CATEGORIES

1. Stuff

2. Rules

THE STUFF of CATEGORIES

OBJECTS

ARROWS

aka

MORPHISMS

Tell us your

CATEGORY

Exercise

Attendee Examples

- in person
- on-line

Rules —

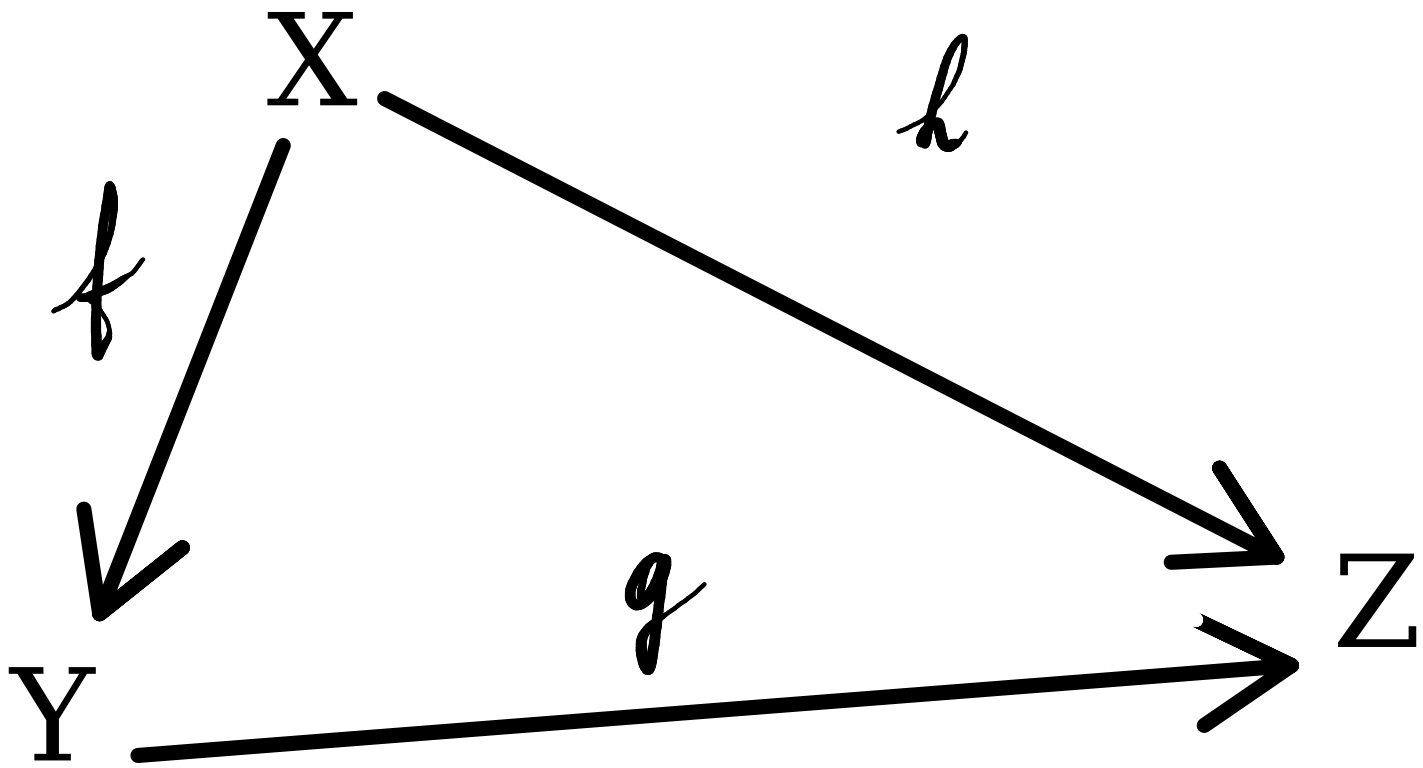
1) Arrows — Compose

2) Arrows — Identify

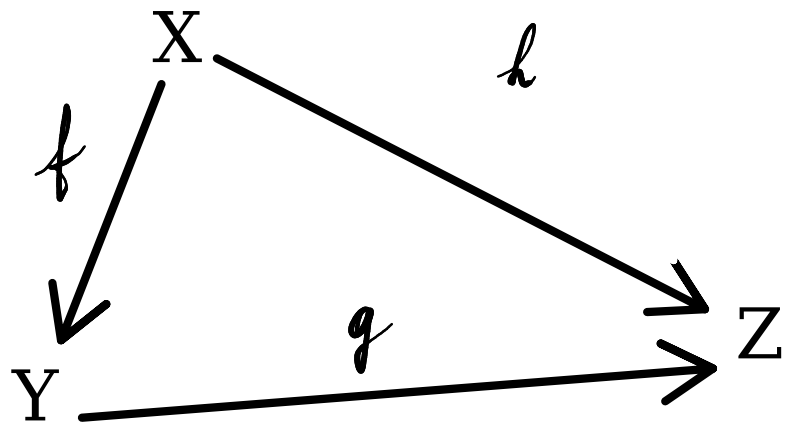
3) Arrows — Associate

C I A

Composition (1)



Composition of ~~Arrows~~ (2)



$$X \xrightarrow{f} Y$$

$$Y \xrightarrow{g} Z$$

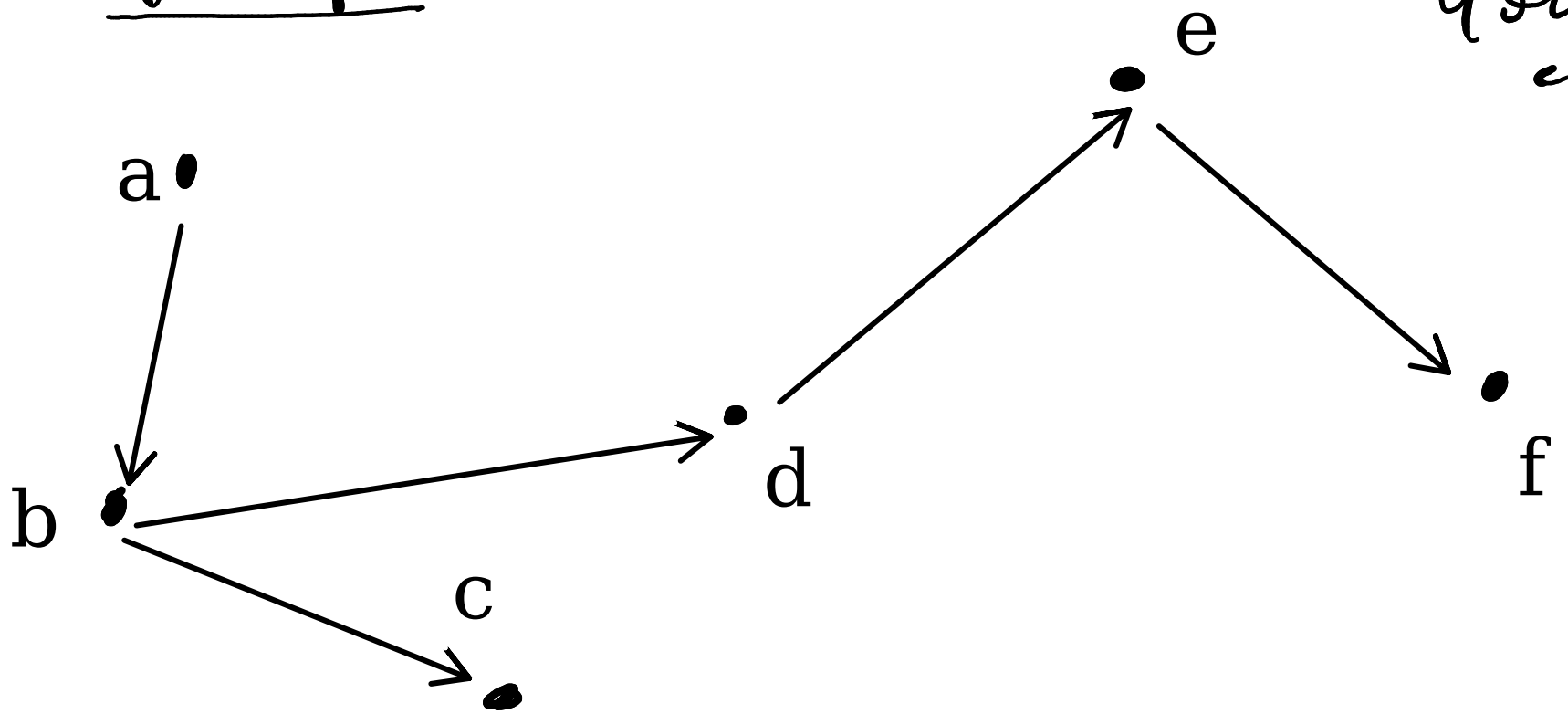
$$X \xrightarrow{h} Z$$

$$g \circ f = h$$

$$f ; g = h$$

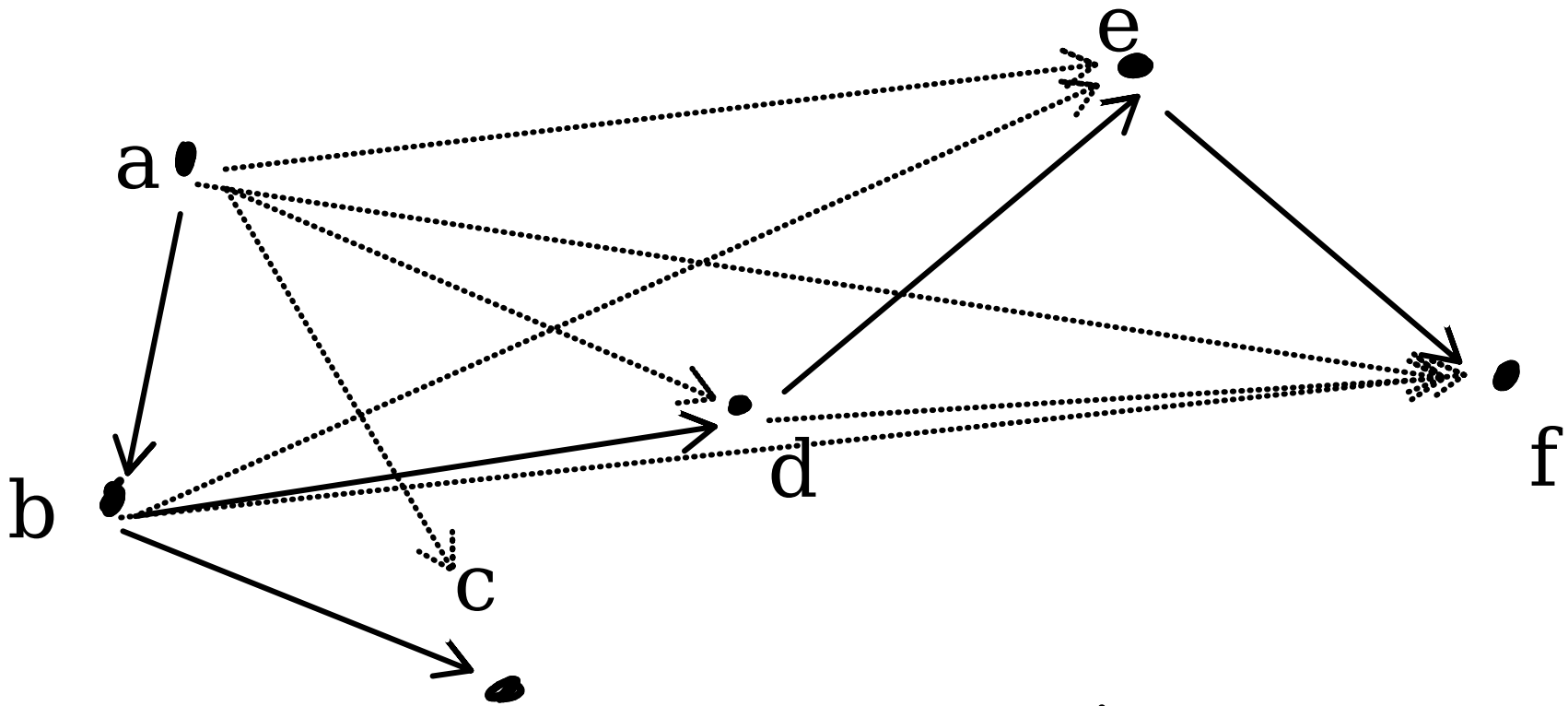
Composition (3)

A graph of ... statistical dependencies! You choose.



Exercise: Is this a category?

Composition (4)

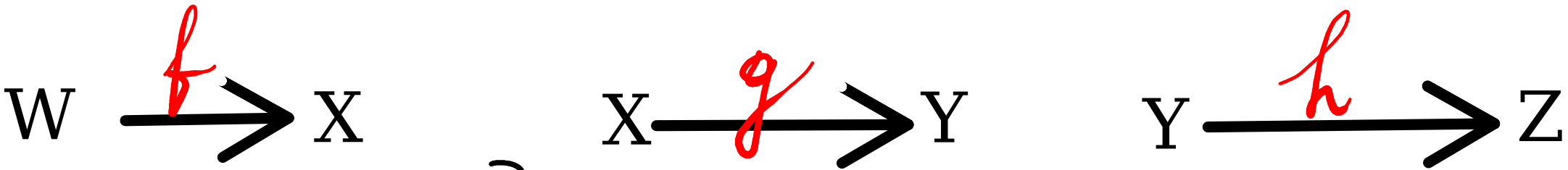


You need to have all the compositions (dotted lines).

Exercise: What is the graph equivalent of the dotted lines?

Associativity

3 arrows f, g, h with compatible domains and co-domains:



Is it $h \circ (g \circ f)$?

Or is it $(h \circ g) \circ f$?

Exercise: Well? And what are the domain/co-domain of f;g;h?

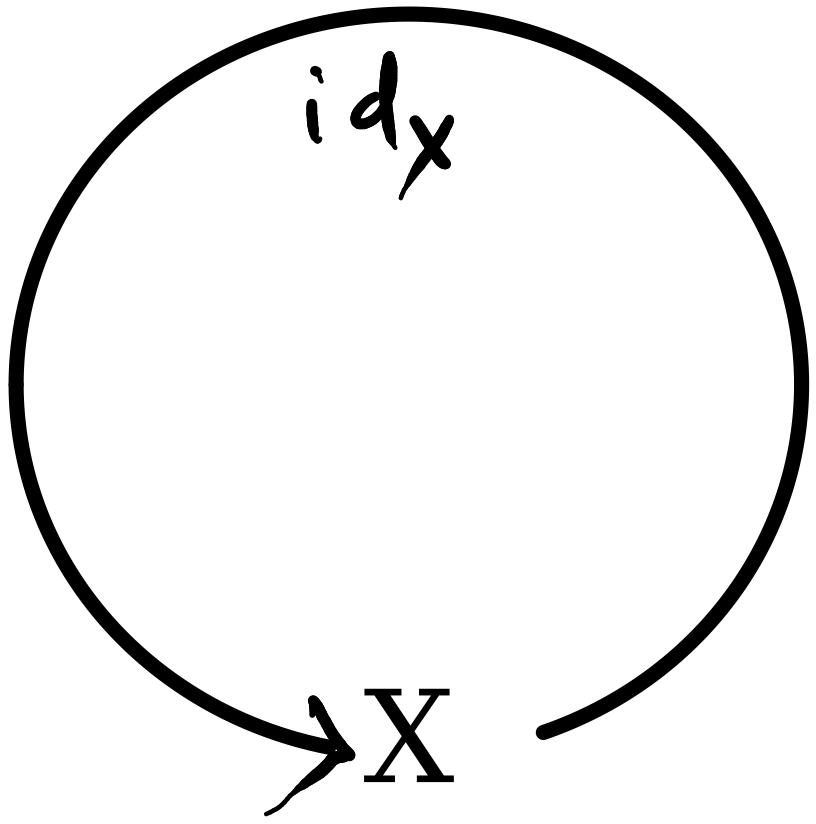
Associativity (2)

$$h \circ (g \circ f) = (h \circ g) \circ f$$

A requirement for something to be a category

IDENTITY

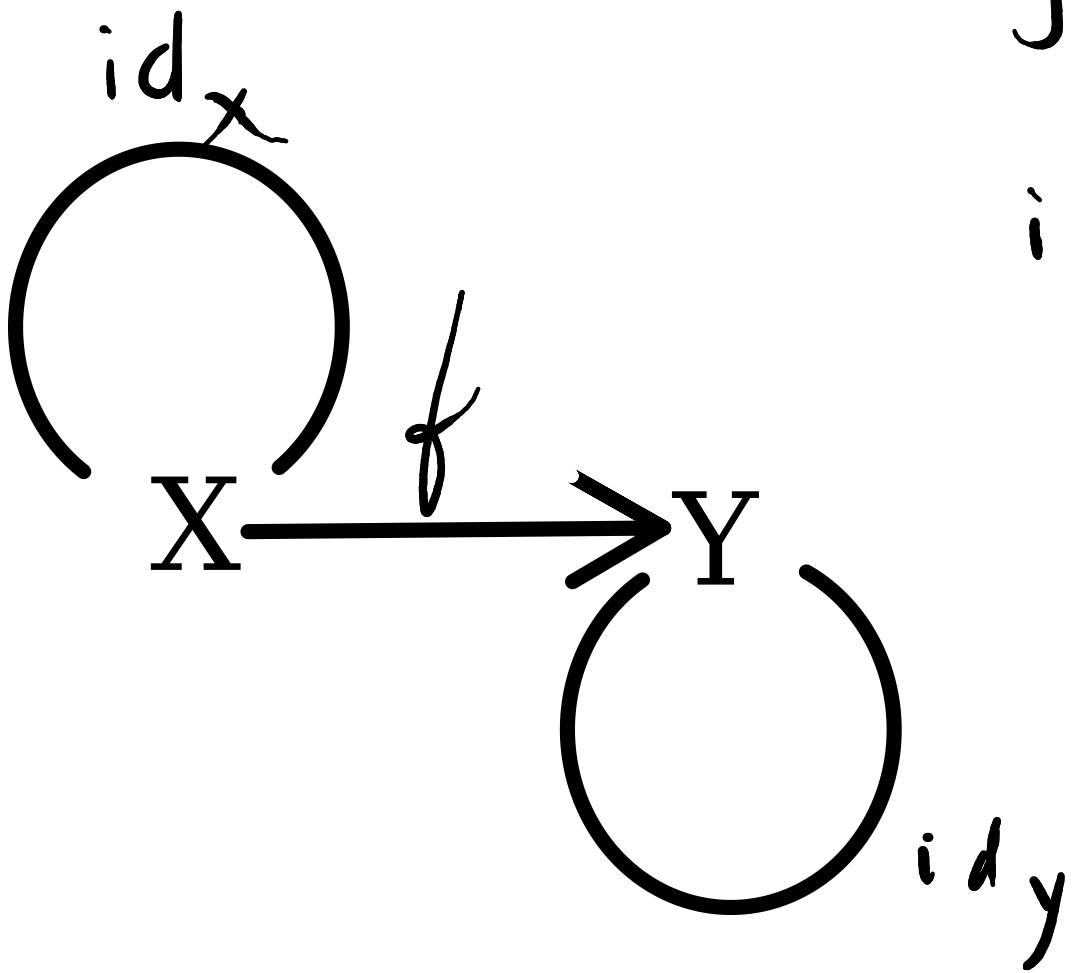
Every object has an identity arrow



Identity (2)

$$f \circ id_X = f$$

$$id_Y \cdot f = f$$



Advice: Judge an arrow by the effect it has on other arrows. For example,

epimorphisms

monomorphisms

isomorphisms.

traps — thinking of arrows as functions and objects as sets. They may be, but often are NOT.

Your category Your choices

DID

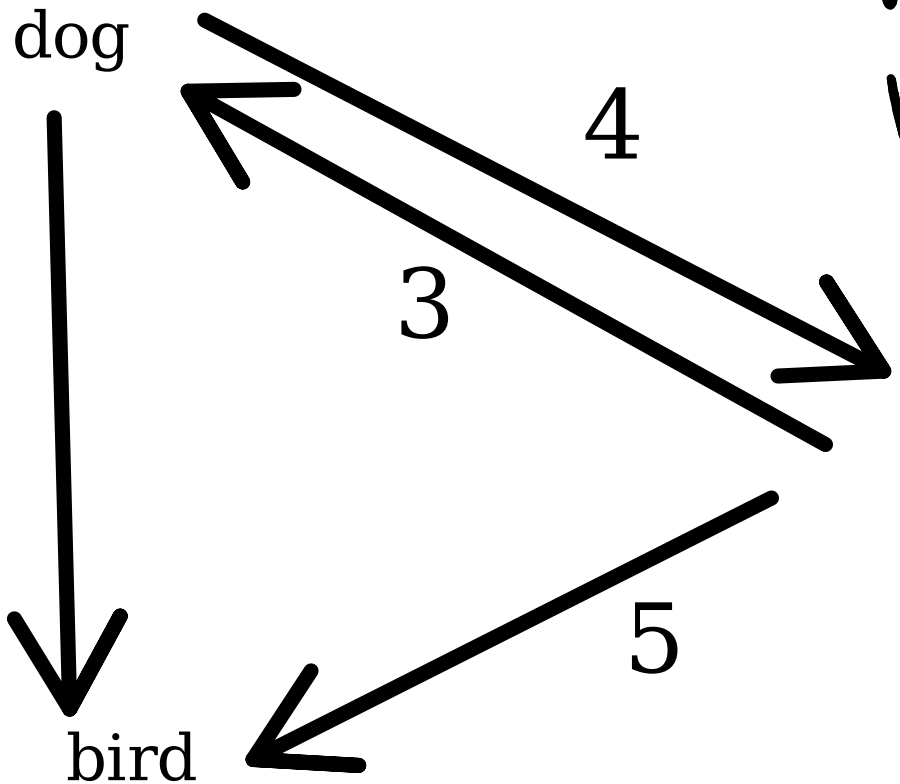
You

MAKE

A

Category?

Is this a category?

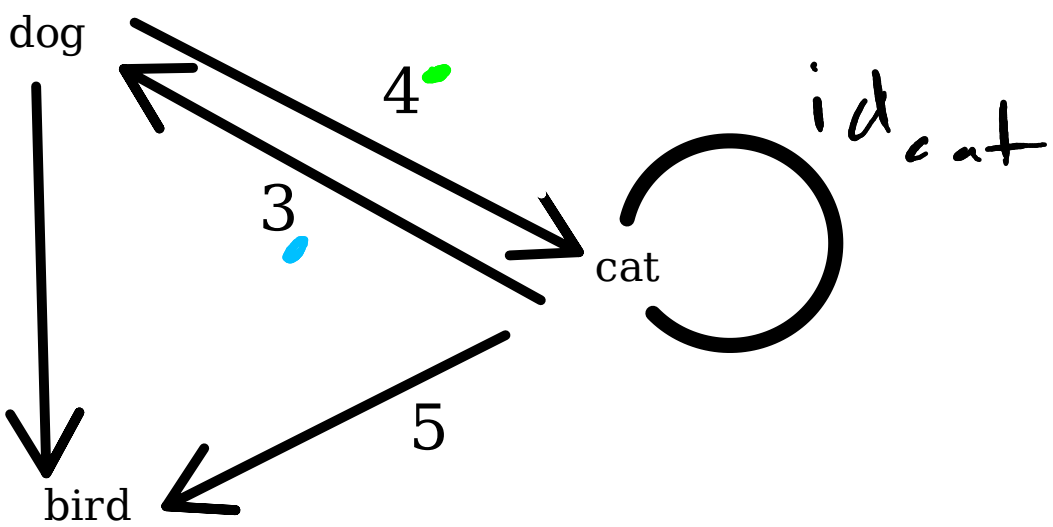


What are our objects?
What are our arrows?
What is composition?
Where are the ids?
Do our choices follow the rules?

"OM" — meditate
"CIA" — the get paranoid

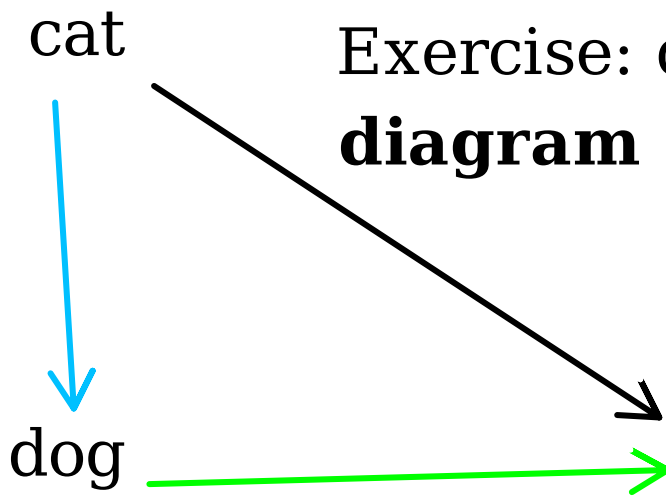
OM — objects and morphisms*
CIA — composition
identity
associativity

*same as arrows



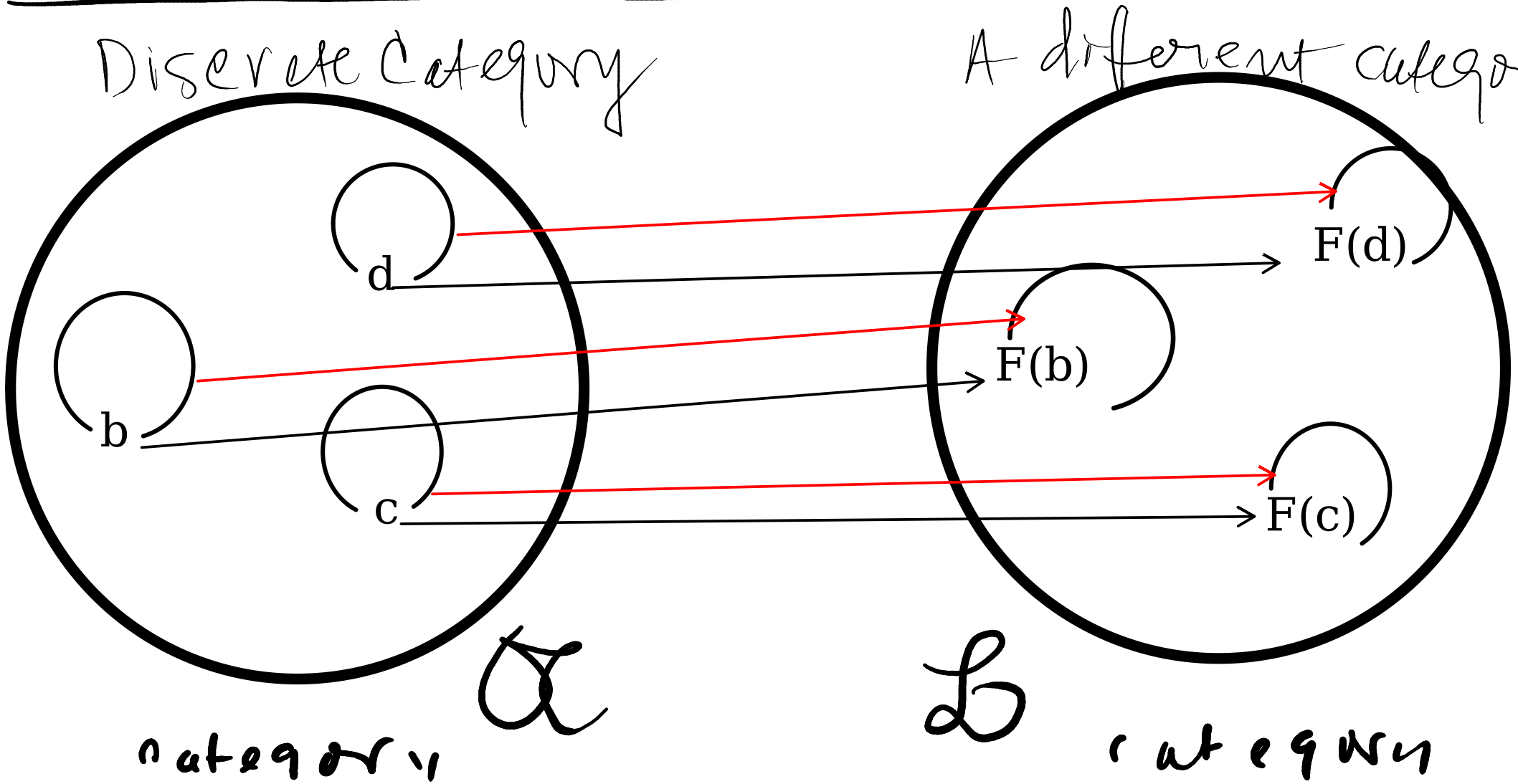
$4 \circ 3 \overset{\text{must}}{=} id_{cat}$

Exercise: does it?



Exercise: does this **diagram** commute ?

Functors (1) - getting to the right space



Functors (2)

Functors take objects from one category to another.

Functors take arrows from one category to another

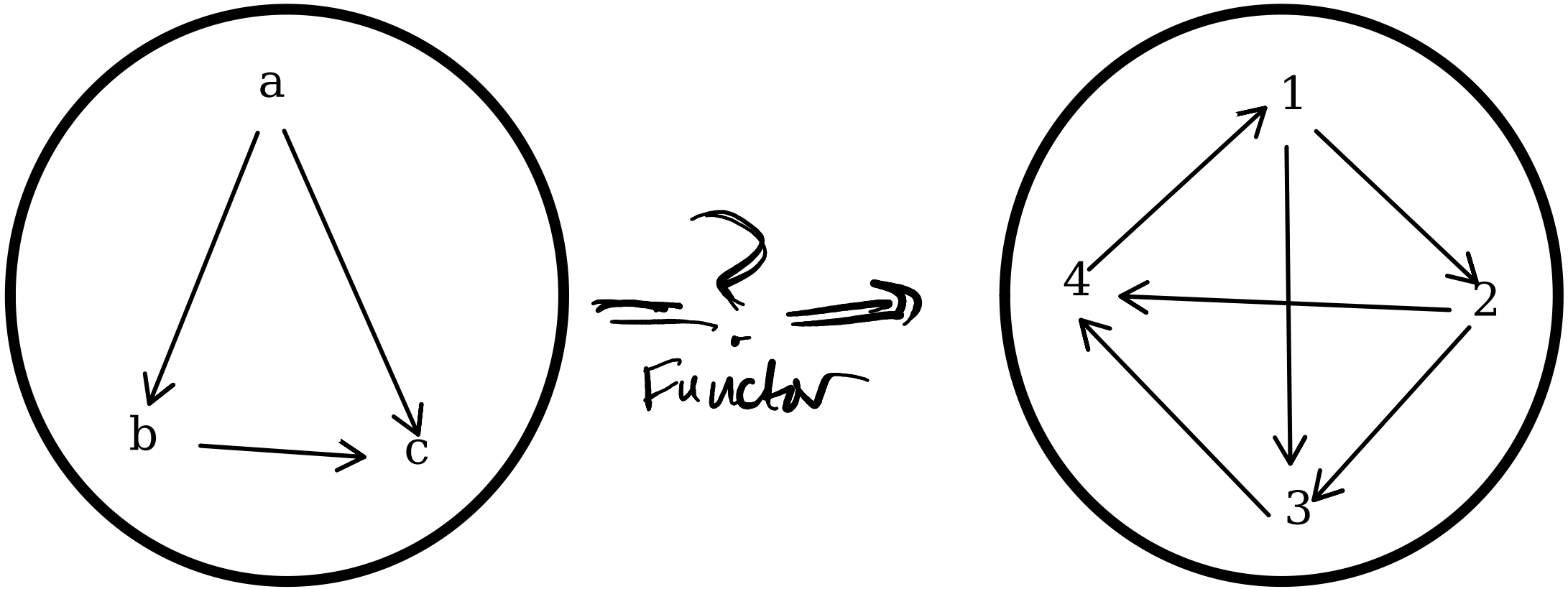
Functors respect **STRUCTURE**

$F(id_X) = id_{F(X)}$

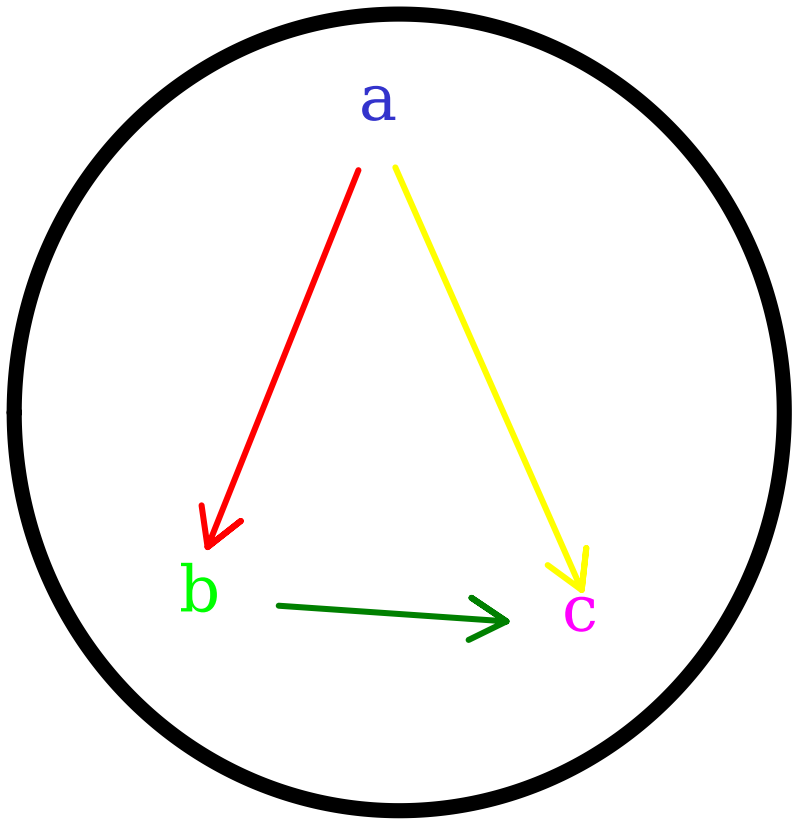
$F(g \circ f) = F(g) \circ F(f)$

Exercise: why did I draw the composition operators in two different colors?

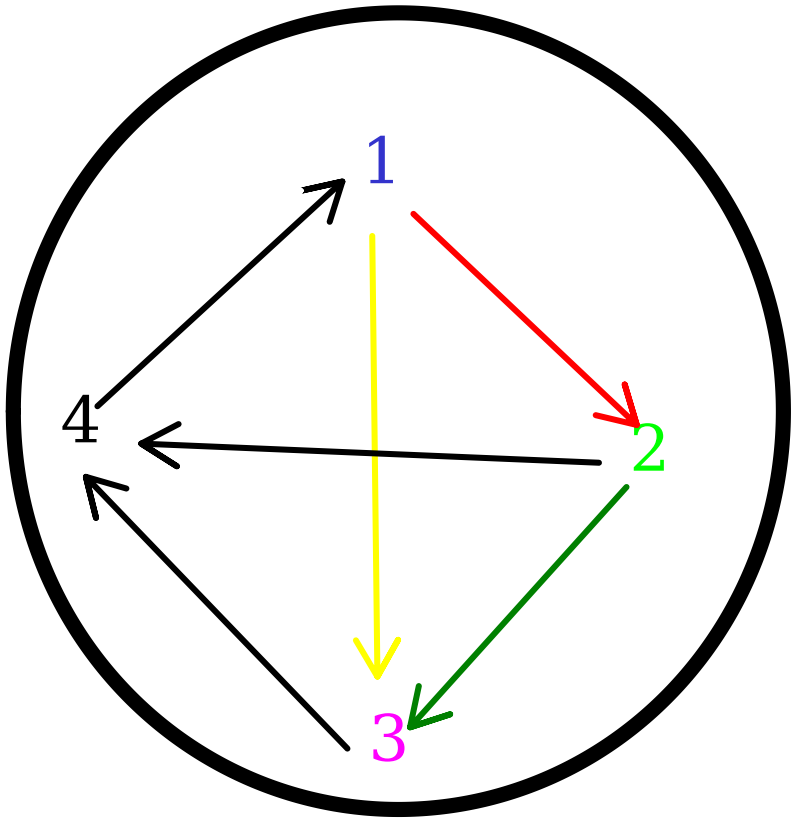
Functors – More than one option (3)



Functor (4)

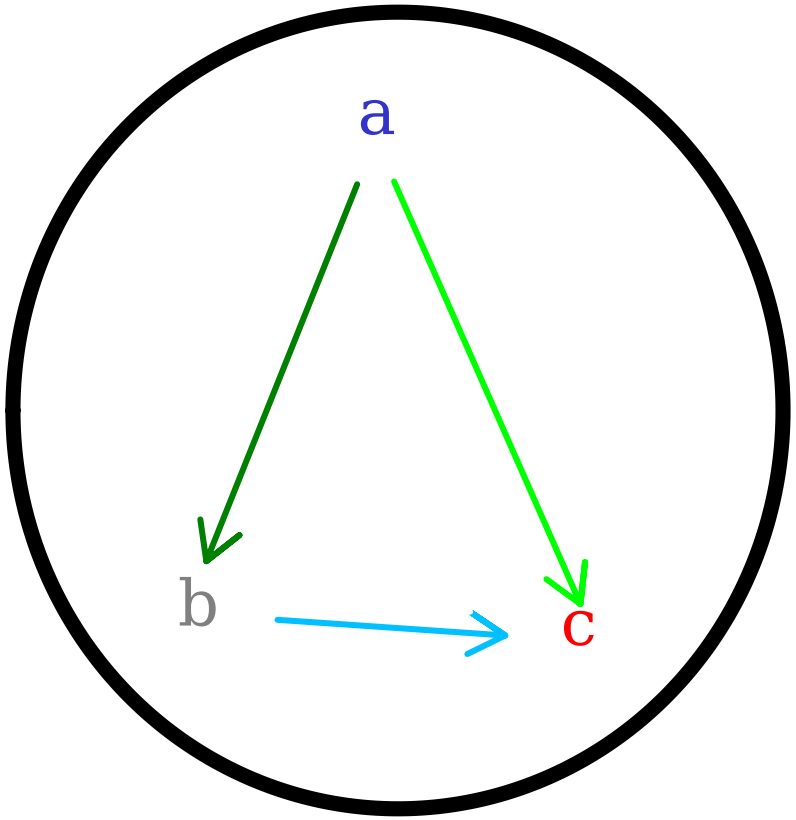


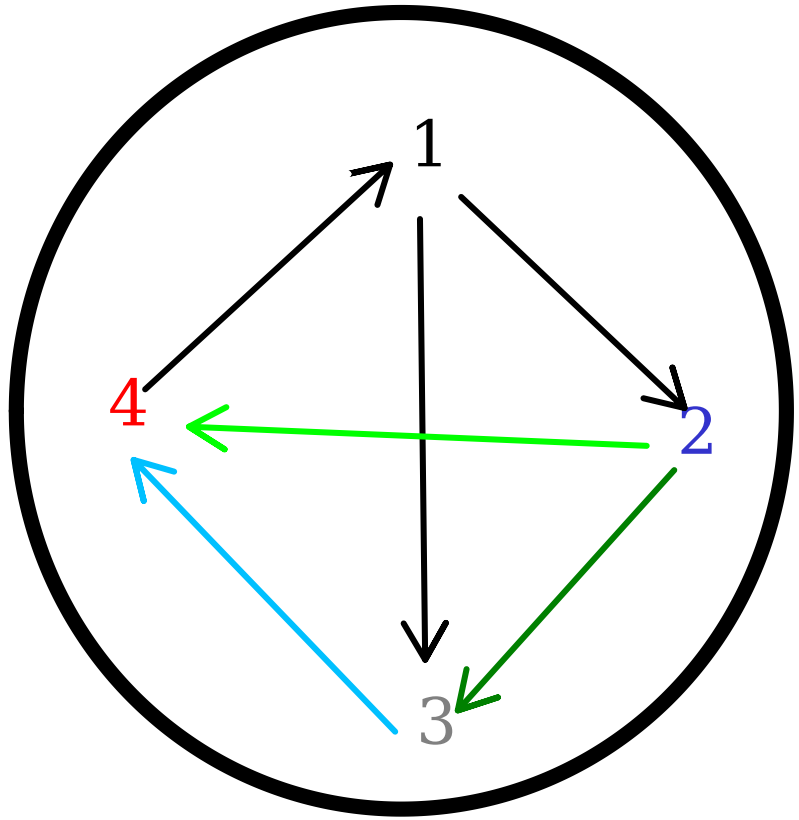
f



Exercise: Is this a Functor?

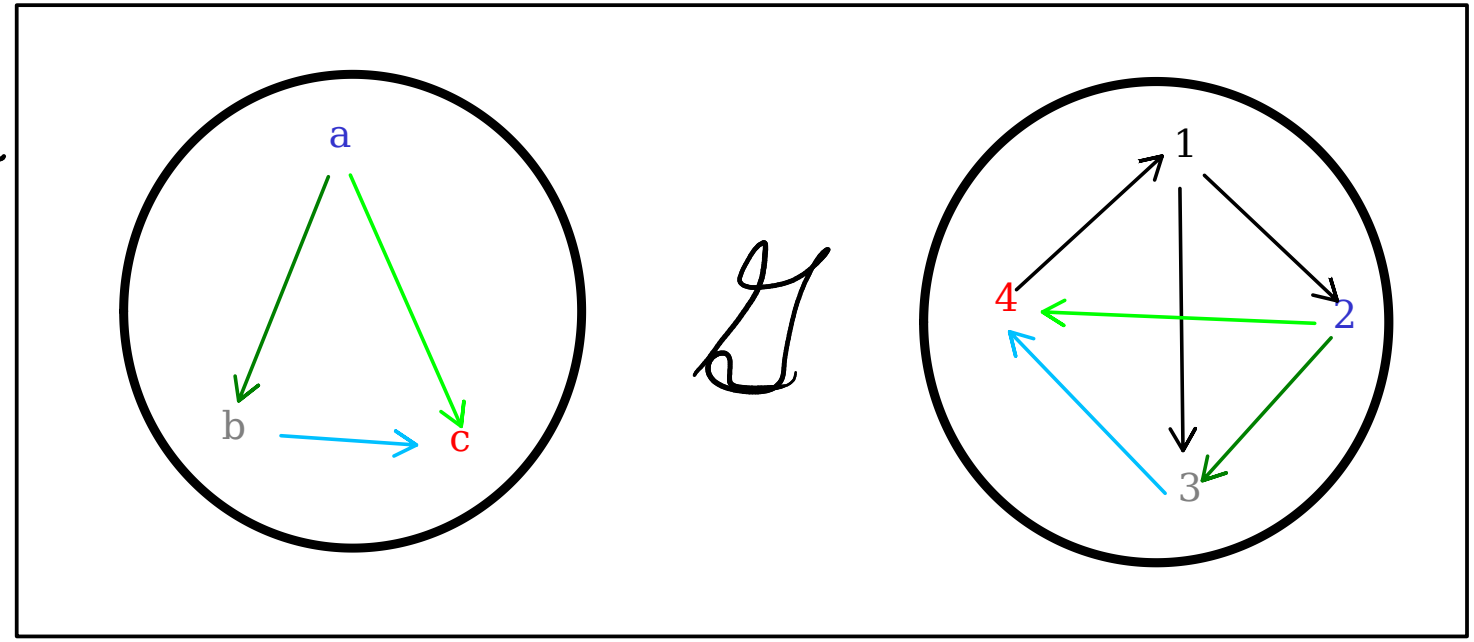
Functor (5)

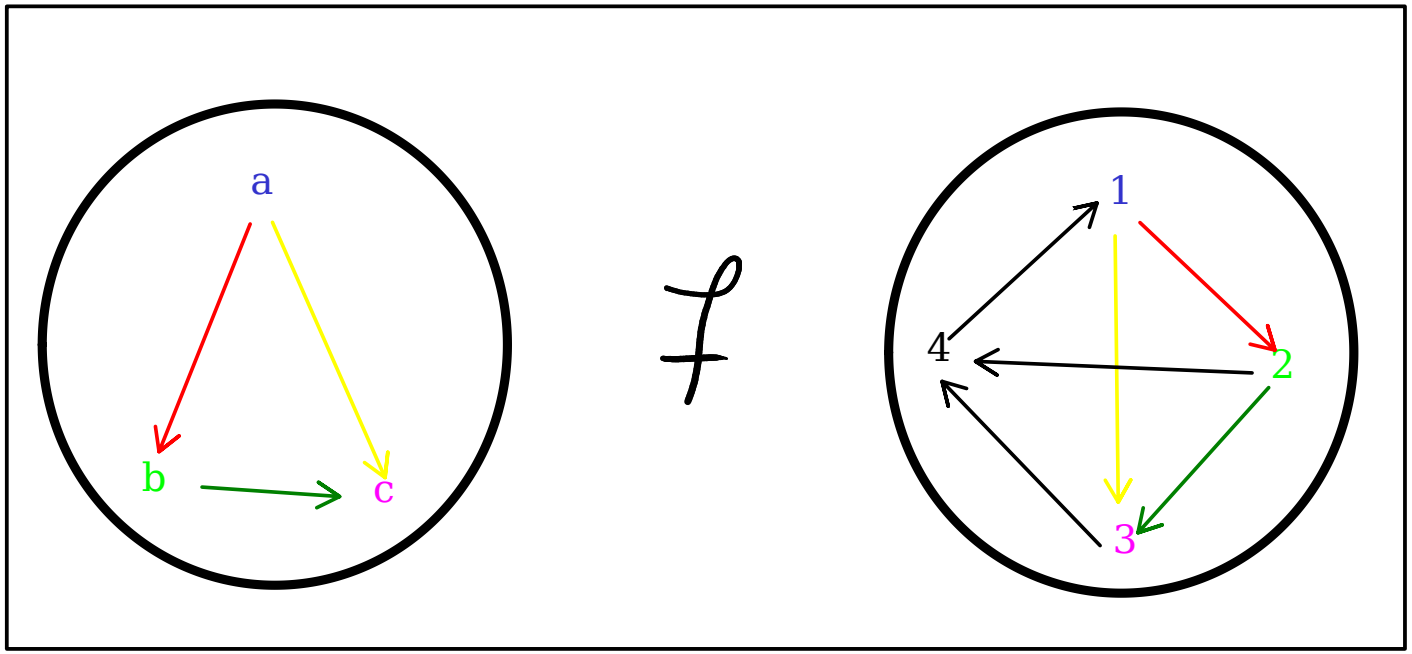




Exercise: Is this a Functor?

Functor (6)



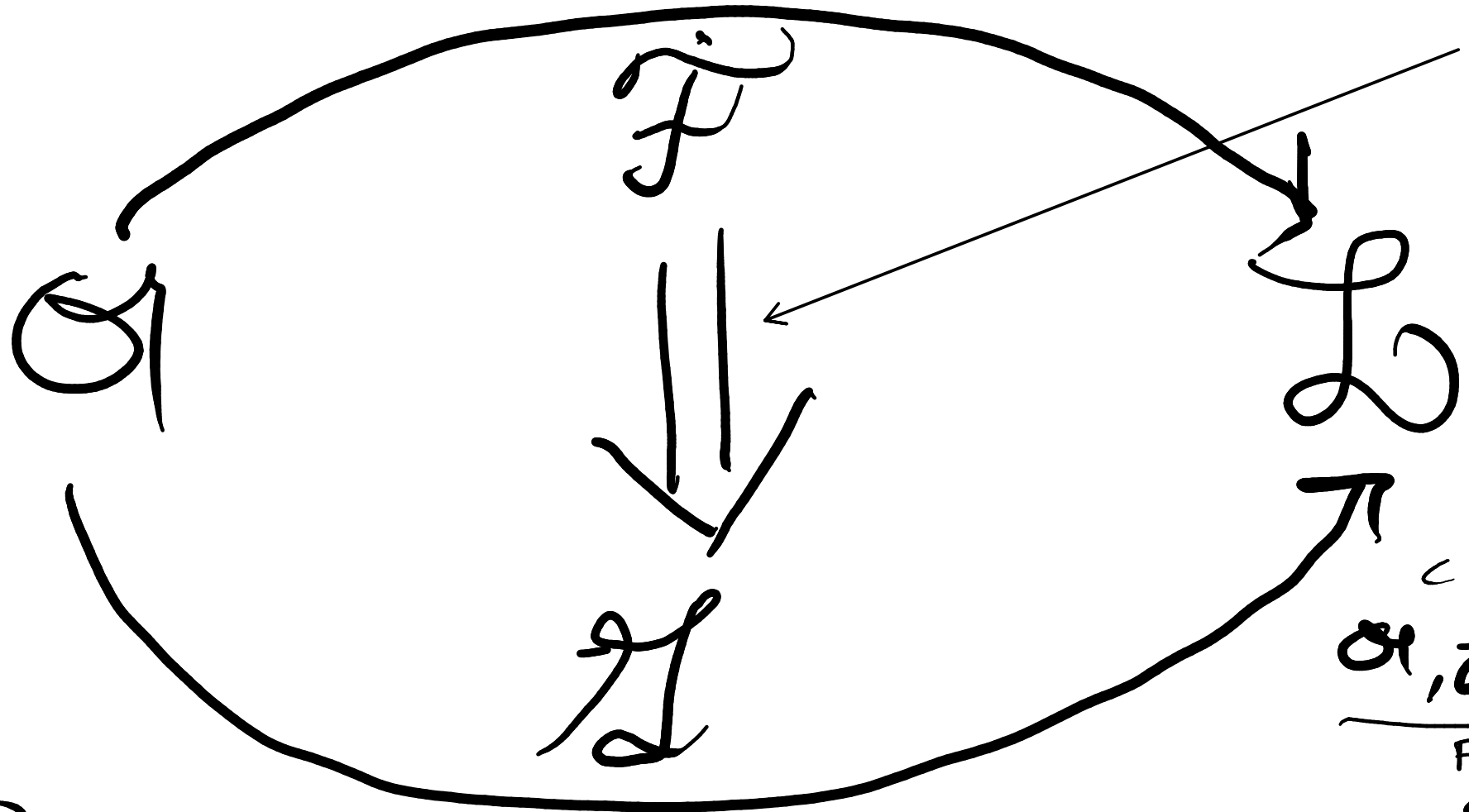


Exercise: Are these the same functor?

What do you mean by same?

Equality	$=$
Isomorphic	$\cong$
Equivalence	$\simeq$

Natural Transformations (1)



CATEGORIES

$\mathcal{A}, \mathcal{B}$

Functors

F, G

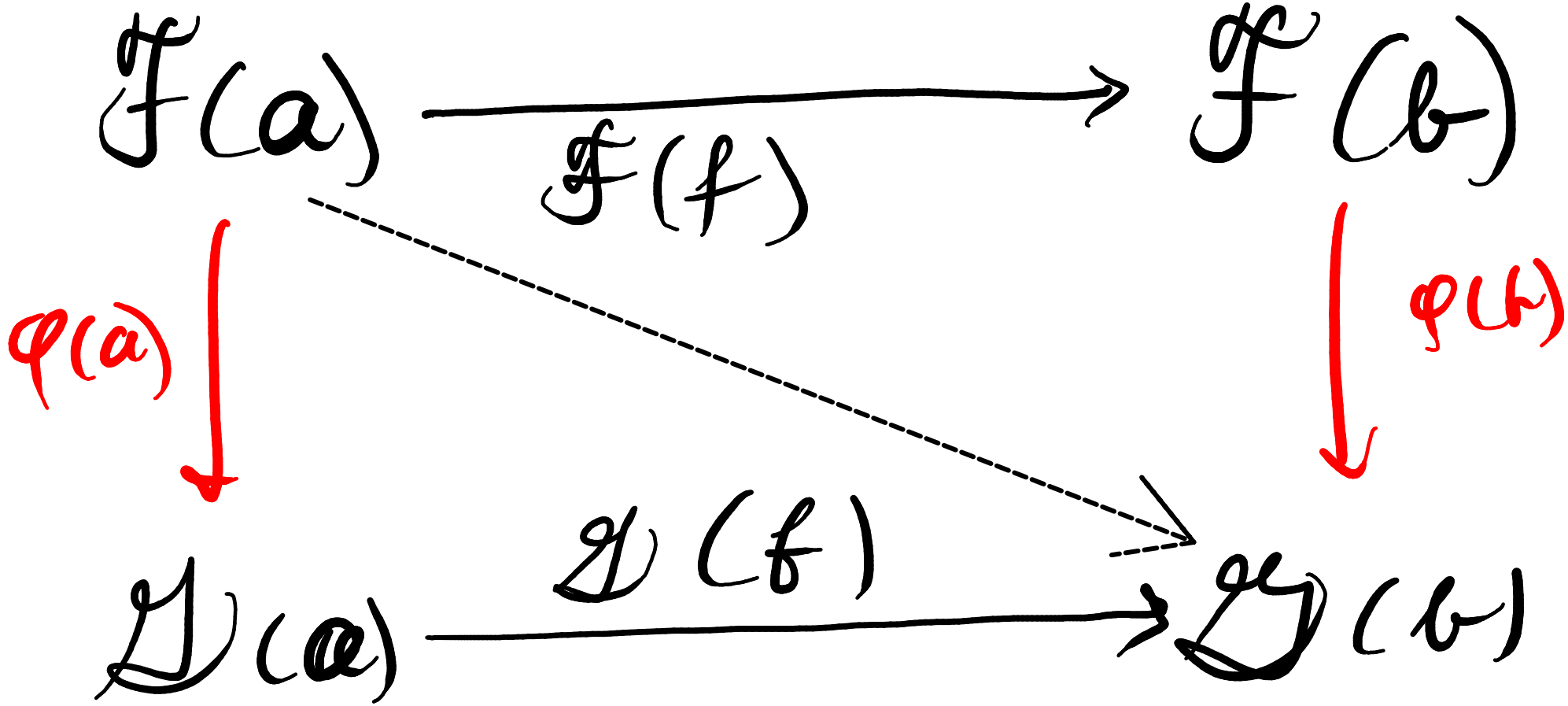
Natural transformations (2)

$$\mathcal{F}(a) \xrightarrow{\mathcal{F}(f)} \mathcal{F}(b)$$

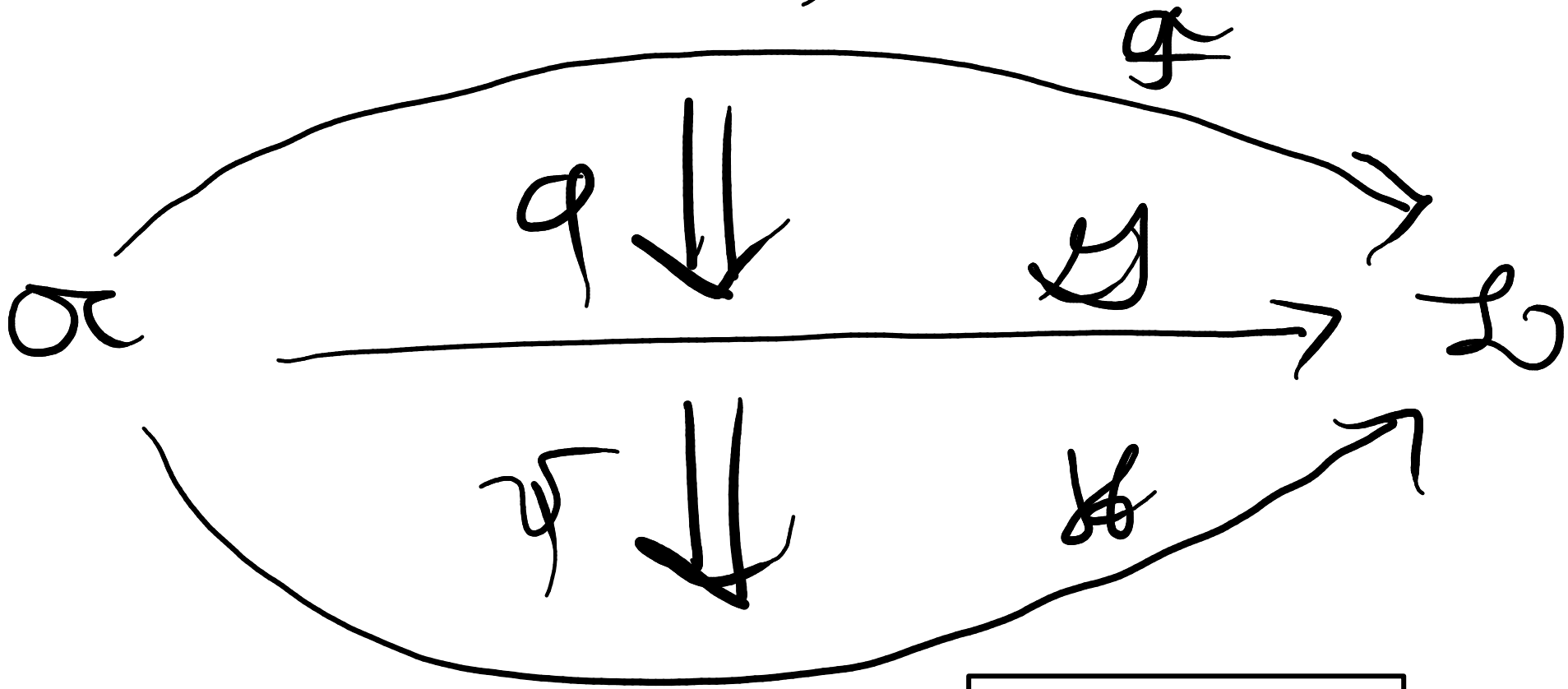
$$\mathcal{G}(a) \xrightarrow{\mathcal{G}(f)} \mathcal{G}(b)$$

$a, b \in$ objects category $\mathcal{C}$ $f \leftarrow$ arrows of $\mathcal{C}$

Natural transformations (3)

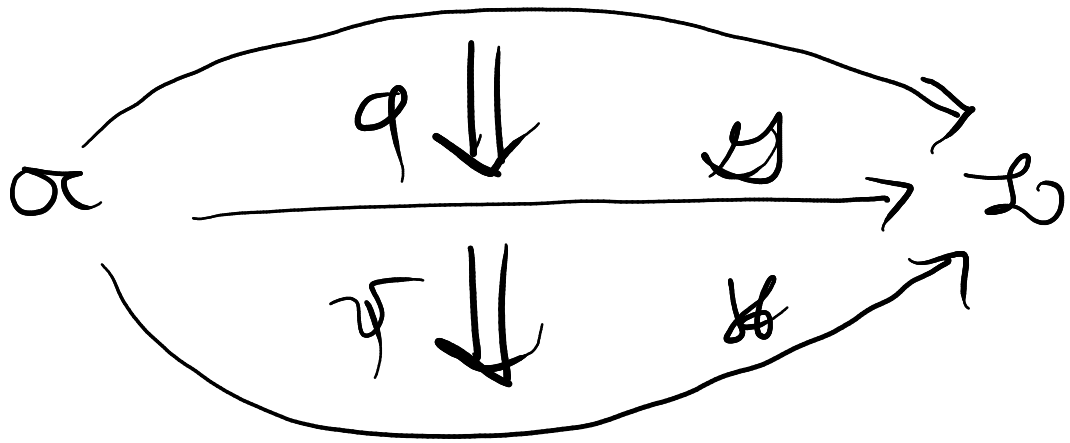


Natural Transformations (4)



Natural Transformations compose $\psi \circ \varphi$

Functor Categories



functors compose $\psi \circ \varphi$

Exercise: can you make a category out of this?

Category
- object
- arrow/morphism
- composition
- identity
- associative

Functor

Natural Trans-formation

Commutativity

Diagrams

Adjoint

Adjunction