# Effects of transmission perturbation in the cultural evolution of language

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#### **Abstract**

Two main factors seem to play a major role in the cultural evolution of language. On the on hand, there is functional pressure towards efficient transfer of relevant information. On the other hand, languages have to be learned repeatedly and will therefore show traces of systematic stochastic perturbations operating during the transmission of linguistic knowledge. While a lot of attention has been paid to the effects of cognitive learning biases on the transmission of language, there is reason to expect that the class of possibly relevant transmission biases is much larger. This paper therefore explores some potential effects of transmission noise due to errors in the observation of states of the world. We look at three case studies on (i) vagueness, (ii) meaning deflation, and (iii) underspecified lexical meaning. These case studies suggest that transmission perturbations other than learning biases might explain attested patterns in the cultural evolution of language and that transmission perturbations due to perceptual noise may even produce effects very similar to learning biases.

**Keywords:** noise; cognitive biases; iterated learning; cultural evolution

## Introduction

Language is shaped by its use and transmission across generations. Linguistic properties therefore need not necessarily arise and stabilize solely due to functional pressure, such as the selection of more communicatively efficient behavior, but may also be influenced and selected for by a pressure for learnability. To put it extremely, an unlearnable language will not make it to the next generation. The effects that (iterated) learning has on language are often seen as stemming from a combination of general learning mechanisms and inductive cognitive biases (e.g. Griffiths & Kalish 2007, Kirby et al. 2014, Tamariz & Kirby 2016). Proposals of biases that shape language acquisition abound. Some prominent examples are mutual exclusivity (Merriman & Bowman 1989, Clark 2009), simplicity (Kirby et al. 2015), regularization (Hudson Kam & Newport 2005), and generalization (Smith 2011, O'Connor 2015).1 But there is good reason to expect that forces other than learning biases may systematically perturb the transmission of linguistic knowledge and therefore additionally contribute to the shaping of language by cultural evolution. In the following we focus on one particular source of perturbation: agents' imperfect perception of the world. The overall goal of this paper is to give a formalism in which to study the effects of such perturbations and apply it to three case studies on (i) vagueness, (ii) meaning deflation, and (iii) underspecified lexical meaning.

## **Iterated Bayesian learning**

We model the transmission of linguistic knowledge as a process of iterated learning (for recent overviews see Kirby et al. 2014, Tamariz & Kirby 2016). More specifically, we focus on iterated Bayesian learning, in which a language learner must infer unobservables, such as the lexical meaning of a word, from the observable behavior of a single teacher, who is a proficient language user (e.g. Griffiths & Kalish 2007, Kirby et al. 2007). The learner observes instances  $\langle s, m \rangle$  of overt language use in context, where s is a world state and m is the message that the teacher used in state s. The learner's task is to infer which latent type t, e.g., which set of lexical meanings, may have produced a sequence of such observations. To do so, the learner considers the posterior probability of type t given a data sequence d of  $\langle s, m \rangle$  pairs:

$$P(t \mid d) \propto P(t) P(d \mid t)$$
,

where P(t) is the learner's prior for type t and  $P(d \mid t) = \prod_{\langle s,m\rangle \in d} P(m \mid s,t)$  is the likelihood of type t producing the observed data d, with  $P(m \mid s,t)$  the probability that a type t produces message m when in world state s. Models of iterated Bayesian learning usually assume that the learner adopts a type with a probability proportional to  $P(t \mid d)^l$ , where  $l \geq 1$  is a parameter that regulates whether learners use probability matching (l = 1) or show tendency towards choosing a maximum of the posterior distribution (l > 1).

The set of possible data a learner may be exposed to is the set  $D_k$  of all sequences d of  $\langle s,m\rangle$  pairs with length k. The lower k, the less information the learner has to recover the true type of the teacher. Putting these components together, a transmission matrix Q captures the probability  $Q_{ji}$  that a learner acquires type i when learning from type j:

$$Q_{ji} \propto \sum_{d \in D_k} P(d \mid t_j) F(t_i \mid d), \text{ where}$$

$$F(t_i \mid d) \propto [P(t_i) P(d \mid t_i)]^l.$$

The prior P(t) can be understood as a encoding learning biases. For example, learners may have a preference for simpler languages over ones with a more complex grammar,

<sup>&</sup>lt;sup>1</sup>Depending on their formulation and the domain(s) they are proposed to apply to, biases may also interact. For instance, a domain-independent bias for simplicity may entail regularization but stand in conflict with mutual exclusivity.

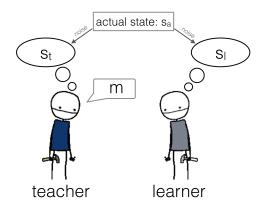


Figure 1: State-noise during observation of language use.

larger or more marked inventories, or cognitively taxing components (c.f. Feldman 2000, Chater & Vitányi 2003, Kirby et al. 2015). Crucially, even weak biases can magnify and have striking effects on an evolving linguistic system. Experimental and mathematical investigations in iterated learning have therefore argued that the linguistic structure evinced by the outcome of this process reflect learners' inductive biases (Kirby et al. 2007; 2014). The role of such biases can be viewed as that of introducing systematic perturbations in the transmission of linguistic knowledge, guiding learners to the convergence on particular evolutionary outcomes.

## **Iterated Bayesian learning with state-noise**

Other stochastic factors beyond learning biases in P(t) can influence the adoption of a linguistic type t based on observation of  $\langle s, m \rangle$  pairs. One further potential source of "transmission noise" are regular stochastic errors in the perception of world states (see Figure 1). Agents may not always perceive a world state perfectly. As a consequence, imperfect perception of world states may lead teachers to produce utterances that deviate from their production behavior – had they witnessed the state correctly. Similarly, learners may mistake utterances as applying to different states than the ones witnessed by the teacher who produced it. For instance, when learning the meaning of a vague adjective such as tall from an utterance like "John is tall," agents may have diverging representations of how tall John actually is, even if he is in a shared perceptual environment. The main idea to be explored here is that regularities in state-misperceptions may have striking and possibly explanatory effects on language evolution.

Let *S* be a set of world states. We denote the probability that the teacher (learner) observes state  $s_t$  ( $s_l$ ) when the actual state is  $s_a$  as  $P_N(s_t \mid s_a)$  ( $P_N(s_l \mid s_a)$ ). The probability that  $s_a$  is the actual state when the learner observes  $s_l$  is therefore:

$$P_N(s_a \mid s_l) \propto P(s_a) P_N(s_l \mid s_a)$$
.

Accordingly, the probability that the teacher observes  $s_t$  when

the learner observes  $s_l$  is:

$$P_N(s_t \mid s_l) = \sum_{s_a} P(s_a \mid s_l) P_N(s_t \mid s_a).$$

The probability that a teacher of type t produces data that is perceived by the learner as a sequence  $d_l$  of  $\langle s_l, m \rangle$  pairs is:

$$P_N(d_l \mid t) = \prod_{\langle s_l, m \rangle \in d_l} \sum_{s_t} P_N(s_t \mid s_l) P(m \mid s_t; t).$$

It is natural to assume that learners, even if they (in tendency) perform rational Bayesian inference on the likely teacher type t based on observation  $\langle s_l, m \rangle$ , do not also reason about statenoise perturbations. In that case, the posterior probability of t given the learner's perceived data sequence  $d_l$  is as before:

$$P(t \mid d_l) \propto P(t) P(d_l \mid t)$$
.

Still, state-noise affects the probability  $Q_{ji}$  that the learner adopts type i given that the teacher is of type j, because it influences the probability of observing a sequence  $d_l$ :

$$Q_{ji} \propto \sum_{d \in D_k} P_N(d_l \mid t_j) F(t_i \mid d),$$

where  $F(t_i \mid d)$  is as before.

Noise free iterated Bayesian learning is obtained as a special case when the perceived state is always the actual state.

In sum, it may be the case that learner and/or teacher do not perceive the actual state as what it is. They are not aware of this, and produce/learn as if what they observed was the actual state. The learner does not reason about noise when she tries to infer the speaker's type. She takes what she observes a state to be as the actual state that the teacher has seen as well and infers which type would have most likely generated the message to this state. This can lead to biases of inferring the "wrong" teacher type if the noise makes some types err in a way that resembles the noiseless behavior of other types. That is, such environmental factors can, in principle, induce transmission perturbations that look as if there was a cognitive bias in favor of a particular type, simply because that type better explains the noise.

### Case studies

In what follows we present three case studies that show how iterated learning under noisy perception of states can lead to the emergence of linguistic phenomena evinced in natural language. Case studies are ordered from more to less obvious examples in which state-noise may help explain phenomena of interest: (i) vagueness, (ii) meaning deflation, and (iii) underspecification in the lexicon. No case study is meant to suggest that state-noise is the definite answer to the question of how these properties arose. Instead, we restrict our attention to minimal settings that deliberately abstract away from aspects not required for our present aim, which is to elucidate the role that transmission perturbations beyond inductive biases may play in shaping the cultural evolution of language.

## **Vagueness**

Many if not most expressions in natural language are vague. Vagueness should be distinguished from imprecision and genuine ambiguity. The hallmark of a vagueness is susceptibility to a Sorites paradox (e.g. Williamson 1994): if a car for one million US\$ is expensive, and if additionally a car that costs a single dollar less than an expensive car is still expensive, then it is also true that a car for one US\$ is expensive. Clearly this is a fallacious reasoning pattern.

Vagueness also poses serious challenges to models of language evolution since functional pressure towards maximal information transfer should, under fairly general conditions, work against vagueness (Lipman 2009). Many have therefore argued that vagueness is intrinsically useful for communication (e.g. van Deemter 2009, de Jaegher & van Rooij 2011, Blume & Board 2014). Others hold that vagueness arises naturally due to limits in perception, memory or information processing (e.g. Franke et al. 2011, Lassiter & Goodman 2015, O'Connor 2014). We follow the latter line of exploration here, arguing that vagueness arises naturally under imperfect observability of states (see Franke & Correia (to appear) for an imitation-based dynamic based on the same idea).

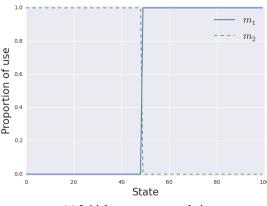
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**Setup.** We analyze the effects noisy perception has on the transmission of a simple language with 100 states,  $s \in [0,99]$ , and two messages  $m \in \{m_1, m_2\}$ . The probability of perceiving the actual state  $s_a$  as  $s_p$  is given by a truncated normal distribution with the actual state as its mean, a standard deviation  $\sigma$  and a truncation range [0,99]. That is,  $P(s_p|s_a) \sim \text{Normal}(s_a, \sigma, s_0, s_{99})$  with  $\sigma$  controlling the degree to which states are confused and  $s_0$  and  $s_{99}$  as boundaries.

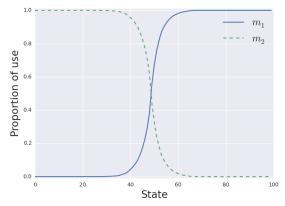
Linguistic behavior is assumed to be uniform across speakers and to depend solely on a type's index  $t_i$ ,  $i \in [0,99]$ . This index indicates which message a type uses in a (perceived) state: If  $s_j$  is the j-th state, then  $P(m_1|s_j,t_i)=1$  iff  $j \geq i$ . Otherwise,  $P(m_2|s_j,t_i)=1$ . In words, if the state is (perceived to be) as large or larger than a type's index, then message  $m_1$ , e.g., tall, is used. Otherwise  $m_2$ , e.g., small, is used.

**Results.** The effects of a single generational turnover under noisy transmission is depicted in Figure 2b. As shown in Figure 2a, this population initially consisted exclusively of type  $t_{50}$ . [TB: Do you think it might be confusing to start speaking about populations here? We now don't have a paragraph mentioning what evolves explicitly] As learners try to acquire this type, even small  $\sigma$  will lead to the emergence of vagueness at the population level. The same outcome is obtained for other values of  $\sigma$ , with higher values leading to more borderline cases that do not clearly fall under either only  $m_1$  or  $m_2$ . Generally, iterated noisy transmission leads to mixed populations and, consequently, to convex areas of the state space not clearly being associated with a particular form. The size of the space devoted to such borderline cases increases over generations with its growth being inversely related to l and k.

As is to be expected, if k is too small to discern even strikingly different types, then iterated learning under noisy perception leads to heterogeneous populations with (almost) no state being exclusively associated with  $m_1$  or  $m_2$ .



(a) Initial non-vague population



(b) Vague population after single generation

Figure 2: Noisy iterated learning with posterior sampling,  $\sigma = 0.4$  and k = 20.

Discussion. Transmission perturbations caused by the noisy perception of states reliably give rise to vagueness even if no borderline cases were initially part of a population's language. As modeled here, vagueness is not evidenced by particular types but at the population level. That is, vagueness is not a property of an individual's language, which makes sharp distinctions inasmuch as perception allows it, but of aggregated heterogeneous linguistic behavior. Of course, the stabilization of a linguistic system or population on a particular vague/clear state partition may reasonably be expected to depend not only on the effects of learning, but also on the functional (dis)advantages that such partition bring about for its users. Therefore, functional pressure may be necessary for borderline cases to be kept in check. Which factors or combination thereof plays a more central role for the emergence of vagueness is an empirical question we can not address here. Instead, we see these results as adding strength to the argument that one way in which vagueness may arise is as a byproduct of interactions between agents that may occasionally err in their perception of the environment – be it in interaction under functional pressure or in acquisition under a pressure for learnability.

## **Deflation**

Meaning deflation is a diachronic process by which a form's once restricted range of applicability broadens. Perhaps the most prominent example is Jespersen's cycle (Dahl 1979), the process by which emphatic negation, such as French *ne* ... *pas*, broadens over time and becomes a marker for standard negation instead. As argued by Bolinger (1981), certain word classes are particularly prone to slight and unnoticed reinterpretation. Consequently, when retrieving their meaning from contextual cues, learners may continuously spread their meaning out. For instance, Bolinger shows how the indefinite quantifier *several* has progressively shifted from meaning *a respectable number* to broader *a few* in American English. We follow this line of reasoning and show how state confusability may lead to meaning deflation through iterated acquisition.

**Setup.** As above, S = [0, 99], each type is associated with an index  $i \in [0,99]$ , and the noise pattern is given by  $P(s_p|s_a) \sim$ Normal( $s_a, \sigma, s_0, s_{99}$ ). However, we now trace the change of a single message m, e.g., several, coupled with linguistic behavior such that  $P(m|s_j,t_i) = 1$  iff  $s_j \ge i$ . Otherwise no message is sent. This behavior causes asymmetry in the learning data as types with high indices will reserve their message only for a small subset of the state space and otherwise remain silent. Consequently, learning also needs to be modified to take such silent observations into account. For simplicity, we assume that learners are aware of k and that  $P(t_i|d) \propto (\prod_{s \in d} P(m|s,t_i)) \times \text{Binom(successes} = k - |d|, \text{trials} = k, \text{succ.prob} = \sum_{j=0}^{i-1} P(s_j))$ . [MF: insert footnote to explain "Fixed k" idea and what the alternative should be] As before, the former factor corresponds to the likelihood of a type producing the perceived data. The latter is the probability of a type not reporting k - |d| events for a total of k events.  $P \in \Delta(S)$  is assumed to be uniform. In words, a long sequence of data consisting of mostly silence gives stronger evidence for the type producing it having a high index, even if the few state-message pairs observed in the sequence may be equally likely to be produced by types with lower indices.

**Results.** The development of a monomorphic population initially consisting only of  $t_{80}$  is shown in Figure 3. In this setup even little noise will cause a message to gradually be applied to larger portions of the state space. As above, the speed by which meaning deflates is regulated by  $\sigma$ , k, and to lesser degree l. In general, more state confusion due to higher  $\sigma$ , shorter sequences, or less posterior maximization will lead to more learners inferring lower types than present in the previous generation.

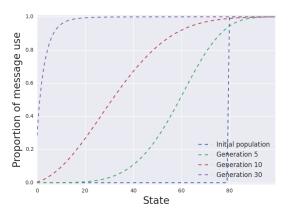


Figure 3: Noisy iterated learning with posterior sampling,  $\sigma = 0.4$  and k = 30.

**Discussion.** In contrast to the previous case study, we now considered the effects of noisy perception under asymmetric data generation in which overt linguistic evidence was not always produced. This setup can be likened to acquisition only from positive linguistic evidence in a world in which not every state is labeled (coupled with the idealized assumption that learners are aware of the amount of silence "produced" by a teacher).

The overall result discerned from this study is similar to that of the previous study. That is, noisy perception can cause transmission perturbations that relax once strict linguistic conventions. In contrast to the case of vagueness, if there are no competing forms, e.g., *small* vs. *tall*, asymmetry in production and noise will iteratively increase the state space that a form carves out. Just as the overuse of a word or difficulties in the retrieval of its meaning from contextual clues may lead to the deflation of its meaning in natural language.

#### **Scalar expressions**

Scalar expressions have been at the center of many studies on pragmatic inference. Examples include quantifiers such as *some* and *most*, adjectives such as *cold* and *big*, and numerals such as *four* and *ten*. Their commonality lies in that their use is often taken to pragmatically convey an upper-bound that these expressions semantically lack (Horn 1972, Gazdar 1979). For instance, while "I ate some of the cookies" is truth-conditionally compatible with a world state in which the speaker ate all of them, this utterance is usually reasoned to convey that the speaker ate *some but not all*. Otherwise, she would have used the stronger expression *all*. In this way, the meaning of a scalar expression lacking an upper-bound is strengthened by interlocutors' mutual reasoning about rational language use (Grice 1975).

To explain the selection for a lack of upper-bounds in these expressions Brochhagen et al. (2016) propose a model that combines functional pressure and iterated learning. Crucially, this account requires the assumption of a prior that favors a lack of upper-bounds. Technically, this assumption is re-

quired to distinguish between a language that rules out the bound pragmatically, as English, and a hypothetical alternative that does so semantically. Let us call the former language  $L_{\rm bound}$  and the latter  $L_{\rm lack}$ . To see the problem posed by  $L_{\rm bound}$  recall that learners need to infer unobservables from overt information. As a consequence, a user of  $L_{\rm bound}$  might be difficult or impossible to tease apart from one using  $L_{\rm lack}$  in tandem with mutual reasoning, i.e., one that conveys the bound through pragmatic reasoning. In the following we focus on only these two languages to elucidate under which conditions noisy perception may lead to the selection of  $L_{\rm lack}$  without a cognitive bias nor functional pressure.

**Setup.** We follow the setup of Brochhagen et al. (2016) with a reduced type space that only considers  $L_{\text{bound}}$  and  $L_{\text{lack}}$ paired with either literal or pragmatic language use. Both lexica specify the truth-conditions of two messages in either of two states. Let us mnemonically label them  $m_{\text{some}}$ ,  $m_{\text{all}}$ ,  $s_{\exists \neg \forall}$  and  $s_{\forall}$ , where the former state is one in which natural language some but not all holds and the latter one where all holds. Consequently, in  $L_{\text{bound}}$   $m_{\text{some}}$  is only true of  $s_{\exists \neg \forall}$  and  $m_{\rm all}$  of  $s_{\forall}$ . In English-like  $L_{\rm lack}$  message  $m_{\rm all}$  is also only true of  $s_{\forall}$ , but the meaning of  $m_{\text{some}}$  is underspecified and lexically holds in both states. Following previous models of probabilistic rational language use, lexica are paired with a linguistic behavior (c.f. Frank & Goodman 2012, Franke & Jäger 2014). This behavior can either be literal or pragmatic, giving rise to the following choice probabilities (see Brochhagen et al. 2016 for details):

where P(m|s,t) corresponds to a cell  $M_{sm}$  of a type's choice matrix. These values, rounded here for better readability, are obtained by combining lexica with linguistic behavior and a rationality parameter  $\lambda$ . Intuitively, higher values of  $\lambda$  increase the speaker's propensity to produce utterances that maximize communicative success. For our purposes, this parameter is fixed to be reasonably high so as to render speaker behavior (mostly) deterministic ( $\lambda = 20$ ). Pragmatic types are obtained through a process of mutual reasoning by which linguistic choice is refined, approximating the informal reasoning spelled out above. In this case a pragmatic  $L_{lack}$  speaker pragmatically associates  $m_{\text{some}}$  with  $s_{\exists \neg \forall}$  because she reasons (that her interlocutor reasons) that, if she wants to convey  $s_\forall$ successfully, she would be better off using  $m_{\rm all}$ . Lastly, and differently from Brochhagen et al.'s noise-free model, noise is introduced by two parameters  $\varepsilon$  and  $\delta$ . The former corresponds to the probability of perceiving the actual state  $s_{\neg \neg}$  as

$$s_{\forall}$$
,  $P(s_{\forall}|s_{\exists\neg\forall}) = \varepsilon$ , and conversely  $P(s_{\exists\neg\forall}|s_{\forall}) = \delta$ .

**Results.** To quantify the effects of the dynamics we ran 50 independent simulations per parameter configuration. Each population was initialized with an arbitrary distribution over types. The mean proportion of pragmatic users of  $L_{\text{lack}}$  under different noise signatures is shown in Figure 4. The results show that when  $\delta$  is small and  $\epsilon$  is high, iterated noisy transmission can lead to populations consisting of mostly, if not exclusively, types that do not lexicalize an upper-bound for *some*-like expressions provided language users are pragmatic. Similar results are obtained for increments in k, l or  $\lambda$ .

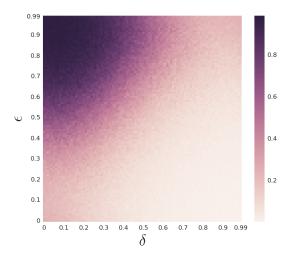


Figure 4: Mean proportion of pragmatic  $L_{lack}$  users after 20 generations with posterior sampling, k = 5 and  $\lambda = 20$ .

**Discussion.** The main goal of this case study was to show that noisy perception can mimic the effect of cognitive biases. In the case of Brochhagen et al. the assumed bias was one for simplicity. Accordingly, learners had an a priori preference for not codifying an upper-bound lexically over codifying it. As noted above, this influenced the propensity of learners to infer pragmatic  $L_{\rm lack}$  over  $L_{\rm bound}$  even if the data witnessed could not tease them apart. Here, we assumed no such bias but nevertheless arrived at an evolutionary outcome that is comparable to the one predicted if the bias were present. Note however that this outcome strongly depends on the types involved. Whether a type thrives under a particular noise signature depends on the proportion of types confused with it during transmission. The addition or extraction of a single type may therefore lead to different results.

At present, it is unclear what role noisy perception should play in the selection of underspecified meaning. Therefore these results should be taken as suggestive but not indicative of a relationship between the two. A possible way to explore this relation may lie in its connection to empirical work on the verification of quantified statements [TB: Reference? Couldn't find any strongly relevant literature after quick search. Will ask Jakub after thinking a bit more about this claim if nothing is to be found.], the idea being that some states are easier to verify, e.g.,  $s_{\forall}$ , and therefore less confusable with other states than others, e.g.,  $s_{\exists \neg \forall}$ .

#### General discussion

• We present three case studies that show how transmission perturbations can lead to the emergence of vagueness, meaning deflation, and a lack of upper-bounds in weak scalar expressions in populations of language users. These results are not meant to suggest noisy perception to be the sole or main determinant of these phenomena. Instead, this investigation's main contribution is conceptual and technical in nature in that it aims to clarify the role of systematic transmission perturbations of linguistic knowledge in language change while showing that such perturbations may stem from other sources, e.g., from learners' noisy perception.

#### Conclusion

[TB: TO DO]

## Acknowledgments

[TB: TO DO]

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