Bounds, Semantics/Pragmatics-Divide, and Evolution

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1 Short exposition

The plan is to expand our work on the semantics/pragmatics divide by means of a more involved & principled study on the lack of upper-bounds in the literal meaning of scalar expressions. The main idea of this extension, other than providing a more detailed treatment as well as a comparison with iterated learning, is to do away with the assumption of communal learning. Potential extensions include more scalar pairs, the addition of an inductive bias towards uniform lexica across scalar pairs, and a variation of state frequencies.

2 Differences to CogSci setup.

(I) No communal learning. Only parental learning is considered

(II) Sequences and atomic observations. Before, the set of all observations was $O = \{\langle\langle s_1, m_i\rangle, \langle s_2, m_j\rangle\rangle \,|m_i, m_j \in M\}$. A member of O encodes that a teacher produced m_i in state s_1 and m_j in s_2, i.e., it encodes one witnessed message for each state. A datum d was a sequence of length k of members of O. Learners witnessed such data sequences. Now, more in line with Griffiths and Kalish [2007], $O = \{\langle s_i, m_j\rangle \,|s_i \in S, m_j \in M\}$ and d is a sequence of length k of members of O. The main difference is that now some d do not provide any production information for some states.

(III) Observations as production. Instead of taking the space of all possible sequences of length k into consideration, we take sample from O k-times according to the production probabilities of each type; $P(o = \langle s, m\rangle \,|t_i) = P(s)P(m|s, t_i)$. n such k-length sequences are sampled for each type. As a consequence, the data used for computing Q_i is not the same as that used for j $(i \neq j)$.

(IV) Parametrized learning $Q_{ij} \propto [\sum_d P(d|t_i)P(t_j|d)]^l$, where $l = 1$ corresponds to probability matching and, as l increases towards infinity, to MAP.

3 Further modifications we may want to keep in mind (but currently don't make use of)

(I) Cost for pragmatic reasoning. At least in the CogSci setup the effect of adding cost to pragmatic reasoning is unsurprising: High cost for pragmatic signaling lowers the prevalence of pragmatic types. Lexica that semantically encode an upper-bound benefit the most from this. However, the cost needed to be substantial to make the pragmatic English-like lexicon stop being the incumbent type (particularly when learning is communal).

(II) Negative learning bias. Instead of penalizing complex semantics (semantic upper-bounds) one may consider penalizing simple semantics (no upper-bounds). This is useful as a sanity check but also yields unsurprising results in the CogSci setup: The more learners are biased against simple semantics, the more prevalent are lexica that semantically encode upper-bounds.

(III) Inductive bias. A second learning bias that codifies the idea that lexica should be uniform, i.e. be biased towards either lexicalizing an upper-bound for all weaker alternatives in a scalar pair or for none.

(IV) Uncertainty. The other advantage of non-upper bounded semantics lies in being non-committal to the negation of stronger alternatives when the speaker is uncertain. Adding this to the model requires the most changes to our present setup and some additional assumptions about the cues available to players to discern the speaker's knowledge about the state she is in.

(V) More scalar pairs. Taking into consideration more than one scalar pair. Preliminary results suggest that this does not influence the results in any meaningful way without further additions, e.g. by (III).

(VI) More lexica. Not necessary. Preliminary results suggest that considering more lexica has no noteworthy effect on the dynamics (tested with all possible 2x2 lexica).

(VII) State frequencies. Variations on state frequencies. This may have an interesting interaction with (III).

(VIII) Reintroduction of communal learning. One possibility: The probably N_{ij} with which a child of t_i adopts t_j could be the weighted sum of Q_{ij} (as before) and a vector we get from learning from all of the population: $L_j = \sum_d P(d|\vec{p})P(t_j|d)$, where $P(d|\vec{p}) = \sum_i P(d|t_i)\vec{p}_i$ is the probability of observing d when learning from a random member of the present population distribution.

4 General setup

4.1 Part 1: Signaling.

We consider a population of players with two signaling behaviors, literal and Gricean (level 0 and 1 below), each equipped with one of 6 lexicons. This yields a total of 12 distinct player types $t \in T$. $|M| = |S| = 2$, i.e., a lexicon is a $(2,2)$-matrix. These are listed in Table 1.

$$L_1 = \begin{pmatrix} 0 & 0 \\ 1 & 1 \end{pmatrix} \quad L_2 = \begin{pmatrix} 1 & 1 \\ 0 & 0 \end{pmatrix} \quad L_3 = \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}$$

$$L_4 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \quad L_5 = \begin{pmatrix} 0 & 1 \\ 1 & 1 \end{pmatrix} \quad L_6 = \begin{pmatrix} 1 & 1 \\ 1 & 0 \end{pmatrix}$$

Table 1: Set of possible $(2, 2)$-matrices, i.e., a lexicon.

As in the CogSci paper, L_4 (semantic upper-bound for m_2) and L_5 (no semantic upper-bound for m_2) are the target lexica. Gricean L_5 users can convey/infer the bound pragmatically, while literal/Gricean L_4 users do so semantically.

Signaling behavior. With $\lambda \geq 1$ (rationality parameter), $\alpha \in [0, 1]$ (pragmatic violations) and $pr \in \Delta(S)$ a common prior over S (uniform so far):

$$R_0(s|m; L) \propto pr(s) L_{sm} \tag{1}$$
$$S_0(m|s; L) \propto \exp(\lambda\ L_{sm}) \tag{2}$$
$$R_1(s|m; L) \propto pr(s) S_0(m|s; L) \tag{3}$$
$$S_1(m|s; L) \propto \exp(\lambda\ R_0(s|m; L)^\alpha) \tag{4}$$

Symmetrized expected utility. With $P \in \Delta(S)$ (uniform so far; therefore $P = pr$):

- $U(t_i, t_j) = [U_S(t_i, t_j) + U_R(t_i, t_j)]/2$
- $U_S(t_i, t_j) = \sum_s P(s)\ \sum_m P_S(m|s; t_i)\ \sum_{s'} P_R(s'|m, t_j)\ \delta(s, s')$, where $\delta(s, s')$ returns 1 iff $s = s'$ and otherwise 0
- $U_R(t_i, t_j) = U_S(t_j, t_i)$

4.2 Part 2: Mutation & replication.

The proportion of players of type i, x_i, is initialized as an arbitrary distribution over T. $p^\star \in \Delta(T)$ is learning a prior over (player) types dependent only on the lexicon of the type.

- $f_i = \sum_j x_j U(x_i, x_j)$
- $\Phi = \sum_i x_i f_i$
- $Q_{ij} \propto [\sum_d P(d|t_i)\ P(t_j|d)]^l$, where $P(t_j|d) \propto p^\star(t_j) P(d|t_j)$, d is a sequence of observations of length k of the form $\langle\langle s_i, m_j\rangle, \ldots \langle s_k, m_l\rangle\rangle$, and $l \geq 1$ is a learning parameter.
- For parental learning (standard RMD): $\dot{x}_i = \sum_j Q_{ji} \frac{x_j f_j}{\Phi}$

4.3 Model parameters & procedure

1. Sequence length k
2. Pragmatic production parameter α

3. Rationality parameter λ
4. Learning prior over types (lexica); cost parameter c. $p^{\star}(t_i) \propto n - c \cdot r$ where n is the total number of states and r that of upper-bounded messages only true of s_1 in t_i's lexicon (if only s_1 is true of a message, then this message encodes an upper-bound). Then the score for L_1, L_3, L_5 is 2, that of L_4 and L_6 is $2 - c$, and that of L_2 is $2 - 2c$; Normalization over lexica scores yields the prior over lexica (which is equal to the prior over types).
5. Prior over meanings (pr). We assume that $pr(s) = \frac{1}{|S|}$ for all s.
6. True state distribution (P). We currently assume that $P = \frac{1}{|S|}$ but it may be interesting to vary this
7. Learning parameter $l \geq 1$ with 1 corresponding to probability matching, and MAP as l approaches infinity
8. n is the sample of sequences of observations of length k sampled from the production probabilities of each type
9. Number of generations g

Procedural description. The game is initialized with some arbitrary distribution over player types. At the game's onset we compute Q once based on the sets of sequences D (one for each parent type). Replicator dynamics are computed based on the fitness of each type in the current population as usual. Q is computed anew for each independent run (of g generations) given that it depends on D, which is sampled from production probabilities.

References

Thomas L. Griffiths and Michael L. Kalish. Language evolution by iterated learning with bayesian agents. *Cognitive Science*, 31(3):441–480, 2007.