Reduce GPS estimated position error in route navigation

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***Abstract*—Reduce estimated position error for route navigation. The purpose of this paper is to provide an application for route navigation using sensor fusion. In this practical example, we obtained sensor raw data using a smartphone application, then process it to the local reference frame and applied estimation algorithms like Kalman filter to obtain a more accurate position. Kalman filtering is very used in navigation systems for estimates uncertain variables using multiple sources, this tends to be more accurate than using a single measurement source. We successfully obtained a more accurate estimated position and reduce drifting from GPS.**

***Index Terms*—sensor fusion, Kalman filter, GPS, navigation.**

I. Introduction

The Global Positioning System (GPS) is a satellite-based navigation system, developed by the Department of Defense of the United States, free and accessible to anyone with a receiver. It is important in many fields like navigation, agriculture, tectonics, robotics, and autonomous vehicles. Consists of a constellation of satellites (space segment) around the orbit of Earth that broadcast radio signals providing information about the location and precise time based on atomic clocks. Those satellites it is maintained by worldwide monitor and control stations (control segment) that track the satellites in their proper orbits and adjust the satellite clocks. Radio signals travel through space and a GPS device receives (user space) these signals noting their exact time of arrival and uses these to calculate the distance from each satellite in view. Once a GPS device knows its distance from at least four satellites it can use geometry to determine its location on Earth in three dimensions: latitude, longitude, and altitude. However, there is not always a direct connection (line of sight) between the satellites and a user receptor. Accuracy depends on additional factors, including satellite availability, multi-path fading, signal blockage, atmospheric conditions, and receiver quality [6] [7].

In this experiment, the GPS and accelerometer data were obtained from a smartphone sensors using an Android app. In specialized geolocation devices, maximum sampling rates of 10 Hz can be obtained, however, the maximum GPS sampling rate that I can achieve on my device (Samsung Galaxy) is 1 Hz [1]. The obtained data expressed in geodetic coordinates (latitude, longitude, and altitude) in degrees were converted to an inertial reference system centered on a starting point since the measurement units of the accelerometer are expressed as a difference in position in meters. Using a script in Python, the system could be modeled using Kalman filtering to obtain a more precise measurement of a path followed by a person. According to the results obtained, the estimation in the position could be improved and the drift error of the GPS sensor could be eliminated.

II. Background

A. Coordinates system

To represent any location on the Earth's surface, three parameters are obtained: latitude (angle ϕ measured between the equatorial plane of the ellipsoid and the line perpendicular to the Earth's surface that passes through the desired location, positive for the north and negative for the south), length (angle λ measured between the base meridian and the meridian plane passing through the desired location, positive for the east and negative for the west) and height (distance h normal to the Earth's surface). Figure 1 shows the three parameters:

It is necessary to convert the coordinates between the geodetic system (latitude, longitude, and altitude) and a local coordinate system (x, y, z) to process the obtained data. This can be implemented in the following way as seen in figure 2:

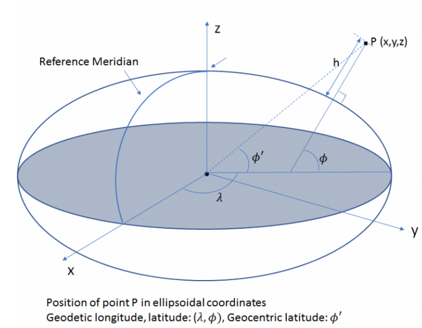

Fig. 1. Geodetic coordinates
https://www.telesens.co/wp-content/uploads/2017/07/img_596cec084e233-768x621.png

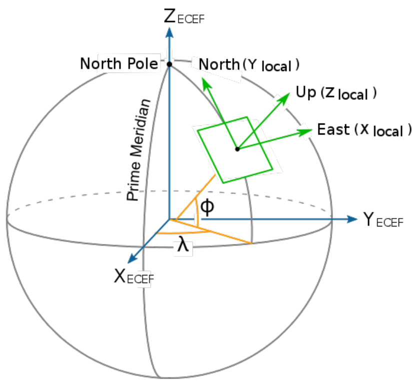

Fig. 2. Geodetic-ECEF-ENU relationship [9]

Geodetic to ECEF (Earth centered, Earth fixed) coordinates:

$$\begin{aligned} X &= (N(\phi) + h)\cos\phi\cos\lambda \\ Y &= (N(\phi) + h)\cos\phi\sin\lambda \\ Z &= \left(\frac{b^2}{a^2}N(\phi) + h\right)\sin\phi \\ N(\phi) &= \frac{a^2}{\sqrt{a^2\cos^2\phi + b^2\sin^2\phi}} \end{aligned}$$

when $a = 6\,378\,137\,\mathrm{m}$ and $b = 6\,357\,002\,\mathrm{m}$ are the equatorial radius (semi-major axis) and the polar radius (semi-minor axis) respectively. The inverse process is the following:

ECEF to Geodetic coordinates:

$$r = \sqrt{X^2 + Y^2}$$
$$e'^2 = \frac{a^2 - b^2}{b^2}$$
$$F = 54b^2Z^2$$
$$G = r^2 + (1 - e^2)Z^2 - e^2(a^2 - b^2)$$
$$c = \frac{e^4Fr^2}{G^3}$$
$$s = \sqrt[3]{1 + c + \sqrt{c^2 + 2c}}$$
$$P = \frac{F}{3(s + s^{-1} + 1)^2G^2}$$
$$Q = \sqrt{1 + 2e^4P}$$
$$r_0 = \frac{-Pe^2r}{1 + Q} + \sqrt{0.5a^2(1 + Q^{-1}) - \frac{P(1 - e^2)Z^2}{Q(1 + Q)} - 0.5Pr^2}$$
$$U = \sqrt{(r - e^2r_0)^2 + Z^2}$$
$$V = \sqrt{(r - e^2r_0)^2 + (1 - e^2)Z^2}$$
$$z_0 = \frac{b^2Z}{aV}$$

$$\begin{aligned} \phi &= \arctan\left(\frac{Z + e'^2z_0}{r}\right) \\ \lambda &= \arctan 2(Y, X) \\ h &= U\left(1 - \frac{b^2}{aV}\right) \end{aligned}$$

To convert from ECEF coordinates to a local reference frame East-North-Up (ENU) is performed by two steps: 1) Convert Geodetic coordinates to ECEF coordinates, 2) Convert ECEF coordinates to ENU coordinates:

ECEF to ENU coordinates:

$$\begin{aligned} x =& -\sin\lambda_0(X - X_0) + \cos\lambda_0(Y - Y_0) \\ y =& -\sin\phi_0\cos\lambda_0(X - X_0) - \sin\phi_0\sin\lambda_0(Y - Y_0) \\ &+ \cos\phi_0(Z - Z_0) \\ z =& \cos\phi_0\cos\lambda_0(X - X_0) + \cos\phi_0\sin\lambda_0(Y - Y_0) \\ &+ \sin\phi_0(Z - Z_0) \end{aligned}$$

when (X_0, Y_0, Z_0) and (ϕ_0, λ_0) are the reference point in local coordinates and geodetic coordinates respectively. The same manner to convert between ENU to ECEF coordinates is the following:

ENU to ECEF coordinates:

$$\begin{aligned} X =& -\sin\lambda_0x - \sin\phi_0\cos\lambda_0y \\ &+ \cos\phi_0\cos\lambda_0z + X_0 \\ Y =& \cos\lambda_0x - \sin\phi_0\sin\lambda_0y \\ &+ \cos\phi_0\sin\lambda_0z + Y_0 \\ Z =& \cos\phi_0y + \sin\phi_0z + Z_0 \end{aligned}$$

B. Multipath propagation

Environmental factors like interference and phenomena such as refraction, signal diffraction caused by obstacles between the transmitter and receiver, cause there may be errors in the position measurement due to the losses of the transmitted power of a signal through space. This is measured by the Path-Loss who are the losses in power density of a signal as it propagates through space and is given by the following formula:

$$L = 20\log_{10}\left(\frac{4\pi Df}{c}\right) \qquad (1)$$

when D is the distance in km, f the frequency in MHz and c is the speed of light in $\mathrm{m/s}$.

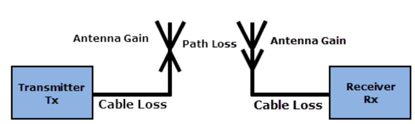

Fig. 3. Path-Loss calculation
https://www.pasternack.com/Images/reference-tools/images/fspl1_pic.png

C. *Kalman Filter [5]*

One of the algorithms most used in navigation is the so-called Kalman filter that allows us to recursively estimate a set of variables such as position and speed that contain uncertainty (noise). Using the Kalman filter to estimate the states of a process given a sequence of noisy observations, one must model the process following the next matrices:

1) $\mathbf{F}_k$: state-transition model
2) $\mathbf{B}_k$: control-input model
3) $\mathbf{u}_k$: control vector
4) $\mathbf{z}_k$: measurement vector
5) $\mathbf{H}_k$: observation model
6) $\mathbf{w}_k$: process noise
7) $\mathbf{Q}_k$: covariance matrix of the process noise $\mathbf{Q}_k$:$\mathbf{w}_k \sim \mathcal{N}(0, \mathbf{Q}_k)$
8) $\mathbf{v}_k$: observation noise
9) $\mathbf{R}_k$: covariance matrix of the observation noise $\mathbf{R}_k$:$\mathbf{v}_k \sim \mathcal{N}(0, \mathbf{R}_k)$

Each state evolved dynamically from the previous state following:

$$\mathbf{x}_k = \mathbf{F}_k \mathbf{x}_{k-1} + \mathbf{B}_k \mathbf{u}_k + \mathbf{w}_k$$

and

$$\mathbf{z}_k = \mathbf{H}_k \mathbf{x}_k + \mathbf{v}_k$$

Kalman filter is computed in two phases:
Predict:

$$\begin{aligned}\hat{\mathbf{x}}_{k|k-1} &= \mathbf{F}_k \hat{\mathbf{x}}_{k-1|k-1} + \mathbf{B}_k \mathbf{u}_k \\ \mathbf{P}_{k|k-1} &= \mathbf{F}_k \mathbf{P}_{k-1|k-1} \mathbf{F}_k^T + \mathbf{Q}_k\end{aligned}$$

Update:

$$\begin{aligned}\tilde{\mathbf{y}}_k &= \mathbf{z}_k - \mathbf{H}_k \hat{\mathbf{x}}_{k|k-1} \\ \mathbf{S}_k &= \mathbf{H}_k \mathbf{P}_{k|k-1} \mathbf{H}_k^T + \mathbf{R}_k \\ \mathbf{K}_k &= \mathbf{P}_{k|k-1} \mathbf{H}_k^T \mathbf{S}_k^{-1} \\ \hat{\mathbf{x}}_{k|k} &= \hat{\mathbf{x}}_{k|k-1} + \mathbf{K}_k \tilde{\mathbf{y}}_k \\ \mathbf{P}_{k|k} &= (\mathbf{I} - \mathbf{K}_k \mathbf{H}_k) \mathbf{P}_{k|k-1} \\ \tilde{\mathbf{y}}_{k|k} &= \mathbf{z}_k - \mathbf{H}_k \hat{\mathbf{x}}_{k|k}\end{aligned}$$

III. Materials and Methods

A. *Materials*

- Samsung Galaxy smartphone with Android
- Phyphox app [2]
- Google Maps API
- Python 3 programming language

B. *Methods*

A smartphone with the Phyphox application installed is used to obtain the data from the sensors during a walk along a previously defined path using Google Maps. The application flow is displayed in figure 4.

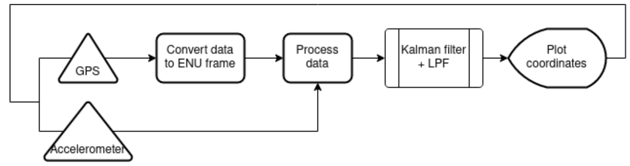

Fig. 4. Application flow

For the system made up of a person walking it can be modeled using the following kinematic equation:

$$\mathbf{r}_k = \mathbf{r}_{k-1} + \dot{\mathbf{r}}_{k-1} \Delta t + \frac{1}{2} \ddot{\mathbf{r}}_k \Delta t^2$$

and the space-state matrices are the following:

$$\mathbf{x}_k = \mathbf{F}_k \mathbf{x}_{k-1} + \mathbf{B}_k \mathbf{u}_k$$

$$\mathbf{x}_k = \begin{bmatrix} x_k \\ y_k \\ \dot{x}_k \\ \dot{y}_k \end{bmatrix} = \begin{bmatrix} 1 & 0 & \Delta t & 0 \\ 0 & 1 & 0 & \Delta t \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_{k-1} \\ y_{k-1} \\ \dot{x}_{k-1} \\ \dot{y}_{k-1} \end{bmatrix} + \begin{bmatrix} \frac{1}{2}(\Delta t)^2 & 0 \\ 0 & \frac{1}{2}(\Delta t)^2 \\ \Delta t & 0 \\ 0 & \Delta t \end{bmatrix} \begin{bmatrix} \ddot{x}_k \\ \ddot{y}_k \end{bmatrix}$$

$$\hat{\mathbf{x}}_{0|0} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\mathbf{P}_{0|0} = \begin{bmatrix} {\sigma_x}^2 & 0 & 0 & 0 \\ 0 & {\sigma_y}^2 & 0 & 0 \\ 0 & 0 & {\sigma_{\dot{x}}}^2 & 0 \\ 0 & 0 & 0 & {\sigma_{\dot{y}}}^2 \end{bmatrix}$$

$$\mathbf{w}_k \sim \mathcal{N}(0, \mathbf{Q})$$

$$\mathbf{Q}_k = \mathbf{B}_k \mathbf{B}_k^T {\sigma_a}^2$$

$$= \begin{bmatrix} \frac{1}{4}(\Delta t)^4 & 0 & \frac{1}{2}(\Delta t)^3 & 0 \\ 0 & \frac{1}{4}(\Delta t)^4 & 0 & \frac{1}{2}(\Delta t)^3 \\ \frac{1}{2}(\Delta t)^3 & 0 & (\Delta t)^2 & 0 \\ 0 & \frac{1}{2}(\Delta t)^3 & 0 & (\Delta t)^2 \end{bmatrix} {\sigma_a}^2$$

$$\mathbf{z}_k = \mathbf{H} \mathbf{x}_k + \mathbf{v}_k$$

$$\mathbf{z}_k = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_k \\ y_k \\ \dot{x}_k \\ \dot{y}_k \end{bmatrix}$$

$$\mathbf{v}_k \sim \mathcal{N}(0, \mathbf{R})$$

$$\mathbf{R} = \begin{bmatrix} {\sigma_x}^2 & 0 \\ 0 & {\sigma_y}^2 \end{bmatrix}$$

Several factors can cause loss of propagation of a signal like multipath fading, diffraction, refraction, etc and affect GPS accuracy as seen in table I. Then calculating the path loss using formula 1, the propagation losses for the GPS signal can be obtained.

TABLE I
GPS SIGNAL PROPAGATION LOSSES [8]

Distance (Km)	20180
Frequency (MHz)	1575.42
Transmitting antenna gain (dB)	13.5
Receiving antenna gain (dB)	3
Satellite transmission power (dBW)	14.25
Radio frequency losses (dB)	1.25
Loss of propagation in the atmosphere (dB)	0.5
Receiver connector losses (dB)	6.7

In figure 5, the propagation losses were obtained considering the free space for the GPS satellite signal at the base frequency (L1C) of $1575.42\,\mathrm{MHz}$. The power received at the receiver is improved through amplifiers in the device around 20dB, which allows better signal reception.

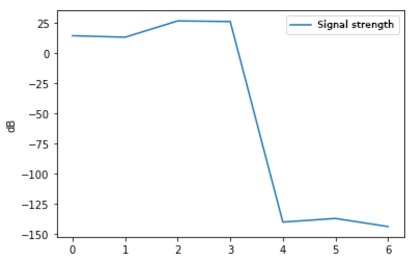

Fig. 5. Signal strength

The accuracy of the GPS signal may be affected due to loss of propagation of the GPS signal through space as seen in figure 6:

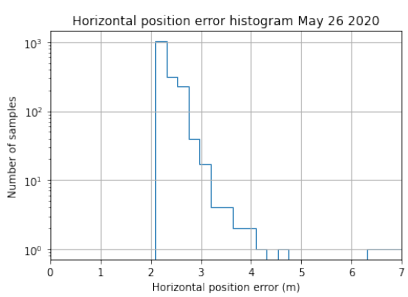

Fig. 6. Static GPS drift error

IV. IMPLEMENTATION

The source code of Pyrouter has been published under the GNU General Public License and is available at: https://github.com/brleoal/pyrouter

V. RESULTS

In figures 7 and 8, it is evident the corrections made in the displacement through different predefined routes, which increases the precision of the measurements in the face of various factors that could affect the reception of the GPS satellite signal.

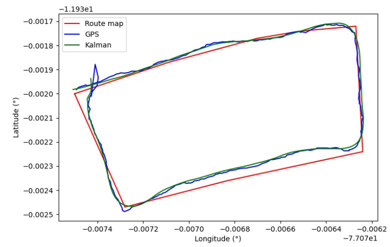

Fig. 7. Result 1 for Pyrouter application

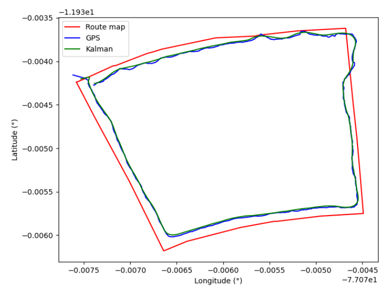

Fig. 8. Result 2 for Pyrouter application

VI. DISCUSSION AND SUMMARY

The proposed analysis could be verified with the obtained graphs. However, several considerations must be taken into account: The covariance values have been estimated by taking measurements of the sensors in a resting state. Kalman filtering is difficult to implement since it requires knowing with great accuracy these values that affect the precision of the estimated values. There are several methods to calculate them, one of which is the Auto-covariance Least Squares (ALS).

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