Álgebra Linear - 1ª avaliação

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Questão 1

$$A = \begin{pmatrix} 1 & 2 & -4 & 0 \\ 0 & -1 & 3 & -9 \\ 10 & -14 & -8 & 16 \\ 0 & 2 & 16 & -12 \end{pmatrix} \qquad b = \begin{pmatrix} -33 \\ -46 \\ 143 \\ 57 \end{pmatrix}$$

RESOLUÇÃO:

$$A_0 = \begin{pmatrix} 1 & 2 & -4 & 0 & -33 \\ 0 & -1 & 3 & -9 & -46 \\ 10 & -14 & -8 & 16 & 143 \\ 0 & 2 & 16 & -12 & 57 \end{pmatrix}$$

$L_3 \leftarrow L_3 - 10L_1$

$$A_1 = \begin{pmatrix} 1 & 2 & -4 & 0 & -33 \\ 0 & -1 & 3 & -9 & -46 \\ 0 & -34 & 32 & 16 & 473 \\ 0 & 2 & 16 & -12 & 57 \end{pmatrix}$$

$L_3 \leftarrow L_3 + 17L_4$

$$A_2 = \begin{pmatrix} 1 & 2 & -4 & 0 & -33 \\ 0 & -1 & 3 & -9 & -46 \\ 0 & 0 & 304 & -188 & 1442 \\ 0 & 2 & 16 & -12 & 57 \end{pmatrix}$$

$$L_3 \leftarrow \frac{1}{2} L_3$$

$$A_3 = \begin{pmatrix} 1 & 2 & -4 & 0 & -33 \\ 0 & -1 & 3 & -9 & -46 \\ 0 & 0 & 152 & -94 & 721 \\ 0 & 2 & 16 & -12 & 57 \end{pmatrix}$$

$$L_4 \leftarrow L_4 + 2L_2$$

$$A_4 = \begin{pmatrix} 1 & 2 & -4 & 0 & -33 \\ 0 & -1 & 3 & -9 & -46 \\ 0 & 0 & 152 & -94 & 721 \\ 0 & 0 & 22 & -30 & -35 \end{pmatrix}$$

$$L_4 \leftarrow L_4 - \frac{22}{152} L_3$$

$$A_5 = \begin{pmatrix} 1 & 2 & -4 & 0 & -33 \\ 0 & -1 & 3 & -9 & -46 \\ 0 & 0 & 152 & -94 & 721 \\ 0 & 0 & 0 & \frac{-623}{38} & \frac{-10591}{76} \end{pmatrix}$$

→ RETROSSUBSTITUIÇÃO

$$\frac{-623}{38} x_4 = \frac{-10591}{76} \rightarrow -17348 x_4 = -402458$$

$$\boxed{x_4 = 17/2}$$

$$152 x_3 - 94 x_4 = 721$$

$$152 x_3 - 94 \left(\frac{17}{2}\right) = 721$$

$$152 x_3 - 799 = 721$$

$$152 x_3 = 1520 \quad \therefore \boxed{x_3 = 10}$$

$$-x_2 + 3x_3 - 9x_4 = -46$$

$$-x_2 + 3(10) - 9\left(\frac{17}{2}\right) = -46$$

$$-x_2 + 30 - \frac{153}{2} = -46$$

$$-x_2 = -76 + \frac{153}{2} \rightarrow -x_2 = -\frac{152}{2} + \frac{153}{2}$$

$$-x_2 = \frac{1}{2}$$

$$\boxed{x_2 = -1/2}$$

$$x_1 + 2x_2 - 4x_3 = -33$$

$$x_1 + 2\left(\frac{-1}{2}\right) - 4(10) = -33$$

$$x_1 - 1 - 40 = -33$$

$$x_1 - 41 = -33 \longrightarrow \boxed{x_1 = 8}$$

Resposta:

$$\begin{cases} x_1 = 8 \\ x_2 = -1/2 \\ x_3 = 10 \\ x_4 = 17/2 \end{cases}$$

Questão 2

$$B = \begin{pmatrix} 1 & 4 & 0 & -2 \\ 2 & 2 & 0 & -3 \\ 0 & -1 & 3 & -2 \\ 0 & 1 & 3 & 0 \end{pmatrix}$$

$$(B|I)_0 = \left(\begin{array}{cccc|cccc} 1 & 4 & 0 & -2 & 1 & 0 & 0 & 0 \\ 2 & 2 & 0 & -3 & 0 & 1 & 0 & 0 \\ 0 & -1 & 3 & -2 & 0 & 0 & 1 & 0 \\ 0 & 1 & 3 & 0 & 0 & 0 & 0 & 1 \end{array}\right)$$

$L_2 \leftarrow L_2 - 2L_1$

$$(B|I)_1 = \left(\begin{array}{cccc|cccc} 1 & 4 & 0 & -2 & 1 & 0 & 0 & 0 \\ 0 & -6 & 0 & 1 & -2 & 1 & 0 & 0 \\ 0 & -1 & 3 & -2 & 0 & 0 & 1 & 0 \\ 0 & 1 & 3 & 0 & 0 & 0 & 0 & 1 \end{array}\right)$$

$L_4 \leftarrow L_4 + L_3$

$$(B|I)_2 = \left(\begin{array}{cccc|cccc} 1 & 4 & 0 & -2 & 1 & 0 & 0 & 0 \\ 0 & -6 & 0 & 1 & -2 & 1 & 0 & 0 \\ 0 & -1 & 3 & -2 & 0 & 0 & 1 & 0 \\ 0 & 0 & 6 & -2 & 0 & 0 & 1 & 1 \end{array}\right)$$

$L_3 \leftarrow L_3 - \frac{1}{6}L_2$

$$(B|I)_3 = \left(\begin{array}{cccc|cccc} 1 & 4 & 0 & -2 & 1 & 0 & 0 & 0 \\ 0 & -6 & 0 & 1 & -2 & 1 & 0 & 0 \\ 0 & 0 & 3 & -\frac{13}{6} & \frac{1}{3} & -\frac{1}{6} & 1 & 0 \\ 0 & 0 & 6 & -2 & 0 & 0 & 1 & 1 \end{array}\right)$$

$$L_4 \leftarrow L_4 - 2L_3$$

$$(B \mid I)_4 = \left(\begin{array}{cccc|cccc} 1 & 4 & 0 & -2 & 1 & 0 & 0 & 0 \\ 0 & -6 & 6 & 1 & -2 & 1 & 0 & 0 \\ 0 & 0 & 3 & -\frac{13}{6} & 1/3 & -1/6 & 1 & 0 \\ 0 & 0 & 0 & 7/3 & -2/3 & 1/3 & -1 & 1 \end{array}\right)$$

$$L_3 = L_3 + \frac{13}{14} L_4$$

$$(B \mid I)_5 = \left(\begin{array}{cccc|cccc} 1 & 4 & 0 & -2 & 1 & 0 & 0 & 0 \\ 0 & -6 & 6 & 1 & -2 & 1 & 0 & 0 \\ 0 & 0 & 3 & 0 & -2/7 & 1/7 & \frac{1}{14} & \frac{13}{14} \\ 0 & 0 & 0 & 7/3 & -2/3 & 1/3 & -1 & 1 \end{array}\right)$$

$$L_4 \leftarrow \frac{3}{7} L_4$$

$$(B \mid I)_6 = \left(\begin{array}{cccc|cccc} 1 & 4 & 0 & -2 & 1 & 0 & 0 & 0 \\ 0 & -6 & 0 & 1 & -2 & 1 & 0 & 0 \\ 0 & 0 & 3 & 0 & -2/7 & 1/7 & 1/14 & 13/14 \\ 0 & 0 & 0 & 1 & -2/7 & 1/7 & -3/7 & 3/7 \end{array}\right)$$

$$L_2 \leftarrow L_2 - L_4$$

$$(B \mid I)_7 = \left(\begin{array}{cccc|cccc} 1 & 4 & 0 & -2 & 1 & 0 & 0 & 0 \\ 0 & -6 & 0 & 0 & -\frac{12}{7} & \frac{6}{7} & 3/7 & -3/7 \\ 0 & 0 & 3 & 0 & -2/7 & 1/7 & 1/14 & 13/14 \\ 0 & 0 & 0 & 1 & -2/7 & 1/7 & -3/7 & 3/7 \end{array}\right)$$

$L_3 \leftarrow \frac{1}{3} L_3$

$L_2 \leftarrow \frac{-1}{6} L_2$

$$(B \mid I)_9 = \left(\begin{array}{cccc|cccc} 1 & 4 & 0 & -2 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & \frac{2}{7} & \frac{-1}{7} & \frac{-1}{14} & \frac{1}{14} \\ 0 & 0 & 1 & 0 & -2/21 & 1/21 & 1/42 & 13/42 \\ 0 & 0 & 0 & 1 & -2/7 & 1/7 & -3/7 & 3/7 \end{array}\right)$$

$L_1 \leftarrow L_1 + 2L_4$

$$(B \mid I)_{10} = \left(\begin{array}{cccc|cccc} 1 & 4 & 0 & 0 & \frac{3}{7} & \frac{2}{7} & \frac{-6}{7} & \frac{6}{7} \\ 0 & 1 & 0 & 0 & \frac{2}{7} & \frac{-1}{7} & \frac{-1}{14} & \frac{1}{14} \\ 0 & 0 & 1 & 0 & \frac{-2}{21} & 1/21 & \frac{1}{42} & \frac{13}{42} \\ 0 & 0 & 0 & 1 & \frac{-2}{7} & \frac{1}{7} & \frac{-3}{7} & \frac{3}{7} \end{array}\right)$$

$L_1 \leftarrow L_1 - 4L_2$

$$(B \mid I)_{11} - \left(\begin{array}{cccc|cccc} 1 & 0 & 0 & 0 & \frac{-5}{7} & \frac{6}{7} & \frac{-4}{7} & \frac{4}{7} \\ 0 & 1 & 0 & 0 & \frac{2}{7} & \frac{-1}{7} & \frac{-1}{14} & \frac{1}{14} \\ 0 & 0 & 1 & 0 & \frac{-2}{21} & \frac{1}{21} & \frac{1}{42} & \frac{13}{42} \\ 0 & 0 & 0 & 1 & \frac{-2}{7} & \frac{1}{7} & \frac{-3}{7} & \frac{3}{7} \end{array}\right)$$

A matriz inversa é: $\left(\begin{array}{cccc} \frac{-5}{7} & \frac{6}{7} & \frac{-4}{7} & \frac{4}{7} \\ \frac{2}{7} & \frac{-1}{7} & \frac{-1}{14} & \frac{1}{14} \\ \frac{-2}{21} & \frac{1}{21} & \frac{1}{42} & \frac{13}{42} \\ \frac{-2}{7} & \frac{1}{7} & \frac{-3}{7} & \frac{3}{7} \end{array}\right)$