Effects of transmission perturbation in the cultural evolution of language

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Abstract

Two main factors seem to play a major role in the cultural evolution of language. On the on hand, there is functional pressure towards efficient transfer of relevant information. On the other hand, languages have to be learned repeatedly and will therefore show traces of systematic stochastic perturbations operating during the transmission of linguistic knowledge. While a lot of attention has been paid to the effects of cognitive learning biases on the transmission of language, there is reason to expect that the class of possibly relevant transmission biases is much larger. This paper therefore explores some potential effects of transmission noise due to errors in the observation of states of the world. We look at three case studies on (i) vagueness, (ii) meaning deflation, and (iii) underspecified lexical meaning. These case studies suggest that transmission perturbations other than learning biases might explain attested patterns in the cultural evolution of language and that transmission perturbations due to perceptual noise may even produce effects very similar to learning biases.

Keywords: noise; cognitive biases; iterated learning; cultural evolution

Introduction

Language is shaped by its use and transmission across generations. Linguistic properties therefore need not necessarily arise and stabilize solely due to functional pressure, such as the selection of more communicatively efficient behavior, but may also be influenced and selected for by a pressure for learnability. In the extreme, an unlearnable language will not make it to the next generation. The effects that (iterated) learning has on language are often seen as stemming from a combination of general learning mechanisms and inductive cognitive biases (e.g. Griffiths & Kalish 2007, Kirby et al. 2014, Tamariz & Kirby 2016). Proposals of biases that shape language acquisition abound. Some prominent examples are mutual exclusivity (Merriman & Bowman 1989, Clark 2009), simplicity (Kirby et al. 2015), regularization (Hudson Kam & Newport 2005), and generalization (Smith 2011, O'Connor 2015). But there is good reason to expect that forces other than learning biases may systematically perturb the transmission of linguistic knowledge and therefore additionally contribute to the shaping of language by cultural evolution. In the following we focus on one particular source of perturbation: agents' imperfect perception of the world. The overall goal of this paper is to give a formalism in which to study the effects of such perturbations and apply it to three case studies on (i) vagueness, (ii) meaning deflation, and (iii) underspecified lexical meaning.

Iterated Bayesian learning

We model the transmission of linguistic knowledge as a process of iterated learning (for recent overviews see Kirby et al. 2014, Tamariz & Kirby 2016). More specifically, we focus on iterated Bayesian learning, in which a language learner must infer unobservables, such as the lexical meaning of a word, from the observable behavior of a single teacher, who is a proficient language user (e.g. Griffiths & Kalish 2007, Kirby et al. 2007). The learner observes instances $\langle s,m \rangle$ of overt language use in context, where s is a world state and m is the message that the teacher used in state s. The learner's task is to infer which latent type t, e.g., which set of lexical meanings, may have produced a sequence of such observations. To do so, the learner considers the posterior probability of type t given a data sequence d of $\langle s,m \rangle$ pairs:

$$P(t \mid d) \propto P(t)\; P(d \mid t)\,,$$

where $P(t)$ is the learner's prior for type t and $P(d \mid t) = \prod_{\langle s,m \rangle \in d} P(m \mid s,t)$ is the likelihood of type t producing the observed data d, with $P(m \mid s,t)$ the probability that a type t produces message m when in world state s. Models of iterated Bayesian learning usually assume that the learner adopts a type with a probability proportional to $P(t \mid d)^l$, where $l \geq 1$ is a parameter that regulates whether learners use probability matching ($l = 1$) or show tendency towards choosing a maximum of the posterior distribution ($l > 1$).

The set of possible data a learner may be exposed to is the set D_k of all sequences d of $\langle s,m \rangle$ pairs with length k. The lower k, the less information the learner has at her disposition to recover the true type of the teacher. Putting these components together in a transmission matrix, Q_{ji} is the probability that a learner acquires type i when learning from type j:

$$Q_{ji} \propto \sum_{d \in D_k} P(d \mid t_j) F(t_i \mid d)\,, \text{where}$$

$$F(t_i \mid d) \propto [P(t_i) P(d \mid t_i)]^l\,.$$

The prior $P(t)$ can be understood as a encoding learning biases. For example, learners may have a preference for simpler languages over ones with a more complex grammar, larger or more marked inventories, or cognitively taxing components (c.f. Feldman 2000, Chater & Vitányi 2003, Kirby et al. 2015). Crucially, even weak biases can magnify and have striking effects on an evolving linguistic system. Experimental and mathematical investigations in iterated learning

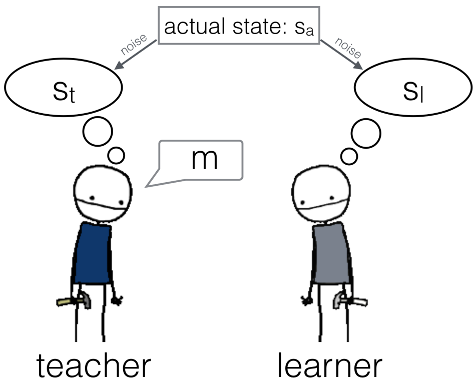


Figure 1: State-noise during observation of language use.

have therefore argued that the linguistic structure evinced by the outcome of this process reflects learners' inductive biases (Kirby et al. 2007; 2014).

Iterated Bayesian learning with state-noise

Other stochastic factors beyond learning biases in $P(t)$ can influence the adoption of a linguistic type t based on the observation of $\langle s,m\rangle$ pairs. One further potential source of "transmission noise" are regular stochastic errors in the perception of world states (see Figure 1). Imperfect perception of world states may lead teachers to produce utterances that deviate from their production behavior had they witnessed the state correctly. Similarly, learners may mistake utterances as applying to different states than the ones witnessed by the teacher who produced them. For instance, when learning the meaning of a vague adjective such as *tall* from an utterance like "John is tall," agents may have diverging representations of how tall John actually is, even if he is in a shared perceptual environment. The main idea to be explored here is that regularities in state-misperceptions may have striking and possibly explanatory effects on language evolution.

Let S be a set of world states. We denote the probability that the teacher (learner) observes state s_t (s_l) when the actual state is s_a as $P_N(s_t \mid s_a)$ ($P_N(s_l \mid s_a)$). The probability that s_a is the actual state when the learner observes s_l is therefore:

$$P_N(s_a \mid s_l) \propto P(s_a)\, P_N(s_l \mid s_a)\,.$$

Accordingly, the probability that the teacher observes s_t when the learner observes s_l is:

$$P_N(s_t \mid s_l) = \sum_{s_a} P(s_a \mid s_l)\, P_N(s_t \mid s_a)\,.$$

The probability that a teacher of type t produces data that is perceived by the learner as a sequence d_l of $\langle s_l,m\rangle$ pairs is:

$$P_N(d_l \mid t) = \prod_{\langle s_l,m\rangle \in d_l} \sum_{s_t} P_N(s_t \mid s_l)\, P(m \mid s_t;t)\,.$$

It is natural to assume that learners, even if they (in tendency) perform rational Bayesian inference on the likely teacher type t based on observation $\langle s_l,m\rangle$, do not also reason about state-noise perturbations. In this case the posterior probability of t given the learner's perceived data sequence d_l is as before:

$$P(t \mid d_l) \propto P(t)\, P(d_l \mid t)\,.$$

Still, state-noise affects the probability Q_{ji} that the learner adopts type i given a teacher of type j, because it influences the probability of observing a sequence d_l:

$$Q_{ji} \propto \sum_{d \in D_k} P_N(d_l \mid t_j) F(t_i \mid d)\,,$$

where $F(t_i \mid d)$ is as before.

Noise free iterated Bayesian learning is obtained as a special case when the perceived state is always the actual state.

In sum, it may be the case that learner and/or teacher do not perceive the actual state as what it is. They are not aware of this, and produce/learn as if what they observed was the actual state. The learner does not reason about noise when she tries to infer the speaker's type. She takes what she observes a state to be as the actual state that the teacher has seen as well and infers which type would have most likely generated the message to this state. This can lead to biases of inferring the "wrong" teacher type if the noise makes some types err in a way that resembles the noiseless behavior of other types. That is, such environmental factors can, in principle, induce transmission perturbations that look as if there was a cognitive bias in favor of a particular type, simply because that type better explains the noise.

Case studies

In what follows we present three case studies that show how iterated learning under noisy perception of states can lead to the emergence of linguistic phenomena found in natural language. Case studies are ordered from more to less obvious examples in which state-noise may help explain phenomena of interest: (i) vagueness, (ii) meaning deflation, and (iii) underspecification in the lexicon. No case study is meant to suggest that state-noise is the definite answer to the question of how this property arose. Instead, we restrict our attention to minimal settings that deliberately abstract away from aspects not required for our present aim, which is to elucidate the role that transmission perturbations beyond inductive biases may play in shaping the cultural evolution of language.

Vagueness

Many if not most expressions in natural language are vague. Vagueness should be distinguished from imprecision and genuine ambiguity. The hallmark of vagueness is susceptibility to a Sorites paradox (e.g. Williamson 1994): if a car for one million US$ is expensive, and if additionally a car that costs a single dollar less than an expensive car is still expensive, then it is also true that a car for one US$ is expensive. Vague expressions fall prey to this fallacious reasoning pattern.

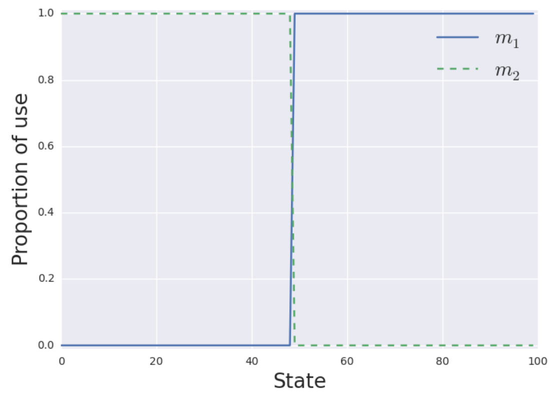
Vagueness also poses serious challenges to models of language evolution since functional pressure towards maximal information transfer should, under fairly general conditions, work against vagueness (Lipman 2009). Many have therefore argued that vagueness is intrinsically useful for communication (e.g. van Deemter 2009, de Jaegher & van Rooij 2011, Blume & Board 2014). Others hold that vagueness arises naturally due to limits in perception, memory, or information processing (e.g. Franke et al. 2011, Lassiter & Goodman 2015, O'Connor 2014). We follow the latter line of exploration here, arguing that vagueness arises naturally under imperfect observability of states (see Franke & Correia (to appear) for an imitation-based dynamic based on the same idea).

[MF: rewrite up to here]

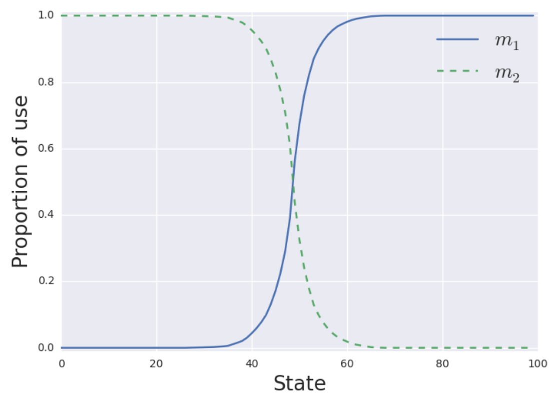
Setup. We analyze the effects noisy perception has on the transmission of a simple language with 100 states, $s \in [0,99]$, and two messages, $m \in \{m_1, m_2\}$. The probability of perceiving the actual state s_a as s_p is given by a truncated normal distribution with the actual state as its mean, a standard deviation σ and a truncation range $[0,99]$. That is, $P(s_p|s_a) \sim \text{Normal}(s_a, \sigma, s_0, s_{99})$ with σ controlling the degree to which states are confused, and as boundaries s_0 and s_{99}. Linguistic behavior is assumed to be uniform across speakers and to depend solely on a type t_i's index, $i \in [0,99]$. This index indicates which message a type uses in a (perceived) state: If s_j is the j-th state, then $P(m_1|s_j,t_i) = 1$ iff $j \geq i$. Otherwise, $P(m_2|s_j,t_i) = 1$. In words, if the state is perceived to be as large or larger than a type's index, then message m_1, e.g., *tall*, is used. Otherwise m_2, e.g., *small*, is used.

Results. The effects of a single generational turnover under noisy transmission is depicted in Figure 2b. As shown in Figure 2a, this population initially consisted exclusively of type t_{50}. [TB: Do you think it might be confusing to start speaking about populations here? We now don't have a paragraph mentioning what evolves explicitly] As learners try to acquire this type, even small σ will lead to the emergence of vagueness. The same outcome is obtained for other values of σ with higher values leading to more borderline cases that do not clearly fall under either only m_1 or m_2. Here, iterated noisy transmission leads to mixed populations and, consequently, to convex areas of the state space not clearly associated with a particular form. The size of the space devoted to such borderline cases increases over generations with its growth being inversely related to l and k. As is to be expected, if k is too small to discern even strikingly different types, then iterated learning under noisy perception leads to heterogeneous populations with (almost) no state being exclusively associated with m_1 or m_2.

Discussion. Transmission perturbations caused by the noisy perception of states reliably give rise to vagueness even if no borderline cases were initially part of a population's language. As modeled here, vagueness is not evidenced by particular types but at the population level. That is, it is not a property of individuals' languages, which make sharp distinctions inasmuch as perception allows it, but of aggregated linguistic behavior. Of course, the stabilization of a linguistic system or population on a particular vague/clear state partition may reasonably be expected to depend not only on the effects of learning, but also on the functional (dis)advantages that this partition brings about for its users. Functional pressure may therefore well be necessary for borderline cases to be kept in check. Which factor or combination thereof plays a more central role for the emergence of vagueness is an empirical question we do not address here. Instead, we see these results as adding strength to the argument that one way in which vagueness may arise is as a byproduct of interactions between agents that may occasionally err in their perception of the environment – be it in interaction under functional pressure or in acquisition under a pressure for learnability.



(a) Initial non-vague population.



(b) Vague population after single generation.

Figure 2: Noisy iterated learning with posterior sampling, $\sigma = 0.4$, and $k = 20$.

Deflation

Meaning deflation is a diachronic process by which a form's once restricted range of applicability broadens. Perhaps the most prominent example is Jespersen's cycle (Dahl 1979), the process by which emphatic negation, such as French *ne*

... *pas*, broadens over time and instead becomes a marker for standard negation. As argued by Bolinger (1981), certain word classes are particularly prone to slight and unnoticed reinterpretation. Consequently, when retrieving their meaning from contextual cues, learners may continuously spread their meaning out. For instance, Bolinger discusses how the indefinite quantifier *several* has progressively shifted from meaning *a respectable number* to broader *a few* in American English. We follow this line of reasoning and show how state confusability may lead to meaning deflation.

Setup. As above, $S = [0,99]$, each type is associated with an index $i \in [0,99]$, and the noise pattern is given by $P(s_p|s_a) \sim \text{Normal}(s_a, \sigma, s_0, s_{99})$. However, we now trace the change of a single message m, e.g., *several*, coupled with linguistic behavior such that $P(m|s_j,t_i) = 1$ iff $j \geq i$. Otherwise no message is sent. This behavior causes asymmetry in the learning data as types with high indices will reserve their message only for a small subset of the state space and otherwise remain silent. Consequently, learning also needs to be modified to take such silent observations into account. For simplicity, we assume that learners are aware of k and that $P(t_i|d_l) \propto (\prod_{s \in d_l} P(m|s,t_i)) \times \text{Binom}(\text{successes} = k - |d_l|, \text{trials} = k, \text{succ.prob} = \sum_{j=0}^{i-1} P(s_j))$.[1] As before, the former factor corresponds to the likelihood of a type producing the perceived data. The latter is the probability of a type not reporting $k - |d|$ events for a total of k events. $P \in \Delta(S)$ is assumed to be uniform. In words, a long sequence of data consisting of mostly silence gives stronger evidence for the type producing it having a high index even if the few state-message pairs observed in the sequence may be equally likely to be produced by types with lower indices.

Results. The development of an initially monomorphic population consisting only of t_{80} is shown in Figure 3. In this setup even little noise will cause a message to gradually be applied to larger portions of the state space. As above, the speed by which meaning deflates is regulated by σ, k, and to lesser degree l. In general, more state confusion due to higher σ, shorter sequences, or less posterior maximization will lead to more learners inferring lower types than present in the previous generation.

Discussion. In contrast to the previous case study, we now considered the effects of noisy perception under asymmetric data generation where overt linguistic evidence is not always produced. This setup can be likened to acquisition only from positive linguistic evidence in a world in which not every state is labeled.

[1] Knowing fixed k allows learners to compute the likelihood of a type not reporting $k - |d_l|$ state observations. A more involved but better justified alternative is to specify a prior over k and for learners to perform a joint inference on k and the teacher's type given a partial observation of only overtly produced data. For simplicity, we opt for the former, albeit admittedly artificial, assumption.

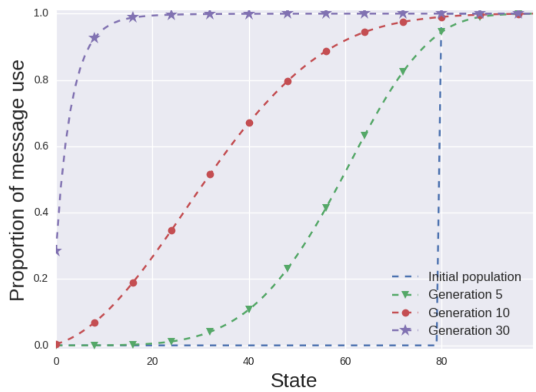


Figure 3: Noisy iterated learning with posterior sampling, $\sigma = 0.4$, and $k = 30$.

The overall result is similar to that of the previous study. Noisy perception can cause transmission perturbations that relax once strict linguistic conventions. In contrast to the case of vagueness, if there are no competing forms, e.g., *small* vs. *tall*, asymmetry in production and noise will iteratively increase the state space that a form carves out. Just as the overuse of a word or difficulties in the retrieval of its meaning from contextual cues may lead to the deflation of its meaning in natural language.

Scalar expressions

Scalar expressions have been at the center of many studies on pragmatic inference. Examples include quantifiers such as *some* and *most*, adjectives such as *cold* and *big*, and numerals such as *four* and *ten*. Their commonality lies in that their use is often taken to pragmatically convey an upper-bound that these expressions semantically lack (Horn 1972, Gazdar 1979). For instance, while "I ate some of the cookies" is truth-conditionally compatible with a world state in which the speaker ate all of them, this utterance is usually reasoned to convey that the speaker ate *some but not all*. Otherwise, she would have used *all*. In this way the meaning of a scalar expression lacking an upper-bound is strengthened by interlocutors' mutual reasoning about rational language use (Grice 1975).

To explain the selection for a lack of upper-bounds in these expressions, Brochhagen et al. (2016) propose a model that combines functional pressure and iterated learning. Crucially, this account requires the assumption of a prior that favors a lack of upper-bounds. Technically, this assumption is required to distinguish between a language that rules out the bound pragmatically, as English, and a hypothetical alternative that does so semantically. Let us call the former language L_{bound} and the latter L_{lack}. To see the problem posed by L_{bound} recall that learners need to infer unobservables from overt information. As a consequence, a user of L_{bound} might be difficult to impossible to tease apart from one using L_{lack} but conveying the bound pragmatically. In the following we focus on only these two languages to elucidate under which conditions noisy perception may lead to the selection of L_{lack} without a cognitive bias nor functional pressure.

Setup. We follow the setup of Brochhagen et al. (2016) with a reduced type space that only considers L_{bound} and L_{lack} paired with either literal or pragmatic language use. Both lexica specify the truth-conditions of two messages in either of two states. Let us mnemonically label them m_{some}, m_{all}, $s_{\exists\neg\forall}$ and $s_{\forall}$, where the former state is one in which natural language *some but not all* holds, and the latter one where *all* holds. Consequently, in L_{bound} message m_{some} is only true of $s_{\exists\neg\forall}$ and m_{all} only of $s_{\forall}$. In English-like L_{lack}, message m_{all} is also only true of $s_{\forall}$, but the meaning of m_{some} is underspecified and lexically holds in both states. Following previous models of probabilistic rational language use lexica are paired with a linguistic behavior (c.f. Frank & Goodman 2012, Franke & Jäger 2014). This behavior can either be literal or pragmatic, giving rise to the following choice probabilities of four different types (see Brochhagen et al. 2016 for details):

$$
\begin{array}{lcc}
 & \underline{L_{\text{bound}}} & \underline{L_{\text{lack}}} \\
 & \begin{array}{ccc} & m_{\text{some}} & m_{\text{all}} \end{array} & \begin{array}{ccc} & m_1 & m_2 \end{array} \\
\text{Literal} \approx & \begin{array}{c} s_{\forall} \\ s_{\exists\neg\forall} \end{array} \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} & \begin{array}{c} s_{\forall} \\ s_{\exists\neg\forall} \end{array} \begin{pmatrix} 0.5 & 0.5 \\ 1 & 0 \end{pmatrix} \\
 & \begin{array}{ccc} & m_{\text{some}} & m_{\text{all}} \end{array} & \begin{array}{ccc} & m_1 & m_2 \end{array} \\
\text{Pragmatic} \approx & \begin{array}{c} s_{\forall} \\ s_{\exists\neg\forall} \end{array} \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} & \begin{array}{c} s_{\forall} \\ s_{\exists\neg\forall} \end{array} \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix},
\end{array}
$$

where $P(m|s,t)$ corresponds to cell M_{sm} of a type's choice matrix. These values, rounded here for better readability, are obtained by combining lexica with linguistic behavior and a rationality parameter λ. Intuitively, higher values of λ increase the speaker's propensity to produce utterances that maximize communicative success. For our purposes this parameter is fixed to be reasonably high so as to render speaker behavior (mostly) deterministic ($\lambda = 20$). Pragmatic types are obtained through a process of mutual reasoning by which linguistic choice is refined, approximating the informal reasoning spelled out above. In this case a pragmatic L_{lack} speaker pragmatically associates $s_{\exists\neg\forall}$ more strongly with m_{some} because she reasons (that her interlocutor reasons) that, if she wants to convey $s_{\forall}$ successfully, she would be better off using m_{all}. Lastly, and differently from Brochhagen et al.'s noise-free model, noise is introduced by parameters ε and δ. The former corresponds to the probability of perceiving the actual state $s_{\exists\neg\forall}$ as $s_{\forall}$, $P(s_{\forall}|s_{\exists\neg\forall}) = \varepsilon$, and $P(s_{\exists\neg\forall}|s_{\forall}) = \delta$.

Results. To quantify the effects of the dynamics we ran 50 independent simulations per parameter configuration. Each population was initialized with an arbitrary distribution over types. The mean proportion of pragmatic users of L_{lack} under different noise signatures is shown in Figure 4. These results show that when δ is small and ε is high, iterated noisy transmission can lead to populations consisting of mostly, if not exclusively, a type that does not lexicalize an upper-bound for *some*-like expressions but conveys it pragmatically. Similar results are obtained for increments in k, l, or λ.

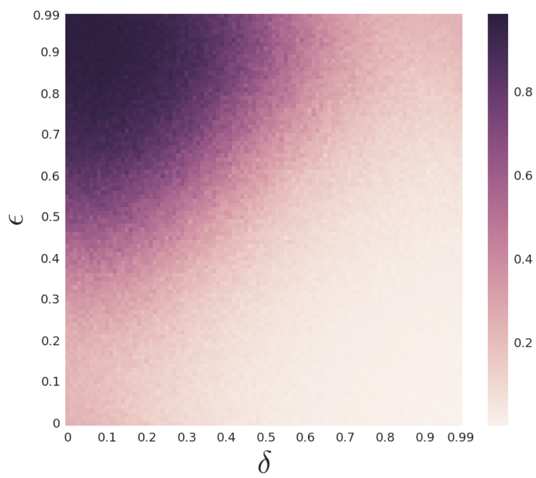


Figure 4: Mean proportion of pragmatic L_{lack} users after 20 generations with posterior sampling, $k = 5$, and $\lambda = 20$.

Discussion. The main goal of this case study was to show that noisy perception may mimic the effect of cognitive biases. In the case of Brochhagen et al. the assumed bias was one for simplicity. Accordingly, learners had an a priori preference for not codifying an upper-bound lexically. This increases the learners' propensity to infer pragmatic L_{lack} over L_{bound} even if the data witnessed could not tease them apart. Here, we assumed no such bias but nevertheless arrived at an evolutionary outcome that is comparable to the one predicted if the bias were present. Note however that this outcome strongly depends on the types involved. Whether a type thrives under a particular noise signature depends on the proportion of types confused with it during transmission. The addition or extraction of a single type may therefore lead to different results.

At present, it is unclear what role noisy perception should play in the selection of underspecified meaning. These results should therefore be taken as suggestive but not indicative of a relationship between the two. A possible way to explore this relation may lie in their connection to empirical work on the verification of quantified statements (see Szymanik 2016 for a recent overview). The idea being that some states are easier to verify, e.g., $s_{\forall}$, and therefore less confusable with other states than others, e.g., $s_{\exists\neg\forall}$.

General discussion

We proposed a model of iterated Bayesian learning that accommodates for systematic noise in agents' perception, giving rise to stochastic perturbations that may influence and explain language change. We investigated the model's predictions in three case studies that show that iterated noisy transmission can lead to outcomes akin to those found in natural language. As stressed before, these results are not meant to suggest noisy perception to be the sole or main determinant of these phenomena. Instead, our aim was mainly conceptual and technical in nature.

Beyond technical aspects, we foregrounded two intertwined issues in the cultural evolution of language. First, the fact that noise signatures may mimic the effects of cognitive biases has consequences for the interpretation of the outcomes of acquisition processes. Care must therefore be exercised in reading off the influence of possible cognitive learning biases from data obtained both in the wild and the laboratory. Second, and more importantly, these results may be seen as complementing and stressing the pivotal role of systematic transmission perturbations as explanatory and predictive devices of language change – independent of the perturbation's source. They thereby strengthen and widen the scope of research on iterated learning by bringing attention to forces beyond inductive biases.

Conclusion

Acquisition is a central force shaping linguistic structure. The consideration of the (imperfect) means by which such knowledge is transmitted is therefore crucial to our understanding of the cultural evolution of language. Here, we focused on one factor that may give rise to explanatory systematic stochastic perturbation in learning; agents' noisy perception of the world, and analyzed its effects in three case studies on (i) vagueness, (ii) meaning deflation, and (iii) underspecified lexical meaning. Our results suggest that the class of relevant perturbation sources reaches beyond the well-studied effects of inductive learning biases. In particular, that some linguistic properties, such as (i), (ii) and more tentatively (iii), may emerge as a byproduct of environmental factors that influence agents' perception of the world.

Acknowledgments

[TB: TO DO]

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