Problem 3 - Convection-Diffusion-Reaction Equation

$\Omega = (0,1) \times (0,1) \in \mathbb{R}^2$

$u: \Omega \to \mathbb{R}$

$$-\nu (u_{xx} + u_{yy}) + \beta u_x + \Gamma u = f(x,y) \quad \text{for } (x,y) \in \Omega$$

Dirichlet Boundary: $u(x,y) = 0$ for $(x,y) \in \partial\Omega$

$\nu > 0$, $\beta \geq 0$, $\Gamma \geq 0$ $\quad f: \Omega \to \mathbb{R}$

3a) Discretisation.

Same mesh as Q2.

Difference Scheme:

$$\left\{ \nu \left[\frac{-u_{i-1,j} - u_{i,j-1} + 4u_{i,j} - u_{i,j+1} - u_{i+1,j}}{h^2} \right] + \beta \left[\frac{u_{i+1,j} - u_{i-1,j}}{2h} \right] + \Gamma u_{i,j} \right\} = f(x_i, y_j)$$

for $1 \leq i \leq m$ and $1 \leq j \leq m$.

Note: I have used a different First Order Difference Approximation to the one supplied in the question as this one produced better results

same boundary nodes as Q2. However the u approximation at the boundary nodes evaluates to zero due to the Dirichlet boundaries here.