1. Runtime analysis

a) so $i = 2$ and the loop exits when $2^k \Rightarrow n$

$\log_2 n$ which means the runtime is $\Theta \log_2(n)$

b)

$$\sum_{i=1}^{n} \Theta \sum_{k=0}^{i} \Theta(1)$$

z will be my dummy variable

	$n=4$	$n=9$	$n=16$
z	2	3	4

$m=2$
$m=4$

$n^{\frac{8}{2}+1}$

$\Downarrow$

$i = z\sqrt{n}\ 3$

$$\sum_{i=1}^{n} \sum_{z=0}^{\sqrt{n}} \sum_{k=0}^{i} \Theta(1)$$

$$= \sum_{i=1}^{n} \sum_{z=0}^{\sqrt{n}} i^3 = \sum_{i=1}^{n} \sqrt{n} \cdot i^3 = \sqrt{n} \cdot (n^4) = \boxed{\Theta n^{9/2}}$$

$$= \sum_{i=1}^{n} \frac{\sqrt{n}(\sqrt{n}+1)}{2} = \sum_{i=1}^{n} n + \sqrt{n} = \sum_{i=1}^{n} n$$

$= \Theta(n^2)$

c)

$$\sum_{i=1}^{n} \sum_{k=1}^{n} \sum_{k=1}^{\log_2 n + 1} \Theta 1$$

this last for loop goes like this:

$m + 2m = 2m$
$2m + 2m = 4m$ $\quad 2^k m$
$4m + 4m = 8m$

$m = 2^k n$

it also runs every time because that is a worst case

Stops when $2^k \cdot m = n$

$2^k = \frac{n}{m}$

$\log_2 \frac{n}{m} = k$

n	2	4	8
~~runtime~~	2	3	4
times ran	n		

$\hookrightarrow = \sum \sum \Theta \log_2 n = \boxed{\Theta n^2 \log_2 n}$

d) when $\left(\frac{3}{2}\right)^k \cdot size = n$ it stops

so $k = \log_{\frac{3}{2}} n$ in the worst case $size = n$

$$Runtime = 2 + \sum_{i=0}^{n} \sum_{k=0}^{\log_{\frac{3}{2}} n} \cancel{5} + \sum_{j=0}^{n} \Theta(1)$$

$$= 2 + \sum_{i=0}^{n} \sum_{k=0}^{\log_{\frac{3}{2}} n} 5 + n$$

$$= \Theta\left(n^2 \log_{\frac{3}{2}} n\right)$$