# 3a) Matrix Equation

$$\underline{\underline{A}}\,\underline{U} = \underline{F}$$

$$\underline{\underline{A}} \in \mathbb{R}^{m^2 \times m^2}, \quad \underline{U} \in \mathbb{R}^{m^2}, \quad \underline{F} \in \mathbb{R}^{m^2}$$

$$\underline{\underline{A}} = \begin{bmatrix} T & D & & \\ D & T & \ddots & \\ & \ddots & \ddots & D \\ & & D & T \end{bmatrix}$$

where

$$T = \begin{bmatrix} \left(\frac{4\nu}{h^2} + r\right) & \left(-\frac{\nu}{h^2} + \frac{\beta}{2h}\right) & & \\ \left(-\frac{\nu}{h^2} - \frac{\beta}{2h}\right) & \ddots & \ddots & \\ & \ddots & \ddots & \ddots \\ & & \ddots & \ddots \end{bmatrix} \in \mathbb{R}^{m \times m}$$

$$D = \begin{bmatrix} -\frac{\nu}{h^2} & & \\ & \ddots & \\ & & -\frac{\nu}{h^2} \end{bmatrix} \in \mathbb{R}^{m \times m}$$

$$\underline{U} = \begin{bmatrix} U^{[1]} \\ U^{[2]} \\ \vdots \\ U^{[m]} \end{bmatrix} = \begin{bmatrix} \begin{pmatrix} u_{1,1} \\ u_{2,1} \\ \vdots \\ u_{m,1} \end{pmatrix} \\ \vdots \\ \begin{pmatrix} u_{1,m} \\ u_{2,m} \\ \vdots \\ u_{m,m} \end{pmatrix} \end{bmatrix}$$

$$\underline{F} = \begin{bmatrix} F^{[1]} \\ F^{[2]} \\ \vdots \\ F^{[m]} \end{bmatrix} = \begin{bmatrix} \begin{pmatrix} f_{1,1} \\ f_{2,1} \\ \vdots \\ f_{m,1} \end{pmatrix} \\ \vdots \\ \begin{pmatrix} f_{1,m} \\ f_{2,m} \\ \vdots \\ f_{m,m} \end{pmatrix} \end{bmatrix}$$

Note: no adjustments needed for boundary nodes since they evaluate to zero.