Exhibit G (ii)

Current Controversies Regarding Option Pricing Models

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Option pricing models (OPMs) are increasingly used to estimate the discount for lack of marketability (DLOM) in the business valuation profession. Some analysts disagree about whether OPMs are applicable for estimating the DLOM. Since OPMs were originally derived to determine option prices for publicly traded securities, many analysts question the merits of applying them to closely held securities. This discussion explores the controversies of applying OPMs to estimate the DLOM for nonmarketable securities.

Introduction

Business valuations prepared for gift and estate purposes usually involve the valuation of a closely held company. That is, the subject company shares are nonmarketable. In these valuations, the valuation analyst typically first estimates the value of the company as if the underlying shares were marketable. Then, the analyst incorporates a discount for lack of marketability (DLOM) to reflect the fact that the underlying shares are nonmarketable.

According to the American Institute of Certified Public Accountants (AICPA) Statement on Standards for Valuation Services (SSVS), the discount for lack of marketability is defined as "an amount or percentage deducted from the value of an ownership interest to reflect the relative absence of marketability."[1]

For purposes of this discussion, the concept of marketability relates to how quickly and with what degree of certainty the investment can be converted into cash at the owner's discretion.

A very liquid security is an ownership interest of an actively traded stock. This security can typically be converted into cash within three business days of the sell decision. This is the typical investment benchmark for a fully marketable security. At the other end of the investment marketability spectrum is an ownership interest in a privately owned company that pays no dividends or other distributions, requires capital contributions, and limits ownership of the company to certain individuals. Of course, there exists a myriad of positions in between these two extremes in the investment marketability spectrum.

Since the 1980s, valuation analysts have used empirical DLOM studies to estimate the DLOM related to a noncontrolling interest in a closely held company. Such studies include (1) studies of price discounts on the sale of restricted shares of publicly traded companies (i.e., the restricted stock studies) and (2) studies of price discounts on private stock sale transactions prior to an initial public offering (i.e., the pre-IPO studies).

In 1993, David Chaffe introduced the concept of using an option pricing model (OPM) to estimate the DLOM. Chaffe wrote that by purchasing a put option to sell nonmarketable securities at the current stock price, the securities' owner has effectively purchased marketability for the shares. And, therefore, the cost of the put option represents the DLOM, with the put option cost divided by the stock price representing the percentage DLOM.

Since Chaffe first introduced the concept of using put options to estimate the DLOM, other option pricing models to estimate the DLOM have been created and promulgated. A large majority of the subsequent literature about OPMs, including new OPM studies and critiques, has focused more on the mechanics of using OPM studies to estimate the

DLOM than on critiquing whether or not a hedging strategy based on stock options is a legitimate way to estimate the DLOM for nonmarketable securities.

This discussion (1) introduces and summarizes commonly used OPMs to estimate the DLOM and (2) describes the current controversies in regard to using OPMs to estimate the DLOM.

Overview of Option Pricing Models

Chaffe European Put Option Model

As mentioned above, David Chaffe authored a 1993 DLOM option pricing study in which he related the cost to purchase a European put option to the DLOM.[2] Chaffe theorized that, "if one holds restricted or non-marketable stock and purchases an option to sell those shares at the free market price, the holder has, in effect, purchased marketability for those shares. The price of that put is the discount for lack of marketability."[3]

In other words, let's assume there are two securities and the only difference between them is that one is marketable with a freely traded price and the other is nonmarketable. Let's suppose someone tried to sell you the security that is not marketable, but also gave you the option to sell it back at its freely traded price at any time in the future.

Under these circumstances, the nonmarketable security is assumed to be equivalent to the fully marketable security. Therefore, the value of the option is equal to the DLOM, and the question becomes how should this hypothetical option be valued?

Chaffe relied on the Black-Scholes-Merton option pricing model (BSM model) to estimate the price of the option in his model. The inputs in the BSM model are:

1. Stock price
2. Strike price
3. Time to expiration
4. Interest rate
5. Volatility

In the Chaffe model, the stock price and strike price equal the marketable value of the private company stock as of the valuation date; the time to expiration equals the time the securities are expected to remain nonmarketable; the interest rate is the cost of capital; and, volatility is a judgmental factor that is often estimated by reference to the volatility of guideline publicly traded stocks.

According to Chaffe, volatility for small privately owned companies is likely to be 60 percent or greater. Chaffe reached this conclusion based on the volatility for small public companies that are traded in the over-the-counter market.

According to the Chaffe study, the appropriate DLOM for a privately held stock with a two-year required holding period and volatility between 60 percent and 90 percent is between 28 percent and 41 percent.

According to Chaffe, "considering that volatility for shares of most smaller, privately held companies fit the 'VOL 60%-70%-80%-90%' curves, a range of put prices of approximately 28% to 41% of the marketable price is shown at the two-year intercept. At the four-year intercept, these ranges are 32% to 49%, after which time increases do not substantially change the put price."

Chaffe noted that his findings are downward biased due to the reliance on European options in his model. Therefore, Chaffe concluded that his findings should be viewed as a minimum applicable DLOM. Exhibit 1 presents representative DLOMs based on the Chaffe model.

According to the Chaffe model, the implied DLOM is between 14.5 percent and 70.4 percent for stocks with (1) volatility between 25 percent and 125 percent and (2) holding periods between 1 year and 4 years, as shown in Exhibit 1.

Although this is a large range for these DLOMs, the results are consistent with other DLOM studies. In order to analyze the reasonableness of the Chaffe model output, this discussion compares the implied DLOM under various scenarios to the results of the FMV Opinions DLOM Study.[5] These results are presented in Exhibit 2.

The FMV Opinions Study breaks down the DLOM by quintile and shows various financial data associated with each quintile. As shown in Exhibit 2, the DLOM increases as volatility increases. According to the Chaffe model, under a one-year holding period

Exhibit 1
Implied DLOM Based on the Chaffe Black-Scholes-Merton Put Option Model

Chaffe Black Scholes Put Option Model

	Volatility				
Holding Period (Years)	σ=25%	σ=50%	σ=75%	σ=100%	σ=125%
1	14.5%	18.3%	27.7%	36.6%	45.0%
2	19.4%	24.6%	37.0%	48.4%	58.4%
3	22.7%	28.8%	43.1%	55.6%	65.9%
4	25.1%	31.9%	47.4%	60.3%	70.4%

and assuming 75 percent volatility, the implied DLOM is 27.7 percent. This DLOM is similar to the fourth quintile DLOM of 27.9 percent in the FMV Opinions study, which reports 80 percent volatility.

In general, it appears from Exhibit 1 and Exhibit 2 that the DLOMs reported in the Chaffe model are reasonable for moderate volatilities, but at higher volatilities the Chaffe model generates high DLOMs.

Longstaff Model

Francis A. Longstaff authored a study that relies on stock options to estimate the DLOM for a private company.[6] Whereas Chaffe based his study on avoiding losses, Longstaff based his study on unrealized gains. Another difference is that the Longstaff study allegedly provides an estimate for the upper limit on the value for marketability.

The Longstaff study is based on the price of a hypothetical "lookback" option. A "lookback" option differs from most other options in that the holder can look back at the end of the option's life and retroactively exercise the option at either the lowest stock price (for a call option) during the holding period or the highest stock price (for a put option) during the holding period.

The Longstaff study assumes an investor has a single-security portfolio, perfect market timing, and trading restrictions that prevent the security from being sold at the optimal time. The value of marketability, based on these assumptions, is the payoff from an option on the maximum value of the security, where the strike price of the option is stochastic.

Longstaff analyzed securities with volatility between 10 percent and 30 percent because, "[t]his range of volatility is consistent with typical stock return volatilities."[7] However, small stocks (such as those traded over-the-counter and analyzed by Chaffe) typically have greater volatility, all else equal.

When comparing the Longstaff model results (Exhibit 3) to the FMV Opinions Study results (Exhibit 2), the Longstaff model reports DLOMs that (1) are far greater than the observed discounts from restricted stock transactions and (2) exceed 100 percent at reasonable levels of volatility.

Transformed Longstaff Discount

The Longstaff model DLOMs exceed 100 percent under certain assumptions—an illogical conclusion.

Exhibit 2
FMV Opinions DLOM Study
Results by Quintile

FMV Opinions Study: Discount for Lack of Marketability [a]
Restricted Stock Transactions

Quintile	1	2	3	4	5
Percentage Discount	1.6%	10.5%	16.7%	27.3%	43.7%
Volatility	68.0%	68.0%	72.0%	80.0%	104.0%
Market Value ($000)	166,358	162,682	110,814	68,824	44,309
Market-to-Book Ratio	3.5	3.9	3.7	5.8	6.0
Total Assets ($000)	66,025	65,752	39,844	18,245	9,434
Revenue ($000)	23,335	29,136	28,811	14,118	7,764
Price Per Share	$12.49	$11.07	$12.02	$8.15	$8.17

[a] Lance S. Hall, "Responding to The IRS DLOM Job Aid," Business Valuation Resources, LLC Webinar, October 12, 2011, slide 42.

However, there is disagreement about whether the Longstaff model, and OPMs in general, conclude a DLOM or a liquidity premium (which needs to be converted to a discount).

Exhibit 3
Longstaff Lookback Put Option Model

Longstaff Lookback Put Option Model

	Volatility				
Holding Period (Years)	σ=25%	σ=50%	σ=75%	σ=100%	σ=125%
1	21.6%	46.6%	75.3%	108.1%	145.2%
2	31.5%	70.1%	116.7%	172.0%	236.9%
3	39.5%	90.0%	153.0%	229.9%	321.9%
4	46.6%	108.1%	186.9%	284.9%	404.0%

Ashok Abbot believes that the Longstaff model results in a premium and not a discount. Abbot suggests the following transformation of the Longstaff model in order to convert the Longstaff model results into a DLOM:

$$DLOM = \frac{\text{Longstaff Discount}}{1 + \text{Longstaff Discount}}$$

After applying this formula, the transformed Longstaff model returns lower DLOMs, as presented in Exhibit 4.

Exhibit 4
Transformed Longstaff Lookback Put Option Model

Transformed Longstaff Lookback Put Option Model

	Volatility				
Holding Period (Years)	σ=25%	σ=50%	σ=75%	σ=100%	σ=125%
1	17.7%	31.8%	43.0%	51.9%	59.2%
2	23.9%	41.2%	53.9%	63.2%	70.3%
3	28.3%	47.4%	60.5%	69.7%	76.3%
4	31.8%	51.9%	65.1%	74.0%	80.2%

Compared to the FMV Opinions Study, the DLOMs under the transformed Longstaff model remain relatively high. For example, a one-year holding period with 75 percent volatility returns a DLOM of 43 percent from the transformed Longstaff model, which is much higher than the FMV Opinion Study's third and fourth quintiles which use 72 percent volatility and 80 percent volatility, respectively.

The DLOMs from the transformed Longstaff model appear more reasonable than the DLOMs that result from the Longstaff model prior to the transformation. The issue of whether the OPM studies conclude a discount or a premium is explored in greater detail later in this discussion.

Finnerty Model

John D. Finnerty conducted an option-pricing study that "tests the relative importance of transfer restrictions on the one hand and information and equity ownership concentration effects on the other in explaining private placement discounts."[8]

The Finnerty option-pricing study is an extension of the Longstaff study. However, unlike Longstaff, Finnerty did not assume that investors have perfect market timing ability. Instead, Finnerty modeled the DLOM as the value of an average strike put option.

As shown in Exhibit 5, the Finnerty model generates DLOMs that are relatively close to the average DLOMs reported in the FMV Opinions study. Assuming 75 percent volatility and a one-year holding period, the Finnerty model returns a DLOM of 16.3 percent. The FMV opinions study shows a DLOM of 16.7 percent using 72 percent volatility.

It appears the Finnerty model works reasonably well at lower volatilities, but yields low DLOMs at higher volatilities when compared to the restricted stock transactions presented in Exhibit 2.

Some OPMs are suitable at certain levels of volatility and produce results that appear reasonable. However, no OPM appears to line up well with the restricted stock transactions at all levels of volatility.

Exhibit 5
Finnerty Average Strike Price Put Option Model

Finnerty Average Strike Put Option Model

	Volatility				
Holding Period (Years)	σ=25%	σ=50%	σ=75%	σ=100%	σ=125%
1	5.7%	11.2%	16.3%	20.9%	24.6%
2	8.0%	15.5%	21.8%	26.6%	29.7%
3	9.8%	18.5%	25.3%	29.5%	31.5%
4	11.2%	20.9%	27.5%	31.0%	32.1%

Long-Term Equity Anticipation Securities (LEAPS) Studies

In September 2003, Robert Trout published a LEAPS study that analyzed LEAPS and marketability discounts.[9] Ronald Seaman updated the Trout LEAPS study in March 2010.[10] Each of these LEAPS studies were conducted using a similar research logic and research design.

A long-term equity anticipation security is essentially a long-term stock option that offers price protection for up to two years into the future. Therefore, an investor who desires protection against stock price declines can purchase a LEAPS put option.

The LEAPS studies examined the cost of buying LEAPS put options and concluded that the cost of the LEAPS put option divided by the stock price is the DLOM.

The authors of the LEAPS studies concluded that the observed DLOMs are appropriately viewed as benchmark minimum price discounts when applied to privately held companies. This is because (1) the underlying securities on which the LEAPS are based are often much larger than the privately held subject company, (2) the underlying securities on which the LEAPS are based are marketable, (3) the LEAPS themselves can be sold at any time during the holding period, and (4) there is a known liquidity event (i.e., the sale of the underlying security) for the LEAPS.

How Common Are OPMs?

It is difficult to determine to what extent OPMs are currently used in the valuation profession. Valuation Products and Services, LLC, presented a webinar on the DLOM. During that webinar, a poll was given that asked participants how they determined the DLOM.[11]

According to the poll, approximately 85 percent of participants used restricted stock benchmark data and 53 percent used IPO benchmark data to help determine a DLOM as of September 2011. Similarly, participants were asked how many people use LEAPS,[12] the Chaffe model, the Longstaff model, or the quantitative marketability discount model (QMDM).[13]

Fourteen percent of participants said they used LEAPS, 8 percent said they used the Chaffe model, 9 percent said they used the Longstaff model, and 25 percent said they used the QMDM. Sixty-one percent said they used none of the above models. Therefore, according to the webinar poll, a meaningful number of listeners were using an option pricing model to help determine a DLOM.[14]

In addition, according to Robert Duffy, most Big 4 accounting firms incorporate the Finnerty put option model into their DLOM analyses.

Current Controversies Regarding Option Pricing Models

There are some controversies regarding the use of OPM studies to estimate the DLOM for nonmarketable securities, including the following:

- Stock options do not exist for nonmarketable securities.
- The cost of put options may understate the DLOM.
- The cost of put options may overstate the DLOM.
- The cost of put options may be unrelated to the marketability of closely held company stock.
- DLOMs from OPM studies may result in a premium, and not a discount.

These five controversies are discussed below.

Options Are Not Available for Owners of Nonmarketable Stock

Given that there is no available option market for closely held shares, an owner of nonmarketable stock cannot purchase a put option to sell his or her shares at a later date. Therefore, an investor cannot really purchase liquidity for his or her interest in nonmarketable stock the way Chaffe theorized in 1993. And, since this strategy is not practically possible, many valuation analysts dismiss the use of OPMs to measure the DLOM for nonmarketable stock.

In addition to being unavailable in the market, an option-based strategy will be unavailable to the owner of closely held company stock if he or she may be contractually restricted from selling the stock. Operating agreements, partnership agreements, or stock transfer agreements often contain provisions that serve to restrict the marketability of the underlying shares of stock. When this is the case, even if the owner of a nonmarketable stock could purchase a put option to sell his or her shares at a later date, provisions in the shareholder agreements might prevent that exercise of the put option.

Issues related to the inability of nonmarketable stock holders to use options to purchase liquidity are criticisms with using OPMs to estimate the DLOM.

Therefore, in order to apply OPMs to closely held shares, two primary assumptions are needed. First, closely held shares combined with a put option are equivalent to marketable securities. Second, the price of the put option measures the DLOM.

While some may question the validity of these assumptions, OPMs may still provide a useful way of thinking conceptually about the DLOM for privately held shares. Just because a theory may initially appear implausible does not mean it is not useful or cannot provide further insight to the issue at hand. There are examples in other fields where some assumptions are used that seem unrealistic or oversimplified in order to achieve a better understanding of a given topic.

For example, some macroeconomic models, such as dynamic stochastic general equilibrium (DSGE) models, use simplified microeconomic assumptions to forecast economic growth, business cycles, and the effects of monetary and fiscal policy.

By analyzing the interaction of agents making microeconomic decisions, the models attempt to better forecast macroeconomic variables. The assumptions are relatively simplified, yet DSGE models have been improved upon and have allowed economists to think about how the economy will evolve over time.

Some economic theories of consumption also use oversimplified assumptions yet offer insight into spending patterns. An example is the permanent income hypothesis where a person's spending reflects both permanent and transitory income. According to the theory, the average propensity to consume depends on the ratio of permanent income to current income.

When current income temporarily rises above permanent income, the average propensity to consume temporarily falls. When current income temporarily falls below permanent income, the average propensity to consume temporarily rises. This model uses very simple assumptions, but over long periods of time one should observe a relatively constant average propensity to consume, which is what the model suggests and the data support.

While these model assumptions are oversimplified approximations of consumer behavior, they still make useful contributions. The general acceptance of these models—in spite of their oversimplified assumptions—suggests that it may be premature to dismiss a model just because the model's assumptions appear simple or unrealistic.

In the case of using OPMs to value a DLOM, the fact that there exists no option market for private shares does not necessarily mean OPMs should not be used. Rather, they may still provide insight or an approximation that is useful.

The Cost of Put Options May Understate the DLOM

It is an interesting theoretical exercise to consider what a put option on nonmarketable shares would look like if it could be underwritten. Chances are it would be much more expensive than a put option on similar marketable shares.

First, the analyst may consider who would underwrite such an option. The potential underwriter would have to be willing and able to take an ownership interest in the underlying securities. Yet the potential underwriter cannot be so interested in owning the underlying securities that he or she would write a put option on the securities rather than simply buying the stock outright. These circumstances greatly reduce the number of potential underwriters for put options on nonmarketable stock.

Let's assume an underwriter was identified. Let's also assume that the subject interest—a 10 percent interest—was recently valued on a marketable basis at $100 and on a nonmarketable basis at $80, and that the goal of the put option was primarily to gain liquidity in two years. Therefore, the security owner would want a put option with a two-year term and a strike price that was close to the current market price of the stock (i.e., $100).

The put option underwriter is probably concerned with one additional type of risk that does not affect put options on marketable securities: the underlying securities are not marketable, but the OPM price is based on marketable securities. Therefore, the underwriter will need to be compensated for the fact that the securities are nonmarketable by incorporating either a lower strike price or a higher premium in the option price.

If the put option price is based on the nonmarketable stock price, and the underwriter agrees with the selected DLOM, then this risk is mitigated.

However, the OPM studies assume that the option is written based on the marketable stock price. If the underwriter writes the option with a strike price equal to the marketable stock price, then the underwriter will likely charge a premium over the formula-derived option price to reflect the fact that the underlying security is not marketable. The OPM studies assume the option price is equal to the price derived from the OPM (i.e., no premium is added).

One way or another, the underwriter will expect to be compensated for the fact that the underlying securities are not marketable. Since OPMs don't compensate the option writer for marketability-related risk, this risk will have to be accounted for another way. Note that this compensation is above and beyond the price that results from a theoretical option pricing model.

Based on the scenario just described, OPM studies may understate the DLOM. This is because the put option on nonmarketable shares does not include a component in the price to provide liquidity to nonmarketable shares. That is, the theoretical price of a put option on nonmarketable shares (the cost to gain liquidity) may be greater than a put option on otherwise similar marketable shares (which already have liquidity). However, some analysts take an opposite view—that the cost of put option overstates the DLOM.

This point is addressed next.

The Cost of the Put Option May Overstate the DLOM

We previously illustrated why some analysts believe the cost of the put option understates the DLOM. If a put option was written on a nonmarketable security, the option underwriter would likely charge a premium relative to the price indicated by an OPM in order to account for the lack of marketability inherent in the underlining security. Based on this argument, the put option cost from an OPM understates the DLOM.

In 2009, Internal Revenue Service analyst Harry Fuhrman critiqued the LEAPS studies (a type of OPM study) for *overstating* the DLOM. Fuhrman reasoned that (1) OPMs estimate the DLOM as the cost to lock-in a price for the subject security and (2) purchasing a put option overstates the cost to lock-in a security's price (the price is not locked in since the investor can still benefit from any appreciation in the price of the stock).

According to Fuhrman, "To 'lock-in' a security's price today, an investor would undertake two courses of action: 1) purchase a put option to protect against any downside risk . . . or 2) have the ability

to sell a call option related to any upside potential in the stock."[15]

By netting the cost to acquire the put option with the income from selling a call option, the investor has eliminated both the downside and upside related to changes in the price of the underlying security. His or her return is certain.

Fuhrman's estimated DLOM using this procedure is calculated as the cost of the put option minus the income from the call option, divided by the stock price.

In the example provided in his critique of the LEAPS studies, Fuhrman illustrates how the DLOM estimated based only on the cost of the put option would range from 20 percent to 28 percent using AT&T put options in January 2009 and would be between 4 percent and 8 percent if the cost of the put option was netted against the income from a call option.

In a response to Fuhrman's critique, Seaman notes that a DLOM "does not attempt to 'lock-in a security's price today,' which Fuhrman states as the objective in his example. A DLOM simply attempts to measure the investor's risk, a major part of which is the risk of loss in value over time. That is precisely the risk measured in an analysis using LEAPS put options."[16]

Fuhrman is not the only one to critique the OPM studies for producing discounts that are too large. In an April/May 2013 article, Jay Fishman and Lester Barenbaum wrote that, "it is our view that use of the cost of put options overstate [sic] the discount for lack of marketability."[17]

Fishman and Barenbaum suggested that a better way to use options to estimate the DLOM would be the cost of a prepaid variable forward (PVF) contract, which involves buying a put option, selling a call option, and borrowing money.

Since the theory of using a prepaid variable forward contract is relatively new, it is unclear how this method to estimate the DLOM will be received by practitioners, courts, and academics. However, the DLOM using prepaid variable forward contracts may be similar to a company's cost of debt.

As noted in the Fishman and Barenbaum article, a PVF contract is "a constructive sale that fully monetizes an asset position with *borrowing cost representing the discount*"[18] [emphasis added]. This result seems unreasonably low.

The Cost of a Put Option May Not Reflect Marketability

A prior section discussed the fact that an underwriter of a put option on nonmarketable stock may charge a premium based on the lack of marketability in the underlying securities. This suggests that option pricing models measure factors other than liquidity.

A distinction between put options and marketability is that a put option becomes more expensive (valuable) as the time to option expiration increases while the nonmarketable stock become less valuable as the expected holding period increases. So, although both options and the DLOM are affected by time horizon and volatility, they are affected in different ways. In this way, stock options aren't analogous to the DLOM; they are more like opposite sides of the same coin.

We next examine the relationship between the cost of a put option and the magnitude of the DLOM by comparing the factors that affect each. This relationship is analyzed to determine if/how the cost of a put option is related to marketability.

Factors that Affect the Option Price

An option provides the holder with the right to buy or sell a specified quantity of an underlying asset at a fixed price at or before the date of expiration. Since it is a right and not an obligation, the holder can choose to not exercise the right and allow the option to expire.

Stock options are bought and sold by three types of investors: (1) hedgers, (2) speculators, and (3) arbitrageurs. Hedgers use options to protect themselves against negative price movements (i.e., an investor that owns shares in Apple, Inc., common stock may want to protect against a near-term decline in the stock price).

Speculators use options to make bets about future price movements (i.e., an investor that wants to bet that shares of Apple common stock will increase can magnify his/her losses and gains using options compared to owning the stock outright).

Arbitrageurs use options to lock in a known profit by taking multiple positions in the stock, options, and futures markets. Arbitrageurs are important to the financial markets because they help establish the prices of stock options. Note that none of the investors that use stock options do so to gain liquidity. Stock options are purchased and sold for reasons completely unrelated to liquidity.

To illustrate how arbitrageurs establish option prices, let's consider a simple portfolio that is (1) long one stock traded at $10/share and (2) short in two call options with a strike price of $11/share. Let's assume there are two possible outcomes for the price of the stock in three months: either $9 or $11.

If the stock ends up at $11/share, then the portfolio is worth $9/share ($11/share × 1 share – 2 options). If the stock ends up at $9/share, then the portfolio is also worth $9 ($9/share × 1 share).

Regardless of what happens to the stock price, the value of the portfolio will always be $9/share. Since a riskless portfolio earns the risk-free rate, one can derive the option price based on (1) the risk-free rate (let's assume it is 3 percent) and (2) the facts in the example above. The present value of the portfolio, based on the above, is $8.93 (i.e., the present value of $9/share).

One can also determine the price of the call option in this scenario. Based on the information outlined above, the price of the call option such that no arbitrage opportunities exist is $1.07.

In this example, if the price of the call option was greater than $1.07, then the portfolio would cost less than $8.93 to set up and would earn more than the risk-free rate of return.

If the price of the call option cost less than $1.07, then shorting the portfolio would provide a way of borrowing money at less than the risk-free rate of return. If the price of the call option was anything other than $1.07, arbitrageurs would invest in such a way to earn a riskless profit and, eventually, supply and demand factors should force the call option price back to $1.07.

The above example provides a simplified method to estimate the price of a call option. In that example, there were only two possible outcomes for the price of the stock and each possibility was just as likely to occur. A more complex and commonly used method to value stock options is the BSM model.

The BSM model is analogous to the no-arbitrage example discussed above. However, in the BSM model, the position that is set up is riskless only for an instantaneously short period of time.

The value of an option using the BSM model is determined primarily by the six variables listed below.[19]

1. *Current value of the underlying asset*: Options derive their value from an underlying asset.
2. *Change in value of the underlying asset (i.e., volatility)*: The buyer of an option has the right to buy or sell the underlying asset at a fixed price. The higher the variance in the expected value of the underlying asset, the greater the value of the option.
3. *Dividends paid on the underlying asset*: The value of the asset can be expected to decrease if dividends are paid on the asset during the life of the option. The value of a call on the asset decreases with the size of the expected dividend payments. The value of a put increases with expected dividend payments due to the cost of delaying exercising options that are in-the-money.[20]
4. *Strike price of option*: The value of a call will decline as the strike price increases and the value of a put will increase as the strike price increases.
5. *Time to expiration on option*: Both calls and puts are more valuable as the time to expiration increases. The long time to expiration provides more time for the value of the underlying asset to change, which increases the value of both types of options.
6. *Risk-free interest rate*: Since a buyer of an option must pay the price up front, purchasing options involves an opportunity cost which depends on the level of interest rates and the time to expiration. The risk-free interest rate also is part of calculating the present value of the exercise price. Increases in the interest rate will increase the value of calls and reduce the value of puts.

Every variable above except volatility is directly observable in the market. When pricing a stock option, the volatility variable in the BSM model can be estimated one of two ways. It can be estimated based on the historical stock price of the underlying security. Or, it can be estimated by calculating the implied volatility from an actively traded stock option on a similar security.

Changing the volatility assumption in the BSM model leads to significant movements in the price of the stock option. It may be appropriate to view the BSM model as a procedure to estimate volatility rather than a model to estimate the price of a stock option. This may be true if (1) the market price of the stock option is determined based on a no-arbitrage condition and not an empirical financial model (that is, supply/demand of arbitrageurs determine the option price) and (2) every variable in the BSM model except volatility is known.

In this scenario, (1) the analyst can solve for volatility and (2) the BSM model is more useful to estimate the market's opinion about the volatility of a stock instead of to determine the price of an option.

The above examples provide a basic theoretical and mathematical understanding of how options are priced. The analyst should understand how a stock option price is determined if he or she will use the option price as a proxy for the DLOM.

Factors That Affect the DLOM

When looking at the relationship between option prices and DLOMs, what are the common elements to suggest an option price is related to the DLOM? When comparing the six option pricing variables listed above to the factors that affect the DLOM, there appear to be some common elements.

In *Mandelbaum v. Commissioner*,[21] the Tax Court listed nine factors to consider when determining the DLOM:

1. Financial statement analysis
2. Dividend policy
3. Nature of the company, its history, its position in the industry, and its economic outlook
4. Management
5. Amount of control in the transferred shares
6. Restrictions on transferability
7. Holding period of the stock
8. Company redemption policy
9. Costs associated with a public offering

For purposes of this discussion let's assume that these nine Mandelbaum factors will affect the magnitude of the DLOM and that no other factor has a material impact on the magnitude of the DLOM.

Some of the factors that affect the magnitude of the DLOM are similar to the variables that affect the price of stock option. For example, both are affected by (1) holding period, (2) dividends, and (3) volatility of the underlying securities. In the BSM model, these three factors are considered directly. Holding period is considered in the DLOM in Mandelbaum factors 6 through 9 (based on the numbering in the above list); dividends are considered in factor 2; and volatility of the underlying securities is considered in factors 1, 3, and 4.

When comparing the variables that affect the price of stock options and the magnitude of the DLOM, it is apparent that there are similarities. This perhaps suggests that OPMs have some relevance for determining DLOMs.

Do OPMs Result in a Price Premium or a Price Discount?

Some analysts theorize that the percentage result from the OPM studies is actually a price premium, and not a price discount. Ashok Abbott wrote that, "Often, however, the value of a put option premium, estimating the cost of liquidity, is presented incorrectly as the discount for lack of liquidity. This is similar to the merger premium being treated as a discount for lack of control. Neglecting to convert the option premium to the applicable discount creates the illusion that the estimated discounts are greater than 100%, an impossible solution."[22]

Margin Greene concurred with this sentiment, saying, "Frequently, appraisers compute the option and assume their result is a discount. In reality, the models produce a premium, which must then be converted to a discount."[23]

There is not universal agreement about whether the OPM studies produce a premium or a discount. Therefore, practitioners who rely on these studies should choose how to use the studies to estimate the DLOM.

Conclusion

There are many issues surrounding the use and applicability of OPMs in determining DLOMs. In some instances, the OPM output can be relatively similar to the ranges found in both restricted stock studies and IPO studies. However, the reasonableness of the output does not necessarily imply the OPM is measuring a DLOM.

Some of the OPMs work relatively well when certain holding periods and volatilities are used as inputs. What needs to be kept in mind is whether output that appears reasonable may be capturing something unrelated to marketability issues. This is because OPMs theoretically have little to do with marketability of the underlying security.

Instead, option prices are determined based on the concept of a no arbitrage condition. The lack of arbitrage opportunities in the market ensures that investors cannot earn more than the risk-free rate of return using a combination of leverage, stock positions, and option positions.

Another issue with using OPMs to estimate the DLOM is that in OPMs, the underlying security is assumed to be marketable. Therefore, some analysts challenge the applicability of OPMs to nonmarketable securities.

A better analogy to the DLOM for private company stock may be from the pre-IPO studies and the restricted stock studies. In both cases, the value of nonmarketable stock is compared to the value of marketable stock. Therefore, the difference in value from the two ownership interests in the pre-IPO studies and restricted stock studies is exclusively related to marketability.

In spite of the differences between how option prices are determined and the factors that affect marketability, some analysts assert that the option price is a useful proxy for the DLOM. In the context of option pricing, the support for this opinion is that both the option price and the DLOM are affected by the following:

1. Volatility of the underlying security
2. Time horizon

Since OPMs incorporate the relationship between volatility and time horizon, they may provide insight in determining an appropriate DLOM.

Notes:

1. *Statement on Standards for Valuation Services* (SSVS1) (New York: AICPA, 2007): 43.
2. "European" options have a single exercise date. In contrast, the holder of an "American" option can exercise the option at any time during the existence of the option.
3. David B.H. Chaffe III, "Option Pricing as a Proxy for Discount for Lack of Marketability in Private Company Valuations," *Business Valuation Review* (December 1993): 182–6.
4. Ibid.: 184.
5. Lance S. Hall, "Responding to the IRS DLOM Job Aid," Business Valuation Resources, LLC Webinar (October 12, 2011): slide 42.
6. Francis A. Longstaff, "How Much Can Marketability Affect Security Values?" *The Journal of Finance* (December 1995): 1767–74.
7. Ibid.: 1771.
8. John D. Finnerty, "The Impact of Transfer Restrictions on Stock Prices." *Analysis Group/Economics* (October 2002).
9. Robert R. Trout, "Minimum Marketability Discounts," *Business Valuation Review* (September 2003): 124–6.
10. Ronald M. Seaman, "Minimum Marketability Discounts—5th Edition" (March 2010), http://www.dlom-info.com/pdf/Full_Report_2009_Study.pdf
11. James Hitchner and Michael Gregory, "Navigating the IRS and the IRS DLOM Job/Practice Aid," ASA Advanced Business Valuation Conference Presentation (October 8, 2012).
12. LEAPS are identical in all respects to short-term options except they have a longer expiration date.
13. The QMDM uses the discounted cash flow method to determine an appropriate DLOM.
14. See Robert Duffy,"Why Finnerty's Put Option Model Is the DLOM Model of Choice," *Financial Valuation Litigation Expert* (August/September 2011): 40.
15. *BVWire* #76-4 (January 28, 2009).
16. Ibid.
17. Jay E. Fishman and Lester Barenbaum, "Do Put Option Models Overstate Discounts for Lack of Marketability?" *Financial Valuation and Litigation Expert* (April/May 2013): 9.
18. Ibid, 11.
19. Aswath Damodaran, "The Promise and Peril of Real Options," Stern School of Business, New York, New York, http://pages.stern.nyu.edu/~adamodar/
20. This is because once a call option is in the money and the holder exercises the option the holder receives the stock and dividends in subsequent periods.
21. Mandelbaum v. Commissioner, T.C. Memo 1995-255 (June 13, 1995).
22. Ashok Abbott, "Discounts for Lack of Liquidity: Understanding and Interpreting Option Models." *Business Valuation Review* 28, No. 3 (Fall 2009): 145.
23. Martin Green, "Do Maximum Strike Price Lookback (Longstaff) and Other Put Option Models Produce a Marketability Premium or a Discount?" *Business Valuation Update* (October 2010): 26.

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