The Real Effects of Capping Bank Leverage

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Abstract

In this paper, I study the effects of bank leverage ratio restrictions in a general equilibrium model of the macroeconomy where lenders can anticipate bank runs. This framework allows the analysis of the tradeoffs associated with bank capital requirements - while unlimited leverage allows capital to flow most freely to its most efficient users, limiting leverage through capital requirements reduces the probability of a bank run. This model enables me to study the general equilibrium effects of these tradeoffs on household welfare to understand characteristics of the optimal bank leverage ratio requirement. I find that the optimal leverage restriction will be time varying across the business cycle. When the household's marginal utility of consumption is highest, the leverage ratio requirement should be the least restrictive. Conversely, when the household's marginal utility approaches its steady state level, the optimal leverage ratio becomes more restrictive.

1 Introduction

After the financial crisis, bank regulation was put in place to increase stability in the financial sector. In 2010 and 2011 the Basel Committee on Banking Supervision (BCBS) agreed upon the reforms, including a leverage ratio requirement for financial institutions, to be introduced in 2013 in the Third Basel Accord (Basel III).[1] Basel III scheduled a phase in of these leverage requirements, which would reach 4.5% common equity to risk weighted assets in 2015. In July 2013 the U.S. Federal Reserve, Federal Deposit Insurance Corp. (FDIC), and the Office of the Comptroller of the Currency proposed a leverage ratio cap for 8 globally systemically important banks of 6% and 5% for their insured bank holding companies, with additional surcharges to be phased in. [2,3] However the question remains what will be the effects of these leverage caps on the real economy?

In this paper, I study whether leverage ratio restrictions can balance the trade-off between increased stability in the banking sector and allocational efficiency losses in a welfare improving way. The model takes a stance on interpreting systemic risk in the economy as an economy wide run on the banking sector. I add an exogenous leverage restriction to an infinite horizon general equilibrium model of the macroeconomy with a banking sector and I study the effects generated by these leverage caps. The banks in the economy need to be highly levered following a crisis in order to buy back their efficient level of capital holdings, however the probability of a bank run is increasing in bank leverage. Bank crises occur endogenously in the model in the form of an economy wide run on the banking sector. Immediately following a crisis, returns on bank assets are high. These high returns loosen the banks' collateral constraint and allow them to take on very high leverage. The probability of a bank run occurring in the model is an increasing function of bank leverage as long as bank assets in a bank run are not sufficient to cover its liabilities. Thus it is during these high leverage periods immediately after a crisis, where the economy is most fragile and susceptible to another crisis. This model is a multiple equilibria model and I follow Gertler and Kiyotaki (2015), in analyzing the equilibrium where the probability of a bank run depends on the amount of leverage in the economy. The banks in this model do not sufficiently internalize the effect of their choice of leverage on the probability of an economy wide bank run and therefore there is a role for leverage restrictions to improve welfare. It is natural to study the effects of leverage caps in a model where leverage is interpreted as the

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[1]Press Release, Basel Committee on Banking Supervision, Group of Governors and Heads of Supervision announces higher global minimum capital standards (Sept. 12, 2010), http://www.bis.org/press/p100912.pdf.

[2]Evan Weinberger, "Leverage Cap Leaves Big Banks with Unpalatable Choices," Law360, Jul. 9, 2013.

[3]Fed and FDIC agree 6% leverage ratio for US Sifis, Central Banking Newsdesk, Jul. 10, 2013.

source of systemic risk in the economy.

In this multiple equilibria model, a bank run equilibrium can arise in part due to a liquidity mismatch in banking caused by the use of short-term liabilities to fund partially illiquid long-term assets, similar to Diamond and Dybvig (1983). However, the inner workings of this model more closely resemble the technical features of the Cole and Kehoe (2000) model of a self-fulfilling sovereign debt crisis. This model studies a crisis like the Financial Crisis, which was a rollover crisis in the same vein as Cole and Kehoe's sovereign debt crisis. The banks in this model correspond to lightly regulated "shadow" banks or net borrowing banks in the unsecured interbank market. These banks were largely funded by short-term unsecured loans, which will correspond to deposits in this model. In the periods directly following an initial bank run, the price of capital is severely depressed and returns on capital are very high as the bankers, who are the efficient users of capital, recover their capital holdings. These high returns increase the bankers' incentive to operate honestly. The depositors understand that bankers have incentive to participate in the game and are therefore willing to increase their lending to banks. However, the probability of a bank run is very high when leverage is high.

The high probability of a bank run at the same time that depositors are eager to lend banks money seems counter intuitive, however this comes from the Cole and Kehoe rollover crisis aspect of the model. If all depositors roll over their deposits, the banks will have enough money to buy the capital, increasing the average efficiency of capital economy wide and driving up its price in the subsequent period. These returns loosen the banker's participation constraint and encourage households to lend to the bankers, since if bankers receive high returns for operating honestly, they will have little incentive to abscond with the deposits. However, if a sunspot occurs and all but one depositor fail to roll over their debt, not enough capital will be purchased by the efficient holders to raise the price to the high forecasted level that warranted the looser banker participation constraint. In this event, the bank franchise value will not be that promised under the scenario with full rollover of deposits and the bankers will prefer to runaway with any single households' deposits that may trickle in. Thus in the model, there is a probability each period that all households in the economy fail to roll over their deposits at the same time.

I add an exogenous leverage restriction to the banking sector and solve for the equilibrium recovery path following a bank run under a leverage cap regime. I then study the effects of the leverage cap on the recovery path the economy takes after a bank run, its implications for the real economy, and how relative levels of economic stability change from the laissez-faire regime with no exogenous leverage restrictions in place. I find that the optimal leverage restriction will be time varying across the business cycle. When the household's marginal

utility of consumption is highest, the leverage ratio requirement should be the least restrictive, conversely, when the household's marginal utility approaches its steady state level, the leverage ratio requirement can be more restrictive. Excessively restricting bank leverage in the periods following a bank run decreases household utility relative to the laissez-faire system since banks are not able to buy back capital from the inefficient households as fast. This decreases production which decreases asset prices and tightens the banks' collateral constraint further, causing bank net worth to remain depressed relative to the laissez-faire system and a persistent decrease in the banks' ability to buy back capital. However a leverage restriction that is time varying and more lenient in the periods where households have a higher marginal utility of consumption offers a welfare improvement relative to the laissez-faire system.

Section 2 begins by outlining the model. In Section 2.4, I add a leverage ratio restriction to the model. Section 3 studies the results of the leverage restriction on the economy's recovery path following a bank run and household welfare implications. In Section 4, I discuss implications for stability of the economy when the economy is simulated for 10,000 periods. Section 5 concludes.

2   The Model

This is an infinite horizon model of the macroeconomy with a banking sector where I enhance the model of Gertler and Kiyotaki (2015) to account for bank regulation in the form of leverage ratio restrictions. Each period, there are two possible states of the world: a bank run state and a no bank run state, and the bank runs are anticipated. There are two types of agents, households and bankers, each type of agent has a continuum of measure unity. The productive technology in the economy is $f(K_t) = ZK_t$. In the bank run state, all of the households run on the entire banking sector. I will focus on the case where if a bank run materializes, the banks do not have sufficient assets to cover their liabilities. This means that the households will receive a fraction of their original deposits and the price of capital during the bank run, Q^*, drops as banks sell their capital at fire sale prices to the inefficient households. These price changes for both deposits and capital affect the household's budget constraint.

This is a two good economy, there is capital, the durable good, and there is the consumption good which is a non-durable good. The paper abstracts from capital accumulation so here there is a fixed supply of capital each period and it does not depreciate:

$$K_t^b + K_t^h = 1$$

4

Both bankers and households have production functions (f^B and f^H respectively). Households require both capital and units of the consumption good inputs in order to produce more units of the consumption good. In other words, the households pay a cost in consumption goods for operating capital. I will suppose that this cost is a convex, increasing function their capital holdings:

$$f^H(K_t^h, f(K_t^h)) = ZK_t^h$$

Where I assume:

$$f(K_t^h) = \frac{\alpha}{2}(K_t^h)^2$$

K_t^h units of capital remain.

The bankers are the efficient users of capital, they only require capital good inputs in order to produce more units of the consumption good.

$$f^B(K_t^b) = ZK_t^b$$

K_t^b units of capital remain.

When households sell more capital to the banks, the amount of consumption goods in the economy increases since the banks are more efficient at producing capital. Therefore, in the absence of financial frictions, banks would intermediate all of the capital stock. However, when the banks are constrained in their ability to borrow funds to purchase the capital, the households will directly hold some of the capital. When the financial constraints tighten on the bank the households will be forced to hold an elevated supply of capital.

However lending to the bank is risky because there is a probability of an economy wide bank run each period. I study the economy in which the probability of a bank run depends on the amount of leverage that the banks have. The probability of a bank run p_t impacts the price of both capital and deposits. It also affects the banker's value function, which is calculated as the banker's return from operating honestly each period in the future given that there is no bank run. When a bank run occurs, banks are liquidated and due to borrowing constraints, once they have zero net worth, they will never be able to take deposits again.

2.1 Households

The households both consume and save. The households can save either by lending funds to the competitive financial institutions, the banks, or by holding the capital directly. Every period, households receive a return on their asset holdings as well as an endowment of the consumption good, ZW^h. This setup allows the household endowment to vary proportionally with the aggregate productivity Z.

Deposits held by the banks are one period bonds. In the no bank run state, these bonds yield a non-contingent rate of return R_t. However, in the bank run state, these assets receive only a fraction x_{t+1} of the promised return. Where x_{t+1} is the total liquidation value of bank asset per unit of promised deposit. So that, the household's return on deposits can be expressed as:

$$
R_t = \begin{cases} \bar{R}_t & \text{if no bank run,} \\ x_{t+1}\bar{R}_t & \text{if bank run occurs} \end{cases}
$$

where $0 \leq x_t < 1$. In the run state, all depositors receive the same pro rata share of liquidated assets. Unlike in Diamond and Dybvig, there is no sequential service constraint on depositor contract that links payoffs in the run state to depositors place in line.

Household utility U_t is given by:

$$
U_t = E_t \left(\sum_{i=0}^{\infty} \beta^i \ln C_{t+i}^h \right)
$$

Where C_t^h is household consumption, $0 < \beta < 1$, and Q_t is the market price of capital. The household chooses consumption, bank deposits D_t, and direct capital holdings K_t^h to maximize expected utility subject to the budget constraint:

$$
C_t^h + D_t + Q_t K_t^h + f(K_t^h) = Z_t W^h + R_t D_{t-1} + (Z_t + Q_t)K_{t-1}^h + (1 - \sigma)N_t
$$

Suppose that p_t is the probability that households assign to an economy wide bank run occurring at time $t + 1$. (A discussion of how p_t is determined will follow.) Since the households anticipate that a bank run will occur with positive probability, the rate of return promised on deposits, R_{t+1}, must satisfy the household's first order condition for deposits:

$$1 = R_{t+1} E_t \left[(1 - p_t)\Lambda_{t,t+1} + p_t \Lambda^*_{t,t+1} x_{t+1} \right]$$

where $\Lambda^*_{t,t+1} = \beta \frac{C^h_t}{C^{h*}_{t+1}}$ is the household's intertemporal marginal rate of substitution conditional on a bank run at $t+1$. The depositor recovery rate, x_{t+1} in the event of a run depends on the rate of return promised on deposits R_{t+1}.

$$x_{t+1} = \min \left[1, \frac{(Q^*_{t+1} + Z_{t+1})k^b_t}{R_{t+1}d_t} \right]$$

In the spirit of the global games approach developed by Morris and Shin (1998) and applied to banks by Goldstein and Panzer (2005), I postulate a reduced form that relates the probability of a bank run, p_t, to the aggregate recovery rate x_{t+1}. In this way, the probability p_t of the "sunspot" bank run outcome depends in a natural way on the fundamental x_{t+1}. In general, the probability that depositors assign to a bank run occurring in the following period is a decreasing function of the recovery rate:

$$p_t = \begin{cases} g(E_t(x_{t+1})) & \text{with} \quad g'(\cdot) < 0 \\ 0 & \text{if } E_t(x_{t+1}) = 1 \end{cases}$$

Where g follows the simple linear form:

$$g(\cdot) = 1 - E_t(x_{t+1})$$

Higher leverage chosen by banks today will decrease the recovery rate tomorrow, which increases the probability of a bank run occurring tomorrow. This increases R_{t+1}, the rate of return households require to hold assets from today until tomorrow. Therefore when the bank is choosing leverage to maximize its value function, the cost of deposits owed at $t+1$, R_{t+1}, will affect the bank's decision on how much leverage to take on. So banks internalize the impact that their choice of leverage has on p_t only indirectly through its affect on R_{t+1}.

2.2   Banks

Banks in this paper correspond to lightly regulated "shadow" banks or net borrowing banks in the unsecured interbank market. These banks hold long-term securities and issue short-

term debt, which makes them vulnerable to bank runs. Each banker manages a financial intermediary. Bankers fund their capital investments by issuing deposits to households as well as by investing their own net worth, n_t.

Bankers may be constrained in their ability to borrow deposits and they will attempt to save their way out of the financial constraints by accumulating their retained earnings. To limit this possibility that bankers will try to move towards one hundred percent equity financing, bankers have a finite expected lifetime and each banker has an i.i.d. probability σ of surviving until the next period and a probability $1 - \sigma$ of exiting at the end of the current period. The expected lifetime of a banker is then $\frac{1}{1-\sigma}$.

Each period, new bankers enter with an endowment w^b which is received only in their first period of life. The number of entering bankers is equal to the number who exit, keeping the total number of bankers constant. Bankers are risk neutral and they will rebate their entire net worth to the households in the period that they exit so that the expected utility of a continuing banker at the end of period t is given by:

$$V_t = E_t \left[\sum_{i=1}^{\infty} \beta^i (1 - \sigma) \sigma^{i-1} \Pi_{t+i} n_{t+i} \right]$$

where $(1 - \sigma)\sigma^{i-1}$ is the probability of a banker exiting at date $t + i$, n_{t+i} is the banker's terminal net worth upon exiting in period $t+i$, and Π_{t+i} is the households marginal utility of consumption in period $t+i$. The bankers take the households marginal utility of consumption a given.

Conditional on the productivity Z, the net worth of the "surviving" bankers is the gross return on assets net the cost of deposits. Banks can only increase net worth using their retained earnings. This friction is a reasonable approximation of banks in reality. In this paper however, I keep Z constant across time, an area for future analysis would be to explore the effects of shocking productivity Z.

$$n_{t+1} = (Z + Q_{t+1}) k_t^b - R_{t+1} d_t$$

Exiting bankers no longer operate their banks and they rebate their net worth to the households in the period that they exit. Each period t, new and surviving bankers finance their asset holdings $Q_t k_t^b$ with newly issued deposits and net worth:

$$Q_t k_t^b = n_t + d_t$$

There is a limit to the amount of deposits that bankers can borrow in a given period. This constraint can be motivated by assuming that a moral hazard problem exists. In time t, after accepting the deposits, but still during the same period, the banker chooses whether to operate "honestly" or to divert the assets for his personal use. Operating honestly requires the banker to invest the deposits, wait until the next period, realize the returns on deposits and meet all deposit obligations. If the banker chooses to divert the assets, he will only be able to liquidate up to the fraction θ of the assets and he will only be able to do so slowly, in order to remain undetected. Therefore the banker must decide whether to divert at time t, before the resolution of uncertainty at time $t + 1$. The cost of diverting assets is that the depositors are able to force the banker into bankruptcy in the next period. Therefore at time t, the bankers decide whether or not to divert the assets by comparing the franchise value of the financial intermediaries that they operate, V_t, to the potential gains from diverting funds $\theta_t \Pi_t Q_t k_t^b$. Where V_t is calculated as the present discounted value of the future payouts from operating the bank honestly every period. Any rational depositor will not lend deposits to a banker who has an incentive to divert funds. Therefore the following incentive constraint on the banker must hold.

$$\theta_t \Pi_t Q_t k_t^b \leq V_t$$

Given that bankers consume their net worth in the period that they exit, their franchise value can be restated recursively as the expected discounted value of the sum of their net worth conditional on exiting in the following period plus their franchise value conditional on continuing in the following period.

$$V_t = E_t \left[\beta(1 - \sigma)\Pi_{t+1} n_{t+1} + \beta\sigma V_{t+1} \right]$$

So that the banker's optimization problem is to choose (k_t^b, d_t) each period to maximize the franchise value subject to the incentive constraint and the balance sheet constraints. As long as the return on bank capital is greater than banks cost of deposits, banks will have incentive to take on the maximum amount of leverage available to them.

$$\phi_t = \frac{\psi_t}{\Pi_t \theta}$$

Since both the banker objective function and constraints are constant returns to scale, the optimization problem can be reduced to choosing the leverage multiple, ϕ_t to maximize the bank's "Tobin's q ratio," $\frac{V_t}{n_t} \equiv \psi_t$.

2.3   Aggregation

Given that the leverage multiple ϕ_t is independent of individual bank-specific factors and given a parameterization where the banker incentive constraint is binding in equilibrium, then the banks can be aggregated to yield the following relationship between total assets held by the banking system and total net worth

$$\theta_t \Pi_t Q_t K_t^b = V_t.$$

The evolution of N_t is given by the sum of surviving and entering bankers as

$$N_{t+1} = \sigma \left[(Z + Q_{t+1})K_t^b - R_{t+1}D_t \right] + W^b.$$

Where $W^b = (1 - \sigma)w^b$ is the total endowment across all entering bankers and the first term is the accumulated net worth of bankers that were operating at period t and survived until period $t + 1$. Conversely, exiting bankers rebate the fraction $(1 - \sigma)$ of accumulated net worth back to the households:

Total output Y_t is equal to the sum of output from capital Z, household endowment ZW^h, and W^b.

$$Y_t = Z + ZW^h + W^b$$

The output is either used to pay capital management costs or for household consumption:

$$Y_t = f(K_t^h) + C_t^h.$$

The household marginal utility of consumption can be defined

$$\Pi_t = \frac{1}{C_t^h}$$

2.4 Adding an Exogenous Leverage Constraint to the Model

I add an exogenous leverage restriction that is more restrictive than then endogenous leverage restriction which arises due to the agency problem that bankers face. I study the effects of this exogenous leverage restriction on the evolution of the economy in the model. Introducing the exogenous leverage restriction to the model adds an additional constraint to the banker's optimization problem. Now bankers must maximize their normalized value function subject to both their endogenous participation constraint as well as the exogenous leverage restriction.

$$\psi_t = \max_{\phi_t} \mathbb{E}_t \left\{ [\beta(1-\sigma)\Pi_t + \beta\sigma\psi_{t+1}] \frac{N_{t+1}}{N_t} \right\} \quad \text{s.t.}$$

$$\theta\Pi_t\phi_t \leq \psi_t$$
$$\phi_t \leq \bar{\phi}, \quad \text{for each } t$$

Taking expectations over the probability that there is no bank run each period and given that the return on bank capital holdings is greater than the cost of deposits,

$$\frac{Z + Q_{t+1}}{Q_t} - R_t \geq 0$$

bankers maximize their value function by choosing the maximum amount of leverage so that, at the optimum, their value function can the which can be written

$$\psi_t = \min\left\{1, \frac{(Z+Q_{t+1}^*)K_t^b}{(\min\{\frac{\psi_t}{\theta\Pi_t},\bar{\phi}\}-1)N_tR_t}\right\} \beta\left((1-\sigma)\Pi_t + \sigma\psi_{t+1}\right)\left[\min\{\frac{\psi_t}{\theta\Pi_t},\bar{\phi}\}\frac{(Z+Q_{t+1})}{Q_t} - (\min\{\frac{\psi_t}{\theta\Pi_t},\bar{\phi}\}-1)R_t\right]$$

Which means that optimal choice of leverage is no longer equal to $\phi_t = \frac{\psi}{\theta\Pi_t}$, it is now equal to the minimum of this value and the exogenous leverage restriction. Therefore, in

order to determine leverage, I must first model the banker's value function in order to know which constraint will bind. The bankers' value function is:

$$V_t = \mathbb{E}_t \left\{ \sum_{i=1}^{\infty} \beta^i (1-\sigma) \sigma^{i-1} \Pi_{t+i} N_{t+i} \right\}$$

Given the law of motion of n_t

$$n_{t+1} = n_t \left[\phi_t \frac{Z + Q_{t+1}}{Q_t} - (\phi_t - 1)R_t \right] \quad \text{and}$$

$$n_{t+i} = n_t \prod_{a=1}^{i} \left[\phi_{t+a-1} \frac{Z + Q_{t+a}}{Q_{t+a-1}} - (\phi_{t+a-1} - 1)R_{t+a-1}) \right]$$

which means that in the aggregate, the banker's normalized value function can be written as

$$\psi_t = \frac{V_t}{N_t}$$

$$\psi_t = \mathbb{E}_t \left\{ \beta(1-\sigma)\Pi_{t+1} \left[\phi_t \frac{(Z + Q_{t+1})}{Q_t} - (\phi_t - 1)R_t \right] \right.$$

$$\left. + \ldots + \beta^{\infty}(1-\sigma)\sigma^{\infty-1}\Pi_{t+\infty} \prod_{a=1}^{\infty} \left[\phi_{t+a-1} \frac{(Z + Q_{t+a})}{Q_{t+a-1}} - (\phi_{t+a-1} - 1)R_{t+a-1} \right] \right\}$$

$$\phi_t = \min\left\{ \frac{\psi_t}{\theta \Pi_t}, \bar{\phi} \right\}$$

I solve for the path that the normalized value function follows to recover from a bank run numerically. Once I have the path for the banker's value function, I can determine the path for capped leverage as a function of the normalized value function.

3 Recovery Following a Bank Run in Economy under Leverage Cap Regime

3.1 Results for Multiple Period Leverage Cap

I study the effects of an exogenous leverage requirement that restricts the maximum amount of leverage that bankers can choose for all periods where banker total assets are greater than

15 times net worth in the uncapped laissez-faire regime. The leverage restriction requires that the amount of deposits banks take on be the minimum of either 90% of the optimal leverage chosen in the laissez-faire regime, or the maximum amount of leverage allowed by their incentive constraint. For this calibration of the model, the leverage cap ceases to bind after period 67 or 16.75 years and the economy reaches the steady state in 120 periods or 30 years. I chose to cap periods with bank leverage above15 times net worth because, taking the reciprocal, this corresponds to net worth equaling 6.67% of total assets. This number is slightly more conservative than the U.S. baseline requirement, proposed in July 2013 by the Federal Reserve Board, FDIC, and Office of the Comptroller of the Currency, that the country's systemically important banks maintain equity capital worth 6% of total assets to be considered well capitalized.
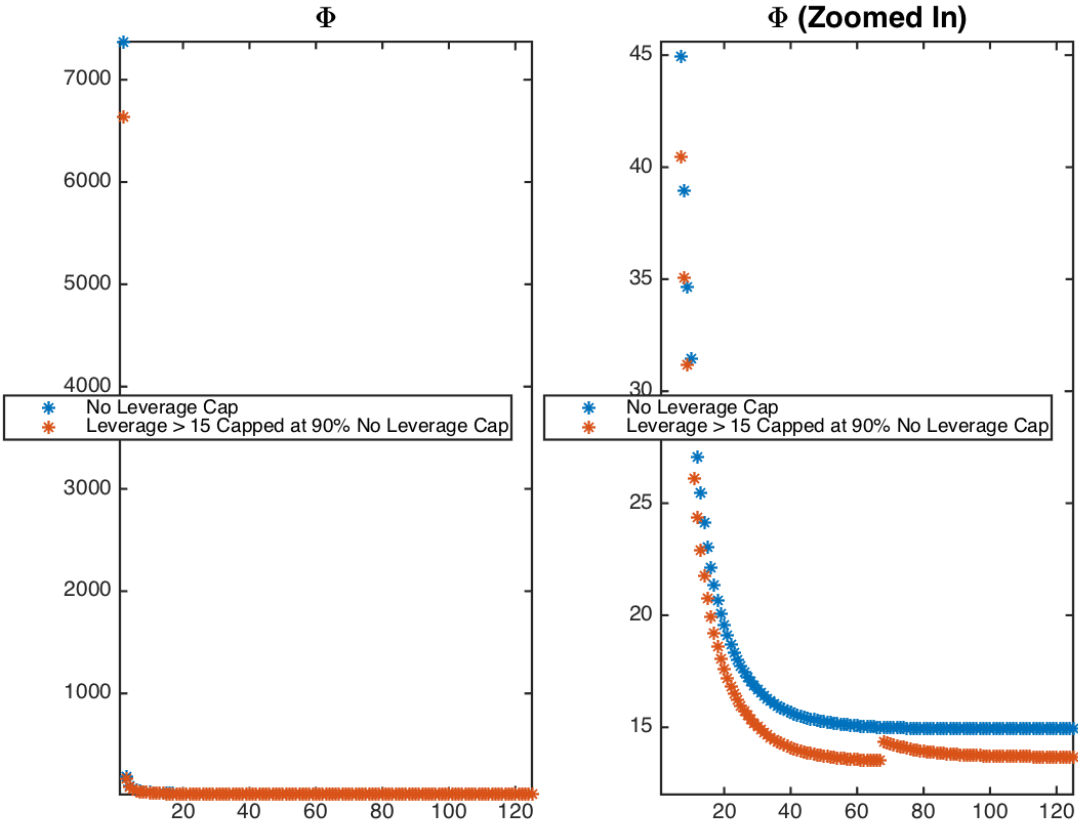

FIGURE 1: Leverage Φ_t with Multiple Period Leverage Cap

Figure 1 illustrates the recovery path that bank leverage in the economy, ϕ_t, follows both with and without the leverage restriction in place. The figure on the right hand side is a zoomed in version that omits the first periods directly following a bank run since these

periods have enormous leverage. For all plots in this section, the x-axis denotes t or the number of periods since the last bank run. The first period, $t = 1$ is the period in which the bank run occurs and the plots illustrate the recovery path that the variable follows from the bank run period to the steady state value ($t = 125$). In the plots, I compare the path that the variables follow in the unrestricted model versus the path that they follow in the model with the same parameter values but with the leverage restriction in place. Each period, there is a probability p_t that a bank run occurs however the plots reflect the variable's trajectory in the case that no subsequent bank run occurs before the economy reaches steady state.
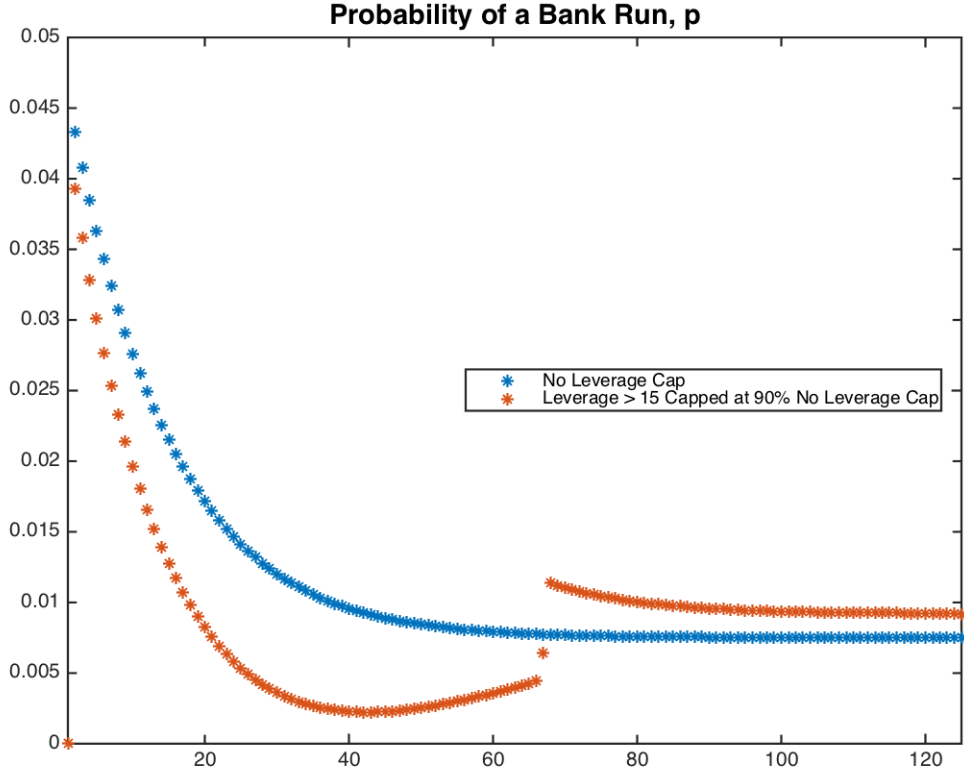

FIGURE 2: Probability of Bank Run p_t with Multiple Period Leverage Cap

Figure 2 plots the path that the probability of a bank run, p_t, follows from the time of a bank run in period one to steady state. The probability of a bank run p_t decreases in the capped model relative to the uncapped model for the first 67 periods. As seen in the formula for p_t the drop in p_t relative to the unrestricted system in the first 66 periods is driven by the decrease in leverage ϕ_t.

14

$$p_t = 1 - \min \left\{ \overbrace{\frac{(Z_{t+1} + Q_{t+1}^*)(1 - K_t^h)}{(\phi_t - 1)N_t R_{t+1}}}^{\text{Recovery Rate, } x_{t+1}}, 1 \right\}$$

There is a feedback loop at work via the recovery rate. The probability of a bank run is inversely related to the recovery rate x_{t+1}. The recovery rate depends not only on leverage but also on the price that capital takes on in the bank run period Q_{t+1}^*. During periods of extremely high leverage following the bank run, the changes in the leverage ratio dominate the effect on p_t. However for periods where the leverage ratio is close to its steady state value, changes in the price of capital in a bank run dominate changes in p_t. However, it is by taking on more leverage that banks can purchase more capital and drive up the price of capital. Therefore forcing banks to take on lower leverage initially after a bank run can inadvertently depress the price of capital to the point that the probability of a bank run increases in the steady state.

The first term in the minimum operator is the recovery rate or the total value of bank assets in the bank run state divided by the total cost of deposits that a bank would owe in the bank run state. The recovery value is driven up as the leverage cap forces banks to take less deposits than households are willing to give them. This decreases the probability of a bank run initially, bringing it to a minimum of of 0.0022% in period 41 or about 10 years after the bank run, if the economy reaches that period without falling into another bank run. However, the probability of a bank run increases after the leverage cap stops binding because the cap causes irreparably low bank capital holdings while the cap was in place, which drive down the price of capital in the bank run state, Q_t^*, as well as the bank's current capital holdings relative to their steady state values in the lassez-faire system. Once the leverage cap ceases to bind, banks are able to increase their net worth due to relatively higher returns on capital when prices are depressed. However the depressed price of capital in the bank run state drives down the steady state probability of a bank run, p_t, tightening banker incentive constraints so that higher values of net worth do not translate into a higher franchise value. Bankers cannot restore their capital holdings to steady state levels because depositors are not willing to lend them enough in the form of deposits. The denominator of the recovery value decreases as the steady state value of leverage decreases, however banks in steady state are slightly larger which offsets the decrease in leverage. The numerator of the recovery value falls by more due to the decrease in Q_t^* and bank capital holdings than the denominator does with a simultaneous fall in ϕ_t and rise in N_t.
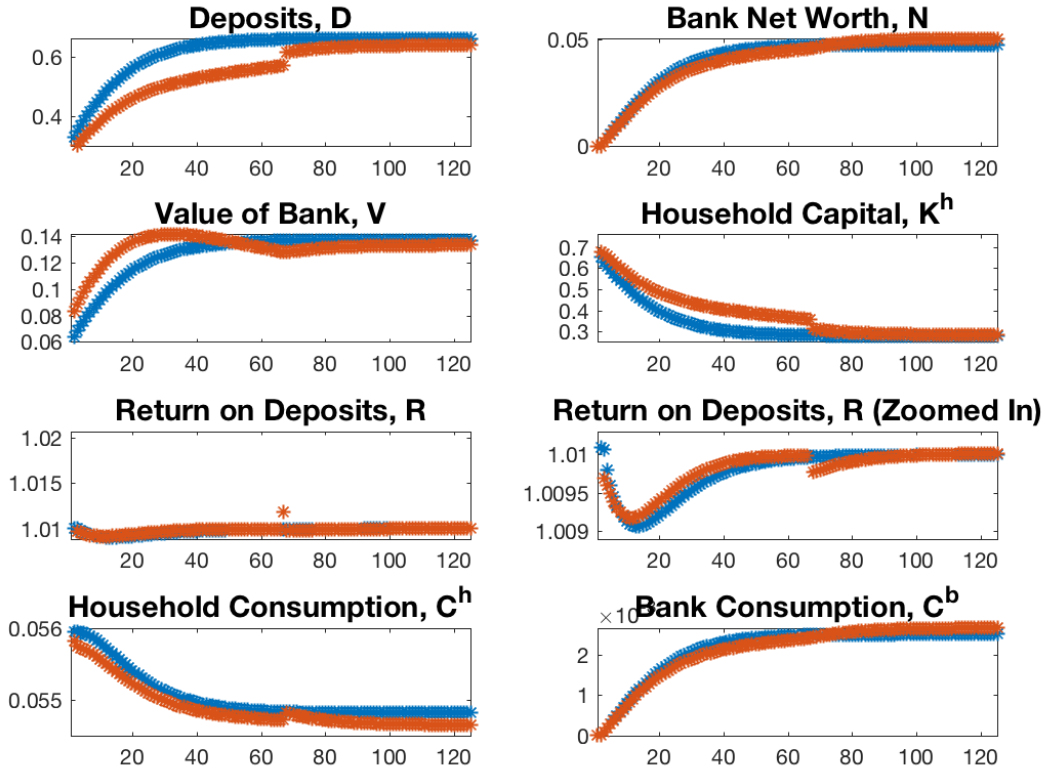
FIGURE 3: Recovery Path for Variables after a Bank Run with and without Multiple Period Leverage Cap (Blue stars indicate: No Leverage Cap while Red stars indicate: Leverage > 15 Capped at 90% No Leverage Cap)

Return on deposits stays relatively similar between the uncapped and capped systems. For the first 6 periods after the bank run, the return on deposits in the capped system is lower than in the uncapped system by maximum of 0.0013 or 0.129% in the period directly following the bank run and then by about 0.000037 or 0.004% for the next 5 periods. After that, it fluctuates between very slight increases and decreases that seem to offset each other, other than a jump in the period where the leverage cap stops binding. This jump is caused by high capital returns as well as a relative increase in the probability of a bank run as banks take on a discontinuous amount of leverage.

The banker net worth N_t is mechanically equal to zero in the period after the bank run and equal to the banker's endowment in the second period in both systems since all existing banks are liquidated in the period that the bank run occurs and banks in the period following the bank run enter with net worth equal only to their endowment. From periods 2 to 67, banks have a smaller net worth in the capped system by on average 0.0023 over this time

period. Once the leverage cap stops binding, banks in the capped system begin increasing their net worth relative to the uncapped system and have a net worth that is greater than the capped system by 0.0029 in the steady state.

When the leverage is capped, bankers are not allowed to take on as many deposits as the households are willing to give to them based on their participation constraint. Since bankers are financially constrained, the households must directly hold the capital themselves. This leads to households holding more capital in the capped model than they do in the uncapped model over the entire recovery path of the economy. From periods 1 to 67 households hold on average 0.0835 units, or 22% of the average in the Laissez-faire system, more capital in the capped system than they do in the uncapped system. From periods 68 to 120, they hold on average 0.0107 more units, or 3.8% of the average, of capital and in steady state, they hold 0.0042 units or 1.5% more capital.
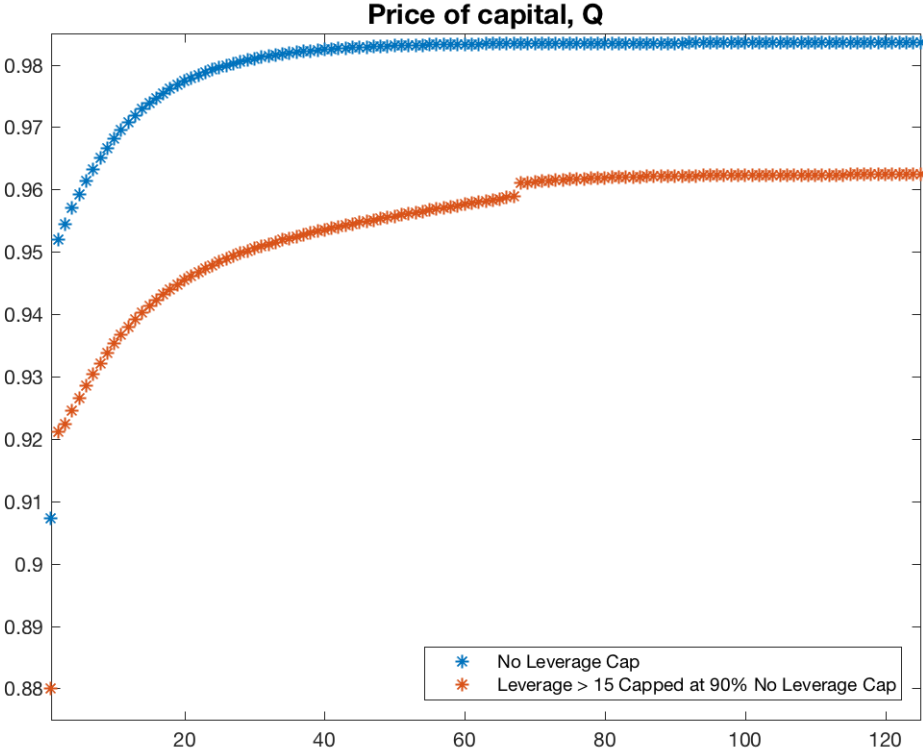

FIGURE 4: Price of Capital Q_t with and without Multiple Period Leverage Cap

The household's first order condition in part helps determine the price of capital, Q_t, when the household is holding any units of capital.

$$1 = E_t \left[(1 - p_t)\beta\Lambda \frac{Z_{t+1} + Q_{t+1}}{Q_t + f'(K_t^h)} + p_t\beta\Lambda^* \frac{Z_{t+1} + Q_{t+1}^*}{Q_t + f'(K_t^h)} \right]$$

Where $\Lambda^* = \frac{C_t^h}{C_{+1}^{h*}}$ is the household's intertemporal marginal rate of substitution conditional on a bank run occurring at time $t+1$ and $f'(K_t^h) = \alpha K_t^h$. The market price of capital tends to be decreasing in household capital, K_t^h holdings since the household's management cost for operating capital is increasing in household capital holdings.

As seen in Figure 4, relative to the system with no leverage cap, the price of capital Q_t is depressed along the economy's entire recovery path after a bank run and remains depressed in steady state. In the system with capped leverage, the price of capital during a bank run is depressed to 0.8800, a decrease of 3% from its laissez fair value of 0.9072. While the leverage cap is in place, from periods 2 through 67, the price of capital is depressed by 3% on average. After the cap is no longer binding, the price of capital remains depressed by 2% on average and stays depressed by 2% in the steady state.

This is because in the bank run period, all banks are liquidated so that their net worth drops to zero. Once a bank has zero net worth, the assumed financial friction that banks can only increase net worth through retained earnings implies that it will never have non-zero net worth at any time in the future. Therefore, in the period following the bank run, only the bankers that enter in that period will have non-zero net worth. These banks are lucky to be born at this time. They enter the economy at a time when the households hold all of the capital in the economy, since households have a convex and increasing management cost associated with operating the capital, this means that the households' management costs are at their maximum. In the laissez-faire regime, the entering bankers are therefore able to extract the total surplus from their advantage in operational efficiency in the form of the maximum returns on bank capital possible. These high returns increase the bankers' value functions, loosening their participation constraints because the households are willing to lend them a lot of money to take advantage of these high returns. These very highly levered periods following a bank run are crucial to allow bankers to purchase as much capital as possible.

Capping leverage in one period decreases the amount of capital that banks are able to purchase from the households. Therefore the households hold relatively more capital and have higher management costs than in the uncapped system. Since households demand similar returns to the uncapped system, the current price of capital decreases as $f'(K_t^h)$ in their first order condition rises. This mechanism causes returns to fall for the first two periods after a bank run relative to the uncapped model since the price at which entering
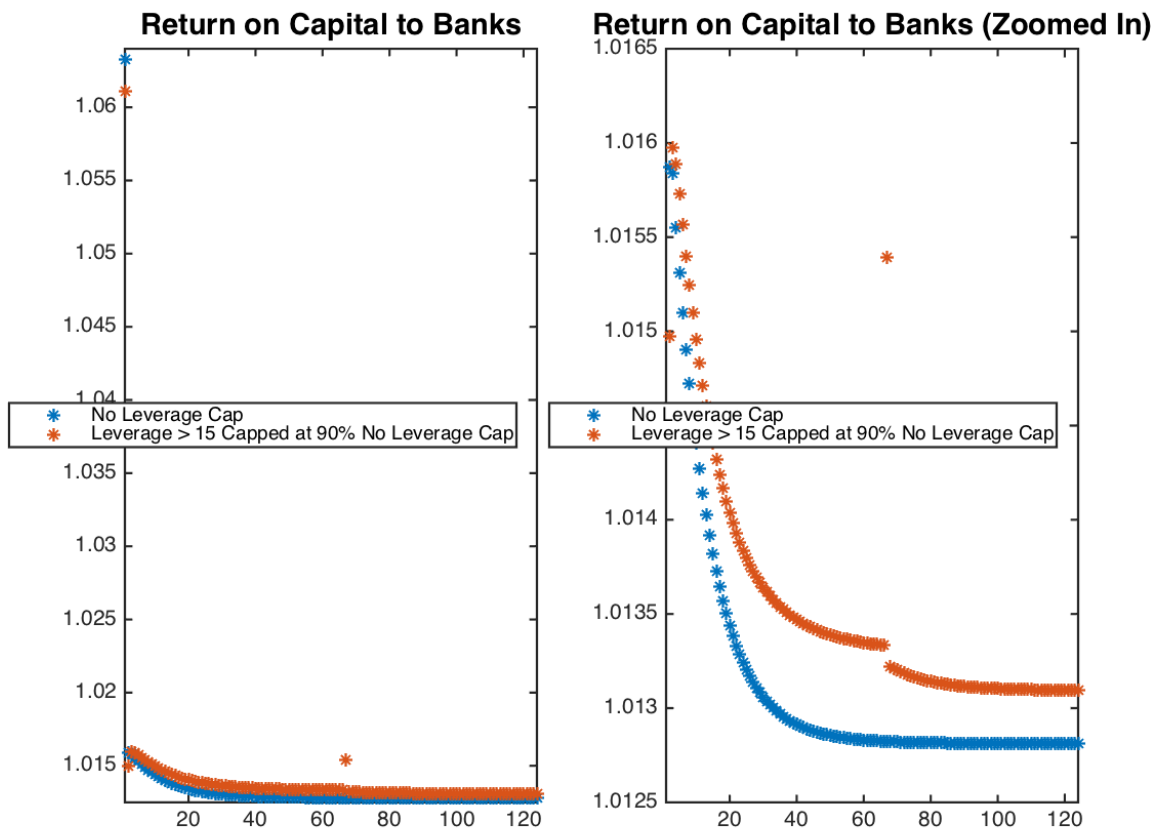
18

FIGURE 5: Return on Bank Assets after a Bank Run with and without Multiple Period Leverage Cap

bankers purchase the capital in the period following the bank run is the same in both models. However beginning in the fourth period, the period returns in the capped system begin to surpass those in the uncapped system. This is because of the convex management costs that households shoulder as they operate more capital. As the leverage cap regime bears on, each period the banks are able to purchase less capital from the households, leaving households to operate incrementally more capital each period than they would in the lassez-faire system. The difference in returns between the capped and uncapped system in Figure 5 reflect the convexity of the management cost. Since the households hold more capital in the capped model than they would in the uncapped model, prices are depressed and the bankers in the capped system are able to purchase the capital at a lower price and extract rents from their advantage in operating efficiency for longer than they would be able to in the uncapped model.

Once the leverage cap ceases to bind, the banks take on the maximum amount of leverage

that their participation constraint allows. As soon as the leverage cap ceases to bind, banks begin to take on the maximum leverage that the depositors are willing to give them. This causes bank returns to jump discontinuously as the banks buy capital for relatively cheap and drive the price of Q_t up discontinuously in this period. This coupled with returns, elevated from uncapped levels, drives up bank net worth. However the increase in net worth relative to the uncapped level does not translate into higher capital holdings by the banks because the higher net worth and depressed price of capital in a bank run state decrease the recovery value, increasing p_t. This increase in p_t decreases the banker value value function and tightened banker participation constraints relative to the uncapped model. Therefore, even though the banks are slightly bigger, they cannot take on enough leverage to buy as much capital from the inefficient households as in model with no leverage caps.

The banker's value function is the sum of all future consumption discounted by the banker's discount rate as well as the probability that the banker reaches a given period. Wrapped into this probability that a banker reaches a given period is the probability that there is no bank run in that given period. In steady state, the banker net worth under the leverage cap increases slightly (which increases banker consumption which is equal to the net worth of the fraction of bankers that exit the economy each period). However the probability of a bank run is increasing as the depressed Q_t^* and bank capital holdings begin to dominate the effect of decreased leverage and higher net worth in the recovery rate x_{t+1}. The increase in p_t increases the discount rate on future values of banker consumption, lowering the bankers value function, tightening the participation constraint and decreasing the amount of deposits that households will lend them. Therefore, even though banker net worth is increasing, the simultaneous decrease in leverage relative to the uncapped system makes the banks unable to purchase as much capital as they can in the uncapped system. This results in households operating elevated levels of capital which directly leads to decreased capital prices throughout the entire recovery path that the economy follows after a bank run. These results seem to imply the leverage cap introduces a wedge in the economy that allow steady state banks to be bigger and generate higher returns. However because banks in the capped system never acquire as much leverage as in the uncapped system, they cannot purchase the lassez-faire value of capital. The wedge therefore forces the inefficient households to operate elevated levels of capital and allows the efficient banks to extract higher operating rents from them each period.

3.2 Welfare Implications of Leverage Restrictions

In this section, I present different types of leverage restrictions and their resulting affects on household utility from the model solved numerically for illustrative purposes. Two factors

drive changes in the household's lifetime expected continuation utility. The first is increased consumption which is increasing in economic productivity so that all else equal, the inefficient households will consume more when the productive bankers operate more of the capital. The second factor driving household utility is the probability of a costly bank run, since if a bank run occurs, the household will be plunged into periods of low consumption.

In the first trio of plots, I present leverage restrictions in the second period (t=2) only. This is the first period after a bank run occurs, since every time a bank run occurs, the economy restarts along its recovery path in period 1 (t=1). The first plot in the series of 3 plots represents a lenient leverage restriction. Leverage is restricted in period 2 only and it is restricted to be at a maximum, 99.99% of the value of leverage that bankers in the unrestricted system, the laissez-faire system, would choose. In a crisis period, all banks are cleared out of deposits and have a net worth equal to zero so that they can never borrow again. Therefore, in period 2 the net worth in the banking sector is very small, since the only banks in the economy with non-zero net worth are the entering banks. Concurrently at this time, the price of capital is severely depressed at its fire sale value. This implies that the return on capital will be at its largest at this time. The small net worth in the banking sector coupled with the large returns on capital allow banks to take on extreme leverage in the periods following a crisis period. This high leverage is necessary because it allows the banks to buy back capital from the inefficient households faster and improve production in the economy. In the trio of figures, the first two plots illustrate a leverage restriction of 99.99% in the second period only. Under this lenient leverage restriction, the household's lifetime expected continuation utility is higher under the leverage cap regime than it is under the no leverage cap regime at every period.

If I make the leverage cap in period 2 even slightly more restrictive and do not restrict leverage in any future period, the household lifetime expected continuation utility falls below the value in the laissez-faire system at every period. This implies that high bank leverage following a crisis is necessary in order to eliminate the largest amount of deadweight losses which are incurred when households are operating all of the capital stock. Further, the initial increase in economic productivity between periods one and two is necessary to set the economy on a higher growth path. Restricting bank leverage too much following a financial crisis can keep capital prices depressed too low for too long and lead to persistently lower household utility.

Conversely, the second set of plots present the household's lifetime expected continuation utility under leverage restrictions in the steady state only, both relative to the household's utility in the world with no exogenous leverage restriction in place. If I restrict bank leverage in the steady state, the household utility is increasing as the leverage cap becomes stricter.
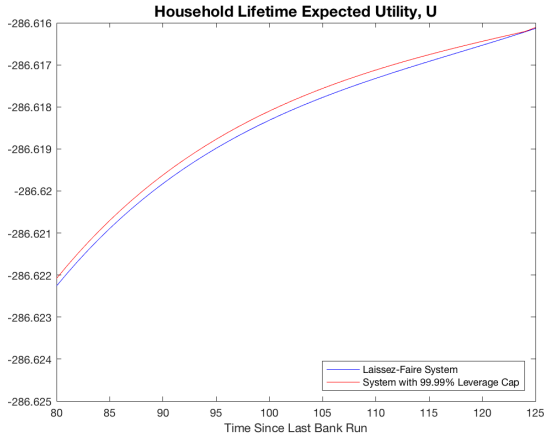
21

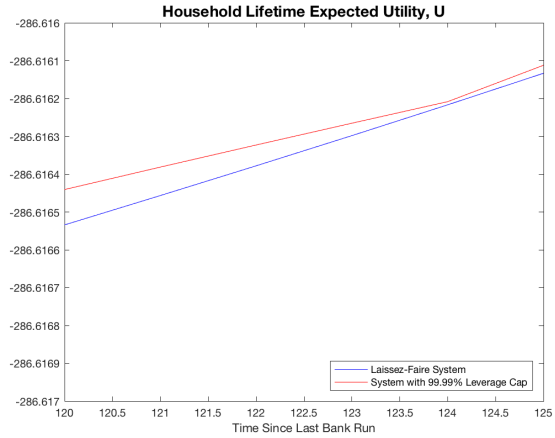FIGURE 6: Household Utility with 99.99% Leverage Cap in Period 2 Only

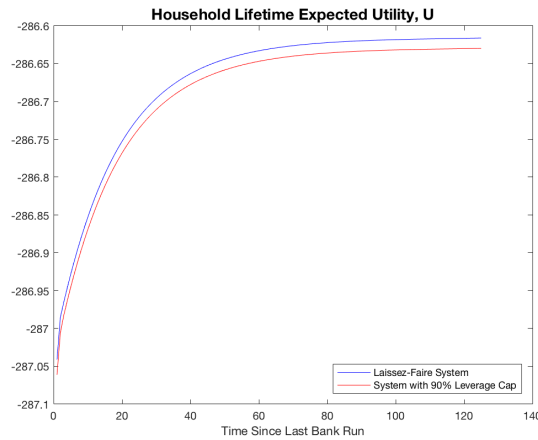FIGURE 7: Household Utility with 99.99% Leverage Cap in Period 2 Only (Zoomed In)

FIGURE 8: Household Utility with 90% Leverage Cap in Period 2 Only

The plot on the left hand side shows a leverage restriction in the steady state only of 98% of the steady state leverage value in the laissez-faire model, while the plot on the left shows a 95% leverage restriction in the steady state. This implies that the benefit of decreasing the probability of a costly bank run in the steady state more than compensates for constraining the productive bank's ability to buy capital. In sum, these results provide evidence in favor of more lenient bank leverage ratio restrictions in periods during an economic downturn when households's marginal utility of consumption is highest and stricter during periods where household's marginal utility of consumption is relatively lower. I am currently solving the Ramsey optimal social planner problem to find the optimal allocation and bank regulation analytically.
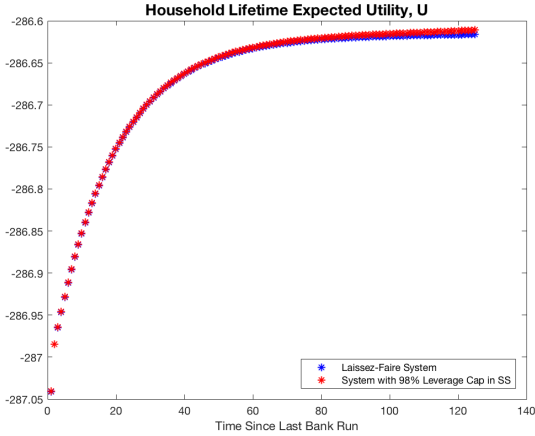
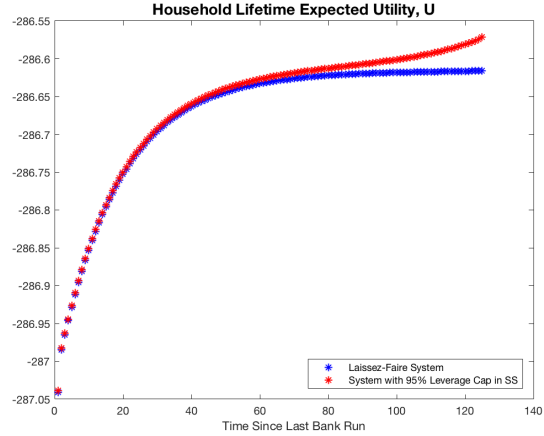FIGURE 9: Household Utility with 98% Leverage Cap in the Steady State Only

FIGURE 10: Household Utility with 95% Leverage Cap in the Steady State Only

4 Implications of Leverage Cap for Simulated Economy

I simulate the economy under both the leverage cap regime in section 3.1 and the uncapped regime for 10,000 periods. The economies both begin in the period following a bank run and are allowed to evolve according to their recovery paths solved for above. Each period, they are subject to a potential run on the banking sector which occurs with probability p_t. Regardless of what period the economy had reached before the bank run, if the economy falls into another bank run, it will need to start at the beginning of its recovery path and begin working sequentially toward its steady state again. Each period, I draw a random number distributed on the unit interval. The stochastic simulation begins at period 1, the period when the bank run occurs. Before the system may evolve to period 2, I first draw a random number. If the number drawn is less than p_2, then the economy is thrown back into a bank run. If not, the economy is allowed to progress to period 2 and I repeat the process, this time checking whether the random number drawn is less than p_3 before allowing the economy to advance to period 3, and so on. In the model, given the banks lose all of their net worth if a bank run occurs, their net worth in period one equals zero. Their participation constraint implies that banks in period one will not be able to take on any deposits given that their net worth is equal to zero, so the probability of a bank run occurring in period two is equal to zero. However, in period three and every future period, there is a positive probability of a bank run occurring. Intuitively, each period with probability $1 - p_t$, the economy evolves along the recovery path plotted in the figures above and with probability p_t a bank run occurs and throws the economy back into period one. p_t is decreasing as the economy moves further away from the bank run in period one. Therefore the economy is

23

the most fragile during the periods immediately following a bank run and may suffer several bank runs that happen in rapid succession and prolong its recovery process after an initial crisis.

After simulating both the economy with a leverage cap in place and the economy with no exogenous leverage cap, I calculate the average number of periods between bank runs, the average number of periods that the economy stays in steady state once it has reached steady state, and the average number of periods that the economy takes to return to steady state after suffering a bank run. I find that on average, a bank run occurs nearly every 81.3 periods or 20.3 years in the uncapped model and nearly every 109.9 periods or 27.5 years in the capped model. The system reaches the steady state about 44.2 periods or 11 years faster when the leverage cap is in place. The longer amount of time between bank runs and the ability for the economy to reach the steady state faster under a leverage cap regime are due to the decreased probability of a bank run, p_t, while the leverage cap binds in the economy with a leverage cap in place. Conditional on reaching steady state however, the system with the leverage cap regime falls out of the steady state into a bank run on average 1.3 years or 5.2 periods earlier than it would without a leverage cap in place. This is due to the fact that p_t is driven up by the decreased price of capital in the bank run state caused by allocational efficiency losses that result from capping bank leverage in the periods directly following a bank run. These preliminary results provide evidence supporting a leverage cap's ability to stabilize the economy. However the results also imply that there could be allocational efficiency losses that increase the risk in the economy if bank leverage is too harshly restricted directly following a bank run as this can permanently slow the economy's ability to recover from a crisis.

TABLE 1: Average Recovery Times in Economy Simulated for 10,000 Periods (Multiple Period Leverage Cap)

Average Number of Periods	No Leverage Cap	Leverage Cap
Between Bank Runs	81.3	109.9
To Reach SS	318.1	273.9
In SS (Conditional on Reaching)	87.1	82.0

5   Conclusion

In this paper I enhance the Gertler Kiyotaki (2015) model to account for bank regulation in the form of leverage ratio restrictions. This is a macroeconomic model with a bank-

ing sector where bank runs can be anticipated. This model integrates two approaches to modeling vulnerabilities in the financial sector - a "macroeconomic" approach stressing the financial accelerator effects and a "microeconomic" approach which stresses the bank liquidity mismatch making banks vulnerable to bank runs. I provide a quantitative method for analyzing welfare effects of bank leverage ratio restrictions over the business cycle in a general equilibrium framework. For all periods where banks take on leverage greater than fifteen times larger than their net worth in the model with no external leverage cap, I restrict the maximum amount of leverage that the banks are able to choose in the system with leverage restrictions in place. While the leverage cap binds, banks can choose leverage to be the minimum of 90% of the amount of leverage that they would optimally choose in the unrestricted system, or the maximum leverage implied by their incentive constraint. In the system with this leverage cap in place, I find that the banking sector becomes less risky, with the probability of a bank run decreasing by 49.8% at its maximum decrease relative to the economy with no exogenous leverage restriction. This point occurs about 5 years after a bank run if the economy can make it that far without falling into another bank run. The probability of a bank run under the leverage cap falls to 0.22% at its lowest point, which occurs 10 years after the bank run again if the economy can reach that point without falling into another bank run. I then simulate the economy for 10,000 periods to calculate the average differences between the systems with and without leverage restrictions in place in the amount of time an economy enjoys between bank runs, the amount of time it takes to return to steady state after a bank run, and the amount of time that the economy spends in steady state following a bank run. On average, the fall in the probability of a bank run along the recovery path while the leverage restriction holds translates into a longer amount of time between bank runs and fewer periods required to reach steady state. These results depend on my parameterization of the model. I am planning to calibrate the model to the data in order to provide economically relevant estimates.

I find that a leverage cap can be welfare improving relative to the laissez-faire system. However this depends on the way that the leverage restriction is structured. During the periods following a bank run, the economy is at a greater risk of suffering another bank run so there may be temptation to harshly restrict bank leverage. However high bank leverage during these times is necessary to increase allocational efficiency of the economy and restricting it too harshly can permanently lower production in the economy. This inhibits the banks' ability to grow their net worth which can make banks in the future riskier since they have a lower baseline net worth. However a very slight leverage restriction during these fragile periods can be welfare improving. In the steady state however, the system can sustain stricter leverage ratio restrictions. At the steady state, banks have already purchased almost

all of the capital back from the households. Therefore the effects of bank leverage restrictions on increasing the deadweight loss associated with increased household capital holdings are minimal at the margin and the benefits achieved by lowering the probability of a bank run more than offset the allocational efficiency losses. This evidence indicates that the optimal leverage restriction will be time varying across the business cycle and will be looser in the states where households have a higher marginal utility of consumption.

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