UP902897

M352 – Advanced Decision Modelling

Coursework

Problem 1

a)

Decision variables:

- $\boldsymbol{x_{ij}}$ – number of products i to produce in j percentage range of demand for product i.

$\mathbf{i} = 1, 2, 3$, where 1, 2, 3 is product A, B, C, respectively.

$\mathbf{j} = 1, 2, 3$, where 1 is 0% to 90% range of demand.

2 is 90% to 100% range of demand.

3 is excess of demand (>100%).

The Total Profit of the company needs to be maximised. It depends on quantity of product i produced (Raw Material and Labour Cost) and their selling price (Total Revenue) which depends on percentage range of demand for products i.

Objective function:

Maximise Total Profit (in £) = Total Revenue – Raw Material Cost – Labour Cost

Total Revenue = sum of the sell price in j percentage range of demand for product i * number of products i in j to produce – penalty cost.

$$\begin{aligned}
\text{Total Revenue} &= 2655(x_{11} + x_{12}) + 0.8(2655)x_{13} - 1000(1650 - x_{11} - x_{12}) \\
&+ 6890(x_{21} + x_{22}) + 0.8(6890)x_{23} - 1000(3068 - x_{21} - x_{22}) \\
&+ 1510(x_{31} + x_{32}) + 0.8(1510)x_{33} - 1000(700 - x_{31} - x_{32}) \\
&= 3655(x_{11} + x_{12}) + 7890(x_{21} + x_{22}) + 2510(x_{31} + x_{32}) + 2124x_{13} \\
&+ 5512x_{23} + 1208x_{33} - 1000(5418)
\end{aligned}$$

Raw Material Cost = sum of the raw material cost to produce one product i * number of products i for all j.

$$\begin{aligned}
\text{Raw Material Cost} &= [200(7) + 20(5)]\sum_{j=1}^{3} x_{1j} + [200(23) + 20(2.5)]\sum_{j=1}^{3} x_{2j} \\
&+ [200(3.5) + 20(7.5)]\sum_{j=1}^{3} x_{3j} \\
&= 1500\sum_{j=1}^{3} x_{1j} + 4650\sum_{j=1}^{3} x_{2j} + 850\sum_{j=1}^{3} x_{3j}
\end{aligned}$$

Labour Cost = sum of the labour cost to produce one product i * overall number of products i.

$$\begin{aligned}\text{Labour Cost} &= \left(30*\frac{1}{6}+25*\frac{1}{3}+37*\frac{3}{4}+25*\frac{1}{2}\right)\sum_{j=1}^{3}x_{1j} \\ &+\left(30*\frac{1}{3}+25*\frac{7}{12}+37+25*\frac{1}{6}\right)\sum_{j=1}^{3}x_{2j} \\ &+\left(30*\frac{1}{4}+25*\frac{5}{12}+37*\frac{1}{3}+25*\frac{1}{3}\right)\sum_{j=1}^{3}x_{3j} \\ &= \frac{643}{12}\sum_{j=1}^{3}x_{1j}+\frac{263}{4}\sum_{j=1}^{3}x_{2j}+\frac{463}{12}\sum_{j=1}^{3}x_{3j}\end{aligned}$$

We could use only 6 variables, namely x_i and y_i for $i = 1, 2, 3$ as we know that 90% of demand must be satisfied (1485 of A, 2762 of B and 630 of C), hence x_i would be production of each product i between 90% and 100% of demand and y_i would be production for each product i in excess of demand (>100%). However, this model is more general and if the guaranteed percentage of demand change, we just change the constraints and the domain for x_{i1}.

Constraints:

Resource constraints: sum of units of resource needed for product i*number of product i.

Stainless steel (kg)

$7\sum_{j=1}^{3}x_{1j} + 23\sum_{j=1}^{3}x_{2j} + 3.5\sum_{j=1}^{3}x_{3j} \leq 150000$

Rust protection (g)

$5\sum_{j=1}^{3}x_{1j} + 2.5\sum_{j=1}^{3}x_{2j} + 7.5\sum_{j=1}^{3}x_{3j} \leq 23000$

Labour constraints: sum of activity hours needed for product i*number of product i.

(hours)

Assemble

$\frac{1}{6}\sum_{j=1}^{3}x_{1j} + \frac{1}{3}\sum_{j=1}^{3}x_{2j} + \frac{1}{4}\sum_{j=1}^{3}x_{3j} \leq 3000$

Edit the steel

$\frac{1}{3}\sum_{j=1}^{3}x_{1j} + \frac{7}{12}\sum_{j=1}^{3}x_{2j} + \frac{5}{12}\sum_{j=1}^{3}x_{3j} \leq 5000$

Chemical process

$\frac{3}{4}\sum_{j=1}^{3}x_{1j} + \sum_{j=1}^{3}x_{2j} + \frac{1}{3}\sum_{j=1}^{3}x_{3j} \leq 8000$

Forge welding

$\frac{1}{2}\sum_{j=1}^{3}x_{1j} + \frac{1}{6}\sum_{j=1}^{3}x_{2j} + \frac{1}{3}\sum_{j=1}^{3}x_{3j} \leq 3000$

Decision variable domain:

Guarantee at least 90% of the demand (j=1):

$$x_{11} = 0.9 * 1650$$

$$x_{21} = 0.9 * 3068$$

$$x_{31} = 0.9 * 700$$

Between 90% and 100% of the demand (j=2): (not 0.1*demand, due to rounding 0.9*3068)

$$x_{12} \leq 1650 - 0.9 * 1650$$

$$x_{22} \leq 3068 - 0.9 * 3068$$

$$x_{32} \leq 700 - 0.9 * 700$$

$$x_{ij} \geq 0 \text{ and integer for i} = 1{,}2{,}3 \text{ , j} = 1{,}2{,}3$$

Extra constraints:

To set production in order is necessarily to make sure that $x_{i1} < \max x_{i1}$ implies $x_{i2} \leq 0$, which is equivalent to

Either-or $x_{i1} \geq \max x_{i1}$, $x_{i2} \leq 0$, so:

$$x_{11} \geq 0.9 * 1650 - Mp$$

$$x_{12} \leq M(1-p)$$

$$x_{21} \geq 0.9 * 3068 - Mr$$

$$x_{22} \leq M(1-r)$$

$$x_{31} \geq 0.9 * 700 - Ms$$

$$x_{32} \leq M(1-s)$$

M must be big enough to not restrict production, we calculate $\max x_{ij}$ for $i, j = 1, 2, 3$.
M=3453 (see calculations in excel file)

Excess of the demand (j=3):

We need to make sure that $x_{i2} < \max x_{i2}$ implies $x_{i3} \leq 0$, which is equivalent to

Either-or $x_{i2} \geq \max x_{i2}$, $x_{i3} \leq 0$, so:

$$x_{12} \geq 0.1 * 1650 - Mt$$

$$x_{13} \leq M(1-t)$$

$$x_{22} \geq (3068 - 0.9 * 3068) - Mu$$

$$x_{23} \leq M(1-u)$$

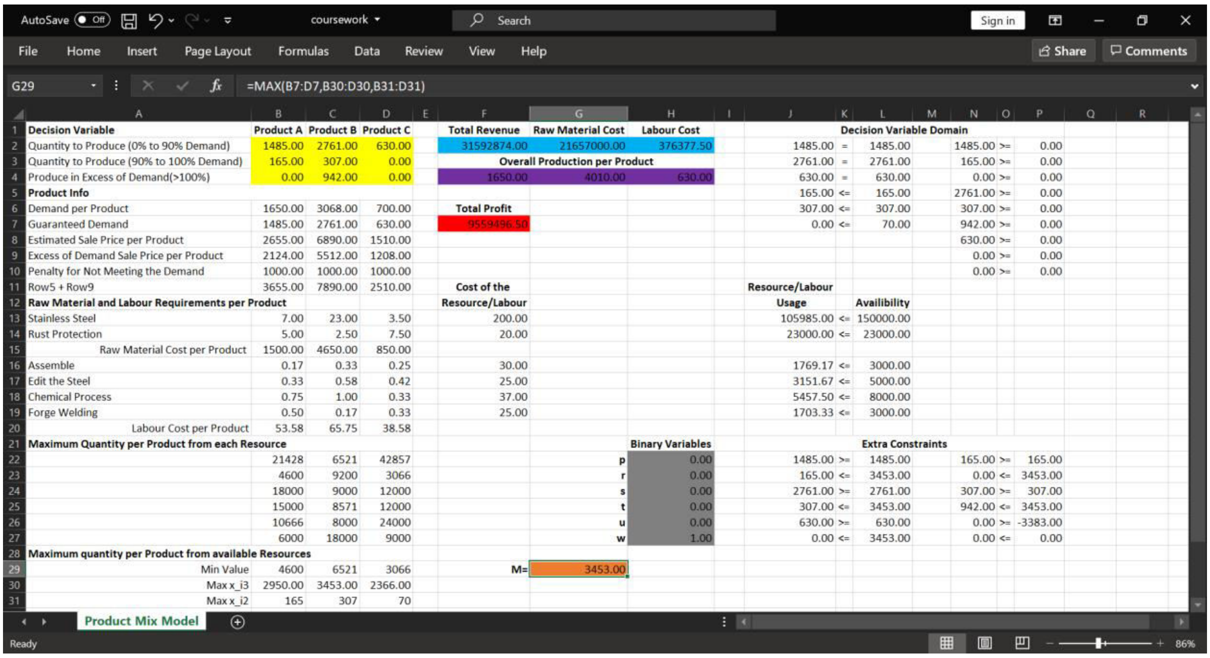
$$x_{32} \geq 0.1 * 700 - Mw$$

$$x_{33} \leq M(1 - w)$$

$$M = 3453.$$

$$p, r, s, t, u, w = binary\ variable(0\ or\ 1)$$

b)



G29 =MAX(B7:D7,B30:D30,B31:D31)

	A	B	C	D	F	G	H	J	K	L	N	O	P
1	**Decision Variable**	**Product A**	**Product B**	**Product C**	**Total Revenue**	**Raw Material Cost**	**Labour Cost**			**Decision Variable Domain**			
2	Quantity to Produce (0% to 90% Demand)	1485.00	2761.00	630.00	31592874.00	21657000.00	376377.50	1485.00	=	1485.00	1485.00	>=	0.00
3	Quantity to Produce (90% to 100% Demand)	165.00	307.00	0.00		**Overall Production per Product**		2761.00	=	2761.00	165.00	>=	0.00
4	Produce in Excess of Demand(>100%)	0.00	942.00	0.00	1650.00	4010.00	630.00	630.00	=	630.00	0.00	>=	0.00
5	**Product Info**							165.00	<=	165.00	2761.00	>=	0.00
6	Demand per Product	1650.00	3068.00	700.00	**Total Profit**			307.00	<=	307.00	307.00	>=	0.00
7	Guaranteed Demand	1485.00	2761.00	630.00	9559496.50			0.00	<=	70.00	942.00	>=	0.00
8	Estimated Sale Price per Product	2655.00	6890.00	1510.00							630.00	>=	0.00
9	Excess of Demand Sale Price per Product	2124.00	5512.00	1208.00							0.00	>=	0.00
10	Penalty for Not Meeting the Demand	1000.00	1000.00	1000.00							0.00	>=	0.00
11	Row5 + Row9	3655.00	7890.00	2510.00	**Cost of the**			**Resource/Labour**					
12	**Raw Material and Labour Requirements per Product**				**Resource/Labour**			**Usage**		**Availibility**			
13	Stainless Steel	7.00	23.00	3.50	200.00			105985.00	<=	150000.00			
14	Rust Protection	5.00	2.50	7.50	20.00			23000.00	<=	23000.00			
15	Raw Material Cost per Product	1500.00	4650.00	850.00									
16	Assemble	0.17	0.33	0.25	30.00			1769.17	<=	3000.00			
17	Edit the Steel	0.33	0.58	0.42	25.00			3151.67	<=	5000.00			
18	Chemical Process	0.75	1.00	0.33	37.00			5457.50	<=	8000.00			
19	Forge Welding	0.50	0.17	0.33	25.00			1703.33	<=	3000.00			
20	Labour Cost per Product	53.58	65.75	38.58									
21	**Maximum Quantity per Product from each Resource**						**Binary Variables**			**Extra Constraints**			
22		21428	6521	42857		p	0.00	1485.00	>=	1485.00	165.00	>=	165.00
23		4600	9200	3066		r	0.00	165.00	<=	3453.00	0.00	<=	3453.00
24		18000	9000	12000		s	0.00	2761.00	>=	2761.00	307.00	>=	307.00
25		15000	8571	12000		t	0.00	307.00	<=	3453.00	942.00	<=	3453.00
26		10666	8000	24000		u	0.00	630.00	>=	630.00	0.00	>=	-3383.00
27		6000	18000	9000		w	1.00	0.00	<=	3453.00	0.00	<=	0.00
28	**Maximum quantity per Product from available Resources**												
29	Min Value	4600	6521	3066	M=	3453.00							
30	Max x_i3	2950.00	3453.00	2366.00									
31	Max x_i2	165	307	70									

Product Mix Model

The optimal solution is to produce 1650 units of product A, 4010 units of product B and 630 units of product C, which according to estimated sale price per product would give £9559496.50 profit. The company would cover demand for product A, make 942 units in excess of demand for product B, and make 630 units, minimum required of product C, paying penalty of £70000.

c)

Answer report:

Objective Cell (Max)

Cell	Name	Original Value	Final Value
$F$7	Total Profit	9559496.50	9559496.50

Variable Cells

Cell	Name	Original Value	Final Value	Integer
$B$2	Quantity to Produce (0% to 90% Demand) Product A	1485.00	1485.00	Integer
$C$2	Quantity to Produce (0% to 90% Demand) Product B	2761.00	2761.00	Integer
$D$2	Quantity to Produce (0% to 90% Demand) Product C	630.00	630.00	Integer
$B$3	Quantity to Produce (90% to 100% Demand) Product A	165.00	165.00	Integer
$C$3	Quantity to Produce (90% to 100% Demand) Product B	307.00	307.00	Integer
$D$3	Quantity to Produce (90% to 100% Demand) Product C	0.00	0.00	Integer
$B$4	Produce in Excess of Demand(>100%) Product A	0.00	0.00	Integer
$C$4	Produce in Excess of Demand(>100%) Product B	942.00	942.00	Integer
$D$4	Produce in Excess of Demand(>100%) Product C	0.00	0.00	Integer

Constraints

Cell	Name	Cell Value	Formula	Status	Slack
$J$18	Chemical Process Usage	5457.50	$J$18<=$L$18	Not Binding	2542.5
$J$17	Edit the Steel Usage	3151.67	$J$17<=$L$17	Not Binding	1848.33333
$J$19	Forge Welding Usage	1703.33	$J$19<=$L$19	Not Binding	1296.66667
$J$16	Assemble Usage	1769.17	$J$16<=$L$16	Not Binding	1230.83333
$J$14	Rust Protection Usage	23000.00	$J$14<=$L$14	Binding	0
$J$13	Stainless Steel Usage	105985.00	$J$13<=$L$13	Not Binding	44015
$B$2	Quantity to Produce (0% to 90% Demand) Product A	1485.00	$B$2>=0	Not Binding	1485.00
$C$2	Quantity to Produce (0% to 90% Demand) Product B	2761.00	$C$2>=0	Not Binding	2761.00
$D$2	Quantity to Produce (0% to 90% Demand) Product C	630.00	$D$2>=0	Not Binding	630.00
$B$3	Quantity to Produce (90% to 100% Demand) Product A	165.00	$B$3>=0	Not Binding	165.00
$C$3	Quantity to Produce (90% to 100% Demand) Product B	307.00	$C$3>=0	Not Binding	307.00
$D$3	Quantity to Produce (90% to 100% Demand) Product C	0.00	$D$3>=0	Binding	0.00
$B$4	Produce in Excess of Demand(>100%) Product A	0.00	$B$4>=0	Binding	0.00
$C$4	Produce in Excess of Demand(>100%) Product B	942.00	$C$4>=0	Not Binding	942.00
$D$4	Produce in Excess of Demand(>100%) Product C	0.00	$D$4>=0	Binding	0.00

Clearly, the most limiting Resource/Activity to the company is amount of Rust Protection, which is indicated by value zero under Slack column and Rust Protection Usage row. The net profit for this solution is £9559496.50 .

Sensitivity report:

Variable Cells

Cell	Name	Final Value	Reduced Cost	Objective Coefficient	Allowable Increase	Allowable Decrease
$B$2	Quantity to Produce (0% to 90% Demand) Product A	1485	0	2101.416667	1E+30	1E+30
$C$2	Quantity to Produce (0% to 90% Demand) Product B	2761	0	3174.25	1E+30	1E+30
$D$2	Quantity to Produce (0% to 90% Demand) Product C	630	0	1621.416667	1E+30	1E+30
$B$3	Quantity to Produce (90% to 100% Demand) Product A	165	0	2101.416667	1E+30	508.9166667
$C$3	Quantity to Produce (90% to 100% Demand) Product B	307	0	3174.25	1E+30	2378
$D$3	Quantity to Produce (90% to 100% Demand) Product C	0	-767.3333333	1621.416667	767.3333333	1E+30
$B$4	Produce in Excess of Demand(>100%) Product A	0	-1022.083333	570.4166667	1022.083333	1E+30
$C$4	Produce in Excess of Demand(>100%) Product B	942	0	796.25	254.4583333	255.7777778
$D$4	Produce in Excess of Demand(>100%) Product C	0	-2069.333333	319.4166667	2069.333333	1E+30

Constraints

Cell	Name	Final Value	Shadow Price	Constraint R.H. Side	Allowable Increase	Allowable Decrease
$J$18	Chemical Process Usage	5457.5	0	8000	1E+30	2542.5
$J$17	Edit the Steel Usage	3151.666667	0	5000	1E+30	1848.333333
$J$19	Forge Welding Usage	1703.333333	0	3000	1E+30	1296.666667
$J$16	Assemble Usage	1769.166667	0	3000	1E+30	1230.833333
$J$14	Rust Protection Usage	23000	318.5	23000	4784.23913	2355
$J$13	Stainless Steel Usage	105985	0	150000	1E+30	44015

The shadow price for Rust Protection is 318.5 which means that extra gram of Rust Protection increases profit by £318.5 and this is valid for any availability between 20645 and 27784 grams. If the Rust Protection amount stays within this limit the current basis remains optimal, which means company should not produce more than 90% of demand of Product C and in excess of demand of product A. However, within this range for Rust Protection we can meet the demand for A and B and minimum required for product C.

If the amount of Rust Protection changes within the above range the production plan is not going to change, company will use the resources to make Product B in excess of demand and cover the demand for A and minimum required for C (reduced cost for all = 0) but total profit is going to change by £796.25(Objective coefficient for excess of demand C) for each change by 2.5 gram in Rust Protection.

d)

i)

We need to add three extra variables, namely d, e, f and add three constraints:

$$\sum_{j=1}^{3} x_{1j} = 1000 * d$$

$$\sum_{j=1}^{3} x_{2j} = 1000 * e$$

$$\sum_{j=1}^{3} x_{3j} = 1000 * f$$

$$d, e, f \geq 0 \; and \; integers$$

ii)

Either-or $x_{13} \leq 0$, $x_{23} \leq 0$, so we need to add two constraints:

$$x_{13} \leq My$$

$$x_{23} \leq M(1 - y)$$

$$M = 3453$$

$$y = binary \; variable \; (0 \; or \; 1)$$

iii)

If $x_{13} > 0.2 * 1650$ then $x_{23} \leq 0.2 * 3068$, which is equivalent to:

Either-or $x_{13} \leq 0.2 * 1650$, $x_{23} \leq 0.2 * 3068$, so we need to add two constraints:

$$x_{13} \leq 330 + My$$

$$x_{23} \leq 613 + M(1 - y)$$

$$M = 3453$$

$$y = binary\ variable\ (0\ or\ 1)$$

iv)

In this case we should change our Total Revenue formula to:

$$\begin{aligned}
&Total\ revenue \\
&\quad = 2655(x_{11} + x_{12} + y_1 + y_2) + 0.8(2655)(x_{13} + y_3) \\
&\quad - 1000(1650 - x_{11} - x_{12} - y_1 - y_2) - 500000s - 1000 \sum_{i=1}^{3} y_i \\
&\quad + +6890(x_{21} + x_{22}) + 0.8(6890)x_{23} - 1000(3068 - x_{21} - x_{22}) \\
&\quad + 1510(x_{31} + x_{32}) + 0.8(1510)x_{33} - 1000(700 - x_{31} - x_{32})
\end{aligned}$$

, where:

y_i - number of products to buy in i percentage range of demand for product A

Also, we should change few decision variable domain constraints, namely:

$$x_{11} + y_1 = 0.9 * 1650$$

$$x_{12} + y_1 \leq 1650 - 0.9 * 1650$$

, add:

$$y_1 \leq 1485 * s$$

$$y_2 \leq 165 * s$$

$$y_3 \leq M * s$$

There is no information about how many products A a third party can produce, neither how big is the budget of the company, M can be infinity.

$$y_i \geq 0\ and\ integer$$

Also, we should change few extra constraints, namely:

$$x_{11} + y_1 \geq 0.9 * 1650 - 1485 * p$$

$$x_{12} + y_2 \leq 165(1 - p)$$

$$x_{12} + y_2 \geq 0.1 * 1650 - 1650 * t$$

$$x_{13} + y_3 \leq M(1 - t)$$

M can be infinity.

Cost of producing product A by the company is £1553.58 (see excel file)

$$1553.58A = 500000 + 1000A$$

$$A = 903.21$$

If the company produce 903 of A, it will cost same as buying those from a third party. As we know minimum requirement for A is 1485 units, hence buying it makes sense, each product bought over 903 units saves roughly £553. Also, they will have more resources to use for products B and C. The company should definitely buy product A from a third party and do not manufacture them on them own as demand is big enough, it is cheaper to buy it and they need resources.

Problem 2

a)

Conceptual Model:

- The Problem Situation:
  Restaurant`s order processing. Orders arrive and go through three service points, first is preparing, second cooking and the third arranging. If any of those is busy, orders wait in a queue. There are considered two queue priority rules: First-In-First-Out (FIFO) and Shortest Expected Processing Time First (SPT).
- Modelling Objectives:
  The objective is to determine the processing rule required that will minimise the average throughput time of an order.
- Model Inputs:
  Orders
- Model Outputs/Responses:
  Average throughput time of an order
- Experimental Factor:
  Queuing priority will be set to FIFO or SPT

- Model Scope

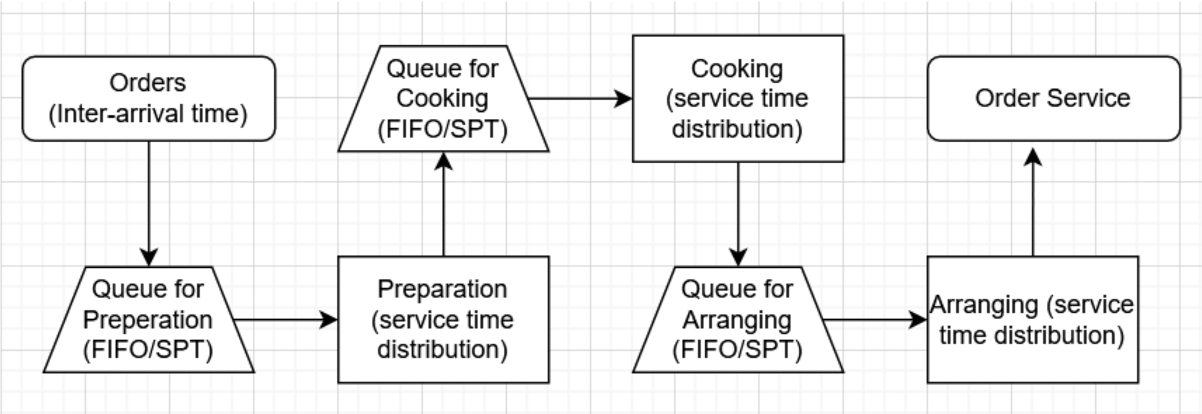
Component	Justification
Entity: Orders, Detail:	Flow through the service process
Activities: Preparation, Cooking, Arranging (Servers)	Required for average throughput time of an order
Queues: Queue for Preparation Queue for Cooking Queue for Arranging	Required for average throughput time of an order and queuing priority experimental factor
Resources: Kitchen personnel (excluded)	Simplification: represented by servers

- Model Level of Detail

Component	Detail	Justification
Entity: Orders	Arrival Pattern: Different mean inter-arrival times which depend on time of the day. Different type of orders distribution. Shift: No arrivals between 10pm and 9am	Required for flow of customers into the system
Activities: Preparation, Cooking, Arranging (Servers)	Service time: Different service time which depends on type of the order	Required for average throughput time of an order
Queues: Queue for Preparation Queue for Cooking Queue for Arranging	Queue discipline: FIFO, SPT	Required for average throughput time of an order and queuing priority experimental factor
Resources: Kitchen personnel (excluded)	N/A	Simplification: represented by servers

- Modelling Assumptions:
  No breakdowns of servers. (Missing ingredients, energy, electricity, cutlery etc)
- Model Simplifications:
  Service personnel (resource) not specifically modelled, but represented by the servers(activities).
  No limit to the number of orders in the queue for service.
  No balking or reneging from the queue.

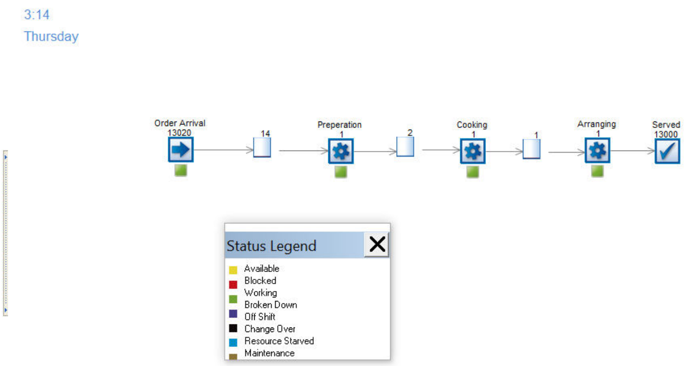
- Process Flow Diagram



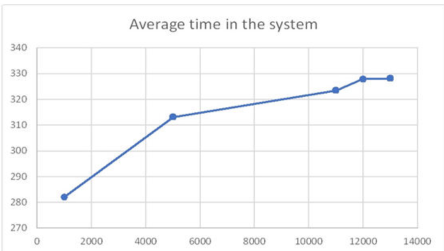
b)

i)

As resources are not considered, I treated each operation (preparing, cooking, arranging) as one server and considered only the service time in the model. The clock runs for 24 hours, 7 days a week and order arrivals are constrained using shift pattern from 9 am to 10pm (after 10 pm restaurant does not accept orders). All the activities (processes) use the timing by label which is assigned to type of order at the start point (also probability profile assigned to the type of order).



Halt at # simulation leaving the system	Average time in the system
1000	282.12
5000	313.22
11000	323.55
12000	327.91
13000	328.2



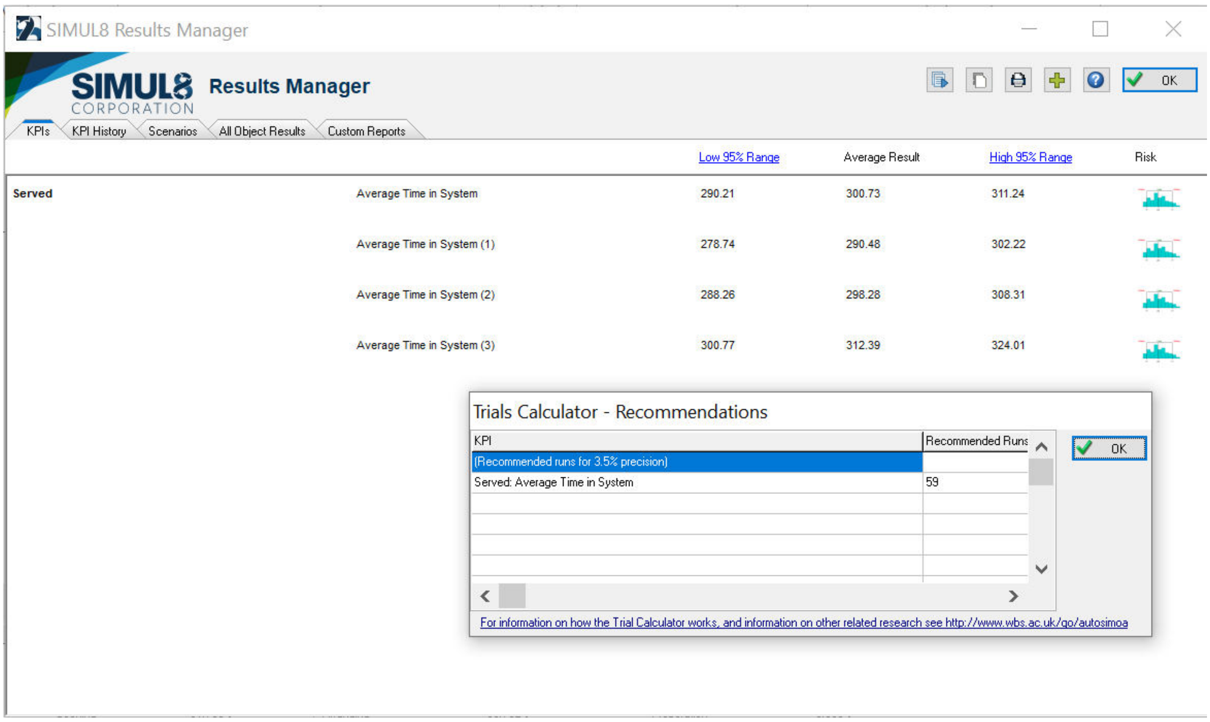
Steady state is being achieved every night after orders stop coming through. Without shift between 10pm and 9 am implemented on order arrival, queueing time in the system increases linearly to infinity.

ii)

Limit used: 1000.

Average throughput time at 1000 limit: 282.12 minutes

Precision needed: $\frac{10}{282.12} = 0.035\ (3.5\%)$



Number of runs required: 59

95% CI for average throughput: (290.21, 311.24)

iii)

Average throughput time of an order of all three types: 300.73 minutes

Average throughput time of an order for cold plates: 290.48 minutes

Average throughput time of an order for hot meals: 298.28 minutes

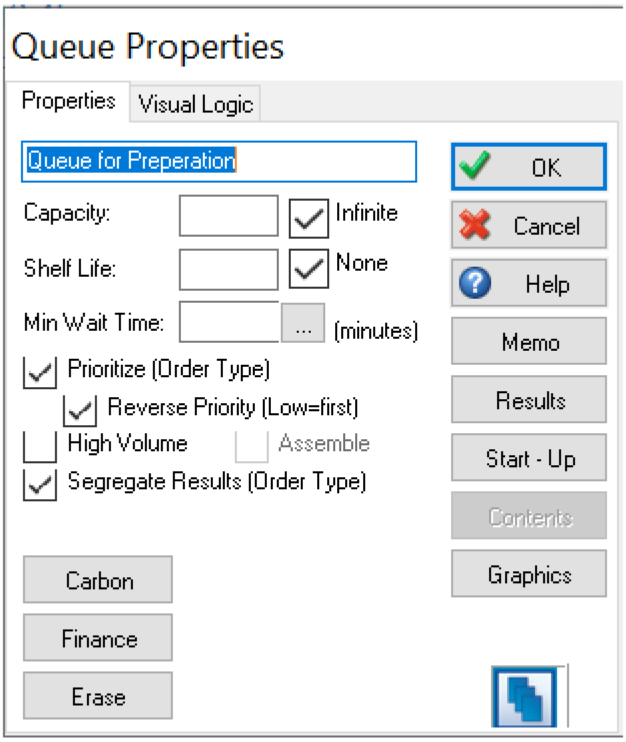
Average throughput time of an order for fish dishes: 312.39 minutes

c)

Expected processing times:

- Cold Meal – 5+4+4=13 minutes
- Hot Meal – 10+12+6=28 minutes
- Fish Meal – 15+23+9=47 minutes

According to SPT rule, we need to prioritise every queue in the system in a way that Cold Meal leaves the queue first and Fish Meal leaves the queue last. By using Reverse Priority, we ensure that order is as required (1-CM, 2-HM, 3-FM, from 1 to 3).

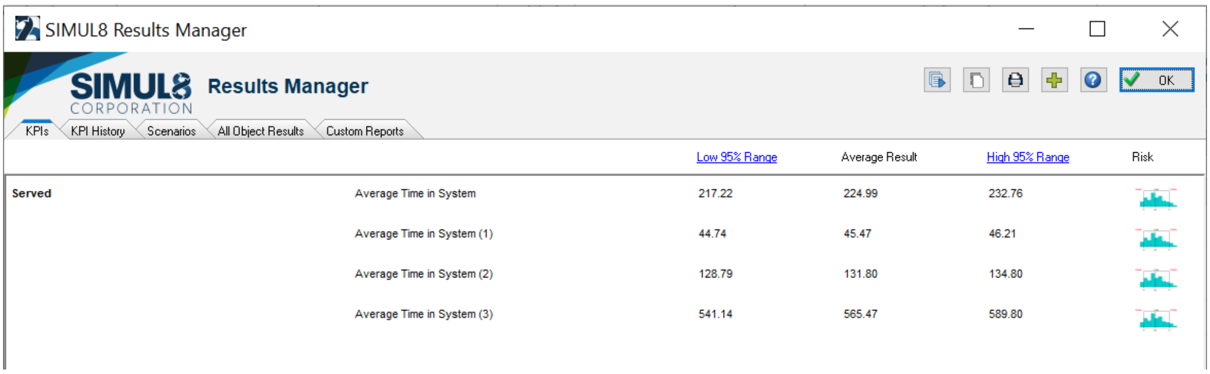


Running our simulation with following parameters:

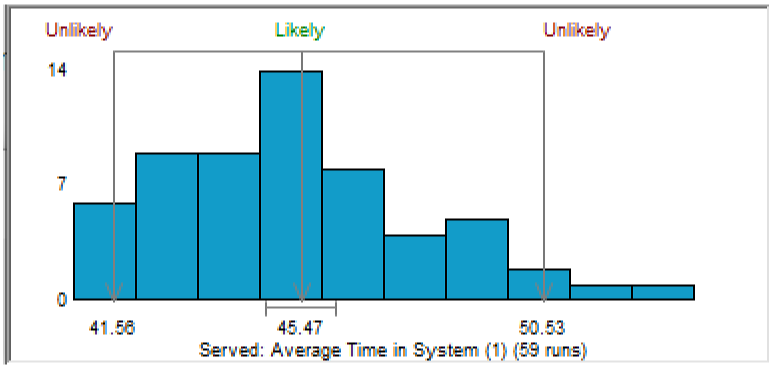
Order limit: 1000

Number of runs in trial: 59

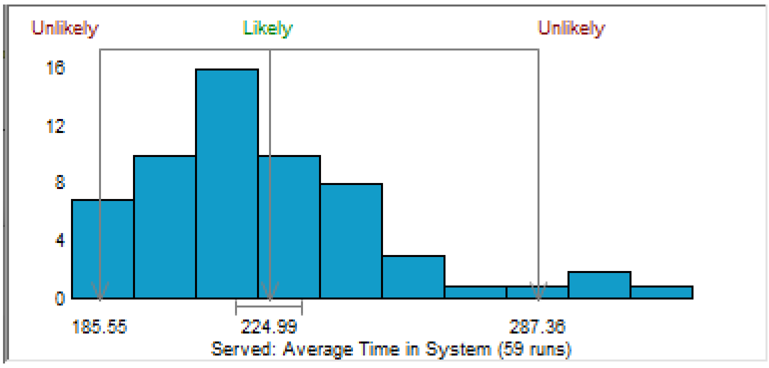
, gives the following results:



Histogram of throughput time of an order for cold plates (59 runs):



Histogram of throughput time of an order for cold plates (59 runs):



So:

Average throughput time of an order of all three types: 224.99 minutes

Average throughput time of an order for cold plates: 45.47 minutes

Average throughput time of an order for hot meals: 131.8 minutes

Average throughput time of an order for fish dishes: 565.47 minutes

Comparing the result with the ones for FIFO rule from b)iii) we can see that average throughput time of an order for cold plate decreased from 290.48 minutes to 45.47 minutes (84.3%) and average throughput time of an order of all three types decreased from 300.73 minutes to 224.99 minutes (25.2%). While average throughput time of an order for hot meals decreased as well, we can observe that average throughput time of an order for fish dishes increased from 312.39 minutes to 546.47 minutes (74.9%).

Based on above calculations I would recommend using SPT queue priority rule.

In addition, the restaurant could use additional Cooking server, as the highest utilised process is Cooking (75.5%). It would significantly decrease order throughput time.

Highest Utilized Activity

Cooking	75.468%
Preperation	65.407%