

$$1. (a) \gcd(132, 84) = \gcd(48, 84) = \gcd(48, 36) = \gcd(12, 36) = 12$$

$$(b) \text{ because } a, b \in \mathbb{N}, \text{ so } \gcd(a, a+b) = \gcd(a, b)$$

$$\text{because } a, b \text{ are co-prime, so } \gcd(a, b) = 1$$

$$\text{so } \Rightarrow \gcd(a, a+b) = \gcd(a, b) = 1$$

$$2. (a) (A \times B) \times (A \times B) = (A \cup B)^c \times (A \cup B)^c = ((A \cup B)^c)^c = A \cup B$$

$$(b) A^c = (A \cup A)^c = A \times A$$

$$(c) A \cap B = ((A \cap B)^c)^c = (A^c \cup B^c)^c = A^c \times B^c = (A \times A) \times (B \times B)$$

$$3. (a) \text{ all possible function: } \begin{array}{l} \textcircled{1} f(a)=0 \quad f(b)=0 \quad f(c)=0 \\ \textcircled{2} f(a)=0 \quad f(b)=0 \quad f(c)=1 \\ \textcircled{3} f(a)=0 \quad f(b)=1 \quad f(c)=0 \\ \textcircled{4} f(a)=0 \quad f(b)=1 \quad f(c)=1 \\ \textcircled{5} f(a)=1 \quad f(b)=0 \quad f(c)=0 \\ \textcircled{6} f(a)=1 \quad f(b)=0 \quad f(c)=1 \\ \textcircled{7} f(a)=1 \quad f(b)=1 \quad f(c)=0 \\ \textcircled{8} f(a)=1 \quad f(b)=1 \quad f(c)=1 \end{array}$$

(b) The 0, 1 in co-domain represent the a, b, c in domain appear or not in element of $\text{pow}(\{a, b, c\})$

$$(c) \textcircled{i} n^m$$

$$\textcircled{ii} 2^{m+n} \quad (\cancel{2^m \cdot 2^n})$$

$$4. (a) \lambda, aaa, aab, aba, abb, baa, bab, bba, bbb$$

$$(b) \textcircled{i}, \textcircled{ii}, \textcircled{iv} \text{ are elements of } R$$

$$(b) \text{ Define } S \subseteq \Sigma^* \times \Sigma^* \text{ as follows: } (w, w') \in S \text{ if there is a } v \in \Sigma^* \text{ such that: either } wv \in L \text{ and } w'v \in L \text{ or } wv \notin L \text{ and } w'v \notin L.$$

(c)

$$\textcircled{1} \text{ Reflexive: if } (w, w') \in S \text{ and } w = w', \text{ so if we have } wv \in L, \text{ then } w'v \in L, \text{ or } wv \notin L, \text{ then } w'v \notin L. \text{ So we proved that } (w, w) \in S, w \in \Sigma^*.$$

So S is reflexive

$$\textcircled{2} \text{ Symmetric (prove if } (x, y) \in S \text{ then } (y, x) \in S)$$

$$\text{That is means that if there is a } v \in \Sigma^*, \text{ such that either } xv \notin L \text{ and } yv \notin L \text{ or } xv \in L \text{ and } yv \in L$$

$$\text{so we have either } yv \notin L \text{ and } xv \notin L \text{ or } yv \in L \text{ and } xv \in L, \text{ too.}$$

$$\text{Therefore, we know that } (y, x) \in S, S \text{ is symmetric}$$

$$\textcircled{3} \text{ Transitive (should prove if } (x, y) \in S, (y, z) \in S, \text{ then } (x, z) \in S$$

$$(x, y) \in S \Rightarrow \text{either } xv \in L \text{ and } yv \in L \text{ or } xv \notin L \text{ and } yv \notin L$$

$$(y, z) \in S \Rightarrow \text{either } yv \in L \text{ and } zv \in L \text{ or } yv \notin L \text{ and } zv \notin L$$

$$\text{so we can get from above two that } \forall v \in \Sigma^*, \text{ either } xv \in L \text{ and } zv \in L, \text{ or } xv \notin L \text{ and } zv \notin L$$

so $(x, z) \in S$, S is transitive.

Because S is reflexive, symmetric and transitive, S is equivalence relation.

(d) S have 3 equivalence classes

$$A \cup B = (A \cup B)^c = (A^c \cap B^c)^c = (A^c \cap B^c)^c = (A^c \cup B^c)^c = (A \cap B)^c = A^c \cup B^c$$

$$A \cap B = (A \cap B)^c = (A^c \cup B^c)^c = (A^c \cap B^c)^c = (A \cup B)^c = A^c \cap B^c$$

$$A \times B = (A \times B)^c = (A^c \times B^c)^c = (A^c \cup B^c)^c = (A \cap B)^c = A^c \cup B^c$$

all possible function:

① $f(a)=0, f(b)=0$	② $f(a)=0, f(b)=1$	③ $f(a)=1, f(b)=0$	④ $f(a)=1, f(b)=1$
⑤ $f(a)=0, f(b)=0$	⑥ $f(a)=0, f(b)=1$	⑦ $f(a)=1, f(b)=0$	⑧ $f(a)=1, f(b)=1$
⑨ $f(a)=0, f(b)=0$	⑩ $f(a)=0, f(b)=1$	⑪ $f(a)=1, f(b)=0$	⑫ $f(a)=1, f(b)=1$
⑬ $f(a)=0, f(b)=0$	⑭ $f(a)=0, f(b)=1$	⑮ $f(a)=1, f(b)=0$	⑯ $f(a)=1, f(b)=1$
⑰ $f(a)=0, f(b)=0$	⑱ $f(a)=0, f(b)=1$	⑲ $f(a)=1, f(b)=0$	⑳ $f(a)=1, f(b)=1$
㉑ $f(a)=0, f(b)=0$	㉒ $f(a)=0, f(b)=1$	㉓ $f(a)=1, f(b)=0$	㉔ $f(a)=1, f(b)=1$
㉕ $f(a)=0, f(b)=0$	㉖ $f(a)=0, f(b)=1$	㉗ $f(a)=1, f(b)=0$	㉘ $f(a)=1, f(b)=1$
㉙ $f(a)=0, f(b)=0$	㉚ $f(a)=0, f(b)=1$	㉛ $f(a)=1, f(b)=0$	㉜ $f(a)=1, f(b)=1$

(b) The 0, 1 in co-domain represent the a, b in domain appear or not in element of $\{a, b, c\}$

(i) $f(a)=0, f(b)=0$
(ii) $f(a)=0, f(b)=1$
(iii) $f(a)=1, f(b)=0$
(iv) $f(a)=1, f(b)=1$

(v) $f(a)=0, f(b)=0$
(vi) $f(a)=0, f(b)=1$
(vii) $f(a)=1, f(b)=0$
(viii) $f(a)=1, f(b)=1$

Define $\Sigma \subseteq \Sigma^* \times \Sigma^*$ as follows: $(w, w') \in \Sigma$ if there is a $v \in \Sigma^*$ such that $w = vw'$ and $w' \neq \epsilon$.

Reflexive: if $(w, w) \in \Sigma$ and $w = w$, so we have $w \neq \epsilon$, then $w \neq \epsilon$ or $w = \epsilon$. So we have $w \neq \epsilon$ or $w = \epsilon$.

So Σ is reflexive.
Symmetric: if $(x, y) \in \Sigma$, then $(y, x) \in \Sigma$.
Transitive: if $(x, y) \in \Sigma$ and $(y, z) \in \Sigma$, then $(x, z) \in \Sigma$.
So we have either $x \neq \epsilon$ or $y \neq \epsilon$ or $z \neq \epsilon$, so $x \neq \epsilon$ or $y \neq \epsilon$ or $z \neq \epsilon$.