Mathematics for Finance Coursework

Portfolio Optimization

- Submission deadline: **23:59, Thursday, 1 April** to the assignment dropbox (Coursework 2 Submission). Late submission (by 23:59, Friday, 30 April) to the other assignment dropbox (Coursework 2 LATE Submission) is allowed but will be capped to 40%.
- Please read the following guideline carefully and do not submit irrelevant materials.

Submission guideline

1. The name of your pdf file must be 'xy.pdf', where 'xy' is the number of the research paper you selected (see Section 2), for instance, '04.pdf'.
2. The frontpage of your pdf file must include the following information *only*
   - The student id number;
   - The number of the research paper to be studied in Section 2;
3. There are two sections for this coursework. You report should follow the requirements mentioned below. Please do not provide unnecessary information.

1 Simulation of the mean-variance model (50%)

Apply the classical mean-variance model to an investment in a real financial market. You are asked to use the following mean-variance optimization model

$$(MV) \quad \begin{array}{ll} \min & \sum_{i,j=1}^{n} w_i w_j \sigma_{ij} (= w^{\mathrm{T}} \Sigma w) \\ \text{s.t.} & \sum_{i=1}^{n} w_i \bar{r}_i = \bar{r} \\ & \sum_{i=1}^{n} w_i = 1 \end{array}$$

to conduct a simple simulation involving several financial products. There is no restriction on programming language/software package, e.g., MATLAB, Python, Mathematica, Excel solver, C, Java. You can also use the methods discussed in the book chapter to compute an optimal

portfolio. The required procedures are listed below for your reference. *Your report should include which programming language you use and the corresponding code, as well as the data/results mentioned below.*

- (10%) Let $n = 5$ (for one-member groups) *or* $n = 10$ (for two-member groups) in the model (MV) and select n financial products, for example, n stocks in a stock exchange. Extract their historical prices for one year on the transaction days between 01/01/2019 and 31/12/2019, as well as their closing prices on 31/12/2020 (if this is not a transaction day, use the previous transaction day). The data can be downloaded from Yahoo Finance (finance.yahoo.com/stock-center) historical data. When you choose the historical prices, please use the *adjusted close* prices which have been adjusted by considering various factors such as dividend since they are helpful in the computation. *You shall report which stock exchange, which n stocks, and their prices on 31/12/2019 and 31/12/2020, respectively.*

- (10%) Based on the historical prices in the year 2019, compute the mean of the return for each stock you selected to get the mean vector $(\bar{r}_1, \bar{r}_2, \ldots, \bar{r}_n) \in \mathbb{R}^n$ and compute the co-variance matrix $\Sigma \in \mathbb{R}^{n\times n}$ for the return of these n stocks. There are many ways to compute the mean and variance of the return of the stocks. *In your report, you should describe the way to compute the mean and the variance as well as your results ($\bar{r}_i$'s and Σ).*

- (20%) Given an initial capital which is one unit, choose an expected return $\bar{r} \in (0, 0.05]$, and solve the model (MV), i.e., to get an optimal portfolio $w \in \mathbb{R}^n$ you shall invest (for each stock it is a proportion of the whole unit) and the corresponding optimal value $w^{\mathrm{T}}\Sigma w$. If one unit is a million, based on the prices on 31/12/2019, compute the number of shares to buy (or short sell if it is negative proportion) for each stock. Fractional shares are allowed. Do the same work by choosing another two expected returns $\bar{r}$, one in $(0.05, 0.1]$ and one in $(0.1, 0.2]$. *For each $\bar{r}$ you choose, report the value of $\bar{r}$, the optimal portfolio w, the variance $w^{\mathrm{T}}\Sigma w$ of the optimal portfolio, and the number of shares to invest on each stock.*

- (10%) Suppose you hold the portfolio until 31/12/2020. *Report the final value of your portfolio on 31/12/2020 for each $\bar{r}$ you choose. Briefly explain your results for different $\bar{r}$'s.*

Remark: There is no transaction fee involved and there is also no risk-free asset to consider. You are free, and in fact encouraged, to discuss with other groups. *However, you should work on your own data and computation (it is zero probability for two groups to choose the exact same stocks) and please avoid copying code.*

2 Research paper analysis (50%)

Select one research paper for you and complete the article analysis (at least one A4 page and no more than two A4 pages in length) as follows:

- (15%) What are the main mathematical/optimization models discussed in the paper? (Please be explicit when you describe models, for instance, using mathematical formulae.)

- (15%) What techniques/methodology/algorithms are used in the paper?

- (10%) What are the differences and advantages in the paper by comparing to the classical mean-variance model?

- (10%) What is the contribution of the paper to the financial optimization field? (You may need to check other research papers and/or media articles where the work of this paper was mentioned.)