Sensitivity-Based Economic NMPC with a Path-Following Approach in Python

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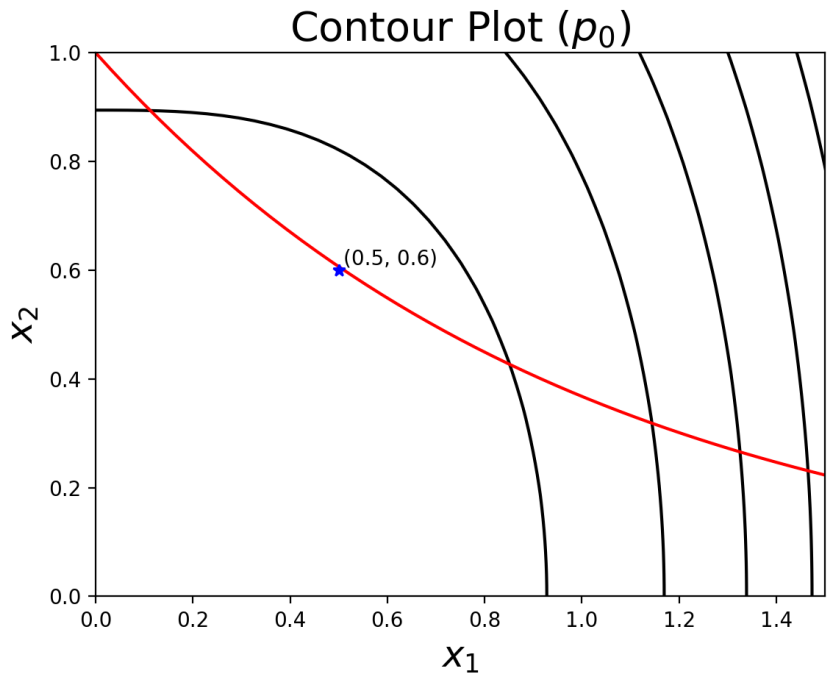
Simple Example Problem

$$\begin{aligned} \min_{x \in \mathbb{R}^2} \quad & p_1 x_1^3 + x_2^2 \\ \text{s.t.} \quad & x_2 - e^{-x_1} \geq 0, \\ & x_1 \geq p_2 \end{aligned}$$

Use approximate solution $x_0 = (0.5, 0.6)$ with $p_0 = (1, -4)$ to trace a path to find an approximate solution for $p_f = (8, 1)$.



CSTR + Distillation Column System

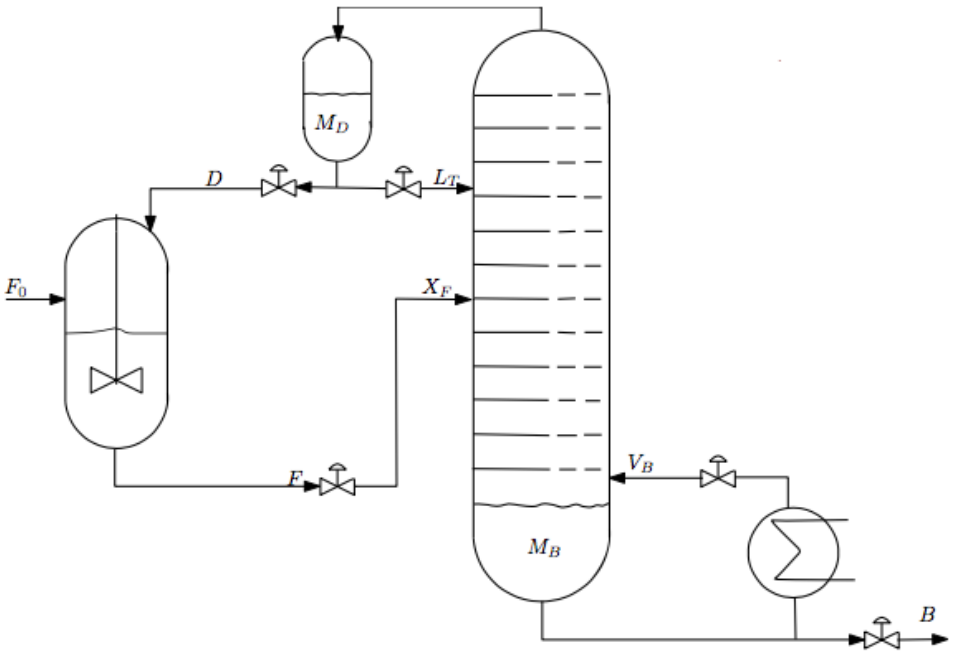


Figure: CSTR + Distillation column

Algorithm 1: General NMPC algorithm.

1. set $k \leftarrow 0$
2. **while** *MPC is running* **do**
3. 1. Measure or estimate x_k.
4. 2. Assign the initial state: set $\mathbf{z}_0 = x_k$.
5. 3. Solve the optimization problem $\mathcal{P}_{nmpc}$ to find $\mathbf{v}_0^*$.
6. 4. Assign the plant input $\mathbf{u}_k = \mathbf{v}_0^*$.
7. 5. Inject $\mathbf{u}_k$ to the plant (1).
8. 6. Set $k \leftarrow k+1$

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Path-following algorithm

Algorithm 2: Path-following algorithm.

Input: initial variables from NLP $\chi^*(\mathbf{p}_0), \lambda^*(\mathbf{p}_0), \mu^*(\mathbf{p}_0)$
fix stepsize Δt, and set $N = \frac{1}{\Delta t}$
set initial parameter value $\mathbf{p}_0$,
set final parameter value $\mathbf{p}_f$,
set $t = 0$,
set constant $0 < \alpha_1 < 1$.
Output: primal variable χ and dual variables λ, μ along the path

```
for k ← 1 to N do
    Compute step Δp = p_k − p_{k−1}
    Solve QP problem ;                          /* to obtain Δχ, Δλ, Δμ  */
    if QP is feasible then
        /* perform update                                                */
        χ ← χ + Δχ;                             /* update primal variables */
        Update dual variables appropriately; using Equations (8) and 9 for the pure-predictor
        method or (14) and (15) for the predictor-corrector method
        t ← t + Δt ;                            /* update stepsize */
        k ← k + 1
    else
        /* QP is infeasible, reduce QP stepsize                          */
        Δt ← α_1 Δt
        t ← t − α_1 Δt
```