Estimating the Marketability Discounts: A Comparison Between Bid-Ask Spreads, and Longstaff's Upper Bound

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This paper contends that the discount for lack of marketability (DLOM) is the difference between the stock price of a liquid company and an equivalent illiquid company and reflects the lack of a free-trading option that is embedded within a company's stock. Longstaff derived a model that views this liquidity swap as a lookback option. We equate this option to the Bid-Ask spread of a stock consistent with the market microstructure literature. We construct a model for the DLOM using the Longstaff (1995) metric and the Bid-Ask spread of Over-the-Counter Bulletin Board stocks as a proxy. We find that our spread-based model does a better job of predicting restricted stock discounts than the Longstaff metric. We include a case study on two companies to illustrate our methodology.

■The discount for lack of marketability (DLOM) is the difference between the stock price of a liquid company and an equivalent illiquid company. In practice, the calculation of the DLOM is a subjective process involving many assumptions and often degenerates to an ad hoc reduction of a certain percentage from the estimated 'marketable' value of the asset without any economic reasoning. The lack of any comprehensive theory on valuation discounts as well as the lack of any legal benchmark has contributed considerably to this state of affairs. The objective of this paper is to construct a model for the DLOM using the Bid-Ask spread of publicly traded liquid stocks and to validate its predictive ability using restricted stock discounts. We offer an alternate strategy for practitioners to estimate the DLOM which is based on existing market microstructure theory and literature.

This paper builds on the notion that the DLOM is essentially the difference between the price of a liquid stock of a company and an equivalent illiquid stock.[1] This difference between the price of the illiquid stock and the price of an equivalent liquid stock arises because of the lack of a free-trading option that implicitly comes along with the latter, Stoll (2000, p. 1482). In the market microstructure literature, the value of this free-trading option, or a liquidity option associated with a publicly traded stock, has been equated to the value of the Bid-Ask spread of a stock (Copeland and Galai, 1983; Stoll, 2000; Bollen, Smith, and Whaley, 2004; Chacko, Jurek, and Stafford, 2006). The Bid-Ask spread, i.e. the cost of making a market in a stock, represents an option written by the market maker that allows a liquidity trader to buy stock at the ask price and to sell stock at the bid price and provides compensation to the market maker for providing liquidity and assuming the fixed costs of order processing, inventory holding costs, and the costs of adverse selection. As also suggested by Kasper (1997), Chipalkatti (2001), and Damodaran (2002, 2005), this paper proposes that the Bid-Ask spread, estimated using data for equivalent public company stock, is a viable market-based proxy for the DLOM of an illiquid company. Consistent with the notion that the spread represents the premium on a free-trading option, we observe that it increases with volatility and trading interval similar to results previously documented in

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[1] Equivalent in this case implies identical cash flow streams and cost of capital.

the market microstructure literature. An alternate model that has received increasing attention as a viable alternative to estimate the DLOM (Damodaran, 2005; Duffy, 2009; Dyl and Jiang, 2008; Stockdale, 2008) is the metric derived by Longstaff (1995). The Longstaff metric models the DLOM using a lookback (call) option that views the DLOM as a liquidity swap.

The objective of this paper is to construct a predictive model for the DLOM using the Bid-Ask spread of Over-the-Counter Bulletin Board (OTCBB) stocks and to calibrate the performance of this model with that of the Longstaff metric using restricted stock discount data. To build a predictive model for the Bid-Ask spread we use data extracted from the OTCBB market which has a high concentration of small market capitalization companies that are relatively illiquid (Levine and Luft, 2004). We find that the spread-based model outperforms the Longstaff metric when it comes to predicting the restricted stock discounts.

This paper provides an alternative to the Longstaff option method of estimating the discount for lack of marketability, DLOM, for restricted stocks.

The paper develops as follows. The next section provides an institutional context to our research. We also review the prior literature relating to the DLOM and the Longstaff metric in this section. Section II presents the intuition behind the notion that spreads and, therefore, the DLOM can be modeled as free-trading options on the underlying stock. In Section III, we construct a predictive model for the DLOM using the Bid-Ask spread of OTCBB stocks. In Section IV, we test the predictive ability of our spread-based model and the Longstaff metric using actual restricted stock discounts to evaluate the performance of the metrics. Our concluding observations are provided in Section V.

I. Background

A. Institutional Context

Most valuation practitioners and the Tax Courts continue to rely on the traditional empirical studies that attempt to estimate the DLOM using data from the public markets. These empirical studies include, (a) pre-initial public offering (pre-IPO) discount studies, (b) studies that compare market multiples of public versus private companies, and (c) restricted stock studies. The pre-IPO studies, popularized by the work of John Emory and summarized by Pratt (2000, p. 408), suggest that the discounts range from 25% to 45%. Using private placement data, however, the work of Bajaj, Denis, Ferris, and Sarin (2001) concluded that the DLOM should be around 7.23%. Block (2007) observes an average discount of 20% to 25% with a low of 9% for financial firms. The age of some of these studies, the range and size of these discounts, the studies' small sample sizes, and the lack of any theory relating to the DLOM have all resulted in considerable friction between the internal revenue service (IRS) and taxpayers about the appropriate valuation discounts. In recent years, the Tax Court has considerably pared down the DLOM to as low as 20 % in the *McCord v. Commissioner* case and has rejected the pre-IPO studies to be biased and unreliable.[2] Primarily relying on the Bajaj et al. (2001) study, the Tax Court in the Gross vs. Commissioner Estate of Heck vs. Commissioner, MCord and Lappocases has significantly pushed back against marketability discounts that are deemed too high (Robak, 2004; McMarrow, 2004) or inadequately supported.[3, 4, 5]

In this post-Mandelbaum environment, Pratt (2000) indicates that there will be a growing expectation that business appraisers develop improved methodologies to quantify marketability discounts.[6] This paper attempts to provide such an improved methodology that is based on market microstructure theory and uses observable market data to model the DLOM.

B. Longstaff Metric

Longstaff (1995) theoretically modeled the DLOM and derived the upper bound as a liquidity swap for a hypothetical investor who owned a lookback option and was assumed to possess perfect market timing. The Longstaff theoretical model holds that the DLOM is positively related to the current value of the security, the length of the marketability restriction imposed on the restricted stock, and variance of stock returns. Finnerty (2002) expanded on Longstaff's model and demonstrated an additional negative relationship with the dividend yield of the stock.

According to Longstaff (1995, p. 1772), the upper bound derived from his model of DLOM also can be viewed as the maximum amount an investor is willing to pay for immediacy in order to liquidate a security position and, hence, provides an endogenous measure of the largest possible transaction cost or the Bid-Ask spread for a security. Longstaff compares this transaction cost to the DLOM for restricted stocks and

[2] McCord v. Commissioner, 120 T.C. No. 13 (2003).

[3] Gross v. Commissioner, No. 99-2239/2257, 6th Cir. (2001).

[4] Estate of Heck, T.C. Memo 2002-34 (2002).

[5] Lappo v. Commissioner, T.C. Memo 2003-258 (2003).

[6] Bernard Mandelbaum, et al. v. Commissioner, T.C. Memo 1995-255 (June 12, 1995).

observes that his metric closely approximates the observed discount for lack of marketability.

II. The Bid-Ask Spread and the DLOM

Stoll (2000) proposes that the spread is a measure of the "friction" in financial markets that measures the difficulty with which an asset is traded; the greater the friction, the greater the spread.[7] He suggests that the spread exists to compensate suppliers of immediacy for the free-trading or liquidity option they grant to the rest of the market. Previously, Copeland and Galai (1983) had theorized that the market maker provides liquidity by giving a prospective trader a call option to buy stock at the asking price and a put option to sell stock at the bid price. Thus, the cost of obtaining immediacy in a publicly-traded stock trade is the size of the Bid-Ask spread, or, the premium on this strangle option position written by the market maker, where the option premium recovers the underlying cost of providing these market making services to the investing public.[8] Bollen et al. (2004) extend Copeland and Galai's (1983) work by assuming that the spread represents the premium on a slightly in-the-money strangle option. More recently Chacko, Jurek, and Stafford (2006) also use an option-based framework to model liquidity. They suggest that when immediate exercise is desired the optimal option strike price is the effective Bid-Ask spread.

We propose that the Bid-Ask spread of publicly traded stock can be used to estimate the value of the DLOM and the theoretical value of the free-trading option in the case of illiquid stock. The DLOM represents the difference that arises between the price of a liquid stock and an equivalent illiquid stock because of the lack of a free-trading option that implicitly comes along with the ownership of a stock that has a liquid market. Assume that PLIQ is the price of liquid stock and PILLQ is the price of illiquid stock of an equivalent company. The relationship between PLIQ , PILLQ, and DLOM, the value of the discount for lack of marketability is:

$$P_{LIQ} = P_{ILLQ} + DLOM. \tag{1}$$

The (relative) illiquidity that comes with such stock may be "eliminated" if the owner holds a call option to buy more of the stock and simultaneously holds a put option to sell the stock. In this case, the holder of the illiquid stock is essentially buying a strangle option position for the underlying stock.[9] This assumes that there is a rational, unbiased agent willing to make a market in the stock and that there is a buying and a selling interest in the stock. The cost of eliminating the illiquidity associated with the illiquid stock is equivalent to the premium on the strangle option position for such a stock which is also equivalent to the hypothetical Bid-Ask spread (BAS) that would be charged on such a stock. In other words:

$$P_{LIQ} = P_{ILLQ} + BAS, \tag{2}$$

Therefore, equating (1) to (2),

$$DLOM = BAS. \tag{3}$$

Consistent with this notion of the BAS as a strangle option, there is extensive documented evidence in the literature of a positive association between the spread and the volatility of the returns and also the trading interval. We use the Bid-Ask spread of OTCBB stocks to estimate the value of the DLOM and the premium on the free-trading option of an illiquid stock because these OTCBB firms are relatively small in size, are illiquid, and have large trading intervals.

In the next section, we outline the methodology used to estimate spreads for OTCBB stocks to estimate the Longstaff DLOM metric.

III. Empirical Analysis

A. Metrics

We estimate the two metrics discussed above: Bid-Ask Spread and the Longstaff Option, as percentages. The Bid-Ask spread is calculated as:

$$BidAsk\% = \left[\frac{(Ask - Bid)}{\frac{(Ask + Bid)}{2}}\right] * 100. \tag{4}$$

The spread is the difference between the price a market maker is willing to pay, the BID, and the price the market maker is willing to accept, the ASK, as a percentage of the spread's midpoint.

The Longstaff metric is estimated as:

[7] The microstructure literature documents that the Bid–Ask spread is set by the market maker to recover three sets of costs: order processing costs, inventory holding costs due to the need to hold sub-optimal levels of inventory and asymmetric information or adverse selection costs of trading with informed investors.

[8] A strangle position is created when a put option and call option are combined, but the exercise prices differ for the call and put options. In this case, the call option exercise price equals the dealer's asking price, and the dealer's bid price defines the exercise price for the put option.

[9] The option's maturity is equal to the holding period of stock. The value of this option position increases with the volatility of the underlying private stock as well as the time it takes to liquidate a position in that stock. The value of the option position is also a function of the price of the underlying stock.

$$F = V\left(2+\frac{\sigma^2 T}{2}\right)N\left(\frac{\sqrt{\sigma^2}T}{2}\right)+V\sqrt{\frac{\sigma^2 T}{2\pi}}\exp\left(-\frac{\sigma^2 T}{8}\right)-V. \quad (5)$$

Where:

F = the value of the discount for lack of marketability,
V = the current price of the asset,
σ^2 = the monthly volatility of the asset price changes,
T = the trading interval expressed in monthly terms.
N( ) = the cumulative normal distribution function.

The Longstaff metric is expressed as a percentage by dividing the option value, F, by the underlying asset value, V.

B. Data

To estimate the metrics, daily data for 1,850 individual OTCBB firms traded between December of 1995 and December of 2000 were obtained from Nasdaq. We did not include OTCBB data reported after December 31st 2000 because the nature of the market changed in 2001. After January 1st, 2001, OTCBB listed firms were required to file their financial statements with the Securities and Exchange Commission (SEC). We felt the OTCBB firms listed through December 31st of 2000 comprised a better benchmark for illiquid firms The inclusion of firms that chose to stay on the OTCBB after this regulation will create a self-selection bias in the sample as improved disclosure requirements enhanced the underlying liquidity of their stock. The data needed to estimate the metrics included the closing Bid price, the closing Ask price, the transaction price, and the trading volume.[10] The monthly number of shares outstanding and any splits or dividends that occurred were obtained from Bloomberg.

Once each firm's trading history had been established from December of 1995 through December of 2000, we transformed the data and computed the percentage spread and the Longstaff metric. We estimated the trading interval for every trade by determining the number of calendar days that elapsed between trades. We then transformed each observation's trading interval into both monthly and annual terms for the computation of the Longstaff metric. Next, we calculated each company's monthly share price volatility via the Garman-Klass-Parkinson (GPK) methodology as modified by Bilson (2003).[11] This methodology is well suited to our data since it allows us to capture the monthly variation in each firm's share price even though many of these firms did not trade on a daily basis. The Garman-Klass-Parkinson volatility was calculated via:

$$GKP\sigma^2 = [\ln(O_t/C_{t-1})]^2 + .5[\ln(H_t/L_t)]^2 - .31[\ln(C_t/O_t)]^2 \quad (6)$$

Where:

$GKP\sigma^2$ = the monthly volatility for the firm's share prices,
O = the firm's opening share price at the beginning of the month,
C = the firm's closing share price at the end of the month,
H = the firm's highest share price during the month,
L = the firm's lowest share price during the month,
t = time.

After estimating each firm's monthly share price volatility, we computed each firm's market capitalization and trading activity for each day between December of 1995 and December of 2000. The market capitalization value was determined by multiplying the firm's closing price on that day by the number of shares outstanding.[12] Monthly trading activity for each firm was computed by summing its daily trading activity during the month and then dividing the total number of shares traded by the total number of shares outstanding at the end of the month.

C. Portfolio Formation

It is well documented in the market microstructure literature (see Stoll (2000) for a comprehensive review), that the Bid-Ask spread is significantly related to trading activity, volatility, market capitalization and trading interval. We assign the OTCBB firms into 24 portfolios based on the firm-specific value for each of these four dimensions relative to the overall sample average. The classification procedure was a four step iterative process which employed the monthly median values for market capitalization, trading activity, share price volatility, and the monthly mean value for trading interval. Firms were identified as High Market Cap firms if their market capitalization value exceeded the

[10] When we examined the data it was obvious that we had three problems. First, there were days when Bid and Ask values were not reported. Second, there were days when the trading volume was zero. Third, there were days where we observed trading volume, a Bid price, an Ask price, but no transaction price. We addressed the first problem by assigning the most recent Bid and Ask values to the observations that were missing Bid and Ask values. If there were several successive days where the Bid and Ask values were not reported, then we continued to use the most recently observed values for the Bid and Ask values. We dealt with the second problem by defining a good trading day as one where trading volume was reported. If no trading volume was reported, then we dropped the observation. The third problem was resolved by using the mid point of the Bid-Ask spread as the transaction price. Thus, we established the trading history for each firm.

[11] This volatility measure is based on the firm's open, high, low, and closing prices over a given trading interval and uses as much of the share price information as possible.

[12] If a trade did not occur on that day, then we used the average of the most recent trading day's Bid-Ask spread as the price in the market capitalization calculation.

Table I. OTCBB Firm Portfolio Classification

Over the Counter Bulletin Board Firm portfolios created via four dimensions: market capitalization, share price volatility, trading activity, and trading interval. Each cell contains the average value the portfolio's dimensions: market capitalization ($100,000); monthly trading activity (Volume / Shares Outstanding) trading interval (days) GKP volatility (monthly sigmas); and the median values for the Bid Ask spread and Longstaff percentages.

		High monthly trading activity			Low monthly trading activity		
		Long trading interval > 5 Days	Medium trading interval 2 -5 Days	Short trading interval < 2 Days	Long trading interval> 5 Days	Medium trading interval 2 -5 Days	Short trading interval < 2 Days
High Market Capitalization	**High σ**						
	Portfolio #	1	2	3	7	8	9
	Size	$27,904	$14,931	$32,905	$37,283	$20,043	$32,415
	Activity	1.39%	0.59%	0.95%	0.03%	0.04%	0.06%
	Interval	14.1	2.1	1.5	13.4	2.4	1.6
	Volatility	95%	43%	37%	59%	38%	34%
	BidAsk%	9%	12%	6%	28%	17%	9%
	Longstaff %	17%	8%	7%	14%	8%	6%
	Low σ						
	Portfolio #	4	5	6	10	11	12
	Size	$24,778	$25,182	$37,901	$116,658	$78,408	$89,460
	Activity	0.56%	0.48%	0.60%	0.04%	0.04%	1.20%
	Interval	9.0	2.4	1.5	8.4	2.5	1.6
	Volatility	7%	11%	15%	7%	10%	13%
	BidAsk%	7%	7%	4%	8%	7%	6%
	Longstaff %	2%	2%	3%	2%	2%	3%
Low Market Capitalization	**High σ**						
	Portfolio #	13	14	15	19	20	21
	Size	$1,713	$2,549	$3,228	$2,104	$2,945	$3,559
	Activity	1.20%	1.52%	5.66%	0.04%	0.05%	0.06%
	Interval	10.0	2.3	1.5	10.3	2.6	1.6
	Volatility	59%	47%	43%	57%	44%	41%
	BidAsk%	39%	22%	12%	47%	26%	17%
	Longstaff %	16%	10%	8%	17%	9%	8%
	Low σ						
	Portfolio #	16	17	18	22	23	24
	Size	$2,751	$3,231	$3,869	$2,927	$3,504	$4,144
	Activity	0.88%	0.84%	1.99%	0.05%	0.05%	0.06%
	Interval	8.2	2.4	1.5	8.0	2.6	1.6
	Volatility	11%	14%	16%	12%	14%	16%
	BidAsk%	18%	12%	9%	21%	14%	12%
	Longstaff %	4%	3%	3%	4%	3%	3%

median and Low Market Cap firms otherwise. The same logic was used to classify firms according to trading activity and volatility.

In the case of trading interval, we employed three categories instead of two. Firms with mean trading intervals that were less than or equal to two days were placed in the Short Trading Interval group and those with mean trading intervals between two days and five days were classified as having Medium Trading Intervals. All others were classified as firms that have Long Trading Intervals. The final results with portfolio averages for each metric are portrayed in Table I.

We examine the behavior of the Bid Ask spread and the Longstaff metric across the four dimensions to confirm that the portfolio median values for the spread and the Longstaff metric behave in a manner that is consistent with the results previously obtained for effects related to firm size, trading activity, trading interval, and volatility. Table II displays the results.[13] We observe that the sample behaves in a manner

[13] We tested the two metrics via the Wilcoxon nonparametric test and rejected the null hypothesis of equal median values at the .0001 level for all the median values associated with the metrics. Thus, the two metrics appear to generate different values across all four dimensions.

Table II. Size, Trading Activity, Volatility and Trading Interval Effects across OTCBB Portfolio Medians for Bid-Ask Spread Percentage Metric and Longstaff Metric

Over the Counter Bulletin Board Firm portfolios created via four dimensions: market capitalization, share price volatility, trading activity, and trading interval. Each cell in the second column contains the average value the portfolios' percentage bid-ask spread, Equation (4), and the cells in the third column present the average values, across the portfolios, for the Longstaff metric, Equation (5).

	Bid-Ask	Longstaff
Compare size effect		
Large cap portfolios 1:12	6.90%	4.42%
Small cap portfolios 13:24	17.47%	6.90%
Compare activity effect		
High activity portfolios 1:6 & 13:18	8.28%	5.73%
Low activity portfolios 7:12 & 19:24	11.92%	4.92%
Compare volatility effect		
High sigma portfolios1:3 7:9 13:15 19:21	13.27%	7.92%
Low sigma portfolios 4:6 10:12 16:18 22:24	7.48%	2.85%
Compare trading interval effect		
Long interval portfolios 1,4,13,16,7,10,19,22	21.39%	6.11%
Medium interval portfolios 2,5,14,17,8,11,20,23	14.60%	5.38%
Short interval portfolios 3,6,15,18,9,12,21,24	7.52%	5.15%

that is consistent with published results in the market microstructure literature.

Large firms exhibit smaller spreads than small firms; and firms that experience higher trading activity have smaller spreads. The spreads for high volatility firms are larger than the spreads for low volatility firms and the longer the trading interval, the greater the spread. The results for Longstaff's metric also are consistent with the market microstructure literature for the size, trading interval, and the volatility effects. However, the trading activity effect is contrary to our expectation that marketability costs will decrease with greater trading volume. The two metrics differ only with respect to their association with the trading activity variable. We model the portfolio averages for the Bid-Ask spread as a function of firm size, trading activity, price volatility, and trading interval. The model is specified as follows:

$$\overline{\ln(\text{BidAsk\%})}_{p,y,m} = \alpha_0 + \alpha_1 \overline{\ln(\text{MC})}_{p,y,m} + \alpha_2 \overline{\ln(\text{L})}_{p,y,m} + \alpha_3 \overline{\ln(\text{GKP}\sigma^2)}_{p,y,m} + \alpha_4 \overline{\ln(\text{I})}_{p,y,m} + \varepsilon. \quad (7)$$

Where:

$\overline{\ln(\text{BidAsk\%})}_{p,y,m}$ = the monthly average value of the log of the percentage Bid-Ask spread, for each portfolio p (1 to 24), in year y (1995 to 2000), for month m (1 to 12).

$\overline{\ln(\text{MC})}_{p,y,m}$ = the monthly average of the log of the market capitalization measured on a daily basis as the current share price times the number of outstanding shares for each portfolio p, in year y, for month m.

$\overline{\ln(\text{GKP}\sigma^2)}_{p,y,m}$ = the monthly average of the log of the Garman-Klass-Parkinson monthly volatility for each portfolio p, year y, for month m.

$\overline{\ln(\text{I})}_{p,y,m}$ = the monthly average of the log of the trading interval expressed as days since the last trade for each portfolio p, in year y, for month m.

$\overline{\ln(\text{L})}_{p,y,m}$ = the monthly average of the log of the daily trading activity calculated as daily number of shares traded as a percentage of outstanding shares for each portfolio p, in year y, for month m.

The results are presented in Table III.

Once again, we observe that our sample behaves in a fashion that is consistent with the relationships documented previously in the market microstructure literature. There is a significant inverse relationship between the percentage Bid-Ask spread and the size and trading activity variables. Also there are significant positive relationships between the percentage Bid-Ask spread and volatility and also trading interval. The explanatory power of this regression, as measured by the adjusted R^2, is 89%.[14]

[14] In his paper, Longstaff suggests that his metric can also be considered to be a measure of the largest Bid-Ask spread for a security. We check

Table III. Regression Model: ln(%Bid Ask Spread)=f(ln(Market Cap), ln(Liquidity),ln(GKP volatility), ln(Trading Interval)

The following regression model was estimated:

$$\overline{\ln(\text{BidAsk}\%)}_{p,y,m} = \alpha_0 + \alpha_1 \overline{\ln(\text{MC})}_{p,y,m} + \alpha_2 \overline{\ln(\text{L})}_{p,y,m} + \alpha_3 \overline{\ln(\text{GKP}\sigma^2)}_{p,y,m} + \alpha_4 \overline{\ln(\text{I})}_{p,y,m} + \varepsilon.$$

$\overline{\ln(\text{BidAsk}\%)}_{p,y,m}$ = the monthly average value of the log of the percentage Bid Ask spread, for each portfolio p (1 to 24), in year y (1995 to 2000), for month m (1 to 12),

$\overline{\ln(\text{MC})}_{p,y,m}$ = the monthly average of the log of the market capitalization measured on a daily basis as the current share price times the number of outstanding shares for each portfolio p, in year y, for month m,

$\overline{\ln(\text{GKP}\sigma^2)}_{p,y,m}$ = the monthly average of the log of the Garman-Klass-Parkinson monthly volatility for each portfolio p, year y, for month m ,

$\overline{\ln(\text{I})}_{p,y,m} + \varepsilon$ = the monthly average of the log of the trading interval expressed as days since the last trade for each portfolio p, in year y, for month m,

$\overline{\ln(\text{MC})}_{p,y,m}$ = the monthly average of the log of the daily trading activity calculated as daily number of shares traded as a percentage of outstanding shares for each portfolio p, in year y, for month m.

Equation (7)

Dependent variable: Monthly average ln (Bid Ask %)$_{p,y,m}$

Over the Counter Bulletin Board firms classified by portfolio, year, and month.

Variable	Coefficient	*t*-statistic
Constant	1.8338	20.87 ***
Average ln(MC)	-.2832	-50.00***
Average ln(L)	-.1129	-25.94***
Average ln(GKPσ^2)	.4067	47.13***
Average ln(I)	.5891	47.63***
Observations	1440	
Adjusted R^2	89%	

***Significant at the 0.01 level

IV. Prediction of Restricted Stock Discounts using the Bid-Ask Spread and the Longstaff Metric

In the previous section, we proposed that we can use the spread of OTCBB stocks as proxies for estimating the DLOM of illiquid stock. The data for the OTCBB Bid-Ask spreads (Table II) provide some insights on DLOM values. The median spread for firms with long trading intervals is about 21% as compared to 7.5 % for those with short trading intervals. The spread for low market capitalization portfolios with low trading activity, portfolios 19 to 24,is approximately 19%. The spread for this category of portfolios with long trading intervals range from 22% to 35% and the spread for portfolios with short trading intervals range from 9% to 15%. In this section we test the predictive performance of the Bid-Ask spread metric as a measure for the DLOM. We mimic Longstaff (1995) and Finnerty (2002) and calibrate the performance of the Longstaff metric and a Bid-Ask spread based metric on a sample of restricted stocks discounts.

The sample of restricted stock transactions was obtained from FMV Opinions Inc. who conduct annual surveys of restricted stock transactions. The FMV price data include

whether using his metric would yield a better predictive model of the spread. We regress the observed Bid-Ask spread as a function of firm size, trading activity, and the Longstaff metric. We use the monthly average value of the log of the Longstaff metric for each portfolio p, in year y, for month m. The results indicate that the percentage Bid Ask spread is significantly and inversely related to firm size and trading activity. The Longstaff metric is also significant and positive. The adjusted R^2, is 81% and less than that obtained using the trading interval and the volatility variables as a linear function. The significant coefficients for Market Capitalization and Trading Activity indicate that the the Longstaff metric the transaction price for the restricted stock's sale; the number of shares outstanding; the monthly trading volume for the exchange traded shares; the monthly Open, Close, High, and Low prices for the exchange traded shares; the prior month's average of the observed high and low values for the restricted stock's exchange traded shares, the stock's annualized volatility measured as the standard deviation of the prior year's weekly returns; and the stock's required holding period, expressed in years, based on the SEC ruling at the time of the restricted stock transaction. We used these data to calculate each restricted stock's Garman-Klass-Parkinson volatility measure and each restricted stock's percentage discount.[15]

We tested the Longstaff metric and our Bid-Ask spread model of the DLOM on a set of restricted stock discount transactions that occurred between March of 1991 and November of 2000. The Longstaff metric and our Bid-Ask spread model were tested twice: once using the FMV volatility, and once using the Garman-Klass-Parkinson volatility measure. The first test, which employed the FMV volatility measure, was performed on a sample of 338 restricted stock discount transactions; the second test used the Garman-Klass-Parkinson volatility measure and was performed on 142 of the 338 restricted stock discount transactions.[16]

We used both the percentage Bid Ask spread and the Longstaff metric (Equation 5) to predict the marketability discount for the restricted stockwhich we then compared to the actual restricted stock discounts.[17] To compute the Longstaff metric for each restricted stock observation, we set V, the underlying asset value, to be equal to the current transaction price of the stock and used monthly volatility.[18] We set T, the trading interval, to be equal to the number of months contained in the required holding period.

In the case of the Bid-Ask spread based model, we obtained the parameters to predict the DLOM by re-estimating Equation (7) after making one adjustment. We dropped all OTCBB observations with trading intervals less than sixty days as this eliminated much of the noise associated with the short trading interval data.[19] We felt that the OTCBB observations with trading intervals that exceeded sixty days more closely matched the long trading interval of either one year or two years of the restricted stock data. To be consistent, we dropped the trading interval variable from our estimating equation. This left us with 48,108 daily trading observations that we had to place in portfolios so that we could calculate monthly averages for the log values of the percentage Bid-Ask spread, market capitalization, trading activity, and volatility. We followed the same portfolio classification procedure as was described previously. Next, we obtained monthly average values for each variable of interest for each portfolio. This procedure yielded 336 monthly portfolio observations, containing 511 firms, which were used in a regression to obtain the parameters we needed to estimate the DLOM for the restricted stock.[20] The regression equation was specified as:

$$\overline{\ln(\text{BidAsk}\%)}_{p,m} = \alpha_0 + \alpha_1 \overline{\ln(\text{MC})}_{p,m} + \alpha_2 \overline{\ln(\text{L})}_{p,m} + \alpha_3 \overline{\ln(\text{GKP}\sigma^2)}_{p,m} + \varepsilon. \tag{8}$$

Where:

$\overline{\ln(\text{BidAsk}\%)}_{p,m}$ = the monthly average value of the log of the percentage Bid-Askspread, for each portfolio p, in month m where there are 8 portfolios, and 42 months.

$\overline{\ln(\text{MC})}_{p,m}$ = the monthly average of the log of the market capitalization measured on a daily basis as the current share price times the number of outstanding shares for each portfolio p, in month m,

$\overline{\ln(\text{GKP}\sigma^2)}_{p,m}$ = the monthly average of the log of the Garman-Klass-Parkinson monthly volatility for each portfolio p, in month m,

$\overline{\ln(\text{L})}_{p,m}$ = the monthly average of the log of the daily trading activity calculated as daily number of shares traded as a percentage of outstanding shares for each portfolio p, in month m.

[15] We calculated the discounts as the percentage difference between the prior month's average of the exchange traded stock's observed high and low values and the restricted stock's transaction price.

[16] Our sample of 142 restricted stock observations was defined by the availability of the price data required to compute the Garman-Klass-Parkinson volatility: monthly Open, Close, High, and Low values for exchange traded shares issued by firms which also completed restricted stock transactions between 1991 and 2000.

[17] Unfortunately, we were unable to obtain financial statement data and dividend data for most of the OTCBB stock. This would have permitted us to include other factors that impact marketability like dividend, profitability, etc. in our spread regression equation. This is the approach taken by Damodaran (2002) and Chipalkatti (2001).

[18] The FMV volatility was used to calculate the discount for all 338 discount observations. The Garman-Klass-Parkinson (GPK) volatility also was used to obtain the discounts for the subset of 142 of observations where the GKP volatility could be obtained.

[19] 130 of the restricted stock observations had one year trade intervals, and the remaining 208 observations had two year trade intervals.

[20] There were 480 possible monthly portfolio observations where trading intervals exceeded five days, 8 portfolios over the five year, 60 month, period; but only 42 months between January of 1995 and December of 2000 contained observations for the eight portfolios with trading intervals greater than 60 days. This yielded 336 monthly portfolio observations with trading intervals greater than 60 days.

Table IV. Regression Results for Monthly Average Natural log of the Percentage Bid Ask Spread for Trading Intervals Greater Than 60 Days: ln(%Bid Ask Spread)=f(ln(Market Cap), ln(GKP volatility), ln(Liquidity)

The following regression model was estimated:

$$\overline{\ln(\text{BidAsk\%})}_{p,m} = \alpha_0 + \alpha_1 \overline{\ln(MC)}_{p,m} + \alpha_2 \overline{\ln(L)}_{p,m} + \alpha_3 \overline{\ln(GKP\sigma^2)}_{p,m} + \varepsilon_{p,m}.$$

$\overline{\ln(\text{BidAsk\%})}_{p,m}$ = the monthly average value of the log of the percentage Bid Ask spread, for each portfolio p, in month m, where there are 8 portfolios and 42 months,

$\overline{\ln(MC)}_{p,m}$ = the monthly average of the log of the market capitalization measured on a daily basis as the current share price times the number of outstanding shares for each portfolio p, in month m,

$\overline{\ln(GKP\sigma^2)}_{p,m}$ = the monthly average of the log of the Garman-Klass-Parkinson monthly volatility for each portfolio p, in month m,

$\overline{\ln(L)}_{p,m}$ = the monthly average of the log of the daily trading activity calculated as daily number of shares traded as a percentage of outstanding shares for each portfolio p, in month m.

Equation (8)
Dependent variable: Monthly average ln (Bid Ask %)$_{p,m}$
Over the Counter Bulletin Board firms classified by portfolio and month.

Variable	Coefficient	*t*-statistic
Constant	3.6906	13.06***
Average ln(MC)	-.3737	-20.42***
Average ln(L)	-.1632	-8.36***
Average ln(GKPσ^2)	. 2921	11.43***
Observations	336	
Adjusted R^2	72%	

***Significant at the 0.01 level

Table IV presents the results from estimating Equation 8 and shows that 72% of the variation in the percentage Bid-Ask spread is explained by the regression. Furthermore, the significant relationships detected earlier in Table III still hold: The percentage Bid-Ask spread is inversely related to both firm size and trading activity and positively related to volatility.

The monthly average of the percentage Bid-Ask spread in equation 8 is our spread-based proxy for the DLOM. The coefficients generated by Equation 8 were then used to estimate the DLOM for the restricted stock discount observations. The spread-based DLOM was also estimated twice, once using the FMV volatility for 338 discounts, and then using the GKP volatility for a sub-sample of 142 discounts for which the latter could be calculated.

Since we need to measure DLOM for sell-side restrictions only, we transformed the calculated DLOMs into half spreads. This transformation makes the calculated DLOMs consistent with Longstaff spreads.[21] These were compared to the restricted stock discounts calculated from the FMV data. The results are presented in Table V.

Table V contains the descriptive statistics for the observed restricted stock discounts and for the estimated discounts generated by the Longstaff model and the Bid-Ask spread model. Table V also reports two measures of forecast accuracy: The Mean Square Error and Theil's U-Statistic for both models. Panel A contains the results for the 338 discount transactions that used only the FMV volatility. Panel B presents the results for the smaller sample of 142 observations with two estimated discount values- one using the FMV volatility measure and the other using the GKP volatility measure.

According to Panel A the median percentage discount for the entire sample was approximately 19%. Furthermore, both the Longstaff model and the Bid-Ask spread model generate higher percentage discounts than the observed FMV percentage discounts. The Bid-Ask spread model yields median DLOM of approximately 28% as compared to 106% obtained by the Longstaff metric. Finally, the Bid –Ask spread model appears to be superior to the Longstaff model as an estimator of the percentage restricted stock's DLOM. The results in Panel B are very similar. These results are not surprising when one considers the fundamental nature of

[21] The spread represents the cost of providing both buy-side and sell-side liquidity. Longstaff's metric, on the other hand, measures the DLOM when there is a restriction to sell. The fact that the spread works out to be almost double the Longstaff metric (See Table II) makes sense in this context.

Table V. Descriptive Statistics for Restricted Stock DLOM Percentages Calculated Via: the FMV Data, the Longstaff Option Model, and the Bid-Ask Regression Model

The statistics are for the percentage half spreads observed in the FMV data, and also estimated via the Longstaff option metric, and Equation (10), where Equation (10) employs the parameters generated by the portfolio Bid Ask regression model, Equation (9). Panel A is the set of 338 observed discounts that employed only the FMV volatility. Panel B is the set of 142 observed discounts that had sufficient data to employ both the GKP and FMV volatilities.

	Observed Restricted Stock Discounts	Longstaff Estimates		Bid Ask Estimates	
		FMV Volatility	GKP Volatility	FMV Volatility	GKP Volatility
Panel A. Restricted Stock Discounts Based on FMV Volatility					
Number of Observations	338	338		338	
Mean	.2277	2.1719		.3592	
Median	.1871	1.0660		.2882	
Sigma	.1621	11.6340		.2219	
Maximum	.7949	205.9746		1.3951	
Minimum	.0005	.2377		.0538	
Mean Square Error		138.0211		.0656	
Theil's U		42.057		.917	
Panel B. Restricted Stock Discounts Based on Both FMV and GKP Volatilities					
Number of Observations	142	142	142	142	142
Mean	.2188	3.1589	3.2421	.3277	.3317
Median	.1733	1.0274	.9927	.2645	.2635
Sigma	.1572	17.8448	13.2215	.2125	.2234
Maximum	.7059	205.9745	114.7837	1.3597	1.2478
Minimum	.0017	.2635	.1662	.0741	.0604
Mean Square Error		323.318	181.2028	.0599	.0637
Theil's U		66.822	50.025	.910	.938

each metric. The percentage Bid-Ask (half) spread is market determined and captures the effects of inventory holding costs, liquidity costs, and asymmetric information costs on a firm's share price. The Longstaff metric is an upper bound for the DLOM and assumes equally well informed parties where the share owner is prohibited from selling until a specific date.

The predictive ability of the estimated DLOMs generated by the two metrics was assessed via Theil's U-statistic. A U-statistic which is less than one indicates that a model performs better than a naive no change extrapolation model.[22]

In Panel A, the U-statistic of 0.917 obtained for the Bid-Ask spread model dominates the U-statistic of 42.015 obtained for Longstaff's model, and leads to the conclusion that the Bid-Ask spread model provides better and more accurate estimates of the discounts than does the Longstaff metric.[23]

[22] Theil's U-statistic is a measure of forecast accuracy that is based on the mean square error and measures the severity of the forecast errors. If all forecasts are perfect, then U equals zero. If U takes a value which is greater than zero, but less than one, then the forecast is not perfect, but it is better than what can be achieved by using a naive no change extrapolation. If U equals one, then the forecast is only as reliable as the no change extrapolation. If U is greater than one, then the forecast is worse than the no change extrapolation.

[23] When the FMV volatility and the GKP volatility results in Panel B are compared for the Bid-Ask spread model, there does not appear to be much difference in the estimated discounts. This is encouraging for the GKP volatility because this estimate is based on the prior month's share price movement: Open, Hi, Low, and Close; and does not require a year's worth of price and return data as does the FMV volatility. This observation was confirmed by a t-test that failed to reject the null hypothesis of equal mean values for the FMV and GKP discounts at the .1 level.

Panel B provides further evidence regarding the performance of the two models. The Theil U-statistics generated for the estimates made using the FMV volatility measure is 0.910 for the Bid-Ask spread model and 66.82 for the Longstaff model. The U-statistics based on the GKP volatility estimate is 0.938 for the Bid-Ask spread model and 50.025 for the Longstaff metric. Both results reinforce the conclusion that the Bid-Ask spread model outperforms the Longstaff metric. Although these results show that the Bid-Ask spread model generated better forecasts of the observed restricted stock DLOM than the Longstaff model, we cannot reject the latter based just on Theil's U-statistic. Hence, we employ Davidson and McKinnon's J-test (1981) for non-nested hypothesis to test for the better, or the most acceptable, model specification for the prediction of restricted stock discount. The J-test is implemented by specifying two hypotheses, H_0 and H_1 and then testing one against the other. For the observed restricted stock discounts, the test specification is:

$$H_0: \ln(2^*\text{Observed Discount}) = \alpha+\beta_1\ln(\text{Size})+\beta_2\ln(\text{Activity}) +\beta_3\ln(\text{Volatility})+\varepsilon, \quad (9)$$[24]

$$H_1: \ln(2^*\text{Observed Discount}) = \beta_1\ln(2^*\text{Longstaff Predicted Value}) +\varepsilon. \quad (10)$$

The J-test can have four possible outcomes. The results may suggest that both models are acceptable or that both are not acceptable. The results may also find one of the two models acceptable and the other model not acceptable.

We estimated both models and generated values for the estimated discount; FMVBAHAT from H_0 and FMVLHAT from H_1. The J-test involves testing the significance of the coefficient on FMVLHAT when included in the equation for H_0 and then, testing the significance of the coefficient on FMVBHAT when included in H_1. If the coefficient on FMVLHAT (FMVBHAT) is not significant, the Bid-Ask spread model (Longstaff model) is acceptable while the alternate model is not acceptable. The slope coefficient on FMVLHAT was not significant (t-statistic of -0.30 with a p-value of 0.764). This indicates that the Bid-Ask spread model was not rejected by H_1, the Longstaff metric. On the other hand, the coefficient on FMVBHAT was significant (t-statistic of 17.93 at $p < 0.0001$). The J-test results indicate that the Bid-Ask spread based model was acceptable while the Longstaff metric was not acceptable as a model for estimating restricted stock DLOM.

In sum, the J-test results and the results for Theil's U indicate that the Bid -Ask spread model is superior to the Longstaff option model for estimating the DLOM.

[24] We multiplied the observed discount by two to convert half spreads in to full spreads to be consistent.

V. Case Study

We illustrate the application of the Bid-Ask spread model using data for two companies. The first company, Carrington Laboratories, was traded on the American Stock Exchange under the ticker symbol CRN. The second firm, Genus Company, was larger than Carrington, and traded Over-the-Counter under the ticker symbol GGNS. The raw data required to apply the model were extracted from the Center for Research in Security Prices (CRSP) database and are presented in Table VI for both companies. The transaction dates represent the first day of the month in which the restricted stock was issued as listed in the FMV database.

We chose these companies to demonstrate the performance (in terms of DLOM) of the Bid-Ask spread model and the Longstaff equation for two different firms based on market capitalizations, trading activity, and volatility. Comparing the data in Table VI reveals that CRN was smaller than GGNS, had a higher share price than GGNS, but had a much lower trading volume than GGNS. The (GKPσ^2) variances also indicate that CRN's share price was less volatile than that of GGNS. Both the Bid-Ask spread model and the Longstaff equation are applied to each firm on the appropriate transaction dates. FMV reports the discounted values for restricted stock issues on the first trading day of the month or the transaction date. Hence, we applied the two models using the raw data as observed in the month prior to the transaction date. We assume that the discounts reflect activity during this month. . The size, activity, and volatility data (see Table VI) are used to generate the values for the model's inputs: Market Capitalization (MC), Trading Activity (L), and Garmon-Klass-Parkinson variance (GKPσ^2). The values for MC were obtained by multiplying the prior month's average price by the number of shares outstanding during the prior month. The values for L were generated by dividing the number of shares outstanding during the prior month by the prior month's trading volume where the number of shares outstanding had to be expressed in hundred share units to be consistent with the monthly trading activity. The GKPσ^2 values were calculated as illustrated using the appropriate share price data.

Table VII provides the details of the calculations used to generate the restricted stock discounts (DLOM) for CRN and GGNS employing the Bid-Ask Spread model and the Longstaff equation. The Bid-Ask spread model's percentage discounts were generated by exponentiating the estimated value of the dependent variable and then dividing by two to obtain the half-spread. The Longstaff discounts were generated by inserting the monthly volatility values for σ^2, with T equal to 24 months, to yield the values for the two terms in the equation. Then, the first two terms were added and one was subtracted from the result to derive the

Table VI. Data for Bid Ask Spread Model Application

Raw data extracted from Center for Research in Security Prices, CRSP, for Carrington Laboratories, CRN, and Genus Company, GGNS. Used to illustrate the application of Bid-Ask spread model and Longstaff metric.

		CRN	GGNS
Company Ticker		CRN	GGNS
Exchange		AMEX	OTC
Transaction Date		10/1/1992	2/1/1995
Holding Period: Months		24	24
Size			
Shares Outstanding in Thousands: Prior Month		6,744.000	12,813.000
Activity			
Trading Volume in hundreds: Prior Month		1,943.00	35,392.00
Share Prices			
O_t Opening Bid Price: First Day Transaction Month		\$11.875	\$7.750
C_t Closing Bid Price: First Day Transaction Month		\$12.000	\$8.125
C_{t-1} Closing Bid Price: Prior Day		\$11.125	\$7.750
H_t Monthly High: Prior Month		\$12.500	9.375
L_t Monthly Low: Prior Month		\$11.125	\$7.500
Average Price: Prior Month		\$11.813	\$8.438
Implied FMV discount		23.81%	18.31%
Market Capitalization MC in thousands \$: Prior Month		\$79,666.872	\$108,116.094
Trading Activity L: Prior Month		.0288	.2762
Monthly Volatility GKPσ^2 : First Day Transaction Month			
GKP$\sigma^2 = [\ln(O_t/C_{t-1})]^2 + .5[\ln(H_t/L_t)]^2 - .31[\ln(C_t/O_t)]^2$			
Term 1:	$[\ln(O_t/C_{t-1})]^2$	.0043	0
Term 2:	$.5[\ln(H_t/L_t)]^2$	.0068	.0249
Term 3:	$.31[\ln(C_t/O_t)]^2$	-.00003	-.0007
GKPσ^2: Monthly Variance		.0111	.0242
GKPσ: Annualized Monthly Standard Deviation		.3649	.5389

percentage discounts. These discounts were applied to the prior month's average share price for CRN and GGNS, (see Table VI) to calculate the discounted price for CRN's and GGNS's restricted stock.

Our discounted price for CRN's restricted stock using the Bid-Ask spread model was \$10.14. This was based on an average share price of \$11.813, and the estimated 14.14% discount.[25] The Longstaff discounted price for CRN's restricted stock was \$6.10 which was based on the average share price of \$11.813 and an estimated 48.28% discount.[26] FMV reported a discounted price of \$9.00, which implies a 23.81% discount from \$11.813, its previous month's average transaction price.

Our discounted price for GGNS's restricted stock was \$7.513 using the Bid-Ask spread model which was based on an average share price of \$8.438 and an estimated discount of 10.96%.[27] The Longstaff discounted price for GGNS's restricted stock was \$1.958; based upon the average share price of \$8.438, and the 76.79% discount.[28] The discounted price reported by FMV was \$6.8924, which implies a 18.31% discount from \$8.438, the average transaction price in the month prior to the transaction date.

While the Bid-Ask spread model produced better discounts than the Longstaff equation, the model's estimated discounts were slightly different than the reported FMV discounts.

[25] \$11.813 (1 - .1414) = \$10.14.

[26] \$11.813 (1 - .4828) = \$6.10.

[27] \$8.438 (1 - .1096) = \$7.513.

[28] \$8.438 (1 - .7679) = \$1.958.

Table VII. Restricted Stock Discount Estimates

Calculated via Bid Ask Spread Model and the Longstaff Estimating Equation.

Bid Ask Spread Model: $\alpha_0 + \alpha_1 \ln(MC) + \alpha_2 \ln(L) + \alpha_3 \ln(GKP\sigma^2)$

	α_0	α_1	α_2	α_3
	3.6909	-.3737	-.1632	.2921
CRN	3.6909	-.3737(ln($79,666.872))	-.1632(ln(.0288))	.2921(ln(.0111))
	3.6906	-4.2174	.5789	-1.3147
	-1.2626			
Model % Discount	14.14%			
Reported FMV Discount	23.81%			
GGNS	3.6909	-.3737(ln($108,116.094))	-.1632(ln(.2762))	.2921ln(.0242))
	3.6906	-4.3315	.2100	-1.0870
	-1.5176			
Model % Discount	10.96%			
Reported FMV Discount	18.31%			

Longstaff Estimating Equation: $$\left(2+\frac{\sigma^2 T}{2}\right)N\left(\frac{\sqrt{\sigma^2}T}{2}\right)+\sqrt{\frac{\sigma^2 T}{2\pi}}\exp\left(-\frac{\sigma^2 T}{8}\right)-1$$

		Term 1	Term 2
CRN			
Longstaff % Discount	48.28%	1.2837	.1991
Reported FMV Discount	23.81%		
GGNS			
Longstaff % Discount	76.79%	1.4852	.2827
Reported FMV Discount	18.31%		

We attribute part of this difference to the discrepancies we observed between the FMV and CRSP share and volume data. However, the differences between our discounted prices, and the observed discounted prices, were approximately one dollar per share, as compared to the Longstaff differences of approximately three and five dollars per share. Thus, we are confident that the model is useful for calculating discounts for lack of marketability on firms that exhibit differences in size, trading activity, and volatility. We believe that the predictive ability of the Bid-Ask model will improve with the incorporation of financial statement data like profitability and other relevant data like dividend yield.

VI. Summary and Conclusion

This paper's goal was to improve the methodology used to determine the discount for lack of marketability for closely held firms, DLOM. We focused on two metrics as proxies for the DLOM: the percentage Bid-Ask spread, and Longstaff's lookback option. We used Over-the-Counter Bulletin Board, OTCBB, Bid-Ask spread data for 1,850 firms that spanned the time period from December of 1995 until January of 2000, and employed accepted empirical models to estimate the Bid-Ask spread and thereby the DLOM.

The estimated DLOM using the Bid-Ask spread can range in value from 7% to 41% depending upon a firm's attributes. We calibrated the performance of the two metrics using actual restricted stock data and concluded that the Longstaff metric did not perform as well as the Bid-Ask spread based metric. We concluded that the percentage Bid-Ask spread is a better proxy than the Longstaff metric for estimating the DLOM of illiquid stock. We believe that the predictive ability of the Bid-Ask model will improve with the incorporation of financial statement data like profitability and other relevant data like dividend yield. Unfortunately such data were not available for our sample period and this is a limitation of this study. Since, both metrics can be estimated easily from firm data that are readily available, our results can be used to determine the DLOM for individual firms based on their size, volatility, and trading activity. Additionally, the spread based metric for DLOM is flexible enough to estimate a discount specific to an industry and a time period for which a valuation is being conducted using appropriately tailored spread data.■

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