

# Calculating DLOM Using the VFC Longstaff Methodology

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Perhaps the biggest conundrum of the business valuation profession has been the estimation of discounts for lack of marketability ("DLOM"). Everyone agrees that such discounts are appropriate for valuing minority interests in privately held businesses, but there has been considerable disagreement regarding how such discounts should be calculated and whether they are appropriate for controlling interests. The rationale for applying DLOM to controlling interests is addressed in my paper entitled, "Restructuring Levels of Value" and is not addressed further here. This paper will focus on the deficiencies of commonly used means of estimating DLOM, and will discuss the theory and application of the VFC Longstaff Method as the preferred alternative.

## How the empirical studies of discounts and liquidity relate to each other

Conventional business valuation has used the well-publicized results of restricted stock studies, pre-IPO studies, and registered versus unregistered stock studies to effectively guess at appropriate DLOM percentages to use in their valuation reports. Understandably, such unspecific means of applying the traditional approaches have been broadly unsatisfactory to the valuation community and the courts.

Figure 1

		IMMEDIATE	INCREASING MARKETING TIME =====>>>>					EXTENDED	
LOW	INCREASING PRICE VOLATILITY =====>>>>	Publicly Traded Stocks	Private Sales of Registered Unrestricted Stocks	Private Sales of Registered Restricted Stocks	Private Sales of Unregistered Stocks	Pre-IPO Control Value	Private Company Control Value	Pre-IPO Minority Value	Private Company Minority Value
HIGH									

Figure 1 presents a relational stratification of the types of empirical studies that researchers have performed to explore the cost of illiquidity. The presentation is instructive in enhancing understanding of what the various studies are measuring, how they relate to publicly traded values, and the extent to which they meet the needs of business valuation. I have attempted to present the studies in relative position based on marketing time and volatility assuming all other aspects of investment were equal:<sup>1</sup>

- Publicly traded companies are the standard against which all of the studies measure results and from which rates of return are calculated. Interests in publicly traded companies are worth more than interests in identical privately held companies because they can be sold immediately to realize gains and to avoid losses. Interests in privately held companies cannot.
- Private sales of publicly registered unrestricted stocks typically involve large blocks of stock that could be sold into the public marketplace but which would materially adversely affect stock prices if the entire block were to be dumped into the market at once. Avoiding that effect results in an extended period of time to liquidate the investment position in the public market during which time the investor is subject to market risk. Negotiating a private sale of the block can accelerate liquidating the position, however the need to find a buyer with the wherewithal to purchase the block restricts the number of potential buyers and represents a diminution of demand for the stock. But though private sales of large blocks of registered unrestricted stocks can somewhat mitigate the market risk, the risk does not go away. The buyer of the block assumes the risks, in turn, of having to sell into a limited pool of buyers or slowly feeding the block into the public market. These risks require compensation by means of a discount (i.e. DLOM).
- Large blocks of publicly registered restricted stocks have the same price risks and sales circumstances as sales of large blocks of registered unrestricted stocks, but have the additional risk of being locked out of the public market for specific periods of time and/or being subject to restrictive “dribble out” rules. Accordingly, large blocks of registered restricted stocks often can only be sold quickly in private sale transactions that logically should take longer to sell than an otherwise equivalent unrestricted block.<sup>2</sup> The result is that a restricted registered stock is worth less (requires more DLOM) than an otherwise equivalent unrestricted registered stock in the same company because of the greater market risk associated with the extended marketing period.
- Unregistered stocks in public companies can only be sold in private transactions, typically involve large blocks of stock, and may have specific sale restrictions. They are worth less (require more DLOM) than equivalently-sized blocks of registered stock (whether unrestricted or restricted) in the same company because of the time and money costs of registration. This cost further restricts the potential number of buyers and extends the marketing period necessary to sell the investment, resulting in relatively greater uncertainty, a relatively longer time to market the interest, and a relatively greater exposure to the price risks of the marketplace than for registered stocks.

Unregistered shares in public companies are somewhat akin to pre-IPO shares because they cannot be traded publicly but could be with registration. But because the company itself is already reporting publicly, its unregistered shares are worth more than a pre-IPO controlling interest in an otherwise equivalent private company.

- Although there are many studies of pre-IPO sales of minority interests, I am not aware of any studies that address discounts observed in sales of controlling interests in pre-IPO companies. Pre-IPO private sales of controlling interests in private companies should have relatively longer marketing periods than private sales of unregistered stocks in public companies, because the pre-IPO company is not yet a public company and the fact and timing of the IPO event can be uncertain. Furthermore, low pre-IPO stock sales prices may reflect compensation for services rendered. These factors cause the pre-IPO controlling interests to be worth less (require more DLOM) than unregistered interests in an otherwise equivalent public company.
- Private sales of controlling interests in a company that has no expectation of going public should be worth less (require more DLOM) than an otherwise identical company with an anticipated IPO event. Despite uncertainty, an anticipated IPO event should result in a shorter marketing period than not anticipating such an event.
- Pre-IPO sales of non-controlling interests in a company planning an IPO event should be worth less (require more DLOM) than the controlling interest in the same company even without the planned IPO. The inability to control whether the planned IPO goes forward should result in greater uncertainty and a longer marketing period to liquidate the investment than would be experienced by the controlling investor. Also, low pre-IPO share prices may reflect compensation for services rendered.
- Non-controlling interests in private companies require greater discounts than all of the preceding circumstances because the relative risks of lacking control cause the period of time to liquidate the position to be potentially much longer than for the controlling interest in the same company or for otherwise comparable minority positions in firms with a planned IPO event.

### **Why the empirical studies are inadequate for estimating DLOM**

Empirical studies of discounts for lack of marketability have typically involved sales of restricted stocks, sales of pre-IPO stocks, or sales of registered and unregistered stocks. Unfortunately, the studies of marketability have limited utility to the appraiser opining on the fair market value of a business interest.

Several authors have noted that most publicly traded firms do not issue restricted stock. This dearth necessitates studies of limited size, in limited industries, with data spread over long periods of time. The result has been substantial standard errors in estimates and difficulty in establishing appropriateness to the valuation subject. The restricted stock analyses are also seldom contemporaneous to the valuation date and thus are not market sensitive. The one-size-fits-all approach of the restricted stock studies usually results in material mismatches of industries, circumstances, and market

conditions. Furthermore, the restricted stock studies measure the difference in value between a publicly traded stock with and without a time restriction on sale. Left unanswered is whether there is a difference between the restricted stock value of a publicly traded company and the value of an interest in that company if the company were not publicly traded at all.

The pre-IPO studies reflect the same essential deficiencies of the restricted stock studies, resulting in substantial standard errors in estimates and mismatches of time, industry, and circumstances. In addition, the pre-IPO studies are distorted by the fact that the studies necessarily are limited to successful IPOs as there are no post-IPO stock prices for failed IPOs. The discounts observed in the pre-IPO studies may also reflect uncertainty about whether the IPO event will actually occur, when the event will occur, at what price the event will occur, and compensation for services rendered. Additionally, most successful IPOs occur when the market is “good.” Given that fact, it is extremely difficult to justify applying a pre-IPO discount in times when the market for public offerings has dried up. Applying “bull” market pre-IPO discounts in “bear” market economies likely materially understates private company values. Logically, the pre-IPO and post-IPO price spreads should narrow in bear markets.

It should also be noted that all of the companies in the restricted stock and pre-IPO studies are, in fact, publicly traded. But few of the privately held companies that are the subject of business valuations have a foreseeable expectation of ever going public. Accordingly, the circumstances of the valuation subjects are highly distinguishable from those of the publicly traded companies that are the subjects of the studies. For all of these reasons the restricted stock and pre-IPO studies are of dubious value for determining the DLOM of privately held companies.

The price of unregistered versus registered stocks was considered by Mukesh Bajaj, David J. Dennis, Stephen P. Ferris and Atulya Sarin in their paper “Firm Value and Marketability Discounts.” They intended to study the value of liquidity by comparing the stock sales of 88 companies that sold both registered and unregistered stock in private offerings. However, the result of the study is not a measure of liquidity. It is instead a measure of the value of registration.<sup>3</sup> And, importantly, the study is not a measure of DLOM as it does not address the discount applicable to the additional time it takes to sell controlling or minority interests in private companies.

### **Problems with some existing analytical methods to measure DLOM**

Many people have offered solutions to the DLOM conundrum. A widely discussed methodology is the “Quantifiable Marketability Discount Model” (“QMDM”) developed by Z. Christopher Mercer. QMDM is a deduction against the marketable value of a business interest that is equal to the present value of the expected future growth of the subject company discounted at the cost of capital estimated by the appraiser. In my opinion, QMDM is seriously defective as an economic concept. First, the theoretical foundation of the basic QMDM model (i.e. that the discounted value of a firm’s future growth represents DLOM) is unexplained. Mercer never provides a theoretical framework for QMDM, relying instead on the unsupported proposition that it simulates the thinking of hypothetical investors.<sup>4</sup> A proper theoretical foundation for QMDM would establish that the method represents a proxy of the opportunity cost of illiquidity.

Second, the results of QMDM depend heavily on what Mercer describes as the “required holding period,”<sup>5</sup> which he equates with the investor’s “probable holding period.”<sup>6</sup> But the “probable” investment holding period is not the proper period over which to measure DLOM. Liquidity represents the ability to sell an investment quickly when the investor decides to sell while illiquidity represents the inability to do so. Therefore, the proper period over which to measure DLOM is the one that commences with the decision to sell the investment and ends with its sale. Even if the QMDM approach of discounting to present value the expected growth in value of an investment made logical sense as a measure of DLOM, it does not address the risks of illiquidity during the marketing period.

Third, calculating DLOM over the probable holding period of the investment is inconsistent with the generally accepted definition of fair market value because it necessarily rests on the expected actions of a particular investor. The fact that the investor may anticipate holding the investment for many years is not relevant. A proper analysis involves an initial hypothetical buyer who must anticipate immediately becoming a seller as of the valuation date. Thus, the appropriate analysis of FMV is always one of “How long after the valuation date will it take to sell this investment?” That time period and the economic risks associated with it represent DLOM.

It has been suggested that the Black-Sholes Option Pricing Model (“BSOPM”) represents a solution to the DLOM conundrum. It does not. BSOPM is not equivalent to DLOM on a theoretical basis. BSOPM is designed to measure European put and call options. European put options represent the right, but not the obligation, to sell stock for a specified price at a specified point in time. European call options represent the right, but not the obligation, to buy stock for a specified price at a specified point in time. DLOM is not the equivalent of either. Instead, DLOM represents the risk of being unable to sell at any price for a specified period of time.

“At the money” put options have also been suggested as a means of estimating DLOM. Such options represent the right, but not the obligation, to sell stock at the current price at a specified future point in time. “At the money” put options do not measure the risk of illiquidity, because the investor is not denied the opportunity to sell for a price that is higher than the put price.

### **The VFC Longstaff Method**

The critical value difference between publicly traded and privately held companies is that publicly traded investments offer liquidity. All other components of business value are shared: earnings and cash flow, growth, industry risk, size risk, and market risk. However, it is not the value of liquidity per se that DLOM seeks to capture. Instead, it is the **risk** associated with **illiquidity**. But first, what is liquidity? It is the ability to sell quickly when the investor decides to sell. Liquidity allows investors to sell investments quickly to realize gains or to avoid losses. DLOM, being the result of **illiquidity**, represents the economic risk associated with failing to realize gains or failing to avoid losses on an investment during the period the investor is trying to sell it.

Logically, DLOM can be reduced to price risk faced by an investor during a particular marketing period. In the market for publicly traded stocks, risk reflects the volatility of stock prices. Conversely, investments with no price volatility or that are immediately marketable have no DLOM. Investments with no price volatility can be arbitrated to

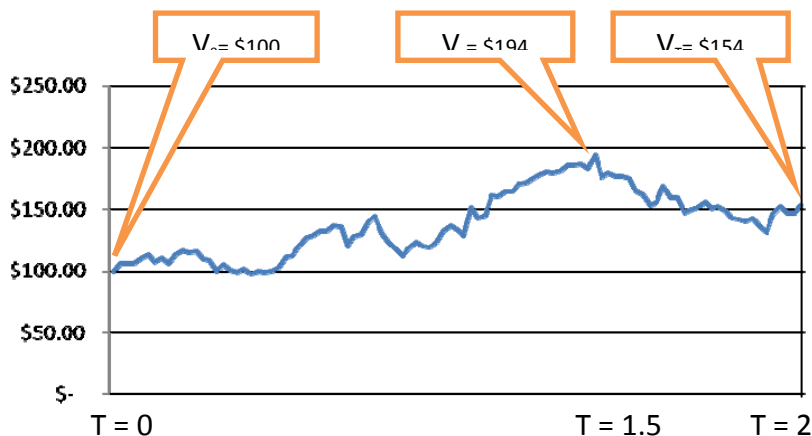
negate the period of restricted marketing, while volatile investments that are immediately marketable can be sold at the current price to avoid future volatility.

In 1995, UCLA professor Francis A. Longstaff published an article in The Journal of Finance<sup>7</sup> that presented a simple analytical upper bound on the value of marketability using “lookback” option pricing theory. Longstaff’s analysis demonstrated that discounts for lack of marketability can be large even when the illiquidity period is very short. Importantly, the results of Longstaff’s formula provide insight into the relationship of DLOM and the length of time of a marketability restriction. Longstaff described the “intuition” behind the results of his formula as follows –

[Consider] a hypothetical investor with perfect market timing ability who is restricted from selling a security for T periods. If the marketability restriction were to be relaxed, the investor could then sell when the price of the security reached its maximum. Thus, if the marketability restriction were relaxed, the incremental cash flow to the investor would essentially be the same as if he swapped the time-T value of the security for the maximum price attained by the security. The present value of this lookback or liquidity swap represents the value of marketability for this hypothetical investor, and provides an upper bound for any actual investor with imperfect market timing ability.

Some have questioned using Longstaff’s formula to calculate DLOM because his description of an investor with perfect market timing does not exist in the real world. However, this criticism is of no legitimate concern because the formula allows the flexibility of using average volatility instead of peak volatility. Using average volatility results in an investor with average market timing instead of perfect market timing.

Figure 2



For this simple path:

- With restriction, present value of T = 2 at T = 0 is  $154 \cdot \exp(-2 \cdot 0.05) = \$139$
- Without restriction, could have  $194 \cdot \exp(-1.5 \cdot 0.05) = \$180$  present value
- Cost of restriction is the difference in present values =  $\$180 - \$139 = \$41$

Figure 2 is a graphic presentation of Longstaff’s description using peak volatility: An investor receives a share of stock worth \$100 at time zero, which he cannot sell for T = 2 years when the stock is worth \$154 (present value at T = 0 discounted at a risk free rate of 5% = \$139). If at its peak value the stock were worth \$194 (present value at T = 0 discounted at a risk free rate of 5% = \$180), then the present value cost of the restriction

to the investor at  $T = 0$  would be \$41, or 41% of his \$100 investment.<sup>8</sup> The mathematical formula of this scenario is:

$$Discount = \left(2 + \frac{\sigma^2 T}{2}\right) N\left(\frac{\sqrt{\sigma^2 T}}{2}\right) + \sqrt{\frac{\sigma^2 T}{2\pi}} \exp\left(-\frac{\sigma^2 T}{8}\right) - 1$$

where

$\sigma$  = volatility

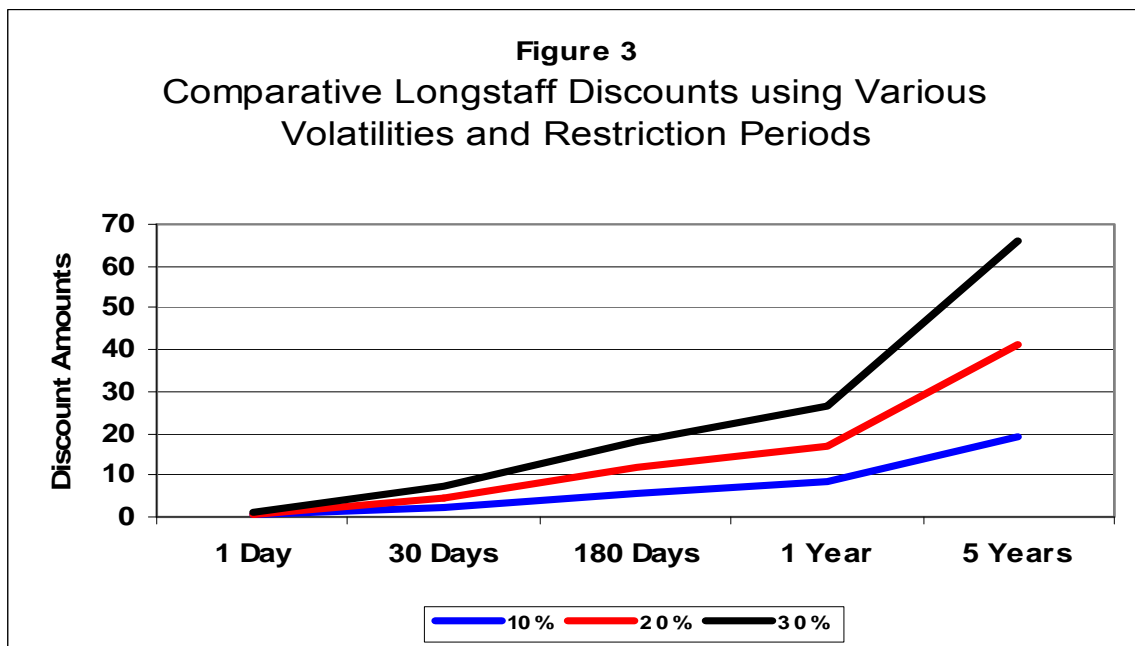
$T$  = Lockout period

$N(\cdot)$  = standard normal cumulative distribution function

The following table presents the amount of discount from 100 (i.e., not the percentage DLOM) resulting from applying the formula to various combinations of volatility and time of restrictions on marketability.

	Volatility		
	10%	20%	30%
	Discount from 100		
1 Day	0.4	0.8	1.3
30 Days	2.3	4.7	7.1
180 Days	5.8	11.8	18.1
1 Year	8.2	17.0	26.3
5 Years	19.1	41.0	65.8

Figure 3 presents the results graphically:



Dr. Longstaff's formula requires a modification to arrive at DLOM, however. The modification is the equivalent of dividing the calculated discount by 1 plus the discount to arrive at a DLOM percentage that can be applied to the marketable value of the subject company. The modified formula is:

Section A   Section B   Section C   Section D

$$\frac{\left(2 + \frac{\sigma^2 T}{2}\right) N\left(\frac{\sqrt{\sigma^2 T}}{2}\right) + \sqrt{\frac{\sigma^2 T}{2\pi}} \exp\left(-\frac{\sigma^2 T}{8}\right) - 1}{\left(2 + \frac{\sigma^2 T}{2}\right) N\left(\frac{\sqrt{\sigma^2 T}}{2}\right) + \sqrt{\frac{\sigma^2 T}{2\pi}} \exp\left(-\frac{\sigma^2 T}{8}\right)} = DLOM \%$$

Or simply: DISCOUNT / (1 + DISCOUNT) = DLOM%

### How to calculate the Longstaff discount and DLOM

Assume the following:

Volatility = 10%

T = lockout period in years = .5

NORMSDIST = normal distribution function in Excel

SQRT = square root function in Excel

EXP = exponential<sup>9</sup> function in Excel

Calculating the sections of the Longstaff Formula:

Section A =

$2 + (\text{Volatility squared} \times T) / 2 =$

$2 + (.1 \times .1 \times .5) / 2 =$

$2 + .005 / 2 =$

2.0025

Section B =

$\text{NORMSDIST} ((\text{SQRT} (\text{Volatility Squared} \times T) / 2)) =$

$\text{NORMSDIST} ((\text{SQRT} (.1 \times .1 \times .5) / 2)) =$

$\text{NORMSDIST} ((\text{SQRT} (.005) / 2)) =$

$\text{NORMSDIST} (.0707107 / 2) =$

$\text{NORMSDIST} .03535535 =$

.5141018

Section C =

$\text{SQRT} ((\text{Volatility Squared} \times T) / (2 \times \pi)) =$



$$\begin{aligned} & \text{SQRT } ((.1 \times .1 \times .5) / (2 \times 3.141592653...)) = \\ & \text{SQRT } (.005 / 6.283185306...) = \\ & \text{SQRT } (.000795775) = \\ & .0282095 \end{aligned}$$

$$\begin{aligned} \text{Section D} = & \\ & \text{EXP } (- ((\text{Volatility Squared} \times T) / 8)) = \\ & \text{EXP } (- ((.1 \times .1 \times .5) / 8)) = \\ & \text{EXP } (- (.005 / 8)) = \\ & \text{EXP } (- (.000625)) = \\ & .9993752 \end{aligned}$$

$$\begin{aligned} \text{Discount calculated} = & \\ & \text{Section A} \times \text{Section B} + \text{Section C} \times \text{Section D} - 1 = \\ & 2.0025 \times .5141018 + .0282095 \times .9993752 - 1 = \\ & 1.0294889 + .0281919 - 1 = \\ & 1.05768 - 1 = .05768 = \text{DISCOUNT} \end{aligned}$$

The following Excel formula calculates the Longstaff discount, and can be entered in the spreadsheet as shown:

$$=(2+A1^2*B1/2)*\text{NORMSDIST}(\text{SQRT}(A1^2*B1)/2)+\text{SQRT}((A1^2*B1)/(2*PI()))*\text{EXP}(-A1^2*B1/8)-1$$

Where: A1 = Annualized volatility  
B1 = Lockout period in years

The final step is calculating the DLOM percentage, which is as follows:

$$\begin{aligned} & \text{DISCOUNT} / (1 + \text{DISCOUNT}) = \\ & .05768 / (1 + .05768) = \\ & .05768 / 1.05768 = .05454 = \% \text{ DLOM} \end{aligned}$$

### **Determining Longstaff formula variables**

At VFC we have been using an application of the Longstaff formula in our business valuations since 2002 (the “VFC Longstaff Methodology”). The VFC Longstaff Methodology calculates a proxy for the subject company’s stock price volatility using appropriately selected guideline companies.<sup>10</sup> These are the same companies that VFC uses to apply the publicly traded guideline valuation method in its valuations.<sup>11</sup> Subject to consideration of observable trends, we calculate the annualized average stock price volatility for each of the guideline companies for an historic period of time equal to the period of time we have concluded it will take to market the interest being valued.<sup>12</sup> We then generally average the calculated volatilities using a simple average.<sup>13 14</sup> As mentioned previously, the use of annualized volatility eliminates the “perfect knowledge” and “upper bound” objections to the Longstaff formula by yielding a DLOM reflective of average knowledge and average volatility. The calculation of annualized average stock price volatility can be done with the following formula:

$$\text{SQRT } (250) \times \text{STDE } (\sigma T \text{ 1-n})$$

Where:  $\text{SQRT}(250)$  = Square root of 250 trading days (i.e. the annualized time period), and  $\text{STDE}(\sigma_T \sqrt{1-n})$  = Standard deviation of the daily stock price volatility of a guideline company for some number of days (day 1 through day “n”).

When applying this formula the issues become the number of days of stock prices to use to estimate volatility and the estimated period of time it will take to sell the investment. The answer to both questions rests with the professional judgment of the appraiser. The sensitivity of DLDM to changes in these parameters is easily tested.

Although some may want to always use the same period of time to estimate volatility (e.g., beta is frequently calculated using 60 months of data) and there is a certain appeal to using a time period equal to the anticipated marketing period for the investment, it is my opinion that appraisers should judgmentally conclude (based on reasoned analysis of information known as of the valuation date) whether there are trends in the volatility of the guideline company's stock prices. Such conclusions might result in the assumption that historic volatility over some period of time will continue for the duration of the marketing period or that some deviation from the past would be appropriate. In either instance, the analytical period for determining future volatility may differ from the expected marketing period of the investment, and may differ from valuation to valuation.

Estimating the time period needed to market the subject interest likewise involves professional judgment. At VFC, we like to supplement our judgment with feedback from business brokers. We have provided redacted summary information of interests we have valued to business brokers to obtain their thoughts on the length of time it would take to sell the subject interest in the subject company. The marketing period is seldom less than a few months, and can be much longer as the following events occur:

- Drafting selling documents
- Developing a marketing strategy
- Implementing the marketing strategy
- Screening buyers
- Conducting site visits
- Assisting buyers in their analysis of the company and the interest being sold
- Drafting letters of intent
- Negotiating with the serious buyers
- Assisting buyers with due diligence
- Drafting the contract of sale
- Participating in arranging financing
- Actually closing the deal

Final estimation of the appropriate marketing period of the subject interest is based on professional judgment. At VFC, we consider the effects of characteristics of the subject company on the time it will take to market the subject interest. Such characteristics include:

- Size
- Financial strength
- Quality and transparency of financial reporting
- The industry

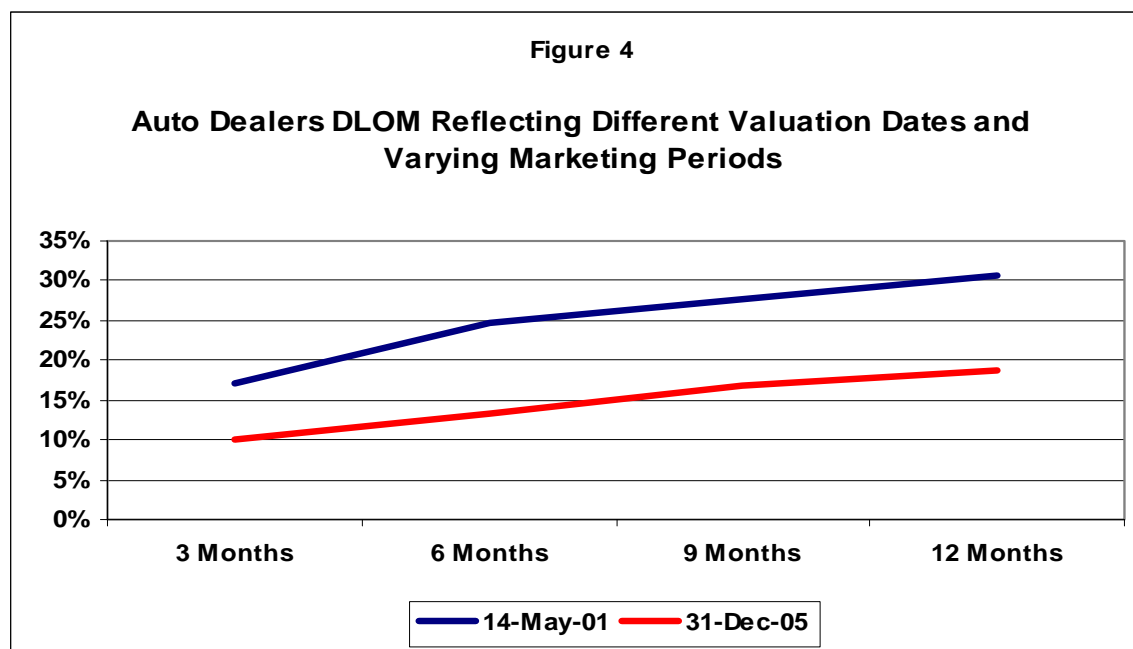
- Competition
- Market fragmentation
- Growth expectations
- How the company operates
- Risk perceptions
- Quality of the product or service
- Depth and experience
- Control or minority interest
- The availability of credit
- Whatever else applies

At VFC, we have applied the VFC Longstaff Methodology to many industries. The following table shows the results from applying it to a hypothetical privately held automobile retailer using the average annualized stock price volatility of five publicly traded automobile retailers<sup>15</sup> as guidelines at two different dates.

**COMPARISON OF AUTO INDUSTRY DLOM USING THE  
VFC LONGSTAFF METHODOLOGY**

<u>Marketing Time</u>	<u>14-May-01</u>	<u>31-Dec-05</u>
3 Months	17.1%	9.9%
6 Months	24.6%	13.2%
9 Months	27.6%	16.8%
12 Months	30.6%	18.6%

Figure 4 graphically presents the results of the table above. From 2001 to 2005 the automobile dealership industry became less volatile, resulting in reduced DLOMs.



**Applying the VFC Longstaff Methodology to sales of large blocks of publicly traded stocks**

The VFC Longstaff Methodology provides an effective means of estimating the discount that should be allowed when valuing large blocks of publicly traded stocks. First, estimate the number of shares of stock expected to be sold each day. For example, this may be quantities equal to the SEC Rule 144 “dribble out” rules. Second, use the VFC Longstaff Methodology to compute a separate discount percentage for each day’s sales. The discount for the first day’s sales may be essentially zero, but the discount for the last day’s sales may be very large depending on the estimated volatility of the stock and the period of time it will take to liquidate the position. Third, multiply each day’s anticipated sales proceeds times one minus the respective day’s DLOM percentage. The result is a declining value curve over time as the successively more extended sales lose value as the risks of price volatility increase over time.

## Conclusion

The VFC Longstaff Methodology offers a scientific means of reducing the speculation of the appropriate discount for lack of marketability. The methodology is solidly grounded in financial risk theory. The methodology results in a unique, supportable estimate of DLOM for each valuation subject as of the applicable valuation date. The results can be tested and replicated.

1. The columns of Figure 1 are intended to indicate that the volatility of a particular level of interest may range from low to high depending on the subject company. The “step” aspect of Figure 1 is based on the assumption for each step that all other things are equal.
2. Some restricted stocks cannot be sold at all for contractually determined periods of time. Such investments have even greater economic risks than those merely subject to the “dribble out” rules.
3. See Section IV.C of “Firm Value and Marketability Discounts.” It is not appropriate to increase the calculated DLOM or otherwise reduce the estimate of FMV for lack of registration. Lack of registration is a factor that is subsumed in the time it takes to market an interest in a private company. Likewise, brokerage and transactions costs should not be deducted from fair market value appraisals. The result of such deductions would be values that no longer represent the price at which the investments change hands between buyers and sellers – a requirement of fair market value.
4. Quantifying Marketability Discounts, Revised Reprint, 2001, 1997, at page 214.
5. Id. at page 218
6. Id. at page 244
7. The Journal of Finance, Volume I, No. 5, December 1995
8. In this instance, the DLOM would be  $41 / (100+41) = 29.1\%$ .
9. The exponential function, also known as Euler’s number, is a mathematical constant that is the base of the natural logarithm. It is approximately equal to 2.718281828.

10. The use of guideline companies to estimate the subject company's stock price volatility is consistent with the requirements of SFAS 123(R) at paragraph 23 and A22.
11. I have seen many valuation reports that do not use the publicly traded guideline method and instead rely on some variation of the single period capitalization method. In my opinion, such valuations are fatally flawed. First, single period capitalization can only logically apply to situations in which the expected earnings and cash flow are linear. This circumstance is never true, but is not unacceptable for theoretical "terminal" values subject to substantial present value discounts. Second, valuations based solely on single period capitalization – or any one method – lack the checks and balances of using different types of methods (e.g. net assets, public company guideline, private sale transactions, and discounted cash flows methods). Third, discount and capitalization rates of valuations that have not been vetted against guideline values often incorporate substantial and wholly subjective "non-systematic" risk factors that allow the appraiser to fabricate an answer. In my experience, it is the rare "non-systematic" risk that cannot be quantified and incorporated into the appraiser's projection of future earnings and cash flow, or benchmarked against an appropriately selected guideline company. Capital structure deviations from industry norms are acceptable specific company risk premiums or discounts because (1) they are true non-systematic circumstances and (2) the effects on discount rates can objectively determined.
12. Subject to possible adjustment described in SFAS 123(R), using the historical volatility of stock over the most recent time period corresponding in length to the expected period of restriction is consistent with the requirements of the pronouncement. See paragraph A21. Nevertheless, at VFC we are continuing to explore the availability and application of forward looking measures of volatility to the VFC Longstaff Methodology.
13. On occasions, we will average the volatilities of the guideline companies using a weighted average that reflects the companies' relative participation in the industry of the subject company.
14. It is my opinion that harmonic average should not be used for this purpose, or generally when calculating valuation multiples. Harmonic averages are useful when one's goal is to create a portfolio of investments that mirrors the market or a segment of the market. That is not the purpose of a business valuation, which seeks the value of a single particular investment.
15. AutoNation, Group 1, Lithia, Sonic, and United Auto