Homework Week 6

MATHEMATICS OF DEEP LEARNING
MASH & IASD 2025

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Instructions: This homework is **due on Monday 03/03/2025**. Please send your solutions in a Python Jupyter notebook file named HW6_NOM_PRENOM.IPYNB to the above address with the subject "*[MATHSDL2025] Homework 6*".

Packages allowed: Numpy, Matplotlib and sklearn. Notebooks with running errors will not be considered.

1 Exercises

Exercise 1.

Reproduce Figure 1 (left & right) from the lecture notes. Your notebook must show both theoretical curves (solid lines) obtained by solving the self-consistent equations and the simulations (crosses) obtaining by evaluating the ridge estimator, both discussed during the lectures.

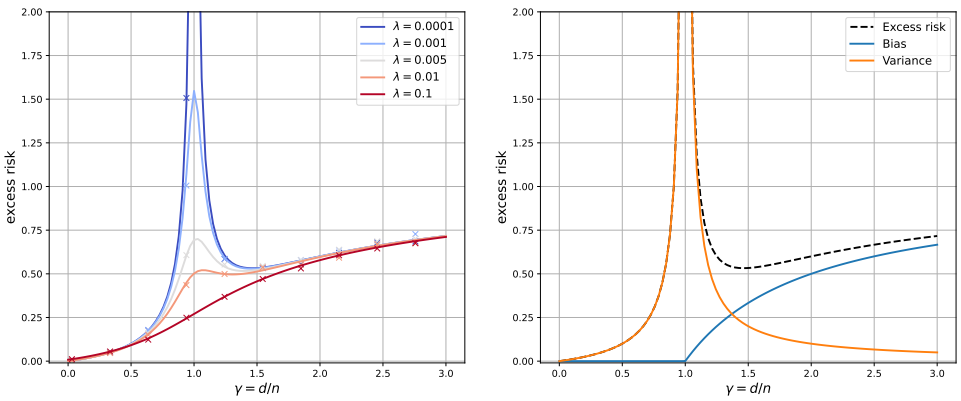

Figure 1: Excess risk of ridge regression as a function of $\gamma = {}^{d}/_{n}$ for $\sigma^2 = 0.1$, $||\boldsymbol{\theta}_\star||_2^2 = 1$ and isotropic covariates $\boldsymbol{\Sigma} = \boldsymbol{I}_d$. (**Left**) Increasing values of λ. (**Right**) Bias-variance decomposition of the excess risk for the ridge interpolator $\lambda = 0^+$ (a.k.a. ordinary least-squares estimator).

Exercise 2.

Reproduce Figures 2 from the lecture notes. Your notebook must show both the histogram of the empirical spectral density and the analytical curve of the Marchenko Pastur law discussed during the lectures.

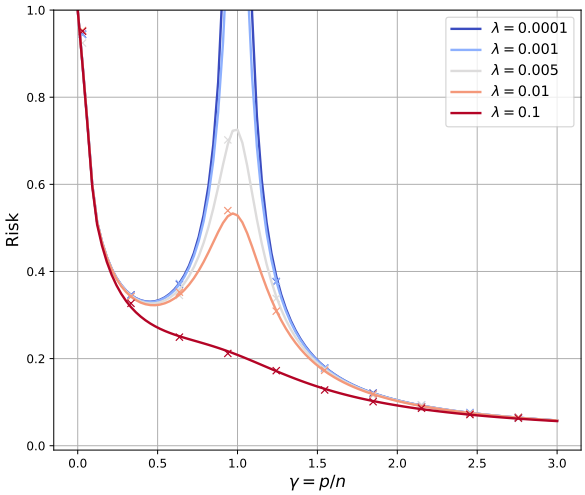

Figure 2: Risk of ridge regression as a function of $\gamma = p/n$ for the latent space model defined in Section 3.4 of the notes for $\tau^2 = 0$, $\alpha = n/d = 10$. Solid curves show the theoretical result, obtained from solving the self-consistent eq.(3.54), and crosses are finite size simulations with $d = 100$.