COMP9318 Assignment 1 Solution

Ran Bai z5187292

2020 March

Question 1:

(1)
The table as follow,

Location	Time	Item	SUM(Quantify)
Sydney	2005	PS2	1400
Sydney	2005	ALL	1400
Sydney	2006	PS2	1500
Sydney	2006	Wii	500
Sydney	2006	ALL	2000
Sydney	ALL	PS2	2900
Sydney	ALL	Wii	500
Sydney	ALL	ALL	3400
Melbourne	2005	XBox 360	1700
Melbourne	2005	ALL	1700
Melbourne	ALL	XBox 360	1700
Melbourne	ALL	ALL	1700
ALL	2005	PS2	1400
ALL	2005	XBox 360	1700
ALL	2005	ALL	3100
ALL	2006	PS2	1500
ALL	2006	Wii	500
ALL	2006	ALL	2000
ALL	ALL	PS2	2900
ALL	ALL	Wii	500
ALL	ALL	XBox 360	1700
ALL	ALL	ALL	5100

(2)
Equivalent SQL statement as below:

```
SELECT Location, Time, Item, SUM(Quantity)
FROM R
GROUP BY Location, Time, Item
UNION ALL
SELECT Location, Time, ALL, SUM(Quantity)
FROM R
GROUP BY Location, Time
UNION ALL
SELECT Location, ALL, Item, SUM(Quantity)
FROM R
GROUP BY Location, Item
UNION ALL
SELECT ALL, Time, Item, SUM(Quantity)
FROM R
GROUP BY Time, Item
UNION ALL
SELECT Location, ALL, ALL, SUM(Quantity)
FROM R
```

```
GROUP BY Location
UNION ALL
SELECT ALL, Time, ALL, SUM(Quantity)
FROM R
GROUP BY Time
UNION ALL
SELECT ALL, ALL, Item, SUM(Quantity)
FROM R
GROUP BY Item
UNION ALL
SELECT ALL, ALL, ALL, SUM(Quantity)
FROM R
```

(3)
The iceberg cube is

Location	Time	Item	SUM(Quantity)
Sydney	2006	ALL	2000
Sydney	ALL	PS2	2900
Sydney	ALL	ALL	3400
ALL	2005	ALL	3100
ALL	2006	ALL	2000
ALL	ALL	PS2	2900
ALL	ALL	ALL	5100

(4)
Use the function
f(Location, Time, Item) = 12 * Location + 4 * Time + Item

Step 1:
Mappings tables

Location	Value	Time	Value	Item	Value
ALL	0	ALL	0	ALL	0
Sydney	1	2005	1	PS2	1
Melbourne	2	2006	2	Xbox 360	2
				Wii	3

Location	Time	Item	SUM(Quantify)	offset
1	1	1	1400	17
1	1	0	1400	16
1	2	1	1500	21
1	2	3	500	23
1	2	0	2000	20
1	0	1	2900	13
1	0	3	500	15
1	0	0	3400	12
2	1	2	1700	30
2	1	0	1700	28
2	0	2	1700	26
2	0	0	1700	24
0	1	1	1400	5
0	1	2	1700	6
0	1	0	3100	4
0	2	1	1500	9

0	2	3	500	11
0	2	0	2000	8
0	0	1	2900	1
0	0	3	500	3
0	0	2	1700	2
0	0	0	5100	0

Step 2:
So the final result is:

ArrayIndex	Value
17	1400
16	1400
21	1500
23	500
20	2000
13	2900
15	500
12	3400
30	1700
28	1700
26	1700
24	1700
5	1400
6	1700
4	3100
9	1500
11	500
8	2000
1	2900
3	500
2	1700
0	5100

Question 2:

The process of performing group average hierarchical clustering on given similarity matrix as below,
Step 1:

	p1	p2	p3	p4	p5
p1	1.00	0.10	0.41	0.55	0.35
p2	0.10	1.00	0.64	0.47	0.98
p3	0.41	0.64	1.00	0.44	0.85
p4	0.55	0.47	0.44	1.00	0.76
p5	0.35	0.98	0.85	0.76	1.00

From table above, we can know that combine p5 with p2 to a new cluster, named p2 instead.

Step 2:
Calculate the new similarity matrix by the given formula,

$$similarity(Cluster_i, Cluster_j) = \frac{\sum_{\substack{p_i, p_j \in Cluster_i \cup Cluster_j \\ p_j \neq p_i}} similarity(p_i, p_j)}{(|Cluster_i| + |Cluster_j|) * (|Cluster_i| + |Cluster_j| - 1)}$$

Cluster now as below,
New p1:{p1}, new p2:{p2, p5}, new p3:{p3}, new p4:{p4}
So, the only change inter-cluster similarity are all about p2,

$$similarity(p_1, p_2) = 2 * \frac{similarity(p_1, p_2) + similarity(p_1, p_5) + similarity(p_2, p_5)}{3*2} = 2 * \frac{0.1 + 0.35 + 0.98}{6} \approx 0.48$$

$$similarity(p_2, p_3) = 2 * \frac{similarity(p_3, p_2) + similarity(p_3, p_5) + similarity(p_2, p_5)}{3*2} = 2 * \frac{0.64 + 0.85 + 0.98}{6} \approx 0.82$$

$$similarity(p_2, p_4) = 2 * \frac{similarity(p_4, p_2) + similarity(p_4, p_5) + similarity(p_2, p_5)}{3*2} = 2 * \frac{0.47 + 0.76 + 0.98}{6} \approx 0.74$$

	p1	p2	p3	p4
p1	1.00	0.48	0.41	0.55
p2	0.48	1.00	0.82	0.74
p3	0.41	0.82	1.00	0.44
p4	0.55	0.74	0.44	1.00

From table above, we know that we should combine p2 with p3, named p2 instead.

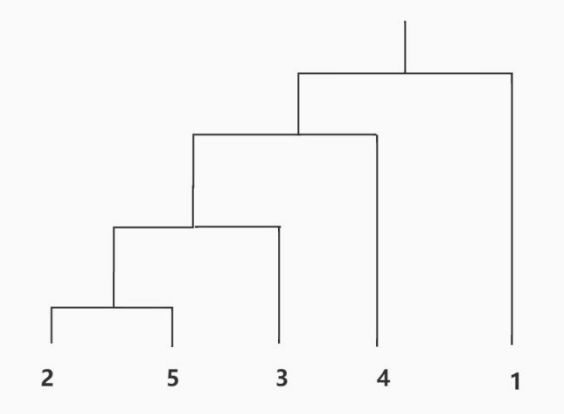
Step 3:
Cluster now are,
New p1:{p1}, new p2:{p2, p3, p5}, new p3:{p4}

$$similarity(p_1, p_2) = 2 * \frac{\sum similarity(p_i, p_j)_{i,j \in (1,2,3,5)}}{4*3} = 2 * \frac{0.1 + 0.41 + 0.35 + 0.64 + 0.98 + 0.85}{12} \approx 0.56$$

$$similarity(p_2, p_3) = 2 * \frac{\sum similarity(p_i, p_j)_{i,j \in (2,3,4,5)}}{4*3} = 2 * \frac{0.64 + 0.47 + 0.98 + 0.44 + 0.85 + 0.76}{12} \approx 0.69$$

	p1	p2	p3
p1	1.00	0.56	0.55
p2	0.56	1.00	0.69
p3	0.55	0.69	1.00

From table above, we can know that it should combine p2 with p3, name p2 instead.
Step 4:
Cluster now are:
New p1:{p1}, new p2:{p2, p3, p4, p5}, and the dendrogram is:



Question 3:

(1)
The stopping criterion is till the algorithm converges to the final k clusters. So the algorithm is as below,

Algorithm 1: k-means(D, k)
1 Initialize k centers C = [c1, c2, . . . , ck];
2 canStop ← false;
3 while canStop = false do

```
4       Initialize k empty clusters G = [g1, g2, . . . , gk];
5       for each data point p ∈ D do
6               cx ← NearestCenter(p, C);
7               gcx .append(p);
        Copy list C to list C'
8       for each group g ∈ G do
9               ci ← ComputeCenter(g);
        If evey center in list C' equal to list C:
            canStop = True;
10 return G;
```

(2)
In every iteration, the k-means algorithm only do two things, update the classification of m points and take the mean point as center of cluster. As below, through these two steps, I will explain why the cost of k cluster as evaluated at the end of each iteration never increases.
1. The n points are assigned to the existing k centers, according to the rules that assign the point to the closest center. Compared with assigning the point with other ways, the method can reduce the cost function. This is because $dist(p,c_i)$ is smaller with the way of clustering the node to the closest center. So $\cos t(g_i)$ and $\cos t(g_1,g_2.....g_i)$ will decrease.

2. Assume a category have N points, and its center is u_1. Now the cost function of this category is $\sum_{i=1}^{N}(x_i-u_1)^2$, and we know $\sum_{i=1}^{N}(x_i-x)^2 \geq \sum_{i=1}^{N}(x_i-u_1)^2$ which x is a random point. So this steps decrease the cost function, too.
In conclusion, the total cost will not increase.

(3)
We know the distance will not less than 0, so $Cost(gi)=\sum_{p\in g_i} dist^2(p,c_i)$ not less than 0. Also, $Cost(g_1,g_2....g_k)$ is not less than 0.
After that, the loop in the algorithm is finite. This is because the possible of cluster is finite(It is k^n, k is number of cluster and n is number of node). Using conclusion of question 2, we can know that the total cost of cluster will never increased.
So the worst case is that it will get the value under k^n decrease. It will get the local minimum which is not less than 0.
In conclusion, it always converges to a local minimum.