The Impact of Transfer Restrictions on Stock Prices

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Abstract

This paper investigates the factors responsible for the discount when unregistered shares of common stock are privately placed based on a sample of 101 private placements of letter stock. The discount depends in part on the stock's volatility, the length of the restriction period, the riskless rate, and the stock's dividend yield, which are consistent with the traditional view that Rule 144 transfer restrictions are responsible for the discount. But it also reflects the information and equity ownership concentration effects that accompany private stock placements, which are cited as the main reasons for the discount in more recent studies. Thus, both sets of factors are important. I also show that the appropriate discount is smaller for relatively low-volatility stocks than the range of marketability discounts investment bankers and stock appraisers usually assume.

The Impact of Transfer Restrictions on Stock Prices

The impact of transfer and other marketability restrictions on stock prices continues to be of both theoretical and practical interest. Prior studies have documented that letter stock, which is not freely transferable because of resale restrictions imposed by Rule 144 under the Securities Act of 1933, is valued at a significant discount to otherwise identical unrestricted stock.[1] The seminal Securities and Exchange Commission (SEC) (1971) study covering 398 letter stock transactions reported an average discount of 25.8 percent.[2] More recently, Wruck (1989) reports an average private placement discount of 13.5 percent; Silber (1991) finds discounts of up to 84 percent and an average discount of 33.75 percent; and Hertzel and Smith (1993) report an average private placement discount of 20.14 percent and a discount attributable to lack of registration amounting to 13.5 percent. The conventional wisdom in the business appraisal field is that the appropriate marketability discount is between 25 and 35 percent for a 2-year restriction period (see Longstaff 1995)). A recent paper by Longstaff (1995) obtains an upper bound on the marketability discount consistent with this range by modeling the value of marketability as the price of a lookback put option.[3] Similarly, Amihud and Mendelson (1991) and Kamara (1994) find that the yields on illiquid Treasury notes average more than 35 basis points higher than the yields on liquid Treasury bills with the same remaining maturity. Boudoukh and Whitelaw (1991) show that the yield spread between the designated benchmark Japanese Government bonds and less liquid non-benchmark Japanese Government bonds of like maturity averages more than 50 basis points. Brenner, Eldor, and Hauser (2001) find that over-the-counter currency options sell for about 21 percent less than similar exchange-traded options.

The entire discount on restricted shares at the time of issue is commonly referred to as a "marketability discount." However, Hertzel and Smith (1993) point out that the private placement discount arises from several factors, of which the Rule 144 restriction on resale is only one and document the significant information and equity ownership concentration effects that accompany a private stock placement. A private placement discount compensates private investors for their due diligence and monitoring costs, reflects the ownership-structure-change effects that result from the direct sale of common stock to a small group of sophisticated investors, and embodies the implied certification effect of the private placement announcement. Discounts of the magnitude observed in private placements of letter stock (averaging 20 percent in their study and as much as 42 percent in others) would provide powerful incentives at the time of the private placement for firms to commit to register the shares promptly following the private sale if they were due solely to the Rule 144 restrictions. Recognizing that there are many large financial institutions with long-dated liabilities, such as life insurance companies and pension funds, that may be less concerned about liquidity than other investors, Hertzel and Smith question whether investors require such large discounts just for agreeing not to resell their shares in the public securities market for two years. A private resale market exists, and the options market provides hedging opportunities. In their cross-sectional analysis, Hertzel and Smith allow for the information and equity ownership concentration effects that accompany a private placement and find that the discount attributable to lack of registration is only 13.5 percent. Hence, transfer restrictions may be far less important than traditionally believed. However, Longstaff (2001) solves the investor's intertemporal portfolio choice problem for an investor who is restricted to trading strategies of bounded variation and obtains values for the shadow cost of illiquidity that are consistent with the apparently large discounts for lack of free transferability that have been measured empirically. His model demonstrates that such large discounts are sustainable in a rational model of investor behavior. This paper tests the relative importance of transfer restrictions on the one hand and information and equity ownership concentration effects on the other in explaining private placement discounts.

The paper is organized as follows. Section I explains the restrictions on hedging that make the Rule 144 transfer restrictions costly. Section II models the transferability discount as the value of an average strike put option and compares the discounts from the model to those from Longstaff's model. Section III describes the sample. Section IV shows that the initial private placement announcement conveys more useful information than the announcement of the terms of the offering, which is consistent with the existence of significant information effects. Section V tests the model's ability to explain the discounts in the private placement sample and the relative importance of transfer restriction and information/ownership concentration effects. Section VI furnishes evidence that actual transferability discounts overstate (understate) the appropriate discounts for low-volatility (high-volatility) stocks. Section VII concludes.

I. Restrictions on Hedging

The persistently large average private placement discounts found in empirical studies over more than 30 years raise an intriguing question: Why don't arbitrageurs purchase restricted shares, hedge their price risk exposure, and capture the discount net of hedging costs as their profit? Such a strategy might not eliminate the transferability discount but it would reduce it to the cost of the hedge plus a return on the arbitrageur's capital.

A stockholder can hedge its price risk exposure in any one of at least three ways: (1) at the time it buys the restricted shares, it could purchase an average strike put option on an equal number of shares with an exercise price equal to the arithmetic average of the forward prices of the unrestricted shares and a time to expiration that matches the restriction period;[4] (2) each day during the resale-restriction period, it could sell short against the box an equal fraction of the block of restricted shares it purchased; or (3) each day it could sell equity swaps covering an equal fraction of the block of restricted shares. The arbitrageur could exercise the put option or cover the short sales after the resale-restriction period expires using the previously restricted shares.[5]

There are legal impediments to the first two hedging strategies for capturing the private placement discount. First, Rule 144 bars purchasing a put option or selling short against the box to hedge the price risk exposure in holding unregistered shares (see Federal securities law reports (1980), Hicks (1998), and Securities and Exchange Commission (1997b)). Specifically, Rule 144 prohibits an option holder who purchased the option before the Rule 144 resale-restriction period expires from exercising the put option after the resale-restriction period terminates and delivering the previously restricted shares to settle the option transaction.[6] In addition, a short seller is prohibited from covering a short position entered into during the resale-restriction period – even when the short seller borrows registered shares to effect the short sale – by delivering the Rule 144 shares; this prohibition applies even if the short covering were to occur after the resale-restriction period expires (see Federal securities law reports (1980), Hicks (1998), and Securities and Exchange Commission (1997b)). In the SEC's view, both hedging strategies are tantamount to the resale of unregistered shares during the resale-restriction period.

The third strategy, equity swaps, faces market as well as potential legal impediments. The equity swap market is relatively new and is generally limited to large-capitalization stocks. Table 2 contains a summary description of a sample of private placements of letter stock that took place between January 1, 1991 and February 3, 1997. The characteristics of this sample suggest that the public companies that issue unregistered shares through private placements tend to be smaller, relatively unprofitable NASDAQ or OTC companies. Equity swaps are unlikely to be available for such stocks. Even in those cases where an equity swap could be arranged, the prohibitions against hedging through option or short-sale transactions would imply that hedging through equity swaps might entail a significant risk that the SEC could successfully challenge the transaction.[7]

Other hedging strategies are available that would appear to involve less legal risk. For example, the purchaser of restricted shares could purchase stock index put options or sell stock index futures short. Both hedging strategies expose the shareholder to basis risk. Nevertheless, to the extent such strategies reduce the price risk exposure inherent in purchasing unregistered shares, they could reduce the size of the discount that investors require when purchasing letter stock. Consequently, transferability discounts may be smaller since the advent of the equity derivatives markets, which postdate the seminal SEC (1971) study, but at least some residual unhedged transfer restriction risk remains.

II. Transferability Discount Model

This section models the transferability discount as the value of an average strike put option. The investor is not assumed to have any special timing ability; instead, it is assumed that the investor would, in the absence of any restrictions, be equally likely to sell the shares anytime during the restriction period.

There are many reasons why the transfer of shares may be restricted. First, as mentioned, transfer may be legally constrained by the resale restrictions imposed by Rule 144 under the Securities Act of 1933. Second, selling restrictions may be imposed by contract. For example, stock lockups in connection with initial public offerings (IPO), which are intended to resolve moral hazard and adverse selection problems, prevent company insiders from selling their shares for 180 days following the IPO (see Field and Hanka, 2001). Third, many firms issue restricted stock as part of their managerial compensation plans. The restriction period ranges from 31 months to 74 months (see Kole, 1997). Fourth, merger agreements often require the target firm's insiders to take restricted stock of the acquiring firm in order to resolve asymmetric information problems and also to align their interests with the interests of the shareholders of the acquiring firm.

A. Assumptions

Let V(t) be the value of a share of common stock without transfer restrictions. The issuer also has restricted shares outstanding, and all the issuer's shares are identical except for the transfer restrictions. Assume the following:

A1. The unrestricted shares trade continuously in a frictionless market.

A2. Transfer restrictions prevent the investor from selling the restricted shares for a period of length T.

A3. Any cash dividends are paid continuously during the time interval [0, T] at a rate $q \geq 0$ that is proportional to V (see Merton (1973)).

A4. The stock price V(t) follows a geometric diffusion process

$$dV = (\mu - q)Vdt + \sigma VdZ \tag{1}$$

where μ and σ are constants and Z is a standard Wiener process.

A5. The riskless interest rate r is constant and the same for all maturities during [0, T].

A6. No shareholder has any special market-timing ability.

The last assumption is consistent with evidence that investors, at least on average, do not have any special ability to outperform the market (see Barber and Odean (2000), Carhart (1997), Chevalier and Ellison (1999), Graham and Harvey (1996), Malkiel (1995), and Odean (1998, 1999)).

Suppose that a stock pays cash dividends ($q > 0$) and that the shareholder can sell the registered shares at t, $0 < t < T$ and reinvest the proceeds in the riskless asset until T. In a risk-neutral world, the investor would be indifferent between selling the share immediately for V(t) and selling it forward for delivery at T with forward price $e^{(r-q)(T-t)}V(t)$. Suppose further that the investor would want to sell the unregistered shares prior to T were it not for the resale restrictions. Since the investor lacks any special timing ability, assume that the investor would be equally likely to sell unrestricted shares at N + 1 discrete points in time and that these points are equally spaced, so that the investor considers selling at $t = 0$, $t = T/N$, $t = 2T/N, \ldots,$ $t = NT/N = T$. Under this assumption, in a risk-neutral world, the investor would be indifferent between holding a registered share and holding an unregistered share plus a series of forward contracts all expiring at T and having an average forward price equal to $\frac{1}{N+1}\sum_{j=0}^{N}[e^{(r-q)T(N-j)/N}V(jT/N)]$. If the investor's transfer restriction risk exposure could be perfectly hedged, or if this risk is idiosyncratic with respect to the investor's securities portfolio, then the unregistered shares would be priced on a risk-neutral basis.

As noted, the securities laws restrict the type of hedging. The investor bears an opportunity cost due to the transfer restrictions if

$$\frac{1}{N+1}\sum_{j=0}^{N}\left[e^{(r-q)T(N-j)/N}V(jT/N)\right] > V(T) \tag{2}$$

but realizes an opportunity gain if the inequality is reversed. If the investor's transfer restriction risk exposure is idiosyncratic, then the gains and losses offset, and so the transferability discount would be zero provided the investor has adequate liquidity from other sources such that the transfer restrictions do not cause the investor to miss any positive-NPV investment opportunities. In effect, the restricted share is a long forward contract for the delivery of an otherwise identical unrestricted share at T when the transfer restrictions expire.

With any unhedged nonidiosyncratic risk exposure, a risk-averse investor would demand a risk premium, and the transferability discount is nonzero. Following Longstaff (1995), inequality (2) suggests that an upper bound on the investor's opportunity cost can be modeled as

$$\max\left\{0, \frac{1}{N+1}\sum_{j=0}^{N}\left[e^{(r-q)T(N-j)/N}V(jT/N)\right] - V(T)\right\} \tag{3}$$

Expression (3) can be interpreted as the payoff function for an average strike put option in which the strike price is the arithmetic average of the forward prices $\frac{1}{N+1}\sum_{j=0}^{N}\left[e^{(r-q)T(N-j)/N}V(jT/N)\right]$.

Expression (3) furnishes an upper bound on this opportunity cost because there are states of nature in which inequality (2) is reversed. However, even in these states the investor may miss positive-NPV investment opportunities unless adequate sources of liquidity are available. If profitable investment opportunities would be missed, then expression (3) would understate the cost of the transfer restrictions if the missed opportunities are worth more than the gains that would result when inequality (2) is reversed. How well this approach to modeling the discount for lack of free transferability explains actual discounts is an empirical question that is addressed later in the paper.

One other factor needs to be considered. The transfer restrictions are costly only if the investor would sell the shares on or before T absent such restrictions. Suppose there is some likelihood p > 0 that the investor would want to hold the stock past T even without resale restrictions. Again assuming that prior to T, any sale would be equally likely to occur anywhere in [0, T], the payoff function becomes

$$\max\left\{0, (1 - p)\left(\frac{1}{N+1}\sum_{j=0}^{N}\left[e^{(r-q)T(N-j)/N} V(jT/N)\right] - V(T)\right)\right.$$

$$= (1 - p)\max\left\{0, \frac{1}{N+1}\sum_{j=0}^{N}\left[e^{(r-q)T(N-j)/N} V(jT/N)\right] - V(T)\right\} \tag{4}$$

In this case, the transferability discount equals 1 - p times the discount calculated assuming the investor would otherwise always sell sometime prior to T.

Expression (3) suggests that the discounts observed in private placements of letter stock will depend on the stock's volatility and other option parameters to the extent transfer restriction risk is priced. If it is not, then information and equity ownership concentration effects should completely explain these discounts. The relative importance of these factors is examined in Section V.

B. The Transferability Discount Model

I obtain a formula for an upper bound on the value of the transferability discount by valuing the average strike put option whose payoff is (3). The appendix derives the following formula for the value of the discount D(T) as:

$$D(T) = V_0\left[e^{(r-q)T} N\left(\frac{r - q}{v}\sqrt{T} + \tfrac{1}{2} v \sqrt{T}\right) - N\left(\frac{r - q}{v}\sqrt{T} - \tfrac{1}{2} v\sqrt{T}\right)\right] \tag{5}$$

$$v^2 = \sigma^2 T + \ln\left[2\left\{e^{\sigma^2 T} - \sigma^2 T - 1\right\}\right] - 2\ln\left[e^{\sigma^2 T} - 1\right] \tag{6}$$

where $N(\cdot)$ is the cumulative normal distribution function. D(T) is proportional to the current share price. It increases with the length of the restriction period T when r > q and with the volatility σ, and it varies inversely with the dividend yield q.

C. The Discounts Implied by the Model

Table 1 reports the sensitivity of the discount D(T) to the length of the restriction period and the stock's volatility. In the dividend-paying case, I assume q = 0.03 because the long-run average dividend yield on the Standard & Poor's 500 portfolio of stocks is about 3 percent. I assume r = 0.05 for the riskless rate. For example, for a non-dividend-paying stock, D(T) increases by approximately 10 percentage points for each additional year in the restriction period when σ is 0.3. If T = 3 years, D(T) = 30.68 percent; if T = 5 years, D(T) = 52.64 percent.

Comparing the discounts for dividend-paying and non-dividend-paying stocks when σ = 0.3, the discount for stocks yielding 3 percent is roughly three-quarters of the discount for otherwise identical non-dividend-paying stocks. For example, with a 5-year restriction period, the discount is 52.64 percent if the stock does not pay dividends but 39.50 percent if the stock provides a 3 percent constant dividend yield. At lower volatilities, the difference between the discounts applicable to dividend-paying and non-dividend-paying stocks is greater for any given restriction period.

Longstaff's (1995) upper bound is based on the assumption that the holder of the restricted shares has perfect timing ability.[8] Figure 1 compares the discounts calculated by applying the Longstaff model and my model, assuming the stock is non-dividend-paying and that the stock's volatility is 0.2, 0.3, or 0.4. The discount should be greater when investors possess special market-timing ability because the transfer restrictions impose a greater opportunity cost, which is evident in Figure 1. For any given restriction period, the difference increases with σ. If the restriction period is T = 0.25 year and $\sigma = 0.3$, the difference is just over 10 percentage points (12.54 percent for perfect timing versus 2.44 percent for no special timing ability). When

T = 2 years, the difference is 18.5 percentage points (38.61 percent versus 20.12 percent). The difference due to timing ability tends to lessen slightly as T increases.

III. Empirical Data and Methodology

I investigated the factors responsible for the discounts observed in new issues of unregistered shares using a sample of 101 private placements of letter stock.

A. Sample Description

I collected a sample of 101 private placements of letter stock that took place between January 1, 1991 and February 3, 1997. The SEC reduced the Rule 144 resale-restriction period to one year from two years on February 20, 1997 (see Securities and Exchange Commission (1997a)).[9] I obtained from Securities Data Company, which regularly collects new issue data, an initial sample of 120 U.S. private placements of unregistered shares of common stock by public U.S. companies. The sample was limited to domestic private placements involving only common stock. Private placements that were accompanied either by a non-U.S. offering or by the simultaneous sale of another class of securities were excluded. I searched the Dow Jones Interactive-Publications Library, the Bloomberg database of company announcements, and the Wall Street Journal Index to confirm the private placement information and identify the earliest announcement date for each offering. Further checking revealed that two of the private issues were accompanied by a simultaneous public offering outside the United States, and four of the issues involved a second class of securities. Thirteen other issues had to be deleted from the sample because of insufficient data to perform either of the studies reported in the paper.

This procedure resulted in a sample of 101 U.S. private placements of letter stock. Six of the firms in the sample conducted two private placements each. In one case, the two placements were two months apart. The second private placement was dropped from the event-study sample.

In all the other cases, the two private placements were at least five months apart. Five private placements had to be dropped from the event study because of insufficient historical trading prices, and 13 others were dropped because a significant corporate announcement unrelated to the private placement occurred between 10 days prior to and five days following the announcement date. This left a reduced sample of 82 private placements for the event study.

Table 2 describes the sample. Panel A reports that the shares of 77 of the 101 firms, or roughly three-quarters of the sample, are quoted in NASDAQ or traded over-the-counter. As reported in Panel B, more than half the private placements occurred during the first half of the study period, and only six took place in 1995, which was a year in which the stock market rose sharply. The relatively small number of letter stock private placements in 1995 could be explained by undervalued firms substituting a public offering for a private placement of common stock because the rising market reduced the undervaluation (Hertzel and Smith (1993) and Lucas and McDonald (1990)).

Panel C describes the issuers. The average market value of firm equity is $153.66 million, which is about 50 percent greater than the average equity market value of the firms in Hertzel and Smith's sample. The average net income and cash flow from operations for the latest fiscal year immediately preceding the offering are both negative. Of the 101 issuers, 69 had negative net income and the same number had negative cash flow from operations for the latest fiscal year. Ten of the 32 with positive net income earned less than $1 million. These characteristics are consistent with Hertzel and Smith's (1993) observation that firms with high financial-distress risk appear more likely to raise funds privately.

Panel D describes the private placements. The average gross proceeds are $9.43 million. I report gross proceeds because the private placement agent's fee was disclosed publicly in only

25 of the 101 offerings. The average proceeds for my sample compare to average proceeds of \$31.5 million for Wruck's (1989) sample of private placements by (larger) exchange-listed firms, \$4.3 million for Silber's (1991) sample, and \$11.4 million for Hertzel and Smith's (1993) sample. The private placements in my sample increased the number of shares outstanding by an average of 16.64 percent, which compares to 19.6 percent for Wruck's (1989) sample, 13.6 percent for Silber's (1991) sample, and 16.0 percent for Hertzel and Smith's (1993) sample.

B. Discount for Lack of Transferability

Panel D reports the mean and median percentage discount calculated with respect to the closing price on the last trading day immediately prior to the date the pricing terms of the private placement are announced and also with respect to the closing price 10 trading days prior to this announcement date. The discount off the prior day's closing price is calculated:

$$\text{Discount (day prior)} = \frac{P_{-1} - P_0}{P_{-1}} \tag{7}$$

where P_{-1} denotes the closing price on the trading day immediately preceding the pricing announcement date and P_0 denotes the private placement offering price.[10]

Because of evidence presented below that the market reacts to information concerning private placements of common stock up to 10 days prior to the pricing announcement date, I also calculate an alternate measure of the transferability discount with respect to the closing price 10 trading days prior to the pricing announcement date, P_{-10}. I adjust P_{-10} to reflect the change in the company's stock price that would be expected to result from changes in market prices generally:

$$\text{Discount (10 days prior)} = \frac{P_{-10}\left[1 + \text{beta}\left(\frac{SAP_{-1}}{SAP_{-10}} - 1\right)\right] - P_0}{P_{-10}\left[1 + \text{beta}\left(\frac{SAP_{-1}}{SAP_{-10}} - 1\right)\right]} \tag{8}$$

where beta is the common stock beta calculated using the Scholes-Williams (1977) procedure. I use the results based on the alternate discount calculation (8) to test the robustness of the results based on (7).

The average Discount (day prior) is 20.13 percent. The average Discount (10 days prior) is 18.41 percent. The difference is not statistically significant. These discounts compare to average discounts of 13.5 percent for unregistered sales reported by Wruck (1989), 33.75 percent for unregistered sales reported by Silber (1991), and 20.14 percent for all private placements reported by Hertzel and Smith (1993).[11]

As Wruck (1989), Silber (1991), and Hertzel and Smith (1993) have all observed, private placement discounts vary widely. In my sample, 28 of the private placements were made at relative-to-prior-day discounts that exceeded 25 percent (25 when the discount is measured relative to the closing price 10 days prior), and one was at a relative-to-prior-day premium that exceeded 10 percent (two when the discount is measured relative to the closing price 10 days prior).[12]

IV. Market Reaction to the Announcements

In testing the information effects of a private stock issue, it is important to consider announcement effects to determine what information is important and how the market reacts to it. Two announcements may occur in connection with a private placement: (1) an initial announcement of the firm's intention to place shares of common stock privately and (2) the firm's announcement that it has completed the offering, which furnishes the terms on which it sold the shares. I refer to the former as the "offering announcement" and to the latter as the "pricing announcement." I refer to the first announcement of either kind as the "initial announcement."

Eighteen issues were initially announced prior to the pricing announcement. The offering announcement and the pricing announcement coincide in the other 64 private placements. A private placement is a best-efforts undertaking – in contrast to a general cash offer, which is usually underwritten – and there is no assurance beforehand that this effort will be successful. Investors will not know that the offering has succeeded until the firm announces its completion and provides the terms on which the shares were sold to investors. In none of the 18 cases did the offering announcement indicate the offering price; it was first disclosed in the pricing announcement. The pricing announcement also disclosed the number of shares sold, and in 25 cases also provided the private placement agent's fee. Thus, both the offering announcement (when one occurs) and the pricing announcement may contain useful information. However, if a private placement announcement signals management's belief that the firm's stock is undervalued, investors should react more strongly to the offering announcement even though there is no assurance the private placement will be successful. The event study results support this hypothesis.

Table 3 reports average cumulative abnormal returns (CARs) and the percentage of positive CARs around the private placement announcements by the 82 firms in the event-study sample. The abnormal daily returns are calculated using the market model and the Scholes-Williams (1977) procedure to estimate beta.[13] I used the CRSP value-weighted index of all NYSE, AMEX, and NASDAQ stocks as the proxy for the market portfolio. Percent positive is the percentage of positive CARs. The comparison period extends from 120 trading days prior through 21 trading days prior to the date the company first publicly announces the private placement.

In Table 3, Panel A reports the market impact of the initial announcement. All seven mean CARs are positive with three significant at the 5 percent level based on a one-tailed test, which is consistent with Wruck (1989) and Hertzel and Smith (1993). The mean CAR for the period (-3, 0) is 1.88%, which is significant at the 5 percent level and is very similar to what Hertzel and Smith (1993) report. The percentage of positive CARs equals or exceeds 50 percent in all seven cases with the period (-3, 3) positive percentage significant at the 1 percent level, the (-5, 5) positive percentage significant at the 5 percent level, and three others significant at the 10 percent level. The significant mean CAR for the period (-10, 0) coupled with the insignificant mean CARs for two of the other periods ending on day 0 may reflect the effect of the pre-announcement marketing of the private placement by firms that refrain from making an announcement (as with 64 of the 82 issues in the sample) until the shares have been successfully placed.

Panel B reports the market impact of the pricing announcement. All seven mean CARs are positive. However, none is statistically significant. The percentage of positive CARs exceeds 50 percent in each case but one, and one of these differences is significant at the 5 percent level. However, the median CAR for the period (-1, 0) is negative, and fewer than 50 percent of the CARs are positive.

Comparing the results in Panel A and Panel B, the initial announcement appears to have the stronger market impact. To try to confirm this conclusion, I performed separate tests of the market reaction to the 18 separate offering and pricing announcements. I report these results in Panels C and D. Any conclusions are tentative due to the small sample sizes. Nevertheless, a comparison of the results in Panels C and D is striking. In panel C, five of the seven mean CARs are positive, and more than 50 percent of the CARs are positive for the periods (-3, 0), (-1, 0), (-3, 3), and (-5, 5). In contrast, in panel D, all seven of the mean CARs are negative and the percentage of positive CARs never exceeds 50 percent, and the negative mean CAR and the percentage of negative CARs for the period (-5, 5) are both significant at the 10 percent level. Taken collectively, the results in Panels A–D suggest that the offering announcement conveys more useful information to investors than the pricing announcement, as predicted by signaling theory.

Finally, in Panel E, the impact of the coincidental announcement of the offering and its pricing is very similar to the impact of the initial announcement reported in Panel A. The mean CARs are generally larger in Panel A but the median CARs and percentage of positive CARs are generally larger in Panel E. The greater percentage of positive CARs may reflect the favorable certification effect that accompanies the successful completion of a common stock private placement.

A tendency for issuers to wait until the terms of the private placement have been negotiated before announcing the new issue could explain why the mean CARs are significant at the 5 percent level for the period (-10, 0) while the percentage of positive CARs is greater for the period (-10, 0) than for all the other pre-announcement periods in Panels A and E. The issuer's intention to place common stock privately becomes known to investors as soon as the firm commences the offering. Prospective purchasers, as well as other market participants, can be expected to react to the firm's attempt to sell common stock as soon as this information becomes available in the market place. If so, then the market's reaction to the offering may already have been incorporated into the issuer's common stock price before the initial announcement occurs when the offering announcement and the pricing announcement coincide.

V. Cross-Sectional Analysis

This section uses regression analysis to test the relative importance of the transfer restrictions and the information/ownership effects in explaining private placement discounts.

A. Factors Indicating the Impact of Transfer Restrictions

In equations (5) and (6), the discount is a function of the volatility of the common stock (σ), the stock's dividend yield (q), the time to expiration of the transfer restrictions (T), and the riskless interest rate (r). I regress the percentage discount on independent variables measuring volatility, dividend yield, and the riskless interest rate and one that proxies for the time to expiration.

Discount (day prior), which is defined by equation (7), measures the percentage discount relative to the closing market price of the issuer's registered stock on the trading day immediately preceding the pricing announcement date, which is the market price that is usually used as the basis for pricing privately placed shares. As an alternate definition of the dependent variable, I also ran the regressions using Discount (10 days prior), which measures the percentage discount relative to the closing price 10 trading days prior to the pricing announcement date, as adjusted for general stock market movements between the two dates. I performed the alternate regression to check whether information about the offering that comes into the market before the pricing announcement date, either because the issuer announces the offering before it is priced (about 1 case in 5) or because of the issuer's marketing efforts, might somehow affect the magnitude of the discount.

Volatility is measured as the annualized standard deviation of the total return on the issuer's common stock during the three-month period ending on the last trading day immediately preceding the initial announcement date. The implied volatility obtained from publicly traded options would be preferable to an historical volatility but none of the stocks in the sample had exchange-traded options around the time of the private placements.[14]

The stock's dividend yield (Yield) is the annualized dividend yield based on the most recently declared cash dividend as of the pricing announcement date and the closing price of the registered shares on the last trading day immediately preceding that date. Only seven of the issuers had declared a cash dividend within the prior 12 months.

A proxy was used for the time to expiration of the transfer restrictions (Time). During the sample period, Rule 144 imposed a minimum two-year restriction period, after which the number of shares that could be sold during any three-month period was limited to the lesser of (1) the average weekly trading volume during the four calendar weeks immediately preceding the filing with the SEC of Form 144 indicating the holder's intention to sell and (2) one percent of the number of outstanding shares. I used the ratio of the number of shares offered to the common stock's trading volume during the three-month period ending on the last trading day immediately preceding the initial announcement date as a proxy for Time. Because of the two-year minimum restriction period under Rule 144, I used the interest rate on two-year Treasury notes as of the pricing announcement date as the measure of the riskless interest rate (Rate).

The discount will vary directly with Volatility and Time because both increase the value of the option. It will vary inversely with Yield because the greater flow of cash during the restriction period reduces the degree of illiquidity. It will also vary inversely with Rate because the nonmarketable stock's higher total return (the riskless rate in a risk-neutral world) partially compensates for the opportunities missed due to the stock's marketability restrictions, reducing the portion of the opportunity cost that must be covered by the discount.

B. Information and Equity Ownership Concentration Effects

I used several proxies to capture information effects and equity ownership concentration effects. Hertzel and Smith (1993) find in their cross-sectional analysis that the incremental impact of the lack of registration amounts to 13.5 percent, which means that about one-third of the overall 20.14 discount they measured can be attributed to factors other than the Rule 144 resale restriction.

I use the book-to-market ratio immediately prior to the pricing announcement date (Book/Market) to proxy for the fraction of the equity market value attributable to intangible assets.[15] The more significant are intangible assets, the smaller the book-to-market ratio, the more difficult it is for investors to assess value, and the greater is likely to be the discount. The book-to-market ratio serves as a measure of the degree of over- or undervaluation. The larger the book-to-market ratio, the greater the degree of undervaluation, and the smaller the discount.[16]

Following Hertzel and Smith (1993), Fraction measures the fraction of the firm's common stock that is outstanding after the new issue due to the private placement. Under their information hypothesis, larger issues involve more intensive information gathering. Since investors have to expend more resources to assess firm value, they require a larger discount when the offering is relatively large. I control for possible economies of scale in the production of information using the logarithm of the private placement's gross proceeds (Log (proceeds)) to measure the size of the private placement. I allow for the possibility that information asymmetries are likely to be more severe for smaller firms, thus making them more costly to evaluate and requiring a larger discount, using the logarithm of one plus the latest fiscal year's total sales measured in millions (Log (1 + sales)).[17]

The costs of information gathering and the degree of difficulty in assessing the issuer's value are also likely to be greater, the weaker the issuer's financial condition. I use two proxies for the financial condition of the issuer at the time of the offering, the ratio of the latest fiscal year's operating cash flow to the book value of total assets (Cash flow/Assets), which proxies for the issuer's profitability, and the ratio of the book value of the issuer's total debt (both short-term and long-term) to the sum of the book value of total debt plus the book value of any preferred stock plus the market value of the issuer's common stock based on the most recent closing price as of the pricing announcement date, which measures the issuer's financial leverage (Leverage).

The discount investors require will also depend on the level of insider ownership and the intensity of monitoring by outside shareholders. Managers of firms with low levels of insider ownership have a greater incentive to invest in unprofitable projects (Jensen and Meckling (1976)). The net proceeds from the new issue give managers more discretionary cash, which increases the risk of non-value-maximizing behavior (Jensen (1986)). Monitoring by institutional investors can mitigate this effect. The greater their percentage ownership, the stronger the incentive institutional shareholders have to protect their investment in the firm's shares. They can achieve this objective by monitoring the use of proceeds to ensure that they are invested productively.[18]

I use the percentage of shares owned by insiders (Inside), as determined from the issuer's most recent proxy statement as of the pricing announcement date, to capture differences across issuers in agency costs due to differences in the degree of alignment between manager and shareholder interests.[19] I use the percentage of shares owned by institutional investors (Instit), as reported by Standard & Poor's Stock Guide, for the month ending immediately prior to the pricing announcement date, to proxy for the intensity of institutional monitoring.

I also measure the intensity of monitoring by the firm's board of directors. Fama and Jensen (1983) argue that outside directors have an incentive to act as effective monitors because they want to protect their reputations for independence.[20] I use one minus the fraction of directors that are also managers (Direct), according to the issuer's most recent proxy statement as of the pricing announcement date, to proxy for the intensity of monitoring by outside board members.

Finally, I use a dummy variable to test the possible significance of exchange listing. The variable Exchange takes on the value 1 if the registered shares are listed on the NYSE or the AMEX at the time of the private placement and zero otherwise.

C. Bivariate Results

Table 4 reports descriptive evidence concerning the bivariate relationships between the discounts and the independent variables in the cross-sectional analysis. All the variables have the predicted signs for both discount measures except Book/Market and Leverage, neither of which is statistically significant for either measure of the discount. Of the four transfer restriction variables, Volatility has the most significant Pearson correlation coefficient in both cases, which is consistent with the option characterization of the transferability discount; Volatility and Time have Pearson correlation coefficients that are significant at the 1 percent level in both cases; the Pearson correlation coefficients for Rate are significant at the 1 percent level in one case and at the 5 percent level in the other; and the Pearson correlation coefficient for Yield is significant at the 10 percent level in one case but not the other, which is not surprising given the small fraction of stocks in the sample that were dividend-paying at the time of the private placement.

Of the 10 information and ownership concentration variables, the Pearson correlation coefficients for Fraction and Log (proceeds) are significant at the 1 percent level in both cases; Log(1+ sales) is significant at the 1 percent level in one case and at the 5 percent level in the other; Direct and Exchange are significant at the 5 percent level in both cases; and none of the other five is significant in either case. Overall, the Pearson correlation coefficients are very similar for the two discount measures.

D. Regression Results

I ran three regression models. Regression model 1 has the four transfer restriction independent variables:

Model 1

$$\text{Discount} = a_0 + a_1 \text{Volatility} + a_2 \text{Yield} + a_3 \text{Time} + a_4 \text{Rate} \tag{9}$$

Regression model 2 has only information and ownership concentration independent variables. There are five information and ownership concentration variables in Table 4 with Pearson correlation coefficients that are significant at the 5 percent level (Fraction, Log(proceeds), Log(1+ sales), Direct, and Exchange). Analysis of the data revealed that Log(proceeds) is highly positively correlated with Log(1 + sales). Since Log(proceeds) is more strongly correlated with Discount, I excluded Log(1 + sales) to eliminate the multicollinearity leaving four independent variables:

Model 2

$$\text{Discount} = a_0 + a_1 \text{Log(proceeds)} + a_2 \text{Fraction} + a_3 \text{Direct} + a_4 \text{Exchange} \tag{10}$$

Regression model 3 contains both types of independent variables. Analysis of the data revealed that Log(proceeds) is highly negatively correlated with Volatility. I employed two-stage least squares. In the first stage, I regressed Log(proceeds) on Volatility and obtained

$$\underset{}{\text{Log(proceeds)}} = \underset{(17.737)}{1.365} - \underset{(-8.667)}{0.797}\ \text{Volatility} \qquad \text{adjusted } R^2 = 0.378,\quad F = 61.22 \tag{11}$$

The heteroskedasticity-consistent t-statistics given in parentheses and the F-statistic are significant at the 1 percent level. I defined the orthogonal variable Adjusted Log(proceeds) to be the residual of Log(proceeds) in equation (11). In addition, Fraction and Exchange are each strongly correlated with three of the transfer restriction variables, and neither variable made a significant incremental contribution to the goodness of fit for Model 2. Consequently, I excluded these two variables. Regression model 3 is:

Model 3

$$\text{Discount} = a_0 + a_1\text{Volatility} + a_2\text{Yield} + a_3\text{Time} + a_4\text{Rate}$$

$$+ a_5\text{Direct} + a_6\text{Adjusted Log(proceeds)} \qquad (12)$$

Table 5 contains the cross-sectional regression results. I ran each model with Discount (day prior) and also with the alternate dependent variable Discount (10 days prior). White's (1980) specification test indicated that the null hypothesis of homoskedasticity could not be rejected at the 10 percent level in Regressions 1, 2, 4, or 5, but the test statistic is barely significant at the 10 percent level in Regressions 3 and 6. I report White heteroskedasticity-consistent estimators and t-statistics to adjust for whatever degree of heteroskedasticity exists in the residuals.

I also applied SYSTAT's Lagrange multiplier nonlinearity test, which tests the joint hypothesis that all possible interactions, including squared regressors, have zero coefficients in a full model and found that I could not reject the null hypothesis that a linear model is the correct specification in all except Regression 3. Finally, the Durbin-Watson statistics indicate that the null hypothesis of zero autocorrelation can not be rejected at the 5 percent level in any of the regressions.

Regressions 2, 3, 5, and 6 have 91 observations. Data were not available for Direct for nine of the 101 issuers, and the SYSTAT regression software indicated that the single private placement with an apparent day-prior private placement premium (Discount (day prior) < 0) is an outlier in the regressions that include the information and ownership concentration variables.[21]

All the coefficients of the transfer restriction variables have the predicted sign in Regressions 1, 3, 4, and 6. The coefficient of Volatility is significant at the 1 percent level in all four regressions, which is consistent with the option characterization of the transferability discount. The coefficient of Time is significant at the 1 percent level in Regressions 1, 3, and 6 and at the 5 percent level in Regression 4. The coefficient of Rate is significant at the 1 percent level in Regression 1 but is not significant at the 10 percent level in Regressions 3, 4, and 6. The coefficient of Yield is significant at the 5 percent level in Regression 6 and at the 10 percent level in Regression 3.

All the coefficients of the information and ownership concentration variables have the predicted sign in Regressions 2, 3, 5, and 6. The coefficient of Log(proceeds) is significant at the 1 percent level in Regressions 2 and 5, and the coefficient of Adjusted Log(proceeds) is significant at the 1 percent level in Regressions 3 and 6. The coefficient of Direct is significant at the 5 percent level in Regressions 5 and 6 and at the 10 percent level in Regression 2. The adjusted R^2 and F statistics indicate that Regressions 2 and 5 have greater explanatory power than Regressions 1 and 4. However, only the coefficient of Log(proceeds) is significant at the 1 percent level in either regression, and it is highly correlated with Volatility, which is the most significant explanatory variable in Regressions 1 and 4.

Combining both sets of variables, Regressions 3 and 6 have similar explanatory power, each explaining roughly half the variation in the discount. The slightly greater adjusted R^2 and F statistics for Regression 6 are consistent with the greater information effect of the offering announcement than the pricing announcement observed in Table 3. The incremental contribution of the transfer restriction variables appears greater than the incremental contribution of the information and equity ownership concentration variables in Regression 3 but the opposite is true in Regression 6, which is also consistent with the relative importance of the pricing and offering announcements. Overall, the regression results confirm the significant explanatory power of both sets of factors.

VI. Comparison of Implied and Predicted Transferability Discounts

I perform three tests of the transferability discount model's ability to explain actual transferability discounts. I compare the mean prediction from the model (5)-(6) (Predicted Discount) to the mean implied transferability discount (Implied Discount) for the sample. Since the coefficients a_5 and a_6 in equation (12) are both negative, the actual private placement discount understates the (implied) transferability discount. I calculate the Implied Discount by adjusting the private placement discounts for the values of the two information and ownership concentration variables in equation (12). Next, I perform a Wilcoxon signed rank test and regress the Implied Discount on the Predicted Discount to investigate whether the model has a systematic tendency to over- or underprice the discount.

Table 6 compares the Implied and Predicted Discounts. I calculate the Implied Discount from the actual private placement discount (Discount):

$$\text{Implied Discount} = \text{Discount} - a_5\ \text{Direct} - a_6\ \text{Adjusted Log}(\text{proceeds}) \tag{13}$$

I use the coefficients from Regression 3 in Table 5 to adjust Discount (day prior) and the coefficients from Regression 6 to adjust Discount (10 days prior).

In Panel A, Implied Discount appears to overstate substantially the Predicted Discount for relatively low-volatility stocks (volatility less than 30 percent). The mean Implied Discount (10 days prior) also exceeds the mean Predicted Discount for volatilities exceeding 120 percent. With the exception of relatively low-volatility stocks and very-high-volatility stocks, the mean Implied Discount (10 days prior) is more consistent with the mean Predicted Discount than the mean Implied Discount (day prior).

In Panel B, the mean Implied Discount (day prior), 28.50 percent, is significantly less than the mean Predicted Discount, 34.64 percent, at the 1 percent level. Only 27 of the 91 differences are positive, which is significant at the 1 percent level. However, the mean Implied Discount (10 days prior) equals the mean Predicted Discount, and 47 of the 91 differences are positive, which is not statistically significant.

The mean and median stock price volatilities for the sample are 74 percent and 67 percent, respectively, which are relatively high volatilities. When the stock price volatility is that high, the model predicts discounts within the 25 to 35 percent range that investment bankers and appraisers customarily apply when there is a two-year restriction period. In particular, the mean Predicted Discount (34.64 percent), the mean Implied Discount (day prior) (28.50 percent), and the mean Implied Discount (10 days prior) (34.64 percent) are all within this range. However, when the restriction period is two years, Table 1 indicates that when $q = 0$, a discount in the range from 25 to 35 percent is appropriate only if the volatility is in the range from 40 to 70 percent. Above (below) this volatility range the discount should be higher (lower).

The result of fitting the regression equation

$$\text{Implied Discount} = a_0 + a_1 \ \text{Predicted Discount} \tag{14}$$

is reported in Panel C. If the implied transferability discounts perfectly track the predicted discounts, then $a_0 = 0$ and $a_1 = 1$. If investment bankers and business appraisers tended to apply discounts within the 25 to 35 percent range during the sample period with little, if any, adjustment for volatility, then I would expect $a_0 > 0$ and $a_1 < 1$ because then the implied discounts would tend to overstate (understate) the predicted discounts for low (high) volatilities. The slope a_1 would indicate how sensitive the discount is to differences in volatility and to variation in the other parameters in the model.

The intercepts a_0 are positive but only the Regression 8 intercept is significant at the 10 percent level. The slope a_1 is significantly different from zero at the 1 percent level and significantly different from one at the 10 percent level in both regressions.

Figure 2 plots the linear equations corresponding to Regression 7 and Regression 8. The Regression 8 line crosses the 45 degree line at Discount = 34.7 percent, which approximates the mean Predicted Discount and the mean Implied Discount for the sample of common stock private placements. It falls within the customary 25 to 35 percent range of discounts and corresponds to a volatility of about 70 percent when q = 0 in Table 1. The Regression 7 line crosses the 45 degree line at Discount = 9.2 percent. Regression 8 (and thus Regression 6 from which the adjustments for the information and ownership concentration effects were derived) appears to fit the implied transferability discounts better than Regression 7 (and underlying Regression 3), which is consistent with the regression results in Table 5.

The regression results in Table 6 imply a tendency for systematic mispricing of the transferability discount in practice in which the discount is overstated for relatively low-volatility stocks (σ under 30 percent) and understated for relatively high-volatility stocks (σ over 70 percent).

VII. Conclusion

The private placement discount results from both the Rule 144 transfer restrictions on unregistered common stock and the information and equity ownership concentration effects that accompany a common stock private placement. The effect of the transfer restrictions can be priced as the value of an average strike put option.

The model (5)-(6) calculates transferability discounts that are consistent with the range of discounts observed empirically in letter-stock private placements for common stocks with volatilities between σ = 30 percent and σ = 70 percent but the implied discounts are greater than (less than) those predicted by the model for lower (higher) volatilities. These differences appear to reflect systematic mispricing of the transferability discount, which is consistent with anecdotal evidence that investment bankers and business appraisers tended to apply a customary range of marketability discounts between 25 and 35 percent during the period covered by the study, which would permit only limited adjustment for volatility. My model implies that when the stock price volatility is under 30 percent, the appropriate discount is smaller than the customary discount range of about 25 to 35 percent. For example, when σ is between 20 percent and 30 percent and there is a two-year restriction period, the proper discount is in the range from 15.76 percent to 20.12 percent for a non-dividend-paying stock and in the range from 11.50 percent to 15.96 percent for a stock yielding 3.0 percent. The halving of the initial restriction period under Rule 144 since February 1997 has roughly halved the transferability discount.

My model can also explain the apparent inconsistency between the range of discounts found in several earlier studies, which concluded that marketability discounts of 25 to 35 percent were appropriate for the 2-year Rule 144 restriction period, and the more recent studies by Wruck (1989) and Hertzel and Smith (1993), which found that this discount is only 13.5 percent. The difference is due in part to the information and ownership concentration effects that accompany a common stock private placement, but may also be due to mispricing of the forgone put option. In any case, the discount varies directly with the stock's volatility.

Appendix

Applying the risk-neutrality transformation of Cox and Ross (1976), the stock price can be described by the geometric diffusion process

$$dV = (r - q)\,Vdt + \sigma VdZ, \tag{A.1}$$

and ln V(T) is normally distributed with mean $\ln V_0 + (r - q - ½\sigma^2)T$ and standard deviation $\sigma\sqrt{T}$, where V_0 is the stock price at t = 0. Similarly, the forward price $F(t) = e^{(r - q)(T - t)}V(t)$ follows the martingale process

$$dF = \sigma\, FdZ \tag{A.2}$$

in a risk-neutral world, and ln F(t) is normally distributed with mean $\ln F_0 - ½\sigma^2 t$ and standard deviation $\sigma\sqrt{t}$ where $F_0 = F(0) = V_0 e^{(r-q)T}$.

In the risk-neutral framework, the strike price is the average of the risk-neutral forward prices, which I will denote A(T):

$$A(T) = \frac{1}{N+1}\sum_{j=0}^{N} F(t_j) \tag{A.3}$$

where $t_j = jT/N$ and the forward prices follow the martingale process (A.2). The payoff function (A.3) contains the sum of a set of correlated lognormal random variables. Although expressions exist for the moment generating function, mean, and variance of the sum of two lognormal random variables, no exact closed-form expression for the density function of the sum of a set of lognormal random variables is known. A sum of independent lognormal random variables can be closely approximated by a lognormal random variable.[22] Levy (1992), Ritchken, Sankarasubramanian, and Vijh (1993), and Turnbull and Wakeman (1991) show that the distribution of the average of a set of correlated lognormal stock prices or exchange rates can be approximated by a lognormal distribution with acceptable accuracy by applying Wilkinson's method, which matches the first and second moments. I use Wilkinson's method to approximate the distribution of A(T).

Assume that ln A(T) is normally distributed with mean $\alpha_1(T)$ and variance $v_1(T)^2$. Since ln V(T) is also normally distributed, assume that X(T) = [ln A(T), ln V(T)] is approximately bivariate normal. An average strike put option can be characterized as the option to exchange a package of forward contracts on a share for the underlying share and evaluated by applying Margrabe's (1978) equation (7).

First, write the moment generating function for X(T) as

$$M(k_1, k_2) = E^*\left[A(T)^{k_1} V(T)^{k_2}\right] = e^{\alpha_1 k_1 + \alpha_2 k_2 + \frac{1}{2}\left(v_1{}^2 k_1{}^2 + 2\rho v_1 v_2 k_1 k_2 + v_2{}^2 k_2{}^2\right)} \tag{A.4}$$

where E* denotes the expectations operator conditional on V_0 and where α_2 = ln V_0+ (r - q - $\frac{1}{2}\sigma^2$)T and $v_2 = \sigma\sqrt{T}$. Since $F_0 = V_0\, e^{(r-q)T}$, write the mean as α_2 = 1n F_0 - ½ $\sigma^2 T$ = ln F_0 - ½ v_2^2.

Obtain expressions for α_l and v_1^2 by substituting (k_l,k_2) = (1,0) and (k_1,k_2) = (2,0) into equation (A.4) and solving the resulting equations for:

$$\alpha_1 = 2 \ln E^*[A(T)] - 1/2\ \ln E^*\left[A(T)^2\right] \tag{A.5}$$

$$v_1^2 = \ln E^*\left[A(T)^2\right] - 2 \ln E^*[A(T)] \tag{A.6}$$

Similar expressions for α_2 and v_2^2 can be obtained by substituting $(k_1, k_2) = (0,1)$ and $(k_1, k_2) = (0,2)$ into equation (A.4). The covariance term, $\rho v_1 v_2$, is obtained by substituting $(k_1, k_2) = (1,1)$:

$$\rho v_1 v_2 = \ln E^*[A(T)V(T)] - (\alpha_1 + \alpha_2) - \tfrac{1}{2}\left(v_1{}^2 + v_2{}^2\right) \tag{A.7}$$

Equations (A.5)-(A.7) require expressions for E*[A(T)], E*[$A(T)^2$], and E*[A(T)V(T)]:

$$E*[A(T)] = E*\left[\frac{1}{N+1}\sum_{j=0}^{N} F(t_j)\right] = \frac{1}{N+1}\sum_{j=0}^{N} E*[F(t_j)] = \frac{1}{N+1}\sum_{j=0}^{N} F_0 = F_0 = V_0 e^{(r-q)T} \tag{A.8}$$

$$E*\left[A(T)^2\right] = E*\left[\left(\frac{1}{N+1}\sum_{i=0}^{N} F(t_i)\right)\left(\frac{1}{N+1}\sum_{j=0}^{N} F(t_j)\right)\right] = \frac{1}{(N+1)^2}\sum_{i=0}^{N}\sum_{j=0}^{N} F_0^2\, e^{\sigma^2 \mathrm{Min}(i,j)T/N} \tag{A.9}$$

since $E*[F(t_i)\, F(t_j)] = F_0^2\, e^{\sigma^2 \mathrm{Min}(i,j)T/N}$. To see this, without loss of generality, suppose Min (i,j) = i. Then

$$E*[F(t_i)F(t_j)] = E*\left[e^{\ln F(t_i)}\, e^{\ln F(t_j)}\right] = F_0^2\, e^{\rho\sigma^2\sqrt{ij}\,T/N} \tag{A.10}$$

Note that $\rho\,\sigma^2\sqrt{ij}\,T/N = \mathrm{Cov}[\ln F(t_i), \ln F(t_j)]$ so that

$$\rho\,\sigma^2\sqrt{ij}\,T/N = \sigma^2 iT/N = \sigma^2 \mathrm{Min}(i,j)T/N \tag{A.11}$$

Substituting equation (A.11) into equation (A.10) justifies equation (A.9). After simplification, equation (A.9) becomes

$$E*\left[A(T)^2\right] = \frac{F_0^{\,2}}{(N+1)^2\left(e^{\sigma^2 T/N}-1\right)}\left\{e^{\sigma^2 T+\sigma^2 T/N} - 1 - 2N + \frac{2\left(e^{\sigma^2 T+\sigma^2 T/N}-1\right)}{e^{\sigma^2 T/N}-1}\right\} \tag{A.12}$$

Finally,

$$E*[A(T)V(T)] = E*\left[\left(\frac{1}{N+1}\sum_{j=0}^{N} F(t_j)\right)V(T)\right] = \frac{F_0^{\,2}}{N+1}\sum_{j=0}^{N} e^{\rho\sigma^2 T\sqrt{j/N}} \tag{A.13}$$

Note that $\rho\sigma^2 T\sqrt{j/N} = \mathrm{Cov}[\ln F(t_j), \ln V(T)]$ so that

$$\rho\sigma^2 T\sqrt{j/N} = \sigma^2 jT/N \tag{A.14}$$

Substituting equation (A.14) into equation (A.13) gives

$$E*[A(T)V(T)] = \frac{F_0^{\,2}}{N+1}\sum_{j=0}^{N}\left[e^{\sigma^2 T/N}\right]^j = \frac{F_0^{\,2}}{N+1}\,\frac{e^{\sigma^2 T(N+1)/N}-1}{e^{\sigma^2 T/N}-1} \tag{A.15}$$

Now assume that investors reevaluate their decision to hold or sell the shares continually.

In that case, the following continuous form expressions can be used to value the option to exchange the package of forward contracts on a share for the underlying share:

$$E^*\left[A(T)\right] = F_0 \tag{A.16}$$

$$\lim_{N\to\infty} E^*\left[A(T)^2\right] = \frac{2F_0^{\,2}}{\left(\sigma^2 T\right)^2}\left\{e^{\sigma^2 T} - \sigma^2 T - 1\right\} \tag{A.17}$$

$$\lim_{N\to\infty} E^*\left[A(T)V(T)\right] = \frac{F_0^{\,2}}{\sigma^2 T}\left\{e^{\sigma^2 T} - 1\right\} \tag{A.18}$$

Substituting equations (A.16)-(A.18) into equations (A.5)-(A.7) and similar expressions for α_2 and v_2^2 gives

$$\alpha_1 = \ln F_0 - \tfrac{1}{2} v_1^{\,2} \tag{A.19}$$

$$v_1^2 = \ln\left[2\left\{e^{\sigma^2 T} - \sigma^2 T - 1\right\}\right] - 2\ln\left[\sigma^2 T\right] \tag{A.20}$$

$$\alpha_2 = \ln F_0 - \tfrac{1}{2}\sigma^2 T \tag{A.21}$$

$$v_2^2 = \sigma^2 T \tag{A.22}$$

$$\rho v_1 v_2 = \ln\left[e^{\sigma^2 T} - 1\right] - \ln\left[\sigma^2 T\right] \tag{A.23}$$

Apply Margrabe's (1978) equation (7) for the value of an option to exchange one asset for another to obtain equations (5) and (6).

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Footnotes

[1] Letter stock is not registered for resale under the Securities Act of 1933 and therefore cannot be freely traded in the public market. It must be placed privately with qualifying (sophisticated) investors. Letter stock gets its name from the practice of requiring the investor to furnish an 'investment letter' confirming that the purchase is for investment and not for resale. This representation is one of the requirements to qualify the shares for the 'private offering exemption' from registration under the Securities Act of 1933 (see Loss and Seligman (1995, p. 332)). The stock bears a legend on the back of the share certificate warning investors that the shares are not registered for public trading. Under Rule 144, prior to its amendment in 1997, a holder of letter stock could not sell the shares for at least two years from the issue date except through another private transaction. Effecting such a transaction would require "an opinion of counsel satisfactory to the company to the effect that the proposed transaction will be exempt from registration [under the Securities Act of 1933]." (See Loss and Seligman (1995).) Neither the issuer nor the stock transfer agent has any obligation to record the transfer of the stock unless the transferee can show that the transfer is exempted from registration (see Loss and Seligman (2000)). The private purchaser would face the same restriction for the remainder of the two-year period. After the two-year period, the shares could be sold in the public market without first registering them but such sales are subject to restrictions on the volume of sales for a period of one year; they could be sold beginning three years after issue without any restrictions. On February 20, 1997, the SEC amended Rule 144 to allow nonaffiliates of the issuer to sell their shares after one year instead of two, followed by the restrictions on the volume of sales just noted, and to sell them after two years without any restrictions. The SEC shortened the holding

period in order to lower the cost of raising equity privately (see Loss and Seligman (2000)). When a company has both registered common stock and letter stock outstanding any difference in price will reflect the impact of these transfer restrictions. The focus of this study is the impact of the transfer restrictions under Rule 144. The results can be used to infer discounts for other types of marketability restrictions by comparing the restrictions to those imposed by Rule 144.

[2] Several small-scale studies in the tax-accounting and appraisal literature have found marketability discounts ranging from 4 percent to nearly 100 percent with an average of 42 percent. Bolten (1990) summarizes the results of nine studies. Many of these date back to the late 1960s or early 1970s, a period that predates the development of the options market.

[3] Longstaff's model provides an upper bound on the marketability discount because it assumes that investors have perfect market-timing ability. At least one earlier paper uses a contingent-claims approach to value the resale restriction as a put option. Alli and Thompson (1991) measure the value of marketability as the Black-Scholes (1973) value of a put option with the share price at the time of issue as the strike price.

[4] Alternatively, the hedge position could consist of a portfolio of put options that correspond to the numbers of shares that could be sold following the expiration of the resale-restriction period subject to Rule 144's trading volume restrictions.

[5] Less frequent hedging transactions might be more cost effective because of transaction costs.

[6] Selling call options before the resale-restriction period expires and delivering the previously restricted shares if the call options are exercised following the expiration of the resale-restriction period is similarly prohibited. See Securities and Exchange Commission (1997b).

[7] The modifications to Rule 144 the SEC made on February 20, 1997 (see Securities and

Exchange Commission (1997b)) note that in some cases, market participants have used equity swaps to arbitrage the private placement discount. Securities lawyers the author contacted explained that they have advised their clients that using an equity swap to arbitrage the discount would expose them to the risk that if the SEC investigates their hedged investment in unregistered shares it might conclude that the resales were in violation of Rule 144.

[8] This ability could be due to valuable private information regarding the issuing company's future prospects. Such shareholders would presumably time their sales of unrestricted shares so as to maximize the sales proceeds. Thus, Longstaff's model can also be interpreted to apply to insiders with valuable private information, such as senior executives. Many publicly traded companies prohibit their executives from buying or selling shares except during limited periods, as for example, a brief period commencing a few days following an earnings announcement. Longstaff's model could be used to measure the impact of such a sale restriction.

[9] The latest offering included in the sample was announced on February 3, 1997 and closed on February 10, 1997.

[10] Discount in equation (7) corresponds to one minus the Offer price/Market price ratio in Wruck's (1989) study.

[11] Hertzel and Smith (1993) measure the discount relative to the share price 10 days after the private placement announcement date. I measure the discount relative to the closing share price immediately prior to the pricing announcement date, which seems more appropriate for calculating the transferability discount for two reasons. First, the issuer and investors can renegotiate the offering price, size of the issue, and other terms right up to the private placement closing date, and under the securities laws, the terms of the offering must be promptly disclosed publicly once they have been finalized. Second, market participants express the discount relative

to the freely traded share price, and a contemporaneous market price is most appropriate for such a comparison.

[12] Only one of the 101 private placements involved a premium as compared to the previous day's closing price, and only 10 involved a premium as compared to the closing price 10 days prior to the offering.

[13] Brown and Warner (1980) describe the event-study procedures and statistical tests of significance used in this paper.

[14] I also ran regressions with annualized six-month and 12-month historical volatilities and obtained results similar to those reported in the paper.

[15] The book-to-market ratio is used instead of the market-to-book ratio because it is better behaved when there are near-zero equity market values, as there are in my sample.

[16] Lehn, Netter, and Poulsen (1990) use the market-to-book ratio as a proxy for Tobin's q; McLaughlin, Safieddine, and Vasudevan (1996) interpret Tobin's q as a proxy for the firm's growth opportunities (Tobin's $q > 1$ signifying a high-growth firm); and Spiess and Affleck-Graves (1995) find that high-growth firms tend to be more prone to overvaluation than low-growth, mature firms.

[17] Since the size of the private placement is positively correlated with firm size, a negative coefficient of Log(proceeds) will also reflect the cost of the greater information asymmetries for smaller firms. Negative coefficients for Log(proceeds) and Log(1 + sales) are both consistent with the information hypothesis. Log(1 + sales) also proxies for the greater information costs and greater difficulty in assessing the prospects of early-stage companies where revenue is typically small. In my sample, 21 of the companies had sales under $1 million and five had zero sales in the latest fiscal year. I add one to avoid taking the logarithm of zero.

[18] McConnell and Servaes (1990) document a significant positive relationship between firm value and the percentage of institutional ownership, and Brous and Kini (1994) find a significant positive relationship between announcement-period abnormal stock returns and the level of institutional ownership. See also Brickley, Lease, and Smith (1988) and Jarrell and Poulsen (1987). These findings are consistent with the efficient-monitoring hypothesis (Pound (1988)): Higher levels of institutional ownership lead to more effective monitoring by outside investors, including the use of proceeds from the new issue.

[19] The effect of insider ownership is complex. Wruck (1989) finds that changes in firm value around the date of the private sale of common stock are positively related to changes in ownership concentration when concentration is high (insiders own 25 percent or more of the common stock after the sale) or low (insiders own 5 percent or less) but that the relationship is negative when ownership concentration is moderate (insiders own between 5 percent and 25 percent). She concludes that within this middle range, increased ownership concentration promotes entrenchment of managers. McConnell and Servaes (1990) find that the relationship between firm value and the percentage of insider ownership is positive for increases in the percentage of insider ownership until this percentage reaches approximately 40 percent to 50 percent, at which point the relationship becomes slightly negative, which is consistent with Stulz (1988). Morck, Shleifer, and Vishny (1988) also note the tendency for higher levels of management ownership to promote managerial entrenchment. However, Hertzel and Smith (1993) do not detect a statistically significant relationship between the discount and either ownership concentration or the change in ownership concentration (see their Table VI). I did not find a significant relationship in my overall sample or in sub-samples based on less than 5 percent, between 5 and 25 percent, or greater than 25 percent insider ownership.

[20] Brickley, Coles, and Terry (1994) find significantly positive abnormal returns among poison-pill-adopting firms when outside directors comprise a majority on the board, but significantly negative returns when outside directors are a minority. Brickley and James (1987), Byrd and Hickman (1992), Rosenstein and Wyatt (1990), and Weisbach (1988) furnish additional empirical evidence of the link between the proportion of outside directors and shareholder wealth.

[21] The SYSTAT regression software identifies studentized residuals greater than 2.0 in absolute value as indicative of outliers. The outlier is a premium of 16.42 percent for Discount (day prior) and 47.17 percent for Discount (10 days prior). Hertzel and Smith's (1993) and Wruck's (1989) samples also include private placements that appeared to occur at a premium. Based on discussions with investment bankers, I suspect that the calculated premia might reflect a measurement problem. If the privately offered shares are priced before the initial announcement date (as sometimes happens) and the market price subsequently decreases sufficiently, the issue will appear to be priced at a premium when the new issue price is compared to the market price just prior to the announcement. Because the pricing date for the one observation that appeared to involve a premium relative to the previous day's closing price could not be determined, I dropped the outlier from the regressions.

[22] For empirical evidence, see Levy (1992) and the references therein. Beaulieu, Abu-Dayya, and McLane (1995) (BAM) describe four methods for analytically approximating the cumulative distribution function (CDF) of a random variable that is the sum of n i.i.d. lognormal random variables and compare these approximations to a numerical simulation of the actual distribution. They do not find that any one approximation dominates the others.

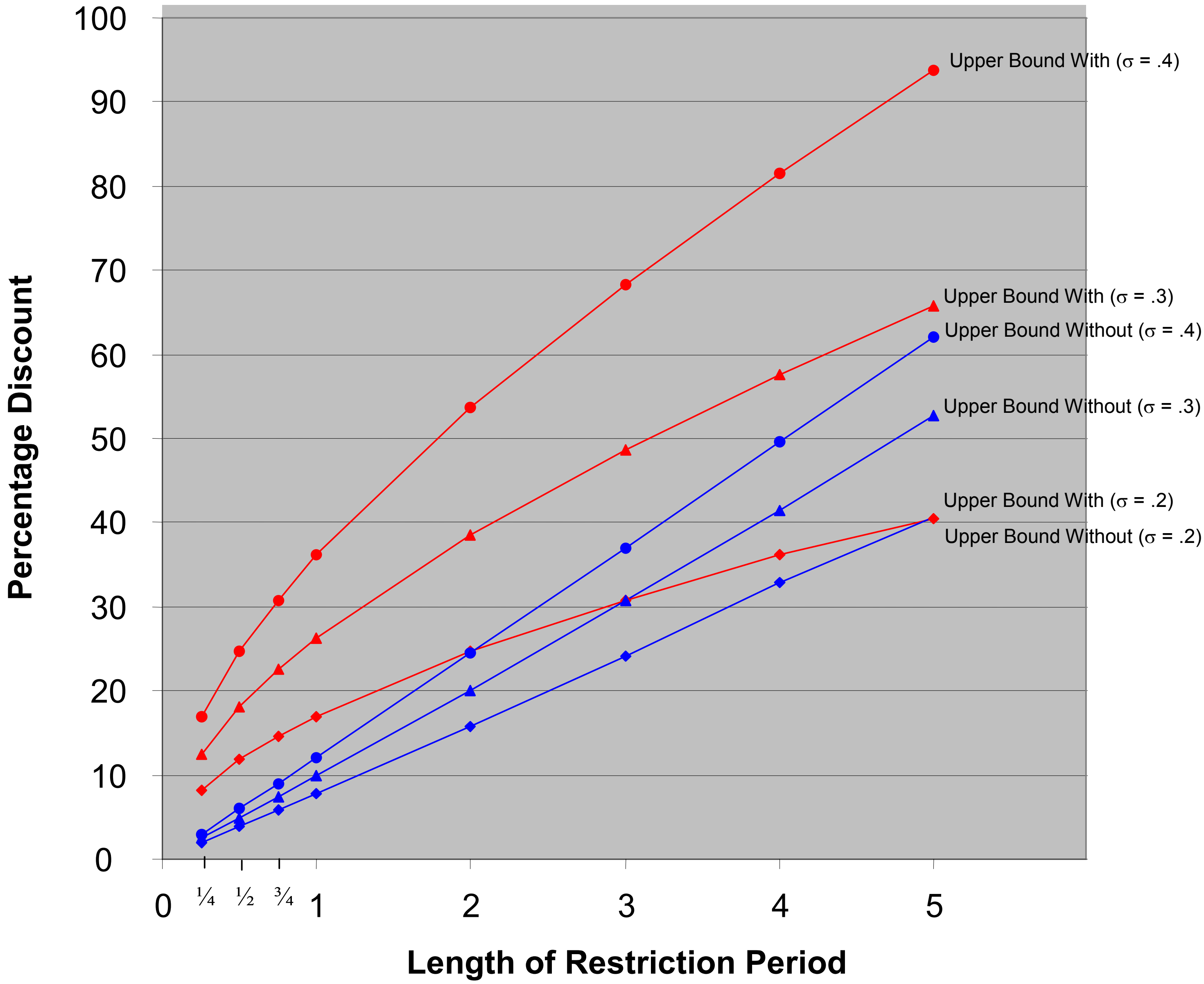

Figure 1. Comparison of the Marketability Discounts for Investors With and Without Special Timing Ability. The figure compares the upper bound on the marketability discount implied by Longstaff's model, which assumes perfect timing ability (Upper Bound With), and the upper bound on the discount obtained from equations (5) and (6), which assumes no special timing ability (Upper Bound Without), when $\sigma = 0.2$, $\sigma = 0.3$, and $\sigma = 0.4$. Since Longstaff's model assumes a non-dividend-paying stock, q is set equal to zero in equation (5).

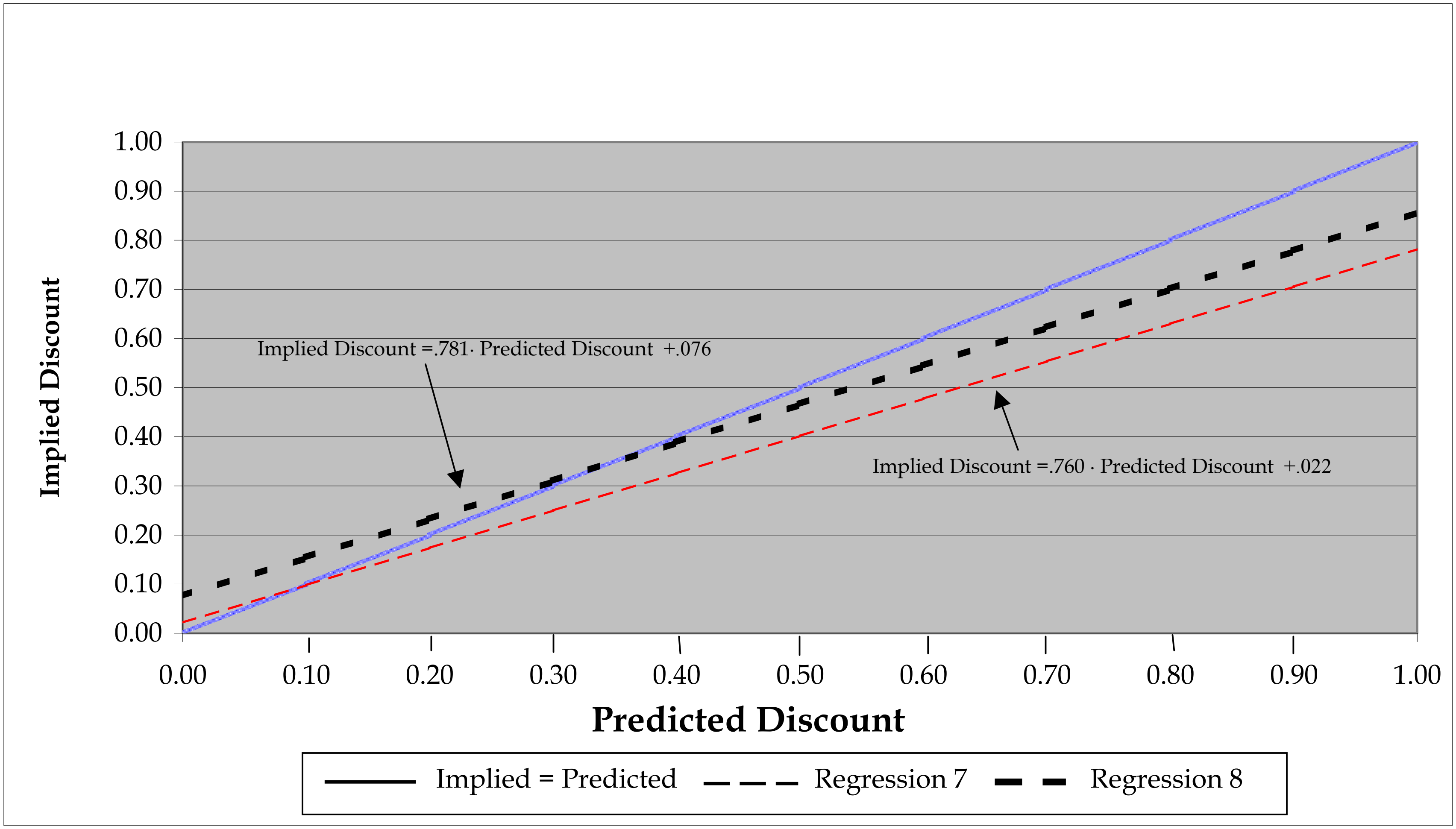

Figure 2. Implied Versus Predicted Discounts. The figure plots the linear equations corresponding to Regression 7 and Regression 8 in Table 6.

Table 1

Marketability Discounts

Equations (5) and (6) are used to calculate the percentage marketability discount for both non-dividend-paying stocks (q = 0) and for stocks that provide a 3% constant percentage dividend yield (q = .03). The range of marketability restriction periods is between 3 months and 5 years, and the range of stock price volatilities is between σ = 0.1 and σ = 0.8. The riskless rate is r = .05.

Restriction	q = 0							
Period	**σ=0.1**	**σ=0.2**	**σ=0.3**	**σ=0.4**	**σ=0.5**	**σ=0.6**	**σ=0.7**	**σ=0.8**
3 months	1.41%	1.89%	2.44%	2.99%	3.55%	4.11%	4.67%	5.23%
6 months	2.84	3.81	4.90	6.01	7.12	8.22	9.31	10.38
9 months	4.29	5.75	7.38	9.04	10.69	12.32	13.91	15.45
1 year	5.76	7.71	9.88	12.09	14.27	16.40	18.45	20.42
2 years	11.81	15.76	20.12	24.43	28.56	32.42	35.97	39.15
3 years	18.16	24.18	30.68	36.93	42.69	47.83	52.25	55.93
4 years	24.83	32.95	41.52	49.52	56.56	62.49	67.25	70.88
5 years	31.84	42.08	52.64	62.13	70.09	76.39	81.07	84.32

Restriction	q = .03							
Period	**σ=0.1**	**σ=0.2**	**σ=0.3**	**σ=0.4**	**σ=0.5**	**σ=0.6**	**σ=0.7**	**σ=0.8**
3 months	0.86%	1.42%	1.99%	2.56%	3.13%	3.69%	4.25%	4.81%
6 months	1.73	2.85	3.98	5.12	6.24	7.35	8.44	9.50
9 months	2.60	4.28	5.98	7.67	9.33	10.95	12.54	14.07
1 year	3.47	5.71	7.98	10.21	12.39	14.51	16.55	18.50
2 years	7.01	11.50	15.96	20.26	24.33	28.12	31.59	34.71
3 years	10.61	17.34	23.90	30.05	35.65	40.62	44.89	48.44
4 years	14.28	23.22	31.76	39.50	46.26	51.91	56.44	59.89
5 years	18.00	29.13	39.50	48.56	56.09	62.01	66.40	69.44

Table 2

Sample Descriptive Statistics

The letter stock sample consists of 101 private placements by NYSE, AMEX, and OTC firms between 1/1/91 and 2/3/97. Market value was obtained from Bloomberg L.P. and is measured as the closing price on the day immediately preceding the announcement for NYSE, AMEX, and NASDAQ-quoted stocks and as the average of the closing bid and ask prices for OTC stocks whose shares are not quoted in NASDAQ. The number of shares outstanding prior to the offering and the issuer's assets, sales, net income, and cash flow as of the latest fiscal year-end were obtained from Compustat. The gross proceeds and number of shares issued were obtained from Securities Data Company. The percentage discount was calculated from equations (7) and (8) using the closing prices on the day prior to the announcement and 10 trading days prior to the announcement of the offering, respectively.

Panel A: Listing Information

NYSE	AMEX	NASDAQ	OTC	Total
14	10	72	5	101

Panel B: Chronological Distribution

1991	1992	1993	1994	1995	1996	1997[a]	Total
12	21	26	16	6	18	2	101

Panel C: Descriptive Statistics for the Issuers

	Mean	Median	Minimum	Maximum	Standard Deviation
Market value of equity (millions)	$153.66	$78.92	$1.70	$1,220.47	$214.58
Total assets[b] (millions)	406.49	25.65	0.42	18,170.30	1,968.28
Total sales[b] (millions)	81.70	10.62	--	1,863.71	261.90
Net income[b] (millions)	-11.93	-1.81	-543.30	47.05	64.61
Cash flow from operations[b] (millions)	-3.82	-1.64	-543.30	108.32	57.83

Table 2 (Continued)

Panel D: Descriptive Statistics for the Private Placements					
	Mean	Median	Minimum	Maximum	Standard Deviation
Gross proceeds[c] (millions)	\$9.43	\$6.89	\$0.37	\$51.00	\$8.83
Fraction issued[d]	16.64%	14.45%	1.23%	81.07%	12.08%
Percentage discount:					
Prior day's close[e]	20.13	15.50	-16.42[g]	64.47	17.89
Day -10 close[f]	18.41	16.74	-47.17[g]	68.31	18.63

[a] Prior to February 20, 1997 when the SEC modified Rule 144 by shortening the resale-restriction period.

[b] For the latest fiscal year immediately preceding the offering.

[c] Before deducting the private placement agent's fee and the other expenses of the offering.

[d] Calculated as the number of shares placed divided by the sum of the number of shares placed and the number outstanding prior to the offering.

[e] Calculated from equation (7).

[f] Calculated from equation (8).

[g] Negative value indicates a premium.

Table 3

Cumulative Abnormal Returns Around the Announcements of

Private Placements of Restricted Stock

Mean and median cumulative abnormal returns (CARs) are calculated using a market model and the Scholes-Williams (1977) procedure to estimate beta. % Positive is the percentage of positive CARs. The sample includes 82 private placements of letter stock that took place in the U.S. between 1/1/91 and 2/3/97. The initial announcement date (the earlier of the offering announcement date, when one occurs, and the pricing announcement date) is day 0 in Panels A, C, and E, and the pricing announcement date is day 0 in Panels B and D. There were 18 private placements in which the announcement of the offering (or the intention to offer) preceded the announcement of the pricing and other terms of the offering, and 64 private placements for which the two announcements coincided. In all five panels, the comparison period extends from 120 trading days through 21 trading days prior to the initial announcement date for the private placement. The t-statistic appears in parentheses beneath the Mean CAR. A Wilcoxon signed rank test was performed on the CARs, and the z-statistic appears in parentheses beneath the % Positive.

Panel A: Initial Announcement (N = 82)[d]

	Period Relative to the Announcement Date (Day 0)						
Statistics	(-10,0)	(-5,0)	(-3,0)	(-1,0)	(-1,1)	(-3,3)	(-5,5)
Mean CAR	3.63%	1.29%	1.88 %	0.39%	1.07%	2.61%	1.71%
(t-statistic)	(2.10[b])	(1.01)	(1.80[b])	(0.53)	(1.18)	(1.89[b])	0.99
Median CAR	1.28	0.29	0.70	0.05	0.64	2.23	2.84
% Positive	54.9	51.2	53.7	50.0	54.9	65.9	61.0
(z-statistic)	(1.51[c])	(0.67)	(1.51[c])	(0.57)	(1.46[c])	(2.89[a])	(1.69[b])

Table 3 (Continued)

Panel B: Pricing Announcement (N = 82)[e]

Statistics	Period Relative to the Announcement Date (Day 0)						
	(-10,0)	(-5,0)	(-3,0)	(-1,0)	(-1,1)	(-3,3)	(-5,5)
Mean CAR	1.98%	0.77%	0.95%	0.17%	0.30%	1.25%	0.76%
(t-statistic)	(1.21)	(0.64)	(0.97)	(0.24)	(0.36)	(0.96)	(0.47)
Median CAR	1.22	0.55	0.56	-0.11	0.80	1.71	1.93
% Positive	54.9	53.7	52.4	48.8	53.7	62.2	56.1
(z-statistic)	(1.18)	(0.60)	(0.90)	(0.28)	(0.61)	(1.69[b])	(0.76)

Panel C: Offering Announcement that Precedes a Pricing Announcement[f] (N = 18)

Statistics	Period Relative to the Announcement Date (Day 0)						
	(-10,0)	(-5,0)	(-3,0)	(-1,0)	(-1,1)	(-3,3)	(-5,5)
Mean CAR	4.43%	-0.03%	2.81%	0.60%	1.90%	3.10%	-0.24%
t-statistic	(1.33[c])	(-0.01)	(1.40[c])	(0.42)	(1.09)	(1.16)	(-0.07)
Median CAR	-0.94	-2.73	3.04	0.55	-0.04	1.22	0.67
% Positive	44.4	38.9	55.6	55.6	50.0	61.1	55.6
(z-statistic)	(-0.28)	(-0.68)	(1.11)	(0.54)	(0.68)	(1.24)	(0.15)

Panel D: Pricing Announcement that Follows an Offering Announcement[f] (N = 18)

Statistics	Period Relative to the Announcement Date (Day 0)						
	(-10,0)	(-5,0)	(-3,0)	(-1,0)	(-1,1)	(-3,3)	(-5,5)
Mean CAR	-3.11%	-2.41%	-1.43%	-0.87%	-1.59%	-3.12%	-4.56%
(t-statistic)	(-0.91)	(-0.96)	(-0.70)	(-0.60)	(-0.90)	(-1.15)	(-1.34[c])
Median CAR	-2.46	-0.35	0.45	-0.25	-2.31	-2.28	-7.84
% Positive	44.4	50.0	50.0	50.0	44.4	44.4	33.3
(z-statistic)	(-1.15)	(-0.68)	(-0.28)	(-0.46)	(-0.89)	(-1.11)	(-1.50[c])

Table 3 (Continued)

Panel E: Offering and Pricing Announcements Coincide[e] (N = 64)							
	Period Relative to the Announcement Date (Day 0)						
Statistics	(-10,0)	(-5,0)	(-3,0)	(-1,0)	(-1,1)	(-3,3)	(-5,5)
Mean CAR	3.41%	1.66%	1.50%	0.33%	0.78%	2.41%	2.26%
t-statistic	(1.78[b])	(1.18)	(1.31[c])	(0.40)	(0.78)	(1.58[c])	(1.18)
Median CAR	2.18	0.74	2.25	-0.11	0.87	2.32	3.22
% Positive	57.8	54.7	53.1	48.4	56.3	67.2	62.5
(z-statistic)	(1.89[b])	(1.14)	(1.08)	(0.39)	(1.14)	(2.54[a])	(1.90[b])

[a,b,c] denote significance (based on a one-tailed test) at the 1%, 5%, and 10% levels, respectively.

[d] The initial announcement of the private placement. Of the 82 announcements, 18 specified an intention to offer common stock privately, and the other 64 indicated specific pricing terms to which investors had agreed.

[e] There were 64 private placements in which the initial announcement coincided with the pricing announcement.

[f] There were 18 private placements for which the pricing announcement followed a separate offering announcement.

Table 4

Bivariate Relationships

Discount (day prior) and Discount (10 days prior) are defined by equations (7) and (8), respectively. The bivariate relationship between each measure of the private placement discount and each of the independent variables is tested based on a Pearson correlation test. The sample includes 101 U.S. private placements of common stock between 1/1/91 and 2/3/97.

			Discount (day prior)		Discount (10 days prior)	
Independent Variable	Predicted Sign	Number of Observations	Pearson Correlation	p-Value[a]	Pearson Correlation	p-Value[a]
Volatility	+	101	0.520	0.000	0.497	0.000
Yield	-	101	-0.174	0.082	-0.156	0.119
Time	+	101	0.301	0.002	0.257	0.010
Rate	-	101	-0.297	0.003	-0.223	0.025
Book/Market	-	101	0.056	0.577	0.081	0.419
Fraction	+	101	0.262	0.008	0.261	0.008
Log (proceeds)	-	101	-0.644	0.000	-0.653	0.000
Log (1 + sales)	-	101	-0.232	0.019	-0.265	0.007
Cash flow/Assets	-	101	-0.087	0.385	-0.107	0.289
Leverage	+	101	-0.006	0.951	-0.036	0.722
Inside	?	88	-0.047	0.660	-0.086	0.428
Instit	-	69	-0.072	0.556	-0.030	0.808
Direct	-	92	-0.220	0.035	-0.252	0.015
Exchange	-	101	-0.236	0.018	-0.251	0.011

[a] Based on a two-tailed test.

Table 5

Multivariate Regression Results

Regression model 1 is

$$\text{Discount}\left(\text{day prior}\right) = a_0 + a_1 \text{ Volatility} + a_2 \text{ Yield} + a_3 \text{ Time} + a_4 \text{ Rate} \tag{9}$$

which includes only the four transfer restriction variables. Regression model 2 is

$$\text{Discount}\left(\text{day prior}\right) = a_0 + a_1 \text{ Log (proceeds)} + a_2 \text{ Fraction} + a_3 \text{ Direct} + a_4 \text{ Exchange} \tag{10}$$

which includes only information and ownership concentration variables. Regression model 3 is

$$\text{Discount}\left(\text{day prior}\right) = a_0 + a_1 \text{ Volatility} + a_2 \text{ Yield} + a_3 \text{ Time} + a_4 \text{ Rate} + a_5 \text{ Direct } + a_6 \text{ Adjusted Log}\left(\text{proceeds}\right) \tag{12}$$

which includes the four transfer restriction variables, Direct, and Adjusted Log(proceeds). This last variable is the residual of Log(proceeds) in equation (11). The dependent variable in Regressions 4, 5, and 6 is Discount (10 days prior). The regressions are fitted using ordinary least squares. The sample includes 100 U.S. private placements between 1/1/91 and 2/3/97 after deleting the observation that SYSTAT identified as an outlier. Nine observations had to be dropped from the regressions that include the variable Direct because the nine firms' historical proxy statements could not be obtained. The predicted sign is provided next to each coefficient. The table reports the heteroskedasticity-consistent estimator for each coefficient, and the t-statistic is given in parentheses beneath the estimated coefficient. In addition to the F statistic for each regression, the change in the F statistic is reported for Regressions 3 and 6 when the information and equity ownership concentration variables Direct and Adjusted Log(proceeds) are added to Model 1 $\left(\Delta F\left(\text{In / Own}\right)\right)$ and when the transfer restriction variables are added to a regression in which Direct and Adjusted Log(proceeds) are the independent variables $\left(\Delta F\left(\text{Option}\right)\right)$.

Table 5 (Continued)

	Regression 1	**Regression 2**	**Regression 3**	**Regression 4**	**Regression 5**	**Regression 6**
Number of Observations	100	91	91	100	91	91
Dependent Variable	Discount (day prior)	Discount (day prior)	Discount (day prior)	Discount (10 days prior)	Discount (10 days prior)	Discount (10 days prior)
Intercept	0.216	0.476	0.236	0.126	0.503	0.200
	(2.171[b])	(5.916[a])	(2.522[b])	(1.187)	(6.799[a])	(2.252[b])
Volatility (+)	0.244		0.224	0.246		0.227
	(5.278[a])		(5.383[a])	(5.009[a])		(5.051[a])
Yield (-)	-0.024		-0.647	-0.159		-0.962
	(-0.080)		(-1.785[c])	(-0.457)		(-2.414[b])
Time (+)	0.004		0.003	0.004		0.002
	(3.197[a])		(5.087[a])	(2.292[b])		(3.681[a])
Rate (-)	-3.851		-2.264	-2.416		-0.394
	(-2.560[a])		(-1.557)	(-1.481)		(-0.256)
Direct (-)		-0.177	-0.138		-0.230	-0.255
		(-1.736[c])	(-1.328)		(-2.508[b])	(-2.439[b])
Log (proceeds) (-)		-0.231			-0.234	
		(-6.538[a])			(-7.263[a])	
Adjusted Log(proceeds) (-)			-0.176			-0.201
			(-3.583[a])			(-4.372[a])
Fraction (+)		0.093			0.107	
		(0.876)			(0.859)	
Exchange (-)		-0.006			-0.023	
		(-0.190)			(-0.729)	
Adjusted R^2	0.361	0.421	0.480	0.319	0.490	0.511
F	15.00[a]	17.35[a]	14.86[a]	12.57[a]	22.59[a]	16.71[a]

Table 5 (Continued)

	Regression 1	Regression 2	Regression 3	Regression 4	Regression 5	Regression 6
<u>F change:</u>						
ΔF(In / Own)[d]			10.868[a]			17.235[a]
ΔF(Option)[e]			12.752[a]			12.622[a]
D.W.	1.83	1.93	1.96	2.07	2.28	2.30

[a,b,c] Denote significance (based on a two-tailed test) at the 1%, 5%, and 10% levels, respectively.
[d] The change in the F statistic from adding the information and ownership concentration variables Direct and Adjusted Log(proceeds) to Regressions 1 and 4.
[e] The change in the F statistic from adding the transfer restriction variables Volatility, Yield, Time, and Rate to a regression model with Direct and Adjusted Log(proceeds) as the only explanatory variables.

Table 6

Test Results for Implied Versus Predicted Transferability Discounts

This table compares the transferability discounts calculated for the reduced sample of 91 common stock private placements after adjusting for information and equity ownership concentration effects to the discounts predicted by the model (5)-(6). Panel A compares the Implied and Predicted Discounts for different stock price volatility ranges. Panel B reports the results of difference of means tests and Wilcoxon signed rank tests comparing the Implied and Predicted Discounts. Panel C reports the results of fitting the regression

$$\text{Implied Discount} = a_0 + a_1 \cdot \text{ Predicted Discount.} \tag{14}$$

The Implied Discount is obtained from equation (13), and the Predicted Discount is obtained from equations (5)-(6).

Panel A: Implied Versus Predicted Transferability Discounts for Different Volatilities

Volatility Range	Mean Predicted Discount	Mean Implied Discount (day prior)	Mean Implied Discount (10 days prior)	Number of Observations
0-29.9%	15.0%	22.1%	28.7%	8
30-59.9	26.7	18.9	24.3	24
60-89.9	36.8	28.0	34.7	40
90-119.9	46.5	40.2	43.7	10
≥ 120	50.5	49.0	56.9	9

Panel B: Difference of Means and Wilcoxon Signed Rank Test Results

	Mean Predicted Discount	Mean Implied Discount	Difference of Means t-statistic[d]	Number of Positive Differences[e]	Wilcoxon Signed Rank Test z-statistic	Number of Observations
Discount (day prior)	34.64%	28.50%	-4.02[a]	27	-3.74[a]	91
Discount (10 days prior)	34.64	34.64	-0.01	47	-0.08	91

Table 6 (Continued)

Panel C: Regression Results		
	Regression 7	Regression 8
Number of Observations	91	91
Dependent Variable	Implied Discount (day prior)	Implied Discount (10 days prior)
a_0	0.022 (0.41)	0.076 (1.546[c])
a_1	0.760	0.781
t-statistics for a_1:		
$H_0 : a_1 = 0$ $\quad H_a : a_1 > 0$	5.20[a]	5.76[a]
$H_0 : a_1 = 1$ $\quad H_a : a_1 < 1$	1.64[c]	1.62[c]
Adjusted R^2	0.225	0.263
F	27.06[a]	33.19[a]
D.W.	1.79	2.28

[a, b, c] denote significance (based on a one-tailed test) at the 1%, 5%, and 10% levels, respectively.

[d] For mean Implied Discount minus mean Predicted Discount.

[e] The difference is Implied Discount minus Predicted Discount.