1. (a) $\gcd(132, 84) = \gcd(48, 84) = \gcd(48, 36) = \gcd(12, 36) = 12$

(b) because $a, b \in N$, so $\gcd(a, a+b) = \gcd(a, b)$

because a, b are co-prime, so $\gcd(a, b) = 1$

so $\Rightarrow$ $\gcd(a, a+b) = \gcd(a, b) = 1$

2. (a) $(A*B)*(A*B) = (A \cup B)^c * (A \cup B)^c = ((A \cup B)^c)^c = A \cup B$

(b) $A^c = (A \cup A)^c = A*A$

(c) $A \cap B = ((A \cap B)^c)^c = (A^c \cup B^c)^c = A^c * B^c = (A*A)*(B*B)$

3. (a) all possible function: ① $f(a)=0$ $f(b)=0$ $f(c)=0$

② $f(a)=0$ $f(b)=0$ $f(c)=1$

③ $f(a)=0$ $f(b)=1$ $f(c)=0$

④ $f(a)=0$ $f(b)=1$ $f(c)=1$

⑤ $f(a)=1$ $f(b)=0$ $f(c)=0$

⑥ $f(a)=1$ $f(b)=0$ $f(c)=1$

⑦ $f(a)=1$ $f(b)=1$ $f(c)=0$

⑧ $f(a)=1$ $f(b)=1$ $f(c)=1$

(b) The 0, 1 in co-domain represent the a, b, c in domain appear or not in element of pow({a, b, c})

(c) (i) n^m

(ii) 2^{m+n} $\cancel{(2^m \cdot 2^n)}$

4. (a) λ, aaa, aab, aba, abb, baa, bab, bba, bbb

(b) (ii) (iii) (iv) are elements of R

(b) Define $S \subseteq \Sigma^* \times \Sigma^*$ as follows: $(w, w') \in S$ if there is a $v \in \Sigma^*$ such that: either $wv \in L$ and $w'v \in L$ or $wv \notin L$ and $w'v \notin L$.

(c)

① Reflexive: if $(w, w') \in S$ and $w = w'$, so if we have $wv \in L$, then $w'v \in L$, or $wv \notin L$, then $w'v \notin L$. So we proved that $(w, w) \in S$, $w \in \Sigma^*$.

So S is reflexive

② Symmetric (prove if $(x, y) \in S$ then $(y, x) \in S$)

That is means that if there is a $v \in \Sigma^*$, such that either $xv \notin L$ and $yv \notin L$ or ~~either~~ $xv \in L$ and $yv \in L$

so we have either $yv \notin L$ and $xv \notin L$ or $yv \in L$ and $xv \in L$, too.

therefore, we know that $(y, x) \in S$, S is symmetric

③ Transitive (should prove if $(x, y) \in S$, $(y, z) \in S$, then $(x, z) \in S$

$(x, y) \in S \Rightarrow$ ~~either~~ $\forall v \in \Sigma^*$. either $xv \in L$ and $yv \in L$ or $xv \notin L$ and $yv \notin L$

$(y, z) \in S \Rightarrow$ $\forall v \in \Sigma^*$, either $yv \in L$ and $zv \in L$ or $yv \notin L$ and $zv \notin L$

so we can get from above two that $\forall v \in \Sigma^*$, either $xv \in L$ and $zv \in L$, or $xv \notin L$ and $zv \notin L$

so $(x,z) \in S$, S is transitive.

Because S is reflexive, symmetric and transitive, S is equivalence relation.

(d) S have 3 equivalence classes

The first one $S_0 = \{ w \in \Sigma^* \mid \text{length}(w) \bmod 3 = 0 \}$

Next one $S_1 = \{ w \in \Sigma^* \mid \text{length}(w) \bmod 3 = 1 \}$

final one $S_2 = \{ w \in \Sigma^* \mid \text{length}(w) \bmod 3 = 2 \}$