COMP3411/9814 Artificial Intelligence 20T0, 2020

Tutorial Solutions - Week 3 tutorial 6

Week 3: Logical Agents (Week 3 Lecture 2)

Tutorial 6: Logic (Activity 7: Logical Agents - Open Learning)

6.1 Three Goddesses in a Temple (Activity 7.1: Three Goddesses in a Temple - Open Learning)

Three goddesses were sitting in an old Indian temple. Their names were Truth, Lie and Wisdom. Truth always told the truth, Lie always lied and Wisdom sometimes told the truth and sometimes lied. A man entered the temple. He first asked the goddess on the left: "Who is sitting next to you?" "Truth," she answered. He then asked the middle one: "Who are you?" "Wisdom." Finally he asked the one on the right: "Who is your neighbor?" "Lie," she replied. Can you say which goddess was which?

The goddess on left cannot be True because she said someone else was True. The middle one cannot be True either, so the one on the right must be True. This means the middle one is Lie and the left goddess is Wisdom.

6.2 Propositional Logic (Activity 7.2: Validity and Satisfiability - Open Learning)

(Exercise 7.10 from R & N)

Decide whether each of the following sentences is valid, satisfiable, or neither. Verify your decisions using truth tables or equivalence and inference rules. For those that are satisfiable, list all the models that satisfy them.

a. Smoke ⇒ Smoke
Valid [implication, excluded middle]

b. Smoke ⇒ Fire Satisfiable

Smoke	Fire	Smoke ⇒ Fire
T	T	T
T	F	F
F	T	T
F	F	T

Models are: {Smoke, Fire}, {Fire}, {} c. (Smoke ⇒ Fire) ⇒ (¬ Smoke ⇒ ¬ Fire)

Satisfiable

Smoke	Fire	Smoke ⇒ Fire	¬ Smoke ⇒ ¬ Fire	KB
T	T	T	T	T
T	F	F	T	T
F	T	T	F	F
F	F	T	T	T

Models are: {Smoke, Fire}, {Smoke}, {}

d. Smoke ∨ Fire ∨ ¬ Fire

Valid

e. ((Smoke ∧ Heat) ⇒ Fire) ⇔ ((Smoke ⇒ Fire) ∨ (Heat ⇒ Fire))

Valid

((S ∧ H) ⇒ F) ⇔ (F ∨ ¬(S ∧ H)) [implication]
⇔ (F ∨ ¬ S ∨ ¬ H) [de Morgan]
⇔ (F ∨ ¬ S ∨ F ∨ ¬ H) [idempotent, commutativity]
⇔ (S ⇒ F) ∨ (H ⇒ F) [implication]

f. (Smoke ⇒ Fire) ⇒ ((Smoke ∧ Heat) ⇒ Fire)

Valid

(S ⇒ F) ⇔ (F ∨ ¬ S) [implication]
⇒ (F ∨ ¬ S ∨ ¬ H) [generalization]
⇒ (F ∨ ¬ (S ∧ H)) [de Morgan]
⇒ ((S ∧ H) ⇒ F) [conditional]

g. Big ∨ Dumb ∨ (Big ⇒ Dumb)

Valid

Big ∨ Dumb ∨ (Big ⇒ Dumb) ⇔ Big ∨ Dumb ∨ Dumb ∨ ¬ Big [implication]
⇔ Big ∨ ¬ Big ∨ Dumb [idempotent]
⇔ TRUE ∨ Dumb [excluded middle]
⇔ TRUE

h. (Big ∧ Dumb) ∨ ¬ Dumb

Satisfiable

Big	Dumb	(Big ∧ Dumb)	(Big ∧ Dumb) ∨ ¬ Dumb
T	T	T	T
T	F	F	T
F	T	F	F
F	F	F	T

Models are: {Big, Dumb}, {Big}, {}

6.3 Inference Rules (Activity 7.3: Resolution and Conjunctive Normal Form - Open Learning)

(Exercise 7.2 from R & N)

Consider the following Knowledge Base of facts:

If the unicorn in mythical, then it is immortal, but if it is not mythical, then it is mortal and a mammal. If the unicorn in either immortal or a mammal, then it is horned. The unicorn is magical if it is horned.

1. Translate the above statements into Propositional Logic.

Myth ⇒ ¬ Mortal
¬Myth ⇒ (Mortal ∧ Mammal)
¬Mortal ∨ Mammal ⇒ Horned
Horned ⇒ Magic

1.

2. Convert this Knowledge Base into Conjunctive Normal Form.

(¬Myth ∨ ¬ Mortal) ∧ (Myth ∨ Mortal) ∧ (Myth ∨ Mammal) ∧ (Mortal ∨ Horned) ∧ (¬Mammal ∨ Horned) ∧ (¬Horned ∨ Magic)

3. Use a series of resolutions to prove that the unicorn is Horned.

Using Proof by Contradiction, we add to the database the negative of what we are trying to prov

¬Horned

We then try to derive the "empty clause" by a series of Resolutions:

¬Horned ∧ (Mortal ∨ Horned)
Mortal

¬Horned ∧ (¬Mammal ∨ Horned)
¬Mammal

Mortal ∧ (¬Myth ∨ ¬Mortal)
¬Myth

¬Myth ∧ (Myth ∨ Mammal)
Mammal

Mammal ∧ ¬Mammal

Having derived the empty clause, the proof (of Horned) is complete.

4. Give all models that satisfy the Knowledge Base. Can you prove that the unicorn is Mythical? How about Magical?

Because of the rule (Horned ⇒ Magic), Magic must also be True. We can construct a truth table for the remaining three variables:

Myth	**Mortal**	**Mammal**	**Myth ⇒ ¬ Mortal**	**¬ Myth ⇒ (Mortal ∧ Mammal)**	**KB**
T	T	T	F	T	F
T	T	F	F	T	F
T	F	T	T	T	T
T	F	F	T	T	T
F	T	T	T	T	T
F	T	F	T	F	F
F	F	T	T	F	F
F	F	F	T	F	F

There are three models which satisfy the entire Knowledge Base:
{Horned, Magic, Myth, Mammal}, {Horned, Magic, Myth}, {Horned, Magic, Mortal, Mammal}.
We cannot prove that the unicorn is Mythical, because of the third model where Mythical is False.

6.4 First Order Logic (Activity 7.4: Sentences in First Order Logic - Open Learning)

Represent the following sentences in first-order logic, using a consistent vocabulary.

f. There is a barber who shaves all men in town who do not shave themselves.

$\exists b\ Barber(b) \wedge \forall m\ (Man(m) \wedge InTown(m) \wedge \neg Shave(m,m) \Rightarrow Shave(b,m))$

g. Politicians can fool some of the people all of the time, and they can fool all of the people some of the time, but they can't fool all of the people all of the time.

$\forall p\ (\ Politician(p) \Rightarrow ((\exists x \forall t\ Fool(p,x,t)) \wedge (\exists t \forall x\ Fool(p,x,t)) \wedge (\neg \forall x \forall t\ Fool(p,x,t))))$

$\exists\, x\ Student(x) \wedge Study(x,French,2016)$

b. Only one student studied Greek in 2015.

$\exists x\ Study(x,Greek,2015) \wedge \forall y\ (Study(y,Greek,2015) \Rightarrow y = x\ )$

sometimes written as

$\exists !x\ Study(x,Greek,2015)$

c. The highest score in Greek is always higher than the highest score in French.

$\forall t\ \exists x\ \forall y\ Score(x,Greek,t) > Score(y,French,t)$

d. Every person who buys a policy is smart.

$\forall x,p\ Person(x) \wedge Policy(p) \wedge Buy(x,p) \Rightarrow Smart(x)$

e. No person buys an expensive policy.

$\neg \exists x,p\ Person(x) \wedge Policy(p) \wedge Expensive(p) \wedge Buy(x,p)$

(OpenLearning solutions week 9 (8))

6.5 Show using the truth table method that the corresponding inferences are valid.

(i) $P \rightarrow Q, \neg Q \models \neg P$
(ii) $P \rightarrow Q \models \neg Q \rightarrow \neg P$
(iii) $P \rightarrow Q, Q \rightarrow R \models P \rightarrow R$

Check your answers using the Python program "tableau_prover.py" .
(19T2 - solutions 5.pdf) (Q3)

(i)

P	Q	$P \rightarrow Q$	$\neg Q$	$\neg P$
T	T	T	F	F
T	F	F	T	F
F	T	T	F	T
F	F	T	T	T

In all rows where both $P \rightarrow Q$ and $\neg Q$ true, $\neg P$ is true. Therefore, valid inference.

(ii)

P	Q	$\neg P$	$\neg Q$	$P \rightarrow Q$	$\neg Q \rightarrow \neg P$
T	T	F	F	T	T
T	F	F	T	F	F
F	T	T	F	T	T
F	F	T	T	T	T

In all rows where both $P \rightarrow Q$ true, $\neg Q \rightarrow \neg P$ is true. Therefore, valid inference.

(iii)

P	Q	R	$P \rightarrow Q$	$Q \rightarrow R$	$P \rightarrow R$
T	T	T	T	T	T
T	T	F	T	F	F
T	F	T	F	T	T
T	F	F	F	T	F
F	T	T	T	T	T
F	T	F	T	F	T
F	F	T	T	T	T
F	F	F	T	T	T

In all rows where both $P \rightarrow Q$ and $Q \rightarrow R$ true, $P \rightarrow R$ is true. Therefore, valid inference.

6.6 Determine whether the following sentences are valid (i.e. tautologies) using truth tables.
(i) ((P∨Q)∧¬P)→Q
(ii) ((P →Q)∧¬(P →R))→(P →Q)
(iii) ¬(¬P ∧P)∧P
(iv) (P ∨Q)→¬(¬P ∧¬Q)

Check your answers using the Python program "tableau prover.py".
(19T2 - solutions 5.pdf) (Q5)

(i)

P	Q	$\neg P$	$P \vee Q$	$(P \vee Q) \wedge \neg P$	$((P \vee Q) \wedge \neg P) \rightarrow Q$
T	T	F	T	F	T
T	F	F	T	F	T
F	T	T	T	T	T
F	F	T	F	F	T

Last column is always true no matter what truth assignment to P and Q. Therefore $((P \vee Q) \wedge \neg P) \rightarrow Q$ is a tautology.

(ii) $S = ((P \rightarrow Q) \wedge \neg(P \rightarrow R)) \rightarrow (P \rightarrow Q)$

P	Q	R	$P \rightarrow Q$	$P \rightarrow R$	$\neg(P \rightarrow R)$	$(P \rightarrow Q) \wedge \neg(P \rightarrow R)$	S
T	T	T	T	T	F	F	T
T	T	F	T	F	T	T	T
T	F	T	F	T	F	F	T
T	F	F	F	F	T	F	T
F	T	T	T	T	F	F	T
F	T	F	T	T	F	F	T
F	F	T	T	T	F	F	T
F	F	F	T	T	F	F	T

Last column is always true no matter what truth assignment to P, Q and R. Therefore $((P \rightarrow Q) \wedge \neg(P \rightarrow R)) \rightarrow (P \rightarrow Q)$ is a tautology.

(iii)

P	$\neg P$	$\neg P \wedge P$	$\neg(\neg P \wedge P)$	$\neg(\neg P \wedge P) \wedge P$
T	F	F	T	T
F	T	F	T	F

Last column is not always true. Therefore $\neg(\neg P \wedge P) \wedge P$ is not a tautology.

(iv) $(P \vee Q) \rightarrow \neg(\neg P \wedge \neg Q)$

P	Q	$\neg P$	$\neg Q$	$P \vee Q$	$\neg P \wedge \neg Q$	$\neg(\neg P \wedge \neg Q)$	$(P \vee Q) \rightarrow \neg(\neg P \wedge \neg Q)$
T	T	F	F	T	F	T	T
T	F	F	T	T	F	T	T
F	T	T	F	T	F	T	T
F	F	T	T	F	T	F	T

Last column is always true no matter what truth assignment to P and Q. Therefore $(P \vee Q) \rightarrow \neg(\neg P \wedge \neg Q)$ is a tautology.