89%
Z-Score	-0.08	0.032588	-2.519890	1.37%
Registration Indicator	-7.23	3.278803	-2.205249	3.02%
Standard Deviation of Returns	3.13	0.799971	3.924254	0.02%
R-squared	35.38%			
Adjusted R-squared	32.27%			

92. Although our linear model is consistent with the received literature (e.g. Hertzel & Smith, *supra note* 37 (1993)), several tests of model specification were conducted and the specification was found to be robust. As part of our sensitivity tests, we considered two alternative specifications for capturing financial health, involving indicator variables for financial distress: (1) A distress indicator that is coded 1 if the Z-Score is below 2.6 (the critical level identified by Altman, *supra note* 84, at 602), and 0 if the Z-Score is at or above 2.6; (2) A distress indicator that is coded 1 if the news story accompanying the private placement and publicly available company filings indicate financial distress, and 0 otherwise. The results were qualitatively very similar to those reported in this paper. In particular, the coefficient of the Registration Indicator is very close to that reported in Table 8 below. Therefore, the results for these alternative specifications are not reported here.

93. Halbert White, *A Heteroskedasticity-Consistent Covariance Matrix Estimator and a Direct Test for Heteroskedasticity*, 48 ECONOMETRICA 817, 817-38 (1980).

The critical test of a factor's explanatory power is that its coefficient is different from zero. The column labeled "Prob." helps identify the degree of certitude with which we can assert this to be true from the sample used for the study. Heuristically, a value of 5% in this column signifies that there is only a 5% probability that the estimated coefficient is different from zero purely by chance. In other words, at a 95% confidence level the coefficient is, indeed, different from zero. Any value of approximately 5% or below is taken to indicate strong statistical significance.

Table 8 reveals that every factor we considered in explaining private placement discount has significant explanatory power. For every percentage point by which the firm increases the amount of shares it issues (relative to the total shares after the issue), the discount rises by 0.40% when other factors remain constant. Similarly, for every unit increase in the Z-Score (indicating increased financial health), the discount decreases by 0.08% when other factors are held constant. For every percent by which the standard deviation of monthly returns increases, indicating greater business risk in the issuer, the discount increases by 3.13%.

Most importantly, Table 8 provides a direct estimate of the discount required for lack of marketability. This estimate comes from the coefficient associated with the Registration Indicator. Table 8 reveals that over the sample period, an issue that was registered (*i.e.* coded as "1") would have required a discount that was 7.23% less than an otherwise identical issue which was unregistered (and hence coded "0"). Therefore, controlling for all other factors influencing private placement discounts, an issuer would have to concede an additional discount of 7.23% simply to compensate the buyer for lack of marketability.

V. SUMMARY AND CONCLUSIONS

The marketability of an asset refers to the degree to which an asset can be converted to cash quickly, without incurring large transaction costs or price concessions. All else equal, the more marketable an asset, the higher the price an investor will be willing to pay for the asset. The lack of marketability of an asset is costly to investors because it potentially causes the investor to miss opportunities to allocate capital to alternative uses, such as liquidity or portfolio rebalancing.

In this Article, we review and critique the various methods that appear in the literature and are used in practice to estimate valuation discounts when an asset lacks marketability. We also present findings from our own original empirical research which offer further insights into the estimation of such discounts.

Current practice recognizes three different methods to estimate valuation discounts due to a lack of marketability. The IPO approach attempts to infer an estimate by comparing post-IPO share prices with transaction prices in the same shares prior to the IPO. The IPO approach is flawed because it samples only successful firms that ultimately go public. Firms that are less profitable and decide not to go public are excluded from consideration. Hence, this approach produces estimates of the marketability discount that are likely to overstate the true discount attributable to a lack of marketability.

The acquisition company approach can provide upper bound estimates for the valuation discounts for private companies (private company discount), in part due to lack

of ready marketability. The restricted stock approach can be used to estimate what part of the private placement discount can be attributed to lack of marketability per se.

Thus, the problem that attorneys and courts face in using the discounts from such studies is that the observed differences in valuation between assets might be due to factors other than differences in marketability. In our opinion, when valuing an operating company that is privately held (or its securities), the appropriate benchmark for discounts is provided by the total private placement discount or the discount observed in the acquisition approach. This is because, whether it is marketability restriction per se or other factors, the relevant analysis aims to determine the total valuation discount. However, when it is appropriate to only consider the effect of marketability restrictions, as is the case in valuation of non-controlling interests in a non-operating partnership which holds assets of known value (*e.g.*, a family limited partnership), the distinction between the total valuation discount and liquidity discount is key. In such cases, the applicable discount is only for the lack of liquidity.

To enhance the usefulness of these methods, we report the findings from our own empirical analyses of the marketability discount. We first update the existing restricted stock studies to include data from the 1990s. Secondly, and perhaps more importantly, we control for a number of other possible factors that might influence the magnitude of the valuation discount. This will allow the courts or attorneys to place greater confidence in our estimates since such discounts should be determined after considering fact circumstances pertinent to the subject valuation.