An Average-Strike Put Option Model of the Marketability Discount

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Stock transfer restrictions, such as the resale restrictions imposed by Rule 144 when unregistered stock is privately placed, reduce shareholders' flexibility to time the sale of their shares and result in a loss of option value in the form of a marketability discount. This article models the marketability discount as the value of an average-strike put option. It shows that the model-predicted marketability discounts are consistent with actual private placement discounts after adjusting for the information, ownership concentration, and overvaluation effects that accompany a stock private placement.

Numerous studies have documented average discounts between 13.5% and 33.75% in private placements of letter stock, which is not freely transferable because of resale restrictions imposed by Rule 144 under the Securities Act of 1933 (Hertzel and Smith [1993] and Hertzel et al. [2002]).[1] Business appraisers typically apply marketability discounts of between 25% and 35% for a two-year restriction period and 15% and 25% for a one-year restriction period (Longstaff [1995]).

Private placement transfer restrictions entail a loss of timing flexibility, which imposes a cost that can be modeled as the value of a forgone put option (Longstaff [1995, 2001] and Kahl, Liu, and Longstaff [2003]). Longstaff [1995] obtains an upper bound on the marketability discount by modeling the value of marketability as the price of a lookback put option.[2] This model assumes that investors have perfect market-timing ability. This assumption is generally consistent with empirical evidence that private information enables insiders to time the market and realize excess returns (Gompers and Lerner [1998]). However, it is inconsistent with evidence that outside investors, at least on average, do not have any special ability to outperform the market (Graham and Harvey [1996] and Barber and Odean [2000]). Thus, while the lookback put option model may be appropriate in the presence of asymmetric information for equity placed with insiders who possess valuable private information, it is likely to overstate the discount when this situation does not exist.

This article models the marketability discount as the value of an average-strike put option. The investor is not assumed to have any special market-timing ability; instead, I assume that the investor would, in the absence of any transfer restrictions, be equally likely to sell the shares anytime during the restriction period. I compare the model-predicted discounts to the discounts observed in a sample of 208 discounted common stock private placements and find that the average-strike put option model discounts are consistent with the observed discounts after adjusting for the information and other effects that accompany a stock private placement.

The rest of the article is organized as follows. I develop the contingent claims model and compare the model-predicted marketability discounts to those implied by actual private placement discounts. I then present my conclusions.

AVERAGE-STRIKE PUT OPTION MARKETABILITY DISCOUNT MODEL

Assumptions

I make the following assumptions:

A1. The firm's unrestricted shares trade continuously in a frictionless market. The firm also has restricted shares outstanding, and all the firm's shares are identical except for the transfer restrictions.

A2. Transfer restrictions prevent the investor from selling the restricted shares for a period of length T.

A3. Any cash dividends are paid continuously during the time interval $[0, T]$ at a rate of $q \geq 0$ that is proportional to V.

A4. $V(t)$ is the value of a share of common stock without transfer restrictions. $V(t)$ follows a geometric diffusion process of the form

$$dV = (\mu - q)\,Vdt + \sigma\, VdZ \qquad (1)$$

where μ and σ are constants and Z is a standard Wiener process.

A5. The riskless interest rate r is constant and the same for all maturities during $[0, T]$.

A6. No shareholder has any special market-timing ability. In the absence of resale restrictions, investors would be equally likely to sell the shares anytime during the restriction period.

Suppose that a stock pays cash dividends ($q > 0$) and that the shareholder can sell the registered shares at t, $0 < t < T$, and reinvest the proceeds in the riskless asset until T. In a risk-neutral world, the investor would be indifferent between selling the share immediately for $V(t)$ and selling it forward for delivery at T with forward price $e^{(r-q)(T-t)}V(t)$. Suppose further that the investor would want to sell the unregistered shares prior to T were it not for the resale restrictions. Since the investor lacks any special timing ability, assume that the investor would be equally likely to sell unrestricted shares at $N+1$ discrete points in time and that these points are equally spaced, so that the investor considers selling at $t = 0$, $t = T/N$, $t = 2T/N$, …, $t = NT/N = T$. In a risk-neutral world, such an investor would be indifferent between holding a registered share and holding an unregistered share plus a series of forward contracts all expiring at T and having an average forward price equal to $\frac{1}{N+1}\sum_{j=0}^{N}[e^{(r-q)T(N-j)/N}V(jT/N)]$. If the investor's transfer restriction risk exposure could be perfectly hedged, or if this risk is idiosyncratic with respect to the investor's securities portfolio, then the unregistered shares would be priced on a risk-neutral basis.

Under Rule 144, a purchaser of restricted shares is permitted to hedge the price risk exposure, but only after paying in full for the shares (U.S. Securities and Exchange Commission [SEC] [2007]). Prehedging is not permitted, so the feasibility and cost of hedging are uncertain at the time the unregistered shares are purchased. After buying unregistered shares, the purchaser can sell short to hedge, if it can borrow unrestricted shares. However, it must comply with Rule 144's restrictions on the short sales and bear the cost of maintaining the hedge.[3] Alternatively, the purchaser could buy put options or sell call options, if there are traded options on the stock, or it could arrange an equity swap, if it can locate a dealer who is willing to take the other side of the swap.[4]

While price risk is hedgeable, liquidity risk is not. Because of the restrictions on hedging, the investor bears an opportunity cost due to the transfer restrictions if

$$\frac{1}{N+1}\sum_{j=0}^{N}[e^{(r-q)T(N-j)/N}V(jT/N)] > V(T) \qquad (2)$$

but realizes an opportunity gain if the inequality is reversed. If the investor's transfer restriction risk exposure is idiosyncratic, then the gains and losses are offset, and so the transferability discount would be zero provided the investor has adequate liquidity from other sources such that the transfer restrictions do not cause the investor to miss any positive-net present value (NPV) investment opportunities. In effect, the restricted share is a long forward contract for the delivery of an otherwise identical unrestricted share at T when the transfer restrictions expire.

With any unhedged nonidiosyncratic risk exposure, a risk-averse investor would demand a risk premium, and the transferability discount would be nonzero. Following

Longstaff [1995], the inequality of Equation (2) suggests that an upper bound on the investor's opportunity cost can be modeled as

$$\max\left\{0, \frac{1}{N+1}\sum_{j=0}^{N}[e^{(r-q)T(N-j)/N}V(jT/N)]-V(T)\right\} \quad (3)$$

Equation (3) is the payoff function for an average-strike put option in which the strike price is the arithmetic average of the forward prices $\frac{1}{N+1}\sum_{j=0}^{N}[e^{(r-q)T(N-j)/N}V(jT/N)]$.

Equation (3) furnishes an upper bound on this opportunity cost because there are states of the world in which the inequality of Equation (2) is reversed. However, even in these states, the investor may miss positive-NPV investment opportunities unless adequate sources of liquidity are available. If profitable investment opportunities would be missed, then Equation (3) would understate the cost of the transfer restrictions if the missed opportunities are worth more than the gains that would result when the inequality of Equation (2) is reversed. How well this approach to modeling the discount for lack of free transferability explains actual discounts is an empirical question that is addressed later in the article.

One other factor needs to be considered. The transfer restrictions are costly only if the investor would sell the shares on or before T absent such restrictions. Suppose there is some likelihood $p > 0$ that the investor would want to hold the stock past T even without resale restrictions. Again, assuming that prior to T, any sale would be equally likely to occur anywhere in [0, T], the payoff function becomes

$$\begin{aligned}&\max\left\{0, (1-p)\times\left(\frac{1}{N+1}\sum_{j=0}^{N}[e^{(r-q)T(N-j)/N}V(jT/N)]-V(T)\right)\right\}\\&=(1-p)\\&\times\max\left\{0, \frac{1}{N+1}\sum_{j=0}^{N}[e^{(r-q)T(N-j)/N}V(jT/N)]-V(T)\right\}\end{aligned} \quad (4)$$

In this case, the transferability discount equals $1-p$ times the discount calculated, assuming the investor would otherwise always sell sometime prior to T.

Average-Strike Put Option Model

I obtain a formula for an upper bound on the value of the marketability discount by valuing the average-strike put option whose payoff is Equation (3). First, applying the risk-neutrality transformation of Cox and Ross [1976], the stock price can be described by the geometric diffusion process

$$dV = (r-q)Vdt + \sigma VdZ \quad (5)$$

$\ln V(T)$ is normally distributed with mean $\ln V_0 + (r-q-½\sigma^2)T$ and standard deviation $\sigma\sqrt{T}$, where V_0 is the stock price at $t = 0$. Similarly, the forward price $F(t) = e^{(r-q)(T-t)}V(t)$ follows the martingale process

$$dF = \sigma FdZ \quad (6)$$

in a risk-neutral world, and $\ln F(t)$ is normally distributed with mean $\ln F_0 - ½\sigma^2 t$ and standard deviation $\sigma\sqrt{t}$ where $F_0 = F(0) = V_0 e^{(r-q)T}$.

In the risk-neutral framework, the expected strike price of the average-strike put option as of time $t = 0$ is the average of the risk-neutral forward prices, which I will denote $E[A(T)]$:

$$E[A(T)] = \frac{1}{N+1}\sum_{j=0}^{N}F(t_j) \quad (7)$$

where $t_j = jT/N$ and the forward prices follow the martingale process in Equation (6). The payoff function in Equation (7) contains the sum of a set of correlated lognormal random variables. Although expressions exist for the moment-generating function, mean, and variance of the sum of two lognormal random variables, no exact closed-form expression for the density function of the sum of a set of lognormal random variables is known. A sum of independent lognormal random variables can be closely approximated by a lognormal random variable.[5] Ritchken, Sankarasubramanian, and Vijh [1993] show that the distribution of the average of a set of correlated lognormal stock prices or exchange rates can be approximated by a lognormal distribution with acceptable accuracy by applying Wilkinson's method, which matches the first and second moments. I use Wilkinson's method to approximate the distribution of $A(T)$.

Assume that $\ln A(T)$ is normally distributed with mean $\alpha_1(T)$ and variance $\nu_1(T)^2$. Since $\ln V(T)$ is also normally distributed, assume that $X(T) = [ln\ A(T), ln\ V(T)]$ is approximately bivariate normal. An average-strike put option can be characterized as the option to exchange a package of forward contracts on a share for the underlying share and evaluated by applying Hull's [2009] generalization of Margrabe's [1978] expression for the value of an option to exchange one asset for another adjusted for a dividend-paying stock. This approach requires expressions for the means α_1 and α_2, the volatilities ν_1 and ν_2, and the correlation between the diffusion processes ρ.

First, write the moment-generating function for $X(T)$ as

$$M(k_1,k_2) = E^*[A(T)^{k_1} V(T)^{k_2}]$$
$$= e^{\alpha_1 k_1 + \alpha_2 k_2 + \frac{1}{2}(\nu_1^2 k_1^2 + 2\rho\nu_1\nu_2 k_1 k_2 + \nu_2^2 k_2^2)} \tag{8}$$

where E* is the expectations operator conditional on V_0 and where $\alpha_2 = ln\ V_0 + (r - q - \frac{1}{2}\sigma^2)T$ and $\nu_2 = \sigma\sqrt{T}$. Since $F_0 = V_0\, e^{(r-q)T}$, write the mean as $\alpha_2 = ln\ F_0 - \frac{1}{2}\sigma^2 T = ln\ F_0 - \frac{1}{2}\nu_2^2$. Next, obtain expressions for α_1 by substituting $(k_1,k_2) = (1,0)$, α_2 by substituting $(k_1,k_2) = (0,1)$, ν_1^2 by substituting $(k_1,k_2) = (2,0)$, ν_2^2 by substituting $(k_1,k_2) = (0,2)$, and the covariance term $\rho\nu_1\nu_2$ by substituting $(k_1,k_2) = (1,1)$ into Equation (8) and rewriting the resulting equations. These equations require expressions for $E^*[A(T)]$, $E^*[A(T)^2]$, and $E^*[A(T)V(T)]$, which can be obtained after some tedious algebra. Substituting into the expressions for the volatilities and the covariance term leads to the following expressions:

$$\nu_1^2 = ln[2\{e^{\sigma^2 T} - \sigma^2 T - 1\}] - 2\,ln[\sigma^2 T]$$
$$\nu_2^2 = \sigma^2 T \tag{9}$$
$$\rho\nu_1\nu_2 = ln[e^{\sigma^2 T} - 1] - ln[\sigma^2 T]$$

Finally, apply Hull's [2009] generalization of Margrabe's [1978] expression for the value of the option to exchange one asset for another when the stock is dividend paying to obtain the following formula for the value of the marketability discount $D(T)$ as:

$$D(T) = V_0 e^{-qT}\left[N\left(\frac{\nu\sqrt{T}}{2}\right) - N\left(-\frac{\nu\sqrt{T}}{2}\right)\right] \tag{10}$$

$$\nu\sqrt{T} = [\sigma^2 T + ln[2\{e^{\sigma^2 T} - \sigma^2 T - 1\}] - 2\,ln[e^{\sigma^2 T} - 1]]^{1/2} \tag{11}$$

where $N(\cdot)$ is the cumulative standard normal distribution function.

$D(T)$ is proportional to the current share price. It is easily shown that the Equations (10)–(11) imply that the marketability discount is:

- Directly related to the stock's price volatility σ.
- Directly related to the length of the transfer restriction period T.
- Inversely related to the stock's dividend yield q.

Note that the discount is independent of the riskless interest rate r, which does not appear in either equation. In addition, if an option model is a valid characterization of the marketability discount, then σ should exert a greater impact on $D(T)$ than T or q.

The Discounts Implied by the Average-Strike Put Option Model

Exhibit 1 reports the sensitivity of $D(T)$ to the length of the restriction period and the stock's volatility. In the dividend-paying case, I assume $q = 0.02$ because the 25-year average dividend yield on the Standard & Poor's 500 portfolio of stocks is about 2.0%. $D(T)$ varies between 2.30% and 23.93% for a one-year restriction period, and it varies between 3.25% and 29.25% for a two-year restriction period when σ is between 10% and 120%. Somewhat smaller discounts apply to dividend-paying stocks. For example, a 2% dividend yield reduces $D(T)$ by only about 3 percentage points when the restriction period is five years and σ exceeds 80%. $D(T)$ is between 15% and 25% for a one-year restriction period when σ is between 70% and 120%, and it is between 25% and 30% for a two-year restriction period when σ is between 90% and 120%. The discounts in Exhibit 1 are at the low end or below the range of discounts estimated by Kahl, Liu, and Longstaff [2003], who model the discount within a continuous-time portfolio choice framework. They find, for example, that when stock representing between 30% and 90% of the holder's wealth is restricted for five years, the discount is between 10.0% and 85.2% when the stock's volatility is 30% (versus 14.78% in Exhibit 1 when $q = 0$) and is between 42.4% and 89.7% when the stock's volatility is 60% (versus 25.79%).

EXHIBIT 1
Marketability Discounts

Restriction Period	σ = 0.1	σ = 0.2	σ = 0.3	σ = 0.4	σ = 0.5	σ = 0.6	σ = 0.7	σ = 0.8	σ = 0.9	σ = 1.0	σ = 1.1	σ = 1.2
Panel A: q = 0												
3 months	1.15%	2.30%	3.45%	4.59%	5.72%	6.85%	7.97%	9.07%	10.16%	11.24%	12.30%	13.34%
6 months	1.63	3.25	4.86	6.46	8.04	9.60	11.13	12.63	14.09	15.50	16.88	18.20
9 months	1.99	3.98	5.94	7.89	9.79	11.66	13.47	15.22	16.91	18.52	20.05	21.49
1 year	2.30	4.59	6.85	9.07	11.24	13.34	15.36	17.30	19.13	20.85	22.45	23.93
2 years	3.25	6.46	9.60	12.63	15.50	18.20	20.68	22.93	24.92	26.63	28.07	29.25
3 years	3.98	7.89	11.66	15.22	18.52	21.49	24.10	26.31	28.10	29.50	30.52	31.23
4 years	4.59	9.07	13.34	17.30	20.85	23.93	26.47	28.47	29.94	30.95	31.58	31.94
5 years	5.13	10.10	14.78	19.03	22.73	25.79	28.16	29.87	31.00	31.66	32.01	32.18
Panel B: q = 0.02												
3 months	1.15%	2.29%	3.43%	4.57%	5.69%	6.82%	7.93%	9.03%	10.11%	11.18%	12.24%	13.27%
6 months	1.61	3.22	4.82	6.40	7.96	9.51	11.02	12.50	13.95	15.35	16.71	18.02
9 months	1.96	3.92	5.86	7.77	9.65	11.48	13.27	14.99	16.66	18.24	19.75	21.17
1 year	2.26	4.50	6.71	8.89	11.02	13.08	15.06	16.96	18.75	20.44	22.01	23.45
2 years	3.12	6.21	9.23	12.13	14.90	17.49	19.87	22.03	23.94	25.59	26.97	28.10
3 years	3.75	7.43	10.98	14.33	17.44	20.24	22.70	24.78	26.47	27.78	28.75	29.42
4 years	4.24	8.37	12.31	15.97	19.25	22.09	24.44	26.28	27.64	28.57	29.15	29.49
5 years	4.64	9.14	13.38	17.22	20.57	23.33	25.48	27.03	28.05	28.65	28.97	29.11

Notes: Equations (10) and (11) are used to calculate the percentage marketability discount for non-dividend-paying stocks (q = 0) and for stocks that provide a 2% constant percentage dividend yield (q = .02). The range of marketability restriction periods is between 3 months and 5 years, and the range of stock price volatilities is between σ = 0.1 and σ = 1.2.

Lookback Put Option Model

Longstaff [1995] develops the following upper bound on the marketability discount for a non-dividend-paying restricted share under the assumption that the holder of the restricted share has perfect market-timing ability:[6]

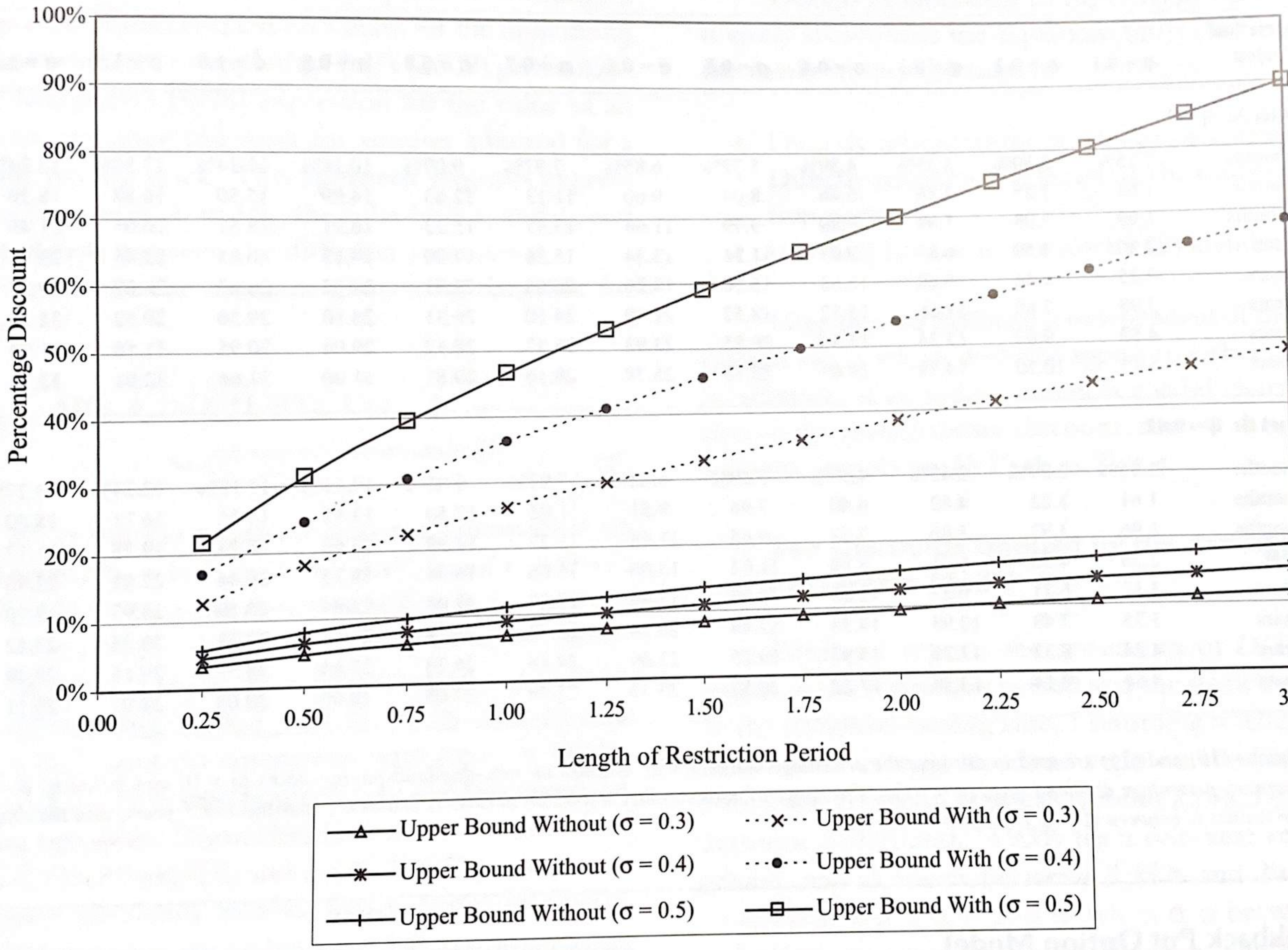
$$D^*(T) = V_0\left[\left(2+\frac{\sigma^2 T}{2}\right)N\left(\frac{\sqrt{\sigma^2 T}}{2}\right) + \sqrt{\frac{\sigma^2 T}{2\pi}}\exp\left(-\frac{\sigma^2 T}{8}\right) - 1\right] \quad (12)$$

where $exp(\cdot)$ is the exponential function and the other variables are as defined in Equations (10) and (11). Exhibit 2 compares the discounts calculated by applying the lookback put option model and the average-strike put option model, assuming the stock is non-dividend-paying and that the stock's volatility is 0.3, 0.4, or 0.5. The discount should be greater when investors possess special market-timing ability because the transfer restrictions impose a greater opportunity cost, which is evident in Exhibit 2. For any given restriction period, the difference increases with σ. If the restriction period is $T = 0.25$ year and $\sigma = 0.3$, the difference is just over 9 percentage points (12.54% for perfect timing versus 3.45% for no special timing ability). When $T = 2$ years, the difference is 29.0 percentage points (38.60% versus 9.60%). The difference due to timing ability increases with T.

COMPARISON OF MODEL-PREDICTED DISCOUNTS TO OBSERVED DISCOUNTS

I collected a sample of 208 private placements of letter stock that took place at a discount to the prevailing market price between April 1, 1991, and March 8, 2007.

EXHIBIT 2
Comparison of the Marketability Discounts for Investors with and without Special Timing Ability



Notes: The exhibit compares the upper bound on the marketability discount implied by the lookback put option model in Equation (12) (Longstaff [1995]), which assumes perfect timing ability (upper bound with), and the upper bound on the discount obtained from the average-strike put option model in Equations (10) and (11), which assumes no special timing ability (upper bound without), when $\sigma = 0.3$, $\sigma = 0.4$, or $\sigma = 0.5$. Since Longstaff's model assumes a non-dividend-paying stock, q is set to zero in Equation (10).

Description of the Sample

I searched on 10kwizard.com by using "private placement of common stock" and similar key words. I identified an initial sample of 403 private placements by public U.S. firms that sold unregistered common shares to U.S. investors for cash. I excluded private placements that were accompanied by a non-U.S. offering (under Regulation S) or by the simultaneous sale of another class of securities (publicly or privately). I also excluded placements by regulated utilities and depository institutions. I searched the *Dow Jones Interactive-Publications Library*, the Bloomberg database of company announcements, and the *Wall Street Journal Index* to confirm the private placement information and identify the offering completion announcement date for each offering. I checked each firm's Form 10-K report for the year of the financing to obtain any reported details concerning the offering.

I dropped from the sample two private placements that were less than three months from an earlier private placement by the same firm; 14 issues with a significant corporate announcement that occurred within 5 trading days prior to and 10 trading days following the pricing announcement in order to isolate the effect of the private placement announcement; and 112 issues that had less than three months of continuous historical stock prices in the Center for Research in Security Prices (CRSP) monthly stock files immediately prior to the announcement date. This left a sample of 275 private placements of which 67 were priced at a premium to the previous day's closing market price. This resulted in a final sample of 208 discounted private placements.

Two-thirds of the sample (144 firms) had their shares listed on the NYSE, AMEX, or NASDAQ National Market. The remainder were quoted in the NASDAQ Small Cap, OTC Bulletin Board, or Pink Sheet markets. Forty-one of the private placements occurred before and the other 167 occurred after the SEC reduced the Rule 144 resale-restriction period to one year from two years on February 20, 1997 (SEC [1997a]). Roughly two-thirds of the sample (114 placements) occurred during 2000–2004, when the PIPE (private investment in public equity) market developed.

The average market value of firm equity is $197.84 million (median market value $101.45 million). The average net income for the latest fiscal year immediately preceding the offering is negative, and the median cash flow from operations is zero. One hundred and fifty-one issuers had negative net income, and 102 had negative cash flow from operations for the latest fiscal year. Fifteen of the 57 with positive net income earned less than $1 million. These sample characteristics are consistent with Hertzel et al.'s [2002] findings that private equity issuers are generally small, young, and unprofitable and issue equity privately following periods of poor operating performance.

Exhibit 3 describes the private placements. The mean gross proceeds are $15.37 million for the pre-February 1997 offerings and $21.73 million for the post-February 1997 offerings. These average proceeds compare to average proceeds between $4.3 million and $31.5 million in previous studies. In the private placements in my sample, the new shareholders purchased an average of 16% of the equity in the pre-February 1997 offerings and an average of 12% in the post-February 1997 offerings, which compares to between 13.6% and 21.2% in previous studies.

Private Placement Discount

Panel B of Exhibit 3 reports the mean and median percentage private placement discount calculated with respect to the closing price on the last trading day immediately prior to the date the pricing terms of the private placement are announced. The discount off the prior day's closing price is calculated:

$$\textit{Discount (day prior)} = \frac{P_{-1} - P_0}{P_{-1}} \tag{13}$$

where P_{-1} is the closing price on the trading day immediately preceding the pricing announcement date and P_0 is the private placement offering price.

The private placement price is usually determined on the closing date (Dresner and Kim [2006] and Morrison & Foerster LLP [2006]). All 208 private placements took place at a discount relative to the closing price the preceding trading day. Exhibit 3 summarizes the private placement discounts. The mean (median) *Discount (day prior)* is 26.27% (20.33)% for the pre-February 1997 offerings and 21.53% (15.61)% for the post-February 1997 offerings. *Discount (day prior)* should be lower after February 1997 because the SEC halved the initial resale-restriction period to one year from two years. Only the median difference for the discounted offerings is significant at the 5% level. This limited significance may be due to the importance of ownership concentration, information, and overvaluation effects, which were not directly affected by the change in Rule 144.

The discounts in Exhibit 3 compare to average discounts of 13.5% in Wruck [1989], 33.75 % in Silber [1991], 20.14% in Hertzel and Smith [1993], and 16.5% in Hertzel et al. [2002]. As previous studies have observed, private placement discounts vary widely. In my sample, 61 of the private placements were made at relative-to-prior-day discounts that exceeded 25%, and 149 were at a relative-to-prior-day discount that exceeded 10%.

Other Explanations for the Private Placement Discount

The marketability hypothesis predicts that the entire discount on restricted shares at the time of issue is due to the Rule 144 transfer restrictions. Accordingly, the private placement discount is often referred to as a "marketability discount." However, Hertzel and Smith [1993] point out that discounts of the magnitude observed in private placements of letter stock (averaging 20% in their study and as much as 42% in earlier studies) would provide powerful incentives at the time of the private placement for firms to commit to register the shares promptly following the private sale if they were due solely to the Rule 144 restrictions. There are many large financial institutions with long-dated liabilities, such as

EXHIBIT 3
Description of the Private Equity Placements

	N	Mean	Median	Minimum	Maximum	Standard Deviation
Panel A: Gross Proceeds and Fraction Issued						
				Pre Feb 1997[d]		
Gross proceeds[e] (millions)	41	15.37	9.00	0.01	98.08	21.40
Fraction issued[f] (%)	41	16%	13%	0%	62%	14%
				Post Feb 1997		
Gross proceeds[e] (millions)	167	21.73	10.33	0.02	235.00	32.09
Fraction issued[f] (%)	167	12%	9%	0%	88%	13%
Panel B: Average Percentage Discount for Shares Placed at a Discount						
Time Period				Pre Feb 1997[d]		
Discount (day prior)[g]	41	26.27	20.33	1.70	68.00	16.89
				Post Feb 1997		
Discount (day prior)[g]	167	21.53	15.61	0.00	85.14	18.60
				Differences between Pre and Post		
Discount (day prior)[g]		4.74 (1.487)	4.72 (2.130[b])	1.70	−17.14	−1.71

Notes: The letter stock sample consists of 208 discounted private placements by NYSE, AMEX, NASDAQ, and OTC firms between April 1, 1991, and March 8, 2007. The number of shares outstanding prior to the offering was obtained from Compustat. The gross proceeds and number of shares issued were obtained from Bloomberg. The Discount (day prior) was calculated from Equation (13) using the closing price on the day prior to the offering date. The t-statistics for the difference in average discounts between pre and post for shares placed at a discount and the z-statistic for the Wilcoxon signed rank test for the difference in median discounts between pre and post for shares placed at a discount are given in parentheses beneath the differences.

[a,b,c]*denote significance (based on a two-tailed test) at the 1%, 5%, and 10% levels, respectively.*

[d]*Prior to February 20, 1997 when the SEC modified Rule 144 by shortening the resale-restriction period.*

[e]*Before deducting the private placement agent's fee and other expenses of the offering.*

[f]*Calculated as the number of shares placed divided by the sum of the number of shares placed and the number outstanding prior to the offering.*

[g]*Calculated from Equation (13).*

life insurance companies and pension funds, that may be less concerned about liquidity than other investors, which brings into question whether investors should require such large discounts just for agreeing not to resell their shares in the public securities market for two years.[7] A private resale market exists, and the options market provides hedging opportunities. When they allow for the information and equity ownership concentration effects that accompany a private placement, Hertzel and Smith [1993] find that the discount attributable to lack of registration is only 13.5%. Hence, transfer restrictions only partly account for the discount.

The finance literature has put forth three alternative explanations for the discount. First, a private placement to a small set of sophisticated investors can decrease ownership concentration and enhance value if it results in more intensive monitoring, and the discount compensates the new private investors for their future monitoring services and expert advice (Wruck [1989]). Second, it also reimburses outside private investors for the cost of information gathering during the private offering's due diligence process (Hertzel and Smith [1993]). Third, if knowledgeable private investors can identify stocks that overoptimistic public investors have overvalued, they will purchase shares privately only at a discount to compensate them for the stock's expected underperformance post-issue (Hertzel et al. [2002]).

Wruck [1989], Hertzel and Smith [1993], Allen and Phillips [2000], and Hertzel et al. [2002] furnish empirical evidence that ownership concentration, information, and overvaluation effects also contribute to the private placement discount. These factors suggest that the private placement discount overstates the marketability discount. However, Longstaff [2001] solves the intertemporal portfolio choice problem for an investor who is restricted to trading strategies of bounded variation and obtains values for the shadow cost of illiquidity that are consistent with a 35% discount for lack of free transferability. His model demonstrates that such large discounts are sustainable in a rational model of investor behavior. Thus, a 'private placement discount' may reflect a 'marketability discount' as well as equity ownership concentration, information gathering, and overvaluation effects, although the relative importance of the transfer restrictions and the other factors is an unresolved issue.

Regression Model

I test the marketability discount model in Equations (10) and (11) by using a regression analysis on the sample of 208 private placement discounts while controlling for the information, ownership concentration, and overvaluation effects that accompany a common stock private placement. The dependent variable is the observed private placement discount, *Discount (day prior)*, which measures the percentage discount relative to the closing market price of the issuer's registered common stock on the trading day immediately preceding the offering completion announcement date. This is the market price that is usually used as the basis for pricing privately placed shares (Dresner and Kim [2006]).

I measure volatility σ as the historical annualized standard deviation of the total return on the issuer's common stock for the period ending on the last trading day immediately preceding the initial announcement date. I restricted the private placement sample to firms with at least three months of historical stock prices, but I used two year's historical stock prices for those private placements issued before February 1997 and one year's historical stock prices for those private placements issued after February 1997, if sufficient data were available.[8]

Previous research (Wruck [1989]; Hertzel and Smith [1993]; and Hertzel et al. [2002]) suggests that the observed private placement discount equals the marketability discount plus adjustments for information, ownership concentration, and overvaluation effects (collectively, placement-related effects):

$$\text{\textit{Discount (day prior)}} = \text{\textit{Marketabiliy Discount}} + \text{\textit{Placement–Related Effects}} \tag{14}$$

I used my private placement sample to estimate the impact of each of these three placement-related effects. Then I subtracted them from each observed private placement discount, *Discount (day prior)*, to calculate the *Implied Marketability Discount* associated with each private placement discount in my sample. I test the average-strike put option model's accuracy by fitting the following equation:

$$\text{\textit{Implied Marketability Discount}} = a_0 + a_1 \text{\textit{Model–Predicted Discount}} \tag{15}$$

Model-Predicted Discount is the marketability discount predicted by the average-strike put option model in Equations (10) and (11). If the model accurately explains the marketability discounts implicit in the private placement discounts in my sample, then I expect $a_0 = 0$ and $a_1 = 1$.

I substituted Equation (15) for *Marketability Discount* in Equation (14) to express the observed private placement discount, *Discount (day prior)*, in terms of the *Model-Predicted Discount* and the private-placement-related effects. I then ran the following cross-sectional regression model to test the average-strike put option model of the marketability discount while controlling for the other factors that affect actual private placement discounts:

$$\begin{aligned} \text{\textit{Discount (day prior)}} = {} & a_0 + a_1 \text{\textit{Model–Predicted Discount}} \\ & + a_2 \text{\textit{Risk}} + a_3 \text{\textit{Direct}} + a_4 \text{\textit{Fraction}} + a_5 \text{\textit{Exchange}} \\ & + a_6 \text{\textit{ Market/Book Deviation}} + a_7 \text{\textit{Post}} + a_8 \text{\textit{Options}} \\ & + a_9 \text{\textit{Registration Rights}} + a_{10} \text{\textit{Registered Stock}} \end{aligned} \tag{16}$$

The regression model in Equation (16) includes the *Model-Predicted Discount* calculated from the marketability discount model in Equations (10) and (11) as an independent variable and nine control variables: *Risk*, *Direct*, *Fraction*, *Exchange*, *Market/Book Deviation*, *Post*, *Options*, *Registration Rights*, and *Registered Stock*. If the model in Equations (10) and (11) accurately explains actual marketability discounts, then the coefficient of *Model-Predicted Discount* should also be one in this expanded model and the intercept should be zero.

I use five proxies to control for the information, ownership concentration, and overvaluation effects. Starting with information effects, *Risk* proxies for the severity of the information asymmetries between the firm and outside investors. I calculated *Risk* as the standard deviation of the daily abnormal returns from fitting the Scholes–Williams beta-adjusted capital asset pricing model (CAPM) (Scholes and Williams [1977]) to stock returns for the period extending from 120 to 21 days prior to the initial announcement.[9] The greater the value of *Risk,* the greater the potential information asymmetry, and the greater the value of the observed private placement discount, *Discount (day prior),* I would expect.

Private placements take place more quickly than general cash offers, which leaves less time for due diligence. As a result, there is a danger that due diligence might be less extensive than in a general cash offer. Larger issues generally require more-intensive information gathering and expose investors to greater asymmetric information costs. I calculated *Fraction* as the number of shares placed divided by the sum of the number of shares placed and the number outstanding prior to the offering (Hertzel and Smith [1993]). Since investors have to expend more resources to assess firm value, they require a larger discount when the offering is relatively large, which suggests a positive coefficient.

Exchange-traded stocks and NASDAQ stocks generally have larger capitalizations, and firm disclosure is governed by exchange or NASDAQ regulations, which result in a freer flow of information to investors. The variable *Exchange* takes on the value one if the registered shares are listed on the NYSE, the AMEX, the NASDAQ National Market, or the NASDAQ Small Cap Market at the time of the private placement and is zero otherwise. I expect a negative coefficient to reflect the value of exchange or NASDAQ listing.

Turning to ownership concentration effects, the discount investors require will depend on the intensity of monitoring by outside shareholders and the firm's board of directors (Wruck [1989]). I use one minus the fraction of directors that are also managers (*Direct*), according to the issuer's most recent proxy statement as of the pricing announcement date, to proxy for the intensity of monitoring by outside directors. *Direct* controls for ownership concentration effects because a private placement to outside institutional investors will dilute the ownership interests of insiders and likely also lead to more intensive monitoring by independent directors on behalf of outside institutional investors or directly by these investors. Outside directors have an incentive to act as effective monitors because they want to protect their reputations for independence. *Direct* should have a positive coefficient to reflect the compensation for more-intensive monitoring on behalf of the outside investors.

Next, turning to the overvaluation/underperformance signaling effect, the ratio of the market value to the book value of the firm's common stock immediately preceding the completion announcement can serve as a measure of the degree of over- or undervaluation. Hertzel et al. [2002] find that firms that issue equity privately have above-average market-to-book ratios and underperform their peers following the offering, which they attribute to overoptimism among public market investors concerning the firm's growth prospects. Since the market-to-book ratio varies across industries, I control for possible overvaluation by calculating *Market/Book Deviation* as the difference between the firm's market-to-book ratio and the average market-to-book ratio for the firm's industrial sector at the time of the private placement (Rhodes-Kropf, Robinson, and Viswanathan [2005]). The industry sectors are the 12 Fama–French industry classifications (Rhodes-Kropf, Robinson, and Viswanathan [2005]). The larger (smaller) the *Market/Book Deviation*, the greater (smaller) the likelihood of over- (under-) valuation, and the greater (smaller) the discount. I expect a positive coefficient based on Hetzel et al.'s [2002] investor overvaluation hypothesis.

I control for the effect of the reduction in the length of Rule 144's initial restriction period to one year from two years in February 1997 by including the dummy *Post*, which takes on the value one following February 1997 and zero before. I expect the sign of *Post* to be negative. To control for private investors' opportunity to hedge price risk, I investigated whether there were options on the firm's stock outstanding at the time of the private placement and included the dummy variable *Options*, which takes on the value one if there were publicly traded options on the stock at the time of the private placement and zero if there were not. *Options* proxies for the existence of hedging opportunities, which should reduce the discount. I expect its sign to be negative.

I also controlled for the existence of registration rights or a commitment to register the stock following the private placement because the discount should be smaller in those cases. *Registration Rights*, which is one for those offerings that provide for mandatory, piggyback, or demand registration rights and is zero otherwise, controls for the effect of private placement investors being able to shorten the restriction period by being able to demand registration of their shares. *Registered Stock*, which is one for those stocks that the firm registered with the SEC within 45 days of the offer date, controls for the even greater shortening of the restriction period when the issuer commits to register the shares quickly as a condition of closing the private placement. I expect both coefficients to be negative (Finnerty [2012]).

The sample includes 208 discounted U.S. private placements. However, 26 observations had to be dropped from the regression that includes the variables *Direct* and *Market/Book Deviation* because nine firms' historical proxy statements could not be obtained, and the book value of equity of 17 other firms was negative at the time of the private placement, which renders the market/book ratio not meaningful.

Regression Results

I examined the bivariate relationships between the observed private placement discount, *Discount (day prior)*, and each independent variable (not reported). All the variables have the predicted signs. First, I fit Equation (16) with the observed private placement discount, *Discount (day prior)*, as the dependent variable and *Model-Predicted Discount* as the sole independent variable. Exhibit 4 contains the cross-sectional regression results. White's [1980] specification test indicates heteroskedasticity, so I report White heteroskedasticity-consistent estimators and t-statistics. The coefficient of *Model-Predicted Discount* is greater than one, but the difference is not significant at the 10% level, and the intercept is also not significantly different from zero at the 10% level.

Next, I ran the full regression in Equation (16). All the regression coefficients have the predicted signs. The coefficient of *Post* has the expected sign but is not significant at the 10% level. Investment bankers and appraisers do not seem to have fully adjusted for the shortening of the Rule 144 restriction period from two years to one year once the effect of all the other factors that can affect private placement discounts are taken into account. The coefficient of *Model-Predicted Discount* has the predicted sign and is less than one, although it is not significantly different from one at the 10% level. The intercept is also not significantly different from zero at the 10% level. Thus, the average-strike put option model discounts are generally consistent with the discounts observed in private placements.[10]

Comparison of the Model-Predicted and Implied Marketability Discounts

Exhibit 5 graphs the relationship between the *Implied Marketability Discount* and the *Model-Predicted Discount*. I calculate the *Implied Marketability Discount* by adjusting the actual observed private placement discounts for the information, ownership concentration, and overvaluation effects by subtracting the values of the control variables on the right-hand side of Equation (16) from the observed private placement discount, *Discount (day prior)*. The average-strike put option *Model-Predicted Discount* understates (overstates) the *Implied Marketability Discount* to the left (right) of the 45 degree line in Exhibit 5. If investment bankers and business appraisers tended to apply discounts within the 25% to 35% range prior to February 1997 and within the 15% to 25% range since February 1997 with little, if any, adjustment for stock price volatility, then I would expect $a_0 > 0$ and $a_1 < 1$ because then the *Model-Predicted Discounts* would tend to understate (overstate) the *Implied Transferability Discounts* for low- (high-) volatility stocks that accordingly have small (large) *Model-Predicted Discounts*. The slope a_1 would indicate how sensitive the *Implied Marketability Discount* is to differences in volatility (and to variation in the other parameters in Equation (16)).

The *Model-Predicted Discounts* tend to understate the *Implied Marketability Discounts* implicit in the private placement discounts in my sample for relatively low-volatility stocks (for which the marketability discount should be small) but overstate them for relatively high-volatility stocks. However, the *Model-Predicted Discounts* are still generally consistent with the *Implied Marketability Discounts* in the sense that the relationship between the two sets of discounts fits the 45 degree line closely enough that the intercept is not significntly different from zero and the slope is not significantly different from one. Investment bankers and appraisers do seem to have adjusted for stock price volatility when pricing stock private placements.

EXHIBIT 4

Cross-Sectional Regression Results

Dependent Variable	Model without Control Variables Discount (day prior)	Model with Control Variables Discount (day prior)
Number of Observations	208	182
Intercept	0.134 (0.0427)	9.190 (1.4098)
Model-Predicted Discount (+)	1.199 (6.5463[a])	0.759 (3.5345[a])
Risk (+)		0.552 (1.7304[c])
Fraction (+)		1.880 (0.1387)
Exchange (–)		–4.666 (–1.2736)
Direct (+)		1.901 (0.3475)
Market / Book Deviation (+)		2.397 (2.3512[b])
Post (–)		–0.170 (–0.0547)
Options (–)		–0.443 (–0.1739)
Registration Rights (–)		–1.821 (–0.8363)
Registered Stock (–)		–6.502 (–2.6832[a])
Adjusted R^2	0.23	0.30
MSE	266.90	213.08
F-value	62.46[a]	8.78[a]
White's Test	13.40[a]	1.01

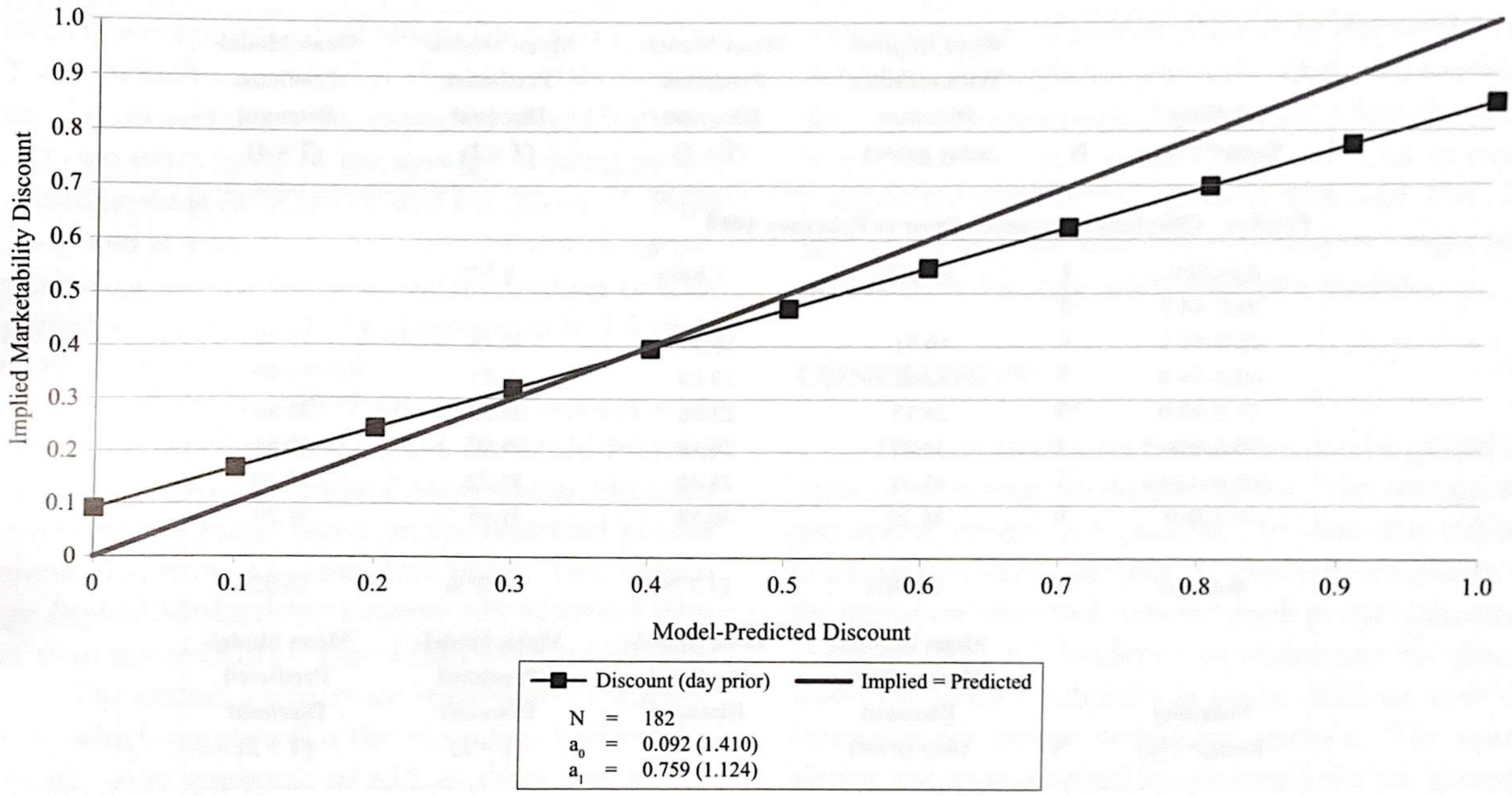
Notes: This exhibit tests the marketability discount model in Equations (10) and (11) while controlling for ownership concentration, information, and overvaluation effects. The regression model is Equation (16), which includes the model-predicted discount calculated from the marketability discount model in Equations (10) and (11) and nine control variables: Risk, Fraction, Exchange, Direct, Market/Book Deviation, Post, Options, Registration Rights, and Registered Stock.

The observed private placement discount, Discount (day prior), is the actual private placement discount, which is calculated using Equation (13). It measures the percentage discount relative to the closing market price of the issuer's registered common stock on the trading day immediately preceding the pricing announcement date. Risk is the standard deviation of the daily abnormal returns from fitting the Scholes–Williams beta-adjusted CAPM to stock returns for the period extending from 120 to 21 days prior to the initial announcement. Fraction is the number of shares placed divided by the sum of the number of shares placed and the number outstanding prior to the offering. Exchange is one for those companies listed on the NYSE, American Stock Exchange, Nasdaq National Market, or Nasdaq Small Cap Market and is zero otherwise. Direct is one minus the fraction of directors that are also managers. Market/Book Deviation, which is the difference between the firm's market-to-book ratio (the market value of equity divided by the book value of equity) and the industrial sector average market-to-book ratio at the time of the private placement, measures the overvaluation/underperformance signaling effect. Post is one for issues that occurred after February 1997 and is zero otherwise. Options is one for those stocks with options during the offer year and zero otherwise. Registration Rights is one for those offerings that provide for mandatory, piggyback, or demand registration rights and is zero otherwise. Registered Stock is one for those stocks that the firm registered with the SEC within 45 days of the offer date.

The regressions are fitted using ordinary least squares. The sample includes 208 U.S. private placements between April 1, 1991, and March 8, 2007, that were priced at a discount. Twenty-six observations had to be dropped from the regression that includes the variables Direct and Market/Book Deviation because 9 firms' historical proxy statements could not be obtained and the book value of equity of 17 other firms is negative. The predicted sign is provided next to each variable. All t-statistics (in parentheses below coefficients) are calculated using heteroskedasticity-consistent standard errors; [a,b,c] denote significance at the 1%, 5%, and 10% levels, respectively.

EXHIBIT 5

Implied Marketability Discount vs. Model-Predicted Discount



Notes: This exhibit plots the linear relationship between the Implied Marketability Discount and the Model-Predicted Discount for private placements of letter stock between April 1, 1991, and March 8, 2007; t-statistics for the separate hypothesis tests $a_0=0$ and $a_1=1$ are given in parentheses next to the estimated slope and intercept coefficients, respectively; [a,b,c]denote significance (based on a two-tailed test) at the 1%, 5%, and 10% levels, respectively.

I performed one additional test of the average-strike put option marketability discount model's ability to explain actual marketability discounts. Rule 144 has a further restriction in addition to the initial restriction. Not adjusting T for this secondary restriction will cause the average-strike put option model to underestimate the marketability discount, as is evident in Exhibit 6. I adjusted T for the further one-year restriction on the volume of sales imposed by Rule 144 and any additional time it might take to sell any remaining shares in the public market after the legal restriction periods lapse. After the minimum holding period, each holder of unregistered shares could sell them in the public market without first registering them, but the number of shares that could be sold during any three-month period was limited to the lesser of (1) the average weekly trading volume during the four calendar weeks immediately preceding the filing with the SEC of Form 144 indicating the holder's intention to sell and (2) 1% of the number of outstanding shares. Prior to February 1997, the minimum holding period for a non-affiliated stockholder was two years; thus, the shares could be sold beginning three years after issue without any Rule 144 restrictions. After February 1997, the SEC amended Rule 144 to allow nonaffiliates to sell their shares after just one year followed by the one-year restriction on the volume of sales; thus, the shares could be sold beginning two years after issue without any Rule 144 restrictions. However, each private placement purchaser would also have to make sure that the sale of any remaining shares did not disrupt the market for the stock.

I compared the *Model-Predicted Discounts* and the *Implied Marketability Discounts* separately for pre-February 1997 placements when the initial Rule 144 restriction period was two years and for post-February 1997 placements when it was one year while allowing for the impact of the secondary restrictions. Exhibit 6 compares the mean *Implied Marketability Discounts* to the mean *Model-Predicted Discounts* predicted by the average-strike put option model in Equations (10) and (11) for various stock volatility ranges and various secondary restriction periods. The secondary restrictions apply to each pri-

Exhibit 6

Implied Marketability Discount vs. Model-Predicted Discount for Different Volatilities

Volatility Range[a] (%)	N	Mean Implied Marketability Discount (day prior)	Mean Model-Predicted Discount (T = 2)	Mean Model-Predicted Discount (T = 3)	Mean Model-Predicted Discount (T = 4)
Panel A: Offerings Announced Prior to February 1997					
0.0–29.9	4	19.47%	7.84%	9.54%	10.95%
30.0–44.9	0	–	–	–	–
45.0–59.9	4	10.51	16.97	20.15	22.55
60.0–74.9	9	13.82	19.18	22.03	23.96
75.0–89.9	10	24.15	23.16	26.52	28.64
90.0–104.9	2	34.97	26.10	29.07	30.64
105.0–120.0	1	61.51	28.40	30.73	31.70
> 120.0	6	44.10	30.88	31.95	32.20
Average:	**36**	24.50%	21.37%	23.97%	25.62%

Volatility Range[a] (%)	N	Mean Implied Marketability Discount (day prior)	Mean Model-Predicted Discount (T = 1)	Mean Model-Predicted Discount (T = 2)	Mean Model-Predicted Discount (T = 3)
Panel B: Offerings Announced After February 1997					
0.0–29.9	7	12.66%	5.22%	6.99%	8.12%
30.0–44.9	19	18.56	7.95	10.75	12.64
45.0–59.9	17	15.92	11.74	16.15	19.22
60.0–74.9	28	19.21	14.83	19.84	23.00
75.0–89.9	25	21.37	17.82	23.50	26.83
90.0–104.9	16	21.61	20.28	26.07	29.05
105.0–120.0	6	24.89	22.74	28.29	30.65
>120.0	28	29.71	27.72	31.20	32.02
Average:	**146**	21.31%	17.02%	21.45%	23.86%

Notes: The Model-Predicted Discount is calculated from the average-strike put option model in Equations (10) and (11), and the Implied Marketability Discount is calculated by subtracting the values of the control variables on the right-hand side of Equation (16) from the observed Discount (day prior) for each sample private placement. Panel A applies to those offerings that were announced prior to February 1997, and Panel B applies to those offerings that were announced after February 1997. The sample includes 182 U.S. private placements between April 1, 1991, and March 8, 2007, that were priced at a discount. Twenty-six observations had to be dropped from the sample of 208 discounted stock private placements because 9 firms' historical proxy statements could not be obtained and the book value of equity of 17 other firms is negative.

vate placement purchaser separately. Firms in my sample seldom identified the number of purchasers publicly. The precise adjustment for secondary restrictions will vary depending on the number of purchasers and how many shares each purchases, and especially the number of shares purchased by the lead investor, who presumably negotiates the discount with the issuer. I gauged the impact of the secondary trading restrictions by considering alternative values for T in Exhibit 6.

For pre-February 1997 private placements with an initial two-year restriction period, the *Model-Predicted Discounts* understate the *Implied Marketability Discounts* based on the observed private placement discount, *Discount (day prior)*, for stock volatilities less than 30% and greater than 75%, but the reverse is true for volatilities between 45% and 75%. The overall average *Implied Marketability Discount* (24.50%) is 3.13% greater than the overall average *Model-Predicted Discount* (21.37%). The overall averages are equal when T is between $T = 3$ and $T = 4$, which suggests that the secondary trading restrictions, on average, were expected to add between one and two extra years to the average holding period for the lead investor (who negotiated the discount). If the shares are sold at a uniform rate after the initial restriction period lapses, the total secondary trading restriction period was expected to be approximately 3.3 years before all the shares were sold.

For post-February 1997 private placements with an initial one-year restriction period, the *Model-Predicted Discounts* understate the *Implied Marketability Discounts* for each volatility range based on the observed private placement discount, *Discount (day prior)*. The overall average *Implied Marketability Discount* (21.31%) is 4.29% greater than the overall average *Model-Predicted Discount* (17.02%). The overall averages are equal when T is about 2.0 years, which suggests that the secondary restrictions, on average, were expected to add an extra year to the average holding period for the lead investor. Again, assuming a uniform rate of share sales after the initial restriction period lapses, the total secondary trading restriction period was expected to be about two years before all the shares were sold. The average-strike put option model generally fits the *Implied Marketability Discounts* better for the one-year restriction period, especially for stocks with volatilities in excess of 90%.

The mean and median stock price volatilities for the sample are 84% and 78%, respectively. When the stock price volatility is that high, the average-strike put option model in Equations (10) and (11) predicts discounts within the 25% to 35% (15% to 25%) range that investment bankers and appraisers customarily apply when there is a two-year (one-year) initial restriction period, if one takes into account the secondary trading restrictions. In particular, when the stock volatility is 78% and $q = 0$, the *Model-Predicted Discount* for a three-year to four-year average restriction period (after adding one to two years for the secondary trading restrictions to the initial two-year period) is 25.90% to 28.11%, which is within the lower end of the 25% to 35% range. The *Model-Predicted Discount* for a two-year average restriction period (after adding one year for the secondary trading restrictions to the initial one-year period) is 22.50%, which is around the middle of the 15% to 25% range.

Combining the statistical results in Exhibits 5 and 6, the average-strike put option marketability discount model appears to fit actual marketability discounts (implicit in actual private placement discounts) reasonably well, although not perfectly. There is a tendency for the model to understate the discount when the volatility is less than 45% and greater than 75% and to overstate it when the volatility is between 45% and 75%. It also appears to be somewhat less accurate for longer restriction periods for very-high-volatility stocks.

CONCLUSION

The cost of transfer restrictions can be priced as the value of an average-strike put option. The average-strike put option model in Equations (10) and (11) calculates marketability discounts that are generally consistent with the discounts observed in letter stock private placements, although there is a tendency to understate the discount when the stock's volatility is under 45% or over 75%, especially for longer restriction periods. The marketability discounts implied by observed private placement discounts reflect differences in stock price volatility, as option theory and the average-strike put option model predict.

ENDNOTES

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[1]Letter stock is not registered for resale under the Securities Act of 1933 and therefore cannot be freely traded in the public market. It must be placed privately with accredited (sophisticated) investors. Under Rule 144, the holder cannot sell the shares during a specified minimum holding period, which is measured from the issue date, except through another exempted transaction. Prior to February 1997, the minimum holding period for a stockholder who was not affiliated with the issuer was two years. The SEC shortened the Rule 144 minimum holding period in 1997 to one year (SEC

[1997a]) and again in December 2007 to six months (SEC [2007]). Longer restriction periods apply if the stockholder is an affiliate of the issuer or if the firm is not current with its SEC filings. After the minimum holding period, the shares can be sold in the public market without first registering them but only after the firm agrees to remove the restrictive stock legend on the share certificate, which usually requires an opinion from counsel that the sale complies with Rule 144.

[2]Longstaff's model provides an upper bound on the marketability discount because it assumes that investors have perfect market-timing ability. At least one earlier article proposed a contingent-claims approach to value the resale restriction as a put option. Alli and Thompson [1991] measure the value of marketability as the Black–Scholes [1973] value of a put option with the share price at the time of issue as the strike price.

[3]Rule 144 under the Securities Act of 1933 restricted the type of hedging a purchaser of restricted shares might employ until the hedging restrictions were relaxed in 1990. Rule 144 barred purchasing a put option, selling a call option, or selling short against the box to hedge the price risk exposure in holding unregistered shares (Federal Securities Law Reports [1980]; Hicks [1998]; and SEC [1997b]).

[4]Only 34 of the stocks in the 208 private placements in my sample had listed options at the time of the offering, and just two of these were long-term equity anticipation securities (LEAPS). The use of equity swaps to hedge the price risk of stocks has fallen out of favor since a 1997 tax law change deemed such transactions a "constructive sale" of the hedged shares (Bettis, Bizjak, and Lemmon [2001]).

[5]For empirical evidence, see Levy [1992] and the references therein. Beaulieu, Abu-Dayya, and McLane [1995] (BAM) describe four methods for analytically approximating the cumulative distribution function of a random variable that is the sum of n i.i.d. lognormal random variables and compare these approximations to a numerical simulation of the actual distribution. They do not find that any one approximation dominates the others.

[6]This ability could be due to valuable private information regarding the issuing firm's future prospects. Such shareholders would presumably time their sales of unrestricted shares so as to maximize the sales proceeds. Thus, Longstaff's [1995] model can also be interpreted to apply to insiders with valuable private information, such as senior executives. Many publicly traded firms prohibit their executives from buying or selling shares except during limited periods, as for example, a brief period commencing a few days following an earnings announcement. Longstaff's model could be used to measure the impact of such blackout period restrictions.

[7]The Rule 144 restriction period was shortened first to one year and then to six months from two years after their study was published.

[8]The SEC further shortened the holding period to six months in December 2007 (SEC [2007]) but this change occurred after the sample period.

[9]I ended the period 21 days prior to the initial announcement to avoid contamination from the private placement marketing that typically precedes the initial public announcement.

[10]In 72 cases, the estimated historical volatility exceeded 90%. As a robustness check, I rescaled estimated volatilities that exceed 90% to lie between 90% and 120% because option market participants often discount very high volatilities estimated from historical share prices when choosing σ. I then reran the cross-sectional regressions. All the regression coefficients have the predicted signs, except for the coefficient of *Post*, although it is not significant at the 10% level. The regression coefficients are slightly less statistically significant, but the results are still generally consistent with the results reported in the article.

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