

The Journal of Entrepreneurial Finance

Volume 17 Issue 1 *Spring 2015*

Article 3

3-2015

Determining Lack of Marketability Discounts: Employing an Equity Collar

Lester Barenbaum LaSalle University

Walter Schubert LaSalle University

Kyle Garcia Financial Research Associates

Follow this and additional works at: https://digitalcommons.pepperdine.edu/jef

Recommended Citation

Barenbaum, Lester; Schubert, Walter; and Garcia, Kyle (2015) "Determining Lack of Marketability Discounts: Employing an Equity Collar," *The Journal of Entrepreneurial Finance*: Vol. 17: Iss. 1, pp. 65-81.

DOI: https://doi.org/10.57229/2373-1761.1254

Available at: https://digitalcommons.pepperdine.edu/jef/vol17/iss1/3

This Article is brought to you for free and open access by the Graziadio School of Business and Management at Pepperdine Digital Commons. It has been accepted for inclusion in The Journal of Entrepreneurial Finance by an authorized editor of Pepperdine Digital Commons. For more information, please contact bailey.berry@pepperdine.edu.

Determining Lack of Marketability Discounts: Employing an Equity Collar **Cover Page Footnote** The authors would like to thank the anonymous reviewers for suggestions that improved both the clarity and focus of our paper.

Determining Lack of Marketability Discounts: Employing an Equity Collar

Lester Barenbaum LaSalle University

Walter Schubert LaSalle University

Kyle Garcia Financial Research Associates

ABSTRACT

A discount for the lack of marketability is the implicit cost of quickly monetizing a non-marketable asset at its current value. These discounts are used in many venues to determine the fair market value of a nonmarketable asset such as a privately-held business. There has been much written on the quantification of the discount for the lack of the marketability which is briefly summarized in this article. Marketability refers to monetizing the non-marketable asset at its cash equivalent current value. Current practice often uses the cost of a put option as a proxy for the discount. A put option insures that the investor will receive no less than the current value of the underlying asset. However, the use of a put also allows the investor to maintain the asset's upside potential. Therefore, the cost of a put overstates the discount for the lack of marketability. We show that the cost of monetizing a non-marketable asset at its current value through a loan, secured by an at-the-money equity collar, more effectively captures the true cost of marketability. When puts and calls cannot be employed to secure the current value on the underlying asset, a portfolio consisting of the non-marketable asset and a stock index, where puts and calls can be written on the index can be constructed. The effectiveness of the portfolio in creating a risk free outcome depends upon the correlation and volatility of the stock index and the non-marketable asset. We demonstrate that, relative to current practice, the use of an equity collar with a loan greatly reduces the implied discount for the lack of marketability.

Keywords: Marketability Discounts, Valuation, Gift & Estate Tax

JEL Codes: G32, H24, K34

Copyright © 2015 Pepperdine Digital Commons and the Academy of Entrepreneurial Finance. All rights reserved. ISSN: 2373-1761.

I. Introduction

Common stock that is restricted from being traded on a public exchange tends to trade at a discount to publicly traded shares of the same corporation. The discount is often referred to as a discount for lack of marketability ("DLOM"). Likewise, the common stock of privately-held corporations, which by definition are non-marketable, are often reduced to their fair market value using a DLOM. This reduction in value can have a significant impact on gift and estate taxes paid. The U.S. Tax Court has accepted discounts for lack of marketability performed for estate and gift tax purposes in the range of 10 to 50 percent¹. Discounts of similar magnitude are also often seen in other venues such as divorce proceedings in the context of equitable distribution of a private corporation. Experts in a judicial setting must be able to defend both the foundation and conclusion of their discount for lack of marketability in order to survive a Daubert (1993) challenge. This paper extends the discussion of option theory used by valuation professionals, and provides the theoretical and empirical basis for determining both the magnitude of, and foundation for, discounts for lack of marketability based upon achieving a cash equivalent value for a non-marketable asset.

We suggest that the cost of a hedging strategy, employing an at the money equity collar along with the interest cost of borrowing funds to monetize the underlying asset, better captures the full cost of providing liquidity to a non-traded asset. The at-the-money equity collar eliminates the downside risk for the investor through a pricing floor set at the current value of the underlying non-marketable asset. This is equivalent to the individual buying an at-the-money put option. At the same time, the upside gain for the individual is eliminated through a pricing cap set at the asset's current value. This is equivalent to the individual writing an at the money call option. The result is that the collar guarantees the current value of the underlying security will be received when the contract ends. A securitized loan it then used to monetize the asset. The interest cost of the loan represents the cost of monetizing the non-marketable asset. Given that both the upside and downside risks of the underlying asset returns have been eliminated through

¹ See Laro and Pratt (2011).

the use of the collar, the cost of the loan correctly captures the discount for the lack of marketability and provides a cash equivalent value.

II. Background and Related Literature

Valuation practitioners have continued to debate the theoretical framework that best supports a pragmatic approach to estimating a discount for lack of marketability (see Reilly and Rotkowski 2007). Studies on observed discounts were first published in 1971 with the SEC Institutional Investor Study. Empirical studies have focused on a lack of marketability discounts based upon surveys of the issuance of restricted stock and IPOs. Restricted stock studies have analyzed the price differential between private placements of restricted stock relative to their freely traded counterpart. Advancements using regression analysis began with Silber (1991) where he found discounts for the lack of marketability had a median value of 33.75 percent. Hertzel and Smith (1993) analyzed 106 private placements during the period from 1980 to 1987. Their regression analysis concluded that an appropriate discount for lack of marketability was 13.5 percent. Bajaj, Denis, Ferris and Sarin (2001) examined 88 private placements during the period 1990 through 1995. Registered shares were issued at a median discount of 9.85 percent while placements of unregistered shares had a median discount of 26.47 percent. A regression analysis indicated that the lack of marketability accounted for 7.23 percent of the overall discount. More recently, Comment (2012) concluded that discounts for lack of marketability should be approximately of 5.5 percent based upon his analysis of over 1,100 private placements that took place between 2004 and 2010. To some extent Comment's results may be due to the shortening of the time period under Rule 144 where trading restrictions were changed from two years to six months. IPO studies have expressed the discount for the lack of marketability as the percentage change between the price at which a private company goes public and the private market price of that firm prior to the IPO. Emory (2002) indicated that the discount for the lack of marketability is between 40 percent and 45 percent based upon transactions up to five months prior to the IPO. IPO studies have not been widely accepted since it is difficult to eliminate other factors that may account for the observed price differential. Pre-IPO transactions generally involve company insiders who may be acquiring stock at discounted prices for a variety of reasons of which a discount for a lack of marketability is only one. In

addition, a company's IPO price may reflect improved expectations of performance relative to when a pre-IPO transaction occurred.

In recent years, option theory has been utilized as both the theoretical and empirical basis for determining discounts for a lack of marketability. Some of the more prominent strategies have used the cost of a put option as a proxy for the discount. For example, Chaffee (1993) proposed using the cost of a put option on the underlying restricted asset as a proxy for the discount for the lack of marketability. According to Chaffee, the owner of a non-marketable asset is exposed to a potential loss due to the asset's lack of marketability, and the put option captures the cost of that potential loss. Based upon the volatility of small publically traded companies, Chaffee concluded that the discount for the lack of marketability for private equity interests with a two year holding period were 28 percent to 41 percent. Longstaff (1995) used option pricing theory to set the upper bound on the value of marketability by using a lookback put. The Longstaff model assumes that an investor has perfect foresight over a security's restriction period, and could sell a security at its maximum price. Longstaff's discount for lack of marketability represents the cost to an investor of giving up the opportunity to sell at that maximum price. Dyl and Jiang (2008) used the lookback put, in conjunction with other methods, to establish the discount for the lack of marketability for publicly traded stock subject to trading volume restrictions. Brooks (2014) provides a general option framework capturing asset maturity, volatility, hedging availability and investor skill. The underlying theory for using the cost of a put option to determine a lack of marketability discount is that the at-the-money put would secure the common shareholder's current price by providing protection from downside risk. If the stock price was lower than the indicated value at the liquidity date, the investor would be able to sell the shares at the put strike price, and be protected from downside risk.

III. Why a Put Option Overstates the Lack of Marketability Discount

The error in estimating the discount using a put relates to the definition of marketability. Marketability refers to immediately receiving exactly the current value of the non-marketable asset, while a put assumes the investor would receive no less than its current value at the end of the restriction period. The essence of a DLOM is quantifying the detriment to value between a non-marketable asset and its freely traded counterpart. A put not only protects the holder from downside risk, it also offers the opportunity for upside potential, a benefit whose cost is included in the put premium. We demonstrate

that the cost of a put option significantly overstates the discount for lack of marketability. In addition, a put option does not capture a critical component of marketability, the cost of monetizing the current value of the underlying asset.

Assume an individual has a share of stock that is restricted from sale for one year. It has a freely traded current value of \$30. The security does not pay dividends, has an expected standard deviation of returns equal to 30 percent, and the risk free rate is 1 percent. Following the Cox, Ross and Rubenstein binomial tree model, the freely traded share price might increase to \$50.44, or fall to \$17.84, as shown on Figure 1.

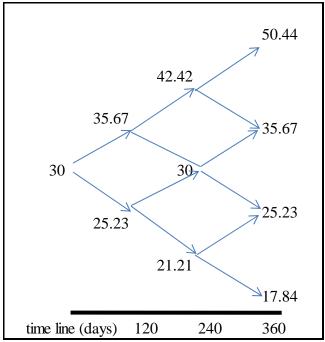


Figure 1. Put Option Example

The individual can eliminate the risk of a loss by purchasing a put option. The put affords the right, but not the obligation, to sell the shares for \$30 any time during the year. The model yields a put option value of \$3.74. Many valuation professionals use the cost of the put to determine the lack of marketability discount. With a cost of \$3.74, the put implies a lack of marketability discount of 12.47 percent (\$3.74 / \$30). However, the cost of the put option is not an appropriate measure of a discount for the lack of marketability for two reasons. First, a put option not only preserves value, but also allows for potential capital appreciation. Second, it does not provide for the current

monetization of the asset's value. The cost of achieving marketability should not include the potential for capital appreciation and it should include the cost of monetization.

A. Estimating the Marketability Discount Using an Equity Collar

We suggest that the cost of a hedging strategy, employing an at the money equity collar along with the interest cost of borrowing funds to monetize the underlying asset, better captures the cost of providing liquidity to a non-traded asset. The result is that the collar guarantees the current value of the underlying security will be received when the contract ends.

Figure 1A shows the results of employing the put option versus using an at the money collar. The put option allows for upside gain while the collar preserves current value.

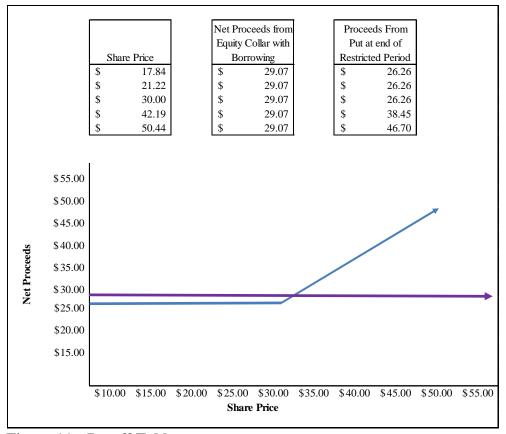


Figure 1A. Payoff Table

The collar hedges both the downside risk of receiving less than \$30, as well as the potential upside to receive more than \$30. The equity collar ensures that the investor will receive the current fair market value of the underlying asset at the end of the collar's term. However, to correctly capture the discount for the lack of marketability one must also determine the cost to monetize the underlying asset at its current value. Given that the holder of the restricted asset is guaranteed to receive \$30 one year from now, plus the earnings of the collar transaction, monetizing the position at a low rate of interest is feasible. Thus, the correct discount for the lack of marketability is the financing cost to monetize the asset less the net proceeds of the equity collar. This is significantly lower than the 12.47 percent implied by the put option. As shown on Figure 1A, the outcome of such a strategy is that the holder of the restricted asset is guaranteed to receive \$29.07². This consists of the \$30 being borrowed at an assumed rate of 4 percent, plus the \$.27 earned on the creation of the collar³ which represents a discount for lack of marketability of 3.1 percent (\$.93 / \$30.00).

In summary, when the underlying asset can be hedged using an at-the-money collar, the DLOM is best measured by the net financing cost to monetize the current value of the underlying asset. In contrast, the cost of a put option represents insurance rather than marketability. Thus the cost of a put significantly overstates the discount and does not account for monetizing the asset. Monetizing an equity collar results in a materially lower discount for the lack of marketability.

B. When the Underlying Asset Cannot be Directly Hedged: The Put Option

In many situations it is unlikely that puts and calls can be employed on a non-marketable asset. Examples include determining a DLOM for an interest in a closely-held firm, and when hedging public stock holdings is not permitted. In these situations, a portfolio can be constructed by combining the non-marketable asset with a traded index or portfolio that allows for put and call contracts. When the non-

² Typically the transaction cost of purchasing a put is not included in the estimated marketability discount.

 $^{^3}$ Under the assumptions employed in Figure 1, the value of the call option is \$4.01. The proceeds of the collar are (\$4.01-\$3.74) * 1.01= \$.27. The cost of borrowing is 4 percent * \$30.00 = \$1.20. Therefore, the net proceeds are: \$30.00 + \$.27 - \$1.20 = \$29.07.

marketable asset and the stock index are not perfectly correlated, the purchase of puts on the stock index will not eliminate all of the downside risk.

Table 1 demonstrates the five basic outcomes that result when an index, which is less than perfectly correlated with the underlying security, is employed to proxy a put option on a non-traded security. For this illustration, we have ignored the cost of the put.

Assume a non-traded security is judged to have a value of \$100. An index in which put options can be traded is employed as the hedge vehicle. A put option with an exercise value of 100 is purchased. Rows A and B represent scenarios in which the non-traded security rises in value to \$110. In case A the Index rises as well to \$120. Since the put option pays off when the index is less than its exercise price, the put is discarded and the client receives \$110. Row B shows the case where the index has fallen to \$80. In this scenario the put is exercised leading the client to end up with a value of \$130 representing the increase in value to \$110 of the underlying asset and the \$20 payoff from the put option. Rows C through E represent scenarios under which the value of the underlying asset falls to \$90. Row C shows the case in which the index rises so that the put option is discarded. The client receives the \$90 for the underlying asset value. In D, the index falls but not as much as did the underlying security. As a result the loss is less than if the put option had not been purchased, but it does not lead to a value equal to the original \$100. Finally line E shows the outcome is which the index falls more than does the underlying asset value. In this scenario, the client ends up with a value greater than the original underlying value since the profit from the put is greater than is the loss attributed to the underlying asset.

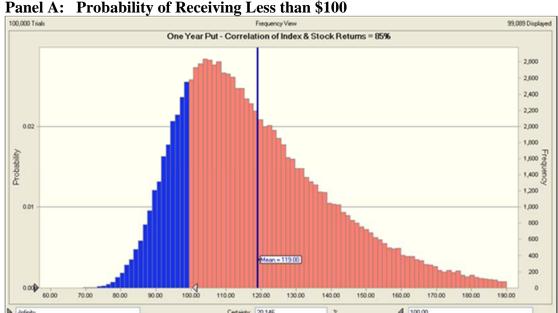
Table 1. Put Portfolio Outcome: Underlying Stock And Stock Index Not Perfectly Correlated

	Endin	g Price:	E	nding		
	Underlying		Price: Stock		Portfolio	
	Stock		Index		Outcome	
A	\$	110	\$	120	\$	110
В	\$	110	\$	80	\$	130
C	\$	90	\$	110	\$	90
D	\$	90	\$	95	\$	95
E	\$	90	\$	85	\$	105

To show this more fully, a Monte-Carlo simulation with 100,000 trials was performed assuming that both an underlying non-marketable asset and a synthetic

stock index have an expected return of 18 percent, a standard deviation of 30 percent, and a risk free rate of 1 percent. The Pearson correlation coefficient between the stock index and the non-marketable asset is assumed to be 85 percent. The cost of the one year put option is \$11.37. Figure 2 Panel A demonstrates that the put option on the index is not capable of providing full downside protection.

Figure 2. Outcome from Purchasing a One Year Put on Stock Index and Liquidating the Underlying Asset at Year End

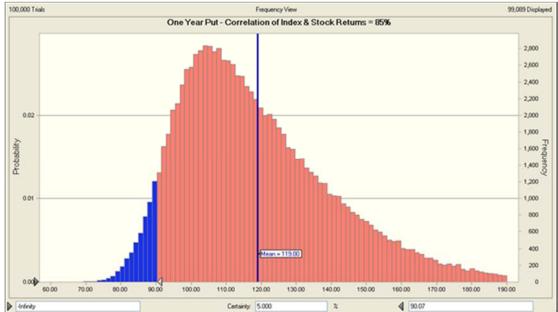


As shown, when the hedge is terminated, there is a 20.15 percent likelihood that the owner of the non-marketable asset will receive less than \$100 at the end of

Figure 2 Panel B shows the process for monetizing the current value of the non-marketable asset. There is a 95 percent level of confidence that \$90.07 or greater will be realized at the termination of the hedge. The owner of the asset can borrow \$90.07, employing the asset as collateral, at a 4 percent interest rate, and the

one year. The mean outcome is \$119 with a range of outcomes from \$70 to \$256.

remaining \$9.93 at a rate of 12 percent.⁴ The weighted average financing cost would be 4.8 percent to monetize the asset. Adding this to the 11.37 percent cost of the put results in a 16.17 percent discount for the lack of marketability.



Panel B: Proceeds at a 95% Confidence Level

C. When the Underlying Asset Cannot be Directly Hedged: The Equity Collar

When options cannot be written on the underlying asset a collar consisting of the underlying non-marketable asset and a publicly traded index can be easily implemented. The use of an index has significant implications for the discounts for lack of marketability used in estate and gift tax valuations as well as other venues. When comparable public companies are used to value the equity of a private

⁴ We have assumed an unsecured loan would have an interest rate of three times the rate of the loan for the secured underlying asset.

company, the group of comparables can provide a basis for creating a hedge instrument.

An appropriate strategy would consist of creating an equity collar by purchasing at the money puts and writing at the money calls on the index. The combination of purchasing puts and selling calls generates a positive cash flow of about 1 percent of the transaction value. Using the same inputs as in the previous example, Figure 3 presents the outcomes for an at the money equity collar. An outcome of at least \$81.97 occurs with a 95% confidence level. The mean outcome is \$102.90. To monetize this position an individual would use the underlying asset as security and borrow \$81.97 at 4 percent and the \$18.03 balance at 12 percent. This results in an overall borrowing rate of 5.4 percent. Combining the cash flow generated by the collar results in a net discount for the lack of marketability of 4.4 percent.

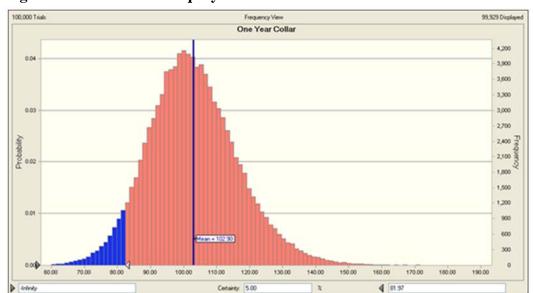


Figure 3. One-Year Equity Collar Proceeds at a 95% Confidence Level

Given that the discount for a lack of marketability is the cost of monetizing the current value of the non-marketable asset, the equity collar with borrowing is the more appropriate instrument. This reflects that the collar eliminates both downside risk and upside potential. The cost of the collar with borrowing is significantly less than the cost of a put option with monetization.

D. Sensitivity Analysis

Table 2 provides insight into the effectiveness of an equity collar with borrowing compared to a put option with borrowing when the underlying asset is not perfectly correlated to the stock index. Restriction periods of six months and eighteen months are compared. The rate of return on the non-marketable asset and the index is assumed to be 18 percent. The standard deviation of returns range from 20 percent to 50 percent for both the underlying asset and the index. The correlation coefficient between the underlying asset and the stock index ranges between 50 percent and 90 percent.

As shown on Table 2, if an asset with a six-month restriction period has a standard deviation of 35 percent, the discount for the lack of marketability, based on the cost of a normal put, would be 9.7 percent. When the correlation between the non-marketable security and a traded index is 70 percent, 82 percent of the downside risk is protected with 95 percent confidence. Therefore, the cost of borrowing would be 2.6 percent (4%*82% + 12%*18%) for six months⁶. The discount for the lack of marketability using the cost of the put (9.7%) plus the cost of borrowing (2.6%) is 12.3 percent. Estimating the discount for the lack of marketability utilizing the put is in error. The put option only removes downside risk and does not provide immediate funds. Further, unless the put can be sold on the underlying asset, even the downside risk may not be fully protected. When the correlation coefficient between the underlying asset and the stock index is reduced to 50 percent the discount for the lack of marketability, using the put plus borrowing, increases to 12.5 percent. In this situation, 77 percent of the downside risk is protected with 95 percent confidence.

The discount for the lack of marketability is reduced significantly when employing an at the money equity collar with borrowing. The reduction in the discount for the lack of marketability properly reflects that quantifying the discount should not include the opportunity to earn a gain on the underlying asset during the restriction period. The cash flow from selling a call and removing this upside

⁵ We have assumed that the expected return and standard deviation of the illiquid asset and the stock index are the same. Simulations indicated that there was very little difference in outcomes with varying expected rates of return and all else held constant.

⁶ The financing cost to monetize the underlying asset was based upon an interest rate of 4 percent for the hedge portfolio outcome at a 95 percent level of confidence based upon running 100,000 simulations. A 12 percent interest rate for the unsecured proportion of asset was employed.

Table 2. Marketability Discounts Based Upon Cost of Hedge and Interest Cost to Monetize

		and in	CI CSt CO	st to Mon	CHZC		
Panel A: Si	ix Month Durat	ion					
				Marketabili	ty Discount		
standard deviation		20%		35%		50%	
	cost of a put		5.5%		9.7%		13.9%
	_	Equity Collar		Equity Collar		Equity Collar	
		with	Put with	with	Put with	with	Put with
correlation	rate of return	Borrowing	Borrowing	Borrowing	Borrowing	Borrowing	Borrowing
0.90		1.9%	7.7%	2.3%	12.0%	2.6%	16.4%
0.70	18%	2.2%	7.8%	2.5%	12.3%	3.0%	16.7%
0.50		2.4%	7.8%	2.9%	12.5%	3.3%	17.0%
PROPORT	ION OF ASSET	HEDGED W	ITH A 95%	CONFIDEN	CE LEVEL		
	Equity Collar hec	lges both upside	and downsic	de price risk			
	Put only hedges	downside price	risk				
S	tandard deviation	20%		35%		50%	
		Equity Collar		Equity Collar		Equity Collar	
		with	Put with	with	Put with	with	Put with
correlation	rate of return	Borrowing	Borrowing	Borrowing	Borrowing	Borrowing	Borrowing
0.90		95%	94%	83%	89%	76%	85%
0.70	18%	86%	91%	78%	82%	64%	74%
0.70		84%	89%	70%	77%	58%	67%
0.50		0.170	02,10				
	ighteen Month		53,70				
	ighteen Month		3,7		ty Discount		
Panel B: E	ighteen Month			Marketabili	•		6
Panel B: E	tandard deviation	Duration		Marketabili	•		6 22.7%
Panel B: E		Duration 20%	ó	Marketabili	ó	50%	
Panel B: E	tandard deviation	Duration	ó	Marketabili	ó		
Panel B: E	tandard deviation	Duration 20% Equity Collar	6 8.57%	Marketabilii 35%	15.7%	50% Equity Collar	22.7% Put with
Panel B: E	tandard deviation cost of a put	Duration 20% Equity Collar with	8.57% Put with	Marketabilii 35% Equity Collar with	15.7% Put with	509 Equity Collar with	22.7% Put with Borrowing
Panel B: E s	tandard deviation cost of a put	Duration 20% Equity Collar with Borrowing	8.57% Put with Borrowing	Marketabilit 35% Equity Collar with Borrowing	15.7% Put with Borrowing	509 Equity Collar with Borrowing	22.7% Put with Borrowing 30.3%
Panel B: E s correlation 0.90	tandard deviation cost of a put rate of return	Duration 20% Equity Collar with Borrowing 5.1%	8.57% Put with Borrowing 14.6%	Marketabilit 35% Equity Collar with Borrowing 6.9%	Put with Borrowing 22.6%	509 Equity Collar with Borrowing 8.2%	22.7% Put with
Panel B: E s correlation 0.90 0.70 0.50	tandard deviation cost of a put rate of return	Duration 20% Equity Collar with Borrowing 5.1% 6.0% 6.5%	8.57% Put with Borrowing 14.6% 14.8% 15.0%	Marketabilit 35% Equity Collar with Borrowing 6.9% 8.3% 8.9%	Put with Borrowing 22.6% 23.6% 24.1%	50% Equity Collar with Borrowing 8.2% 10.0%	Put with Borrowing 30.3% 31.7%
Panel B: E s correlation 0.90 0.70 0.50	tandard deviation cost of a put rate of return 18%	Duration 20% Equity Collar with Borrowing 5.1% 6.0% 6.5%	8.57% Put with Borrowing 14.6% 14.8% 15.0%	Marketabilit 35% Equity Collar with Borrowing 6.9% 8.3% 8.9%	Put with Borrowing 22.6% 23.6% 24.1%	50% Equity Collar with Borrowing 8.2% 10.0%	22.7% Put with Borrowing 30.3% 31.7%
Panel B: E s correlation 0.90 0.70 0.50	tandard deviation cost of a put rate of return 18%	Equity Collar with Borrowing 5.1% 6.0% 6.5%	8.57% Put with Borrowing 14.6% 14.8% 15.0%	Marketabilit 35% Equity Collar with Borrowing 6.9% 8.3% 8.9%	Put with Borrowing 22.6% 23.6% 24.1%	50% Equity Collar with Borrowing 8.2% 10.0%	Put with Borrowing 30.3% 31.7%
correlation 0.90 0.70 0.50	rate of return 18% ION OF ASSET Equity Collar hec Put only hedges	Equity Collar with Borrowing 5.1% 6.0% 6.5%	8.57% Put with Borrowing 14.6% 14.8% 15.0% ITH A 95% and downsie	Marketabili 35% Equity Collar with Borrowing 6.9% 8.3% 8.9% CONFIDENCE de price risk	15.7% Put with Borrowing 22.6% 23.6% 24.1% CE LEVEL	50% Equity Collar with Borrowing 8.2% 10.0% 10.8%	22.7% Put with Borrowing 30.3% 31.7% 32.5%
Panel B: E s correlation 0.90 0.70 0.50 PROPORT	rate of return 18% ION OF ASSET Equity Collar hece	Equity Collar with Borrowing 5.1% 6.0% 6.5%	8.57% Put with Borrowing 14.6% 14.8% 15.0% ITH A 95% and downsie	Marketabili 35% Equity Collar with Borrowing 6.9% 8.3% 8.9% CONFIDENCE de price risk	15.7% Put with Borrowing 22.6% 23.6% 24.1% CE LEVEL	50% Equity Collar with Borrowing 8.2% 10.0% 10.8%	22.7% Put with Borrowing 30.3% 31.7% 32.5%
Panel B: E s correlation 0.90 0.70 0.50 PROPORT	rate of return 18% ION OF ASSET Equity Collar hec Put only hedges	Equity Collar with Borrowing 5.1% 6.0% 6.5% THEDGED W. dges both upside downside price 20% Equity Collar	8.57% Put with Borrowing 14.6% 15.0% ITH A 95% e and downsic	Marketabilio 35% Equity Collar with Borrowing 6.9% 8.3% 8.9% CONFIDENCE de price risk 35% Equity Collar	15.7% Put with Borrowing 22.6% 23.6% 24.1% CE LEVEL	50% Equity Collar with Borrowing 8.2% 10.0% 10.8%	22.7% Put with Borrowing 30.3% 31.7% 32.5%
correlation 0.90 0.70 0.50 PROPORT	rate of return 18% ION OF ASSET Equity Collar hece Put only hedges of tandard deviation	Equity Collar with Borrowing 5.1% 6.0% 6.5% THEDGED Water Both upside downside price 20% Equity Collar with	9.57% Put with Borrowing 14.6% 15.0% THA 95% and downsic risk Put with	Marketabilion 35% Equity Collar with Borrowing 6.9% 8.3% 8.9% CONFIDENCE Price risk 35% Equity Collar with	15.7% Put with Borrowing 22.6% 23.6% 24.1% CE LEVEL	50% Equity Collar with Borrowing 8.2% 10.0% 10.8% 50% Equity Collar with	22.7% Put with Borrowing
Panel B: E s correlation 0.90 0.70 0.50 PROPORT	rate of return 18% ION OF ASSET Equity Collar hec Put only hedges	Equity Collar with Borrowing 5.1% 6.0% 6.5% THEDGED Wilges both upside downside price 20% Equity Collar with Borrowing	Put with Borrowing 14.6% 15.0% ITH A 95% and downsiansk Put with Borrowing	Marketabilit 35% Equity Collar with Borrowing 6.9% 8.3% 8.9% CONFIDENCE risk 35% Equity Collar with Borrowing	15.7% Put with Borrowing 22.6% 23.6% 24.1% CE LEVEL	50% Equity Collar with Borrowing 8.2% 10.0% 10.8% 50% Equity Collar with Borrowing	Put with Borrowing 30.3% 31.7% 32.5%
correlation 0.90 0.70 0.50 PROPORT	rate of return 18% ION OF ASSET Equity Collar hece Put only hedges of tandard deviation	Equity Collar with Borrowing 5.1% 6.0% 6.5% THEDGED Water Both upside downside price 20% Equity Collar with	9.57% Put with Borrowing 14.6% 15.0% THA 95% and downsic risk Put with	Marketabilion 35% Equity Collar with Borrowing 6.9% 8.3% 8.9% CONFIDENCE Price risk 35% Equity Collar with	15.7% Put with Borrowing 22.6% 23.6% 24.1% CE LEVEL	50% Equity Collar with Borrowing 8.2% 10.0% 10.8% 50% Equity Collar with	22.7% Put with Borrowing 30.3% 31.7% 32.5%

potential reduces the cost of marketability. The cost of the equity collar is also reduced as the call component of the collar yields slightly more cash flow than the cost of the put component. Increases in volatility cause moderate increases in the discount for the lack of marketability because a smaller proportion of the underlying asset value can be financed at a low rate.

Unlike the put option, the equity collar is only moderately impacted when the duration of the option contracts are extended from six to eighteen months, as the put option and call option cash flows offset each other. On Table 2, a put option with financing, with a 90 percent correlation between the underlying non-marketable asset and the index, and a 35 percent standard deviation, has a discount for lack of marketability of 12 percent. Increasing the restriction period from 6 to 18 months, increases the discount for the lack of marketability by 10.6 percentage points to 22.6 percent. The increase in the collar plus financing with the same characteristics leads to an increase of 4.6 percentage points.

IV. Conclusion

A marketability discount represents the cost of eliminating all price risk and monetizing the current value of a non-marketable asset. The cost of a put contract to proxy the lack of marketability discount is inappropriate because, while it reduces downside risk, it continues to allow for upside potential. In addition, the put does not capture the cost of monetizing the non-marketable asset. An equity collar, coupled with a loan to monetize the asset, properly values the discount for lack of marketability, and results in a lower discount than a put option. In addition, our research implies that the discount for lack of marketability employing the collar is similar to the average discounts found in more recent studies of restricted stock discounts (Bajaj, Denis, Ferris and Sarin (2001), Comment (2012)).

In several estate and gift tax cases, Tax Court has criticized the use of hedging instruments to determine an appropriate DLOM based upon the feasibility of employing the strategy.⁷ However, it is well established that customized collar contracts are created by commercial and investment banks (see Bajaj, Denis, Ferris

⁷ Litman v. United States, 78 Fed, Cl. 90, (2007) and Murphy v. United States, 94923, 2009-2

and Sarin 2001).⁸ In addition, Tax Court in other venues has recently found that Variable Prepaid Forward Contracts ("VFPC") represent a current sale of the underlying asset which makes the cost of a VFPC a mechanism to estimate a discount for the lack of marketability. (See Rubinger and LePree 2011). The equity collar in conjunction with borrowing is similar to the current practice of constructing VFPCs for individuals with concentrated equity positions.⁹ VPFCs are currently used by individuals to monetize security positions for a variety of reasons (Welch 2000). As pointed out by Jagolinzer, Matsunaga and Yeung (2007) in their analysis of 201 VPFCs, the payment may be the result of the individual meeting an immediate liquidity need. VPFC transactions combine an equity collar with borrowing to provide the holder of an non-marketable asset an immediate cash payment in exchange for a pledge to deliver the current value of the non-marketable asset at a future date as pointed out by Bettis, Bizjak and Kalpathy (2013) in their analysis of 983 VPFC and other similar derivative instruments used by corporate insiders.

In some circumstances puts and calls cannot be used on the underlying asset. In those cases a portfolio consisting of the non-marketable asset and a stock index allows for the creation of an effective hedge. As shown on Table 2 the effectiveness of the equity collar depends upon the correlation between the non-marketable asset and the index used to create the hedge. A potential weakness of both the put and the equity collar methods is that as the correlation falls, less of the position can be effectively hedged and a lender may become less willing to monetize the asset. While it is not possible to define a specific cut off between the correlation of the underlying asset and an available index that would lead a lender to refuse financing, it is clear that higher correlation leads to both lower discounts for marketability and an increased likelihood that a loan can be obtained. However, in cases where no loan is possible, we suggest restricted stock studies be given more weight in determining an appropriate DLOM.

⁸ In a recent engagement, an investment bank was willing to collar an illiquid asset comprised of public and private securities that was restricted from sale for an eighteen month period. The bank was willing to lend the holder of the asset 91% of its current value.

⁹ Unlike the at-the-money collar, VPFCs are typically not issued at-the-money to avoid being characterized as a constructive sale by the IRS.

REFERENCES

- Bajaj, M., D.J. Denis, S.P. Ferris, and A. Sarin. 2001. "Firm Value and Marketability Discounts", *Journal of Corporation Law*, vol. 27, no. 1 (Fall):89-115.
- Bettis, C., Bizjak, J., and S. Kalpathy. 2013. "Why Do Insiders Hedge Their Ownership? An Empirical Examination", *SSRN:1364810*, June 2013.
- Brooks, Robert. 2014. "A General Option Valuation Approach to Discount for Lack of Marketability", Working Paper, University of Alabama.
- Chafee, David. 1993. "Option Pricing as a Proxy for Discount for Lack of Marketability for Private Company Valuations", *Business Valuation Review*, December: 182:186.
- Comment, R. 2012. "Revisiting the Illiquidity Discount for Private Companies: A New (and Skeptical) Restricted Stock Study", *Journal of Applied Corporate Finance*, vol. 24 no. 1, (Winter): 80-92.
- Cox, John C., Stephen A. Ross, and Mark Rubinstein. 1979. "Option Pricing: A Simplified Approach", *Journal of Financial Economics*, 7 no. 3 September: 229-264.
- Daubert v. Merrell Dow Pharmaceuticals, Inc. 1993, 509 U.S. 579
- Dyl, E.A., and G. Jiang. 2008. "Valuing Illiquid Common Stock", *Financial Analysts Journal*, vol. 64, no. 34. (Jul/Aug 2008): 44-47.
- Emory, John D. 2002. "Discounts for Lack of Marketability, Emory Pre-IPO Studies 1980-2000 as Adjusted October 10, 2002", *Business Valuation Review*, Vol. 21 No. 4. (December 2002).
- Engineering/Valuation Program DLOM Team. 2009. Discount for Lack of Marketability: Job Aid for IRS Professionals, *Internal Revenue Service*.
- Hertzel, M., and R.L. Smith. 1993. "Market Discounts and Shareholder Gains from Placing Equity Privately." *Journal of Finance*, vol.48, no.2 (June): 459-485.
- Jagolinzer, A., S. Matsunaga, and E Yeung. 2007. An Analysis of Insiders' Use of Prepaid Variable Forward Transactions, *Journal of Accounting Research*, vol. 45 no. 5, Dec 2007: 1055-1079.
- Laro, D., and S.P. Pratt. 2011. *Business Valuation and Taxes: Procedure, Law and Perspective*", John Wiley & Sons, Inc. 2nd edition.
- Litman v. United States, 78 Fed, Cl. 90, (2007).

- Longstaff, F.A. 1995. "How Much Can Marketability Affect Security Values? "Journal of Finance, vol. 50, no. 5 (December): 1767-1774.
- Murphy v. United States, 94923, 2009-2.
- Reilly, R., and A. Rotkowski. 2007. "The Discount for Lack of Marketability: Update on Current Studies and Analysis of Current Controversies", *The Tax Lawyer*, vol. 61 no. 1 (Fall): 241-286.
- Rubinger, J., L. LePree and A. Summer 2011. "Tax Court Finds Variable Prepaid Forward Contract Gives Rise to Current Sale", *Journal of Taxation*, vol. 114 no. 1, (Jan.): 36-46.
- Silber, W.L. 1991. "Discounts on Restricted Stock: The Impact of Illiquidity on Stock Prices. "Financial Analysts Journal, vol. 47, no. 4 (July/August): 60-64.
- Welch, Scott. 2000. "Managing Single-Stock Risk: Unlocking Liquidity with High Net Worth Clients", *Journal of Financial Service Professionals*, vol. 54, no. 5 (Sep 2000): 78-85.