15.1_Numpy(2)

April 24, 2023

1 Introduction to Python for Open Source Geocomputation

- Instructor: Dr. Wei Kang
- Class Location and Time: ENV 336, Mon & Wed 12:30 pm - 1:50 pm

Content:

- Numpy
- A new data type: `numpy.array`
  - How to create an array
  - Array operations

2 What is Numpy?

- The fundamental package for scientific computing with Python
- Nearly every scientist working in Python draws on the power of NumPy.
- NumPy brings the **computational power** of languages like C and Fortran to Python, a language much easier to learn and use. With this power comes **simplicity: a solution in NumPy is often clear and elegant**.
- Essential in many different realms:
  - NumPy lies at the core of a rich ecosystem of **data science** libraries
  - NumPy forms the basis of powerful **machine learning** libraries like scikit-learn, SciPy, TensorFlow, and PyTorch
  - NumPy is an essential component in the burgeoning Python **visualization landscape**, which includes Matplotlib, Seaborn, Plotly, Altair, Bokeh, Holoviz, Vispy, Napari, and PyVista, to name a few.

2.1 What makes Numpy so important?

arrays: A very powerful data type essential to numerical computing: * sequences of data all of the *same type* * behave a lot like lists, except for the constraint in the type of their elements. * There is a huge efficiency advantage when you know that **all elements of a sequence are of the same type**—so equivalent methods for arrays execute a lot **faster** than those for lists.

2.2 Numpy `Array` (or `ndarray`)

- homogeneous multidimensional array
  - a table of elements (usually numbers), all of the same type, indexed by a tuple of non-negative integers
    - for the data types accepted in Numpy. Read the docs: Data type objects.
  - dimensions are called *axes*
- An Example: points' coordinates
  - one single point: one-dimensional array: `np.array([1,2])`
  - two or more points: two-dimensional array:
    - two points: `np.array([[1,2], [3,4]])`
    - five points: `np.array([[1,2], [3,4],[5,6], [7,8], [9,10]])`

[1]:
```
import numpy as np
```

[2]:
```
a1 = np.array([1,2])
a1
```

[2]:
```
array([1, 2])
```

[3]:
```
a2 = np.array([[1,2], [3,4],[5,6], [7,8], [9,10]])
a2
```

[3]:
```
array([[ 1,  2],
       [ 3,  4],
       [ 5,  6],
       [ 7,  8],
       [ 9, 10]])
```

2.2.1 Motivation (1): What can a Numpy array used for?

- An array can contain:
  - values of an experiment/simulation at discrete time steps, e.g., income, air pollution, crime rate, animal/plant occurrence
  - pixels of an image, grey-level or colour
  - signal recorded by a measurement device, e.g. sound wave
  - 3-D data measured at different X-Y-Z positions, e.g. MRI scan, digital elevation model

2.2.2 Motivation (2): Efficiency of Numpy array - an example

- Problem description: Write a python program that calculate the square of each number in a list, such that $x_i = i^2$, for $0 \leq i < n$.

Two data types: * Python built-in type: list * Numpy array

We use `%timeit` to calculate the time execution of a Python statement or expression.

[4]:
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L = list(range(1000)) #produce a list of integers from 0 to 999
L
```

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[5]:
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%timeit -n 1000 [i**2 for i in L]
```

```
186 µs ± 5.03 µs per loop (mean ± std. dev. of 7 runs, 1,000 loops each)
```

[6]:
```
import numpy as np
a = np.arange(1000) #produce an array of integers from 0 to 999
a
```

[6]:
```
array([  0,   1,   2,   3,   4,   5,   6,   7,   8,   9,  10,  11,  12,
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       988, 989, 990, 991, 992, 993, 994, 995, 996, 997, 998, 999])
```

[7]:
```
%timeit -n 1000 a**2
```

```
1.14 µs ± 469 ns per loop (mean ± std. dev. of 7 runs, 1,000 loops each)
```

3 Importing Numpy

```
import numpy as np
```

[8]:
```
import numpy as np
```

[9]:
```
dir(np) #function dir gives you the package's attributes and functions.
```

[9]:
```
['ALLOW_THREADS',
 'AxisError',
 'BUFSIZE',
 'CLIP',
 'ComplexWarning',
 'DataSource',
 'ERR_CALL',
 'ERR_DEFAULT',
 'ERR_IGNORE',
 'ERR_LOG',
 'ERR_PRINT',
 'ERR_RAISE',
 'ERR_WARN',
 'FLOATING_POINT_SUPPORT',
 'FPE_DIVIDEBYZERO',
 'FPE_INVALID',
 'FPE_OVERFLOW',
 'FPE_UNDERFLOW',
 'False_',
 'Inf',
 'Infinity',
 'MAXDIMS',
 'MAY_SHARE_BOUNDS',
```

```
'MAY_SHARE_EXACT',
'ModuleDeprecationWarning',
'NAN',
'NINF',
'NZERO',
'NaN',
'PINF',
'PZERO',
'RAISE',
'RankWarning',
'SHIFT_DIVIDEBYZERO',
'SHIFT_INVALID',
'SHIFT_OVERFLOW',
'SHIFT_UNDERFLOW',
'ScalarType',
'Tester',
'TooHardError',
'True_',
'UFUNC_BUFSIZE_DEFAULT',
'UFUNC_PYVALS_NAME',
'VisibleDeprecationWarning',
'WRAP',
'_CopyMode',
'_NoValue',
'_UFUNC_API',
'__NUMPY_SETUP__',
'__all__',
'__builtins__',
'__cached__',
'__config__',
'__deprecated_attrs__',
'__dir__',
'__doc__',
'__expired_functions__',
'__file__',
'__getattr__',
'__git_version__',
'__loader__',
'__name__',
'__package__',
'__path__',
'__spec__',
'__version__',
'_add_newdoc_ufunc',
'_distributor_init',
'_financial_names',
'_globals',
```

```
'_mat',
'_pyinstaller_hooks_dir',
'_pytesttester',
'_version',
'abs',
'absolute',
'add',
'add_docstring',
'add_newdoc',
'add_newdoc_ufunc',
'all',
'allclose',
'alltrue',
'amax',
'amin',
'angle',
'any',
'append',
'apply_along_axis',
'apply_over_axes',
'arange',
'arccos',
'arccosh',
'arcsin',
'arcsinh',
'arctan',
'arctan2',
'arctanh',
'argmax',
'argmin',
'argpartition',
'argsort',
'argwhere',
'around',
'array',
'array2string',
'array_equal',
'array_equiv',
'array_repr',
'array_split',
'array_str',
'asanyarray',
'asarray',
'asarray_chkfinite',
'ascontiguousarray',
'asfarray',
'asfortranarray',
```

```
'asmatrix',
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'atleast_2d',
'atleast_3d',
'average',
'bartlett',
'base_repr',
'binary_repr',
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'bitwise_and',
'bitwise_not',
'bitwise_or',
'bitwise_xor',
'blackman',
'block',
'bmat',
'bool8',
'bool_',
'broadcast',
'broadcast_arrays',
'broadcast_shapes',
'broadcast_to',
'busday_count',
'busday_offset',
'busdaycalendar',
'byte',
'byte_bounds',
'bytes0',
'bytes_',
'c_',
'can_cast',
'cast',
'cbrt',
'cdouble',
'ceil',
'cfloat',
'char',
'character',
'chararray',
'choose',
'clip',
'clongdouble',
'clongfloat',
'column_stack',
'common_type',
'compare_chararrays',
'compat',
```

```
'complex128',
'complex256',
'complex64',
'complex_',
'complexfloating',
'compress',
'concatenate',
'conj',
'conjugate',
'convolve',
'copy',
'copysign',
'copyto',
'core',
'corrcoef',
'correlate',
'cos',
'cosh',
'count_nonzero',
'cov',
'cross',
'csingle',
'ctypeslib',
'cumprod',
'cumproduct',
'cumsum',
'datetime64',
'datetime_as_string',
'datetime_data',
'deg2rad',
'degrees',
'delete',
'deprecate',
'deprecate_with_doc',
'diag',
'diag_indices',
'diag_indices_from',
'diagflat',
'diagonal',
'diff',
'digitize',
'disp',
'divide',
'divmod',
'dot',
'double',
'dsplit',
```

```
'dstack',
'dtype',
'e',
'ediff1d',
'einsum',
'einsum_path',
'emath',
'empty',
'empty_like',
'equal',
'error_message',
'errstate',
'euler_gamma',
'exp',
'exp2',
'expand_dims',
'expm1',
'extract',
'eye',
'fabs',
'fastCopyAndTranspose',
'fft',
'fill_diagonal',
'find_common_type',
'finfo',
'fix',
'flatiter',
'flatnonzero',
'flexible',
'flip',
'fliplr',
'flipud',
'float128',
'float16',
'float32',
'float64',
'float_',
'float_power',
'floating',
'floor',
'floor_divide',
'fmax',
'fmin',
'fmod',
'format_float_positional',
'format_float_scientific',
'format_parser',
```

```
'frexp',
'from_dlpack',
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'fromiter',
'frompyfunc',
'fromregex',
'fromstring',
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'full_like',
'gcd',
'generic',
'genfromtxt',
'geomspace',
'get_array_wrap',
'get_include',
'get_printoptions',
'getbufsize',
'geterr',
'geterrcall',
'geterrobj',
'gradient',
'greater',
'greater_equal',
'half',
'hamming',
'hanning',
'heaviside',
'histogram',
'histogram2d',
'histogram_bin_edges',
'histogramdd',
'hsplit',
'hstack',
'hypot',
'i0',
'identity',
'iinfo',
'imag',
'in1d',
'index_exp',
'indices',
'inexact',
'inf',
'info',
'infty',
```

```
'inner',
'insert',
'int0',
'int16',
'int32',
'int64',
'int8',
'int_',
'intc',
'integer',
'interp',
'intersect1d',
'intp',
'invert',
'is_busday',
'isclose',
'iscomplex',
'iscomplexobj',
'isfinite',
'isfortran',
'isin',
'isinf',
'isnan',
'isnat',
'isneginf',
'isposinf',
'isreal',
'isrealobj',
'isscalar',
'issctype',
'issubclass_',
'issubdtype',
'issubsctype',
'iterable',
'ix_',
'kaiser',
'kron',
'lcm',
'ldexp',
'left_shift',
'less',
'less_equal',
'lexsort',
'lib',
'linalg',
'linspace',
'little_endian',
```

```
'load',
'loadtxt',
'log',
'log10',
'log1p',
'log2',
'logaddexp',
'logaddexp2',
'logical_and',
'logical_not',
'logical_or',
'logical_xor',
'logspace',
'longcomplex',
'longdouble',
'longfloat',
'longlong',
'lookfor',
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'mask_indices',
'mat',
'math',
'matmul',
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'matrixlib',
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'maximum_sctype',
'may_share_memory',
'mean',
'median',
'memmap',
'meshgrid',
'mgrid',
'min',
'min_scalar_type',
'minimum',
'mintypecode',
'mod',
'modf',
'moveaxis',
'msort',
'multiply',
'nan',
'nan_to_num',
'nanargmax',
'nanargmin',
```

```
'nancumprod',
'nancumsum',
'nanmax',
'nanmean',
'nanmedian',
'nanmin',
'nanpercentile',
'nanprod',
'nanquantile',
'nanstd',
'nansum',
'nanvar',
'nbytes',
'ndarray',
'ndenumerate',
'ndim',
'ndindex',
'nditer',
'negative',
'nested_iters',
'newaxis',
'nextafter',
'nonzero',
'not_equal',
'numarray',
'number',
'obj2sctype',
'object0',
'object_',
'ogrid',
'oldnumeric',
'ones',
'ones_like',
'os',
'outer',
'packbits',
'pad',
'partition',
'percentile',
'pi',
'piecewise',
'place',
'poly',
'poly1d',
'polyadd',
'polyder',
'polydiv',
```

```
'polyfit',
'polyint',
'polymul',
'polynomial',
'polysub',
'polyval',
'positive',
'power',
'printoptions',
'prod',
'product',
'promote_types',
'ptp',
'put',
'put_along_axis',
'putmask',
'quantile',
'r_',
'rad2deg',
'radians',
'random',
'ravel',
'ravel_multi_index',
'real',
'real_if_close',
'rec',
'recarray',
'recfromcsv',
'recfromtxt',
'reciprocal',
'record',
'remainder',
'repeat',
'require',
'reshape',
'resize',
'result_type',
'right_shift',
'rint',
'roll',
'rollaxis',
'roots',
'rot90',
'round',
'round_',
'row_stack',
's_',
```

```
'safe_eval',
'save',
'savetxt',
'savez',
'savez_compressed',
'sctype2char',
'sctypeDict',
'sctypes',
'searchsorted',
'select',
'set_numeric_ops',
'set_printoptions',
'set_string_function',
'setbufsize',
'setdiff1d',
'seterr',
'seterrcall',
'seterrobj',
'setxor1d',
'shape',
'shares_memory',
'short',
'show_config',
'sign',
'signbit',
'signedinteger',
'sin',
'sinc',
'single',
'singlecomplex',
'sinh',
'size',
'sometrue',
'sort',
'sort_complex',
'source',
'spacing',
'split',
'sqrt',
'square',
'squeeze',
'stack',
'std',
'str0',
'str_',
'string_',
'subtract',
```

```
'sum',
'swapaxes',
'sys',
'take',
'take_along_axis',
'tan',
'tanh',
'tensordot',
'test',
'testing',
'tile',
'timedelta64',
'trace',
'tracemalloc_domain',
'transpose',
'trapz',
'tri',
'tril',
'tril_indices',
'tril_indices_from',
'trim_zeros',
'triu',
'triu_indices',
'triu_indices_from',
'true_divide',
'trunc',
'typecodes',
'typename',
'ubyte',
'ufunc',
'uint',
'uint0',
'uint16',
'uint32',
'uint64',
'uint8',
'uintc',
'uintp',
'ulonglong',
'unicode_',
'union1d',
'unique',
'unpackbits',
'unravel_index',
'unsignedinteger',
'unwrap',
'use_hugepage',
```

```
 'ushort',
 'vander',
 'var',
 'vdot',
 'vectorize',
 'version',
 'void',
 'void0',
 'vsplit',
 'vstack',
 'w',
 'warnings',
 'where',
 'who',
 'zeros',
 'zeros_like']
```

3.0.1 Creating a Numpy Array

- create an array from a regular Python list or tuple using the `array` function.

`np.array(list/tuple)`

- functions from Numpy to create special arrays
  - `np.arange()`: create evenly spaced values within a given interval.
  - `np.linspace(start, stop, num=50)`: create evenly spaced numbers over a specified interval.
  - `np.ones(shape)`: create new array of given shape and type, filled with ones.
  - `np.zeros(shape)`: create a new array of given shape and type, filled with zeros.
  - `np.eye(N)`: create a 2-D array with ones on the diagonal and zeros elsewhere.

[10]:
```
a1 = np.array([1,2])
a1
```

[10]:
```
array([1, 2])
```

[11]:
```
type(a1)
```

[11]:
```
numpy.ndarray
```

[12]:
```
a1.size
```

[12]:
```
2
```

`array.size` gives the number of items in the array.

[13]:
```
len(a1)
```

[13]:
```
2
```

`len(array)` gives the same result to `array.size`

[14]:
```
a1.ndim
```

[14]:
```
1
```

`array.ndim` gives the number of axes (dimensions) of the array.

[15]:
```
a1.shape
```

[15]:
```
(2,)
```

`array.shape` gives the dimensions of the array. This is a tuple of integers indicating the **size** of the array in each dimension. For a matrix with n rows and m columns, shape will be (n,m). The length of the shape tuple is therefore the number of axes, ndim.

[16]:
```
a1.dtype
```

[16]:
```
dtype('int64')
```

`array.dtype` returns an object describing the type of the elements in the array

[17]:
```
a_str = np.array([1.0,2,"1"])
a_str
```

[17]:
```
array(['1.0', '2', '1'], dtype='<U32')
```

[18]:
```
a_str.dtype #32-character  string
```

[18]:
```
dtype('<U32')
```

[19]:
```
a2 = np.array([[1,2], [3,4]])
a2
```

[19]:
```
array([[1, 2],
       [3, 4]])
```

[20]:
```
a2.ndim
```

[20]:
```
2
```

[21]:
```
a2.size
```

[21]:
```
4
```

[22]:
```
len(a2)
```

[22]:
```
2
```

`len(array)` gives the number of rows or the size of the first dimension when encountering a 2-dimensional array

[23]:
```
a2.shape
```

[23]:
```
(2, 2)
```

[24]:
```
a2.dtype
```

[24]:
```
dtype('int64')
```

[25]:
```
a3 = np.array([[1,2], [3,4],[5,6], [7,8], [9,10]])
a3
```

[25]:
```
array([[ 1,  2],
       [ 3,  4],
       [ 5,  6],
       [ 7,  8],
       [ 9, 10]])
```

[26]:
```
a3.ndim
```

[26]:
```
2
```

[27]:
```
len(a3)
```

[27]:
```
5
```

[28]:
```
a3.size
```

[28]:
```
10
```

[29]:
```
a3.shape
```

[29]:
```
(5, 2)
```

[30]:
```
a3.dtype
```

[30]:
```
dtype('int64')
```

We can create a 3-dimensional array

[31]:
```
import numpy as np
```

[32]:
```
c = np.array([[[1,1], [2,2]], [[3,23], [4,5]], [[5,3], [9,10]]])
```

[33]:
```
c
```

[33]:
```
array([[[ 1,  1],
        [ 2,  2]],

       [[ 3, 23],
        [ 4,  5]],

       [[ 5,  3],
        [ 9, 10]]])
```

[34]:
```
c.ndim
```

[34]:
```
3
```

[35]:
```
c.shape
```

[35]:
```
(3, 2, 2)
```

[36]:
```
c.size
```

[36]:
```
12
```

[37]:
```
len(c)
```

[37]:
```
3
```

[38]:
```
a = np.array(1, 2, 3, 4)
```

```
---------------------------------------------------------------------------
TypeError                                 Traceback (most recent call last)
Cell In[38], line 1
----> 1 a = np.array(1, 2, 3, 4)

TypeError: array() takes from 1 to 2 positional arguments but 4 were given
```

The input needs to be an ordered sequence data type: list or tuples

[39]:
```
a = np.array([1, 2, 3, 4])
a
```

[39]:
```
array([1, 2, 3, 4])
```

[40]:
```
a = np.array((1, 2, 3, 4))
a
```

[40]:
```
array([1, 2, 3, 4])
```

[41]:
```
a = np.array((1, 2, 3, 4), (1, 2, 3, 4))
a
```

```
---------------------------------------------------------------------------
TypeError                                 Traceback (most recent call last)
Cell In[41], line 1
----> 1 a = np.array((1, 2, 3, 4), (1, 2, 3, 4))
      2 a

TypeError: Tuple must have size 2, but has size 4
```

[42]:
```
a = np.array(((1, 2, 3, 4), (1, 2, 3, 4)))
a
```

[42]:
```
array([[1, 2, 3, 4],
       [1, 2, 3, 4]])
```

[43]:
```
a = np.array([[1, 2, 3, 4], [1, 2, 3, 4]])
a
```

[43]:
```
array([[1, 2, 3, 4],
       [1, 2, 3, 4]])
```

3.1 Numpy functions to generate special arrays

3.1.1 `numpy.arange()`

`numpy.arange()` gives an array of evenly spaced values in a defined interval. Similar to `range()`

Syntax:

`numpy.arange(start, stop, step)`

where `start` by default is zero, `stop` is not inclusive, and the default for `step` is one.

[44]:
```
list(range(3))
```

[44]:
```
[0, 1, 2]
```

[45]:
```
np.arange(10) # 0 .. n-1 (!)
```

[45]:
```
array([0, 1, 2, 3, 4, 5, 6, 7, 8, 9])
```

[46]:
```
np.arange(2, 6)
```

[46]:
```
array([2, 3, 4, 5])
```

[47]:
```
np.arange(2, 6, 2) # start, end (exclusive), step
```

[47]:
```
array([2, 4])
```

[48]:
```
np.arange(2, 6, 0.5) # start, end (exclusive), step
```

[48]:
```
array([2. , 2.5, 3. , 3.5, 4. , 4.5, 5. , 5.5])
```

3.1.2 `numpy.linspace()`

`numpy.linspace()` is similar to `numpy.arange()`, but uses number of samples instead of a step size. It returns an array with evenly spaced numbers over the specified interval.

Syntax:

`numpy.linspace(start, stop, num)`

`stop` is included by default (it can be removed, read the docs), and `num` by default is 50.

[49]:
```
np.linspace(2.0, 3.0)
```

[49]:
```
array([2.        , 2.02040816, 2.04081633, 2.06122449, 2.08163265,
       2.10204082, 2.12244898, 2.14285714, 2.16326531, 2.18367347,
       2.20408163, 2.2244898 , 2.24489796, 2.26530612, 2.28571429,
       2.30612245, 2.32653061, 2.34693878, 2.36734694, 2.3877551 ,
       2.40816327, 2.42857143, 2.44897959, 2.46938776, 2.48979592,
       2.51020408, 2.53061224, 2.55102041, 2.57142857, 2.59183673,
       2.6122449 , 2.63265306, 2.65306122, 2.67346939, 2.69387755,
       2.71428571, 2.73469388, 2.75510204, 2.7755102 , 2.79591837,
       2.81632653, 2.83673469, 2.85714286, 2.87755102, 2.89795918,
       2.91836735, 2.93877551, 2.95918367, 2.97959184, 3.        ])
```

[50]:
```
len(np.linspace(2.0, 3.0))
```

[50]:
```
50
```

[51]:
```
np.linspace(0, 1, 6) # start, end, num of points
```

[51]:
```
array([0. , 0.2, 0.4, 0.6, 0.8, 1. ])
```

[52]:
```
np.linspace(-1, 1, 9)
```

[52]:
```
array([-1.  , -0.75, -0.5 , -0.25,  0.  ,  0.25,  0.5 ,  0.75,  1.  ])
```

We can also create special arrays using Numpy functions

[53]:
```
a = np.ones(3) # creating a 1-D array full of 1s
```

[54]:
```
a
```

[54]:
```
array([1., 1., 1.])
```

[55]:
```
a = np.ones((3, 2)) # (3,2) is the shape of the array we want to create, which␣
↪needs to be a tuple
```

[56]:
```
a
```

[56]:
```
array([[1., 1.],
       [1., 1.],
       [1., 1.]])
```

[57]:
```
b = np.zeros(10) # creating a 1-D array full of 0s
```

[58]:
```
b
```

[58]:
```
array([0., 0., 0., 0., 0., 0., 0., 0., 0., 0.])
```

[59]:
```
b = np.zeros((3, 3))
```

[60]:
```
b
```

[60]:
```
array([[0., 0., 0.],
       [0., 0., 0.],
       [0., 0., 0.]])
```

[61]:
```
np.eye(5)
```

[61]:
```
array([[1., 0., 0., 0., 0.],
       [0., 1., 0., 0., 0.],
       [0., 0., 1., 0., 0.],
       [0., 0., 0., 1., 0.],
       [0., 0., 0., 0., 1.]])
```

[62]:
```
np.eye(2)
```

[62]:
```
array([[1., 0.],
       [0., 1.]])
```

[63]:
```
np.empty((2, 3))
```

[63]:
```
array([[1., 1., 1.],
       [1., 1., 1.]])
```

3.2 Arithmetic operations on arrays

- Arithmetic operators on arrays apply elementwise
- Different from the application of Arithmetic operators to lists

[64]:
```
a = np.array([20, 30, 40, 50])
b = np.array([1,2,3,4])
```

[65]:
```
c = a + b
c
```

[65]:
```
array([21, 32, 43, 54])
```

[66]:
```
d = np.array([1,2,3])
```

[67]:
```
a + d
```

```
---------------------------------------------------------------------------
ValueError                                Traceback (most recent call last)
Cell In[67], line 1
----> 1 a + d

ValueError: operands could not be broadcast together with shapes (4,) (3,)
```

[68]:
```
list_a = list(a)
list_b = list(b)
```

[69]:
```
type(list_a)
```

[69]:
```
list
```

[70]:
```
list_a
```

[70]:
```
[20, 30, 40, 50]
```

[71]:
```
list_b
```

[71]:
```
[1, 2, 3, 4]
```

[72]:
```
list_a + list_b
```

[72]:
```
[20, 30, 40, 50, 1, 2, 3, 4]
```

[73]:
```
d = a - b
d
```

[73]:
```
array([19, 28, 37, 46])
```

[74]:
```
list_d = list_a - list_b
list_d
```

```
---------------------------------------------------------------------------
TypeError                                 Traceback (most recent call last)
Cell In[74], line 1
----> 1 list_d = list_a - list_b
      2 list_d

TypeError: unsupported operand type(s) for -: 'list' and 'list'
```

[75]:
```
a
```

[75]:
```
array([20, 30, 40, 50])
```

[76]:
```
a ** 2
```

[76]:
```
array([ 400,  900, 1600, 2500])
```

[77]:
```
list_a ** 2
```

```
---------------------------------------------------------------------------
TypeError                                 Traceback (most recent call last)
Cell In[77], line 1
----> 1 list_a ** 2

TypeError: unsupported operand type(s) for ** or pow(): 'list' and 'int'
```

[78]:
```
a
```

[78]:
```
array([20, 30, 40, 50])
```

[79]:
```
a * 2
```

[79]:
```
array([ 40,  60,  80, 100])
```

[80]:
```
list_a * 2
```

[80]:
```
[20, 30, 40, 50, 20, 30, 40, 50]
```

[81]:
```
a
```

[81]:
```
array([20, 30, 40, 50])
```

[82]:
```
b
```

[82]:
```
array([1, 2, 3, 4])
```

[83]:
```
a < b
```

[83]:
```
array([False, False, False, False])
```

[84]:
```
list_a
```

[84]:
```
[20, 30, 40, 50]
```

[85]:
```
list_b
```

[85]:
```
[1, 2, 3, 4]
```

[86]:
```
list_a < list_b
```

[86]:
```
False
```

[87]:
```
a/b
```

[87]:
```
array([20.        , 15.        , 13.33333333, 12.5       ])
```

[88]:
```
list_a/list_b
```

```
---------------------------------------------------------------------------
TypeError                                 Traceback (most recent call last)
Cell In[88], line 1
----> 1 list_a/list_b

TypeError: unsupported operand type(s) for /: 'list' and 'list'
```

[89]:
```
a.shape
```

[89]:
```
(4,)
```

[90]:
```
a
```

[90]:
```
array([20, 30, 40, 50])
```

[91]:
```
a + 1
```

[91]:
```
array([21, 31, 41, 51])
```

Broadcasting with scalar numerical data type

[92]:
```
list_a + 1
```

```
---------------------------------------------------------------------------
TypeError                                 Traceback (most recent call last)
```

```
Cell In[92], line 1
----> 1 list_a + 1

TypeError: can only concatenate list (not "int") to list
```

[93]:
```
a
```

[93]:
```
array([20, 30, 40, 50])
```

[94]:
```
a < 30
```

[94]:
```
array([ True, False, False, False])
```

[95]:
```
list_a < 30
```

```
---------------------------------------------------------------------------
TypeError                                 Traceback (most recent call last)
Cell In[95], line 1
----> 1 list_a < 30

TypeError: '<' not supported between instances of 'list' and 'int'
```

[96]:
```
c = np.array([10,15,20])
```

[97]:
```
a
```

[97]:
```
array([20, 30, 40, 50])
```

[98]:
```
a + c
```

```
---------------------------------------------------------------------------
ValueError                                Traceback (most recent call last)
Cell In[98], line 1
----> 1 a + c

ValueError: operands could not be broadcast together with shapes (4,) (3,)
```

Shape mismatches

Arithmetic operation on 2-D arrays

[99]:
```
X = np.array([[1, 2], [3, 4]])
print(X)
```

```
[[1 2]
 [3 4]]
```

[100]:
```
Y = np.array([[1, -1], [0, 1]])
print(Y)
```

```
[[ 1 -1]
 [ 0  1]]
```

[101]:
```
X + Y
```

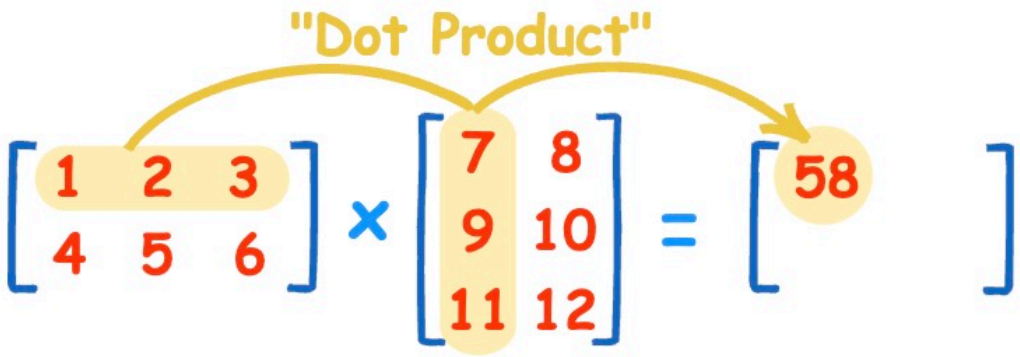
[101]:
```
array([[2, 1],
       [3, 5]])
```

[102]:
```
X * Y
```

[102]:
```
array([[ 1, -2],
       [ 0,  4]])
```

The multiplication using the `'*'` operator is element-wise.

What if we want to do matrix multiplication? Using the `'@'` operator:



[103]:
```
X @ Y
```

[103]:
```
array([[1, 1],
       [3, 1]])
```

Or equivalently we can use `np.dot()`:

[104]:
```
np.dot(X, Y)
```

[104]:
```
array([[1, 1],
       [3, 1]])
```

[105]:
```
Z = np.arange(12)
Z
```

[105]:
```
array([ 0,  1,  2,  3,  4,  5,  6,  7,  8,  9, 10, 11])
```

[106]:
```
Z.reshape(3,4) #reshape() change the shape of an array
```

[106]:
```
array([[ 0,  1,  2,  3],
       [ 4,  5,  6,  7],
       [ 8,  9, 10, 11]])
```

[107]:
```
Z.reshape((3,4)) #reshape() change the shape of an array
```

[107]:
```
array([[ 0,  1,  2,  3],
       [ 4,  5,  6,  7],
       [ 8,  9, 10, 11]])
```

[108]:
```
Z = Z.reshape(3,4)
Z
```

[108]:
```
array([[ 0,  1,  2,  3],
       [ 4,  5,  6,  7],
       [ 8,  9, 10, 11]])
```

[109]:
```
Z.sum()
```

[109]:
```
66
```

[110]:
```
Z.max()
```

[110]:
```
11
```

[111]:
```
Z.min()
```

[111]:
```
0
```

[112]:
```
Z
```

[112]:
```
array([[ 0,  1,  2,  3],
       [ 4,  5,  6,  7],
       [ 8,  9, 10, 11]])
```

[113]:
```
Z.sum(axis=0) # sum of each column
```

[113]:
```
array([12, 15, 18, 21])
```

[114]:
```
Z.sum(axis=1) # sum of each row
```

[114]:
```
array([ 6, 22, 38])
```

[115]:
```
Z.mean(axis=1) # average of each row
```

[115]:
```
array([1.5, 5.5, 9.5])
```

[116]:
```
Z.mean(axis=0) # average of each column
```

[116]:
```
array([4., 5., 6., 7.])
```

3.3 Indexing, Slicing and Iterating

- 1-dimensional arrays can be indexed, sliced and iterated over, much like lists and other Python sequences.
- Multidimensional arrays have one index per axis. These indices are given in a tuple separated by commas

[117]:
```
a = np.arange(10)**3
a
```

[117]:
```
array([  0,   1,   8,  27,  64, 125, 216, 343, 512, 729])
```

[118]:
```
a[0]
```

[118]:
```
0
```

[119]:
```
a[3]
```

[119]:
```
27
```

[120]:
```
a[2:5]
```

[120]:
```
array([ 8, 27, 64])
```

[121]:
```
for i in a:
    print(i)
```

```
0
1
8
27
64
125
216
343
512
729
```

[122]:
```
b = np.arange(12).reshape(3,4)
b
```

[122]:
```
array([[ 0,  1,  2,  3],
       [ 4,  5,  6,  7],
       [ 8,  9, 10, 11]])
```

[123]:
```
list_b = []
for i in b:
    list_b.append(list(i))
list_b
```

[123]:
```
[[0, 1, 2, 3], [4, 5, 6, 7], [8, 9, 10, 11]]
```

[124]:
```
list_b[0][0]
```

[124]:
```
0
```

[125]:
```
b
```

[125]:
```
array([[ 0,  1,  2,  3],
       [ 4,  5,  6,  7],
       [ 8,  9, 10, 11]])
```

[126]:
```
b[0,0]
```

[126]:
```
0
```

[127]:
```
b
```

[127]:
```
array([[ 0,  1,  2,  3],
       [ 4,  5,  6,  7],
       [ 8,  9, 10, 11]])
```

[128]:
```
b[2,3]
```

[128]:
```
11
```

[129]:
```
b[:2,0]
```

[129]:
```
array([0, 4])
```

[130]:
```
b[:,0]
```

[130]:
```
array([0, 4, 8])
```

[131]:
```
b
```

[131]:
```
array([[ 0,  1,  2,  3],
       [ 4,  5,  6,  7],
       [ 8,  9, 10, 11]])
```

[132]:
```
b[0]
```

[132]:
```
array([0, 1, 2, 3])
```

[133]:
```
b[0, :]
```

[133]:
```
array([0, 1, 2, 3])
```

The missing indices are considered complete slices: `b[0]` is equivalent to `b[0,:]`

Exercise:

`b = np.arange(12).reshape(3,4)`

- Obtain each column in the second and third row of b
- Obtain the first three rows and columns of b

[134]:
```
b = np.arange(12).reshape(3,4)
b
```

[134]:
```
array([[ 0,  1,  2,  3],
       [ 4,  5,  6,  7],
       [ 8,  9, 10, 11]])
```

[135]:
```
b[1:3,]
```

[135]:
```
array([[ 4,  5,  6,  7],
       [ 8,  9, 10, 11]])
```

[136]:
```
b[1:3,:]
```

[136]:
```
array([[ 4,  5,  6,  7],
       [ 8,  9, 10, 11]])
```

[137]:
```
b[1:3]
```

[137]:
```
array([[ 4,  5,  6,  7],
       [ 8,  9, 10, 11]])
```

[138]:
```
b[0:3,0:3]
```

[138]:
```
array([[ 0,  1,  2],
       [ 4,  5,  6],
       [ 8,  9, 10]])
```

[139]:
```
b[:3,:3]
```

[139]:
```
array([[ 0,  1,  2],
       [ 4,  5,  6],
       [ 8,  9, 10]])
```

[140]:
```
b
```

[140]:
```
array([[ 0,  1,  2,  3],
       [ 4,  5,  6,  7],
       [ 8,  9, 10, 11]])
```

[141]:
```
for i in b:
    print(i)
```

```
[0 1 2 3]
[4 5 6 7]
[ 8  9 10 11]
```

Iterating over multidimensional arrays is done with respect to the first axis: row by row

More flexible indexing - fancy indexing

- Indexing with Arrays of Indices
- Indexing with Boolean Arrays

[142]:
```
a = np.arange(12)**2  # the first 12 square numbers
a
```

[142]:
```
array([  0,   1,   4,   9,  16,  25,  36,  49,  64,  81, 100, 121])
```

[143]:
```
np.__version__
```

[143]:
```
'1.23.4'
```

[144]:
```
i = np.array([1, 1, 3, 8, 5])  # an array of indices
i
```

[144]:
```
array([1, 1, 3, 8, 5])
```

[145]:
```
a[i]  # the elements of `a` at the positions `i`
```

[145]:
```
array([ 1,  1,  9, 64, 25])
```

[146]:
```
j = np.array([[3, 4], [9, 7]])  # a bidimensional array of indices
j
```

[146]:
```
array([[3, 4],
       [9, 7]])
```

[147]:
```
a
```

[147]:
```
array([  0,   1,   4,   9,  16,  25,  36,  49,  64,  81, 100, 121])
```

[148]:
```
a[j] # the same shape as `j`
```

[148]:
```
array([[ 9, 16],
       [81, 49]])
```

What if a is multidimensional?

[149]:
```
a = a.reshape(4,3)
a
```

[149]:
```
array([[  0,   1,   4],
       [  9,  16,  25],
       [ 36,  49,  64],
       [ 81, 100, 121]])
```

[150]:
```
i = np.array([[2, 1],  # indices for the first dim of `a`
              [3, 3]])
```

[151]:
```
j = np.array([[0, 1],  # indices for the second dim of `a`
              [1, 2]])
```

[152]:
```
a[i,j]
```

[152]:
```
array([[ 36,  16],
       [100, 121]])
```

[153]:
```
a
```

[153]:
```
array([[  0,   1,   4],
       [  9,  16,  25],
       [ 36,  49,  64],
       [ 81, 100, 121]])
```

[154]:
```
b = a> 14
b
```

[154]:
```
array([[False, False, False],
       [False,  True,  True],
       [ True,  True,  True],
       [ True,  True,  True]])
```

[155]:
```
a[b]  # 1d array with the selected elements
```

[155]:
```
array([ 16,  25,  36,  49,  64,  81, 100, 121])
```

use boolean arrays that have the same shape as the original array

[156]:
```
a[b] = 0  # All elements of `a` higher than 14 become 0
a
```

[156]:
```
array([[0, 1, 4],
       [9, 0, 0],
       [0, 0, 0],
       [0, 0, 0]])
```

3.4 More on Shape manipulation

[157]:
```
a = np.arange(20)
a
```

[157]:
```
array([ 0,  1,  2,  3,  4,  5,  6,  7,  8,  9, 10, 11, 12, 13, 14, 15, 16,
       17, 18, 19])
```

[158]:
```
a.shape
```

[158]:
```
(20,)
```

[159]:
```
a.shape = (4,5)
```

[160]:
```
a
```

[160]:
```
array([[ 0,  1,  2,  3,  4],
       [ 5,  6,  7,  8,  9],
       [10, 11, 12, 13, 14],
       [15, 16, 17, 18, 19]])
```

[161]:
```
a.shape = (2,4)
```

```
---------------------------------------------------------------------------
ValueError                                Traceback (most recent call last)
Cell In[161], line 1
----> 1 a.shape = (2,4)

ValueError: cannot reshape array of size 20 into shape (2,4)
```

[162]:
```
a.shape = (2,10)
```

[163]:
```
a
```

[163]:
```
array([[ 0,  1,  2,  3,  4,  5,  6,  7,  8,  9],
       [10, 11, 12, 13, 14, 15, 16, 17, 18, 19]])
```

[164]:
```
a.transpose() # Transpose of the array
```

[164]:
```
array([[ 0, 10],
       [ 1, 11],
       [ 2, 12],
       [ 3, 13],
       [ 4, 14],
       [ 5, 15],
       [ 6, 16],
       [ 7, 17],
       [ 8, 18],
       [ 9, 19]])
```

[165]:
```
a.T # Transpose of the array
```

[165]:
```
array([[ 0, 10],
       [ 1, 11],
       [ 2, 12],
       [ 3, 13],
       [ 4, 14],
       [ 5, 15],
       [ 6, 16],
       [ 7, 17],
       [ 8, 18],
       [ 9, 19]])
```

[166]:
```
a.reshape(4,5)
```

[166]:
```
array([[ 0,  1,  2,  3,  4],
       [ 5,  6,  7,  8,  9],
       [10, 11, 12, 13, 14],
       [15, 16, 17, 18, 19]])
```

[167]:
```
a.reshape(4,-1)
```

[167]:
```
array([[ 0,  1,  2,  3,  4],
       [ 5,  6,  7,  8,  9],
       [10, 11, 12, 13, 14],
       [15, 16, 17, 18, 19]])
```

If in a reshaping operation a dimension is given as -1, it is automatically calculated to correspond to the other dimensions.

[168]:
```
a.reshape(7,-1)
```

```
---------------------------------------------------------------------------
ValueError                                Traceback (most recent call last)
Cell In[168], line 1
----> 1 a.reshape(7,-1)

ValueError: cannot reshape array of size 20 into shape (7,newaxis)
```

[169]:
```
mean_row = a.mean(axis=1)
mean_row
```

[169]:
```
array([ 4.5, 14.5])
```

[170]:
```
mean_row.shape
```

[170]:
```
(2,)
```

[171]:
```
mean_row + a
```

```
---------------------------------------------------------------------------
ValueError                                Traceback (most recent call last)
Cell In[171], line 1
----> 1 mean_row + a

ValueError: operands could not be broadcast together with shapes (2,) (2,10)
```

[172]:
```
a.shape
```

[172]:
```
(2, 10)
```

[173]:
```
mean_row = mean_row.reshape(2, -1)
mean_row
```

[173]:
```
array([[ 4.5],
       [14.5]])
```

[174]:
```
mean_row.shape
```

[174]:
```
(2, 1)
```

[175]:
```
mean_row + a
```

[175]:
```
array([[ 4.5,  5.5,  6.5,  7.5,  8.5,  9.5, 10.5, 11.5, 12.5, 13.5],
       [24.5, 25.5, 26.5, 27.5, 28.5, 29.5, 30.5, 31.5, 32.5, 33.5]])
```

3.4.1 Broadcasting for 2-d arrays

How NumPy treats arrays with different shapes during arithmetic operations * One dimension has the same size * The other dimension is of size 1

[176]:
```
a.flatten() # turn the array into 1-d
```

[176]:
```
array([ 0,  1,  2,  3,  4,  5,  6,  7,  8,  9, 10, 11, 12, 13, 14, 15, 16,
       17, 18, 19])
```

Stacking arrays together

[177]:
```
a
```

[177]:
```
array([[ 0,  1,  2,  3,  4,  5,  6,  7,  8,  9],
       [10, 11, 12, 13, 14, 15, 16, 17, 18, 19]])
```

[178]:
```
a.shape
```

[178]:
```
(2, 10)
```

[179]:
```
b=np.arange(200).reshape(-1,10)
b
```

[179]:
```
array([[  0,   1,   2,   3,   4,   5,   6,   7,   8,   9],
       [ 10,  11,  12,  13,  14,  15,  16,  17,  18,  19],
       [ 20,  21,  22,  23,  24,  25,  26,  27,  28,  29],
       [ 30,  31,  32,  33,  34,  35,  36,  37,  38,  39],
       [ 40,  41,  42,  43,  44,  45,  46,  47,  48,  49],
       [ 50,  51,  52,  53,  54,  55,  56,  57,  58,  59],
       [ 60,  61,  62,  63,  64,  65,  66,  67,  68,  69],
       [ 70,  71,  72,  73,  74,  75,  76,  77,  78,  79],
       [ 80,  81,  82,  83,  84,  85,  86,  87,  88,  89],
       [ 90,  91,  92,  93,  94,  95,  96,  97,  98,  99],
       [100, 101, 102, 103, 104, 105, 106, 107, 108, 109],
       [110, 111, 112, 113, 114, 115, 116, 117, 118, 119],
       [120, 121, 122, 123, 124, 125, 126, 127, 128, 129],
       [130, 131, 132, 133, 134, 135, 136, 137, 138, 139],
       [140, 141, 142, 143, 144, 145, 146, 147, 148, 149],
       [150, 151, 152, 153, 154, 155, 156, 157, 158, 159],
       [160, 161, 162, 163, 164, 165, 166, 167, 168, 169],
       [170, 171, 172, 173, 174, 175, 176, 177, 178, 179],
       [180, 181, 182, 183, 184, 185, 186, 187, 188, 189],
       [190, 191, 192, 193, 194, 195, 196, 197, 198, 199]])
```

[180]:
```
b.shape
```

[180]:
```
(20, 10)
```

[181]:
```
np.vstack((a,b)) #Stack arrays in sequence vertically (row wise): number of␣
↪columns have to match
```

[181]:
```
array([[  0,   1,   2,   3,   4,   5,   6,   7,   8,   9],
       [ 10,  11,  12,  13,  14,  15,  16,  17,  18,  19],
       [  0,   1,   2,   3,   4,   5,   6,   7,   8,   9],
       [ 10,  11,  12,  13,  14,  15,  16,  17,  18,  19],
       [ 20,  21,  22,  23,  24,  25,  26,  27,  28,  29],
       [ 30,  31,  32,  33,  34,  35,  36,  37,  38,  39],
       [ 40,  41,  42,  43,  44,  45,  46,  47,  48,  49],
       [ 50,  51,  52,  53,  54,  55,  56,  57,  58,  59],
       [ 60,  61,  62,  63,  64,  65,  66,  67,  68,  69],
       [ 70,  71,  72,  73,  74,  75,  76,  77,  78,  79],
       [ 80,  81,  82,  83,  84,  85,  86,  87,  88,  89],
       [ 90,  91,  92,  93,  94,  95,  96,  97,  98,  99],
       [100, 101, 102, 103, 104, 105, 106, 107, 108, 109],
       [110, 111, 112, 113, 114, 115, 116, 117, 118, 119],
       [120, 121, 122, 123, 124, 125, 126, 127, 128, 129],
       [130, 131, 132, 133, 134, 135, 136, 137, 138, 139],
       [140, 141, 142, 143, 144, 145, 146, 147, 148, 149],
       [150, 151, 152, 153, 154, 155, 156, 157, 158, 159],
       [160, 161, 162, 163, 164, 165, 166, 167, 168, 169],
       [170, 171, 172, 173, 174, 175, 176, 177, 178, 179],
       [180, 181, 182, 183, 184, 185, 186, 187, 188, 189],
       [190, 191, 192, 193, 194, 195, 196, 197, 198, 199]])
```

[182]:
```
np.hstack((a,b)) #Stack arrays in sequence horizontally (column wise): : number␣
↪of rows have to match
```

```
---------------------------------------------------------------------------
ValueError                                Traceback (most recent call last)
Cell In[182], line 1
----> 1 np.hstack((a,b))

File <__array_function__ internals>:180, in hstack(*args, **kwargs)

File ~/opt/anaconda3/lib/python3.9/site-packages/numpy/core/shape_base.py:345,␣
↪in hstack(tup)
    343     return _nx.concatenate(arrs, 0)
    344 else:
--> 345     return _nx.concatenate(arrs, 1)

File <__array_function__ internals>:180, in concatenate(*args, **kwargs)
```

```
ValueError: all the input array dimensions for the concatenation axis must matc
  ↪exactly, but along dimension 0, the array at index 0 has size 2 and the array
  ↪at index 1 has size 20
```

3.5 Other array functions

[183]:
```
a=np.array( [[ 7,2, 1], [4,3, 8] ])
a
```

[183]:
```
array([[7, 2, 1],
       [4, 3, 8]])
```

[184]:
```
np.sort?
```

[185]:
```
np.sort(a)
```

[185]:
```
array([[1, 2, 7],
       [3, 4, 8]])
```

[186]:
```
np.sort(a,axis=1)
```

[186]:
```
array([[1, 2, 7],
       [3, 4, 8]])
```

[187]:
```
np.sort(a,axis=0)
```

[187]:
```
array([[4, 2, 1],
       [7, 3, 8]])
```

3.6 Further reading

- read Numpy tutorial to learn more about numpy functionalities