Additional tutorials - conversion into CNF - Solutions

1. Convert in the following formulae into Conjunctive Normal Form (CNF).

$(p \wedge q) \vee (p \wedge \neg q)$

Apply [Distribute $\vee$ over $\wedge$]

1. $(p \wedge q) \vee (p \wedge \neg q)$
   $\equiv ((p \wedge q) \vee p) \wedge ((p \wedge q) \vee \neg q)$
   $\equiv ((p \vee p) \wedge (q \vee p)) \wedge ((p \vee \neg q) \wedge (q \vee \neg q))$

2. Convert the following formulae into Conjunctive Normal Form (CNF).

(i) $P \rightarrow Q$
(ii) $(P \rightarrow \neg Q) \rightarrow R$
(iii) $\neg(P \wedge \neg Q) \rightarrow (\neg R \vee \neg Q)$

(i) $P \rightarrow Q$
$\neg P \vee Q$ [Remove $\rightarrow$]

(ii) $(P \rightarrow \neg Q) \rightarrow R$
$\neg(\neg P \vee \neg Q) \vee R$ [Remove $\rightarrow$]
$(\neg\neg P \wedge \neg\neg Q) \vee R$ [De Morgan]
$(P \wedge Q) \vee R$ [Double Negation]
$(P \vee R) \wedge (Q \vee R)$ [Distribute $\vee$ over $\wedge$]

(iii) $\neg(P \wedge \neg Q) \rightarrow (\neg R \vee \neg Q)$
$\neg\neg(P \wedge \neg Q) \vee (\neg R \vee \neg Q)$ [Remove $\rightarrow$]
$(P \wedge \neg Q) \vee (\neg R \vee \neg Q)$ [Double Negation]
$(P \vee \neg R \vee \neg Q) \wedge (\neg Q \vee \neg R \vee \neg Q)$ [Distribute $\vee$ over $\wedge$]

This can be further simplified to:
$(P \vee \neg R \vee \neg Q) \wedge (\neg Q \vee \neg R)$

and even further simplified to
$\neg Q \vee \neg R$, since $\neg Q \vee \neg R$ subsumes $P \vee \neg R \vee \neg Q$