2. $1 - \frac{14}{15}\cdot\frac{13}{14}\cdot\frac{12}{13}\cdot\frac{11}{12}\cdot\frac{10}{11}\cdot\frac{9}{10}\cdot\frac{8}{9}\cdot\frac{7}{8}$

$= 53\%$

2. numbers with 5 digits

$5\cdot 4\cdot 7\cdot 6\cdot 5$

$= 4200$

with 4 digits

$5\cdot 4\cdot 7\cdot 5 = 700$

with 3 digits

$5\cdot 4\cdot 5 = 100$

} 5000 numbers

Probability for only one $5000/1000000$

0.05

$\binom{6}{5}(0.05)^5\cdot(1-0.05)^1$

$= 1.5\cdot 10^{-7}$

3. Event A

4 4
4 5
4 6
5 4
5 5
5 6
6 4
6 5
6 6

Event B

1 1 1
2 2 2
3 3 3
4 4 4
5 5 5
6 6 6

$P(A) = \frac{6}{24} = \frac{1}{36}$

$P(A) = \left(\frac{1}{2}\right)^3\cdot {}_3C_3 + \left(\frac{1}{2}\right)^3\cdot {}_3C_2$

$P(B)\cdot P(B) = \frac{1}{72}$

$P(A)\cap P(B) = \frac{3}{216} = \frac{1}{72}$

they are independent

$P(A)\cdot P(B) = P(A)\cap P(B)$

7. Probability of getting a flush

$\binom{13}{5}\cdot 4 = 5148$

because there are 4 ranks

$= \frac{5148}{\binom{52}{5}} = 0.00198$

$\frac{1}{0.00198} = 505$ hands have to be played

2. B = winning 4/5 games

F = superstar played

$\bar{F}$ = superstar didn't play

$P(E|F) = \binom{5}{4}(0.7)^4(1-0.7) = 0.36$

$P(E|\bar{F}) = \binom{5}{4}(0.5)^4\cdot 0.5 = 0.16$

$P(F|E) = \frac{P(E|F)\cdot P(F)}{P(E|F)\cdot P(F) + P(E|\bar{F})\cdot P(\bar{F})}$

$= \frac{0.36\cdot 0.75}{0.36\cdot 0.75 + 0.16\cdot 0.25}$

$= \boxed{0.87}$