**The connection to Lagrange polynomials**

To establish this connection, we should first understand how the rectangle rule, the trapezoidal rule, and the barrel rule (simpson’s method) are related to Newton-Cotes formulas.

Newton-Cotes formulas:

These formulas are used to approximate with polynomials, more specifically with Lagrange polynomials. We are interested in the first three, closed Newton-Cotes formulas, which are:

Let

* the rectangle rule: (Polynomial of Degree 0)
* the trapezoidal rule: (Polynomial of Degree 1)
* the barrel rule: (Polynomial of Degree 2)

We Proof here the trapezoidal rule using Lagrange polynomials.

Lagrange polynomial is the [polynomial](https://mathworld.wolfram.com/Polynomial.html)  of degree  that passes through  points, and is given by

where,

Proof of the connection between the Trapezoidal Rule and Lagrange Polynomials:

We want to prove that:

By using the Lagrange polynomial

Let and

If we let, we get:

q.e.d.

For the barrel rule, one just needs to use a Lagrange polynomial of degree 2 and the rest of the proof works very similar.

This shows the connection between Lagrange polynomials and Numerical integration in 1D.

Sources:

https://mathworld.wolfram.com/LagrangeInterpolatingPolynomial.html

https://mathworld.wolfram.com/Newton-CotesFormulas.html

https://www.math-linux.com/mathematics/numerical-integration/article/newton-cotes-formulas