Week 3: Logical Agents (Week 3 Lecture 2)

Tutorial 6: Logic (Activity 7: Logical Agents - Open Learning)

6.1 Three Goddesses in a Temple (Activity 7.1: Three Goddesses in a Temple - Open Learning)

Discuss your answers from the activity on the Logic and Entailment page:
Three goddesses were sitting in an old Indian temple. Their names were Truth, Lie and Wisdom. Truth always told the truth, Lie always lied and Wisdom sometimes told the truth and sometimes lied. A man entered the temple. He first asked the goddess on the left: "Who is sitting next to you?" "Truth," she answered. He then asked the middle one: "Who are you?" "Wisdom." Finally he asked the one on the right: "Who is your neighbor?" "Lie," she replied. Can you say which goddess was which?

6.2 Propositional Logic (Activity 7.2: Validity and Satisfiability - Open Learning)

Discuss your answers from the activity on the Propositional Logic page:
Decide whether each of the following sentences is valid, satisfiable, or unsatisfiable. Verify your decisions using truth tables or logical equivalence and inference rules. For those that are satisfiable, list all the models that satisfy them.

a. Smoke ⇒ Smoke
b. Smoke ⇒ Fire
c. (Smoke ⇒ Fire) ⇒ (¬ Smoke ⇒ ¬ Fire)
d. Smoke ∨ Fire ∨ ¬ Fire
e. ((Smoke ∧ Heat) ⇒ Fire) ⇔ ((Smoke ⇒ Fire) ∨ (Heat ⇒ Fire))
f. (Smoke ⇒ Fire) ⇒ ((Smoke ∧ Heat) ⇒ Fire)
g. Big ∨ Dumb ∨ (Big ⇒ Dumb)
h. (Big ∧ Dumb) ∨ ¬ Dumb

6.3 Inference Rules (Activity 7.3: Resolution and Conjunctive Normal Form - Open Learning)

Discuss your answers from the activity on the Inference Rules page:
Consider the following Knowledge Base of facts:

> *If the unicorn in mythical, then it is immortal, but if it is not mythical, then it is mortal and a mammal. If the unicorn in either immortal or a mammal, then it is horned. The unicorn is magical if it is horned.*

1. Translate the above statements into Propositional Logic, using the symbols

Myth	The unicorn is Mythical
Mortal	The unicorn is Mortal
Mammal	The unicorn is a Mammal
Horned	The unicorn is Horned
Magic	The unicorn is Magical

2. Convert this Knowledge Base into Conjunctive Normal Form.

3. Use a series of resolutions to prove that the unicorn is Horned.
Hint: add ¬Horned to the KB and then try to derive the empty clause.

4. Give all models that satisfy the Knowledge Base. Does it follow from the KB that the unicorn is Mythical? How about Magical?

6.4 First Order Logic (Activity 7.4: Sentences in First Order Logic - Open Learning)

Discuss your answers from the activity on the First Order Logic page:

Represent the following sentences in first-order logic, using a consistent vocabulary.

a. Some students studied French in 2015.
b. Only one student studied Greek in 2014.
c. The highest score in Greek is always higher than the highest score in French.
d. Every person who buys a policy is smart.
e. No person buys an expensive policy.
f. There is a barber who shaves all men in town who do not shave themselves.
g. Politicians can fool some of the people all of the time, and they can fool all of the people some of the time, but they can't fool all of the people all of the time. (Use *Fool(p,x,t)* to mean that *p* fools *x* at time *t)*.

6.5 Show using the truth table method that the corresponding inferences are valid.

(i) $P \rightarrow Q, \neg Q \models \neg P$
(ii) $P \rightarrow Q \models \neg Q \rightarrow \neg P$
(iii) $P \rightarrow Q, Q \rightarrow R \models P \rightarrow R$

Check your answers using the Python program “tableau_prover.py” .

6.6 Determine whether the following sentences are valid (i.e. tautologies) using truth tables.

(i) $((P \vee Q) \wedge \neg P) \rightarrow Q$
(ii) $((P \rightarrow Q) \wedge \neg(P \rightarrow R)) \rightarrow (P \rightarrow Q)$
(iii) $\neg(\neg P \wedge P) \wedge P$
(iv) $(P \vee Q) \rightarrow \neg(\neg P \wedge \neg Q)$

Check your answers using the Python program “tableau prover.py”.