Quantitative Methods in Social Research

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Day 2 – Outline

1. Understanding the Basics of Trends and Associations

2. Correlation

3. The Linear Regression Model

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- Association is a very general relationship: one variable provides information about another.In other words, two variables are associated if there is a pattern in the scatter-plot, which is too strong to be likely to arise simply by chance.
- Correlation measures a specific form of association: two variables are correlated when they display an increasing or decreasing trend. This is relevant to relationships with a linear trend.
- We can find patterns that look like one variable (X) is correlated with another (Y), even when that one variable does not **cause** the other variable.

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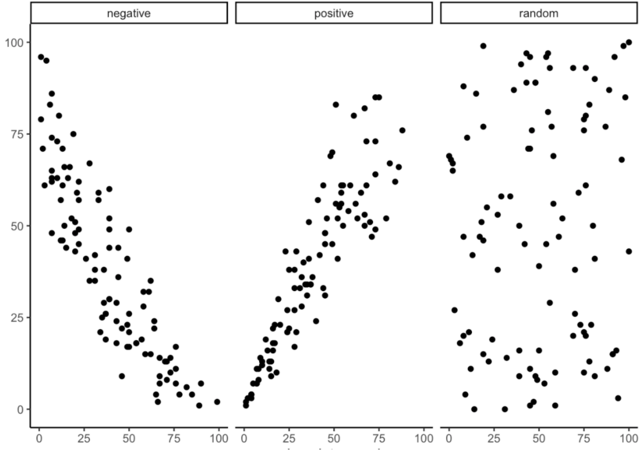
Positive, Negative, and No-Correlation

Imagine the scatter plot would look like if the relationship between variables X and Y was negative and positive or if there was no relationship.



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- Statistically speaking, co-variance is the multiplication of the deviations in X from the mean of X, and the deviation in Y from the mean of Y.

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- When we look at a co-variation statistic, we can see what direction it points, positive or negative. Yet, we don't know how big or small it is compared to the maximum or minimum possible value, so we don't know the relative size, which means we cannot say how strong the correlation is!

Pearson's Correlation r

- Therefore, for quantitative and ordinal data, we often use a certain measure of correlation, which is called Pearson's correlation (r). It measures linear trends!

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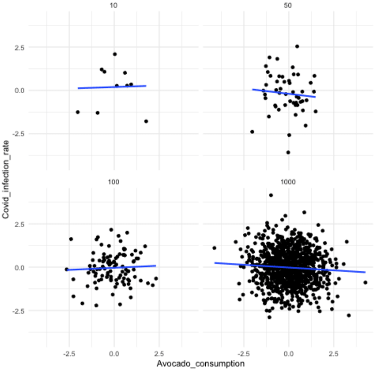
- Pearson's r is the co-variance of X and Y, divided by the product of the standard deviation of X and the standard deviation of Y. Why are we dividing the co-variance by the product of the standard deviations? Because this operation has the effect of normalising the co-variance into the range -1 to 1.

Spurious correlation

- This simulation illustrates some conundrums in interpreting correlations. They can be produced by random chance. This means that you can find a positive or negative correlation between two measures, even when they have absolutely nothing to do with one another.



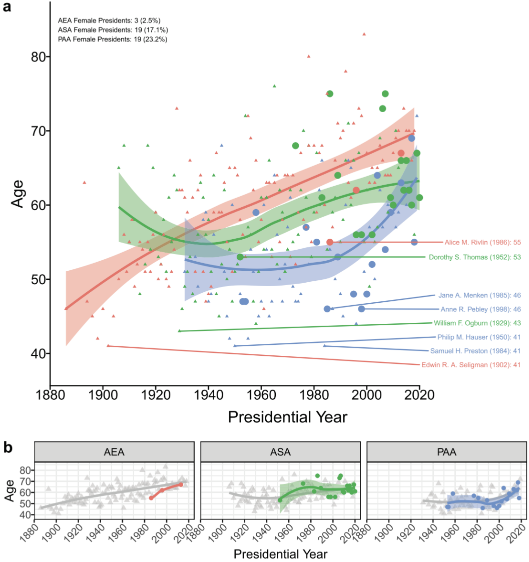
No correlation doesn’t mean no causality

- Just because there is no observable relationship does not mean there is no causal one!

No correlation doesn’t mean no causality.

CAUSAL INFERENCE: THE MIXTAPE, SCOTT CUNNINGHAM

Example: The Trends in Presidential Ages (Hauer, 2019)



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- Demography presidents have tended to be considerably younger than both sociology and economic presidents, though PAA presidents have aged considerably since the year 2000, approaching parity with the ASA only recently.
- The first female presidents for ASA and PAA appear almost simultaneously in the early 1950s, but female association presidents for ASA and PAA were still relatively rare until the mid-1980s.

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- What type of data is used in this research?
- What is the analytical strategy in this research?
- What are the main methodological strengths and limitations of the study?
- Methodologically speaking, how would you approach these questions?
- Do young academics lack the credentials of previous generations; is the age composition of association membership also aging; does the increasing number of female presidents reflect the slow erosion of patriarchy in sociology and demography; why has this shift not occurred in economics?

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- A model is a simplified abstraction of reality
- All models are wrong, but some are useful
- We will be using statistical models which will always be ‘wrong’, but some will be useful
- Simply put, a linear regression model is an approximation of the relationship between our independent variable X and our response variable Y

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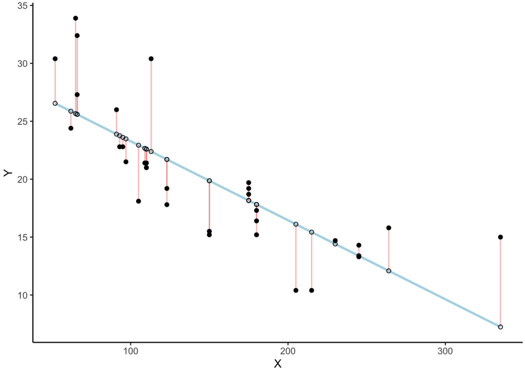
- By observing a vast number of individuals with varied combinations of characteristics, we seek to isolate the impact of one characteristic one is interested in on the outcome by looking at the outcome of groups that are similar on all counts except for the characteristic one is interested in

Linear Relationships



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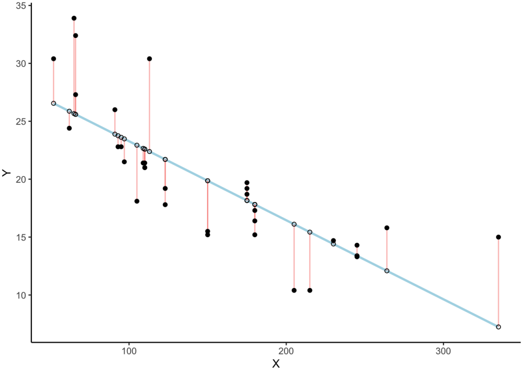

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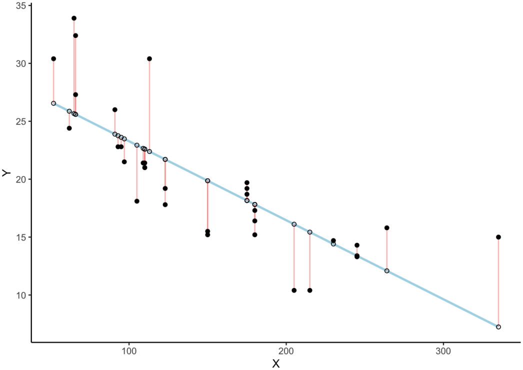

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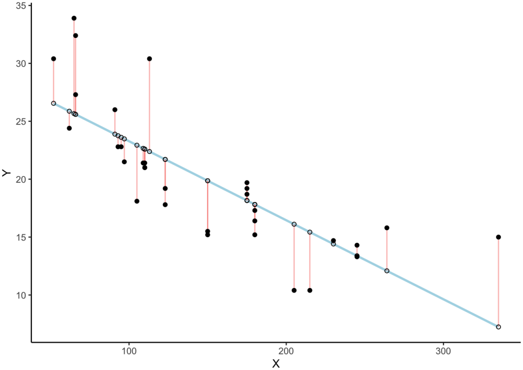

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- The slope (β) is the slant of the line, and the y-intercept is where the line crosses the y-axis when X is zero
- β is the amount that Y increases when X increases by one unit

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- What's is even the best line? It is the one that has the least error.
- Our goal is to estimate the line that 'best' fits our data

The Linear Regression Model

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where:

- Observations i = 1, . . . , n
- Y is the dependent variable.
- X is the independent variable.
- β_0 is the intercept or constant.
- β_1 is the slope.
- u_i is the error term or residuals.
- β_0 and β_1 are known as the coefficients of the regression line

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- The final term is the random part of the equation u_i represents all the other factors aside from X that determine the value of Y
- We want to know the population value of β_1 and β_0. We must estimate the values using a sample from the population

Ordinary Least Squares

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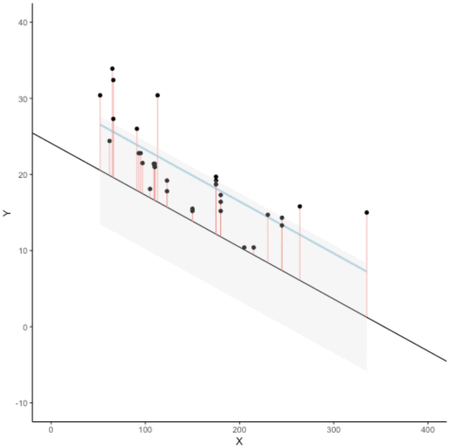
- The OLS estimator chooses the regression coefficients so that the estimated line is "as close as possible" to the data.
- It minimises the sum of the squared differences between the actual values of each observation (Y_i) and the predicted value of each value based on the estimated line ($\hat{Y}_i$).

The best fit line – regression

- For this line, the sum of the squared distances between Y_i and $\hat{Y_i}$:

$$\sum_{i=1}^{n}[Y_i - (\hat{\beta}_0 + \beta_1\hat{}X_i]^2 \tag{4}$$



Residuals

- These lines drop down from each dot, and they land on the line. Each of these little lines is called a residual. They show you how far off the line is for different dots. It's measure of error, it shows us just how wrong the line is. This means the line does not actually represent all of the dots. The best fit line is the least wrong of all the wrong lines.

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- If we wanted to know how wrong this line was, we could simply gather up all the red lines, measure how long they are, and then add all the wrongness together. This would give us the total amount of wrongness.
- What we will actually be doing with the red lines, is computing the sum of the squared deviations from the line. That sum is the total amount of error. Now, this blue line here minimizes the sum of the squared deviations. Any other line would produce a larger total error.

Regression analysis

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- Using more than one explanatory variables (continuous and discrete) forms multiple regression model.

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- A one-unit increase in X is associated with a $\hat{\beta}_1$ change in Y, on average
- A one year increase in the age of job candidates is associated with a decrease of 0.45 in hiring favourability, on average, holding constant other variables constant

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- **Normality of errors**: The regression assumption that is generally least important is that the errors are normally distributed. In fact, for the purpose of estimating the regression line (as compared to predicting individual data points), the assumption of normality is barely important at all. Even if there's no clear-cut consensus, we do not recommend diagnostics of the normality of regression residuals.

Model Fit

- How does our model perform?
- How tightly are the observations clustered around the line?
- What proportion of the variation in the dependent variable can be explained by the explanatory variable?

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- You can show mathematically that SSE + SSR is equal to the following expression, which is referred to as the total sum of squares (TSS) $TSS = \sum_{i=1}^{n}(\bar{y}_i - y_i)^2$

Hypothesis Test: Multiple Linear Regression

- More than one explanatory variables, continuous or discrete. Coefficients describe partial associations of explanatory variable with response variable, controlling for the other explanatory variable(s) in the model.
- H_0: in the population, there is no partial association between X_j and Y, controlling for the other explanatory variable(s) in the model.
- For a continuous variable, null hypothesis is that the slope or beta is 0.
- For a dummy variable, null hypothesis is no difference between the two categories.