The effect of liquidity on non-marketable securities

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Abstract

We generalize the prevailing theoretical models that estimate the discount on securities for lack of marketability, by considering the discrete trading frequency of the securities. The generalization shows that accounting for the illiquidity of securities may significantly reduce their non-marketability discount.

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1. Introduction

A non-marketability discount is the compensation that investors demand for being unable to trade a security compared to an all-else-equal security with no such constraint. Legal and contractual restrictions on trading are common in financial markets. Examples include stock lockups in IPOs (Brav and Gompers, 2003; Ding, Getmansky, Liang, and Wermers, 2009), merger and acquisition agreements, equity-based compensation (Abudy and Benninga, 2013), and private placements of publicly traded companies, under Rule 144 of the Securities and Exchange Commission (SEC). Longstaff (1995) calculates the upper bound for the non-marketability discount by assuming that without trading restrictions, an investor with perfect market-timing ability can sell a security at its maximum price during the restricted trading period. Thus, the upper bound for the non-marketability discount is priced as the difference between this maximum price during the restricted trading period and the security price at the end of this period. Finnerty (2012a) assumes that in the absence of trading restrictions, an investor is equally likely to sell a security at any time during the restricted trading period. Hence, the discount is calculated relative to the average value of the security during this period.[1]

Both Finnerty's (2012a) and Longstaff's (1995) models make the unrealistic assumption that securities are continuously traded in frictionless markets. However, in practice, in most cases in which non-marketability is involved securities are illiquid.[2] We bridge this gap by adjusting the non-marketability discount for illiquidity risk. First, we relate to the *frequency of trading,* where a security may be traded in various trading frequencies

[1] The paper refers to the marketability discount of the firm's shares, and therefore the terms security and stock are used interchangeably. For the case of a corporate bond a structural model should be introduced as in Abudy and Raviv (2016).

[2] For example, in the sample analyzed by Finnerty (2012b), one-third of the placements were quoted in the National Association of Securities Dealers Automated Quotation (NASDAQ) small cup, the OTC Bulletin Board, or Pink Sheets markets.

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(Kalay, Sade and Wohl, 2004). Exogenous factors, such as the trading mechanism, could result in lower frequency trading, which may affect the marketability discount (Kalay, Wei and Wohl, 2002). Allen, Gottesman and Peng (2012) find that the presence of dual market makers in the equity market and in the over-the-counter (OTC) loan market improves the liquidity of the more competitive and transparent equity market, but widens the spread of the less competitive OTC market. Second, we introduce a *limit on the traded quantity*, where the informed investor with perfect market-timing ability has to sell his holding in several transactions to avoid a potential impact on its fundamental price, as described in Dyl and Jiang (2008) and Albuquerque and Schroth (2015).[3] In this respect, the result of Jacoby and Zheng (2010) is consistent with the idea that higher ownership dispersion improves market liquidity, indicating that stocks concentrated ownership firms are more illiquid.

We show that when illiquidity is introduced, the contributions of Longstaff (1995) and Finnerty (2012a) are special, corner solutions of our general method. In the case where there are no limits on the traded quantity, a perfect-timer investor, in the absence of trading restrictions, will sell a security at its maximum price during the restricted trading period. For this case, Longstaff (1995) provides an upper bound for the non-marketability discount. At the other extreme, if an investor needs to sell his holding in a security in an infinite number of transactions, the solution converges to the model of Finnerty (2012a), and the method provides a lower bound for the non-marketability discount. While these two models may accurately describe the non-marketability discount of a security with very specific liquidity conditions, our method covers a wider array of liquidity conditions that can fit a variety of real-world circumstances. Moreover, our approach presents the difference between

[3] Our approach considers infrequent trading and not trade that results from asymmetric information. In fact, one can view Longstaff (1995) as relating to an extreme case of illiquidity, since the investor with perfect market-timing ability is perfectly informed while all the other traders in the market have the same information. This is the reason that Longstaff's model yields an upper bound for the non-marketability discount.

Longstaff's (1995) and Finnerty's (2012a) discounts as a result of different assumptions on the security's liquidity and not due to the timing abilities of the investors.

Using our general method, one can accurately estimate the non-marketability discount while considering the liquidity of a security. For example, Dyl and Jiang (2008) analyze a case study of an estate owning 396,000 unregistered shares of the Elite Financial Corporation, which had a mean daily trading volume of 4,113 shares during the study period. They estimate that the non-marketability discount using Longstaff's (1995) model is 59%. However, as we shall see below, applying our method and considering the limit on the traded quantity and on the frequency of trading yields a substantially lower non-marketability discount of 36.6%.

An upper bound for non-marketability discount

Our method generalizes the prevailing models, which assume that securities are traded continuously in frictionless markets, to a discrete process. In a continuous framework, the value of the firm's security V_T is governed by a geometric Brownian motion process under a risk-neutral measure, as in Equation (1) below:

$$dV_T = rV_T dt + \sigma V_T dW_T \tag{1}$$

where r is the risk-free rate, σ is the volatility of the firm's security, and W_T is a standard Brownian motion process. Longstaff (1995) assumes a hypothetical investor who is restricted from trading a specific security during a predetermined period t. With no trading restrictions, an investor with perfect market-timing ability will sell the security at its maximum price during the restricted trading period, at time τ, and will reinvest the proceeds at the risk-free rate till the end of the period. Hence, the payoff to the perfect-timer investor at the end of the period denoted by M_t is

$$M_t = \underset{0 \le \tau \le t}{Max}\left(e^{r(t-\tau)} \cdot V_\tau\right) \qquad (2)$$

where V_τ denotes the maximum price of the security at time τ during the restricted trading period *t* (i.e., $0 \le \tau \le t$).

If the perfect-timer investor is restricted from trading, the result is a loss that is equal to the incremental value $Max\left(0, M_t - V_t\right) \equiv M_t - V_t$ at time *t*. The present value of this cash flow is the upper bound for the non-marketability discount:

$$e^{-rt} \cdot \left(E\left[M_t - V_t\right]\right) \qquad (3)$$

The impact of illiquidity on the non-marketability discount

A limit on the *frequency of trading* is introduced by shifting from the continuous process as in Equation (1) to a discrete setting, where $V_{T,}$ is expressed as

$$\Delta V_T = rV_T \Delta t + \sigma \Delta V_T W_T \qquad (4)$$

where Δ stands for a discrete frequency of trading. For example, assuming that the security is traded twice daily, then $\Delta t = \frac{1}{2 \cdot s}$, where *s* is the number of trading days per year. The drift and volatility of the stock are set accordingly.

A *limit on the traded quantity* accounts for the case where an investor who trades a large quantity may impact the security price or may not be able to execute the entire quantity in a single trade. Therefore, we consider the case where the trade is divided into several transactions. An investor with perfect market-timing ability will choose to sell at the points of time that have the highest price during the restricted trading period. The payoff, at the end of the period, is expressed as

$$M_t = E\left[\underset{0\le\bar{\tau}\le t}{Max}\left(e^{r(t-\bar{\tau})}\cdot V_{\bar{\tau}}\right)\right] \quad (5)$$

where $\bar{\tau}$ denotes a vector from 1 to n that includes the times of the highest n values of V during the restricted trading period (e.g., n can be regarded as the number of transactions), and $V_{\bar{\tau}}$ is a vector that includes the maximum values of the security in this period. For example, if the entire trade splits into three transactions, and the security receives its maximum price at times τ_1 τ_2 τ_3, then $\bar{\tau} = \{\tau_1, \tau_2, \tau_3\}$, and $V_{\bar{\tau}} = \{V_{\tau_1}, V_{\tau_2}, V_{\tau_3}\}$ includes the maximum values of the security. The payoff at the end of the period is $M_t = E\left[e^{r(t-\tau_1)}\cdot V_{\tau_1} + e^{r(t-\tau_2)}\cdot V_{\tau_2} + e^{r(t-\tau_3)}\cdot V_{\tau_3}\right]$. Since there is no closed-form solution for the pricing equations, we solve them by using a Monte Carlo simulation.[4]

Figure 1 demonstrates the two trading effects of the perfect timer investor. The figure presents one specific realization of the value of a security during a restricted trading period of 60 days. Panel A relates to the case of a limit on the frequency of trading and Panel B relates to the case of a limit on both the frequency and the traded quantity. In Panel A, the investor can execute the trade in a single transaction. However, since we assume that the stock is traded in a daily frequency, the maximum value of the stock is below its continuous maximum and is equal to $34.49. Since the value of the security at the end of the period is $30.00, the potential loss of the perfect timer investor is equal to $4.49. In Panel B, which demonstrates the effect of a limit on the traded quantity as well, the investor must divide the trade into six different transactions, each made on a different trading day. Therefore, instead of having a single maximum, there are six different transactions executed on these six days with the highest stock price. The average price of the security on these days is $33.86, and the potential loss of the investor on this price path is $3.86, which is 14% less than in the case of

[4] We run at least 50,000 simulation paths, till we reach a stable solution in the first four digits.

a single transaction. Note that the example shows only one simulation path of the security, and to calculate the effect of illiquidity on the marketability discount one should simulate a large enough number of realizations over the restricted trading period.

The previously mentioned contributions of Longstaff (1995) and Finnerty (2012a) are special cases of our general method. First, with continuous trading in the security, where $\Delta t \rightarrow dt$, and an investor who can execute the sell in a single transaction, $n=1$, the model converges to the solution of Longstaff (1995). Second, with continuous trading, $\Delta t \rightarrow dt$, and an investor who has to sell his holding in an infinite number of transactions, $n \rightarrow \infty$, the model converges to the solution of Finnerty (2012a). All other cases, which represent different trading frequencies, are in between the upper bound of Longstaff (1995) and the lower bound of Finnerty (2012a).

Applying the method: A real world case study

The effect of the limit on the traded quantity and on the frequency of trading is demonstrated by referring to a case study analyzed by Dyl and Jiang (2008), who estimate the discount due to the lack of marketability of the unregistered shares of the Elite Financial Corporation for tax estate purposes in 1998.[5] The estate owned 396,000 common shares of the firm. In 1998, the stock was traded on NASDAQ, with 3.6 million outstanding shares, and with a mean (median) daily trading volume of 4,113 (1,900) shares.[6] Given the SEC regulation on unregistered stocks, the maximum number of shares that the estate was allowed to sell was 36,000 shares per quarter. Dyl and Jiang (2008) estimate that the average time that the estate's shares were restricted from trading was 1.375 years and that the stock's volatility was

[5] Unregistered shares are subject to selling limits under SEC Rule 144. The Rule sets that the number of restricted shares that can be sold during any three-month window cannot exceed more than 1% of the firm's outstanding shares, and sets the average weekly trading volume of the stock (calculated over the 4 weeks prior to the week in which the sale occurs).

[6] Considering the market price of Elite in September 1998, the total value of the estate's shares was slightly more than $6 million.

60.5%. Using Longstaff's (1995) model the authors calculate a non-marketability discount of 59%.

The use of Longstaff's (1995) model overestimates the non-marketability discount since it assumes that the entire holdings of the estate can be sold in a single transaction; however, under the SEC regulation and given the low daily trading in the stock, a sale of 396,000 securities would need to be divided into several transactions. With an average daily volume of 4,113 shares, this means that at least 9 transactions per quarter would be needed to sell the entire holdings of the estate, which leads to a non-marketability discount of 36.6% (see Table 1). If the stock is traded four times a day, then 36 transactions would be needed per quarter, which yields a discount of 37.0%. These levels are significantly lower than the discount of 59% calculated by Longstaff (1995) and significantly higher than the 15% discount calculated by Finnerty (2012a), where, in both cases, the estate's holdings are sold in an infinite number of transactions.

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Table 1 The non-marketability discount as a function of the frequency of trading and the limit on the traded quantity

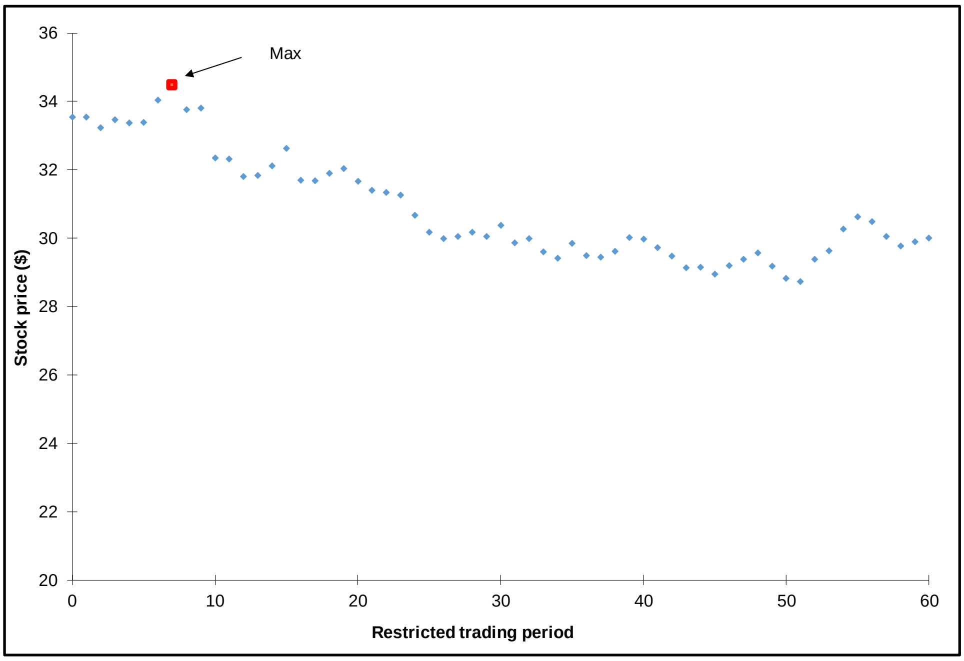
The table presents the non-marketability discount of an illiquid security as a function of the security's frequency of trading and the limit on the traded quantity. The calculations use the real-world case of Dyl and Jiang (2008).

		Number of daily transactions of the security					
		1	2	4	8	24	48
Number of quarterly transactions of investor during the non-marketability period	1	49.7%	51.8%	53.3%	54.0%	54.9%	55.8%
	2	47.3%	50.1%	52.1%	53.2%	54.5%	55.4%
	3	45.0%	48.5%	51.0%	52.4%	54.0%	55.1%
	4	43.4%	47.4%	50.3%	51.9%	53.7%	54.9%
	5	41.7%	46.2%	49.4%	51.3%	53.4%	54.7%
	6	40.4%	45.3%	48.8%	50.9%	53.2%	54.5%
	7	39.0%	44.3%	48.1%	50.4%	52.9%	54.3%
	8	37.9%	43.5%	47.6%	50.0%	52.7%	54.2%
	9	36.6%	42.6%	47.0%	49.6%	52.4%	54.0%
	10	35.6%	41.9%	46.5%	49.3%	52.2%	53.9%
	18	28.1%	36.7%	42.9%	46.8%	50.8%	52.9%
	27	21.3%	31.9%	39.7%	44.6%	49.6%	52.0%
	36	15.5%	28.0%	37.0%	42.7%	48.5%	51.2%

Figure 1

Simulating security price with one transaction each trading day
The figure plots the stochastic process of a security price. The security has one transaction each trading day. The simulation is executed using a Geometric Brownian Motion volatility of 30%. Panel A shows the highest price of the security. Panel B shows the six days with the highest security prices.

Panel A:



Panel B: