Week 4: Reasoning with Uncertainty

Tutorial 8: Reasoning with Uncertainty

8.1 (Activity 10.1: Conditional Probability - Open Learning)

Only 4% of the population are colour blind, but 7% of men are colour blind.

What percentage of colour blind people are men?   Explain your answer...

8.2 (Activity 10.2: Enumerating Probabilities - Open Learning)

Complete the activity from the Conditional Independence page (Exercise 13.8 from Russell & Norvig):

1 Given the full joint probability distribution shown above, calculate the following:

a. $P(\texttt{toothache} \wedge \neg\texttt{catch})$
b. $P(\texttt{catch})$
c. $P(\texttt{cavity} \mid \texttt{catch})$
d. $P(\texttt{cavity} \mid \texttt{toothache} \vee \texttt{catch})$

2 Verify the conditional independence claimed above, by showing that

$$P(\texttt{catch} \mid \texttt{toothache} \wedge \texttt{cavity}) = P(\texttt{catch} \mid \texttt{cavity})$$

8.3 Show how to derive Bayes' Rule from the definition

$$P(A \wedge B) = P(A|B).P(B).$$

8.4 Suppose you are give the following information

Mumps causes fever 75% of the time
The chance of a patient having mumps is $\frac{1}{15000}$
The chance of a patient having fever is $\frac{1}{1000}$

Determine the conditional probability of a patient suffering from mumps given that they don't have a fever, i.e.

$$P(Mumps|\neg Fever).$$

8.5 Consider the following statements

Headaches and blurred vision may be the result of sitting too close to a monitor. Headaches may also be caused by bad posture. Headaches and blurred vision may cause nausea. Headaches may also lead to blurred vision.

(i) Represent the causal links in a Bayesian network. Let H stand for "headache", B for "blurred vision", S for "sitting too close to a monitor", P for "bad posture" and N for "nausea". In terms of conditional probabilities, write a formula for the event that all five variables are true, i.e. $P(H \wedge B \wedge S \wedge P \wedge N)$.

(ii) Suppose the following probabilities are given

$$\begin{array}{ll}
P(H|S,P) = 0.8 & P(H|\neg S,P) = 0.4 \\
P(H|S,\neg P) = 0.6 & P(H|\neg S,\neg P) = 0.02 \\
P(B|S,H) = 0.4 & P(B|\neg S,H) = 0.3 \\
P(B|S,\neg H) = 0.2 & P(B|\neg S,\neg H) = 0.01 \\
P(S) = 0.1 & \\
P(P) = 0.2 & \\
P(N|H,B) = 0.9 & P(N|\neg H,B) = 0.3 \\
P(N|H,\neg B) = 0.5 & P(N|\neg H,\neg B) = 0.7
\end{array}$$

Furthermore, assume that some patient is suffering from headaches but not from nausea. Calculate joint probabilities for the 8 remaining possibilities (that is, according to whether S, B, P are true or false).

(iii) What is the probability that the patient suffers from bad posture given that they are suffering from headaches but not from nausea?

8.6 Consider the "burglar alarm" Bayesian network from the lectures. Derive, using Bayes' Rule, an expression for P(Burglary|Alarm) in terms of the conditional probabilities represented in the network. Then calculate the value of this probability.

Is this number what you expected? Explain what is going on.

Check your answer using the AIPython program probVE.py. Answers to method calls such as bn4v.query(B, {A:True}) are in a list form [F,T] giving the probability that B is false and the probability that B is true, in conjunction with the list of conditions (here that A is true). The desired answer is then calculated by normalization. The Bayesian network is encoded as bn4 in probGraphicalModels.py.

8.7 Prove the conditional version of Bayes' Rule:

$$P(B|A,C) = \frac{P(A|B,C)P(B|C)}{P(A|C)}.$$

Here C is an added condition to all terms in the original version of Bayes' Rule.