Co-evolution of lexical meaning & pragmatic use

Abstract

According to standard linguistic theory, the meaning of an utterance is the product of conventional semantic meaning and general pragmatic rules on language use. To investigate how cultural evolution of language plays out under this picture of the semantics-pragmatics interface, we present a game-theoretic model of the competition between types of language users, each endowed with a selection of lexical representations and a particular pragmatic disposition to act on them. Our model traces two evolutionary forces and their interaction: (i) pressure towards communicative efficiency and (ii) transmission perturbations during the acquisition of linguistic knowledge. We illustrate the model based on a case study on scalar implicatures, which suggests the relationship between underspecified semantics and pragmatic inference to be one of co-evolution. The maintenance of underspecified semantics depends on pragmatic reasoning for their enrichment. In the opposite direction, complex semantics that can do away with mutual reasoning may not foster pragmatic reasoning, and may even be encumbered by it.

1   Introduction

What is conveyed usually goes beyond what is said. A request for a blanket can be politely veiled by uttering "I'm cold;" a temporal succession of events can be communicated by the order in which conjuncts appear as in "I traveled to Paris and got married;" an invitation can be declined by saying "I have to work." An influential explanation of the relation between the literal meaning of expressions and what they may convey in context is due to Grice (1975), who characterizes pragmatic use and interpretation as a process of mutual reasoning about rational language use. For instance, under the assumption that the speaker is cooperative and relevant, "I have to work" may be interpreted as providing a reason why the speaker will not be able to accept an invitation, going beyond its literal meaning. Some of these enrichments are rather *ad hoc*. Others show striking regularities, such as the use of ability questions for polite requests ("Could you please . . . ?"), or certain enrichments of lexical meanings such as *and* to convey *and then*.

A particularly productive and well studied class of systematic pragmatic enrichments are scalar impli-

catures (Horn 1984, Hirschberg 1985, Levinson 1983, Geurts 2010). Usually, the utterance of a sentence like "I own some of Johnny Cash's albums" will be taken to mean that the speaker does not own all of them. This is because, if the speaker instead owned them all, she could have used the word *all* instead of *some* in her utterance, thereby making a more informative statement. Scalar implicatures, especially the inference from *some* to *some but not all*, have been studied extensively, both theoretically (e.g., Sauerland 2004, Chierchia et al. 2012, van Rooij and de Jager 2012) as well as experimentally (e.g., Bott and Noveck 2004, Huang and Snedeker 2009, Grodner et al. 2010, Goodman and Stuhlmüller 2013a, Degen and Tanenhaus 2015). This makes them particularly suitable candidates for the study of the evolution of regular pragmatic inferences. While there is much dispute in this domain about many details, a position endorsed by a clear majority is that a scalar item like *some* is underspecified to mean *some and maybe all* and that the enrichment to *some but not all* is part of some regular process with roots in pragmatics.

If this majority view is correct, the question arises how such a division of labor between semantics and pragmatics could have evolved; why it would be so pervasive across natural languages; and why it is that some expressions systematically draw from it while others semantically conventionalize.

Models of language evolution abound. There are simulation-based models studying populations of communicating agents (Hurford 1989, Steels 1995, Lenaerts et al. 2005, Steels and Belpaeme 2005, Baronchelli et al. 2008, Steels 2011, Spike et al. 2016) and there are mathematical models of language evolution, many coming from game theory (Wärneryd 1993, Blume et al. 1993, Nowak and Krakauer 1999, Nowak 2006, Huttegger 2007, Skyrms 2010). Much of this work has focused on explaining basic properties such as compositionality and combinatoriality (e.g., Batali 1998, Nowak and Krakauer 1999, Nowak et al. 2000, Kirby and Hurford 2002, Kirby 2002, Smith et al. 2003, Gong 2007, Kirby et al. 2015, Verhoef et al. 2014, Franke 2016), but little attention has been paid to the interaction between conventional meaning and pragmatic use. What is more, many mathematical models explain evolved meaning as a regularity in the overt behavior of agents, abstracting from complex interactions between semantic representations and pragmatic use. In contrast, we will here look at language users with a richer linguistic make-up. More precisely, we spell out a model of the evolution of conventional meaning and pragmatic reasoning. The objects of replication and selection are pairs consisting of a set of lexical meanings and a manner of pragmatic behavior, which together define agents' linguistic choices. Pragmatic behaviors are represented using probabilistic models of pragmatic language use (Frank and Goodman 2012, Franke and Jäger 2016, Goodman and Frank 2016). Replication and selection are described by the *replicator mutator dynamic*, a general and established model of evolutionary change in large and homogeneous populations (Hofbauer 1985, Nowak et al. 2000; 2001, Hofbauer and Sigmund 2003, Nowak 2006). This

approach allows us to study the interaction between (i) pressure towards communicative efficiency and (ii) infidelity in the transmission of linguistic knowledge, caused by factors such as inductive learning biases and sparse learning data (Kirby and Hurford 2002, Smith et al. 2003, Kirby et al. 2014). Considering the effects that the transmission of linguistic knowledge through (iterated) learning have is important because neither semantic meaning nor pragmatic usage patterns are directly observable. Instead, language learners have to infer these unobservables from the observable behavior in which they result. We formalize this process as a form of Bayesian inference. Our approach thereby contains a well-understood model of iterated Bayesian learning (Griffiths and Kalish 2005; 2007), but combines it with functional selection, here formalized as the most versatile dynamic from evolutionary game theory, the replicator dynamic (Taylor and Jonker 1978). Section 2 introduces this model.

Section 3 applies this model to a case study on scalar implicatures. We discuss a setting in which the majority view of underspecified lexical meanings and systematic pragmatic enrichments emerges if selection and transmission infidelity are combined. In particular, we show that inductive learning biases of Bayesian learners that favor simpler lexical meanings may play a role in preventing the lexicalization of scalar inferences and thereby lead to the emergence of Gricean-like pragmatic reasoning types. The results of this study are critically assessed in the light of the assumptions that feed our model in Section 4.

2   A model of evolving lexical representations and pragmatic behavior

2.1   Communicative success and learnability

The emergence and change of linguistic structure is influenced by many intertwined factors. These range from biological and socio-ecological to cultural ones (Benz et al. 2005, Steels 2011, Tamariz and Kirby 2016). Social and ecological pressures determine communicative needs, while biology determines the architecture that enables and constrains the means by which they can be fulfilled. In the following, our focus lies on cultural aspects, wherein processes of linguistic change are viewed as shaped by language use and its transmission, i.e., as a result of a process of cultural evolution (Pagel 2009, Thompson et al. 2016).

The idea that language is an adaptation to serve a communicative function is fundamental to many synchronic and diachronic analyses, at least since Zipf's (1949) explanation of word frequency rankings as a result of competing hearer and speaker preferences (e.g., in Martinet 1962, Horn 1984, Jäger and van Rooij 2007, Jäger 2007a, Piantadosi 2014, Kirby et al. 2015). If processes of selection, such as conditional imitation or reinforcement, favor more communicatively efficient types of behavior, languages are driven

towards semantic expressivity (e.g., Nowak and Krakauer 1999, Skyrms 2010). But pressure towards communicative efficiency is not the only force that shapes language. Learnability is another, as natural languages need to be learnable to survive their faithful transmission across generations. Furthermore, the effects of even small learning biases implicit in acquisition can build up and have quite striking impact on an evolving language in a process of iterated learning (Kirby and Hurford 2002, Smith et al. 2003, Kirby et al. 2014).

While natural languages are pressured for both communicative efficiency and learnability, these forces may pull in opposite directions (Christiansen and Chater 2008:§7). Their opposition becomes particularly clear when considering the extreme (Kemp and Regier 2012, Kirby et al. 2015). A language consisting of a single form-meaning association is easy to learn but likely bad for communication with peers. Conversely, a language that lexicalizes distinct forms for a large number of different meanings may be better for communication but challenging to acquire.

2.2 The replicator mutator dynamic

An elegant formal approach to capture the interaction between communicative efficiency and learnability is the *replicator mutator dynamic* (Hofbauer 1985, Nowak et al. 2000; 2001, Hofbauer and Sigmund 2003, Nowak 2006). The RMD is a population dynamic. It describes the average change in the composition of a community of agents of different types. The dynamic assumes a virtually infinite population, so that it is able to abstract away from what individual agents do and how they change their individual behavior over time. The result is an abstract description of the most probable trajectory of change in the population vector $\vec{x}$, which codifies relative frequencies of types, so that x_i is the relative frequency of type i and $\sum_i x_i = 1$. In its simplest discrete-time formulation, the RMD defines the frequency x_i' of each type i in the population at the next time step as a function of: (i) the frequency x_i of each type i before the update step, (ii) the fitness f_i of each type i before the update, and (iii) the probability Q_{ji} that an agent who observes the behavior of an agent with type j ends up acquiring type i:

$$x_i' = \sum_j \frac{x_j f_j Q_{ji}}{\Phi} \,, \tag{1}$$

where $\Phi = \sum_k x_k f_k$ is the average fitness in the population.

The RMD consists of two components: fitness-based selection and transmission biases encoded in the so-called mutation matrix Q. This becomes most transparent when we consider an equivalent formulation in terms of a step-wise application of the discrete-time replicator dynamic (Taylor and Jonker 1978) on

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the initial population vector $\vec{x}$ and its subsequent multiplication with the mutation matrix Q:

$$x_i' = (\mathrm{M}(\mathrm{RD}(\vec{x})))_i\,, \tag{2}$$

where

$$(\mathrm{RD}(\vec{x}))_i = \frac{x_i f_i}{\Phi} \quad \text{and} \quad (\mathrm{M}(\vec{x}))_i = (\vec{x} \cdot Q)_i = \left(\sum_j x_j Q_{ji}\right)_i.$$

If there is no mutation, so that types are always replicated faithfully with $Q_{ji} = 1$ whenever $j = i$, the dynamic reduces to the replicator dynamic. If fitness is the same for all types, $f_i = f_j$ for all i and j, the dynamics reduces to a process of iterated mutation. Appendix A provides an abstract minimal example of how the replicator mutator dynamic works.

2.3 Interpreting the replicator mutator dynamic

The replicator mutator dynamic is an abstract description of population-level changes. It abstracts away from specific assumptions about how agents interact. To understand this better, the following looks at each component in isolation: first replication, then mutation.

According to the replicator dynamic the relative frequency x_i of type i increases proportional to its average fitness in the population. This dynamic is popular and versatile because it can be derived from many abstract processes of biological and cultural transmission and selection (for overview and several derivations see Sandholm 2010). Under a biological interpretation, the model describes population-level changes when the number of offspring under single-parent preproduction is proportional to the parent's fitness. Under a cultural interpretation, examples of concrete agent-level processes that lead to population-level changes as described by the replicator dynamic include conditional imitation (e.g., Helbing 1996, Schlag 1998) or reinforcement learning (e.g., Börgers and Sarin 1997, Beggs 2005). In conditional imitation, for instance, each agent carries a fixed type i which it may change to a type j when it happens to observe another agent using type j. Imitation occurs with a probability that is a monotonic function of how well j fares against the population: the better the behavior j, the more likely it is to be imitated.[1]

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[1] A crucial assumption in all of the derivations of the replicator dynamic as a biological or cultural process of evolution is that agents themselves are rather limited in their strategic capacities. They are, for example, not able to innovate a strategy, i.e., to just choose a type, that is optimal given the current population state. They are also not able to strategically adapt their behavior to any particular agent they are interacting with on some given occasion. Such minimalism about the strategic capacities of agents is a conservative methodological choice. It allows us to study —as the formal thought experiment that any such model is— how fitness-based selection changes a population in the absence of optimal agent-level reasoning and it abstracts away from

If all types have equal fitness at all times, the replicator mutator dynamic reduces to a process of iterated mutation. Mutation can be interpreted as biological or cultural as well. Under a biological interpretation where types are genetically encoded behavioral traits, Q_{ji} is the probability by which offspring of type j genetically mutates into type i. Under a cultural interpretation where types are transmitted by observation or learning, such as in conditional imitation, Q_{ji} is the probability that an agent trying to learn, adopt or imitate the behavior of an agent with type j acquires type i. As detailed below, in this way, the process in (1), and its equivalent in (2), can contain a model of iterated learning (Griffiths and Kalish 2005; 2007).

In the following application to language evolution, we prefer a cultural interpretation of the mutator component as the perturbation induced by how easy different linguistic traits are learnable from observation of language use. Nothing much hinges on whether the replicator component is interpreted as a biological process or a cultural process. Fitness-based selection may either be thought of as the outcome of actual reproductive success (increased number of offspring which learn a language from their single parent), or as a process of cultural evolution, such as conditional imitation for instance. We believe that this flexibility of interpretation is an asset. It abstracts away from specific assumptions and focuses on a general, albeit abstract description of fitness-based selection in general. Still, to have something concrete in mind when interpreting the following application, it is also important to be concrete. Appendix B therefore derives the replicator mutator dynamic from a specific scheme of agent-level conditional imitation.

2.4   Fitness & learnability of lexical meanings & pragmatic strategies

Moving beyond an abstract example, our goal is to apply the RMD to investigate the evolution of lexical representations and pragmatic behavior. To do so, we need to fix three things: (i) what the relevant types are that populate x, (ii) how fitness derives from communicative success and (iii) how the mutation matrix Q is computed. These issues are addressed, one by one, in the following.

2.4.1   Types: Lexica and pragmatic strategies

Types are what evolution operates on. They define an agent's fitness, usually through a payoff accrued in single interactions with other agents. Often types can be identified as the possible acts in a game, e.g., either cooperating or defecting in a prisoner's dilemma. In other cases, they may be thought of as general properties of an agent that influence her fitness, such as being positive or negative in our previous

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specific assumptions about how fitness-based selection might be realized concretely.

example (whatever that means). For our present purposes, types are identified by specific assumptions about their cognitive make-up. More concretely, since we are interested in the evolutionary competition between different lexical representations and ways of deploying them in communication, a type is here defined as a pair consisting of a lexicon and a linguistic strategy of language use.

Agents play signaling games, in which the speaker wants to communicate a world state s with a message m to a hearer who receives m but does not know s (e.g. Lewis 1969, Skyrms 2010). A lexicon associates each message with a (possibly fuzzy) set of states. A linguistic behavior specifies a probabilistic speaker rule (a probabilistic choice of message for each state) and a probabilistic hearer rule (a probabilistic choice of state for each message) given a lexicon. There are many ways of making these general notions concrete. There are models of rational language use coming from a game-theoretic tradition (e.g., Benz 2006, Benz and van Rooij 2007, Jäger 2007b, Franke and Jäger 2014) and ones from a Bayesian tradition (e.g., Frank and Goodman 2012, Goodman and Stuhlmüller 2013b, Goodman and Frank 2016). We will not focus on the particular design choices that make up each approach (on this see Qing and Franke 2015). Instead, we will stress how the approach, common to all of these models, of pragmatic language use as *rational language use* can be embedded within the replicator mutator dynamic to answer evolutionary questions at the semantics-pragmatics interface. Here is what we will assume in the remainder of this paper.[2]

Lexica codify the truth-conditions of expressions. A convenient way to represent lexica is by $(|S|, |M|)$-Boolean matrices, where S is a set of states (meanings) and M a set of messages (forms available in the language). For example, suppose that there are two relevant world states $S = \{s_{\exists\neg\forall}, s_\forall\}$. In state $s_{\exists\neg\forall}$ Chris owns some but not all of Johnny Cash's albums while in $s_\forall$ Chris owns them all. Suppose that there are two messages $M = \{m_{\text{some}}, m_{\text{all}}\}$ where m_{some} is short for a sentence like *Chris owns some of Johnny Cash's albums* and m_{all} for the same sentence with *some* replaced by *all*. Lexica for this case would assign a Boolean truth value, either 0 for false or 1 for true, to each state-message pair. The following two lexica are minimal examples for the distinction between a lexicalized upper-bound for *some*, in L_{bound}, and the widely assumed logical semantics with only a lower-bound, in L_{lack}.

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[2]Readers more familiar with the Bayesian tradition than the game-theoretic one should note that the definitions below follow the latter in assuming rational sender choice to consist in sending messages that maximize expected utility, rather than message informativity (see Qing and Franke 2015 for discussion).

$$L_{\text{bound}} = \begin{array}{c} \\ s_{\exists\neg\forall} \\ s_{\forall} \end{array} \begin{array}{cc} m_{\text{some}} & m_{\text{all}} \\ \left[\begin{array}{cc} 1 & 0 \\ 0 & 1 \end{array}\right] \end{array} \qquad L_{\text{lack}} = \begin{array}{c} \\ s_{\exists\neg\forall} \\ s_{\forall} \end{array} \begin{array}{cc} m_{\text{some}} & m_{\text{all}} \\ \left[\begin{array}{cc} 1 & 0 \\ 1 & 1 \end{array}\right] \end{array}$$

Linguistic strategies define dispositions to produce and interpret messages given a lexicon. We distinguish between two kinds strategies. *Literal interlocutors* produce and interpret messages literally, being guided only by their lexica. *Pragmatic interlocutors* instead engage in mutual reasoning to inform their choices. Recent probabilistic models of rational language use (Frank and Goodman 2012, Franke and Jäger 2016, Goodman and Frank 2016) capture different types of pragmatic behavior in a reasoning hierarchy. The hierarchy's bottom, level 0, corresponds to literal language use, as in Equations (3) and (4). Pragmatic language users of level $n + 1$ act (approximately) rational with respect to level-n behavior of their interlocutors, as in Equations (5) and (6).

$$H_0(s \mid m; L) \quad \propto pr(s)L_{[s,m]} \tag{3}$$

$$S_0(m \mid s; L) \quad \propto \exp(\lambda\, L_{[s,m]}) \tag{4}$$

$$H_{n+1}(s|m; L) \propto pr(s)S_n(m|s; L) \tag{5}$$

$$S_{n+1}(m|s; L) \quad \propto \exp(\lambda\, H_n(s|m; L)) \tag{6}$$

According to (3), a literal hearer's interpretation of a message depends on her lexicon and her prior over states, $pr \in \Delta(S)$, which is here assumed flat for simplicity. Literal interpreters thereby choose an arbitrary true interpretation for each message according to their lexicon. Pragmatic hearers, defined in (5), instead use Bayes' rule to weigh interpretations based on a conjecture about speaker behavior. Speaker behavior is regulated by a soft-max parameter $\lambda \geq 0$ (Luce 1959, Sutton and Barto 1998). As λ increases, choices approximate strict maximization of expected utilities. Expected utility of a message m in state s for a level $n+1$ speaker is here defined as $H_n(s|m; L)$, the probability that the hearer will assign to or choose the correct meaning. For literal speakers, utility only tracks truthfulness. Literal speakers choose any true message with equal probability but may send false messages as well with a probability dependent on λ.

In words, an agent's linguistic behavior – what message to send when or which message to interpret how – is defined by (i) her reasoning level and (ii) her lexicon. In what follows, this is what we iden-

tify an agent's type with, in order to analyze under which conditions combinations of (i) and (ii) emerge through cultural evolution. Intuitively, literal behavior (level-0 reasoning) results from unreflected language use. Such agents produce and comprehend solely based on the truth-conditions of their lexicon. In a Gricean spirit, pragmatic behavior (level-$n + 1$ reasoning) results from mutual reasoning about rational language use. Such agents produce and comprehend by reasoning about their interlocutor's likely interpretation/production patterns. Higher order behavior is therefore, even if in tendency rational, quite simple. It does not assume agents to know each other's types when interacting (e.g., what lexicon the interlocutor is using); they simply act based on a conjecture of their own behavior if they were in the role of their interlocutor.

The following examples illustrate these behaviors numerically. A literal interpreter with lexicon L_{bound} assigns $s_{\exists\neg\forall}$ a probability of $H_0(s_{\exists\neg\forall} \mid m_{\text{some}}; L_{\text{bound}}) = 1$ after hearing m_{some}, while a literal interpreter with L_{lack} has $H_0(s_{\exists\neg\forall} \mid m_{\text{some}}; L_{\text{lack}}) = 0.5$:

$$
H_0(\cdot \mid \cdot, L_{\text{bound}}) = \begin{array}{c} \\ m_{\text{some}} \\ m_{\text{all}} \end{array} \begin{array}{cc} s_{\exists\neg\forall} & s_{\forall} \\ \left[\begin{array}{cc} 1 & 0 \\ 0 & 1 \end{array}\right] \end{array} \qquad H_0(\cdot \mid \cdot, L_{\text{lack}}) = \begin{array}{c} \\ m_{\text{some}} \\ m_{\text{all}} \end{array} \begin{array}{cc} s_{\exists\neg\forall} & s_{\forall} \\ \left[\begin{array}{cc} .5 & .5 \\ 0 & 1 \end{array}\right] \end{array}
$$

By contrast, pragmatic hearers of level 1 have the following interpretative behavior for $\lambda = 1$:

$$
H_1(\cdot \mid \cdot, L_{\text{bound}}) \approx \begin{array}{c} \\ m_{\text{some}} \\ m_{\text{all}} \end{array} \begin{array}{cc} s_{\exists\neg\forall} & s_{\forall} \\ \left[\begin{array}{cc} .73 & .27 \\ .27 & .73 \end{array}\right] \end{array} \qquad H_1(\cdot \mid \cdot, L_{\text{lack}}) \approx \begin{array}{c} \\ m_{\text{some}} \\ m_{\text{all}} \end{array} \begin{array}{cc} s_{\exists\neg\forall} & s_{\forall} \\ \left[\begin{array}{cc} .59 & .41 \\ .35 & .65 \end{array}\right] \end{array}
$$

This is the outcome of reasoning about their level-0 speaker counterparts with $\lambda = 1$:

$$
S_0(\cdot \mid \cdot, L_{\text{bound}}) \approx \begin{array}{c} \\ s_{\exists\neg\forall} \\ s_{\forall} \end{array} \begin{array}{cc} m_{\text{some}} & m_{\text{all}} \\ \left[\begin{array}{cc} .73 & .27 \\ .27 & .73 \end{array}\right] \end{array} \qquad S_0(\cdot \mid \cdot, L_{\text{lack}}) \approx \begin{array}{c} \\ s_{\exists\neg\forall} \\ s_{\forall} \end{array} \begin{array}{cc} m_{\text{some}} & m_{\text{all}} \\ \left[\begin{array}{cc} .73 & .27 \\ .5 & .5 \end{array}\right] \end{array}
$$

With low λ speakers choose true messages with more slack. Reasoning over this behavior therefore also results in a weaker association of messages with only true states in receivers, but also in a slightly stronger association of m_{some} with $s_{\exists\neg\forall}$ over $s_{\forall}$ for L_{lack} users because they reason that $S_0(m_{\text{some}}|s_{\exists\neg\forall}; L_{\text{lack}}) >$

$S_0(m_{\text{some}}|s_\forall; L_{\text{lack}})$. For $\lambda = 20$, there will be less slack in literal speaker behavior:

$$S_0(\cdot \mid \cdot, L_{\text{bound}}) \approx \begin{array}{c} \\ s_{\exists\neg\forall} \\ s_\forall \end{array} \begin{array}{cc} m_{\text{some}} & m_{\text{all}} \\ \left[\begin{array}{cc} 1 & 0 \\ 0 & 1 \end{array}\right] \end{array} \qquad S_0(\cdot \mid \cdot, L_{\text{lack}}) \approx \begin{array}{c} \\ s_{\exists\neg\forall} \\ s_\forall \end{array} \begin{array}{cc} m_{\text{some}} & m_{\text{all}} \\ \left[\begin{array}{cc} 1 & 0 \\ .5 & .5 \end{array}\right] \end{array}$$

And accordingly less slack in level 1 pragmatic interpretation:

$$H_1(\cdot \mid \cdot, L_{\text{bound}}) \approx \begin{array}{c} \\ m_{\text{some}} \\ m_{\text{all}} \end{array} \begin{array}{cc} s_{\exists\neg\forall} & s_\forall \\ \left[\begin{array}{cc} 1 & 0 \\ 0 & 1 \end{array}\right] \end{array} \qquad H_1(\cdot \mid \cdot, L_{\text{lack}}) \approx \begin{array}{c} \\ m_{\text{some}} \\ m_{\text{all}} \end{array} \begin{array}{cc} s_{\exists\neg\forall} & s_\forall \\ \left[\begin{array}{cc} 0.67 & 0.33 \\ 0 & 1 \end{array}\right] \end{array}$$

Lastly, turning to types that have no bearing on the choices of hearers of level 1, with $\lambda = 1$ pragmatic speakers of level 1 have:

$$S_1(\cdot \mid \cdot, L_{\text{bound}}) \approx \begin{array}{c} \\ s_{\exists\neg\forall} \\ s_\forall \end{array} \begin{array}{cc} m_{\text{some}} & m_{\text{all}} \\ \left[\begin{array}{cc} .73 & .27 \\ .27 & .73 \end{array}\right] \end{array} \qquad S_1(\cdot \mid \cdot, L_{\text{lack}}) \approx \begin{array}{c} \\ s_{\exists\neg\forall} \\ s_\forall \end{array} \begin{array}{cc} m_{\text{some}} & m_{\text{all}} \\ \left[\begin{array}{cc} .62 & .38 \\ .38 & .62 \end{array}\right] \end{array}$$

For $\lambda = 20$, pragmatic speaker behavior of level 1 is instead as follows:

$$S_1(\cdot \mid \cdot, L_{\text{bound}}) \approx \begin{array}{c} \\ s_{\exists\neg\forall} \\ s_\forall \end{array} \begin{array}{cc} m_{\text{some}} & m_{\text{all}} \\ \left[\begin{array}{cc} 1 & 0 \\ 0 & 1 \end{array}\right] \end{array} \qquad S_1(\cdot \mid \cdot, L_{\text{lack}}) \approx \begin{array}{c} \\ s_{\exists\neg\forall} \\ s_\forall \end{array} \begin{array}{cc} m_{\text{some}} & m_{\text{all}} \\ \left[\begin{array}{cc} 1 & 0 \\ 0 & 1 \end{array}\right] \end{array}$$

There are two particularly important things to note. First, in contrast to their literal counterparts of level 0, pragmatic agents of level 1 using L_{lack} associate m_{some} preferentially with $s_{\exists\neg\forall}$. This association is not perfect, and usually less strong than what agents with a lexicalized upper bound in L_{bound} can achieve – with or without pragmatic reasoning. Higher order reasoning beyond level 1 leads to stronger associations of m_{some} and $s_{\exists\neg\forall}$ also for the receiver. Still, the case study presented in Section 3 will consider sender and receiver behavior at levels 0 and 1, as the latter are the simplest pragmatic reasoning types which show a tendency to communicatively attuned pragmatic enrichment. Using only level 1 reasoning and possibly small λ is therefore a conservative choice that works against the fitness-based

selection of pragmatic language use for a notion of fitness defined as communicative success, which is introduced next. Second, when it comes to competition between types of use of lexicon L_{bound}, pragmatic reasoning at level 1 is not advantageous. The reason for this is that literal use of L_{bound} already endows agents with a behavioral strategy that associates a single state with a single message (in tendency; depending on λ for senders). For L_{bound}-receivers of level 1, reasoning over stochasticity introduced at $S_0(\cdot \mid \cdot, L_{\text{bound}})$ will generally decrease the association of one state with one message. This decrease is only slight if λ is high, but nevertheless present. That is to say, level-1 reasoning does not necessarily confer a functional advantage. For some types, such as users of L_{bound}, literal signaling is preferable.

2.4.2 Fitness & fitness-based selection based on communicative success

Under the replicator dynamic the proportion of type i in a population will increase or decrease as a function of its relative fitness f_i. In the context of language evolution, fitness is usually associated with the ability to successfully communicate with other language users from the same population (e.g., Nowak and Krakauer 1999, Nowak et al. 2000; 2002). Under a biological interpretation the assumption is that organisms have a higher chance of survival and reproduction if they are able to share and receive useful information via communication with peers. Under a cultural interpretation the picture is that agents themselves strive towards communicative success and therefore occasionally adapt or revise their behavior to achieve higher communicative success (see Benz et al. 2005:§3.3 for discussion).

The replicator equation gives us the means to make the ensuing dynamic precise, without necessarily committing to a biological or cultural interpretation. As above, the proportion of types in a given population is codified in a vector $\vec{x}$, where x_i is the proportion of type i. The fitness of type i is its average expected communicative success, or *expected utility* (EU), given the frequencies of types in the current population:

$$f_i = \sum_j x_j \text{EU}(t_i, t_j).$$

In signaling games, the expected utility $\text{EU}(t_i, t_j)$ for type i when communicating with type j is the average success of i when talking or listening to j. If, as standardly assumed, agents are speakers half of the time, this yields:

$$\text{EU}(t_i, t_j) = \text{\textonehalf} \, \text{EU}_S(t_i, t_j) + \text{\textonehalf} \, \text{EU}_H(t_i, t_j),$$

where $\text{EU}_S(t_i, t_j)$ and $\text{EU}_H(t_i, t_j)$ are the expected utilities for i as a speaker and as a hearer when communicating with j, defined as follows, where n_i and n_j are type i's and type j's pragmatic reasoning types and L_i and L_j are their lexica:

$$\text{EU}_S(t_i, t_j) = \sum_s P(s) \sum_m S_{n_i}(m \mid s; L_i) \sum_{s'} H_{n_j}(s' \mid m; L_j) \delta(s, s'),$$

$$\text{EU}_H(t_i, t_j) = \text{EU}_S(t_j, t_i).$$

As usual, we assume that agents are cooperative: $\delta(s, s') = 1$ iff $s = s'$ and 0 otherwise.

In words, expected utility $\text{EU}(t_i, t_j)$ quantifies how successful communication between agents of types i and j is, with each type's behavior resulting from a combination of a lexicon and a reasoning level (Section 2.4.1). Fitness f_i indicates how well type i fares in a population given what other types are found in it: it is type i's weighted expected utility.

2.4.3 Learnability

Languages are shaped not only by functionalist forces towards greater communicative success. Another important factor is the fidelity with which linguistic knowledge is transmitted. Among others, linguistic production can be prone to errors, states or messages may be perceived incorrectly, and multiple languages may be compatible with the data learners are exposed to. These sources of uncertainty introduce variation in the transmission of linguistic knowledge from one generation to the next. In particular, learning biases in the iterated transmission process can influence language evolution substantially.

In biological evolution, where types are expressed genetically, transmission infidelity comes into the picture through infrequent and mostly random mutation and genetic drift (Kimura 1983). However, an agent's lexicon and pragmatic reasoning behavior is likely not inherited genetically. They need to be learned from observation. Concretely, when agents learn from type j, they observe the overt linguistic behavior of type j and need to infer the covert type that most likely produced the observed behavior. As in biology, this transmission process is not perfectly accurate.

Iterated learning is a process in which languages are learned repeatedly from the observation of linguistic behavior of agents who have themselves acquired their behavior from observation and inference. In the simplest case there is a single teacher and a single learner in each generation (e.g., Kirby 2001, Brighton 2002). After sufficient training the learner becomes a teacher and produces behavior that serves as input for a new learner. Due to the pressure towards learnability it exerts, iterated learning alone generally leads to simpler and more regular languages (see Kirby et al. 2014 and Tamariz and Kirby

2016 for recent surveys).

Following Griffiths and Kalish (2005; 2007) we model language acquisition as a process of Bayesian inference in which learners combine the likelihood of a type producing the witnessed learning input with prior inductive biases. In a Bayesian setting these biases can be codified in a prior $P \in \Delta(T)$, which reflects the amount of data a learner requires to faithfully acquire the language of the teacher (cf. Griffiths and Kalish 2007:450). Put differently, types favored by the prior are easier to learn because they require less data than less favored types to be inferred, even if they are both compatible with the learning data at hand. The extent of the prior's influence has been shown to heavily depend on the learning strategy assumed to underlie the inference process. On the one hand, early simulation results suggested that weak biases could be magnified by exposing learners to only small data samples (e.g. in Brighton 2002). On the other, Griffiths and Kalish's (2007) mathematical characterization showed that iterated learning converges to the prior in the limit, i.e., that the resulting distribution over types corresponds to the learners' prior distribution and is not influenced by the amount of input given to them. This difference in predictions can be traced back to differences in the selection of hypotheses from the posterior. Griffith & Kalish's convergence to the prior holds for learners that sample from the posterior. More deterministic strategies such as the adoption of the type with the highest posterior probability, so-called *maximum a posterior estimation* (MAP), increase the influence of both the prior and the data (Griffiths and Kalish 2007, Kirby et al. 2007). In the following, we use a parameter $l \geq 1$ to modulate between posterior sampling and the MAP strategy. When $l = 1$ learners sample from the posterior. The learners' propensity to maximize the posterior grows as l increases.

The way the posterior over types is formed and how learners employ it are important design choices. There are other alternatives as well. For instance, instead of adopting a single type, learners could keep their uncertainty over types and base their production behavior on the posterior over types they form (Burkett and Griffiths 2010). A straightforward operationalization of this idea would then see production defined as a two-step process wherein a speaker first samples from her posterior and then produces utterances based on the type she sampled, as done in Kirby et al. 2015. Beyond adding complexity to the model, a disadvantage of this approach is that it could happen that a speaker that first uttered a Hungarian utterance speaks Quechua in the next, so to speak. We stick to the simpler assumption of a single type being acquired, which we now define.

Let D be the set of possible data that learners may be exposed to. This set contains all sequences of state-message pairs of length k, e.g., $\langle \langle s_1, m_1 \rangle, \ldots, \langle s_k, m_k \rangle \rangle$. As k increases, learners have more data to base their inference on and so tend to recover the true types that generated a given sequence with

Figure 1: Sketch of dynamics on a two-dimensional type space with four types. The x-axis represents the proportion of types with L_{lack}; the y-axis that of pragmatic ones. Arrows sketch population trajectories under functional pressure (replicator steps) and learnability pressure (mutator steps) with either a flat prior or one that favors L_{lack}.

higher probability. The mutation matrix Q of the replicator mutator dynamic in (1) can then be defined as follows: Q_{ji} is the probability that a learner acquires type i when learning from an agent of type j. The learner observes length-k sequences d of state-message pairs, but the probability $P(d \mid t_j)$ with which sequence $d = \langle \langle s_1, m_1 \rangle, \ldots, \langle s_k, m_k \rangle \rangle$ is observed depends on type j's linguistic behavior:

$$P(d = \langle \langle s_1, m_1 \rangle, \ldots, \langle s_k, m_k \rangle \rangle \mid t_j) = \prod_{i=1}^{k} S_{n_j}(m_i \mid s_i; L_j),$$

where, as before, n_j is j's pragmatic reasoning type and L_j is j's lexicon. For a given observation d, the probability of acquiring type i is $F(t_i \mid d)$, so that:

$$Q_{ji} \propto \sum_{d \in D} P(d \mid t_j) F(t_i \mid d).$$

The acquisition probability $F(t_i \mid d)$ given datum d is obtained by probability matching $l = 1$ or a tendency towards choosing the most likely type $l > 1$ from the posterior distribution $P(\cdot \mid d)$ over types given the data. This is calculated by Bayes' rule:

$$F(t_i \mid d) \propto P(t_i \mid d)^l \text{ and}$$
$$P(t_i \mid d) \propto P(t_i) P(d \mid t_i).$$

2.5 Example

The case study in Section 3 shows how these components and their interaction can shed light on the evolution of scalar implicatures. Before moving on, this section illustrates the model's workings in a constrained type space to better trace the effects that pressure for communicative success and learnability can have on an evolving population. To this end, we focus only on the four types from Section 2.4.1. Recall that these types are made up of a combination of either lexicon L_{lack} or L_{bound} and either literal (level-0) or pragmatic (level-1) behavior.

A population will be made up of frequencies of these four types. We may think of such a population as a point in a two-dimensional space. The grey dotted square in Fig. 1 outlines this space. The space's

first dimension determines the proportion of types that lexicalize ambiguous m_{some} (the x-axis in Fig. 1). That is, the proportion of types with L_{lack}. The second dimension determines the proportion of pragmatic language users (the y-axis in Fig. 1). Populations at the corners of this space are monomorphic; all agents in them are of a single type. If we move from away from edges toward the center of the space, then this two-dimensional representation underspecifies the exact proportions of types found in a population. The space's center, for instance, could be composed 50% of pragmatic L_{lack} and of an equal proportion of literal L_{bound}. But it could equally well be a population that has each type with the same frequency. This level of granularity will nevertheless be sufficient for our purposes, as we are only interested in sketching rough trajectories after replicator or mutator steps.

From Section 2.4.1, we already know some things about this type space. When it comes to communicative success: types with L_{bound} have a functional advantage over L_{lack}; literal L_{bound} has a slight advantage over pragmatic L_{bound}; all have a sizable advantage over literal L_{lack}. The red arrows in Fig. 1 sketch out these relationships. They depict the rough trajectories we can expect if there is only pressure for communicative success. How much each replicator step moves a population through this space ultimately depends on the rationality parameter λ. In particular, note that functional differences decrease as λ increases: if λ is sufficiently high, all types except for literal L_{lack} speakers will show a strong tendency to associate $s_{\exists\neg\forall}$ with m_{some} and $s_\forall$ with m_{all}. As a consequence, when it comes to learnability and transmission fidelity, the likelihood that any of these three types produces a given datum d will be approximately equal. In other words, if λ is high the likelihood offers little differentiation between pragmatic L_{lack} and any disposition to use L_{bound} for learners. We can therefore expect that, if the prior is flat, no type is passed on very faithfully in this space. This would lead populations to gravitate toward the space's center after mutator steps. This is sketched by the blue dash-dotted arrows in Fig. 1. Neither prior nor likelihood alone, but their combination, are what determines the effects of (iterated) Bayesian learning. If, instead of a flat prior, there is a prior that favors, say, L_{lack} over L_{bound} we can expect mutator steps to be skewed toward the upper-half of the space. This is suggested by the purple arrows in Fig. 1. Taking stock, the effects of iterated learning will depend not only on λ, affecting the likelihood, but also on the prior (how strong of a bias there is) and the degree to which learners tend to maximize the posterior (controlled by parameter l).

Fig. 2 shows actual evolutionary trajectories in this type space. The replicator steps make the functional disadvantage of L_{lack} against L_{bound} apparent, particularly for its literal users and/or low λ. They also highlight that the functional difference between literal and pragmatic L_{bound} is small. Under functional pressure only, populations move toward the lower part of the square but the steps become small as
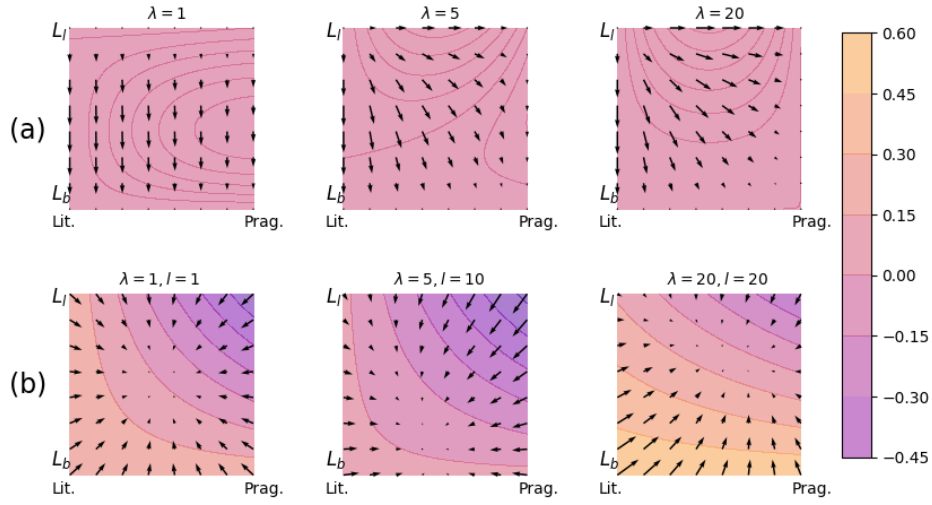
Figure 2: Dynamics on a two-dimensional type space with four types. The x-axis represents the proportion of types with L_{lack}; the y-axis that of pragmatic ones. Arrows indicate directionality of trajectories after (a) replicator steps and (b) mutator steps with a prior that favors types with L_{lack} by a factor of 1.05 over those with L_{bound}. Colored contours show the proportion of pragmatic L_{lack} users after a step [TB: I don't think the contours are adding much, so they could be removed. Then again, I think they are rather non-intrusive, look nice, and illustrate how the step-size, depending on where we are in the space, varies.]

the lower edge, representing populations composed only of L_{lack} users, are reached. As for the mutator diagrams, there are two main things to note. As expected, a small bias for a type will lead to larger proportions to adopt it, provided that likelihood offers little to no differentiation between types. This is the situation the right-most space in Fig. 2 depicts. However, the bias is far from absolute. If, instead, certain types are more likely to be recovered based on the linguistic input they produce, they will be passed on more faithfully. This is why, for instance, populations move toward a majority of L_{lack}-types for $\lambda = 5$ and $l = 10$.

Fig. 1 and 2 depict replicator and mutator steps in isolation to better showcase their effects. Their combination will yield different outcomes, which we now turn to in a case study on scalar implicatures. [TB: Also: In this type space RMD way too similar to MD to make it worth the ink. It raises more questions than it answers so I left it out as the plots look identical to those in Fig. 3b]

2.6 Model summary

Communicative success and learnability are central to the cultural evolution of language. These components can be modelled, respectively, as replication based on a measure of fitness in terms of communicative efficiency and iterated Bayesian learning. Their interaction is described by the discrete time

replicator mutator dynamic in (1), repeated here:

$$x_i' = \sum_j Q_{ji} \frac{x_j f_j}{\sum_h x_h f_h}\,.$$

This equation defines the frequency x_i' of type i at the next time step, based on its frequency x_i before the step, its fitness f_i, and the probability that a learner infers i when observing the behavior of a type-j agent. Fitness-based selection is here thought of not as biological (fitness as expected relative number of offspring) but cultural (fitness as likelihood of being imitated or repeated) evolution, since the types that the dynamic operates on are pairs consisting of a lexicon and a pragmatic use pattern. A type's communicative success depends on how well it communicates within a population while its learnability depends on the fidelity by which it is inferred by new generations of learners. The learners' task is consequently to perform a joint inference over types of linguistic behavior and lexical meaning.

The model has three parameters: λ regulates the degree to which speakers choose messages that appear optimal from the point of view of the agent's own utility measure (which may be unrelated to the expected utility when communicating with a given population); k is the length of observations received by each language learner; l regulates where the learners' inference behavior lies on a spectrum from probability matching to acquisition of the most likely teacher type.

3   Case study: scalar implicatures

The model of the previous section formalizes the evolutionary competition between different sets of lexical representations and ways of using them. This section looks at a case study on scalar implicatures. It engages in a formal thought experiment to address the question: if a population of language users could freely combine different lexica with different dispositions to act on them, what are conditions under which the majority view of scalar implicatures could have evolved?

Recall that the majority view is that scalar implicatures are non-lexicalized pragmatic enrichments. Scalar implicature triggers like *some*, *warm* or *may* are semantically weak expressions for which logically stronger expressions are salient, e.g., *all*, *hot* or *must*. For instance, *some* is entailed by *all*. If the sentence "Chris owns all of Johnny Cash's albums" is true, then "Chris owns some of Johnny Cash's albums" is also true. However, while weaker expressions such as *some* are truth-conditionally compatible with stronger alternatives such as *all*, this is not what their use is normally taken to convey. Instead, the use of a less informative expression when a more informative one could have been used can license a defea-

sible inference that stronger alternatives do not hold (cf. Horn 1972, Gazdar 1979). In this way, "Chris owns some of Johnny Cash's albums" is strengthened to convey that she owns *some but not all* albums. According to the majority view, this is a pragmatic inference, not part of the conventional meaning.

In the following we consider a specific application of the model from Section 2 which allows us to address the question if or when scalar inferences might (not) lexicalize. We consider what is perhaps one of the simplest non-trivial setups that speak to this matter and reflect on its limitations in Section 4. The setup is introduced in Section 3.1. Section 3.2 describes simulations and their results.

3.1 Setup

To fill the model from Section 2 with life, we need to specify the sets of states, messages and lexica we consider (Section 3.1.1). Additionally, we want to explore the effects of a learning bias in favor of simple lexical representations. One way of motivating and formalizing such a bias is introduced in Section 3.1.2.

3.1.1 States, messages, lexical representations, and lexica

Consider a state space with three states $S = \{s_\emptyset, s_{\exists\neg\forall}, s_\forall\}$ and think of it as a partition of possible worlds into cells where none, some or all of the As are Bs, for some arbitrary fixed predicates A and B. Enlarging the setup from Section 2.4.1 by another state in this way enables us to discern the difference between three kinds of lexica which we will use to unravel the model's dynamics (see below). Eight lexical representations can be distinguished based on their truth or falsity in three world states, six of which are not contradictory or tautological (see Table 2 below).

A lexicon L is a mapping $M \to R$ from messages to representations. With three messages there are $6^3 = 216$ possible lexica. Some assign the same representations to more than one message and others lexicalize the same representations but associate them with different messages. Out of these possible lexica, three kinds are of particular relevance. First, lexica that assign the same lexical representations to more than one message. Such lexica lack in expressivity but may be favored by particular learning biases nonetheless (see below). Second, lexica that conventionalize upper-bounds to realize a one-to-one mapping of messages to states. Finally, lexica that do not lexicalize an upper bound but allow it to be conveyed pragmatically due to the presence of a stronger lexical item. There are six lexica of the second

kind and six of the third. The following three lexica exemplify each kind:

$$
\begin{array}{c}
\underline{L_{\text{all}}} \\
\begin{array}{c@{\;}ccc}
 & m_{\text{none}} & m_{\text{some}} & m_{\text{all}} \\
s_{\emptyset} & \left[\begin{array}{ccc} 0 & 0 & 0 \\ \end{array}\right. \\
s_{\exists\neg\forall} & 0 & 0 & 0 \\
s_{\forall} & \left. 1 & 1 & 1 \end{array}\right]
\end{array}
\qquad
\begin{array}{c}
\underline{L_{\text{bound}}} \\
\begin{array}{ccc}
m_{\text{none}} & m_{\text{some}} & m_{\text{all}} \\
\left[\begin{array}{ccc} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{array}\right]
\end{array}
\end{array}
\qquad
\begin{array}{c}
\underline{L_{\text{lack}}} \\
\begin{array}{ccc}
m_{\text{none}} & m_{\text{some}} & m_{\text{all}} \\
\left[\begin{array}{ccc} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 1 & 1 \end{array}\right]
\end{array}
\end{array}
$$

Lexicon L_{all} is clearly bad for communication: all message and interpretation choices will be equally likely; no information about the observed world state will be conveyed by its users. In contrast, users of L_{bound} can communicate world states perfectly, no matter whether they are literal or pragmatic users. Users of L_{lack} can also communicate information about the actual world state but need pragmatic language use to approximate a one-to-one mapping between message use and states (see Section 2.4.1).

Recall that types are a combination of a lexicon and a manner of language use. We analyze the model's predictions in populations of types with one of the two behaviors introduced earlier: literal or pragmatic. The former correspond to level-0 reasoners and the latter to level 1. Accordingly, there is a total of 432 types. Six types are pragmatic language users with L_{lack}-like lexica. We refer to these as *target types* because they represent lexica and language use that conform to the majority view of scalar implicatures. Twelve types are either literal or pragmatic users of lexica of the L_{bound} kind. We refer to these as *competitor types*, because they are expected to be the target types' main contenders in evolutionary competition.

Note that while different types may lexicalize the same representations, they may nevertheless map different states to different overt messages. More informally, they speak different languages that lexicalize the same concepts. Consequently, more often than not different types of the same kind fail to understand each other.

3.1.2 An inductive learning bias for semantic simplicity

There is a growing effort to develop empirically testable representational languages that allow for the measure of semantic complexity. For instance, so-called *languages of thought* (LOTs) have been put to test in various rational probabilistic models that show encouraging results (see, e.g., Katz et al. 2008, Piantadosi et al. under review; 2012, and Piantadosi and Jacobs 2016 for recent discussion). At its core, a LOT defines a set of operations and composition rules from which lexical representations can be

$$R \rightarrow_2 R \wedge R \qquad R \rightarrow_2 \neg R$$
$$R \rightarrow_1 X \subseteq X \qquad R \rightarrow_1 X \neq \emptyset \qquad R \rightarrow_1 X = \emptyset$$
$$X \rightarrow_1 \{A, B\} \qquad X \rightarrow_1 X \cap X \qquad X \rightarrow_1 X \cup X$$

Table 1: Toy grammar in a set-theoretic LOT with weighted rules.

intuitive name	$s_\emptyset$	$s_{\exists\neg\forall}$	$s_\forall$	least complex formula	complexity
"all"	0	0	1	$A \subseteq B$	3
"some but not all"	0	1	0	$A \cap B \neq \emptyset \wedge A \neq \emptyset$	8
"some"	0	1	1	$A \cap B \neq \emptyset$	4
"none"	1	0	0	$A \cap B = \emptyset$	4
"none or all"	1	0	1	$\neg(A \cap B \neq \emptyset \wedge A \neq \emptyset)$	10
"not all"	1	1	0	$\neg(A \subseteq B)$	5

Table 2: Available lexical representations and their minimal derivation cost.

derived. As a first approximation and for the sake of concreteness, we follow this approach to motivate and formalize a preference of learners for simpler semantic representations (Feldman 2000, Chater and Vitányi 2003, Piantadosi et al. 2012, Kirby et al. 2015, Piantadosi et al. under review). In a weighted generative LOT a representation's complexity is a function of its derivation cost.

Our toy grammar of lexical representations is given in Table 1. This grammar uses basic set-theoretic operations to form expressions which can be evaluated as true or false in states $s_\emptyset$, $s_{\exists\neg\forall}$, or $s_\forall$ from above. Applications of generative rules have a cost attached to them. Here we simply assume that the formation of Boolean combinations of representations incurs 2 cost units, while all other rule applications incur only 1 cost unit. Table 2 lists all six lexical representations relevant here, their truth conditions and the simplest formula that expresses this representation in the grammar in Table 1.

A complexity measure for lexical representations from Table 2 is used to define a learning bias that favors simpler representations over more complex ones. The prior probability of a type is just the prior probability of its lexicon. The prior of a lexicon is a function of the complexity of the lexical representations in its image set. Lexica with simpler representations accordingly have a higher prior. One simple way of defining such priors over lexica (and thereby types) is:

$$P(L) \propto \prod_{r \in Im(L)} P(r) \text{, with} \qquad P(r) \propto \max_{r'} Compl(r') - Compl(r) + 1 \,,$$

where $Compl(r)$ is the complexity of the minimal derivation cost of representation r according to the LOT-grammar (see Table 2). Applied to our space of lexica, this construal assigns the highest probability

to a lexicon of type L_{all}, which only uses the simplest lexical representation "all" for all messages. Lexica of type L_{lack} are less likely, but more likely than L_{bound}.

The menu of inductive biases argued to shape language acquisition is steadily being refined. Apart from simplicity, prominent examples include mutual exclusivity (Merriman and Bowman 1989, Clark 2009), regularization (Hudson Kam and Newport 2005), and generalization (Smith 2011). Even when these biases are considered in isolation, there are many ways in which they can be translated into priors over types. The key assumption here, common to all simplicity-biased priors, is that simple representational expressions should be favored over more complex ones (see, e.g., Goodman et al. 2008, Piantadosi et al. 2012, Kirby et al. 2015). In view of the menu of possible operationalizations, other biases and their potential interplay, we should stress that these details – from the generative grammar to its complexity measure – are to be regarded as one convenient operationalization of one general approach to explicating learning biases; this is not a commitment that this general approach is necessarily superior or that, within it, this particular instrumentalization is the single most plausible. We return to this issue in Section 4.

3.2 Simulation results

Recall that there are three parameters held constant across types: soft-max parameter λ affects how strongly speakers favor messages that appear best from their subjective point of view; the bottleneck size k influences how faithfully learners can identify their teacher type; l defines the learners' disposition towards choosing the most likely teacher type from the posterior distribution. We expect that competitor types (types with lexica of the kind L_{bound}) have a fitness advantage over target types (pragmatic types with lexica of the kind L_{lack}), especially for very low levels of λ (see Fig 1). Selection based on fitness alone may therefore not lead to prevalence of target types in the population. On the other hand, lexica of type L_{lack} are simpler than those of type L_{bound} by the postulated measure from above. This may make them more likely to be adopted by learners, especially when l is high. Still, lexica of the kind L_{all} are in turn even more likely *a priori* than lexica of the kind L_{lack}. In any case, as showcased in Section 2.5, the prior is not all that counts for learnability. Simulation results will shed light on the question whether target types can emerge, and for which parameter constellations.

To better understand the workings of a pressure towards communicative efficiency and a pressure towards learnability, we look at the behavior of the replicator and mutator step first in isolation, and then in combination. All simulation runs are initialized with an arbitrary distribution over types, constituting a population's first generation. All reported results are the outcome of 50 update steps. These outcomes correspond to developmental plateaus in which change is, if not absent, then at least very slow. In other
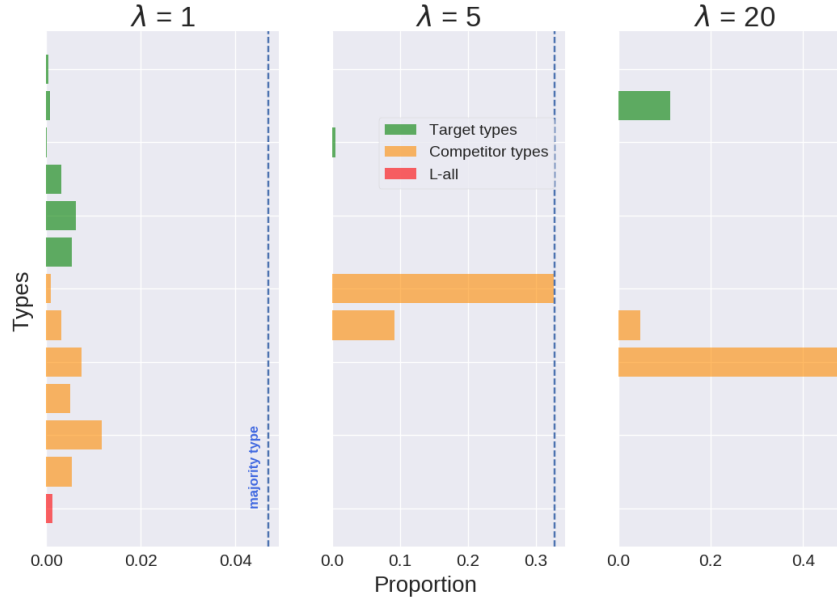
Figure 3: Proportion of target types, the six competitor types with the highest proportion, the highest type of the L_{all}-kind, and the population's majority type, in representative populations after 50 generations under only a pressure for communicative success.

words, even if the resulting states do not correspond to an eventual attracting state, they characterize population states in which the system remains for a long time. As specified in Section 2.4.3, the mutation matrix Q can be obtained by considering all possible state-message sequences of length k. Given that this is intractable for large k, the sets of data which learners are exposed to are approximated by sampling 250 k-length sequences from each type's production probabilities.

3.2.1 Replication only: selection based on communicative success

Selection based on communicative success is sensitive to λ since λ influences signaling behavior, which in turn determines communicative success. This is showcased in Fig. 3, which shows the proportion of target types, the highest competitor types and the highest type with L_{all} in a representative population after 50 replicator steps. The plot also indicates the proportion of the *majority type*: the type with the highest proportion in the population. With low λ many types have very similar behavior, so that evolutionary selection lacks grip and becomes very slow. The result is a very long transition with near stagnancy in a rather homogeneous population with many types. Conversely, higher λ promotes less-stochastic linguistic behavior, widening the gap in expressivity between some types and promoting more homogeneous populations. As suggested by Fig. 3, the majority in most populations is not one of the six pragmatic L_{lack}-style types. That is, a pressure only for communicative success does not lead to a prevalence of target types under any λ-value. For instance, with $\lambda = 20$ 1000 independent populations
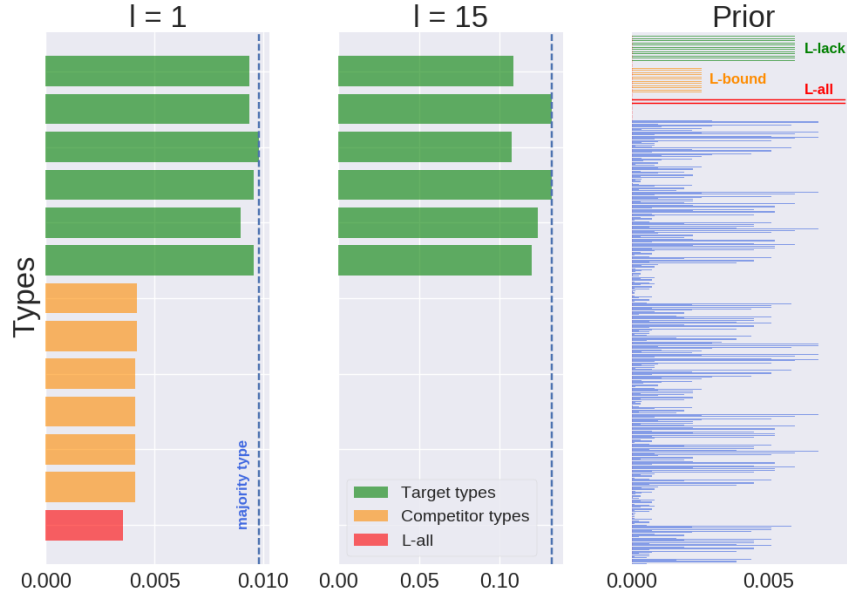
22

Figure 4: Proportion of target types, the six competitor types with the highest proportion, the highest type of the L_{all}-kind, and the population's majority type in representative populations after 50 generations under only a pressure for learnability ($\lambda = 20, k = 5$). The learning prior is shown in the right-most plot with top-most groupings corresponding to types, literal and pragmatic, with lexica of kinds L_{lack}, L_{bound}, and L_{all}.

only had 11 cases in which the target type was the majority type, corresponding to a mean proportion of .003 across populations. By contrast, in 913 cases the majority types had L_{bound} with, close to an even share between literal (454) and pragmatic types (459), corresponding to a mean proportion of about .48 taken together.

3.2.2 Iterated learning only: transmission fidelity and learnability

The effect of iterated learning without a pressure for communicative success using either posterior sampling ($l = 1$) or a stronger tendency towards posterior maximization ($l = 15$) is shown in Fig. 4 together with the prior over types. The prior shows that while users of L_{lack} are not the most favored by the inductive bias (compared, e.g., to L_{all}) they are nevertheless more advantaged than others, such as L_{bound}, in virtue of the relatively simple semantics they conventionalize (see Section 3.1.2). Crucially, L_{lack} enables its users to convey each state with a single message when combined with pragmatic reasoning and sufficiently high λ. This makes it less likely to be confused with other types if the learning data is not too sparse ($k \geq 5$). What is more, learners have a propensity to infer pragmatic L_{lack} when the teacher's type produces very similar data, such as when using L_{bound} (see Section 2.5). Moreover, L_{lack} is less likely to be confused with types with different observable behavior because its pragmatic use approximates a one-to-one form-meaning mapping. As a consequence, a stronger propensity to maximize the posterior

increases their proportion in the population.

However, in contrast to a pressure only for communicative success with high λ (see Fig. 3), learnability alone does not succeed in selecting for a single prevalent type; all six target types tend to coexist at roughly equal proportion. Each is passed on to the next generation with the same faithfulness and, differently from a pressure for communicative success, they do not stand in competition with each other. In 1000 independent populations with $\lambda = 20$ all majority types were target types, with each reaching approximately the same proportion of users in the population.

As with a pressure only for communicative success, low values of λ make the differences in observable behavior across types less pronounced. This makes differences in the likelihood of particular types having generated a learning input less pronounced. Therefore, low λ leads populations to reflect the learners' inductive bias more faithfully. This favors functionally deficient but a priori preferred types such as those that use L_{all}. A pressure for learnability alone may consequently lead to a spread of communicatively suboptimal types that lexicalize simpler semantics. In the extreme, when $l = 1$ and $\lambda = 1$ all of 1000 independent populations had users of L_{all} as majority types. For higher λ, it becomes clear that a high prior is not the only thing that counts when it comes to learnability (see Section 2.5). As soon as there is information for learners to discern whether one type is more likely to have generated the data (depending on λ and k), it becomes paramount for types to produce data that makes them easily identifiable if they are to be inferred more often.

3.2.3 Combining pressures of communicative success and learnability

Pressure for communicative success and learnability are not sufficient on their own to have a single target type dominate the population. When pressured for communicative success, the communicative advantage of L_{bound} users leads to their prevalence. When pressured for learnability, pragmatic L_{lack} is promoted over functionally similar but semantically more complex alternatives such as L_{bound}. However, learnability alone does not foment the propagation of a single target type across the population.

Fig. 5 illustrates the combined effects of both pressures for a sample of λ and l values. These results show that an inductive learning bias for simpler semantics in tandem with functional pressure can lead to the selection of a single target type, and so to a division of labor between semantics and pragmatics. The proportion of a single majority target type increases with λ and l. Pressure for communicative success magnifies the effects of iterated learning and dampens the proliferation of multiple types of a kind, equal in expressivity and learnability. A pressure towards learnability favors the transmission of simpler semantics and thereby indirectly selects for pragmatic language use in these types. Pragmatic
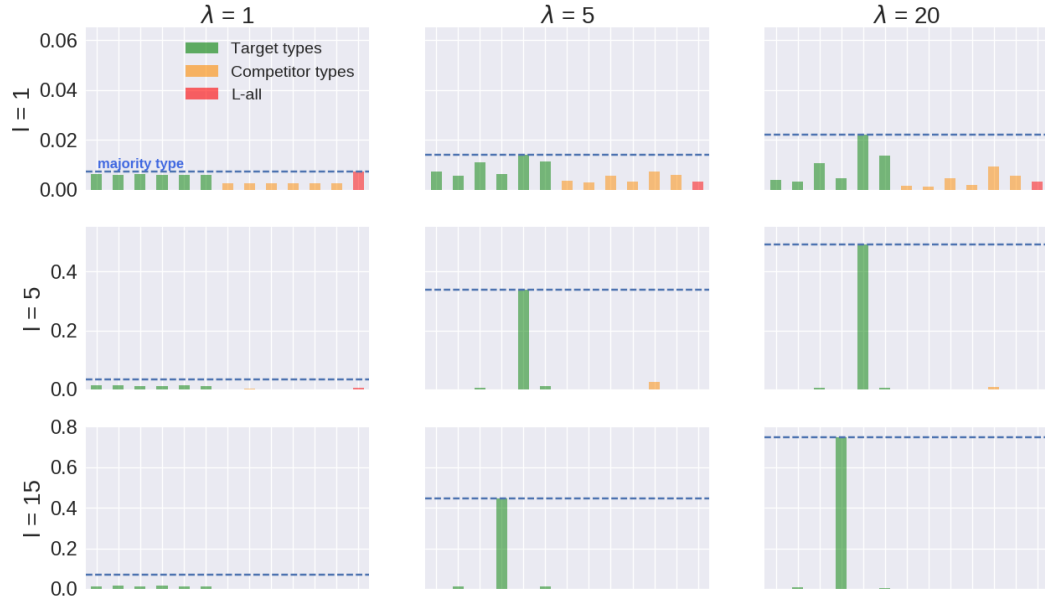
Figure 5: Proportion of target types, the six competitor types with the highest proportion, the highest type of the L_{all}-kind, and the population's majority type in representative populations after 50 generations under both pressures ($k = 5$).

behavior, however, is not necessarily selected for in types that can do away without pragmatic reasoning such as L_{bound} users (see Section 2.5).

As before, low λ and l lead to the prevalence of communicatively suboptimal types that are a priori favored, such as L_{all}. An increase in λ leads to the selection of target types but does not lead to monomorphic populations if learners sample from the posterior (see the uppermost row in Fig. 5). Finally, a combination of high λ and l leads to increasing proportions of a single majority target type. This joint influence is summarized in Fig. 6, which shows the mean difference between the highest target type and the highest proportion of a type of a different kind in 1000 independent populations across λ and l values. Higher values of λ and l increase the prevalence of a single target type, whereas lower values lead to less pronounced differences, with a valley resulting from low λ and high l (cf. Fig. 5 with $\lambda = 1$ and $l = 15$).

Effects of manipulating the sequence length k have not been addressed so far, but are rather predictable: small values lead to more heterogeneous populations that reflect the learner's prior more faithfully. This is due to the fact that the likelihood that a small sequence was produced by any type is relatively uniform (modulo prior). By contrast, larger values increasingly allow learners to differentiate types with different signaling behaviors.

When pressured for communicative success only, target types are outcompeted by competitor types. When pressured for learnability, populations are polymorphic due to lack of competition; minimally
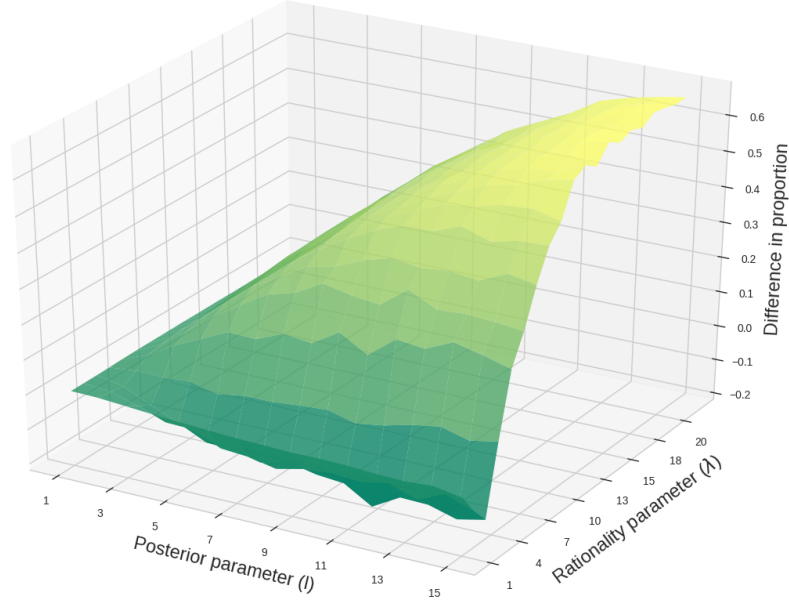
Figure 6: Mean difference between proportion of highest target type and highest other type in 1000 independent populations after 50 generations under both pressures ($k = 5$).

among types of a kind. However, due to the prior, learners that are exposed to the input of either targets or competitors tend to acquire the former. The consequence of combining both pressures is then that the (slight) functional disadvantage of targets is counterbalanced by an advantage in learnability. This leads to large proportions of targets in a population and, due to the competition among types applied by each replicator steps, to more monomorphic populations.

To recapitulate, other than the involvement of pressure on both communicative success and learnability, the resulting proportion of pragmatic L_{lack} speakers primarily hinges on three factors. First, the degree, captured by λ, to which agents try to maximize communicative success from their own subjective point of view. Second, the inductive bias, which leads learners to prefer simpler over more complex semantic representations in acquisition. Lastly, the learning behavior, captured by parameter l, where approximating a MAP strategy magnifies the effects of the learning bias in tandem with replication.

More broadly, target types, which represent the majority view of scalar implicatures, can come to dominate the population if three assumptions are met: (i) language is pressured toward both communicative success *and* learnability; (ii) pragmatic language use is an option; (iii) learners prefer simpler over more complex lexical representations and exhibit a tendency towards the acquisition of the type that best explains the learning data.

4   General discussion

The approach introduced here combines game-theoretic models of functional pressure towards successful communication (Nowak and Krakauer 1999), effects of transmission perturbations on (iterated) language learning (Griffiths and Kalish 2007), probabilistic speaker and listener types of varied degrees of pragmatic sophistication (Frank and Goodman 2012, Franke and Jäger 2014) as well as reasoning about unobservable lexical representations (Bergen et al. 2012; 2016). This allows for a conceptual investigation of the (co-)evolution of conventional meaning and pragmatic language use. Main contributions of the model are (i) its modular separation of communicative success and learnability on evolutionary trajectories, (ii) the characterization of language learning as a joint inference over linguistic behavior and lexical meaning, and (iii) the possibility to trace the co-evolution of conventional semantics and pragmatic use.

With respect to (i), Kirby et al. (2015) propose a comparable model of the interaction between a lexicon's expressivity and its learnability. A main difference is that here we considered communicative pressure for mutual understanding. This pressure can indirectly select for expressive types, those that can convey states unequivocally, whereas Kirby et al. only consider the bearing that the latter ability has on the production of learnable data. We see three main reasons for considering communicative success rather than just expressivity, and for looking at communication and learning rather than just learning. First, learning alone can promote populations with non-negligible proportions of functionally defective types. This is true both of simulations, e.g., our L_{all}-types in Fig. 4, as well as of laboratory experiments with human subjects (see Kirby et al. 2008, Silvey et al. 2014, and see Fay and Ellison 2013 for review of laboratory results). Second and more importantly, types may be equally expressive but their performance as means of information transfer crucially depend not only on themselves but on the population they find themselves in (compare the competition of target types in Fig. 5 and their lack of competition in Fig. 4). That is, we contend that adopting an expressive type that generates learnable data does not, in itself, capture a type's arguably central communicative function of transferring information to peers. Taking communication into consideration allows the model to be responsive to the task for which language is learned. Lastly, chains of iterated learning alone do not put types in direct competition. Accordingly, learning leads to polymorphous populations in which multiple types of a kind coexist (Nowak 2006).

The main result of our case study is that types that correspond to the majority view of scalar implicatures – scalar readings are non-lexicalized pragmatic enrichments – can come to dominate a population. This can happen if pragmatic language use is recruited indirectly by a preference for simpler lexical representations (relative to more complex ones that lead to comparable overt linguistic behavior without

pragmatic language use). Under this view, semantics and pragmatics play a synergic role: pragmatic use allows maintenance of simpler representations; pressure towards representational simplicity indirectly promotes pragmatic over literal language use. As a consequence, iterated transmission and use of language lead to a regularization that may explain the lack of lexicalization of systematic pragmatic enrichments.

While the results of this case study are interesting, they also raise a number of critical issues. First of all, while many favorable parameter settings exist which lead to a prevalence of target types, other types are usually represented in non-negligible proportions (see Fig. 5 and 6). This may just be a technical quirk of the mutator step; but there is a related issue of empirical importance. Several experimental studies on scalar implicatures suggest that participants can be classified as either semantic or pragmatic users of, in particular, *some* (e.g., Bott and Noveck 2004, Nieuwland et al. 2010, Degen and Tanenhaus 2015). The former consistently accept *some* where *all* would be true as well, the latter do not. Interestingly, in our simulations when a target type is the majority type an inflated proportion of the population uses compatible lexica with a lexicalized upper bound. Particularly in those parameter settings where the prior influences the outcome less. In other words, we do find a tendency toward a similar co-existence of semantic and pragmatic types. Whether this analogy has any further explanatory value is an interesting path for future exploration.

Another important issue that is not addressed in the model are potential costs associated with pragmatic reasoning. Here, we simply assumed that literal and pragmatic reasoning strategies exist from the start and are equally costly to apply. In contrast, empirical results suggest that the computation of a scalar implicature may involve additional cognitive effort (e.g., Breheny et al. 2006, Neys and Schaeken 2007, Huang and Snedeker 2009, Tomlinson Jr. et al. 2013). Extensions of the model presented here to include processing costs for pragmatic language use would be interesting future work. It seems plausible that effects of reasoning cost may trade off with the frequency with which a given scalar expression is used. It may be that frequently drawn scalar implicatures lexicalize to avoid cost, whereas infrequent ones are derived on-line to avoid more complex lexical representations during acquisition. Such a prediction would lend itself to empirical testing in line with a recent interest in differences between various scalar implicature triggers (van Tiel et al. 2016).

Our case study could be criticized as follows: all it shows is that scalar implicatures do not lexicalize because upper bounds are dispreferred lexical representations. This criticism would be too superficial and highly unjust. Dispreferred lexical representations can thrive under evolutionary selection. Lexicalized upper-bounds can dominate a population because they may boost communicative success. But they do

not have to. Moreover, as stressed throughout this investigation, even without selective pressure for communicative success, it is not necessarily the case that the types that are most likely *a priori* will dominate. The dynamics of iterated learning are not that trivial. Iterated learning does not necessarily promote the *a priori* more likely type, but tends to promote a type t based on a gradient of how many other types might likely mutate into t, so to speak. Taken together, without an explicit model of the interaction between pressure for communicative success and learnability, it is far from trivial to judge whether or when preferred or dispreferred representations can be adaptive. This is why a major contribution of this paper is the arrangement of many different ingredients into a joined model of the (co-)evolution of lexical meaning and pragmatic use.

What is more, it is not that we just assumed a prior disadvantage of lexicalized upper bounds. We tried to motivate and formalize a general assumption about lexical representations' complexity with a concrete, albeit provisional proposal. The specification of a learning bias in terms of a "grammar of representations" can and should be seen critically, however. Much depends on the primitives of such a grammar. For instance, the lexical representation "none or all" is the most complex in Table 2. But consider adding a new primitive relation between sets $A \smile B$ which is true if and only if $\neg(A \cap B \neq \emptyset \wedge A \neq \emptyset)$. The lexical representation "none or all" would then be one of the simplest. Clearly, further research, empirical and conceptual, into the role of representational complexity, processing costs and learning biases is needed. The model here makes a clear and important contribution nonetheless: it demonstrates how simplicity of representations can interact with use and evolutionary selection and shows that for simple representations to emerge it may require pragmatic strategies to compensate their potential expressive deficiencies. Hence a model of co-evolving semantics and pragmatics is needed. Future work should also include the possibility that representational simplicity may itself be a notion that is subject to evolutionary pressure (cf. Thompson et al. 2016), as well for the evolution of elements that define the agents' cognitive make-up; λ and l.

Finally, this case study should not be interpreted as a proposal for a definite explanation of how scalar implicatures evolved. Other factors should be considered eventually even if they will lead to much more complex modeling. One such factor is the observation that non-lexicalized upper bounds allow a broader range of applicability. For example, when the speaker is not certain as to whether *all* is true. This may suggest an alternative and purely functionalist argument for why upper-bounded meanings do not conventionalize: should contextual cues provide enough information to the hearer to identify whether a bound is intended to be conveyed pragmatically, then this is preferred over expressing it overtly through longer expressions. For example, by saying *some but not all* explicitly. Importantly, although morphosyntactic

disambiguation may be dispreferred due to its relative length and complexity (Piantadosi et al. 2012), it allows speakers to enforce an upper-bound and override contextual cues that might otherwise mislead the hearer. In a nutshell, this explanation posits that scalar implicatures fail to lexicalize because, all else being equal, speakers prefer to communicate as economically as possible and pragmatic reasoning enables them to do so. What this alternative argument does not explain is why functional pressure does not lead to the emergence of different, equally costly lexical items to express different knowledge states of the speaker (Horn 1984:252-267, Horn 1972, Traugott 2004, van der Auwera 2010). For instance, to the emergence of two expressions for each weak scalar expression; one with and one lacking an upper-bound. That is, this argument does not explain why English and other languages do not have a monomorphemic dual for, e.g., *some* that lexicalizes an upper-bound. If this hypothetical expression existed, it could be deployed to signal that the speaker knows that *some but not all* holds, and unbounded *some* could exclusively signal epistemic uncertainty. Looking at pressure from learnability might come in again.

Beyond scalar implicatures, the model can generate predictions about likely lexicalization trajectories of pragmatic inferences, or a lack thereof. In this realm an interesting issue is whether proposed principles, such as the semantic conventionalization of once highly context-dependent inferences if they become regular enough (Levinson 2000, Traugott 2004), can be given a formal rationale and inform postulated directionalities of change. The present investigation made a first start and gave a framework for exploring these issues systematically.

5 Conclusion

The cultural evolution of meaning is influenced by intertwined pressures. We set out to investigate this process by putting forward a model that combines pressure toward successful information transfer with perturbations that may arise in the transmission of linguistic knowledge in acquisition. Its objects of selection and replication are pairs of lexical meanings and patterns of language use. This allows the model to trace the evolutionary interaction between conventional meaning and pragmatic use. Additionally, it takes the challenge seriously of neither semantics nor pragmatics being directly observable. Instead, learners need to infer these unobservables from overt data that results from their combination. These components and their mutual influence were highlighted in a case study on the lack of lexical upper-bounds in weak scalar expression. This study showed that, when pressured for learnability and communicative success, the former force can drive for simpler semantic representations inasmuch as
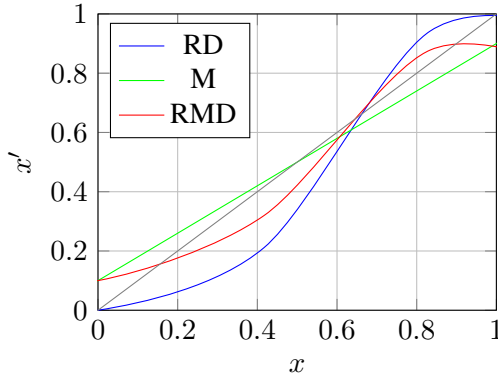
(a) Update functions: the population state x is mapped onto x' in one update step.
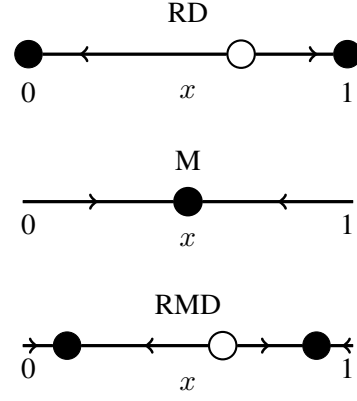
(b) Phase portraits for RD, M and RMD: unstable rest points are hollow, attractors are solid.

Figure 7: Example.

pragmatics can compensate for lack of expressivity in use. That is, the relative learning advantage of simpler semantics in combination with functional pressure in use may offer an answer to why natural languages fail to lexicalize systematic pragmatic inferences. And, more broadly, lead to a division of labor between semantics and pragmatics.

A A minimal example of the replicator mutator dynamic

Consider a simple and abstract coordination game. Agents are of two types: positive or negative. If agents of different type interact with each other, they obtain a payoff of 0. If negative meets negative, each receives a payoff of 1. If positive meets positive, they get a payoff of 2. A population state is completely characterized by the proportion x of negatives. The fitness of negatives in population state x is $f_n(x) = x$, that of positives is $f_p(x) = 2-2x$. The average fitness is $\Phi(x) = xf_n(x)+(1-x)f_p(x) = 3x^2 - 4x + 2$. Without mutation, the replicator dynamic will update x to $\mathrm{RD}(x) = {f(x)x}/{\Phi(x)} = {x^2}/{\Phi(x)}$. The update function $\mathrm{RD}(x)$ of the replicator step is plotted in Fig. 7a as the blue line. Rest points, for which $\mathrm{RD}(x) = x$, are at $x = 0$, $x = 1$ and $x = {2}/{3}$. The former are attractors; nearby points converge to them. Points near $x = {2}/{3}$ also move towards 0 or 1. This is schematically pictured in the topmost phase portrait in Fig. 7b. Adding mutation changes the dynamic and its rest points. Let us assume that $Q_{ji} = .9$ when $j = i$. This is the proportion of types that are replicated faithfully. Conversely, a proportion of .1 will change their type from positive to negative, and vice-versa. The update effect of mutation on its own are described by $\mathrm{M}(x) = .9x + .1(1-x) = .8x + .1$, plotted as the linear green line in Fig. 7a. As shown in Fig. 7b, in this example mutation alone has only one stable rest point. It is located at $x = .5$. If we first take the replicator step and then the mutation step in sequence, we obtain the replicator mutator

dynamic $\mathrm{RMD}(x) = \mathrm{M}(\mathrm{RD}(x)) = {}^{.9x^2 - .2x + .2}/{3x^2 - 4x + 2}$, which is plotted in red in Fig. 7a. The rest

points are at $x = .121$, $x = .903$ and $x = .609$. The former two are attractors (see Fig. 7b).

B Deriving the replicator mutator dynamic from conditional imitation

Consider a huge, virtually infinite population of agents. At any point in time each agent has a fixed type i which determines its fitness f_i at that moment in time when playing with an arbitrary member of the population. The relative frequency of type i is x_i with $\sum_i x_i = 1$. The average population fitness is $\Phi = \sum_k x_k f_k$. The mutation probability from j to i is Q_{ji}, with $\sum_k Q_{jk} = 1$. The proportion of type i at the next time step x_i' is given by the discrete time replicator dynamic:

$$ x_i' = \sum_j \frac{x_j f_j Q_{ji}}{\Phi}, \tag{7} $$

We will derive this dynamic from an agent-level update scheme of conditional imitation. There are many variants on this derivation. Our choice is to be as simple as possible. Sandholm (2010) provides much more detail and discusses many interesting variants.

A simple conditional imitation dynamic runs as follows. Every now and then an random agent is selected at random from the population and given a chance to possibly change its behavior. Call this agent (perhaps slightly misleadingly) *learner L*. The learner chooses a random agent from the population to possibly imitate. Call this agent (somewhat more misleadingly) *teacher T*. Whether L actually chooses to imitate the behavior of T depends on how good T's behavior is at the moment. For simplicity, we simply assume that L adopts T's type with a probability given by f_j, where j is the type of T.[3] If L chooses to imitate T's behavior, it may not be able to directly observe T's actual type. Rather it may have to infer T's latent type from its overt behavior. The probability that L then actually acquires type k when it chose to imitate a teacher of type j is Q_{jk}. In sum, we assume that learners have (at least roughly) an estimate of how good another agent's behavior is, decide to imitate that behavior conditionally on how good it actually is and the adopt whatever mechanism (type) they infer may cause the observable behavior.

This model of occasional conditional imitation gives rise to expected changes in relative frequencies in the population. Given a huge population, the expected probability $P(k \rightarrow i)$ that some agent of type k switches by conditional imitation to type i is the product of probabilities for the following four events:

---

[3] We assume here that $f_i \in [0; 1]$ for all i. This is actually the case for the definition of fitness as communicative success from Section 2.4. See Sandholm (2010) for more complex alternative conditional update rules that still derive the same population dynamic.

1. a learner L of type k is sampled, which happens with probability x_k

2. a teacher T of type j is sampled, which happens with probability x_j

3. L chooses to imitate T, which happens with probability f_j

4. L infers T's type to be i, which happens with probability Q_{ji}

The average expected change $\dot{x}_i$ in proportion of type i is then:

$$
\begin{aligned}
\dot{x}_i &= \sum_k P(k \to i) - \sum_k P(i \to k) \\
&= \sum_k x_k \sum_j x_j f_j Q_{ji} - \sum_k x_i \sum_j x_j f_j Q_{jk} \\
&= \sum_j x_j f_j Q_{ji} - x_i \sum_j x_j f_j \\
&= (x'_i - x_i)\Phi
\end{aligned}
$$

In this way, the discrete-time replicator mutator equation in (7) captures the expected difference in relative frequency of type i from one discrete time step to the next (relative to current baseline fitness Φ, which is constant for all types, and so only governs the absolute speed of change).

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