inzva BTS Problems
DP, Knapsack, STL

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1 DP

1. You are obliged to create an array a of length n, consisting of only integers. This array has to satisfy the conditions given below:

   - $a[0] = 0$
   - $a[n-1] \leq K$
   - $a[i-1] \leq a[i] \leq a[i-1] + 1 \ (\forall i > 0)$

   where K is a non-negative integer and $a[i]$ stands for the i^{th} element (0–indexed) in the array, from left. Then, how many different a arrays can you create? **Two arrays are considered different if and only if the elements of a same index differ in those arrays.**
   Constraint
   $n * K \leq 10^6$

2. How many arrays a of length n, containing only non-negative integers exist satisfying the following condition?

   $$\sum_{j=0}^{i} a[j] = a[0] + a[1] + \cdots + a[i] \leq i \text{ for all } 0 \leq i < n$$

   In English, for all indices i, sum of all elements until i is not greater than i.
   Constraint
   $n \leq 5000$

3. Consider a set S containing only the integers from 1 to n (formally, $S = \{x : 1 \leq x \leq n \wedge x \in \mathcal{Z}\}$. Your task is to find out how many ways there are to partition this set into two disjoint subsets such that each element in S rests in exactly one of these subsets and the sum of the elements in the subsets are equal. For instance if $n = 3$, there are 2 ways of partitioning: $(\{1, 2\}, \{3\})$ and vice versa.
   Constraint
   $n \leq 1000$

4. How many arrays consisting of positive integers can be formed, such that the sum of elements equals n?
   Constraint
   $n \leq 500/1000$?
   Bonus (The answer is 2^n.)
   $n \leq 10^6$
   Extra Bonus
   $n \leq 10^{18}$

5. A sequence of brackets ('(' and ')') is considered *balanced* if for every element, the number of closing brackets does not exceed the number of opening brackets and the total number of opening and closing brackets are the same. So, how many different *balanced* bracket sequences of length n exist? (n can be either odd or even.)
   Constraint
   $n \leq 5000$
   Bonus (Catalan Numbers)
   $n \leq 10^6$

2 Knapsack

You have an equal-arm balance ("eşit kollu terazi" in Turkish) and n positive integer weights (not necessarily different). If you are able to put any weight on either arms, how many different positive weights are you able to exactly weigh?
For example with weights $2, 3$ and 5, you can weigh:

- $1 : 3 - 2$
- $2 : 2$
- $3 : 3$
- $4 : 2 + 5 - 3$
- $5 : 5$
- $6 : 3 + 5 - 2$
- $7 : 2 + 5$
- $8 : 3 + 5$
- $10 : 2 + 3 + 5$

Constraint

$$n * \sum_{i=0}^{n-1} weight[i] \leq 10^6$$

3 STL

1. Given an array a consisting of n integers, determine whether there exist at least one pair of different elements (not necessarily different integers), sum of which is divisible by P.
   Constraints
   $n \leq 3 * 10^5$
   $|a[i]| \leq 10^{13}$
   $1 \leq P \leq 10^{18}$

2. You have a pool (e.g. array, multiset) of strings, initially empty. You are going to perform one of the following 3 operations each time:

   (a) 1 s: Add the given string s to the pool. Note that there can be multiple identical strings in the pool.

   (b) 2 s: Remove a single instance of the given string s from the pool. For example if there are 3 strings identical to s in the pool, there are going to be 2 of them after the operation. If there are not any strings identical to s, ignore the operation.

   (c) 3 s: Check whether there is a string in the pool of which the given string s is a prefix. If there more than one such strings in the pool, print the lexicographically smallest one. If there is not any such string, print "-1" without quotes.

   The total number of operations is Q. It is guaranteed that there will be at least one operation of the third type.
   Constraints
   $Q \leq 3 * 10^5$
   Length of any string in the input (s) is at most 20.