COMP3411/9814 Artificial Intelligence 20T0, 2020

Tutorial Solutions - Week 5 tutorial 9

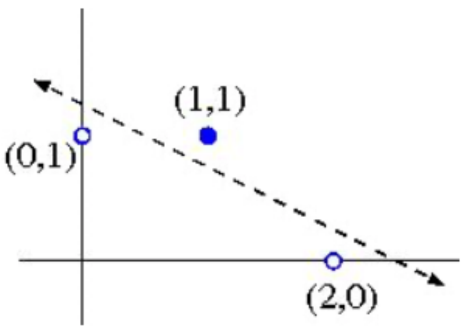
9.1 (Activity 8.3: Activity 8.1: Decision Trees - Perceptron Learning -Open Learning)

1) Construct by hand a Perceptron which correctly classifies the following data; use your knowledge of

plane geometry to choose appropriate values for the weights w_0, w_1 and w_2.

Training Example	x_1	x_2	Class
a.	0	1	-1
b.	2	0	-1
c.	1	1	+1

The first step is to plot the data on a 2-D graph, and draw a line which separates the positive from the negative data points:



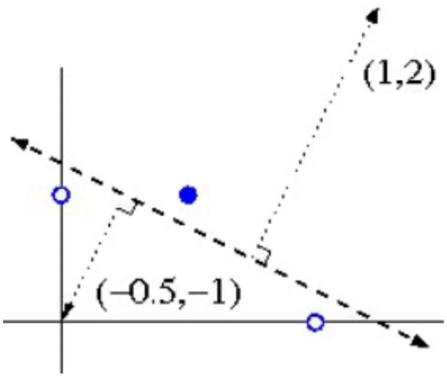
This line has slope -1/2 and x_2-intersect 5/4, so its equation is:

$x_2 = 5/4 - x_1/2$,
i.e. $2x_1 + 4x_2 - 5 = 0$.

Taking account of which side is positive, this corresponds to these weights:

$$w_0 = -5$$
$$w_1 = 2$$
$$w_2 = 4$$

Alternatively, we can derive weights w_1=1 and w_2=2 by drawing a vector normal to the separating line, in the direction pointing towards the positive data points:



The bias weight w_0 can then be found by computing the dot product of the normal vector with a perpendicular vector from the separating line to the origin. In this case $w_0 = 1(-0.5) + 2(-1) = -2.5$

Note: these weights differ from the previous ones by a normalising constant, which is fine for a Perceptron.

2) Demonstrate the Perceptron Learning Algorithm on the above data, using a learning rate of 1.0 and initial weight values of

Iteration	w_0	w_1	w_2	Example	x_1	x_2	Class	$w_0 + w_1x_1 + w_2x_2$	Action
1	-0.5	0	1	a	0	1	$-$	$+0.5$	Subtract
2	-1.5	0	0	b	2	0	$-$	-1.5	None
3	-1.5	0	0	c	1	1	$+$	-1.5	Add
4	-0.5	1	1	a	0	1	$-$	$+0.5$	Subtract
5	-1.5	1	0	b	2	0	$-$	$+0.5$	Subtract
6	-2.5	-1	0	c	1	1	$+$	-3.5	Add
7	-1.5	0	1	a	0	1	$-$	-0.5	None
8	-1.5	0	1	b	2	0	$-$	-1.5	None
9	-1.5	0	1	c	1	1	$+$	-0.5	Add
10	-0.5	1	2	a	0	1	$-$	$+1.5$	Subtract
11	-1.5	1	1	b	2	0	$-$	$+0.5$	Subtract
12	-2.5	-1	1	c	1	1	$+$	-2.5	Add
13	-1.5	0	2	a	0	1	$-$	$+0.5$	Subtract
14	-2.5	0	1	b	2	0	$-$	-2.5	None
15	-2.5	0	1	c	1	1	$+$	-1.5	Add
16	-1.5	1	2	a	0	1	$-$	$+0.5$	Subtract
17	-2.5	1	1	b	2	0	$-$	-0.5	None
18	-2.5	1	1	c	1	1	$+$	-0.5	Add
19	-1.5	2	2	a	0	1	$-$	$+0.5$	Subtract
20	-2.5	2	1	b	2	0	$-$	$+1.5$	Subtract
21	-3.5	0	1	c	1	1	$+$	-2.5	Add
22	-2.5	1	2	a	0	1	$-$	-0.5	None
23	-2.5	1	2	b	2	0	$-$	-0.5	None
24	-2.5	1	2	c	1	1	$+$	$+0.5$	None

Multi-Layer Neural Networks to Compute Logical Functions

Explain how each of the following could be constructed:

1. Perceptron to compute the OR function of *m* inputs Set the bias weight to -½, all other weights to 1.
   The OR function is almost always True. The only way it can be False is if all inputs are 0. Therefore, we set the bias to be slightly less than zero for this input.

2. Perceptron to compute the AND function of *n* inputs

   Set the bias weight to (½ - *n*), all other weights to 1.

   AND function is almost always False. The only way it can be True is if all inputs are 1. Therefore, we set the bias so that, when all inputs are 1, the combined sum is slightly greater than 0.

3. 2-Layer Neural Network to compute any (given) logical expression, assuming it is written in Conjunctive Normal Form.

   Each hidden node should compute one disjunctive term in the expression. The weights should be -1 for items that are negated, $+1$ for the others. The bias should be $k - 1/2$ where k is the number of items that are negated. The output node then computes the conjunction of all the hidden nodes, as in part (ii).

   For example, here is a network that computes $(A \vee B) \wedge (\neg B \vee C \vee \neg D) \wedge (D \vee \neg E)$.