

## 2a) Matrix Equation

$$\underline{A} \underline{U} = \underline{F}$$

$\swarrow \quad \searrow \quad \searrow$   
 $\mathbb{R}^{m^2 \times m^2} \quad \mathbb{R}^{m^2} \quad \mathbb{R}^{m^2}$

$$\underline{A} = \frac{1}{h^2} \begin{bmatrix} T & D & & & \\ & D & T & D & \\ & & \ddots & \ddots & \ddots \\ & & & D & T \\ & & & & D & T \end{bmatrix}$$

where  $T = \begin{bmatrix} 4 & -1 & & \\ & \ddots & \ddots & \\ & & 4 & -1 \\ & & & \ddots & \ddots \\ & & & & 4 \end{bmatrix} \in \mathbb{R}^{m \times m}$

$$D = \begin{bmatrix} -1 & & & \\ & \ddots & & \\ & & -1 & \\ & & & \ddots & \ddots \\ & & & & -1 \end{bmatrix} \in \mathbb{R}^{m \times m}$$

$$\underline{U} = \begin{bmatrix} U^{[1]} \\ U^{[2]} \\ \vdots \\ U^{[m]} \end{bmatrix} = \begin{bmatrix} (u_{1,1}) \\ (u_{2,1}) \\ \vdots \\ (u_{m,1}) \\ \vdots \\ (u_{1,m}) \\ (u_{2,m}) \\ \vdots \\ (u_{m,m}) \end{bmatrix}$$

$$\underline{F} = \begin{bmatrix} F^{[1]} \\ F^{[2]} \\ \vdots \\ F^{[m]} \end{bmatrix} = \begin{bmatrix} \left( \begin{matrix} f_{1,1} + \boxed{\frac{g(0,h_j)}{h^2}} \\ f_{2,1} \\ \vdots \\ f_{m,1} + \boxed{\frac{g((m+1)h,h_j)}{h^2}} \end{matrix} \right) + \frac{g(h_i,0)}{h^2} \\ \vdots \\ \left( \begin{matrix} f_{1,m} + \boxed{\frac{g(0,h_j)}{h^2}} \\ f_{2,m} \\ \vdots \\ f_{m,m} + \boxed{\frac{g((m+1)h,h_j)}{h^2}} \end{matrix} \right) + \frac{g(h_i,(m+1)h)}{h^2} \end{bmatrix}$$

Note:    added to all  $f_{i,j}$   
and    added to all  $f_{m,j}$