1. (a) $\gcd(132,84) = \gcd(48,84) = \gcd(48,36) = \gcd(12,36) = 12$

(b) because $a,b \in \mathbb{N}$, so $\gcd(a, a+b) = \gcd(a,b)$

because a,b are co-prime, so $\gcd(a,b)=1$

so $\Rightarrow$ $\gcd(a,a+b) = \gcd(a,b) = 1$

2. (a) $(A*B)*(A*B) = (A\cup B)^c * (A\cup B)^c = ((A\cup B)^c)^c = A\cup B$

(b) $A^c = (A\cup A)^c = A*A$

(c) $A\cap B = ((A\cap B)^c)^c = (A^c \cup B^c)^c = A^c * B^c = (A*A)*(B*B)$

3. (a) all possible function: ① $f(a)=0$ $f(b)=0$ $f(c)=0$

② $f(a)=0$ $f(b)=0$ $f(c)=1$

③ $f(a)=0$ $f(b)=1$ $f(c)=0$

④ $f(a)=0$ $f(b)=1$ $f(c)=1$

⑤ $f(a)=1$ $f(b)=0$ $f(c)=0$

⑥ $f(a)=1$ $f(b)=0$ $f(c)=1$

⑦ $f(a)=1$ $f(b)=1$ $f(c)=0$

⑧ $f(a)=1$ $f(b)=1$ $f(c)=1$

(b) The 0, 1 in co-domain represent the a, b, c in domain appear or not in element of $pow(\{a,b,c\})$

(c) (i) n^m

(ii) 2^{m+n} ~~$(2^m \cdot 2^n)$~~

4. (a) λ, aaa, aab, aba, abb, baa, bab, bba, bbb

(b) (ii) (iii) (iv) are elements of R

(b) Define $S \subseteq \Sigma^* \times \Sigma^*$ as follows: $(w,w') \in S$ if there is a $v \in \Sigma^*$ such that: either $wv \in L$ and $w'v \in L$ or $wv \notin L$ and $w'v \notin L$.

(c)

① Reflexive: if $(w,w') \in S$ and $w=w'$, so if we have $wv \in L$, then $w'v \in L$, or $wv \notin L$, the $w'v \notin L$. So we proved that $(w,w) \in S$, $w \in \Sigma^*$.

So S is reflexive

② symmetric (prove if $(x,y) \in S$ then $(y,x) \in S$)

That is means that if there is a $v \in \Sigma^*$, such that either $xv \notin L$ and $yv \notin L$ or ~~either~~ $xv \in L$ and $yv \in L$

so we have either $yv \notin L$ and $xv \notin L$ or $yv \in L$ and $xv \in L$, too.

Therefore, we know that $(y,x) \in S$, S is symmetric

③ Transitive (should prove if $(x,y) \in S$, $(y,z) \in S$, then $(x,z) \in S$

$(x,y) \in S \Rightarrow$ ~~either~~ $\forall v \in \Sigma^*$. either $xv \in L$ and $yv \in L$ or $xv \notin L$ and $yv \notin L$

$(y,z) \in S \Rightarrow$ $\forall v \in \Sigma^*$, either $yv \in L$ and $zv \in L$ or $yv \notin L$ and $zv \notin L$

so we can get from above two that $\forall v \in \Sigma^*$, either $xv \in L$ and $zv \in L$, or $xv \notin L$ and $zv \notin L$

So $(x,z) \in S$, S is transitive.

Because S is reflexive, symmetric and transitive. S is equivalence relation.

(d) S have 2 equivalence classes

According to the rule of S, every elements in S ~~can be~~ such as $(w,w') \in S$ can be divided into 2 equivalence classes. The first one is the length of w, w' is $3k$ $(k \in N)$. This is mean that $3 \mid \text{length}(w)$, $3 \mid \text{length}(w')$. The second one is the length of w, w' satisfies $3 \nmid \text{length}(w)$, $3 \nmid \text{length}(w')$.