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Q1.

a) Xavier Ltd is the long party and Darrow Investments is the short party. The long party holds an option, Xavier Ltd buys the options from Darrow Investments.

b) The underlying asset of each option is ~~the~~ one share in Exillon Energy PLC.

c) Xavier Ltd buys 400 calls at price $C_0 = 3.5$ pounds each, so Xavier Ltd pays the setup costs.

Cashflows are:

- Xavier Ltd : $400(-C_0) = 400(-3.5) = -1400$ £
- Darrow Investments: $400(+C_0) = 400(3.5) = 1400$ £

d)

1) We have $S_T = 11.25$ and $K = 32.25$ $(K > S_T)$. Xavier Ltd has the right but no obligation to buy 400 shares, so it does nothing, giving payoff zero. Buying shares would give negative payoff $(S_T - K)$, -21£ per call.

2) Payoff for one call is $C_T = \max(S_T - K, 0)$
$= \max(11.25 - 32.25, 0) = \max(-21, 0) = 0$

Xavier Ltd is long 400 calls, giving payoff:

$+400\, C_T = 400 \cdot 0 = 0$

Darrow Investments is short 400 calls, giving payoff:

$-400\, C_T = -400 \cdot 0 = 0$

(p2)

3. Profit = Payoff + initial Cashflow

For Xavier Ltd we have profit $C_T - C_0$ per call, giving:

$$400(C_T - C_0) = 400(0 - 3.5) = -1400 \text{ £}$$

(net loss)

For Darrow Investments we have profit $-C_T + C_0$ per call, giving:

$$400(-C_T + C_0) = 400(0 + 3.5) = 1400 \text{ £}$$

(net gain)

e) Here, $S_T = 39.75$ and $K = 32.25$

1. $(S_T > K)$ Xavier Ltd uses the calls to buy 400 shares at 32.25 each and immediately sells them to the market at 39.75, getting payoff 7.5 per call $(S_T - K)$.

2. Payoff for one call is $C_T = \max(S_T - K, 0)$

$$= \max(39.75 - 32.25, 0) = \max(7.5, 0) = 7.5$$

Xavier Ltd is long 400 calls, giving payoff:

$$+400\, C_T = 400 \cdot 7.5 = 3000 \text{ £}$$

Darrow Investments is short 400 calls, giving payoff:

$$-400\, C_T = -400 \cdot 7.5 = -3000 \text{ £}$$

3. For Xavier Ltd we have profit $C_T - C_0$ per call:

$$400(7.5 - 3.5) = 1600 \text{ £}$$

For Darrow Investments we have profit $-C_T + C_0$ per call:

$$400(-C_T + C_0) = 400(-7.5 + 3.5) = -1600 \text{ £}$$

Q2

a) $S_t \approx 3 + \frac{1}{11} \approx 3.1$ pence

$S_T \approx 7 + \frac{6.5}{11} \approx 7.6$ pence

$K = 6$ pence

$\max(\underline{S}) \approx 9 + \frac{3.5}{11} \approx 9.3$ pence

$\min(\underline{S}) \approx 3 - \frac{1}{11} \approx 2.9$ pence

1) Payoff for one European call option:

$C_T = \max(S_T - K, 0) = \max(7.6 - 6, 0) = 1.6$ pence per call

2) Payoff for one European put option:

$P_T = \max(K - S_T, 0) = \max(6 - 7.6, 0) = 0$ pence per put

3) Payoff for one Lookback Call:

$\max(\max(\underline{S}) - K, 0) = \cancel{9.3} \max(9.3 - 6, 0) = 3.3$ pence per call

4) Payoff for one Lookback Put:

$\max(K - \min(\underline{S}), 0) = \max(6 - 2.9, 0) = 3.1$ pence per put

b) Let Π_T be the value of a BVCS portfolio at expiry time T

one short call with strike price K_2

$\Pi_T = \underbrace{+C_{K_1,T}}_{\text{one long call with strike price } K_1} - C_{K_2,T}$

c) $\Pi_T = \max(S_T - K_1, 0) - \max(S_T - K_2, 0)$

d) We have been given that $0 < K_1 < K_2$, which for different possible values of S_T, breaks down into the following three cases.

Q4

$$\Pi_T = \begin{cases} 0-0=0, & 0 \leq S_T \leq K_1 \\ \max(S_T-K_1,0)-0 = S_T-K_1, & K_1 \leq S_T \leq K_2 \\ \max(S_T-K_1,0)-\max(S_T-K_2,0) = \\ = S_T-K_1-S_T+K_2 = K_2-K_1, & K_2 \leq S_T \end{cases}$$

Q3

a) Payoff of the option A can be replicated using a portfolio $V = C + nD$ if V has the same payoff in all possible future scenarios. We need to show that:

$$V_T = A_T$$

$$\Rightarrow C_T + n D_T = A_T$$

There are two cases:

1) $S_T > K$
2) $S_T \leq K$

1) $S_T > K$

$$C_T = S_T - K, \quad D_T = 1, \quad A_T = S_T$$

$$S_T - K + n = S_T$$

$$\Rightarrow S_T - S_T + n = K$$

$$\Rightarrow n = K$$

2) $S_T \leq K$

$$C_T = 0, \quad D_T = 0, \quad A_T = 0$$

$$0 + n \cdot 0 = 0$$

$$\Rightarrow n \in R$$

So, $n = K$ for both cases,

and $V_T = C_T + KD_T$ replicates the payoff of the option A (A_T).

b) $C_0 = S_0 N(d_1) - Ke^{-rT} N(d_2)$, $D_0 = e^{-rT} N(d_2)$

No arbitrage $\Rightarrow$ $A_0 = V_0 = C_0 + K D_0$

$$A_0 = S_0 N(d_1) - Ke^{-rT} N(d_2) + K e^{-rT} N(d_2)$$

$$A_0 = S_0 N(d_1)$$

where $d_1 = \dfrac{\ln(S_0/K) + (r + \sigma^2/2)T}{\sigma\sqrt{T}}$, $t=0$

c) In case where $S_T > K$, we have:

$R_T = 0$, $A_T = S_T \Rightarrow S_T - A_T = 0$

so $R_T = S_T - A_T$

In case where $S_T \leq K$, we have:

$R_T = S_T$, $A_T = 0$

$\Rightarrow S_T - R_T = 0$

So $A_T = S_T - R_T$

$\Rightarrow R_T = S_T - A_T$

Hence, R can be replicated using S and A.

Assuming no arbitrage $\Rightarrow$ $R_0 = S_0 - A_0$

So, $R_0 = S_0 - S_0 N(d_1) = S_0 (1 - N(d_1))$

$= S_0 N(-d_1)$

where $d_1 = \dfrac{\ln(S_0/K) + (r + \sigma^2/2)T}{\sigma\sqrt{T}}$, $t=0$

(p6)

d) $R_T = \begin{cases} 0 & , S_T > K \\ S_T & \text{otherwise} \end{cases}$

We will put $\mu = r$ and use

$$\frac{R_0}{B_0} = \mathbb{E}_Q\left[\frac{R_T}{B_T} \middle| S_0\right]$$

$$\Rightarrow R_0 = B_0\, \mathbb{E}_Q\left[\frac{R_T}{B_T} \middle| S_0\right] = \cancel{R_0}$$

$$R_0 = \frac{B_0}{B_T}\, \mathbb{E}_Q\left[R_T | S_0\right]$$

$$R_0 = e^{-rT}\, \mathbb{E}_Q\left[R_T | S_0\right]$$

$$\cancel{R_0 = e^{-rT} \int_{-\infty}^{+\infty} R_T\, n(z)\, dz}$$

To compute the expectation via integral form:

$$I = \mathbb{E}_Q\left[R_T | S_0\right] = \int_{z=\infty}^{+\infty} R_T\, n(z)\, dz$$

$$\cancel{R_0 = e^{-rT} \int_{-\infty}^{+\infty} R_T\, n(z)\, dz}$$

$$I = \int_{-\infty}^{-d_2} R_T\, n(z)\, dz + \int_{-d_2}^{\infty} R_T\, n(z)\, dz$$

Under the risk neutral measure, $S_T > K \iff z > -d_2$
and using the payoff of the ~~function~~ option:

$$I = \int_{-\infty}^{-d_2} S_T\, n(z)\, dz + \underbrace{\int_{-d_2}^{\infty} 0\, n(z)\, dz}_{\to 0}$$

$$= \int_{-\infty}^{-d_2} e^{m+sz}\, \frac{e^{-z^2/2}}{\sqrt{2\pi}}\, dz$$

$$= \int_{-\infty}^{-d_2} e^{m} \cdot e^{sz}\, \frac{e^{-z^2/2}}{\sqrt{2\pi}}\, dz$$

$$= e^{m} \int_{-\infty}^{-d_2} \frac{e^{sz - z^2/2}}{\sqrt{2\pi}} \, dz$$

Completing the square in the exponent:

$$= e^{m} \int_{-\infty}^{-d_2} \frac{e^{-\frac{1}{2}(z-s)^2 + \frac{1}{2}s^2}}{\sqrt{2\pi}} \, dz$$

$$= e^{m} \int_{-\infty}^{-d_2} \frac{e^{-\frac{1}{2}(z-s)^2} e^{\frac{1}{2}s^2}}{\sqrt{2\pi}} \, dz$$

$$= e^{m} \cdot e^{\frac{1}{2}s^2} \int_{-\infty}^{-d_2} \frac{e^{-\frac{1}{2}(z-s)^2}}{\sqrt{2\pi}} \, dz$$

Changing variables:

$$u = z - s \Rightarrow du = dz$$

$$z = -\infty \Rightarrow u = -\infty$$

$$z = -d_2 \Rightarrow \cancel{-d_2} \cdot u = -d_2 - s = -d_2 - \sigma\sqrt{T-t} =$$

$$= -d_1 + \sigma\sqrt{T-t} - \sigma\sqrt{T-t} = -d_1$$

$$= e^{m} e^{\frac{1}{2}s^2} \int_{-\infty}^{-d_1} \frac{e^{-\frac{1}{2}u^2}}{\sqrt{2\pi}} \, du$$

$$= e^{m + \frac{1}{2}s^2} \int_{-\infty}^{-d_1} n(u) \, du = e^{m + \frac{1}{2}s^2} \left[N(u) \right]_{-\infty}^{-d_1}$$

$$= e^{m + \frac{1}{2}s^2} \left(N(-d_1) - N(-\infty) \right) = e^{m + \frac{1}{2}s^2} \left(N(-d_1) - 0 \right)$$

$$= e^{m + \frac{1}{2}s^2} \cdot N(-d_1)$$

$$m = \ln S_t + (r - \tfrac{1}{2}\sigma^2)(T-t)$$

$$s = \sigma\sqrt{(T-t)}$$

$$= e^{\ln S_t + (r - \sigma^2/2)(T-t) + \sigma^2 (T-t)/2} = e^{\ln S_t + r(T-t)}$$

$$= S_t e^{r(T-t)} N(-d_1)$$

(8)

So:

$$R_0 = e^{-rT} I$$

$$\Rightarrow R_0 = e^{-rT} \cdot S_0 e^{rT} N(-d_1)$$

$$\Rightarrow R_0 = S_0 N(-d_1)$$

By performing the integration, we obtained the same result as in answer c, namely:

$$R_0 = S_0 N(-d_1)$$