COMP3411/9814 Artificial Intelligence 20T0, 2020

Tutorial Solutions - Week 4 tutorial 8

Week 4: Reasoning with Uncertainty

Tutorial 8: Reasoning with Uncertainty

8.1 (Activity 10.1: Conditional Probability - Open Learning)

Only 4% of the population are colour blind, but 7% of men are colour blind.

What percentage of colour blind people are men?   Explain your answer..

Only 4% of the population are color blind, but 7% of men are color blind. What percentage of color blinc people are men?

> If we assume 50% of the population are men, then the fraction of "color blind men" is 0.5 * 0.07 = 0.035
> This means the fraction of "color blind women" is 0.04 - 0.035 = 0.005
> Therefore, the fraction of color blind people who are men is 0.035 / 0.04 = 87.5%.

8.2 (Activity 10.2: Enumerating Probabilities - Open Learning

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(Exercise 13.8 from Russell & Norvig)

	toothache		¬toothache	
	catch	¬catch	catch	¬catch
cavity	.108	.012	.072	.008
¬cavity	.016	.064	.144	.576

1. Given the full joint distribution shown in Figure 13.3 (also on page 17 of the Uncertainty lecture slides), calculate the following:

   a. $P(\text{toothache} \wedge \neg \text{catch}) = 0.012 + 0.064 = 0.076$

   b. $P(\text{catch}) = 0.108 + 0.016 + 0.072 + 0.144 = 0.34$

   c. $P(\text{cavity} \mid \text{catch}) = P(\text{cavity} \wedge \text{catch}) / P(\text{catch})$
   $= (0.108 + 0.072)/(0.108 + 0.072 + 0.016 + 0.144) = 0.18 / 0.34 = 0.53$

   d. $P(\text{cavity} \mid \text{toothache} \vee \text{catch}) =$
   $P(\text{cavity} \wedge (\text{toothache} \vee \text{catch})) / P(\text{toothache} \vee \text{catch})$
   $= (0.108 + 0.012 + 0.072)/(0.108 + 0.012 + 0.072 + 0.016 + 0.064 + 0.144)$
   $= 0.192 / 0.416 = 0.46$

2. Verify the conditional independence claimed in the lecture slides by showing that P(catch | toothache, cavity) = P(catch | cavity)

P(catch | toothache ∧ cavity) =

P(catch ∧ toothach ∧ cavity) / P(toothache ∧ cavity)
= 0.108 /(0.108 + 0.012) = 0.108 / 0.12 = 0.9

P(catch | cavity) = P(catch ∧ cavity) / P(cavity)
= (0.108 + 0.072)/(0.108 + 0.012 + 0.072 + 0.008) = 0.18 / 0.2 = 0.9

8.3 Show how to derive Bayes' Rule from the definition

$P(A \wedge B) = P(A|B).P(B)$
$P(B \wedge A) = P(B|A).P(A)$
Now $P(A \wedge B) = P(B \wedge A)$ [why exactly?]
Therefore $P(A|B).P(B) = P(B|A).P(A)$
Rearranging gives Bayes' Rule $P(A|B) = \frac{P(B|A).P(A)}{P(B)}$ if $P(B) > 0$

8.4 Suppose you are give the following information

Mumps causes fever 75% of the time
The chance of a patient having mumps is $\frac{1}{15000}$
The chance of a patient having fever is $\frac{1}{1000}$

Determine the conditional probability of a patient suffering from mumps given that they don't have a fever, i.e.

$P(Mumps|\neg Fever)$.

$$
\begin{aligned}
P(Mumps|\neg Fever) &= \frac{P(\neg Fever|Mumps).P(Mumps)}{P(\neg Fever)} \\
&= \frac{(1-P(Fever|Mumps)).P(Mumps)}{1-P(Fever)} \\
&= \frac{(1-\frac{3}{4}).\frac{1}{15000}}{1-\frac{1}{1000}} \\
&= 0.0000167
\end{aligned}
$$

8.5 Consider the following statements

Headaches and blurred vision may be the result of sitting too close to a monitor. Headaches may also be caused by bad posture. Headaches and blurred vision may cause nausea. Headaches may also lead to blurred vision.

(i) Represent the causal links in a Bayesian network. Let H stand for "headache", B for "blurred vision", S for "sitting too close to a monitor", P for "bad posture" and N for "nausea". In terms of conditional probabilities, write a formula for the event that all five variables are true, i.e. $P(H \wedge B \wedge S \wedge P \wedge N)$.

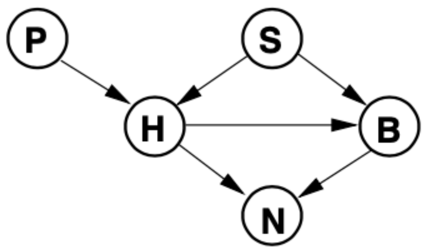
(ii) Suppose the following probabilities are given

$P(H|S,P) = 0.8$ $\quad P(H|\neg S,P) = 0.4$
$P(H|S,\neg P) = 0.6$ $\quad P(H|\neg S,\neg P) = 0.02$
$P(B|S,H) = 0.4$ $\quad P(B|\neg S,H) = 0.3$
$P(B|S,\neg H) = 0.2$ $\quad P(B|\neg S,\neg H) = 0.01$
$P(S) = 0.1$
$P(P) = 0.2$
$P(N|H,B) = 0.9$ $\quad P(N|\neg H,B) = 0.3$
$P(N|H,\neg B) = 0.5$ $\quad P(N|\neg H,\neg B) = 0.7$

Furthermore, assume that some patient is suffering from headaches but not from nausea. Calculate joint probabilities for the 8 remaining possibilities (that is, according to whether S, B, P are true or false).

(iii) What is the probability that the patient suffers from bad posture given that they are suffering from headaches but not from nausea?

(i)



$$P(H \wedge B \wedge S \wedge P \wedge N) = P(H|P \wedge S).P(B|S \wedge H).P(S).P(P).P(N|H \wedge B)$$

(ii) $P(H \wedge B \wedge S \wedge P \wedge \neg N) = P(H|S \wedge P).P(B|H \wedge S).P(S).P(P).P(\neg N|H \wedge B)$
$= 0.8 \times 0.4 \times 0.1 \times 0.2 \times 0.1$
$= 0.00064$

$P(H \wedge \neg B \wedge S \wedge P \wedge \neg N) = P(H|S \wedge P).P(\neg B|H \wedge S).P(S).P(P).P(\neg N|H \wedge \neg B)$
$= 0.8 \times 0.6 \times 0.1 \times 0.2 \times 0.5$
$= 0.00480$

$P(H \wedge B \wedge \neg S \wedge P \wedge \neg N) = P(H|\neg S \wedge P).P(B|H \wedge \neg S).P(\neg S).P(P).P(\neg N|H \wedge B)$
$= 0.4 \times 0.3 \times 0.9 \times 0.2 \times 0.1$
$= 0.00216$

$P(H \wedge \neg B \wedge \neg S \wedge P \wedge \neg N) = P(H|\neg S \wedge P).P(\neg B|H \wedge \neg S).P(\neg S).P(P).P(\neg N|H \wedge \neg B)$
$= 0.4 \times 0.7 \times 0.9 \times 0.2 \times 0.5$
$= 0.02520$

$P(H \wedge B \wedge S \wedge \neg P \wedge \neg N) = P(H|S \wedge \neg P).P(B|H \wedge S).P(S).P(\neg P).P(\neg N|H \wedge B)$
$= 0.6 \times 0.4 \times 0.1 \times 0.8 \times 0.1$
$= 0.00192$

$P(H \wedge \neg B \wedge S \wedge \neg P \wedge \neg N) = P(H|S \wedge \neg P).P(\neg B|H \wedge S).P(S).P(\neg P).P(\neg N|H \wedge \neg B)$
$= 0.6 \times 0.6 \times 0.1 \times 0.8 \times 0.5$
$= 0.0144$

$P(H \wedge B \wedge \neg S \wedge \neg P \wedge \neg N) = P(H|\neg S \wedge \neg P).P(B|H \wedge \neg S).P(\neg S).P(\neg P).P(\neg N|H \wedge B)$
$= 0.02 \times 0.3 \times 0.9 \times 0.8 \times 0.1$
$= 0.000432$

$P(H \wedge \neg B \wedge \neg S \wedge \neg P \wedge \neg N) = P(H|\neg S \wedge \neg P).P(\neg B|H \wedge \neg S).P(\neg S).P(\neg P).P(\neg N|H \wedge \neg B)$
$= 0.02 \times 0.7 \times 0.9 \times 0.8 \times 0.5$
$= 0.00504$

(iii) $P(P|H \wedge \neg N) = \frac{P(P \wedge H \wedge \neg N)}{P(H \wedge \neg N)} = \frac{0.0328}{0.054592} = 0.60082$

Note:

$P(P \wedge H \wedge \neg N) = \Sigma_{b,s} P(H \wedge b \wedge s \wedge P \wedge \neg N) = 0.00064 + 0.00480 + 0.00216 + 0.02520$

$P(H \wedge \neg N) = \Sigma_{b,s,p} P(H \wedge b \wedge s \wedge p \wedge \neg N) = 0.00064 + 0.00480 + 0.00216 + 0.02520 + 0.00192 + 0.0144 + 0.000432 + 0.00504 = 0.05452$

We could also use direct inference, though this provides no advantage in this example. To do this, we need a conditional version of Bayes' Rule: $P(B|A, C) = \frac{P(A|B,C)P(B|C)}{P(A|C)}$

Then $P(P|H, \neg N) = \frac{P(\neg N|H,P).P(P|H)}{P(\neg N|H)}$

Now $P(\neg N|H, P) = P(\neg N|H, B, P).P(B|H, P) + P(\neg N|H, \neg B, P).P(\neg B|H, P)$
$= P(\neg N|H, B).P(B|H, P) + P(\neg N|H, \neg B).P(\neg B|H, P)$
$= P(\neg N|H, B).(P(B|H, S, P).P(S|H, P) + P(B|H, \neg S, P).P(\neg S|H, P)) +$
$P(\neg N|H, \neg B).(P(\neg B|H, S, P).P(S|H, P) + P(\neg B|H, \neg S, P).P(\neg S|H, P))$
$= [P(\neg N|H, B).(P(B|H, S).P(H|S, P).P(S|P) + P(B|H, \neg S).P(H|\neg S, P).P(\neg S|P)) +$
$P(\neg N|H, \neg B).(P(\neg B|H, S).P(H|S, P).P(S|P) + P(\neg B|H, \neg S).P(H|\neg S, P).P(\neg S|P))]/P(H|P)$
$= [P(\neg N|H, B).(P(B|H, S).P(H|S, P).P(S) + P(B|H, \neg S).P(H|\neg S, P).P(\neg S)) +$
$P(\neg N|H, \neg B).(P(\neg B|H, S).P(H|S, P).P(S) + P(\neg B|H, \neg S).P(H|\neg S, P).P(\neg S))]/P(H|P)$

Also $P(P|H) = P(H|P).P(P)/P(H)$, so cancelling $P(H|P)$
$P(\neg N|H, P).P(P|H)$
$= [P(\neg N|H, B).(P(B|H, S).P(H|S, P).P(S) + P(B|H, \neg S).P(H|\neg S, P).P(\neg S)) +$
$P(\neg N|H, \neg B).(P(\neg B|H, S).P(H|S, P).P(S) + P(\neg B|H, \neg S).P(H|\neg S, P).P(\neg S))].P(P)/P(H)$

This generates four terms exactly as above, and similarly $P(\neg N|H)$ generates eight terms as above. The extra $P(H)$ cancel each other out.

8.6 Consider the "burglar alarm" Bayesian network from the lectures. Derive, using Bayes' Rule, an expression for P(Burglary|Alarm) in terms of the conditional probabilities represented in the network. Then calculate the value of this probability.
Is this number what you expected? Explain what is going on.

Let A stand for "Alarm", B for "Burglary" and E for "Earthquake".

Then by Bayes' Rule
$P(B|A) = P(A|B).P(B)/P(A) = (P(A|B \wedge E).P(E).P(B) + P(A|B \wedge \neg E).P(\neg E).P(B))/P(A$
and as in lectures
$P(A) = P(A|B \wedge E).P(E).P(B) + P(A|B \wedge \neg E).P(\neg E).P(B) + P(A|\neg B \wedge E).P(E).P(\neg B) +$
$P(A|\neg B \wedge \neg E).P(\neg E).P(\neg B)$

So $P(B|A) = (0.95 \times 0.002 \times 0.001 + 0.94 \times 0.998 \times 0.001)/P(A)$ and
$P(A) = 0.95 \times 0.002 \times 0.001 + 0.94 \times 0.998 \times 0.001 + 0.29 \times 0.002 \times 0.999 +$
$0.001 \times 0.998 \times 0.999$

Thus $P(B|A) = 0.00094002/0.002516442 = 0.3735512$

Intuitively, the "true positives" (when there really is a burglary) account for roughly only 10/26 of the cases when the alarm is ringing (around 0.001 of the time), while the "false positives" account for 16/26 cases (6/26 when the alarm is ringing because of an earthquake, due to a false positive rate around 0.3 and prior of 0.002, so around 0.0006 of the time, and 10/26 when there is neither a burglary nor an earthquake, due to a false positive rate of 0.001 and a prior close to 1, so around 0.001 of the time). The rough calculation is $10/26 = 0.001/(0.001 + 0.0006 + 0.001)$. That is, the false positives significantly outweigh the true positives in this scenario.

8.7 Prove the conditional version of Bayes' Rule:

$$P(B|A, C) = \frac{P(A|B,C)P(B|C)}{P(A|C)}.$$

Here C is an added condition to all terms in the original version of Bayes' Rule.

By the Chain Rule, $P(A \wedge B \wedge C) = P(B|A, C).P(A|C).P(C) = P(A|B, C).P(B|C).P(C)$. So, provided $P(C) \neq 0$ and $P(A|C) \neq 0$, the conditional version of Bayes' Rule follows.