Numerical Optimal Control
Lecture 10: Parametric Optimization

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S2, Chalmers

NTNU PhD course

Outline

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1. Parametric Optimization
2. Continuity & differentiability
3. Sensitivity in Newton
4. Sensitivity with inequality constraints
5. Path Following Methods

Parametric NLPs

Problem:

$$\begin{aligned} \mathbf{w}(\mathbf{p}) = \arg\min_{\mathbf{w}} \quad & \Phi(\mathbf{w}, \mathbf{p}) \\ \text{s.t.} \quad & \mathbf{g}(\mathbf{w}, \mathbf{p}) = 0 \\ & \mathbf{h}(\mathbf{w}, \mathbf{p}) \leq 0 \end{aligned}$$

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- Predictors: what does $\mathbf{w}(\mathbf{p})$, tell us about $\mathbf{w}(\mathbf{p}+\Delta\mathbf{p})$?
- Robust optimization
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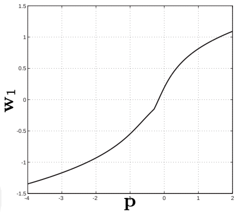

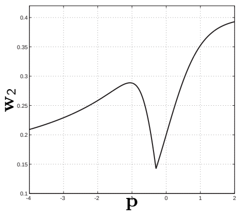

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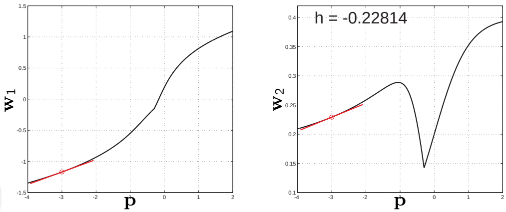

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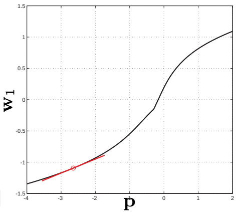

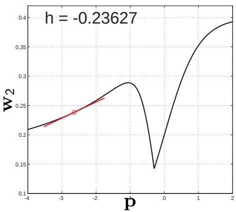

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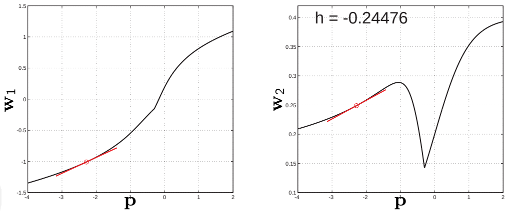

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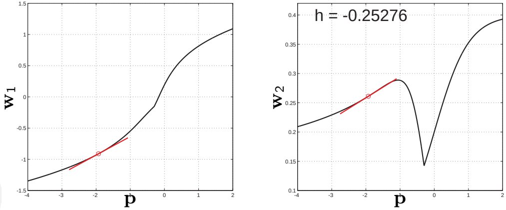

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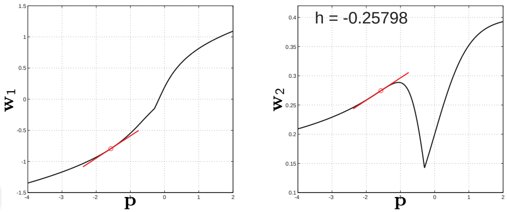

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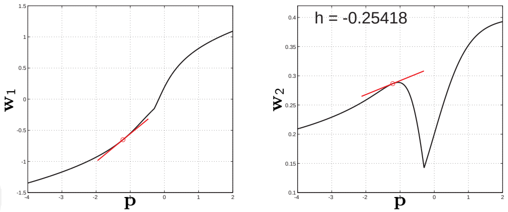

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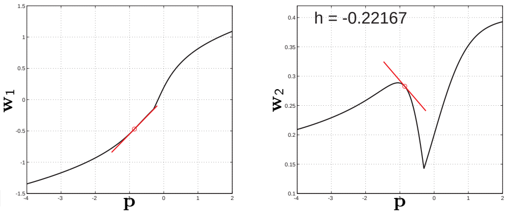

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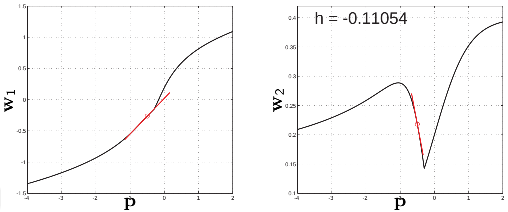

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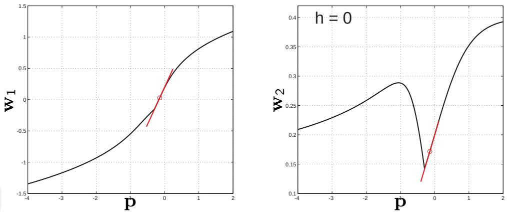

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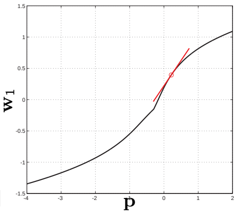

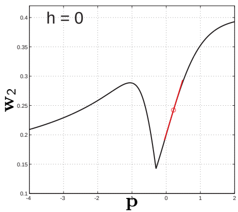

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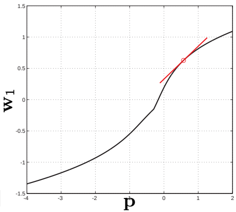

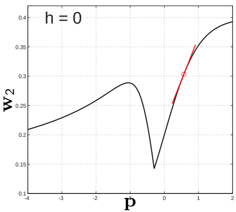

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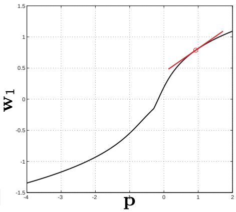

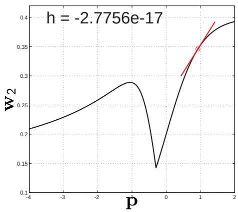

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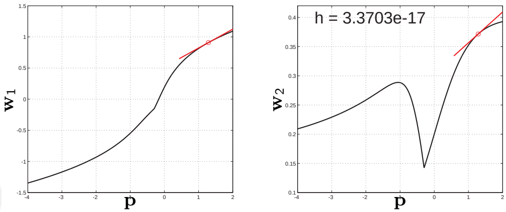

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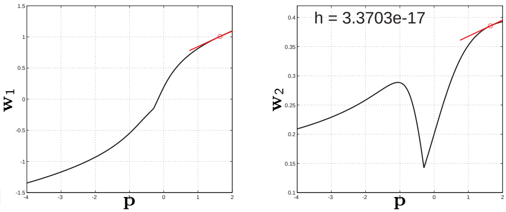

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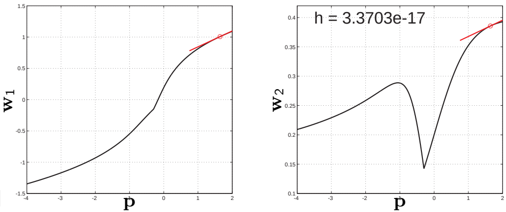

Changes of active set yield non-smooth points in the solution manifold $\mathbf{w}(\mathbf{p})$.

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One ought to understand

$$\mathbf{w}(\mathbf{p}) : \mathbf{p} \in \mathbb{R}^p \mapsto \mathbf{w} \in \mathbb{R}^n$$

as a genuine function defined by the parametric NLP

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Questions:

- Continuity & differentiability ?
- Sensitivities $\frac{\partial \mathbf{w}}{\partial \mathbf{p}}$?
- Predictors: using $\mathbf{w}(\mathbf{p}_0)$, what can I say about $\mathbf{w}(\mathbf{p})$?

$$\mathbf{w}(\mathbf{p}) \approx \mathbf{w}(\mathbf{p}_0) + \frac{\partial \mathbf{w}}{\partial \mathbf{p}}(\mathbf{p} - \mathbf{p}_0)$$

- Path-following: how to keep track of $\mathbf{w}(\mathbf{p})$ for a (continuously) changing $\mathbf{p}$?

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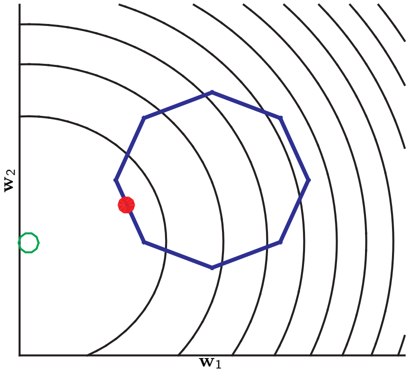
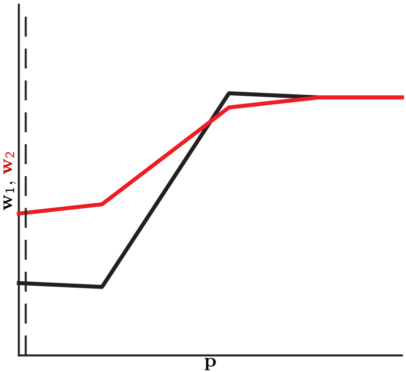
E.g. convex QP: SOSC holds everywhere (positive def. Hessian)





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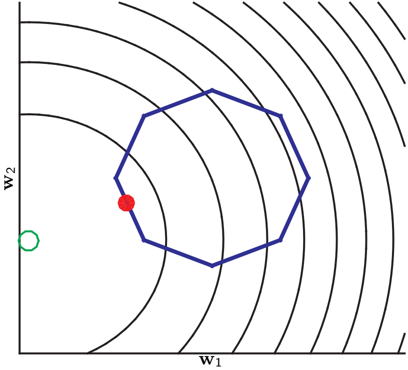

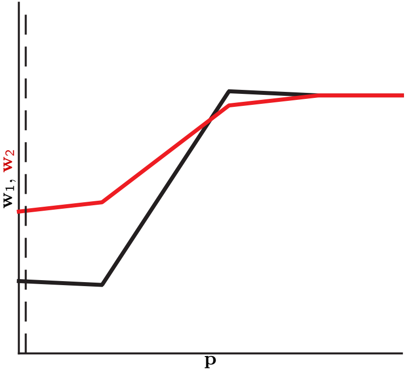

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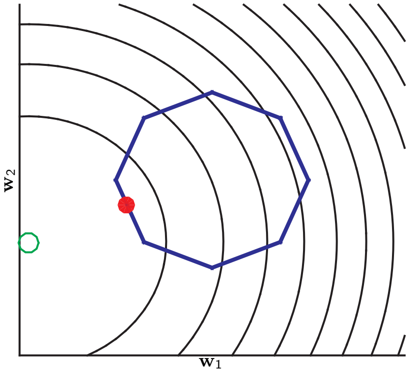

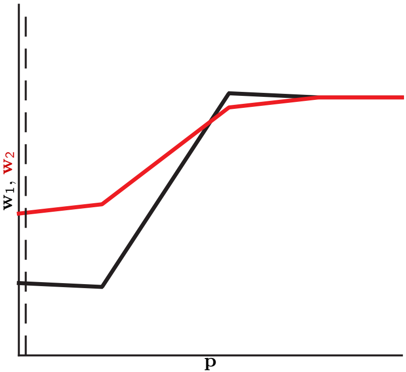

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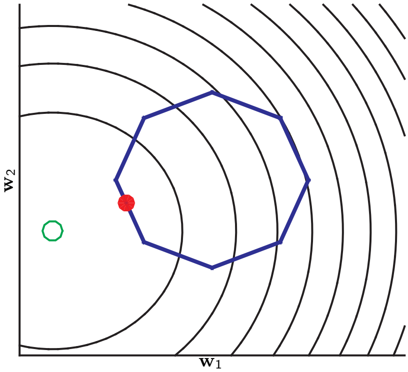

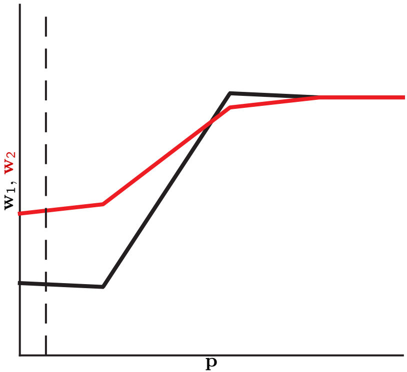

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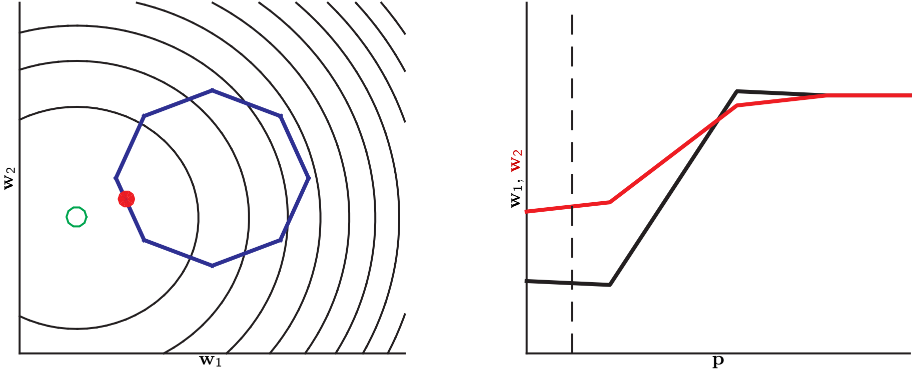

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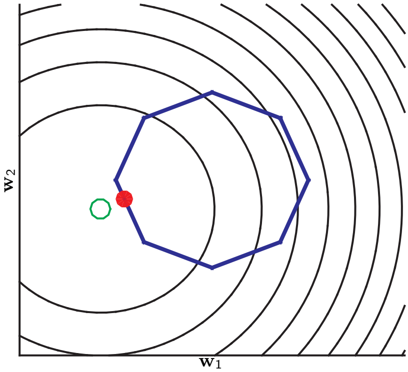

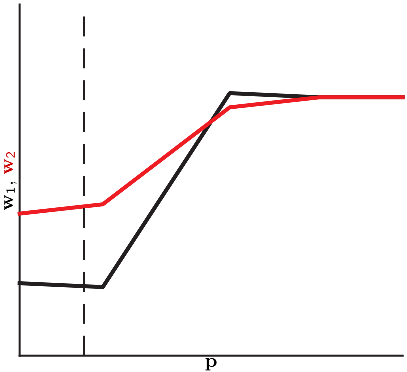

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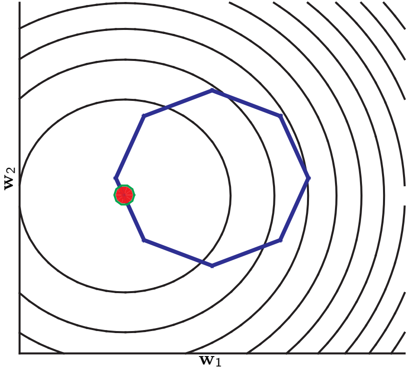

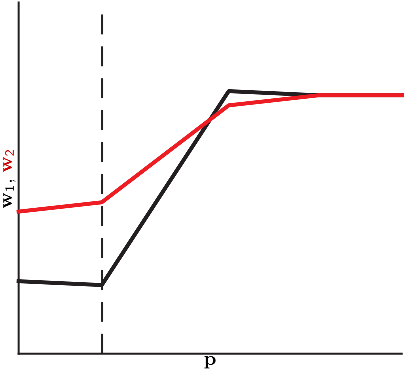

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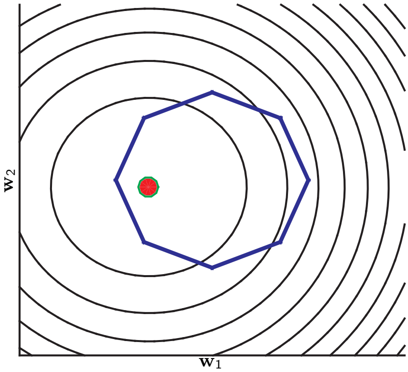

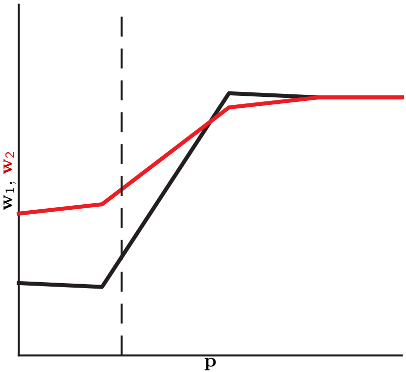

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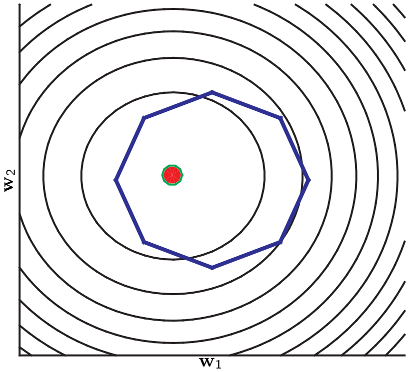

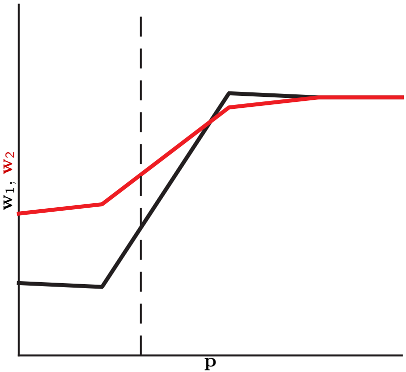

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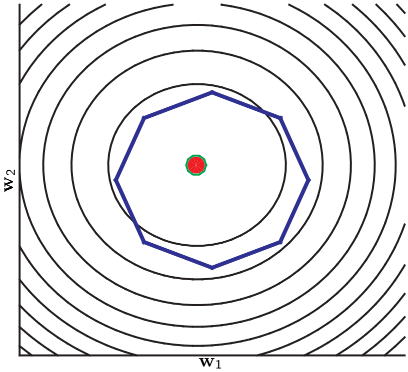

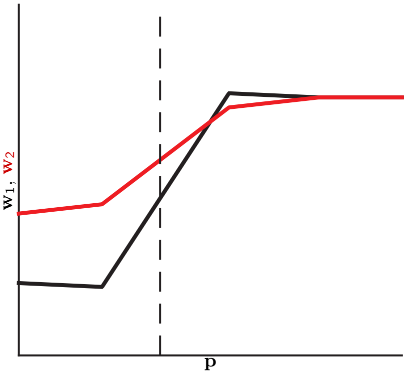

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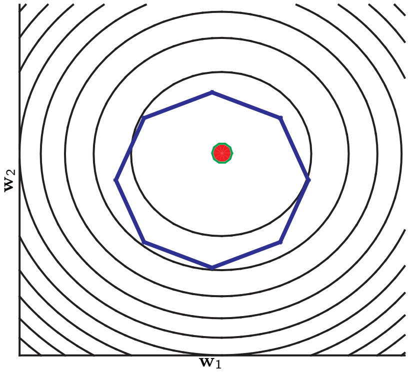

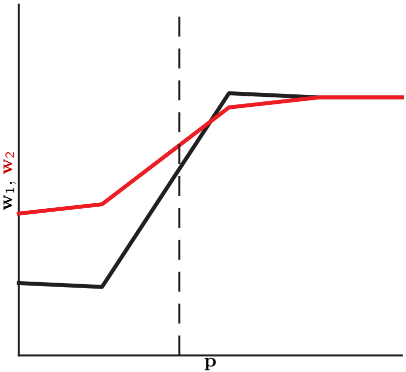

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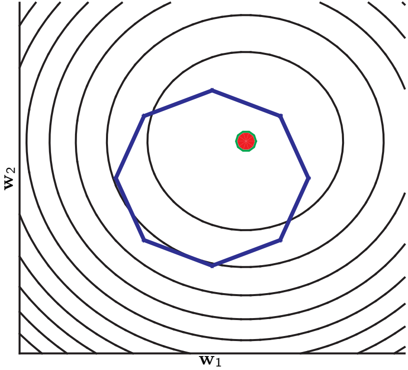

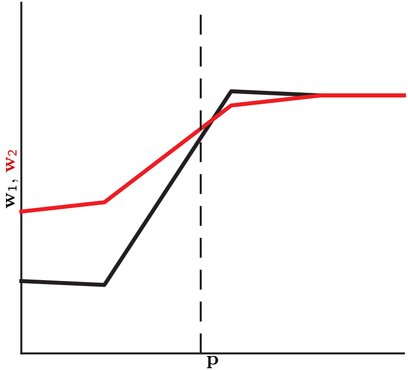

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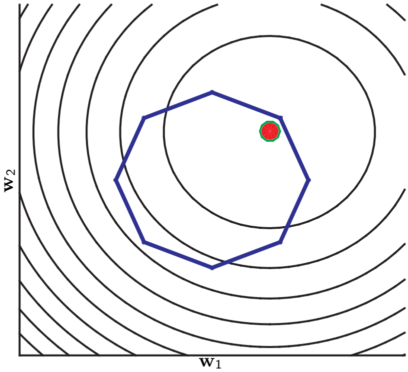

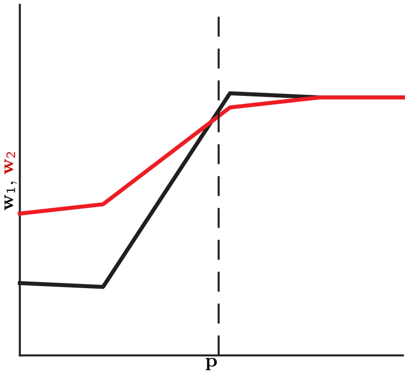

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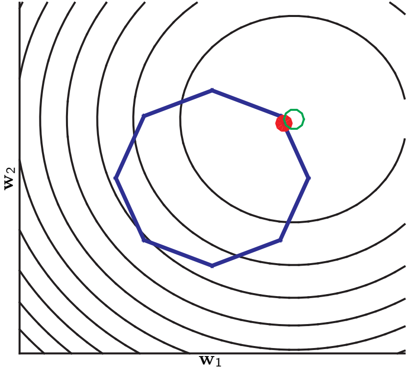

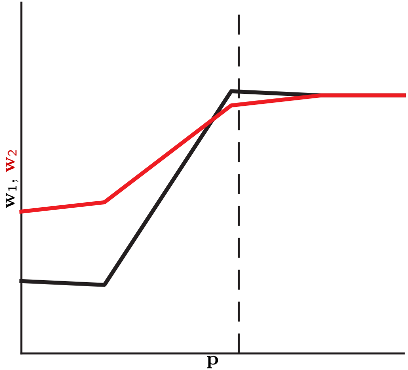

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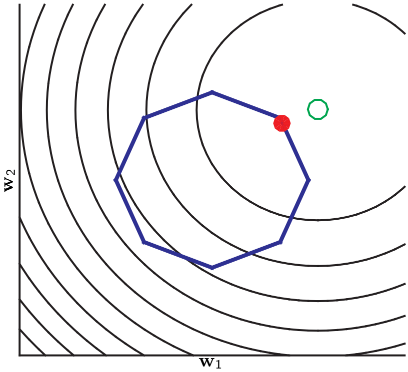

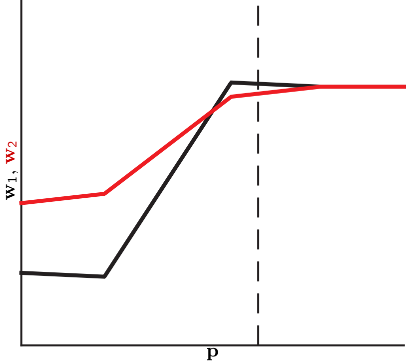

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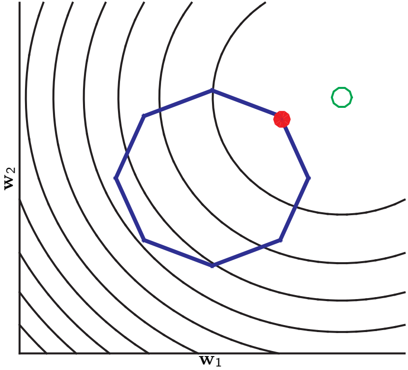

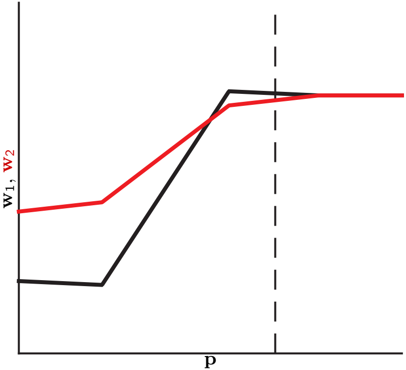

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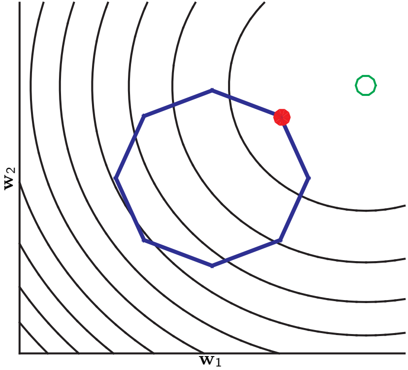

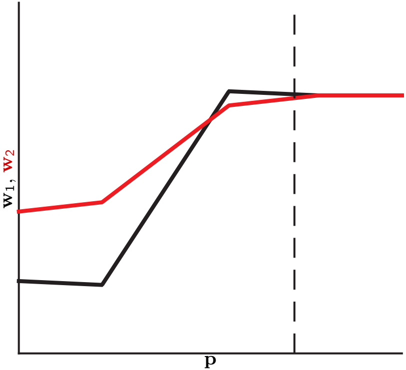

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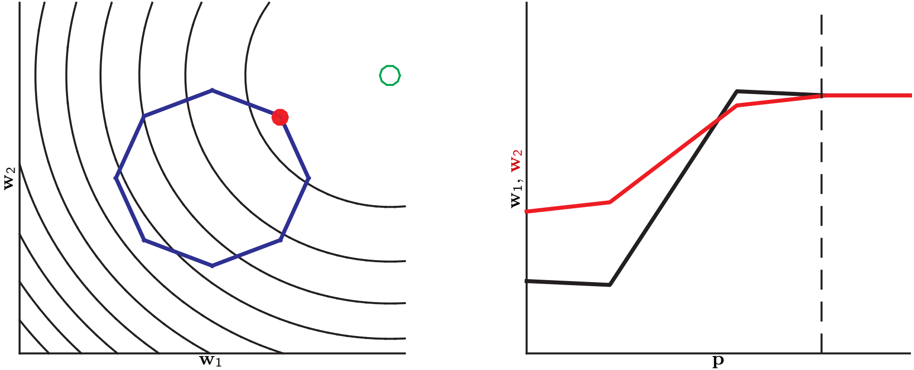

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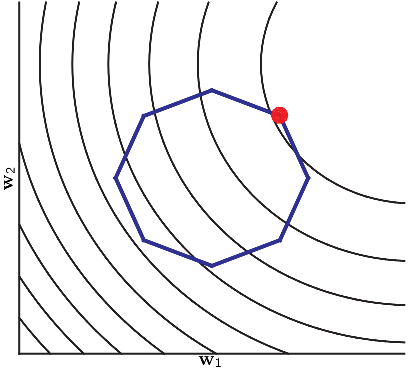

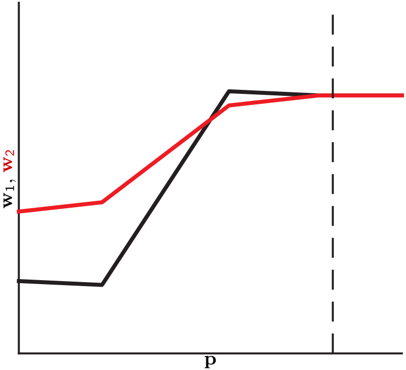

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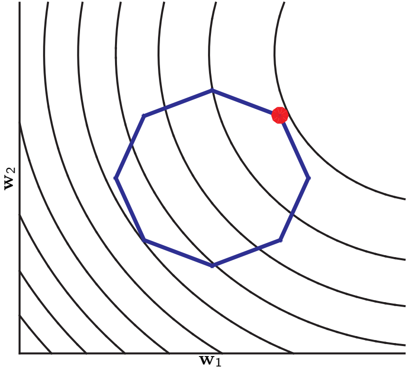

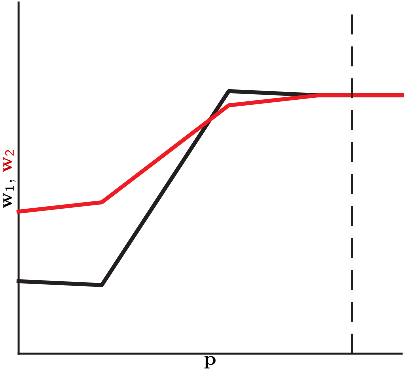

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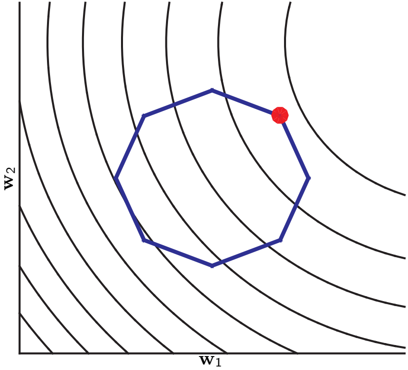

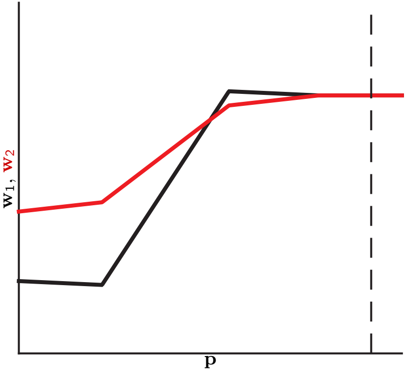

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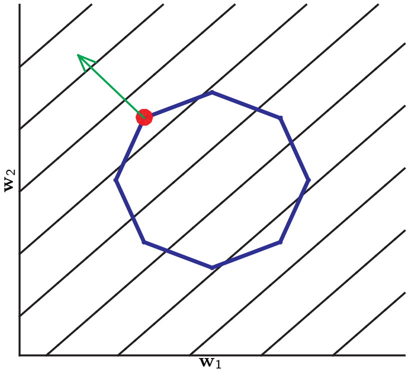

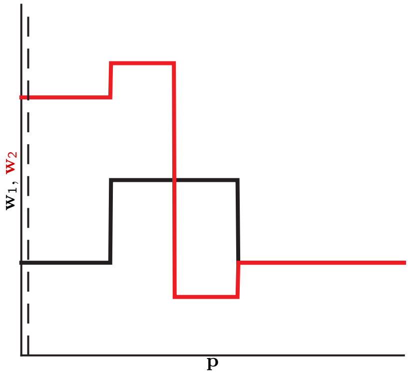

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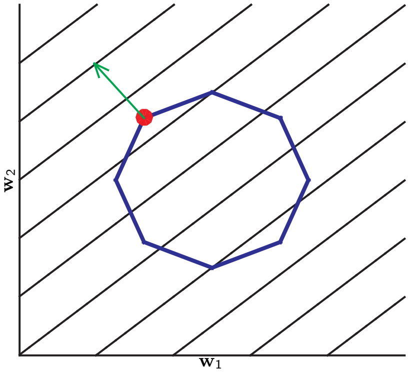

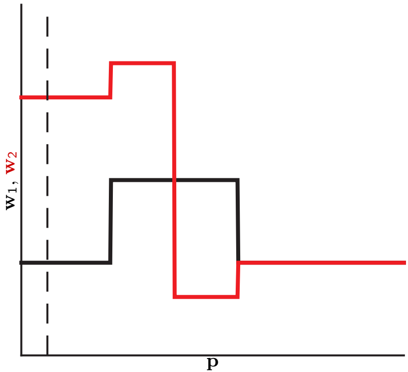

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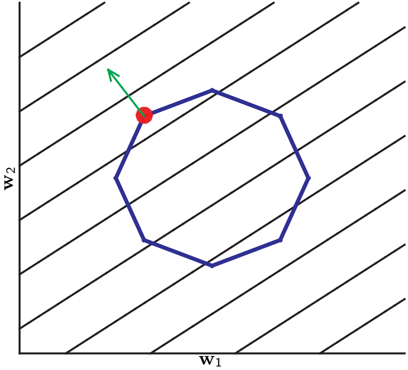

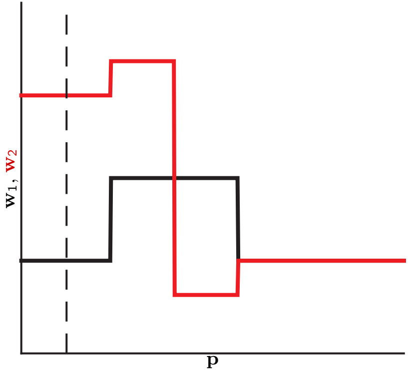

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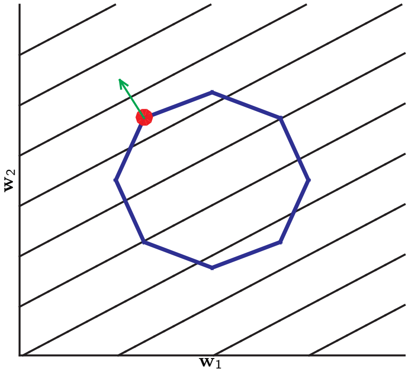

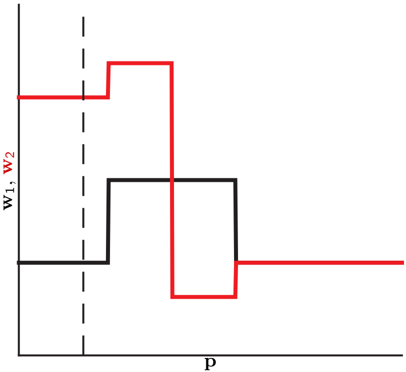

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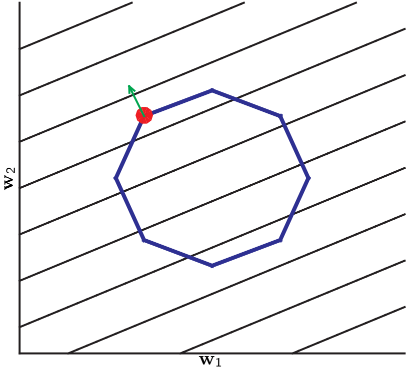

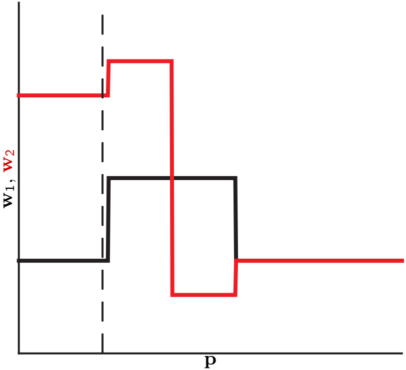

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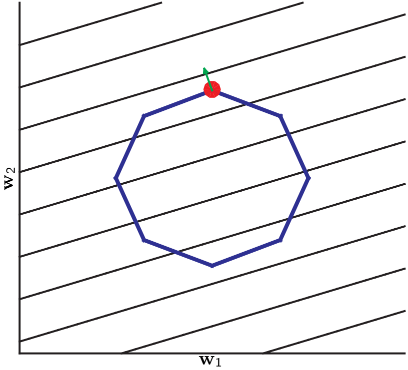

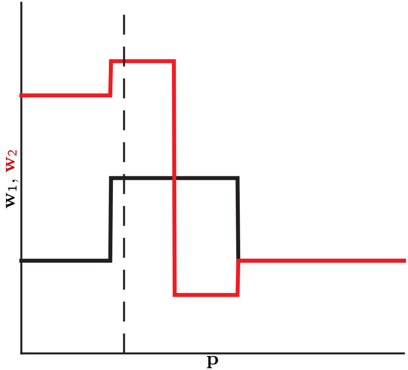

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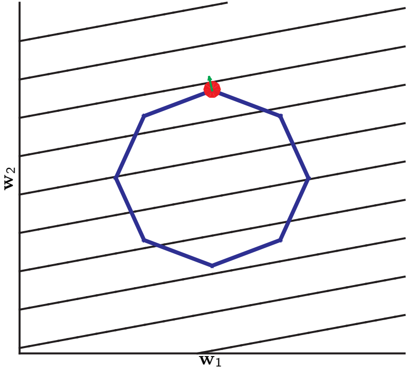

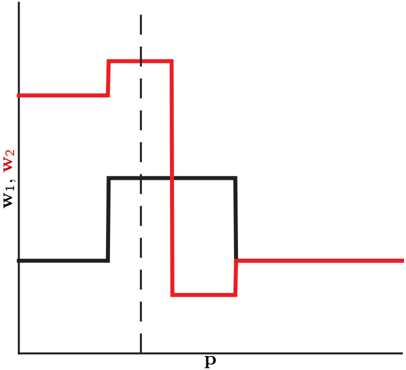

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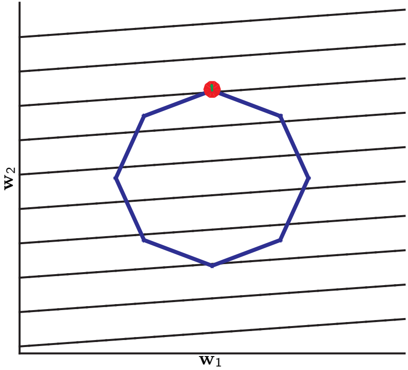

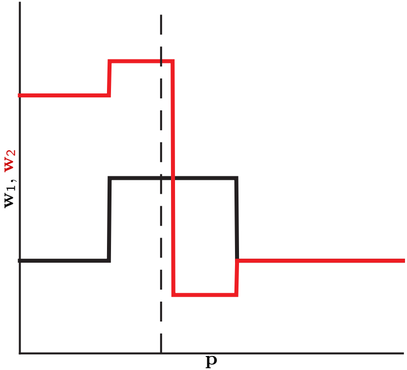

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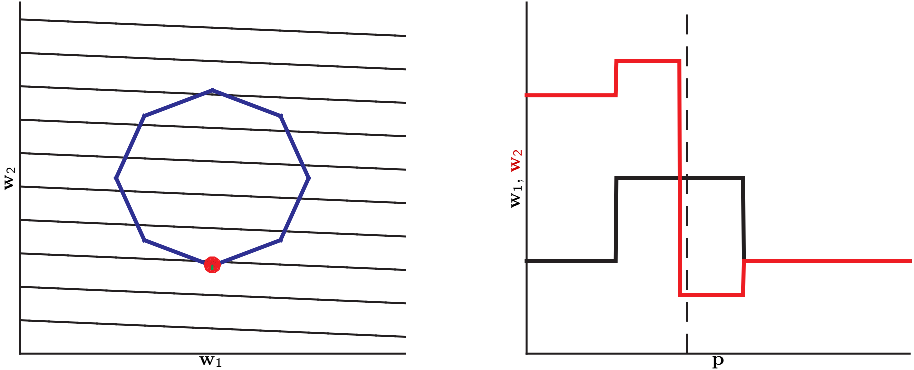

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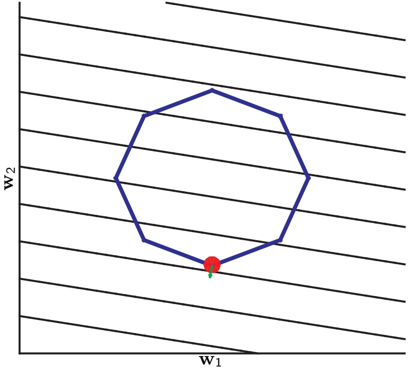

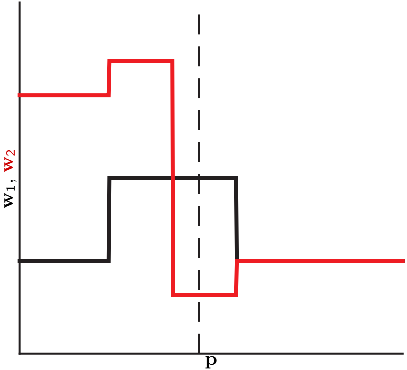

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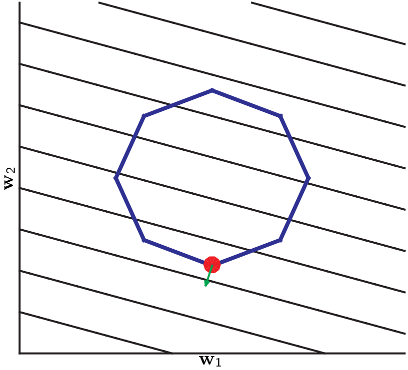

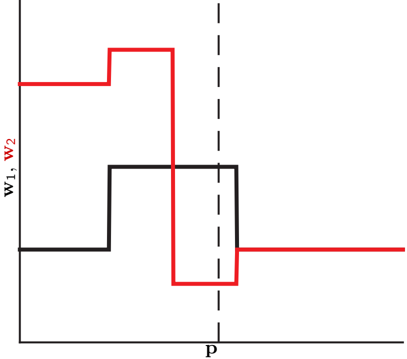

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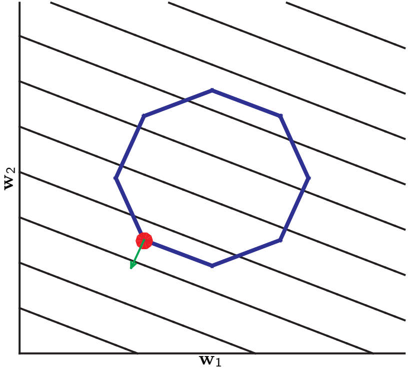

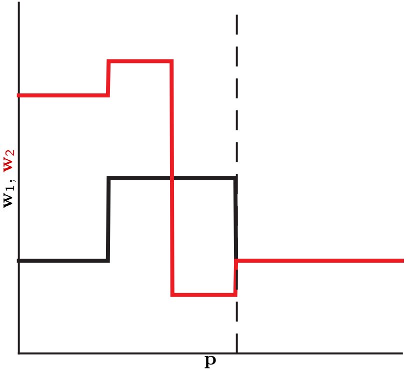

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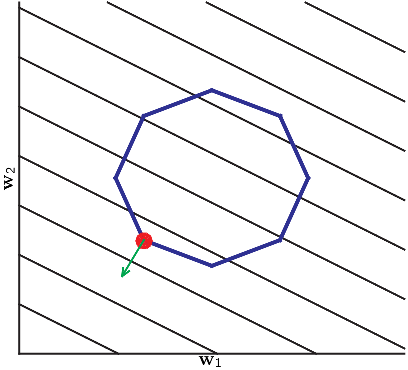

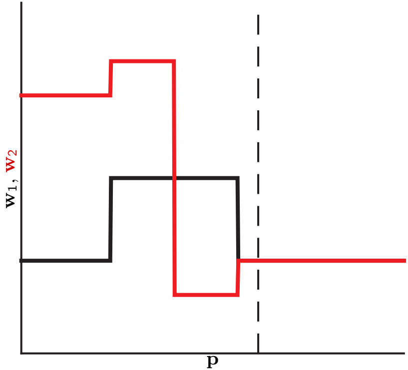

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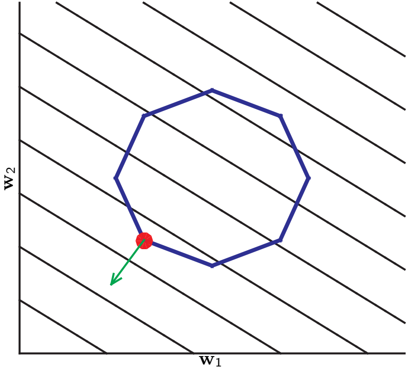

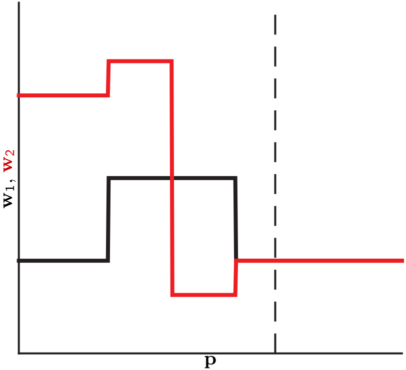

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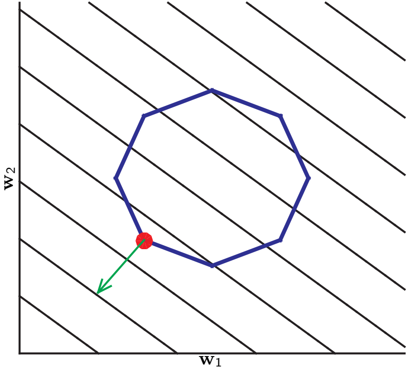

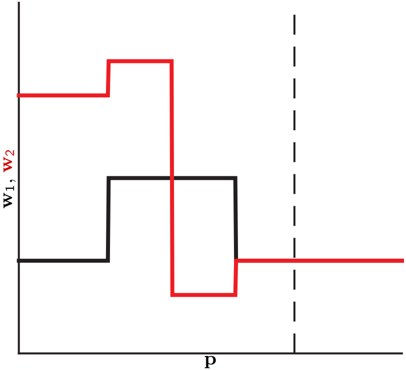

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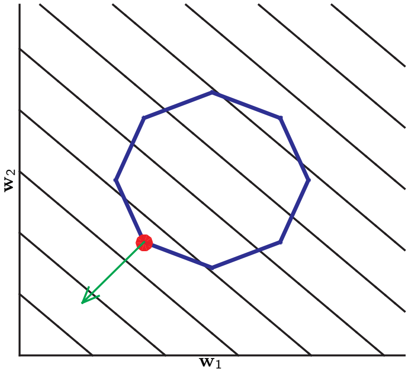

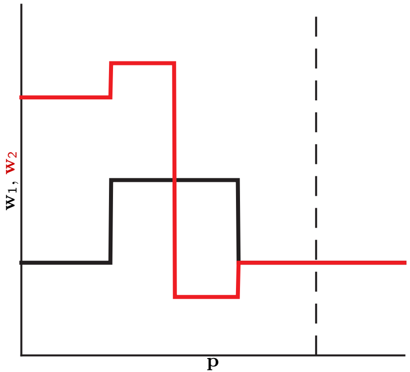

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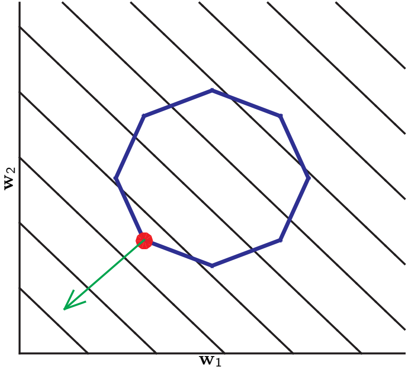

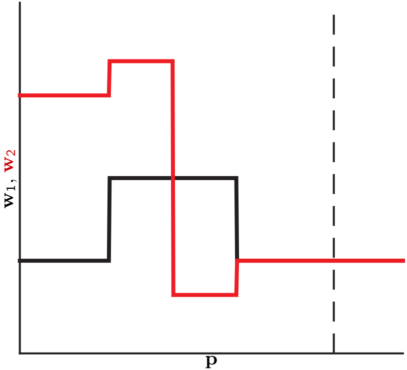

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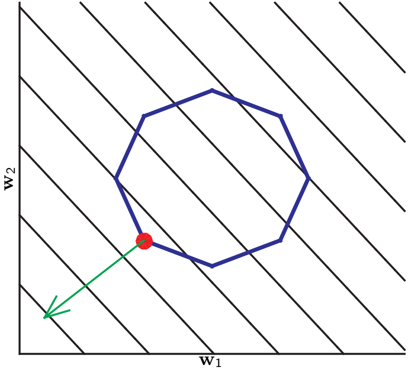

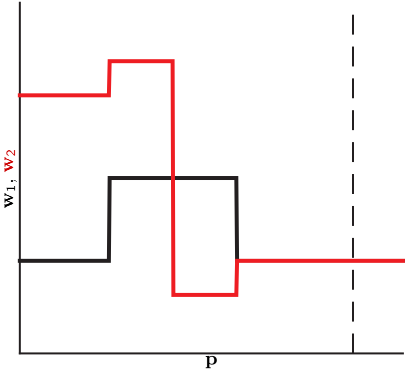

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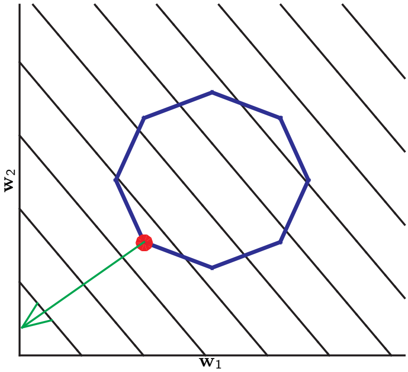

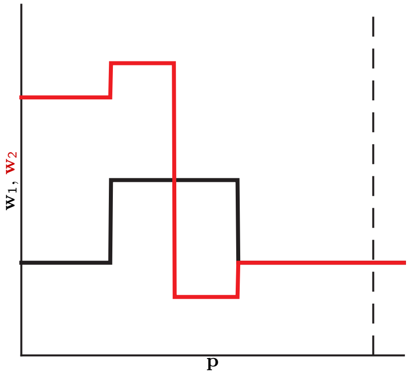

Differentiability

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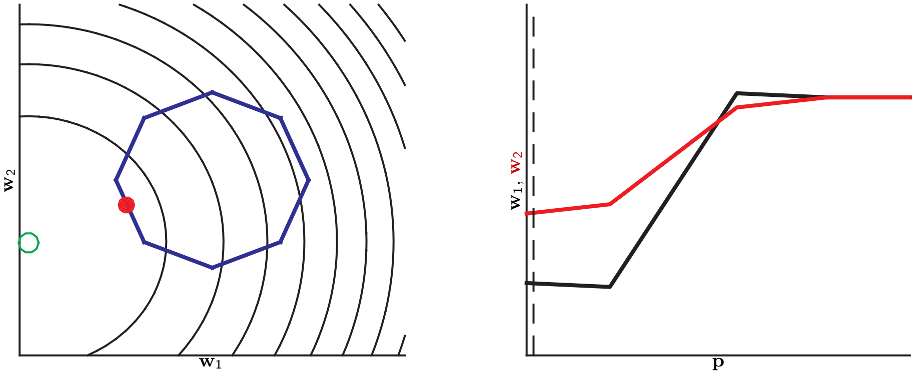

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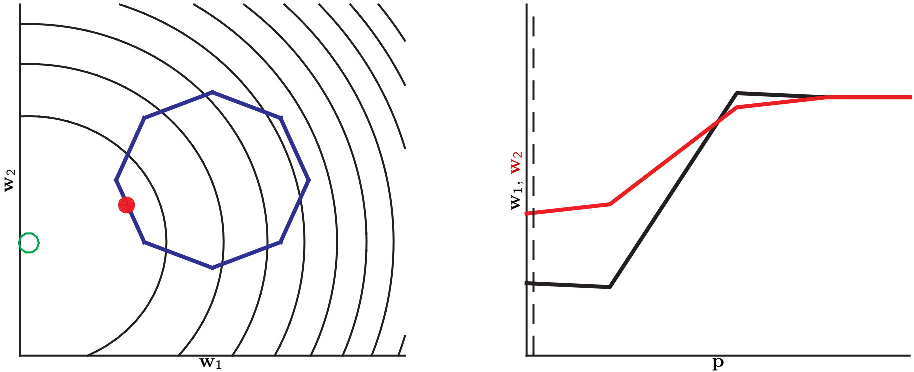

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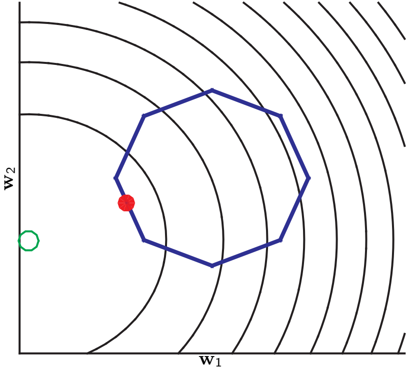

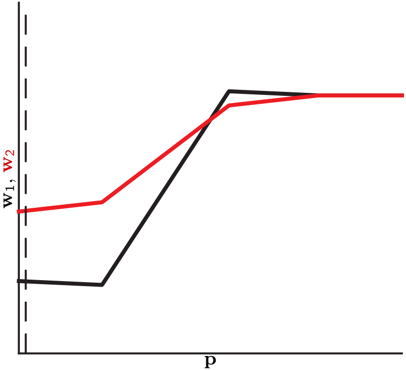

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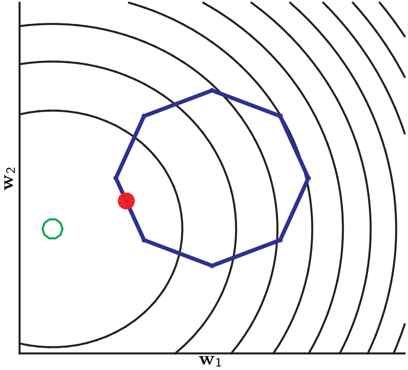

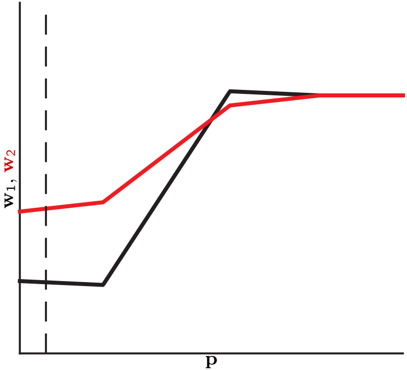

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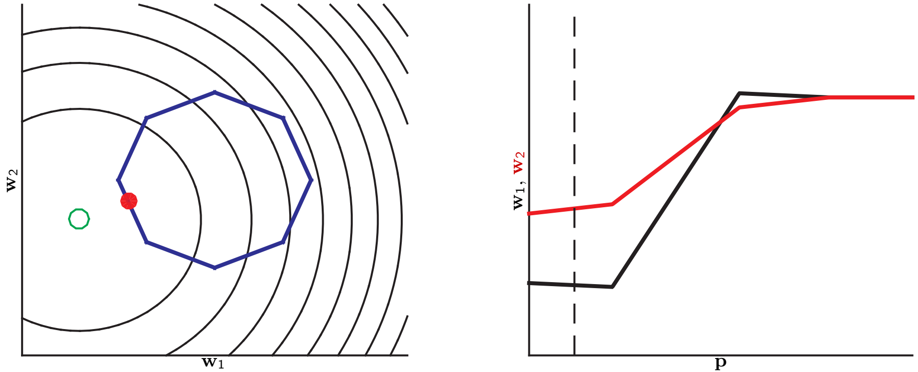

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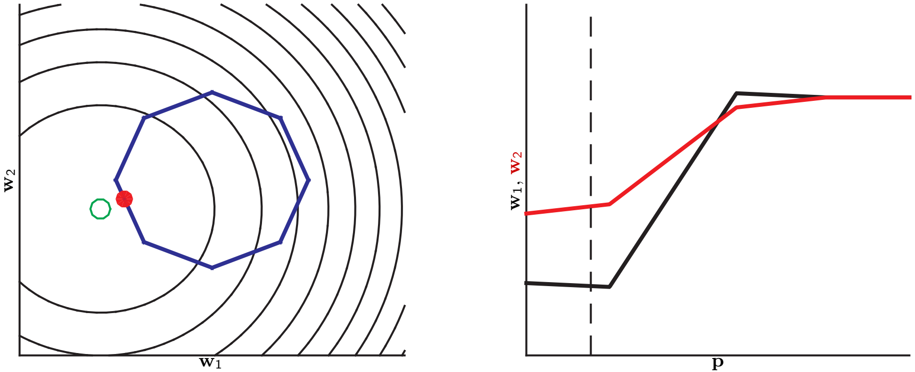

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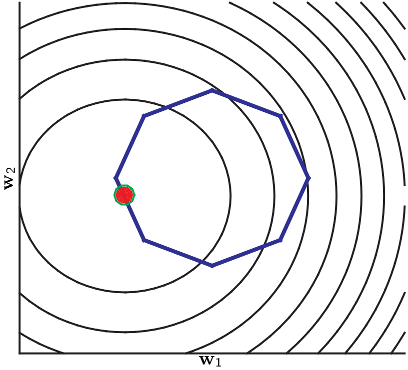

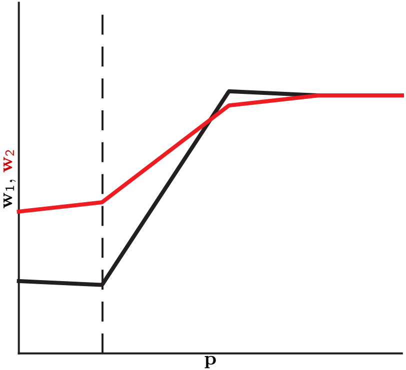

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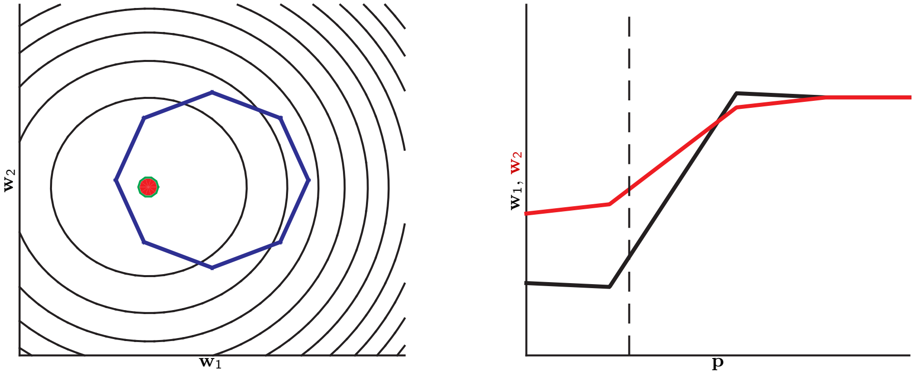

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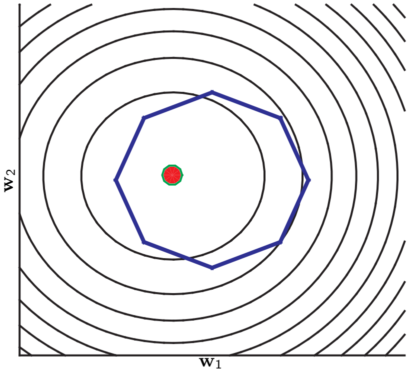

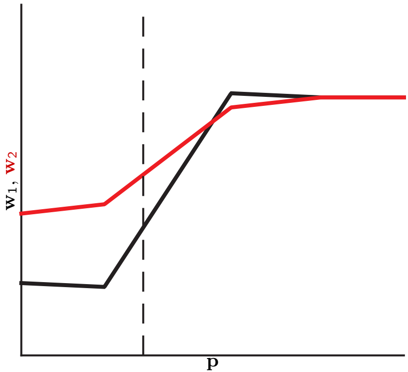

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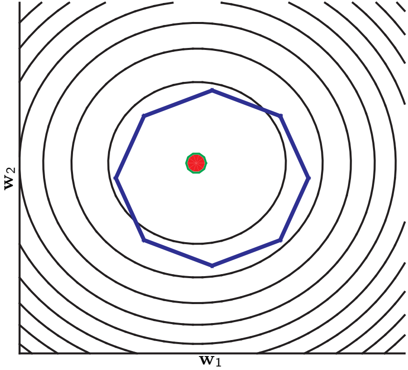

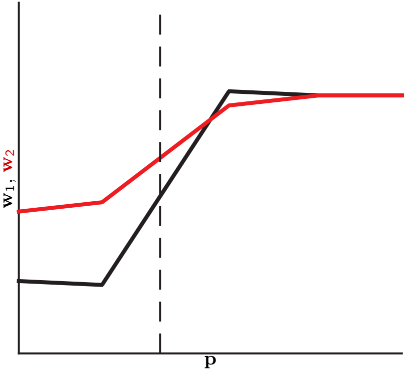

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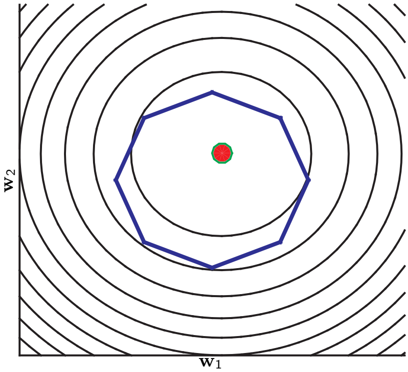

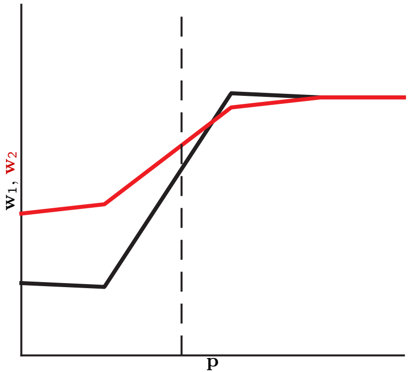

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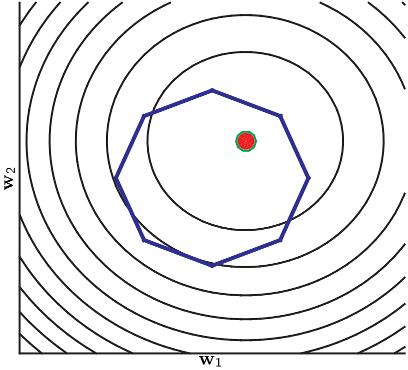

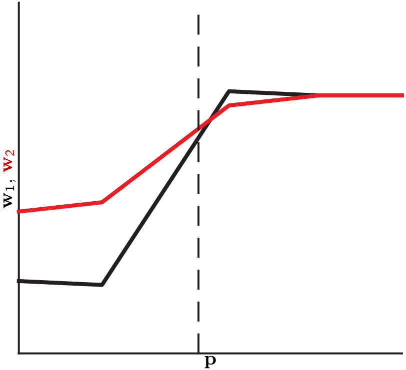

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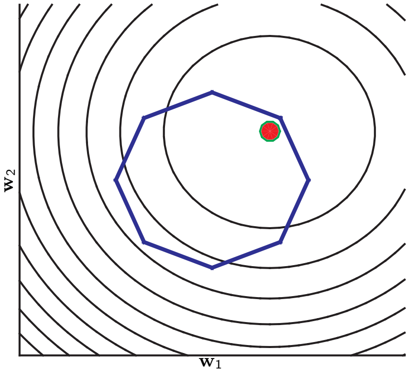

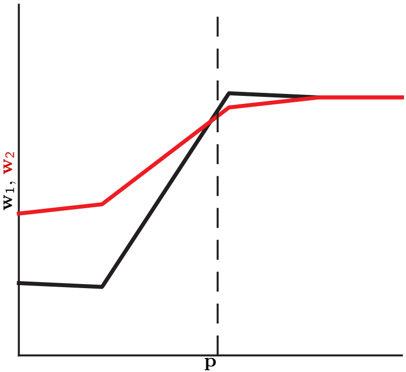

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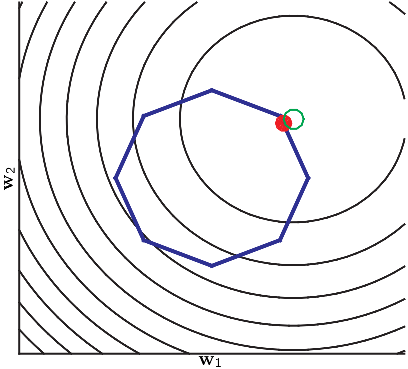

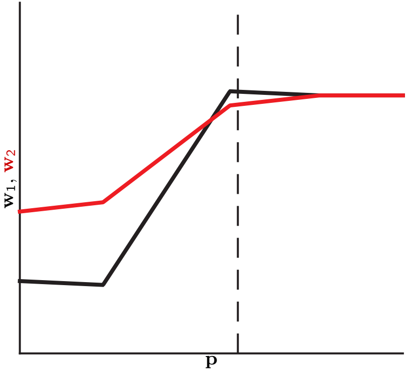

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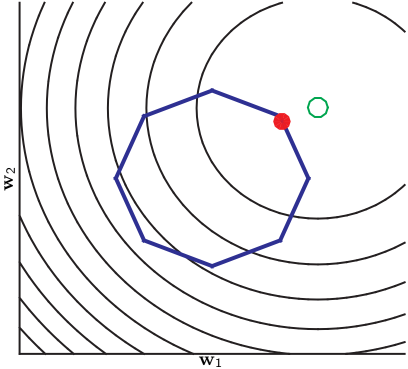

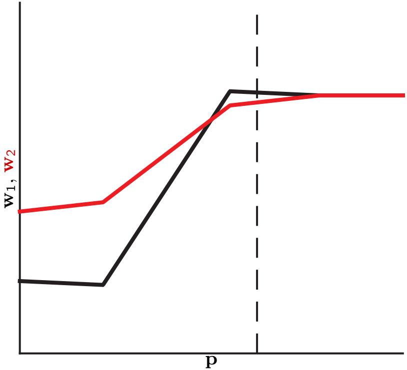

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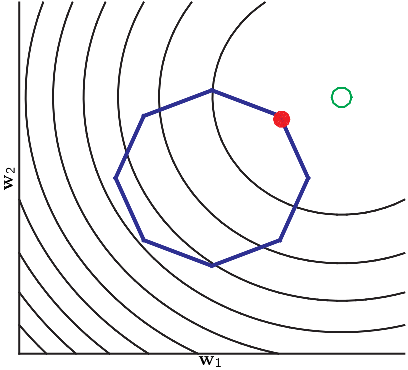

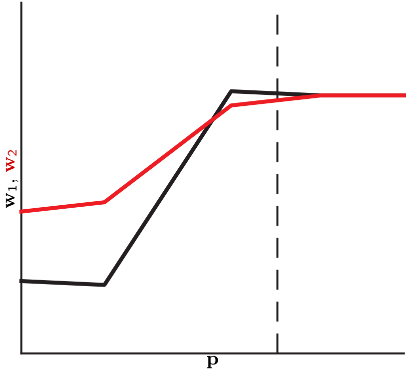

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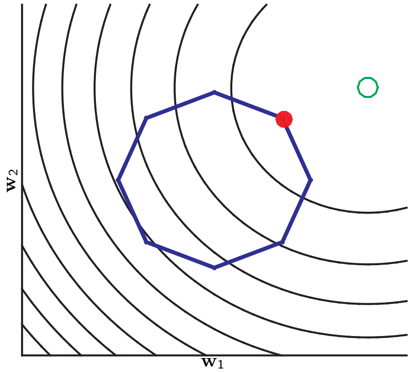

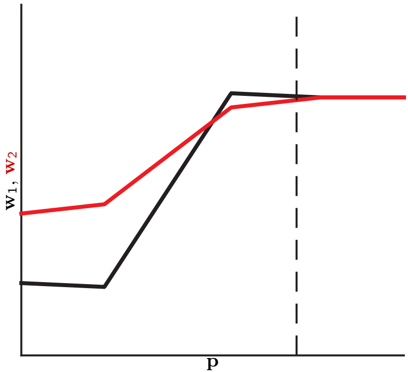

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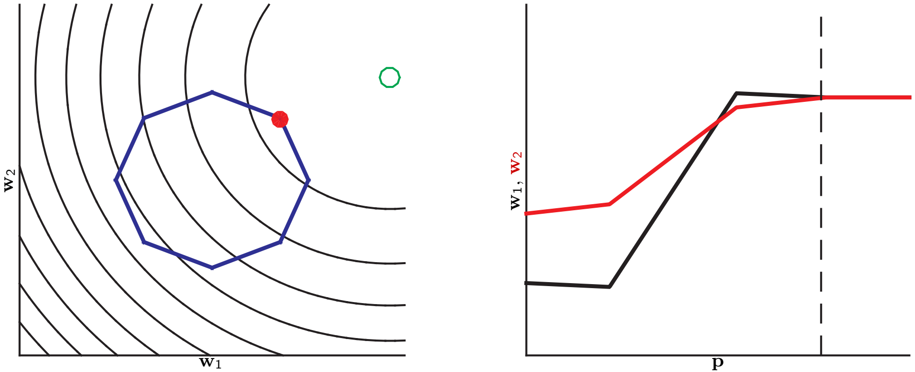

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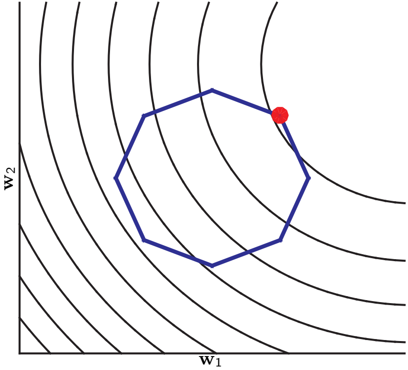

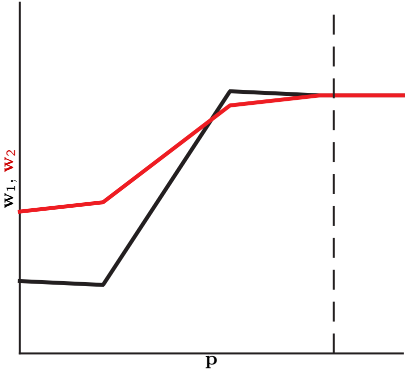

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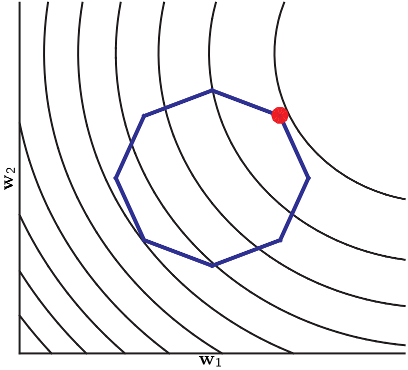

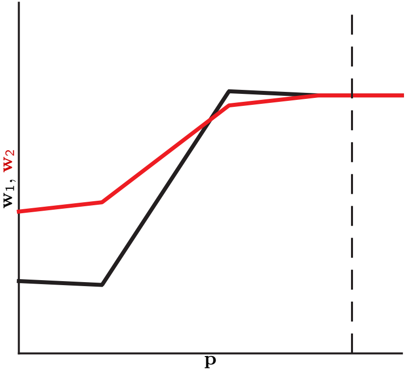

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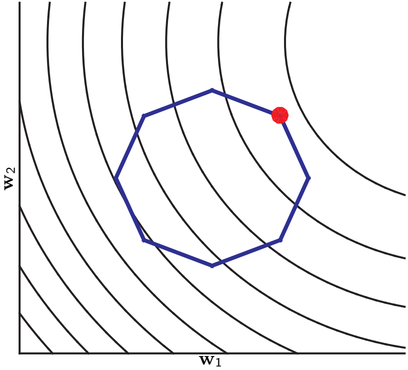

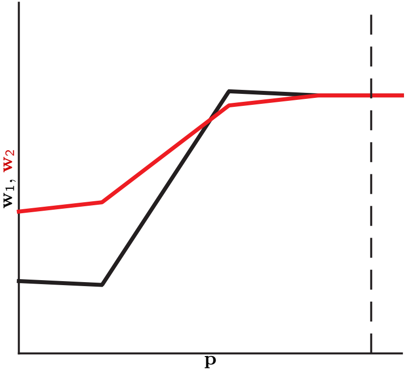

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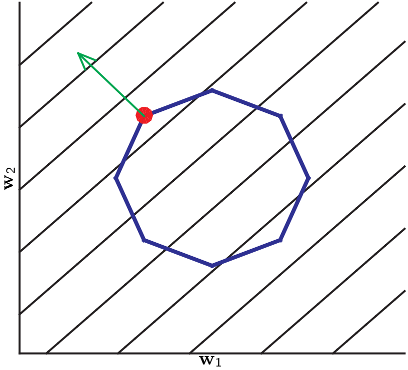

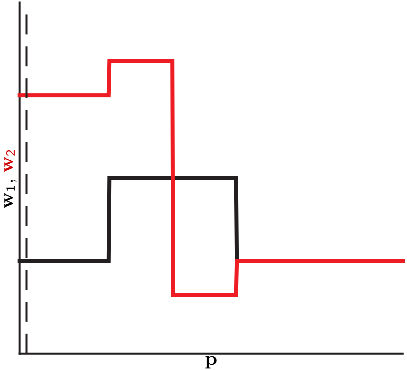

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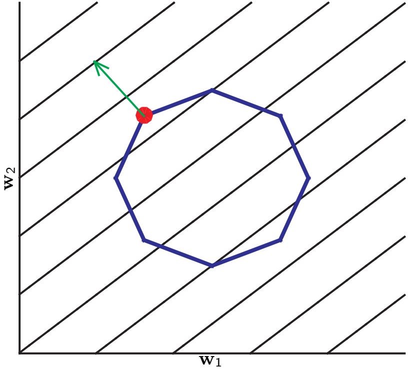

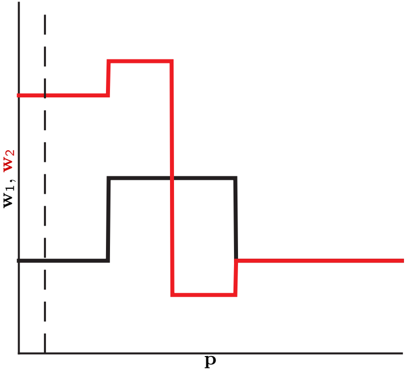

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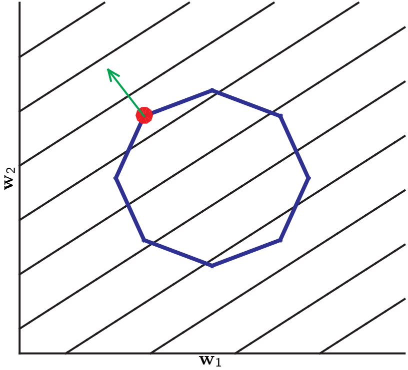

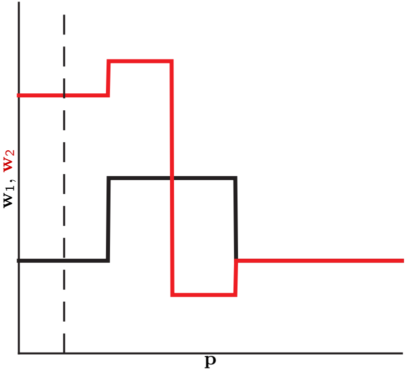

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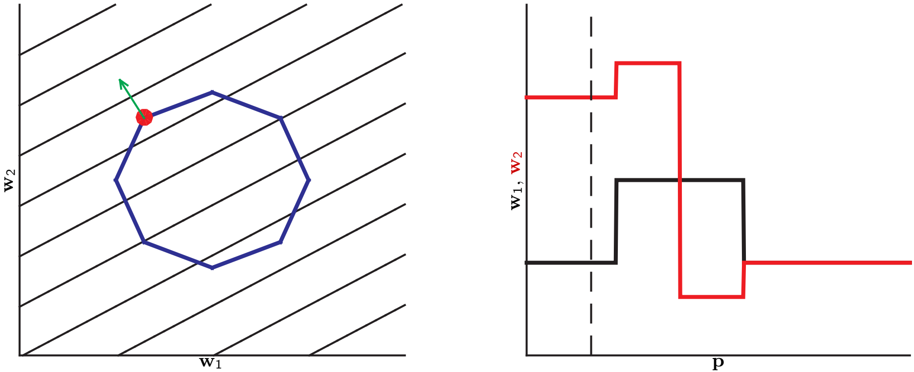

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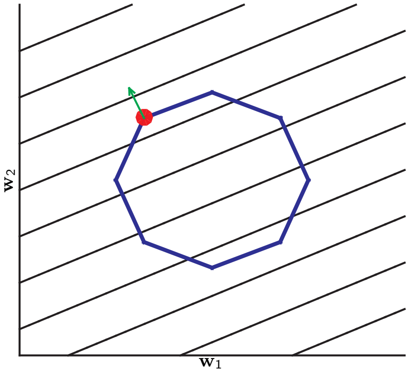

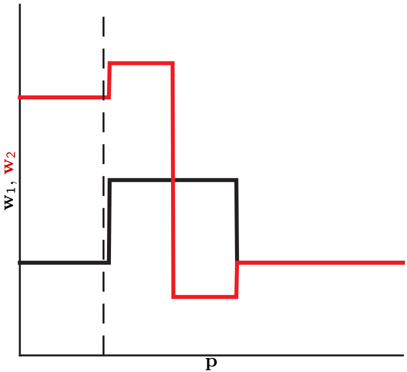

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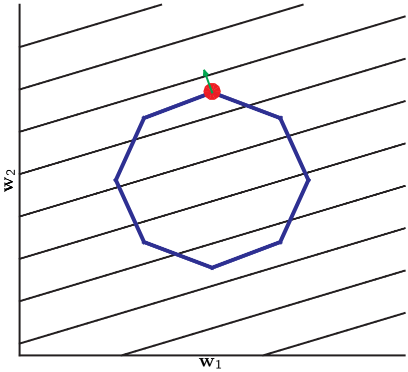

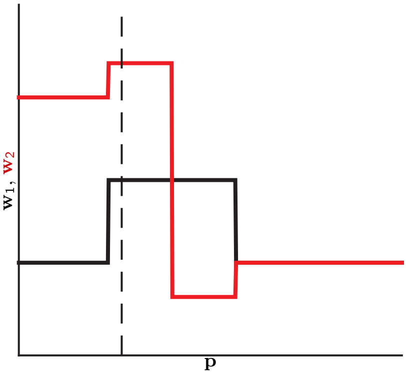

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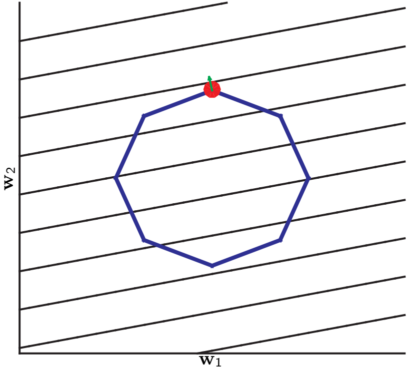

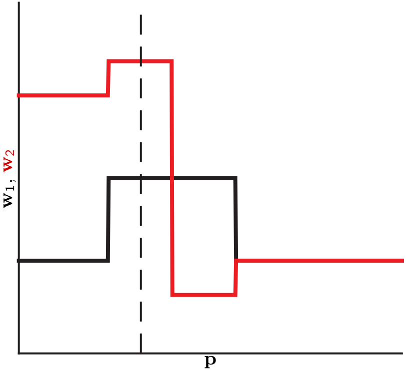

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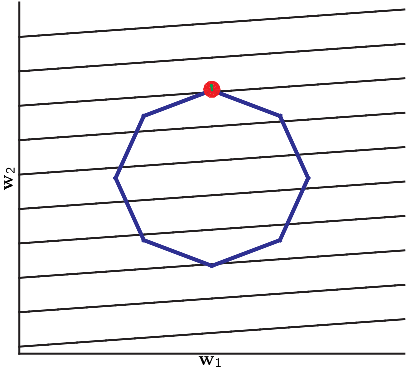

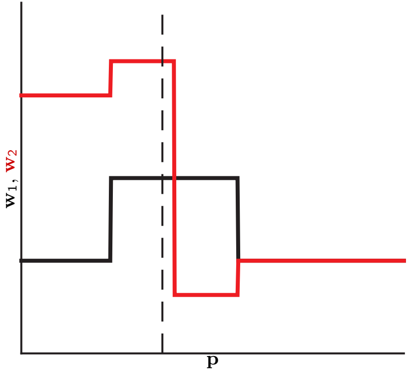

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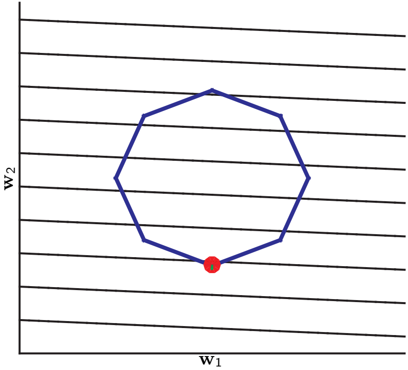

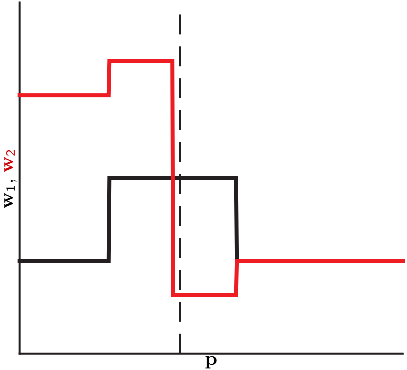

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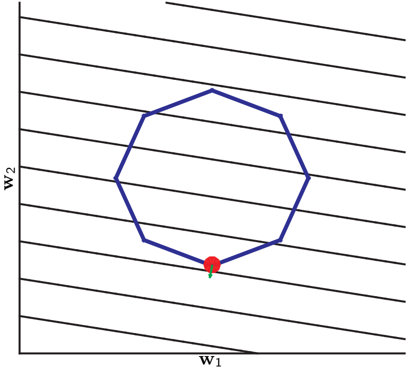

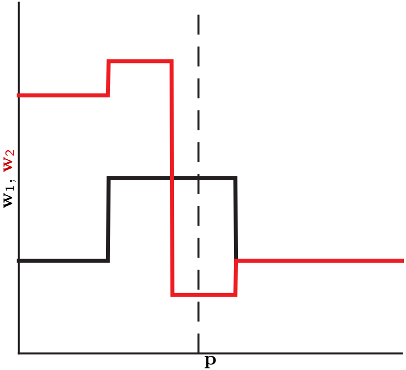

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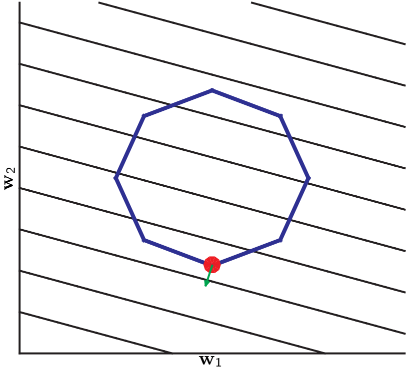

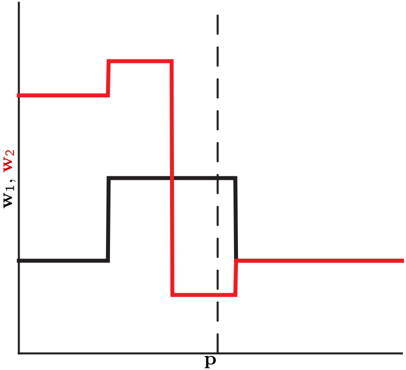

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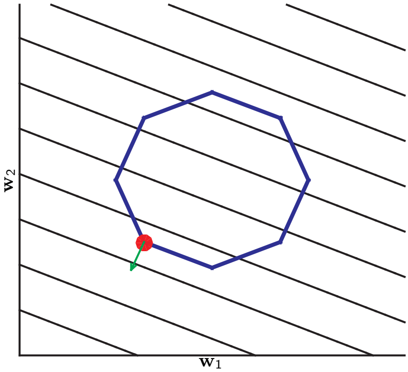

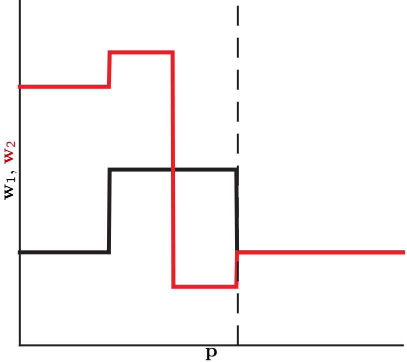

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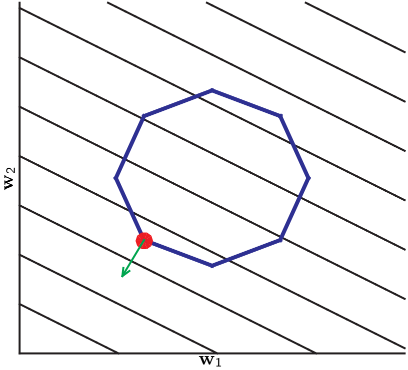

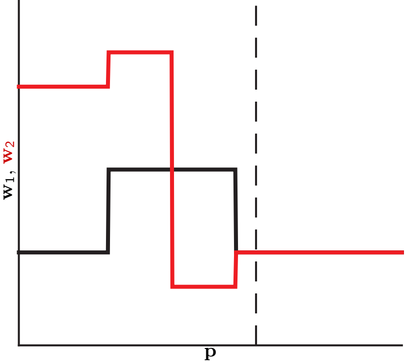

Differentiability

Parametric NLP:

$$\mathbf{w}(\mathbf{p}) = \arg\min_{\mathbf{w}} \quad \Phi(\mathbf{w}, \mathbf{p})$$
$$\mathbf{g}(\mathbf{w}, \mathbf{p}) = 0$$
$$\mathbf{h}(\mathbf{w}, \mathbf{p}) \leq 0$$

Theorem: consider $\mathbf{w}(\mathbf{p})$ at a given $\mathbf{p}$, with

- LICQ & strict SOSC
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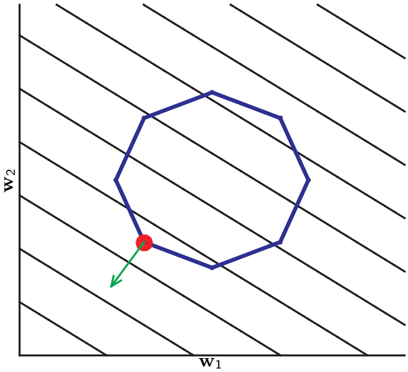

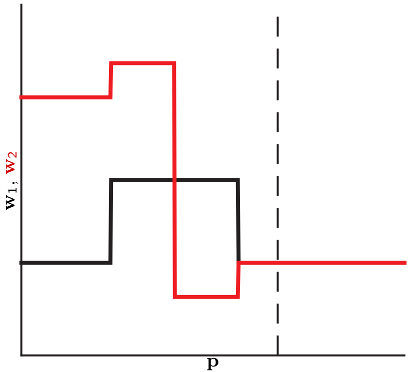

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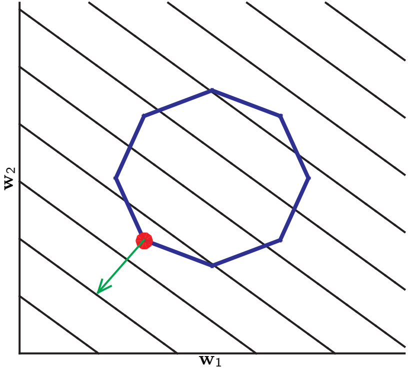

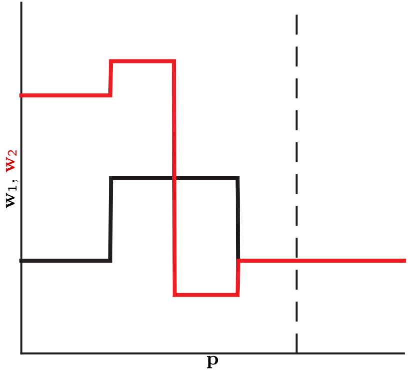

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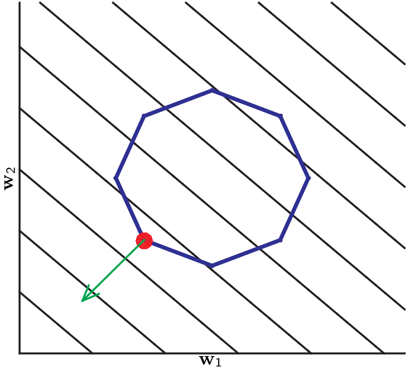

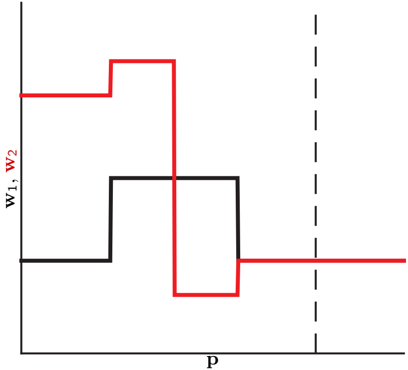

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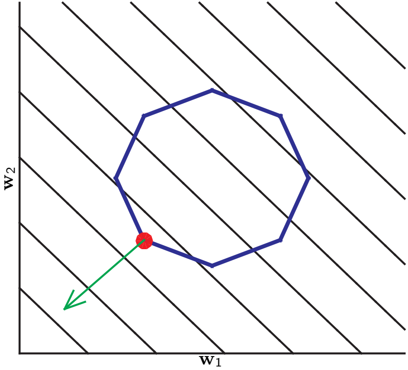

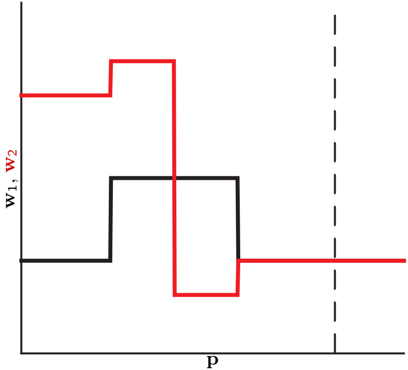

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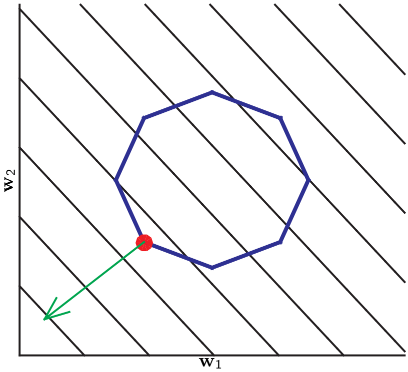

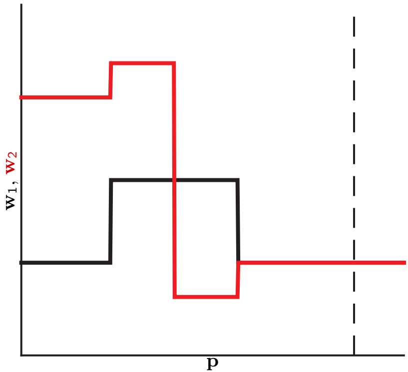

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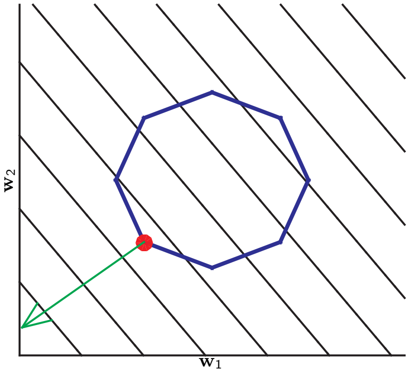

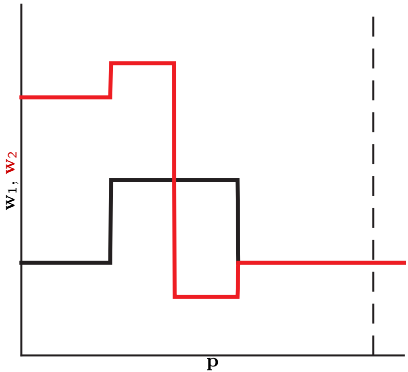

Outline

Newton as an implicit function

Parametric NLP:

$$\begin{aligned} \min_{\mathbf{w}} \quad & \Phi\left(\mathbf{w},\mathbf{p}\right) \\ & \mathbf{g}\left(\mathbf{w},\mathbf{p}\right) = 0 \end{aligned}$$

Newton as an implicit function

Parametric NLP: Solution $\mathbf{w}(\mathbf{p})$, $\boldsymbol{\lambda}(\mathbf{p})$ implicitly given by the KKT conditions:

$$\min_{\mathbf{w}} \quad \Phi(\mathbf{w},\mathbf{p})$$
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Let $\mathbf{z}$ be implicitly given by the $\mathcal{C}^1$ function:

$$\mathbf{R}(\mathbf{z},\mathbf{p}) = 0$$

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$$\mathbf{z} = \begin{bmatrix} \mathbf{w} \\ \boldsymbol{\lambda} \end{bmatrix}$$

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$$\mathbf{R}(\mathbf{z}, \mathbf{p}) = 0, \quad \text{with} \quad \nabla_{\mathbf{z}} \mathbf{R}(\mathbf{z}, \mathbf{p}_0) \text{ full rank}$$

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Then for any $\mathbf{p}_0$ there is a $\mathcal{C}^1$ function $\xi(\mathbf{p})$ such that:

$$\mathbf{R}(\xi(\mathbf{p}), \mathbf{p}) = 0$$

holds in a neighbourhood of $\mathbf{p}_0$.

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That means we have $\mathbf{z}(\mathbf{p}) = \xi(\mathbf{p})$ around $\mathbf{p}_0$.

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$$\mathbf{z} = \begin{bmatrix} \mathbf{w} \\ \boldsymbol{\lambda} \end{bmatrix}$$
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$$\mathbf{R} = \begin{bmatrix} \nabla_{\mathbf{w}}\mathcal{L}(\mathbf{w},\boldsymbol{\lambda},\mathbf{p}) \\ \mathbf{g}(\mathbf{w},\mathbf{p}) \end{bmatrix}$$

In other words...

... our $\mathbf{z}(\mathbf{p})$ is **locally well defined and differentiable** if $\nabla_{\mathbf{z}}\mathbf{R}(\mathbf{z},\mathbf{p})$ exists and is full rank

Implicit functions

Parametric NLP:

$$\min_{\mathbf{w}} \quad \Phi(\mathbf{w})$$
$$\mathbf{g}(\mathbf{w},\mathbf{p}) = 0$$

Solution $\mathbf{w}(\mathbf{p})$, $\boldsymbol{\lambda}(\mathbf{p})$ implicitly given by the KKT conditions:

$$\nabla_{\mathbf{w}}\mathcal{L}(\mathbf{w},\boldsymbol{\lambda},\mathbf{p}) = 0$$
$$\mathbf{g}(\mathbf{w},\mathbf{p}) = 0$$

Let's check $\nabla_{\mathbf{z}}\mathbf{R}(\mathbf{z},\mathbf{p})$ for the KKT conditions:

$$\mathbf{R}(\mathbf{z},\mathbf{p}) = \begin{bmatrix} \nabla_{\mathbf{w}}\mathcal{L}(\mathbf{w},\mathbf{p},\boldsymbol{\lambda}) \\ \mathbf{g}(\mathbf{w},\mathbf{p}) \end{bmatrix} \quad \text{with} \quad \mathbf{z} = \begin{bmatrix} \mathbf{w} \\ \boldsymbol{\lambda} \end{bmatrix}$$

Implicit functions

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$$\mathbf{R}(\mathbf{z},\mathbf{p}) = \left[\begin{array}{c} \nabla_{\mathbf{w}}\mathcal{L}(\mathbf{w},\mathbf{p},\boldsymbol{\lambda}) \\ \mathbf{g}(\mathbf{w},\mathbf{p}) \end{array}\right] \quad \text{with} \quad \mathbf{z} = \left[\begin{array}{c} \mathbf{w} \\ \boldsymbol{\lambda} \end{array}\right]$$

We get:

$$\nabla_{\mathbf{z}}\mathbf{R}(\mathbf{z},\mathbf{p}) = \left[\begin{array}{cc} \nabla^2_{\mathbf{w}}\mathcal{L}(\mathbf{w},\boldsymbol{\lambda},\mathbf{p}) & \nabla_{\mathbf{w}}\mathbf{g}(\mathbf{w},\mathbf{p}) \\ \nabla_{\mathbf{w}}\mathbf{g}(\mathbf{w},\mathbf{p})^{\mathsf{T}} & 0 \end{array}\right]$$

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This is the KKT matrix providing the Newton step, remember:

$$\begin{bmatrix} \nabla^2_{\mathbf{w}}\mathcal{L}(\mathbf{w},\boldsymbol{\lambda},\mathbf{p}) & \nabla_{\mathbf{w}}\mathbf{g}(\mathbf{w},\mathbf{p}) \\ \nabla_{\mathbf{w}}\mathbf{g}(\mathbf{w},\mathbf{p})^\top & 0 \end{bmatrix} \begin{bmatrix} \Delta\mathbf{w} \\ \boldsymbol{\lambda} \end{bmatrix} = -\begin{bmatrix} \nabla\Phi(\mathbf{w}) \\ \mathbf{g}(\mathbf{w},\mathbf{p}) \end{bmatrix}$$

Implicit functions

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Theorem

The parametric solution $\mathbf{z}(\mathbf{p})$ is well defined and $\mathcal{C}^1$ in a neighbourhood of $\mathbf{p}$ if the KKT matrix is full rank at $\mathbf{p}$

Computing the Sensitivities

Parametric NLP: Solution $\mathbf{w}(\mathbf{p}), \boldsymbol{\lambda}(\mathbf{p})$ implicitly given by the KKT conditions:

$$\min_{\mathbf{w}} \quad \Phi(\mathbf{w},\mathbf{p}) \qquad\qquad \nabla_{\mathbf{w}}\mathcal{L}(\mathbf{w},\mathbf{p},\boldsymbol{\lambda}) = 0$$

$$\mathbf{g}(\mathbf{w},\mathbf{p}) = 0 \qquad\qquad \mathbf{g}(\mathbf{w},\mathbf{p}) = 0$$

Differentiating implicit functions

Let $\mathbf{z}$ be implicitly given by the $\mathcal{C}^1$ function:

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then $\mathbf{z}(\mathbf{p})$ is well defined and $\mathcal{C}^1$.

Where:

$$\mathbf{z} = \begin{bmatrix} \mathbf{w} \\ \boldsymbol{\lambda} \end{bmatrix}$$

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Sensitivity $\frac{\partial}{\partial \mathbf{p}}\mathbf{z}(\mathbf{p})$ is given by

$$\frac{d\mathbf{R}(\mathbf{z},\mathbf{p})}{d\mathbf{p}} = \frac{\partial \mathbf{R}(\mathbf{z},\mathbf{p})}{\partial \mathbf{z}}\frac{\partial \mathbf{z}}{\partial \mathbf{p}} + \frac{\partial \mathbf{R}(\mathbf{z},\mathbf{p})}{\partial \mathbf{p}} = 0$$

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i.e.

$$\frac{\partial \mathbf{z}}{\partial \mathbf{p}} = -\frac{\partial \mathbf{R}(\mathbf{z},\mathbf{p})}{\partial \mathbf{z}}^{-1}\frac{\partial \mathbf{R}(\mathbf{z},\mathbf{p})}{\partial \mathbf{p}}$$

Where:

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Differentiating the optimal solution

Sensitivity given by:

$$\frac{\partial \mathbf{z}}{\partial \mathbf{p}} = -\frac{\partial \mathbf{R}(\mathbf{z}, \mathbf{p})}{\partial \mathbf{z}}^{-1} \frac{\partial \mathbf{R}(\mathbf{z}, \mathbf{p})}{\partial \mathbf{p}}$$

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With

$$\frac{\partial \mathbf{R}(\mathbf{z},\mathbf{p})}{\partial \mathbf{z}} = \begin{bmatrix} H(\mathbf{w},\boldsymbol{\lambda},\mathbf{p}) & \nabla_{\mathbf{w}}\mathbf{g}(\mathbf{w},\mathbf{p}) \\ \nabla_{\mathbf{w}}\mathbf{g}(\mathbf{w},\mathbf{p})^{\mathsf{T}} & 0 \end{bmatrix}$$

Where:

$$\mathbf{z} = \begin{bmatrix} \mathbf{w} \\ \boldsymbol{\lambda} \end{bmatrix}$$

$$\mathbf{R} = \begin{bmatrix} \nabla_{\mathbf{w}}\mathcal{L}(\mathbf{w},\boldsymbol{\lambda},\mathbf{p}) \\ \mathbf{g}(\mathbf{w},\mathbf{p}) \end{bmatrix}$$

Computing the Sensitivities

Parametric NLP: Solution $\mathbf{w}(\mathbf{p})$, $\boldsymbol{\lambda}(\mathbf{p})$ implicitly given by the KKT conditions:

$$\min_{\mathbf{w}} \quad \Phi(\mathbf{w},\mathbf{p})$$
$$\mathbf{g}(\mathbf{w},\mathbf{p}) = 0$$

$$\nabla_{\mathbf{w}}\mathcal{L}(\mathbf{w},\mathbf{p},\boldsymbol{\lambda}) = 0$$
$$\mathbf{g}(\mathbf{w},\mathbf{p}) = 0$$

Differentiating the optimal solution

Sensitivity given by:

$$\frac{\partial \mathbf{z}}{\partial \mathbf{p}} = -\frac{\partial \mathbf{R}(\mathbf{z},\mathbf{p})}{\partial \mathbf{z}}^{-1}\frac{\partial \mathbf{R}(\mathbf{z},\mathbf{p})}{\partial \mathbf{p}}$$

With

$$\frac{\partial \mathbf{R}(\mathbf{z},\mathbf{p})}{\partial \mathbf{z}} = \begin{bmatrix} H(\mathbf{w},\boldsymbol{\lambda},\mathbf{p}) & \nabla_{\mathbf{w}}\mathbf{g}(\mathbf{w},\mathbf{p}) \\ \nabla_{\mathbf{w}}\mathbf{g}(\mathbf{w},\mathbf{p})^{\mathsf{T}} & 0 \end{bmatrix}$$

$$\frac{\partial \mathbf{R}(\mathbf{z},\mathbf{p})}{\partial \mathbf{p}} = \begin{bmatrix} \nabla_{\mathbf{w},\mathbf{p}}\mathcal{L}(\mathbf{w},\boldsymbol{\lambda},\mathbf{p}) \\ \nabla_{\mathbf{p}}\mathbf{g}(\mathbf{w},\mathbf{p})^{\mathsf{T}} \end{bmatrix}$$

Where:

$$\mathbf{z} = \begin{bmatrix} \mathbf{w} \\ \boldsymbol{\lambda} \end{bmatrix}$$

$$\mathbf{R} = \begin{bmatrix} \nabla_{\mathbf{w}}\mathcal{L}(\mathbf{w},\boldsymbol{\lambda},\mathbf{p}) \\ \mathbf{g}(\mathbf{w},\mathbf{p}) \end{bmatrix}$$

Computing the Sensitivities

Parametric NLP: Solution $\mathbf{w}(\mathbf{p}), \boldsymbol{\lambda}(\mathbf{p})$ implicitly given by the KKT conditions:

$$\min_{\mathbf{w}} \quad \Phi(\mathbf{w}, \mathbf{p}) \qquad \mathbf{g}(\mathbf{w}, \mathbf{p}) = 0$$

$$\nabla_{\mathbf{w}} \mathcal{L}(\mathbf{w}, \mathbf{p}, \boldsymbol{\lambda}) = 0$$
$$\mathbf{g}(\mathbf{w}, \mathbf{p}) = 0$$

Differentiating the optimal solution

Sensitivity given by:

$$\frac{\partial \mathbf{z}}{\partial \mathbf{p}} = -\frac{\partial \mathbf{R}(\mathbf{z}, \mathbf{p})}{\partial \mathbf{z}}^{-1} \frac{\partial \mathbf{R}(\mathbf{z}, \mathbf{p})}{\partial \mathbf{p}}$$

With

$$\frac{\partial \mathbf{R}(\mathbf{z}, \mathbf{p})}{\partial \mathbf{z}} = \begin{bmatrix} H(\mathbf{w}, \boldsymbol{\lambda}, \mathbf{p}) & \nabla_{\mathbf{w}} \mathbf{g}(\mathbf{w}, \mathbf{p}) \\ \nabla_{\mathbf{w}} \mathbf{g}(\mathbf{w}, \mathbf{p})^{\mathsf{T}} & 0 \end{bmatrix}$$

$$\frac{\partial \mathbf{R}(\mathbf{z}, \mathbf{p})}{\partial \mathbf{p}} = \begin{bmatrix} \nabla_{\mathbf{w},\mathbf{p}} \mathcal{L}(\mathbf{w}, \boldsymbol{\lambda}, \mathbf{p}) \\ \nabla_{\mathbf{p}} \mathbf{g}(\mathbf{w}, \mathbf{p})^{\mathsf{T}} \end{bmatrix}$$

Where:

$$\mathbf{z} = \begin{bmatrix} \mathbf{w} \\ \boldsymbol{\lambda} \end{bmatrix}$$

$$\mathbf{R} = \begin{bmatrix} \nabla_{\mathbf{w}} \mathcal{L}(\mathbf{w}, \boldsymbol{\lambda}, \mathbf{p}) \\ \mathbf{g}(\mathbf{w}, \mathbf{p}) \end{bmatrix}$$

- If $\mathbf{p}$ enters linearly in $\mathbf{g}(\mathbf{w}, \mathbf{p})$, then $\frac{\partial \mathbf{R}(\mathbf{z}, \mathbf{p})}{\partial \mathbf{p}} = \begin{bmatrix} 0 \\ \text{Cst.} \end{bmatrix}$
- Sensitivities are for free since a factorisation of the KKT matrix is available from the Newton algorithm

Computing the Sensitivities - Implementation

Parametric NLP:

$$\min_{\mathbf{w}} \quad \Phi(\mathbf{w}, \mathbf{p})$$
$$\mathbf{g}(\mathbf{w}, \mathbf{p}) = 0$$

Remarks:

- M re-used in the sensitivities, computationally cheap !!
- Sensitivities are inexact if Newton is not properly converged
- Must use $\nabla_{\mathbf{w},\mathbf{p}}\mathcal{L}$ and not $\nabla_{\mathbf{w},\mathbf{p}}\Phi$ in sensitivities !

Algorithm: NLP solution with sensitivities

Input: $\mathbf{w}$, $\boldsymbol{\lambda}$, $\mathbf{p}$

while *not converged* **do**

Compute:

$$M = \begin{bmatrix} H & \nabla_{\mathbf{w}}\mathbf{g} \\ \nabla_{\mathbf{w}}\mathbf{g}^\top & 0 \end{bmatrix}^{-1}$$

Newton step

$$\begin{bmatrix} \Delta\mathbf{w} \\ \boldsymbol{\lambda}^+ \end{bmatrix} = -M \begin{bmatrix} \nabla_{\mathbf{w}}\Phi \\ \mathbf{g} \end{bmatrix}$$

Update: $\mathbf{w} \leftarrow \mathbf{w} + t\Delta\mathbf{w}, \quad \boldsymbol{\lambda} \leftarrow t\boldsymbol{\lambda}^+ + (1-t)\boldsymbol{\lambda}_k$

Compute sensitivities at the solution:

$$\frac{\partial}{\partial \mathbf{p}} \begin{bmatrix} \mathbf{w} \\ \boldsymbol{\lambda} \end{bmatrix} = -M \begin{bmatrix} \nabla_{\mathbf{w},\mathbf{p}}\mathcal{L} \\ \nabla_{\mathbf{p}}\mathbf{g}^\top \end{bmatrix}$$

return $\mathbf{w}$, $\boldsymbol{\lambda}$, $\frac{\partial \mathbf{w}}{\partial \mathbf{p}}$, $\frac{\partial \boldsymbol{\lambda}}{\partial \mathbf{p}}$

Outline

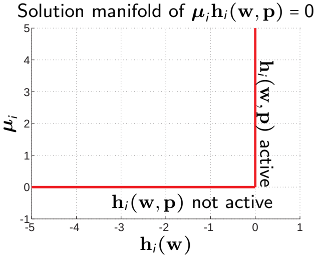
Sensitivity with inequality constraints

Parametric NLP: Solution $\mathbf{w}(\mathbf{p}), \boldsymbol{\lambda}(\mathbf{p}), \boldsymbol{\mu}(\mathbf{p})$ given by the KKT conditions:

$$\min_{\mathbf{w}} \quad \Phi(\mathbf{w},\mathbf{p})$$
$$\mathbf{g}(\mathbf{w},\mathbf{p}) = 0$$
$$\mathbf{h}(\mathbf{w},\mathbf{p}) \leq 0$$

$$\nabla_{\mathbf{w}}\mathcal{L}(\mathbf{w},\mathbf{p},\boldsymbol{\lambda},\boldsymbol{\mu}) = 0$$
$$\mathbf{g}(\mathbf{w},\mathbf{p}) = 0$$
$$\mathbf{h}(\mathbf{w},\mathbf{p}) \leq 0$$
$$\boldsymbol{\mu}_i \mathbf{h}_i(\mathbf{w},\mathbf{p}) = 0$$

Now we have non-smooth conditions...



Sensitivity with inequality constraints

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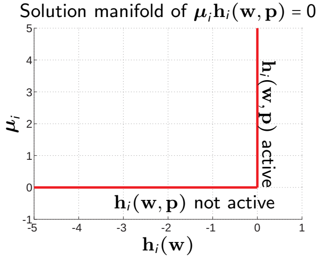


... however, they are piecewise smooth !!

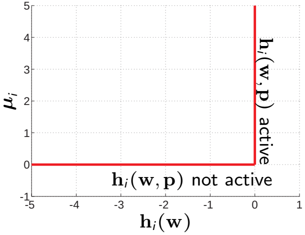
Sensitivity with inequality constraints

Parametric NLP: Solution $\mathbf{w}(\mathbf{p}), \boldsymbol{\lambda}(\mathbf{p}), \boldsymbol{\mu}(\mathbf{p})$ given by the KKT conditions:

$$\begin{aligned} \min_{\mathbf{w}} \quad & \Phi(\mathbf{w}, \mathbf{p}) \\ & \mathbf{g}(\mathbf{w}, \mathbf{p}) = 0 \\ & \mathbf{h}(\mathbf{w}, \mathbf{p}) \leq 0 \end{aligned}$$

$$\begin{aligned} \nabla_{\mathbf{w}} \mathcal{L}(\mathbf{w}, \mathbf{p}, \boldsymbol{\lambda}, \boldsymbol{\mu}) &= 0 \\ \mathbf{g}(\mathbf{w}, \mathbf{p}) &= 0 \\ \mathbf{h}(\mathbf{w}, \mathbf{p}) &\leq 0 \\ \boldsymbol{\mu}_i \mathbf{h}_i(\mathbf{w}, \mathbf{p}) &= 0 \end{aligned}$$

Now we have non-smooth conditions...



... however, they are piecewise smooth !!

Let $\mathbb{A}$ be the active set, then we have:

$$\begin{aligned} \nabla_{\mathbf{w}} \mathcal{L}(\mathbf{w}, \mathbf{p}, \boldsymbol{\lambda}, \boldsymbol{\mu}) &= 0 \\ \mathbf{g}(\mathbf{w}, \mathbf{p}) &= 0 \\ \mathbf{h}_{\mathbb{A}}(\mathbf{w}, \mathbf{p}) &= 0 \\ \boldsymbol{\mu}_{\bar{\mathbb{A}}} &= 0 \end{aligned}$$

and $\mathbf{h}_{\bar{\mathbb{A}}}(\mathbf{w}, \mathbf{p}) < 0$, $\boldsymbol{\mu}_{\mathbb{A}} > 0$

Sensitivity with inequality constraints

Parametric NLP: Solution $\mathbf{w}(\mathbf{p}), \boldsymbol{\lambda}(\mathbf{p}), \boldsymbol{\mu}(\mathbf{p})$ given by the KKT conditions:

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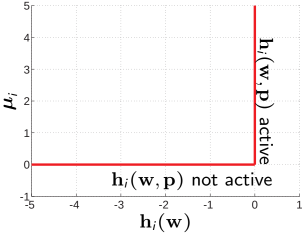
Now we have non-smooth conditions...

Solution manifold of $\boldsymbol{\mu}_i \mathbf{h}_i(\mathbf{w}, \mathbf{p}) = 0$



... however, they are piecewise smooth !!

Let $\mathbb{A}$ be the active set, then we have:

$$\begin{aligned} \nabla_{\mathbf{w}} \mathcal{L}(\mathbf{w}, \mathbf{p}, \boldsymbol{\lambda}, \boldsymbol{\mu}) &= 0 \\ \mathbf{g}(\mathbf{w}, \mathbf{p}) &= 0 \\ \mathbf{h}_{\mathbb{A}}(\mathbf{w}, \mathbf{p}) &= 0 \\ \boldsymbol{\mu}_{\bar{\mathbb{A}}} &= 0 \end{aligned}$$

and $\mathbf{h}_{\bar{\mathbb{A}}}(\mathbf{w}, \mathbf{p}) < 0$, $\boldsymbol{\mu}_{\mathbb{A}} > 0$

Conditions are smooth as long as all constraints are **strictly active**. Then we avoid the "corner" of the complementarity slackness manifold.

Sensitivity with inequality constraints

Parametric NLP:

$$\min_{\mathbf{w}} \quad \Phi\left(\mathbf{w}\right)$$

$$\mathbf{g}\left(\mathbf{w},\mathbf{p}\right) = 0$$

$$\mathbf{h}\left(\mathbf{w},\mathbf{p}\right) \leq 0$$

Let $\mathbb{A}$ be the (strictly) active set, then we have:

$$\begin{aligned}
\nabla_{\mathbf{w}}\Phi\left(\mathbf{w}\right) + \nabla_{\mathbf{w}}\mathbf{g}\left(\mathbf{w},\mathbf{p}\right)\boldsymbol{\lambda} + \nabla_{\mathbf{w}}\mathbf{h}_{\mathbb{A}}\left(\mathbf{w},\mathbf{p}\right)\boldsymbol{\mu}_{\mathbb{A}} &= 0\\
\mathbf{g}\left(\mathbf{w},\mathbf{p}\right) &= 0\\
\mathbf{h}_{\mathbb{A}}\left(\mathbf{w},\mathbf{p}\right) &= 0\\
\boldsymbol{\mu}_{\bar{\mathbb{A}}} &= 0
\end{aligned}$$

and $\mathbf{h}_{\bar{\mathbb{A}}}\left(\mathbf{w},\mathbf{p}\right) < 0$, $\boldsymbol{\mu}_{\mathbb{A}} > 0$

Sensitivity with inequality constraints

Parametric NLP:

$$
\begin{aligned}
\min_{\mathbf{w}} \quad & \Phi(\mathbf{w}) \\
& \mathbf{g}(\mathbf{w},\mathbf{p}) = 0 \\
& \mathbf{h}(\mathbf{w},\mathbf{p}) \leq 0
\end{aligned}
$$

Let $\mathbb{A}$ be the (strictly) active set, then we have:

$$
\begin{aligned}
\nabla_{\mathbf{w}}\Phi(\mathbf{w}) + \nabla_{\mathbf{w}}\mathbf{g}(\mathbf{w},\mathbf{p})\boldsymbol{\lambda} + \nabla_{\mathbf{w}}\mathbf{h}_{\mathbb{A}}(\mathbf{w},\mathbf{p})\boldsymbol{\mu}_{\mathbb{A}} &= 0 \\
\mathbf{g}(\mathbf{w},\mathbf{p}) &= 0 \\
\mathbf{h}_{\mathbb{A}}(\mathbf{w},\mathbf{p}) &= 0 \\
\boldsymbol{\mu}_{\bar{\mathbb{A}}} &= 0
\end{aligned}
$$

and $\mathbf{h}_{\bar{\mathbb{A}}}(\mathbf{w},\mathbf{p}) < 0$, $\boldsymbol{\mu}_{\mathbb{A}} > 0$

Let's define:

$$
\mathbf{R}(\mathbf{z},\mathbf{p}) = \begin{bmatrix} \nabla_{\mathbf{w}}\Phi(\mathbf{w}) + \nabla_{\mathbf{w}}\mathbf{g}(\mathbf{w},\mathbf{p})\boldsymbol{\lambda} + \nabla_{\mathbf{w}}\mathbf{h}_{\mathbb{A}}(\mathbf{w},\mathbf{p})\boldsymbol{\mu}_{\mathbb{A}} \\ \mathbf{g}(\mathbf{w},\mathbf{p}) \\ \mathbf{h}_{\mathbb{A}}(\mathbf{w},\mathbf{p}) \end{bmatrix}, \qquad \mathbf{z} = \begin{bmatrix} \mathbf{w} \\ \boldsymbol{\lambda} \\ \boldsymbol{\mu}_{\mathbb{A}} \end{bmatrix}
$$

Sensitivity with inequality constraints

Parametric NLP:

$$\min_{\mathbf{w}} \quad \Phi(\mathbf{w})$$

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$$\mathbf{R}(\mathbf{z},\mathbf{p}) = \begin{bmatrix} \nabla_{\mathbf{w}}\Phi(\mathbf{w}) + \nabla_{\mathbf{w}}\mathbf{g}(\mathbf{w},\mathbf{p})\boldsymbol{\lambda} + \nabla_{\mathbf{w}}\mathbf{h}_{\mathbb{A}}(\mathbf{w},\mathbf{p})\boldsymbol{\mu}_{\mathbb{A}} \\ \mathbf{g}(\mathbf{w},\mathbf{p}) \\ \mathbf{h}_{\mathbb{A}}(\mathbf{w},\mathbf{p}) \end{bmatrix}, \quad \mathbf{z} = \begin{bmatrix} \mathbf{w} \\ \boldsymbol{\lambda} \\ \boldsymbol{\mu}_{\mathbb{A}} \end{bmatrix}$$

Sensitivity given by $\frac{\partial \mathbf{z}}{\partial \mathbf{p}} = -\frac{\partial \mathbf{R}}{\partial \mathbf{z}}^{-1}\frac{\partial \mathbf{R}}{\partial \mathbf{p}}$ with:

$$\frac{\partial \mathbf{R}}{\partial \mathbf{z}} = \begin{bmatrix} H & \nabla_{\mathbf{w}}\mathbf{g} & \nabla_{\mathbf{w}}\mathbf{h}_{\mathbb{A}} \\ \nabla_{\mathbf{w}}\mathbf{g}^{\mathsf{T}} & 0 & 0 \\ \nabla_{\mathbf{w}}\mathbf{h}_{\mathbb{A}}^{\mathsf{T}} & 0 & 0 \end{bmatrix}$$

Sensitivity with inequality constraints

Parametric NLP:

$$\min_{\mathbf{w}} \quad \Phi(\mathbf{w})$$
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Let's define:

$$\mathbf{R}(\mathbf{z},\mathbf{p}) = \left[\begin{array}{c} \nabla_{\mathbf{w}}\Phi(\mathbf{w}) + \nabla_{\mathbf{w}}\mathbf{g}(\mathbf{w},\mathbf{p})\boldsymbol{\lambda} + \nabla_{\mathbf{w}}\mathbf{h}_{\mathbb{A}}(\mathbf{w},\mathbf{p})\boldsymbol{\mu}_{\mathbb{A}} \\ \mathbf{g}(\mathbf{w},\mathbf{p}) \\ \mathbf{h}_{\mathbb{A}}(\mathbf{w},\mathbf{p}) \end{array}\right], \qquad \mathbf{z} = \left[\begin{array}{c} \mathbf{w} \\ \boldsymbol{\lambda} \\ \boldsymbol{\mu}_{\mathbb{A}} \end{array}\right]$$

Sensitivity given by $\frac{\partial \mathbf{z}}{\partial \mathbf{p}} = -\frac{\partial \mathbf{R}}{\partial \mathbf{z}}^{-1}\frac{\partial \mathbf{R}}{\partial \mathbf{p}}$ with:

$$\frac{\partial \mathbf{R}}{\partial \mathbf{z}} = \left[\begin{array}{ccc} H & \nabla_{\mathbf{w}}\mathbf{g} & \nabla_{\mathbf{w}}\mathbf{h}_{\mathbb{A}} \\ \nabla_{\mathbf{w}}\mathbf{g}^{\top} & 0 & 0 \\ \nabla_{\mathbf{w}}\mathbf{h}_{\mathbb{A}}^{\top} & 0 & 0 \end{array}\right]$$

Matrix $\frac{\partial \mathbf{R}}{\partial \mathbf{z}}$ is factorized inside Active Set QP solvers (c.f. Chapter "QP solvers"). I.e. we get the sensitivities for free† when using SQP !!

Sensitivity with inequality constraints

Parametric NLP:

$$\begin{aligned} \min_{\mathbf{w}} \quad & \Phi(\mathbf{w}) \\ & \mathbf{g}(\mathbf{w},\mathbf{p}) = 0 \\ & \mathbf{h}(\mathbf{w},\mathbf{p}) \leq 0 \end{aligned}$$

Let $\mathbb{A}$ be the (strictly) active set, then we have:

$$\begin{aligned} \nabla_{\mathbf{w}}\Phi(\mathbf{w}) + \nabla_{\mathbf{w}}\mathbf{g}(\mathbf{w},\mathbf{p})\boldsymbol{\lambda} + \nabla_{\mathbf{w}}\mathbf{h}_{\mathbb{A}}(\mathbf{w},\mathbf{p})\boldsymbol{\mu}_{\mathbb{A}} &= 0 \\ \mathbf{g}(\mathbf{w},\mathbf{p}) &= 0 \\ \mathbf{h}_{\mathbb{A}}(\mathbf{w},\mathbf{p}) &= 0 \\ \boldsymbol{\mu}_{\bar{\mathbb{A}}} &= 0 \end{aligned}$$

and $\mathbf{h}_{\bar{\mathbb{A}}}(\mathbf{w},\mathbf{p}) < 0$, $\boldsymbol{\mu}_{\mathbb{A}} > 0$

Let's define:

$$\mathbf{R}(\mathbf{z},\mathbf{p}) = \begin{bmatrix} \nabla_{\mathbf{w}}\Phi(\mathbf{w}) + \nabla_{\mathbf{w}}\mathbf{g}(\mathbf{w},\mathbf{p})\boldsymbol{\lambda} + \nabla_{\mathbf{w}}\mathbf{h}_{\mathbb{A}}(\mathbf{w},\mathbf{p})\boldsymbol{\mu}_{\mathbb{A}} \\ \mathbf{g}(\mathbf{w},\mathbf{p}) \\ \mathbf{h}_{\mathbb{A}}(\mathbf{w},\mathbf{p}) \end{bmatrix}, \quad \mathbf{z} = \begin{bmatrix} \mathbf{w} \\ \boldsymbol{\lambda} \\ \boldsymbol{\mu}_{\mathbb{A}} \end{bmatrix}$$

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$$\frac{\partial \mathbf{R}}{\partial \mathbf{z}} = \begin{bmatrix} H & \nabla_{\mathbf{w}}\mathbf{g} & \nabla_{\mathbf{w}}\mathbf{h}_{\mathbb{A}} \\ \nabla_{\mathbf{w}}\mathbf{g}^{\top} & 0 & 0 \\ \nabla_{\mathbf{w}}\mathbf{h}_{\mathbb{A}}^{\top} & 0 & 0 \end{bmatrix}$$

Matrix $\frac{\partial \mathbf{R}}{\partial \mathbf{z}}$ is factorized inside Active Set QP solvers (c.f. Chapter "QP solvers"). I.e. we get the sensitivities for free† when using SQP !!

†however this factorisation may be hidden deep inside a code you don't have access to... ☹☹☹. Lobby for free access to the factorisation of the KKT matrix !!

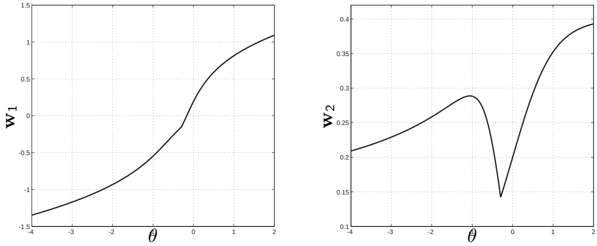
Linear Predictor

Parametric NLP:

$$\begin{aligned} \min_{\mathbf{w}} \quad & \Phi\left(\mathbf{w},\mathbf{p}\right) \\ & \mathbf{g}\left(\mathbf{w},\mathbf{p}\right) = 0 \\ & \mathbf{h}\left(\mathbf{w},\mathbf{p}\right) \leq 0 \end{aligned}$$

Example

$$\begin{aligned} \min_{\mathbf{w}} \quad & \left\|\mathbf{w}\right\|^2 \\ \text{s.t.} \quad & \mathbf{w}_2 - \mathbf{w}_1\left(1+\mathbf{w}_1^2\right) + \theta = 0 \\ & \frac{1}{5}\left(\tanh\theta + 1\right) - \mathbf{w}_2 \leq 0 \end{aligned}$$



Linear Predictor

Parametric NLP:

$$\min_{\mathbf{w}} \quad \Phi(\mathbf{w},\mathbf{p})$$
$$\mathbf{g}(\mathbf{w},\mathbf{p}) = 0$$
$$\mathbf{h}(\mathbf{w},\mathbf{p}) \leq 0$$

Constraint $\mathbf{h}$ inactive, $\boldsymbol{\mu} = 0$, $\mathbb{A} = \emptyset$ and:

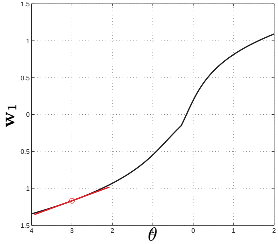
$$\frac{\partial}{\partial \mathbf{p}} \begin{bmatrix} \mathbf{w} \\ \boldsymbol{\lambda} \end{bmatrix} = -\begin{bmatrix} H & \nabla_{\mathbf{w}} \mathbf{g} \\ \nabla_{\mathbf{w}} \mathbf{g}^{\top} & 0 \end{bmatrix}^{-1} \frac{\partial}{\partial \mathbf{p}} \begin{bmatrix} \nabla_{\mathbf{w}} \mathcal{L} \\ \mathbf{g} \end{bmatrix}$$

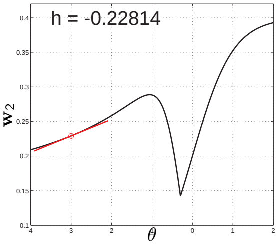
$$\frac{\partial \boldsymbol{\mu}}{\partial \mathbf{p}} = 0$$

Example

$$\min_{\mathbf{w}} \quad \|\mathbf{w}\|^2$$
$$\text{s.t.} \quad \mathbf{w}_2 - \mathbf{w}_1\left(1 + \mathbf{w}_1^2\right) + \theta = 0$$
$$\frac{1}{5}\left(\tanh\theta + 1\right) - \mathbf{w}_2 \leq 0$$

Check $\frac{\partial \mathbf{z}}{\partial \theta} = -\frac{\partial \mathbf{R}}{\partial \mathbf{z}}^{-1} \frac{\partial \mathbf{R}}{\partial \mathbf{p}}$





Linear Predictor

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$$\frac{\partial \boldsymbol{\mu}}{\partial \mathbf{p}} = 0$$

Example

$$\min_{\mathbf{w}} \quad \|\mathbf{w}\|^2$$
$$\text{s.t.} \quad \mathbf{w}_2 - \mathbf{w}_1\left(1 + \mathbf{w}_1^2\right) + \theta = 0$$
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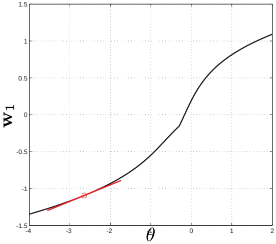
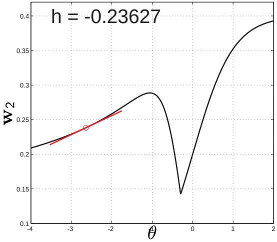
Check $\frac{\partial \mathbf{z}}{\partial \theta} = -\frac{\partial \mathbf{R}}{\partial \mathbf{z}}^{-1}\frac{\partial \mathbf{R}}{\partial \mathbf{p}}$





Linear Predictor

Parametric NLP:

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$$\mathbf{g}(\mathbf{w}, \mathbf{p}) = 0$$
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Constraint $\mathbf{h}$ inactive, $\boldsymbol{\mu} = 0$, $\mathbb{A} = \varnothing$ and:

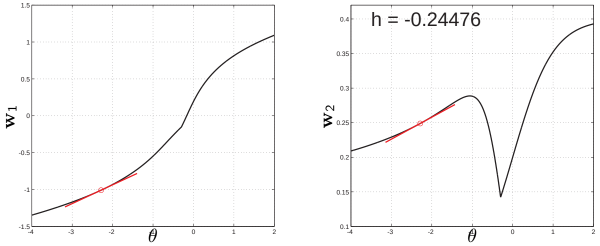
$$\frac{\partial}{\partial \mathbf{p}} \begin{bmatrix} \mathbf{w} \\ \boldsymbol{\lambda} \end{bmatrix} = - \begin{bmatrix} H & \nabla_{\mathbf{w}} \mathbf{g} \\ \nabla_{\mathbf{w}} \mathbf{g}^\top & 0 \end{bmatrix}^{-1} \frac{\partial}{\partial \mathbf{p}} \begin{bmatrix} \nabla_{\mathbf{w}} \mathcal{L} \\ \mathbf{g} \end{bmatrix}$$

$$\frac{\partial \boldsymbol{\mu}}{\partial \mathbf{p}} = 0$$

Example

$$\min_{\mathbf{w}} \quad \|\mathbf{w}\|^2$$
$$\text{s.t.} \quad \mathbf{w}_2 - \mathbf{w}_1 \left(1 + \mathbf{w}_1^2\right) + \theta = 0$$
$$\frac{1}{5}(\tanh\theta + 1) - \mathbf{w}_2 \leq 0$$

Check $\frac{\partial \mathbf{z}}{\partial \theta} = -\frac{\partial \mathbf{R}}{\partial \mathbf{z}}^{-1} \frac{\partial \mathbf{R}}{\partial \mathbf{p}}$



Linear Predictor

Parametric NLP:

$$\min_{\mathbf{w}} \quad \Phi(\mathbf{w},\mathbf{p})$$
$$\mathbf{g}(\mathbf{w},\mathbf{p}) = 0$$
$$\mathbf{h}(\mathbf{w},\mathbf{p}) \leq 0$$

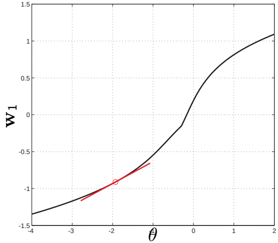
Constraint $\mathbf{h}$ inactive, $\boldsymbol{\mu} = 0$, $\mathbb{A} = \emptyset$ and:

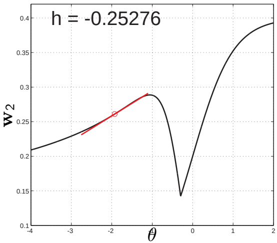
$$\frac{\partial}{\partial \mathbf{p}}\begin{bmatrix} \mathbf{w} \\ \boldsymbol{\lambda} \end{bmatrix} = -\begin{bmatrix} H & \nabla_{\mathbf{w}}\mathbf{g} \\ \nabla_{\mathbf{w}}\mathbf{g}^\top & 0 \end{bmatrix}^{-1} \frac{\partial}{\partial \mathbf{p}}\begin{bmatrix} \nabla_{\mathbf{w}}\mathcal{L} \\ \mathbf{g} \end{bmatrix}$$
$$\frac{\partial \boldsymbol{\mu}}{\partial \mathbf{p}} = 0$$

Example

$$\min_{\mathbf{w}} \quad \|\mathbf{w}\|^2$$
$$\text{s.t.} \quad \mathbf{w}_2 - \mathbf{w}_1\left(1 + \mathbf{w}_1^2\right) + \theta = 0$$
$$\frac{1}{5}\left(\tanh\theta + 1\right) - \mathbf{w}_2 \leq 0$$

Check $\frac{\partial \mathbf{z}}{\partial \theta} = -\frac{\partial \mathbf{R}}{\partial \mathbf{z}}^{-1}\frac{\partial \mathbf{R}}{\partial \mathbf{p}}$





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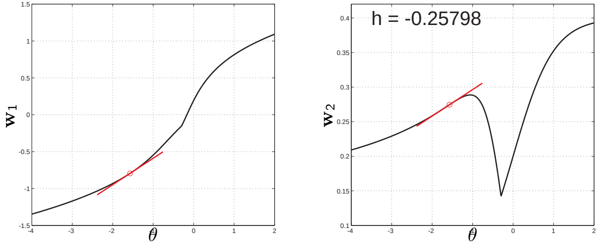

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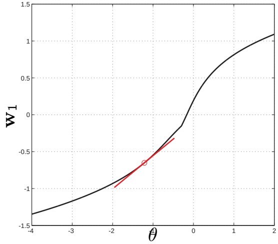

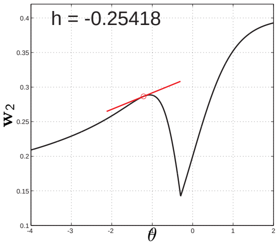

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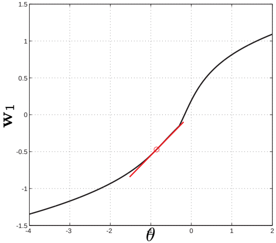

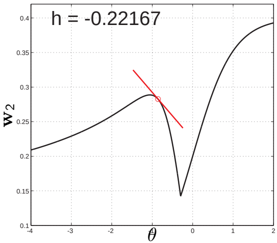

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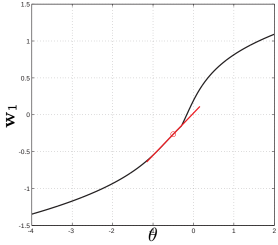

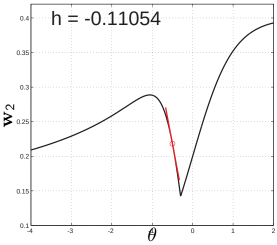

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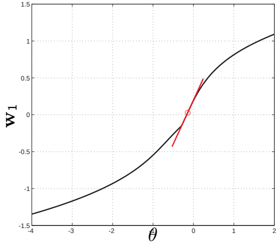

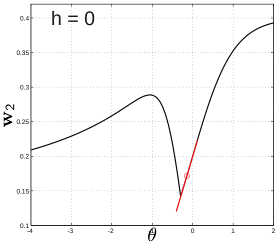

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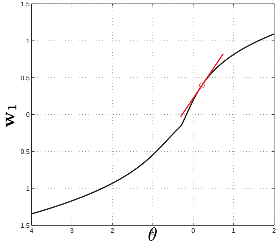

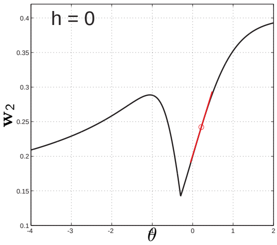

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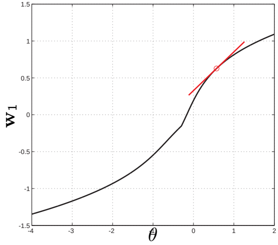

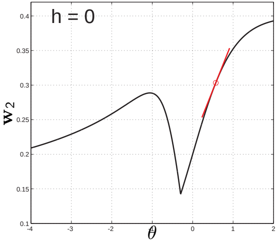

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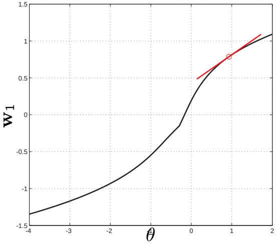

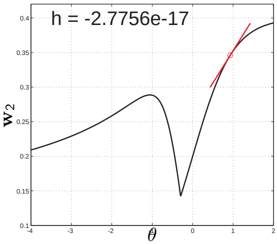

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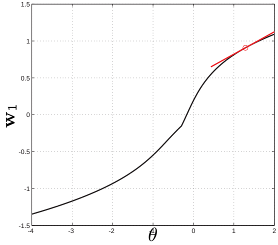

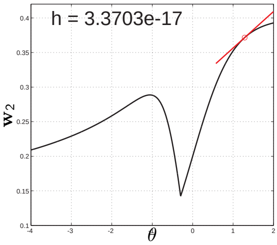

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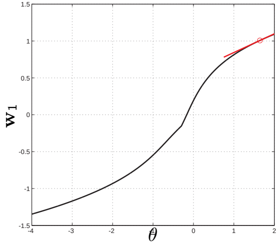

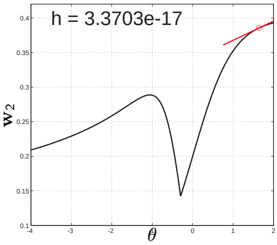

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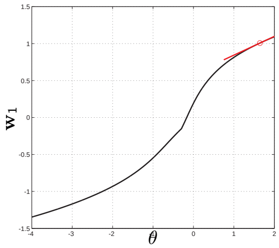

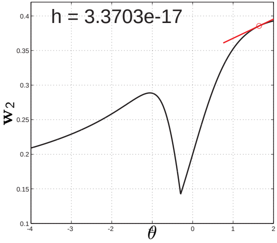

At the "corner", derivative does not exist. But **directional derivatives** do !! I.e.

$$\lim_{\mathbf{p}\to\mathbf{p}_-} \frac{\partial \mathbf{z}}{\partial \mathbf{p}} \quad \text{and} \quad \lim_{\mathbf{p}\to\mathbf{p}_+} \frac{\partial \mathbf{z}}{\partial \mathbf{p}} \quad \text{exist}$$

The non-smooth "linear" approximator - QP approximation

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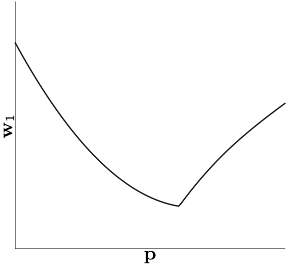
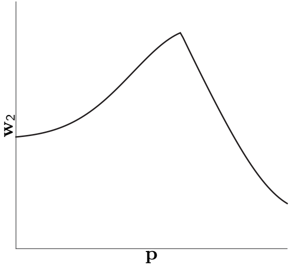
Sensitivities

$$\frac{\partial}{\partial \mathbf{p}} \begin{bmatrix} \mathbf{w} \\ \boldsymbol{\lambda} \\ \boldsymbol{\mu} \end{bmatrix} = - \begin{bmatrix} H & \nabla_{\mathbf{w}} \mathbf{g} & \nabla_{\mathbf{w}} \mathbf{h} \\ \nabla_{\mathbf{w}} \mathbf{g}^\top & 0 & 0 \\ \nabla_{\mathbf{w}} \mathbf{h}^\top & 0 & 0 \end{bmatrix}^{-1} \frac{\partial}{\partial \mathbf{p}} \begin{bmatrix} \nabla_{\mathbf{w}} \mathcal{L} \\ \mathbf{g} \\ \mathbf{h} \end{bmatrix}$$





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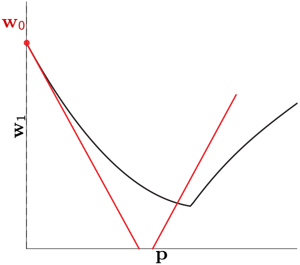
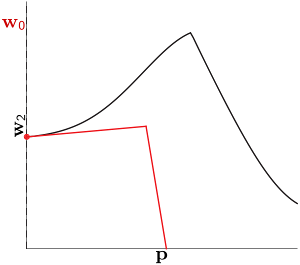
Approximating QP

$$\begin{aligned} \min_{\Delta\mathbf{w}} \quad & \frac{1}{2}\Delta\mathbf{w}^\top H \Delta\mathbf{w} + \Delta\mathbf{p}^\top \nabla_{\mathbf{pw}}\mathcal{L}\Delta\mathbf{w} \\ & \mathbf{g} + \nabla_{\mathbf{w}}\mathbf{g}^\top \Delta\mathbf{w} + \nabla_{\mathbf{p}}\mathbf{g}^\top \Delta\mathbf{p} = 0 \\ & \mathbf{h} + \nabla_{\mathbf{w}}\mathbf{h}^\top \Delta\mathbf{w} + \nabla_{\mathbf{p}}\mathbf{h}^\top \Delta\mathbf{p} \leq 0 \end{aligned}$$

where

$$\begin{aligned} \Delta\mathbf{w} &= \mathbf{w} - \mathbf{w}_0 \\ \Delta\mathbf{p} &= \mathbf{p} - \mathbf{p}_0 \end{aligned}$$





Parametric NLP:

$$\min_{\mathbf{w}} \quad \Phi(\mathbf{w},\mathbf{p})$$
$$\mathbf{g}(\mathbf{w},\mathbf{p}) = 0$$
$$\mathbf{h}(\mathbf{w},\mathbf{p}) \leq 0$$

Approximating QP

$$\min_{\Delta\mathbf{w}} \quad \frac{1}{2}\Delta\mathbf{w}^\top H \Delta\mathbf{w} + \Delta\mathbf{p}^\top \nabla_{\mathbf{pw}}\mathcal{L}\Delta\mathbf{w}$$
$$\mathbf{g} + \nabla_{\mathbf{w}}\mathbf{g}^\top \Delta\mathbf{w} + \nabla_{\mathbf{p}}\mathbf{g}^\top \Delta\mathbf{p} = 0$$
$$\mathbf{h} + \nabla_{\mathbf{w}}\mathbf{h}^\top \Delta\mathbf{w} + \nabla_{\mathbf{p}}\mathbf{h}^\top \Delta\mathbf{p} \leq 0$$

where

$$\Delta\mathbf{w} = \mathbf{w} - \mathbf{w}_0$$
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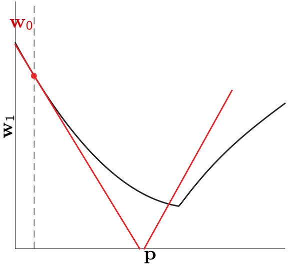
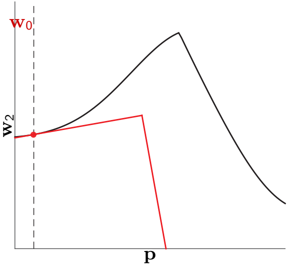




The non-smooth "linear" approximator - QP approximation

Parametric NLP:

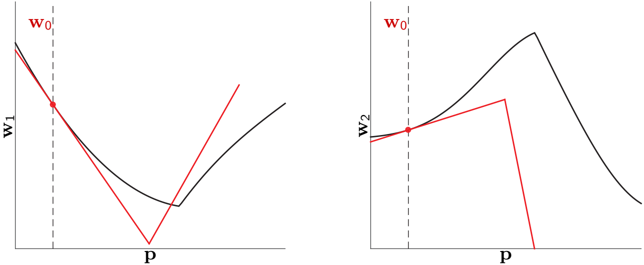
$$\min_{\mathbf{w}} \quad \Phi(\mathbf{w}, \mathbf{p})$$
$$\mathbf{g}(\mathbf{w}, \mathbf{p}) = 0$$
$$\mathbf{h}(\mathbf{w}, \mathbf{p}) \leq 0$$

Approximating QP

$$\min_{\Delta \mathbf{w}} \quad \frac{1}{2}\Delta \mathbf{w}^\top H \Delta \mathbf{w} + \Delta \mathbf{p}^\top \nabla_{\mathbf{pw}} \mathcal{L} \Delta \mathbf{w}$$
$$\mathbf{g} + \nabla_{\mathbf{w}} \mathbf{g}^\top \Delta \mathbf{w} + \nabla_{\mathbf{p}} \mathbf{g}^\top \Delta \mathbf{p} = 0$$
$$\mathbf{h} + \nabla_{\mathbf{w}} \mathbf{h}^\top \Delta \mathbf{w} + \nabla_{\mathbf{p}} \mathbf{h}^\top \Delta \mathbf{p} \leq 0$$

where

$$\Delta \mathbf{w} = \mathbf{w} - \mathbf{w}_0$$
$$\Delta \mathbf{p} = \mathbf{p} - \mathbf{p}_0$$



The non-smooth "linear" approximator - QP approximation

Parametric NLP:

$$\min_{\mathbf{w}} \quad \Phi(\mathbf{w},\mathbf{p})$$
$$\mathbf{g}(\mathbf{w},\mathbf{p}) = 0$$
$$\mathbf{h}(\mathbf{w},\mathbf{p}) \leq 0$$

Approximating QP

$$\min_{\Delta\mathbf{w}} \quad \frac{1}{2}\Delta\mathbf{w}^\top H \Delta\mathbf{w} + \Delta\mathbf{p}^\top \nabla_{\mathbf{pw}}\mathcal{L}\Delta\mathbf{w}$$
$$\mathbf{g} + \nabla_{\mathbf{w}}\mathbf{g}^\top\Delta\mathbf{w} + \nabla_{\mathbf{p}}\mathbf{g}^\top\Delta\mathbf{p} = 0$$
$$\mathbf{h} + \nabla_{\mathbf{w}}\mathbf{h}^\top\Delta\mathbf{w} + \nabla_{\mathbf{p}}\mathbf{h}^\top\Delta\mathbf{p} \leq 0$$

where

$$\Delta\mathbf{w} = \mathbf{w} - \mathbf{w}_0$$
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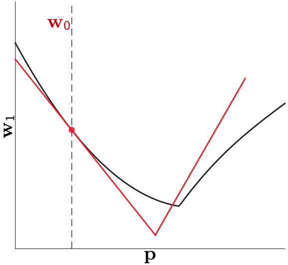

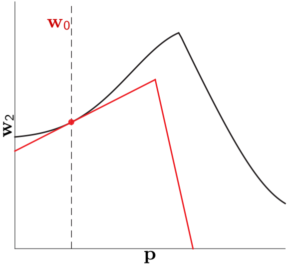

The non-smooth "linear" approximator - QP approximation

Parametric NLP:

$$\begin{aligned}\min_{\mathbf{w}} \quad & \Phi(\mathbf{w},\mathbf{p}) \\ & \mathbf{g}(\mathbf{w},\mathbf{p}) = 0 \\ & \mathbf{h}(\mathbf{w},\mathbf{p}) \leq 0\end{aligned}$$

Approximating QP

$$\begin{aligned}\min_{\Delta\mathbf{w}} \quad & \frac{1}{2}\Delta\mathbf{w}^\top H \Delta\mathbf{w} + \Delta\mathbf{p}^\top \nabla_{\mathbf{pw}}\mathcal{L}\Delta\mathbf{w} \\ & \mathbf{g} + \nabla_{\mathbf{w}}\mathbf{g}^\top\Delta\mathbf{w} + \nabla_{\mathbf{p}}\mathbf{g}^\top\Delta\mathbf{p} = 0 \\ & \mathbf{h} + \nabla_{\mathbf{w}}\mathbf{h}^\top\Delta\mathbf{w} + \nabla_{\mathbf{p}}\mathbf{h}^\top\Delta\mathbf{p} \leq 0\end{aligned}$$

where

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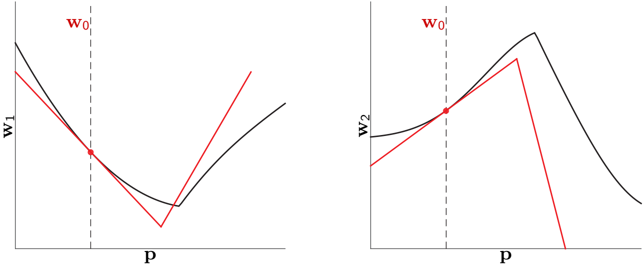

The non-smooth "linear" approximator - QP approximation

Parametric NLP:

$$\begin{aligned}\min_{\mathbf{w}} \quad & \Phi(\mathbf{w},\mathbf{p}) \\ & \mathbf{g}(\mathbf{w},\mathbf{p}) = 0 \\ & \mathbf{h}(\mathbf{w},\mathbf{p}) \leq 0\end{aligned}$$

Approximating QP

$$\begin{aligned}\min_{\Delta\mathbf{w}} \quad & \frac{1}{2}\Delta\mathbf{w}^\top H \Delta\mathbf{w} + \Delta\mathbf{p}^\top \nabla_{\mathbf{pw}}\mathcal{L}\Delta\mathbf{w} \\ & \mathbf{g} + \nabla_{\mathbf{w}}\mathbf{g}^\top\Delta\mathbf{w} + \nabla_{\mathbf{p}}\mathbf{g}^\top\Delta\mathbf{p} = 0 \\ & \mathbf{h} + \nabla_{\mathbf{w}}\mathbf{h}^\top\Delta\mathbf{w} + \nabla_{\mathbf{p}}\mathbf{h}^\top\Delta\mathbf{p} \leq 0\end{aligned}$$

where

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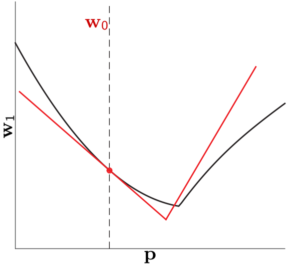

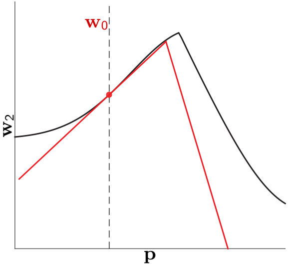

The non-smooth "linear" approximator - QP approximation

Parametric NLP:

$$\begin{aligned}\min_{\mathbf{w}} \quad & \Phi(\mathbf{w},\mathbf{p}) \\ & \mathbf{g}(\mathbf{w},\mathbf{p}) = 0 \\ & \mathbf{h}(\mathbf{w},\mathbf{p}) \leq 0\end{aligned}$$

Approximating QP

$$\begin{aligned}\min_{\Delta\mathbf{w}} \quad & \frac{1}{2}\Delta\mathbf{w}^\top H \Delta\mathbf{w} + \Delta\mathbf{p}^\top \nabla_{\mathbf{pw}}\mathcal{L}\Delta\mathbf{w} \\ & \mathbf{g} + \nabla_{\mathbf{w}}\mathbf{g}^\top \Delta\mathbf{w} + \nabla_{\mathbf{p}}\mathbf{g}^\top \Delta\mathbf{p} = 0 \\ & \mathbf{h} + \nabla_{\mathbf{w}}\mathbf{h}^\top \Delta\mathbf{w} + \nabla_{\mathbf{p}}\mathbf{h}^\top \Delta\mathbf{p} \leq 0\end{aligned}$$

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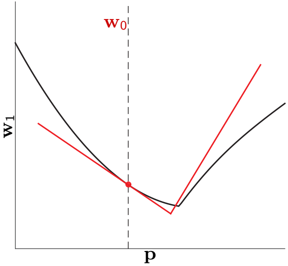

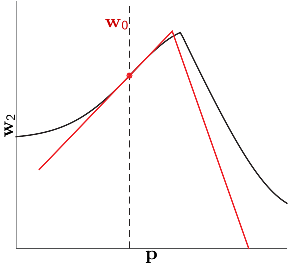

The non-smooth "linear" approximator - QP approximation

Parametric NLP:

$$\min_{\mathbf{w}} \quad \Phi\left(\mathbf{w},\mathbf{p}\right)$$
$$\mathbf{g}\left(\mathbf{w},\mathbf{p}\right) = 0$$
$$\mathbf{h}\left(\mathbf{w},\mathbf{p}\right) \leq 0$$

Approximating QP

$$\min_{\Delta\mathbf{w}} \quad \frac{1}{2}\Delta\mathbf{w}^\top H \Delta\mathbf{w} + \Delta\mathbf{p}^\top \nabla_{\mathbf{pw}}\mathcal{L}\Delta\mathbf{w}$$
$$\mathbf{g} + \nabla_{\mathbf{w}}\mathbf{g}^\top\Delta\mathbf{w} + \nabla_{\mathbf{p}}\mathbf{g}^\top\Delta\mathbf{p} = 0$$
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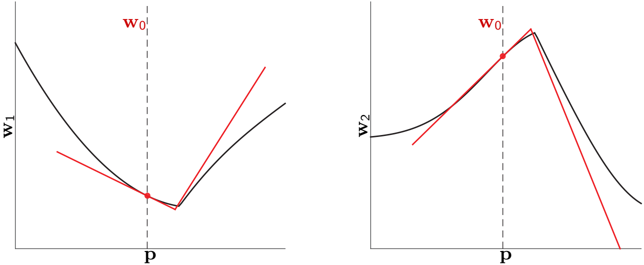

The non-smooth "linear" approximator - QP approximation

Parametric NLP:

$$\min_{\mathbf{w}} \quad \Phi(\mathbf{w},\mathbf{p})$$
$$\mathbf{g}(\mathbf{w},\mathbf{p}) = 0$$
$$\mathbf{h}(\mathbf{w},\mathbf{p}) \leq 0$$

Approximating QP

$$\min_{\Delta\mathbf{w}} \quad \frac{1}{2}\Delta\mathbf{w}^\top H \Delta\mathbf{w} + \Delta\mathbf{p}^\top \nabla_{\mathbf{pw}}\mathcal{L}\Delta\mathbf{w}$$
$$\mathbf{g} + \nabla_{\mathbf{w}}\mathbf{g}^\top \Delta\mathbf{w} + \nabla_{\mathbf{p}}\mathbf{g}^\top \Delta\mathbf{p} = 0$$
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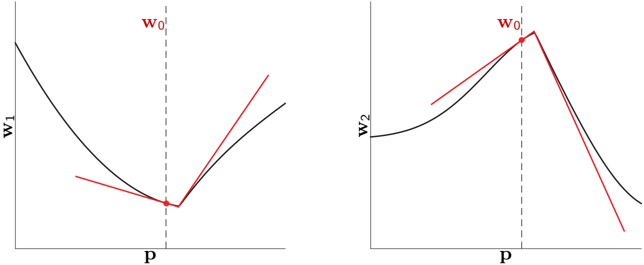

The non-smooth "linear" approximator - QP approximation

Parametric NLP:

$$\min_{\mathbf{w}} \quad \Phi(\mathbf{w},\mathbf{p})$$
$$\mathbf{g}(\mathbf{w},\mathbf{p}) = 0$$
$$\mathbf{h}(\mathbf{w},\mathbf{p}) \leq 0$$

Approximating QP

$$\min_{\Delta\mathbf{w}} \quad \frac{1}{2}\Delta\mathbf{w}^\top H \Delta\mathbf{w} + \Delta\mathbf{p}^\top \nabla_{\mathbf{pw}}\mathcal{L}\Delta\mathbf{w}$$
$$\mathbf{g} + \nabla_{\mathbf{w}}\mathbf{g}^\top \Delta\mathbf{w} + \nabla_{\mathbf{p}}\mathbf{g}^\top \Delta\mathbf{p} = 0$$
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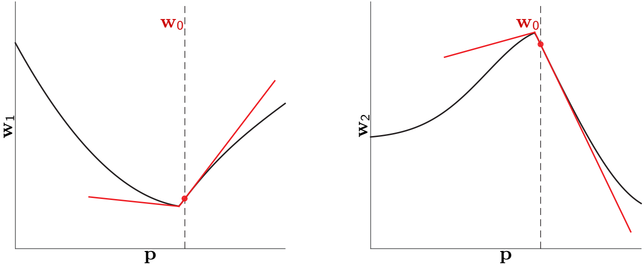

The non-smooth "linear" approximator - QP approximation

Parametric NLP:

$$\min_{\mathbf{w}} \quad \Phi(\mathbf{w}, \mathbf{p})$$
$$\mathbf{g}(\mathbf{w}, \mathbf{p}) = 0$$
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Approximating QP

$$\min_{\Delta \mathbf{w}} \quad \frac{1}{2}\Delta \mathbf{w}^\top H \Delta \mathbf{w} + \Delta \mathbf{p}^\top \nabla_{\mathbf{pw}}\mathcal{L}\Delta \mathbf{w}$$
$$\mathbf{g} + \nabla_{\mathbf{w}}\mathbf{g}^\top \Delta \mathbf{w} + \nabla_{\mathbf{p}}\mathbf{g}^\top \Delta \mathbf{p} = 0$$
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$$\Delta \mathbf{w} = \mathbf{w} - \mathbf{w}_0$$
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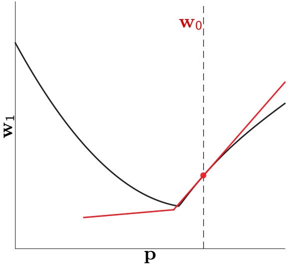

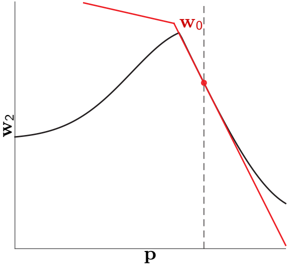

The non-smooth "linear" approximator - QP approximation

Parametric NLP:

$$\min_{\mathbf{w}} \quad \Phi(\mathbf{w},\mathbf{p})$$
$$\mathbf{g}(\mathbf{w},\mathbf{p}) = 0$$
$$\mathbf{h}(\mathbf{w},\mathbf{p}) \leq 0$$

Approximating QP

$$\min_{\Delta\mathbf{w}} \quad \frac{1}{2}\Delta\mathbf{w}^\top H \Delta\mathbf{w} + \Delta\mathbf{p}^\top \nabla_{\mathbf{pw}}\mathcal{L}\Delta\mathbf{w}$$
$$\mathbf{g} + \nabla_{\mathbf{w}}\mathbf{g}^\top\Delta\mathbf{w} + \nabla_{\mathbf{p}}\mathbf{g}^\top\Delta\mathbf{p} = 0$$
$$\mathbf{h} + \nabla_{\mathbf{w}}\mathbf{h}^\top\Delta\mathbf{w} + \nabla_{\mathbf{p}}\mathbf{h}^\top\Delta\mathbf{p} \leq 0$$

where

$$\Delta\mathbf{w} = \mathbf{w} - \mathbf{w}_0$$
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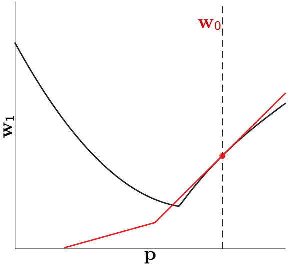
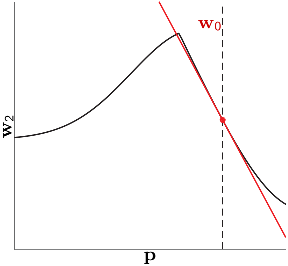




The non-smooth "linear" approximator - QP approximation

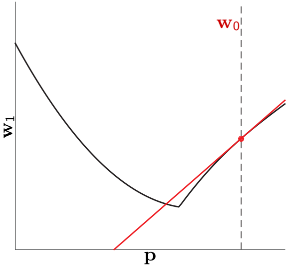
Parametric NLP:

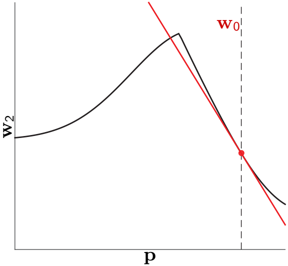
$$\begin{aligned} \min_{\mathbf{w}} \quad & \Phi(\mathbf{w}, \mathbf{p}) \\ & \mathbf{g}(\mathbf{w}, \mathbf{p}) = 0 \\ & \mathbf{h}(\mathbf{w}, \mathbf{p}) \leq 0 \end{aligned}$$

Approximating QP

$$\begin{aligned} \min_{\Delta\mathbf{w}} \quad & \frac{1}{2}\Delta\mathbf{w}^\top H \Delta\mathbf{w} + \Delta\mathbf{p}^\top \nabla_{\mathbf{pw}}\mathcal{L}\Delta\mathbf{w} \\ & \mathbf{g} + \nabla_{\mathbf{w}}\mathbf{g}^\top \Delta\mathbf{w} + \nabla_{\mathbf{p}}\mathbf{g}^\top \Delta\mathbf{p} = 0 \\ & \mathbf{h} + \nabla_{\mathbf{w}}\mathbf{h}^\top \Delta\mathbf{w} + \nabla_{\mathbf{p}}\mathbf{h}^\top \Delta\mathbf{p} \leq 0 \end{aligned}$$

where





$$\Delta\mathbf{w} = \mathbf{w} - \mathbf{w}_0$$
$$\Delta\mathbf{p} = \mathbf{p} - \mathbf{p}_0$$

The non-smooth "linear" approximator - QP approximation

Parametric NLP:

$$\begin{aligned} \min_{\mathbf{w}} \quad & \Phi(\mathbf{w}, \mathbf{p}) \\ & \mathbf{g}(\mathbf{w}, \mathbf{p}) = 0 \\ & \mathbf{h}(\mathbf{w}, \mathbf{p}) \leq 0 \end{aligned}$$

Approximating QP

$$\begin{aligned} \min_{\Delta\mathbf{w}} \quad & \frac{1}{2}\Delta\mathbf{w}^\top H \Delta\mathbf{w} + \Delta\mathbf{p}^\top \nabla_{\mathbf{pw}}\mathcal{L}\Delta\mathbf{w} \\ & \mathbf{g} + \nabla_{\mathbf{w}}\mathbf{g}^\top \Delta\mathbf{w} + \nabla_{\mathbf{p}}\mathbf{g}^\top \Delta\mathbf{p} = 0 \\ & \mathbf{h} + \nabla_{\mathbf{w}}\mathbf{h}^\top \Delta\mathbf{w} + \nabla_{\mathbf{p}}\mathbf{h}^\top \Delta\mathbf{p} \leq 0 \end{aligned}$$

where

$$\Delta\mathbf{w} = \mathbf{w} - \mathbf{w}_0$$

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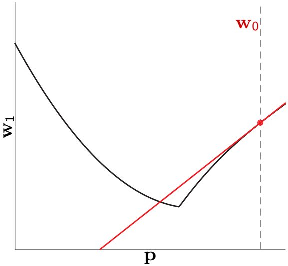

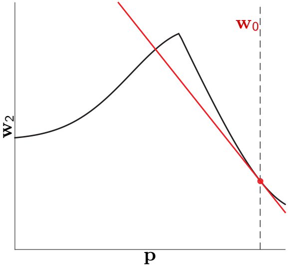

The non-smooth "linear" approximator - QP approximation

Parametric NLP:

$$\min_{\mathbf{w}} \quad \Phi(\mathbf{w}, \mathbf{p})$$
$$\mathbf{g}(\mathbf{w}, \mathbf{p}) = 0$$
$$\mathbf{h}(\mathbf{w}, \mathbf{p}) \leq 0$$

Approximating QP

$$\min_{\Delta\mathbf{w}} \quad \frac{1}{2}\Delta\mathbf{w}^\top H \Delta\mathbf{w} + \Delta\mathbf{p}^\top \nabla_{\mathbf{pw}}\mathcal{L}\Delta\mathbf{w}$$
$$\mathbf{g} + \nabla_{\mathbf{w}}\mathbf{g}^\top \Delta\mathbf{w} + \nabla_{\mathbf{p}}\mathbf{g}^\top \Delta\mathbf{p} = 0$$
$$\mathbf{h} + \nabla_{\mathbf{w}}\mathbf{h}^\top \Delta\mathbf{w} + \nabla_{\mathbf{p}}\mathbf{h}^\top \Delta\mathbf{p} \leq 0$$

where

$$\Delta\mathbf{w} = \mathbf{w} - \mathbf{w}_0$$
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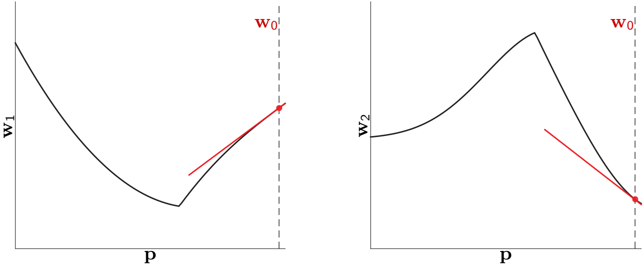
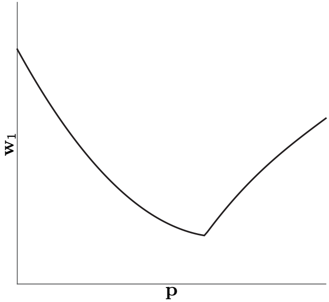


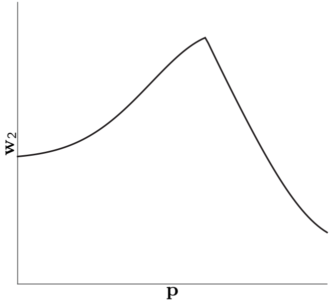
QP predictor holds the sensitivities implicitly (current active set), but also catches a linear approximation of the "kinks" resulting from changes of active set

Parametric NLP:

$$\begin{aligned} \min_{\mathbf{w}} \quad & \Phi(\mathbf{w},\mathbf{p}) \\ & \mathbf{g}(\mathbf{w},\mathbf{p}) = 0 \\ & \mathbf{h}(\mathbf{w},\mathbf{p}) \leq 0 \end{aligned}$$

- solved at $\mathbf{p}_0$ yields $\mathbf{w}_0$
- new parameter $\mathbf{p}$, perturbation $\Delta\mathbf{p} = \mathbf{p} - \mathbf{p}_0$ inserted in the QP
- predicted solution $\mathbf{w} = \mathbf{w}_0 + \Delta\mathbf{w}$ from the QP





The non-smooth "linear" predictor - Implementation

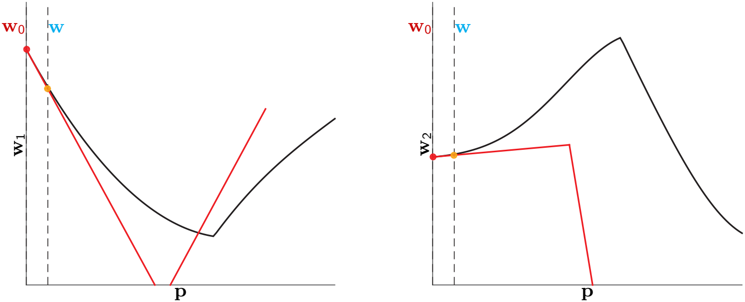
Parametric NLP:

$$\begin{aligned}\min_{\mathbf{w}} \quad & \Phi\left(\mathbf{w},\mathbf{p}\right) \\ & \mathbf{g}\left(\mathbf{w},\mathbf{p}\right) = 0 \\ & \mathbf{h}\left(\mathbf{w},\mathbf{p}\right) \leq 0\end{aligned}$$

Approximating QP

$$\begin{aligned}\min_{\Delta\mathbf{w}} \quad & \frac{1}{2}\Delta\mathbf{w}^\top H\Delta\mathbf{w} + \Delta\mathbf{p}^\top \nabla_{\mathbf{pw}}\mathcal{L}\Delta\mathbf{w} \\ & \mathbf{g} + \nabla_{\mathbf{w}}\mathbf{g}^\top\Delta\mathbf{w} + \nabla_{\mathbf{p}}\mathbf{g}^\top\Delta\mathbf{p} = 0 \\ & \mathbf{h} + \nabla_{\mathbf{w}}\mathbf{h}^\top\Delta\mathbf{w} + \nabla_{\mathbf{p}}\mathbf{h}^\top\Delta\mathbf{p} \leq 0\end{aligned}$$

- solved at $\mathbf{p}_0$ yields $\mathbf{w}_0$
- new parameter $\mathbf{p}$, perturbation $\Delta\mathbf{p} = \mathbf{p} - \mathbf{p}_0$ inserted in the QP
- predicted solution $\mathbf{w} = \mathbf{w}_0 + \Delta\mathbf{w}$ from the QP



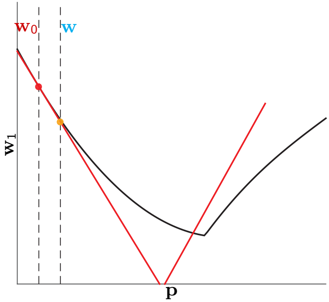
Parametric NLP:

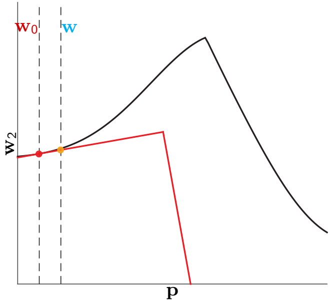
$$\begin{aligned} \min_{\mathbf{w}} \quad & \Phi\left(\mathbf{w},\mathbf{p}\right) \\ & \mathbf{g}\left(\mathbf{w},\mathbf{p}\right) = 0 \\ & \mathbf{h}\left(\mathbf{w},\mathbf{p}\right) \leq 0 \end{aligned}$$

Approximating QP

$$\begin{aligned} \min_{\Delta\mathbf{w}} \quad & \frac{1}{2}\Delta\mathbf{w}^\top H \Delta\mathbf{w} + \Delta\mathbf{p}^\top \nabla_{\mathbf{pw}}\mathcal{L}\Delta\mathbf{w} \\ & \mathbf{g} + \nabla_{\mathbf{w}}\mathbf{g}^\top\Delta\mathbf{w} + \nabla_{\mathbf{p}}\mathbf{g}^\top\Delta\mathbf{p} = 0 \\ & \mathbf{h} + \nabla_{\mathbf{w}}\mathbf{h}^\top\Delta\mathbf{w} + \nabla_{\mathbf{p}}\mathbf{h}^\top\Delta\mathbf{p} \leq 0 \end{aligned}$$

- solved at $\mathbf{p}_0$ yields $\mathbf{w}_0$
- new parameter $\mathbf{p}$, perturbation $\Delta\mathbf{p} = \mathbf{p} - \mathbf{p}_0$ inserted in the QP
- predicted solution $\mathbf{w} = \mathbf{w}_0 + \Delta\mathbf{w}$ from the QP





The non-smooth "linear" predictor - Implementation

Parametric NLP:

$$\begin{aligned} \min_{\mathbf{w}} \quad & \Phi(\mathbf{w},\mathbf{p}) \\ & \mathbf{g}(\mathbf{w},\mathbf{p}) = 0 \\ & \mathbf{h}(\mathbf{w},\mathbf{p}) \leq 0 \end{aligned}$$

Approximating QP

$$\begin{aligned} \min_{\Delta\mathbf{w}} \quad & \frac{1}{2}\Delta\mathbf{w}^\top H \Delta\mathbf{w} + \Delta\mathbf{p}^\top \nabla_{\mathbf{pw}}\mathcal{L}\Delta\mathbf{w} \\ & \mathbf{g} + \nabla_{\mathbf{w}}\mathbf{g}^\top\Delta\mathbf{w} + \nabla_{\mathbf{p}}\mathbf{g}^\top\Delta\mathbf{p} = 0 \\ & \mathbf{h} + \nabla_{\mathbf{w}}\mathbf{h}^\top\Delta\mathbf{w} + \nabla_{\mathbf{p}}\mathbf{h}^\top\Delta\mathbf{p} \leq 0 \end{aligned}$$

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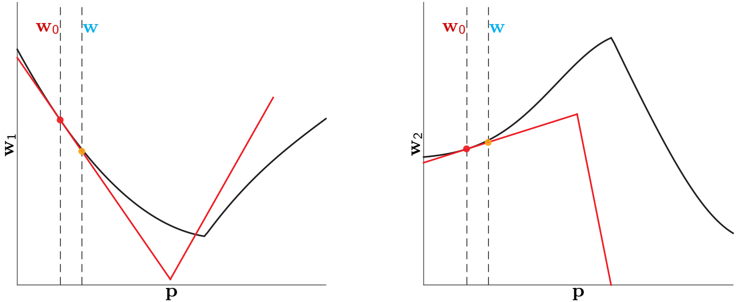

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Approximating QP

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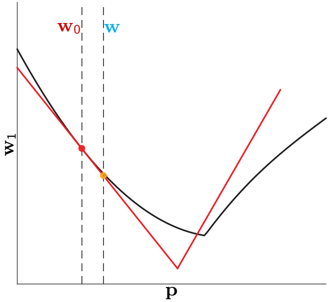

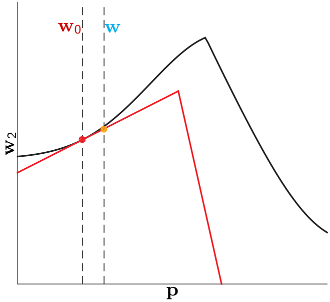

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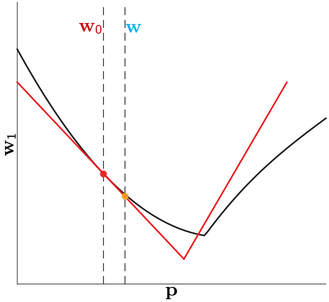

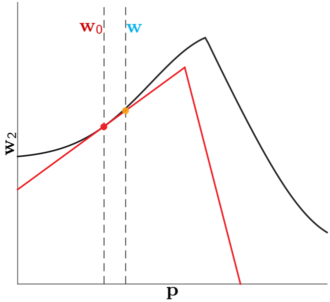

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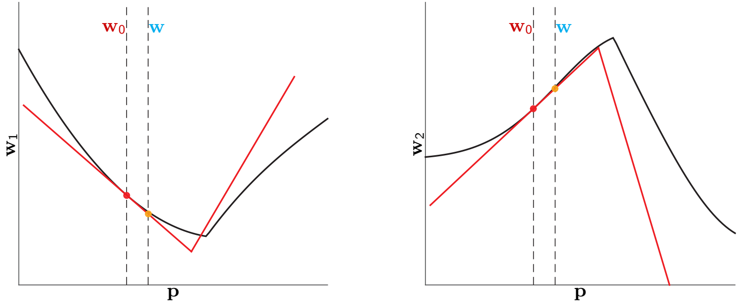

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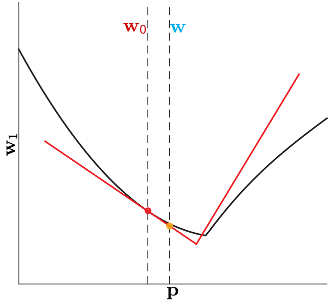

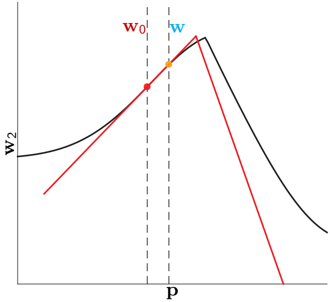

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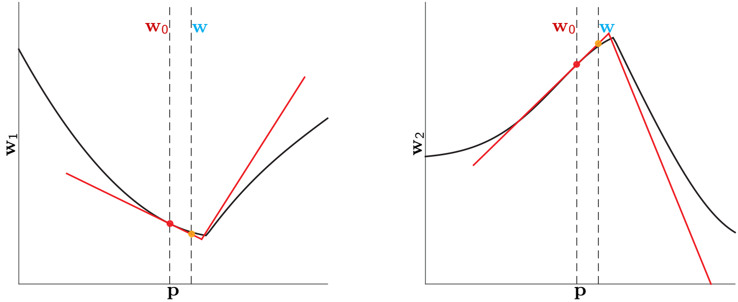

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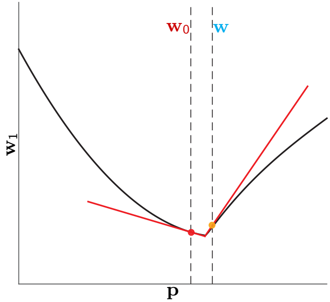

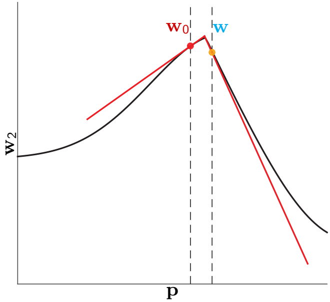

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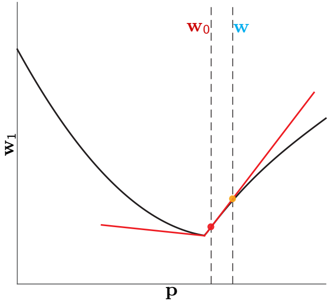

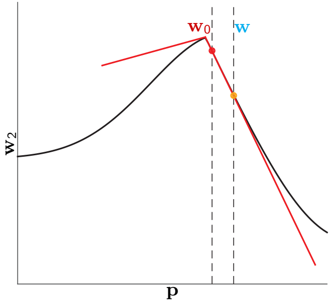

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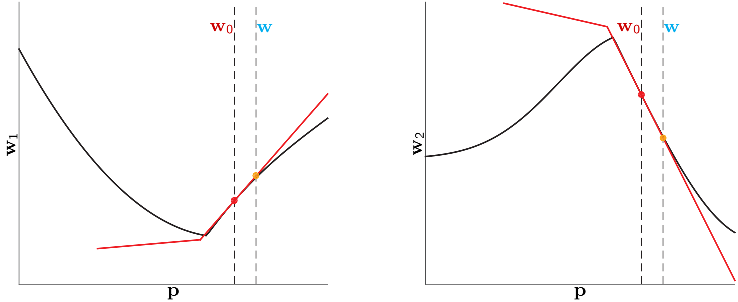

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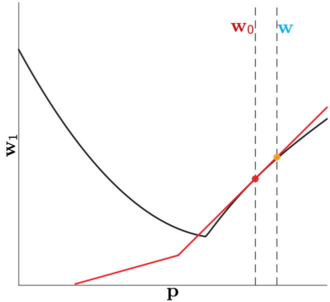

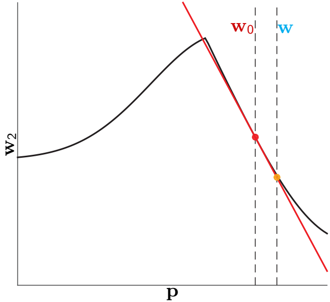

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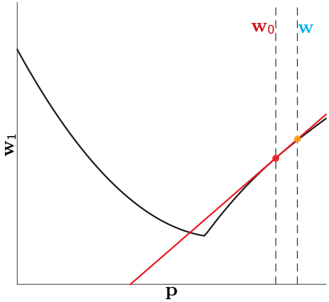

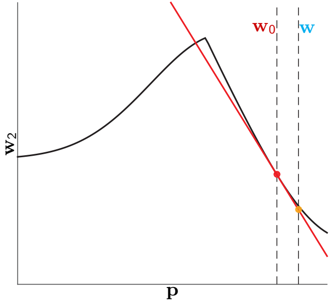

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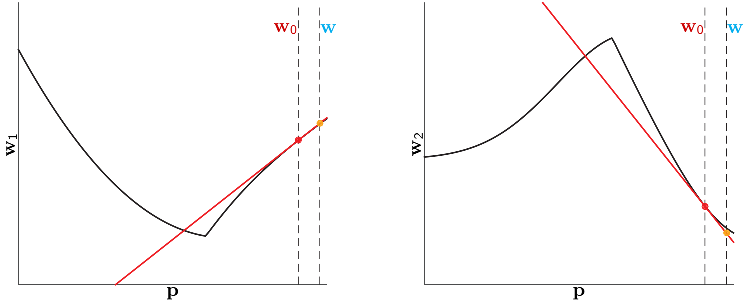

Outline

Off-line Path Following

Follow a solution path $\mathbf{w}(\mathbf{p})$, for a given "parameter trajectory" $\mathbf{p}$?

Algorithm: SQP

Input: Solution $\mathbf{w}$, $\boldsymbol{\lambda}$, $\boldsymbol{\mu}$ at $\mathbf{p}$, and $\mathbf{p}^+$
Use initial guess:

$$\mathbf{w},\ \boldsymbol{\lambda},\ \boldsymbol{\mu}$$

to solve NLP at $\mathbf{p}^+$, new solution:

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return $\mathbf{w}$, $\boldsymbol{\lambda}$, $\boldsymbol{\mu}$

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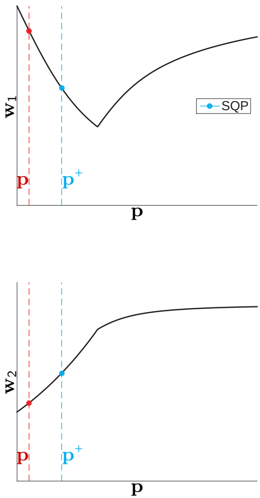
1^{st} Newton step of SQP



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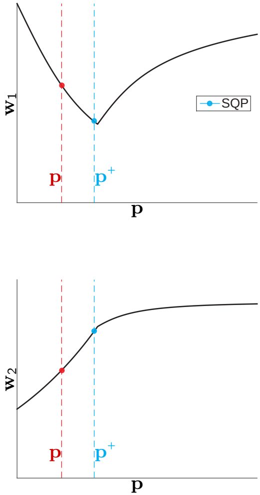

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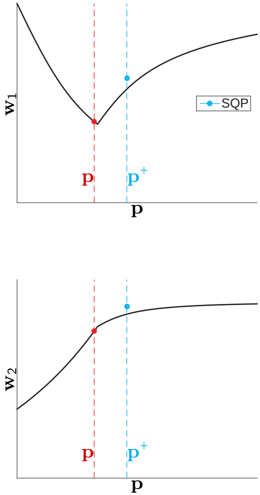

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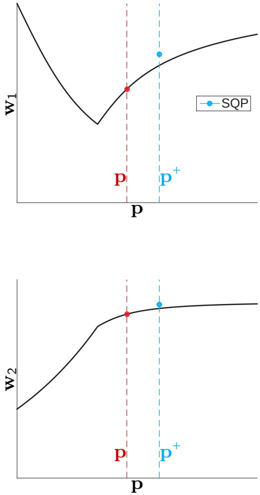

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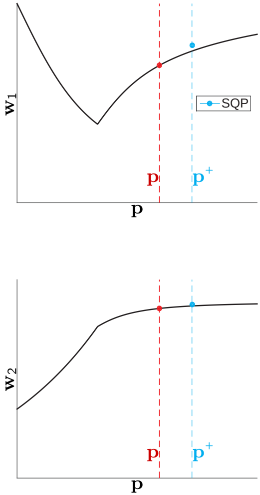

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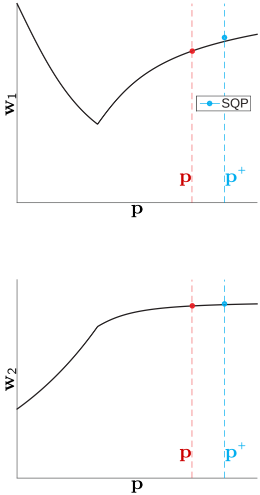

Off-line Path Following

Follow a solution path $\mathbf{w}(\mathbf{p})$, for a given "parameter trajectory" $\mathbf{p}$?

Algorithm: Path-following SQP

Input: Solution $\mathbf{w}$, $\boldsymbol{\lambda}$, $\boldsymbol{\mu}$ at $\mathbf{p}$, and $\mathbf{p}^+$
and $H, \mathbf{g}, \nabla\mathbf{g}, \ldots$

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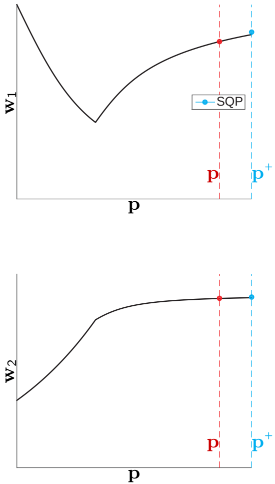

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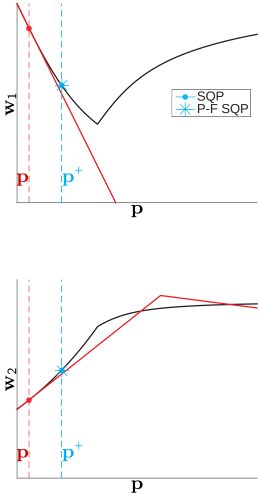
1st Newton step of path-following SQP



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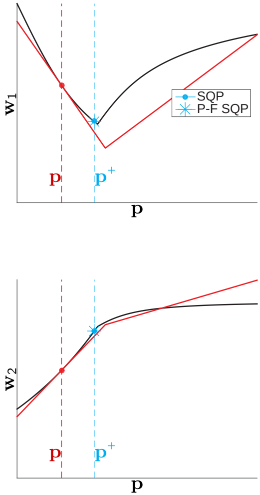

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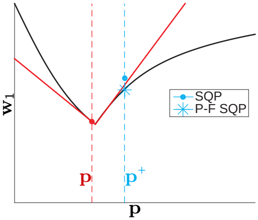

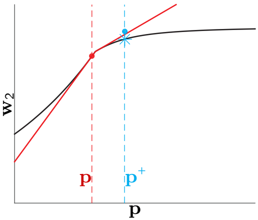

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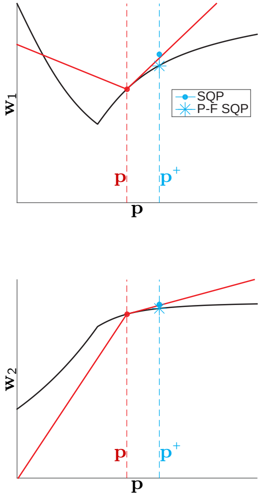

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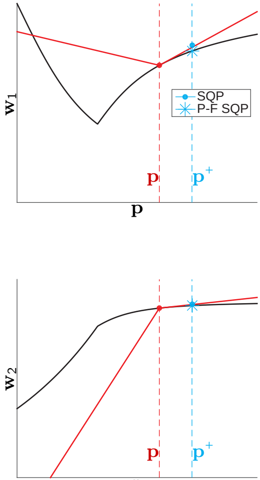

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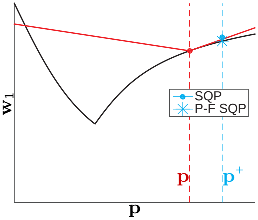

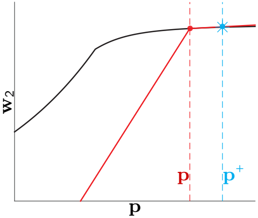

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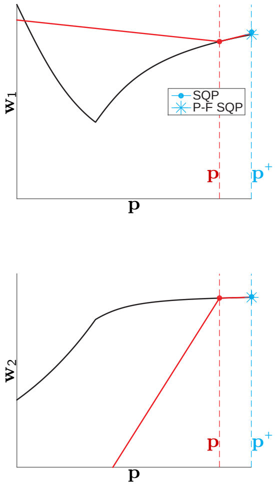

Off-line Path Following

Follow a solution path $\mathbf{w}(\mathbf{p})$, for a given "parameter trajectory" $\mathbf{p}$?

Predictor-Corrector

QP predictor

$$\min_{\Delta \mathbf{w}} \quad \frac{1}{2}\Delta \mathbf{w}^\top H \Delta \mathbf{w} + \left(\mathbf{p}^+ - \mathbf{p}\right)^\top \nabla_{\mathbf{pw}} \mathcal{L} \Delta \mathbf{w}$$
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Predicts solution for a parametric change

SQP (corrector)

$$\min_{\Delta \mathbf{w}} \quad \frac{1}{2}\Delta \mathbf{w}^\top H \Delta \mathbf{w} + \nabla \Phi^\top \Delta \mathbf{w}$$
$$\mathbf{g} + \nabla_{\mathbf{w}} \mathbf{g}^\top \Delta \mathbf{w} = 0$$
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Corrects solution until KKT satisfied

Predictor-Corrector:

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Implementation: solution $\mathbf{w}$ at $\mathbf{p}$ known, together with $H, \mathbf{g}, \nabla \mathbf{g}, \ldots$. For new parameter $\mathbf{p}^+$

1. Solve Predictor-Corrector, using $H, \mathbf{g}, \nabla \mathbf{g}, \ldots$ (already known). Predicts next solution + correct $\mathbf{w}$ if KKT not yet fulfilled !
2. Update $\mathbf{p} \leftarrow \mathbf{p}^+$, re-evaluate $H, \mathbf{g}, \nabla \mathbf{g}, \ldots$, back to **1**. Correct solution until KKT satisfied.

Predictor-Corrector and Parametric Embedding

Parametric NLP:

$$\mathbf{w}(\mathbf{p}) = \arg\min_{\mathbf{w}} \quad \Phi(\mathbf{w},\mathbf{p})$$
$$\mathbf{g}(\mathbf{w},\mathbf{p}) = 0$$
$$\mathbf{h}(\mathbf{w},\mathbf{p}) \leq 0$$

NLP with parametric embedding is equivalent to the "Path-following SQP" algorithm. This is a "cheap" way of implementing the algorithm.

Resulting QP: is a predictor-correct QP

$$\min_{\Delta\mathbf{w},\Delta\theta} \quad \frac{1}{2}\Delta\mathbf{w}^\top H\Delta\mathbf{w} + \nabla\Phi^\top\Delta\mathbf{w} + \Delta\theta^\top\nabla_{\theta\mathbf{w}}\mathcal{L}\Delta\mathbf{w}$$
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- If $\mathbf{p} \neq \mathbf{p}^+$, QP does predictor-corrector step, and sets $\mathbf{p} \leftarrow \mathbf{p}^+$ (full SQP step)
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Predictor-Corrector and Parametric Embedding

NLP with parametric embedding:

$$
\begin{aligned}
\mathbf{w}\left(\mathbf{p}^{+}\right) = \arg\min_{\mathbf{w},\mathbf{p}} \quad & \Phi\left(\mathbf{w},\mathbf{p}\right) \\
& \mathbf{g}\left(\mathbf{w},\mathbf{p}\right) = 0 \\
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\end{aligned}
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Real-time Path Following - The real-time dilemma

Suppose $\mathbf{p}(t)$ is **continuously changing with time** t and continuously measured.

Real-time Path Following - The real-time dilemma

Suppose $\mathbf{p}(t)$ is **continuously changing with time** t and continuously measured.

- **NMPC**: $\mathbf{p}\left(\mathbf{t}\right)$ is a state estimation, solution $\mathbf{w}$ contains the control
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Algorithm: Real-time path-following

Input: Solution $\mathbf{w}$, $\boldsymbol{\lambda}$, $\boldsymbol{\mu}$ and $H, \mathbf{g}, \nabla\mathbf{g}, \ldots$ at $\mathbf{p}$,
new parameter $\mathbf{p}^+$ (measurement)

Take single full SQP step on NLP:

$$\mathbf{w}\left(\mathbf{p}^+\right) = \arg\min_{\mathbf{w},\mathbf{p}} \quad \Phi(\mathbf{w},\mathbf{p})$$

$$\mathbf{g}(\mathbf{w},\mathbf{p}) = 0, \qquad \mathbf{p} - \mathbf{p}^+ = 0$$

$$\mathbf{h}(\mathbf{w},\mathbf{p}) \leq 0$$

using $H, \mathbf{g}, \nabla\mathbf{g}, \ldots$ at $\mathbf{p}$

Update $H, \mathbf{g}, \nabla\mathbf{g}, \ldots$

return $\mathbf{w}$, $\boldsymbol{\lambda}$, $\boldsymbol{\mu}$ and $H, \mathbf{g}, \nabla\mathbf{g}, \ldots$

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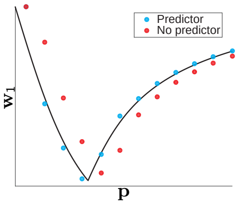

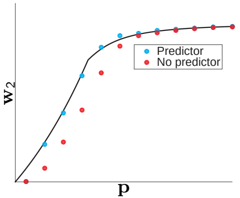

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Application - Real-Time Iteration (RTI) for NMPC

NMPC at physical time i is an **NLP** with

$$\mathbf{w} = \left\{\mathbf{x}_0, \mathbf{u}_0, \ldots, \mathbf{x}_{N-1}, \mathbf{u}_{N-1},\ \mathbf{x}_N\right\}$$

$$\mathbf{w}\left(\hat{\mathbf{x}}_i\right) = \arg\min_{\mathbf{w}} \quad \Phi\left(\mathbf{w}\right)$$

$$\text{s.t.} \quad \mathbf{g}\left(\mathbf{w}, \hat{\mathbf{x}}_i\right) = \left[\begin{array}{c} \hat{\mathbf{x}}_i - \mathbf{x}_0 \\ \mathbf{f}\left(\mathbf{x}_0, \mathbf{u}_0\right) - \mathbf{x}_1 \\ \ldots \\ \mathbf{f}\left(\mathbf{x}_{N-1}, \mathbf{u}_{N-1}\right) - \mathbf{x}_N \end{array}\right] = 0$$

$$\mathbf{h}\left(\mathbf{w}\right) = \left[\begin{array}{c} \mathbf{h}\left(\mathbf{x}_0, \mathbf{u}_0\right) \\ \ldots \\ \mathbf{h}\left(\mathbf{x}_{N-1}, \mathbf{u}_{N-1}\right) \\ \mathbf{h}\left(\mathbf{x}_N\right) \end{array}\right] \leq 0$$

Application - Real-Time Iteration (RTI) for NMPC

NMPC at physical time i is an **NLP** with

$$\mathbf{w} = \{\mathbf{x}_0, \mathbf{u}_0, ..., \mathbf{x}_{N-1}, \mathbf{u}_{N-1}, \ \mathbf{x}_N\}$$

$$\mathbf{w}(\hat{\mathbf{x}}_i) = \arg\min_{\mathbf{w}} \quad \Phi(\mathbf{w})$$

$$\text{s.t.} \quad \mathbf{g}(\mathbf{w}, \hat{\mathbf{x}}_i) = \begin{bmatrix} \hat{\mathbf{x}}_i - \mathbf{x}_0 \\ \mathbf{f}(\mathbf{x}_0, \mathbf{u}_0) - \mathbf{x}_1 \\ \ldots \\ \mathbf{f}(\mathbf{x}_{N-1}, \mathbf{u}_{N-1}) - \mathbf{x}_N \end{bmatrix} = 0$$

$$\mathbf{h}(\mathbf{w}) = \begin{bmatrix} \mathbf{h}(\mathbf{x}_0, \mathbf{u}_0) \\ \ldots \\ \mathbf{h}(\mathbf{x}_{N-1}, \mathbf{u}_{N-1}) \\ \mathbf{h}(\mathbf{x}_N) \end{bmatrix} \leq 0$$

Initial conditions $\hat{\mathbf{x}}_i$ at time i are already **embedded**

Application - Real-Time Iteration (RTI) for NMPC

NMPC at physical time i is an **NLP** with
$$\mathbf{w} = \left\{\mathbf{x}_0, \mathbf{u}_0, \dots, \mathbf{x}_{N-1}, \mathbf{u}_{N-1},\ \mathbf{x}_N\right\}$$

$$\mathbf{w}\left(\hat{\mathbf{x}}_i\right) = \arg\min_{\mathbf{w}} \quad \Phi\left(\mathbf{w}\right)$$

$$\text{s.t.} \quad \mathbf{g}\left(\mathbf{w}, \hat{\mathbf{x}}_i\right) = \begin{bmatrix} \hat{\mathbf{x}}_i - \mathbf{x}_0 \\ \mathbf{f}\left(\mathbf{x}_0, \mathbf{u}_0\right) - \mathbf{x}_1 \\ \dots \\ \mathbf{f}\left(\mathbf{x}_{N-1}, \mathbf{u}_{N-1}\right) - \mathbf{x}_N \end{bmatrix} = 0$$

$$\mathbf{h}\left(\mathbf{w}\right) = \begin{bmatrix} \mathbf{h}\left(\mathbf{x}_0, \mathbf{u}_0\right) \\ \dots \\ \mathbf{h}\left(\mathbf{x}_{N-1}, \mathbf{u}_{N-1}\right) \\ \mathbf{h}\left(\mathbf{x}_N\right) \end{bmatrix} \leq 0$$

Initial conditions $\hat{\mathbf{x}}_i$ at time i are already **embedded**

Perform between $\hat{\mathbf{x}}_{i-1}$ and $\hat{\mathbf{x}}_i$:

Algorithm: Preparation phase

Input: Solution $\mathbf{w}$, $\boldsymbol{\lambda}$, $\boldsymbol{\mu}$ and at $\mathbf{p}$

Compute $H, \mathbf{h}, \nabla\mathbf{h}, \nabla\mathbf{g}, \nabla\Phi$ and

$$\bar{\mathbf{g}} = \begin{bmatrix} \mathbf{f}\left(\mathbf{x}_0, \mathbf{u}_0\right) - \mathbf{x}_1 \\ \vdots \\ \mathbf{f}\left(\mathbf{x}_{N-1}, \mathbf{u}_{N-1}\right) - \mathbf{x}_N \end{bmatrix}$$

return $H, \mathbf{h}, \nabla\mathbf{h}, \nabla\mathbf{g}, \nabla\Phi$ and $\bar{\mathbf{g}}$

Application - Real-Time Iteration (RTI) for NMPC

NMPC at physical time i is an **NLP** with
$$\mathbf{w} = \{\mathbf{x}_0, \mathbf{u}_0, \dots, \mathbf{x}_{N-1}, \mathbf{u}_{N-1}, \ \mathbf{x}_N\}$$

$$\mathbf{w}(\hat{\mathbf{x}}_i) = \arg\min_{\mathbf{w}} \quad \Phi(\mathbf{w})$$

$$\text{s.t.} \quad \mathbf{g}(\mathbf{w}, \hat{\mathbf{x}}_i) = \begin{bmatrix} \hat{\mathbf{x}}_i - \mathbf{x}_0 \\ \mathbf{f}(\mathbf{x}_0, \mathbf{u}_0) - \mathbf{x}_1 \\ \dots \\ \mathbf{f}(\mathbf{x}_{N-1}, \mathbf{u}_{N-1}) - \mathbf{x}_N \end{bmatrix} = 0$$

$$\mathbf{h}(\mathbf{w}) = \begin{bmatrix} \mathbf{h}(\mathbf{x}_0, \mathbf{u}_0) \\ \dots \\ \mathbf{h}(\mathbf{x}_{N-1}, \mathbf{u}_{N-1}) \\ \mathbf{h}(\mathbf{x}_N) \end{bmatrix} \leq 0$$

Initial conditions $\hat{\mathbf{x}}_i$ at time i are already **embedded**

Perform between $\hat{\mathbf{x}}_{i-1}$ and $\hat{\mathbf{x}}_i$:

Algorithm: Preparation phase

Input: Solution $\mathbf{w}$, $\boldsymbol{\lambda}$, $\boldsymbol{\mu}$ and at $\mathbf{p}$
Compute $H, \mathbf{h}, \nabla\mathbf{h}, \nabla\mathbf{g}, \nabla\Phi$ and

$$\bar{\mathbf{g}} = \begin{bmatrix} \mathbf{f}(\mathbf{x}_0, \mathbf{u}_0) - \mathbf{x}_1 \\ \vdots \\ \mathbf{f}(\mathbf{x}_{N-1}, \mathbf{u}_{N-1}) - \mathbf{x}_N \end{bmatrix}$$

return $H, \mathbf{h}, \nabla\mathbf{h}, \nabla\mathbf{g}, \nabla\Phi$ and $\bar{\mathbf{g}}$

Perform upon receiving $\hat{\mathbf{x}}_i$

Algorithm: Feedback phase

Input: $\hat{\mathbf{x}}_i$, $\mathbf{w}$ and $H, \mathbf{h}, \nabla\mathbf{h}, \nabla\mathbf{g}, \nabla\Phi$ and $\bar{\mathbf{g}}$
Form
$$\mathbf{g}(\mathbf{w}, \hat{\mathbf{x}}_i) = \begin{bmatrix} \hat{\mathbf{x}}_i - \mathbf{x}_0 \\ \bar{\mathbf{g}} \end{bmatrix}$$

Solve QP gives $\Delta\mathbf{w}$, $\boldsymbol{\lambda}$, $\boldsymbol{\mu}$
Update $\mathbf{w} \leftarrow \mathbf{w} + \Delta\mathbf{w}$
return $\mathbf{w}$, $\boldsymbol{\lambda}$, $\boldsymbol{\mu}$

Application - Real-Time Iteration (RTI) for NMPC

NMPC at physical time i is an **NLP** with
$$\mathbf{w} = \{\mathbf{x}_0, \mathbf{u}_0, \ldots, \mathbf{x}_{N-1}, \mathbf{u}_{N-1},\ \mathbf{x}_N\}$$

$$\mathbf{w}(\hat{\mathbf{x}}_i) = \arg\min_{\mathbf{w}} \quad \Phi(\mathbf{w})$$

$$\text{s.t.} \quad \mathbf{g}(\mathbf{w}, \hat{\mathbf{x}}_i) = \begin{bmatrix} \hat{\mathbf{x}}_i - \mathbf{x}_0 \\ \mathbf{f}(\mathbf{x}_0, \mathbf{u}_0) - \mathbf{x}_1 \\ \ldots \\ \mathbf{f}(\mathbf{x}_{N-1}, \mathbf{u}_{N-1}) - \mathbf{x}_N \end{bmatrix} = 0$$

$$\mathbf{h}(\mathbf{w}) = \begin{bmatrix} \mathbf{h}(\mathbf{x}_0, \mathbf{u}_0) \\ \ldots \\ \mathbf{h}(\mathbf{x}_{N-1}, \mathbf{u}_{N-1}) \\ \mathbf{h}(\mathbf{x}_N) \end{bmatrix} \leq 0$$

Initial conditions $\hat{\mathbf{x}}_i$ at time i are already **embedded**

RTI reduces control delay by moving the linearization "out of the way", into Preparation phase. Feedback phase boils down to solving a QP.

Perform between $\hat{\mathbf{x}}_{i-1}$ and $\hat{\mathbf{x}}_i$:

Algorithm: Preparation phase

Input: Solution $\mathbf{w}$, $\boldsymbol{\lambda}$, $\boldsymbol{\mu}$ and at $\mathbf{p}$
Compute $H, \mathbf{h}, \nabla\mathbf{h}, \nabla\mathbf{g}, \nabla\Phi$ and

$$\bar{\mathbf{g}} = \begin{bmatrix} \mathbf{f}(\mathbf{x}_0, \mathbf{u}_0) - \mathbf{x}_1 \\ \vdots \\ \mathbf{f}(\mathbf{x}_{N-1}, \mathbf{u}_{N-1}) - \mathbf{x}_N \end{bmatrix}$$

return $H, \mathbf{h}, \nabla\mathbf{h}, \nabla\mathbf{g}, \nabla\Phi$ and $\bar{\mathbf{g}}$

Perform upon receiving $\hat{\mathbf{x}}_i$

Algorithm: Feedback phase

Input: $\hat{\mathbf{x}}_i$, $\mathbf{w}$ and $H, \mathbf{h}, \nabla\mathbf{h}, \nabla\mathbf{g}, \nabla\Phi$ and $\bar{\mathbf{g}}$
Form
$$\mathbf{g}(\mathbf{w}, \hat{\mathbf{x}}_i) = \begin{bmatrix} \hat{\mathbf{x}}_i - \mathbf{x}_0 \\ \bar{\mathbf{g}} \end{bmatrix}$$

Solve QP gives $\Delta\mathbf{w}$, $\boldsymbol{\lambda}$, $\boldsymbol{\mu}$
Update $\mathbf{w} \leftarrow \mathbf{w} + \Delta\mathbf{w}$
return $\mathbf{w}$, $\boldsymbol{\lambda}$, $\boldsymbol{\mu}$