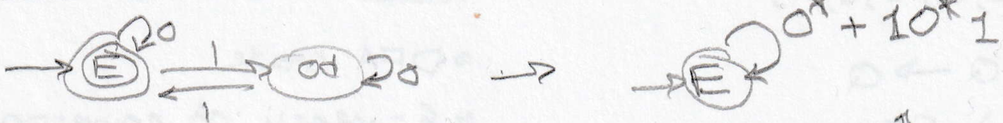
CFL example

$\{0^n 1^n, n \geq 0\} = L$

$S \rightarrow 01 \mid 0S1$

Regex example / DFA example



Accepting whenever:

$(0^* + 10^*1)^*$

Example of state removal

Inductive Proof:

- Base case
- Inductive Hypothesis
- Inductive Step

Closure Properties

- Regular languages are closed over:

$\cup, \cap, +, *, -$, compliment

Countability argument

- All languages = uncountably inf.
- Regex = countably inf.

equivalence class / Differentiability

- strings w, x are equivalent over language L, $w \equiv_L x$ whenever $w \in L \wedge x \in L$ or $w \notin L \wedge x \notin L$.
- Differentiable whenever adding a suffix makes the two strings no longer equivalent

Myhill-Nerode Theorem

For any language L following are equivalent:

(a) The minimum-number states in a DFA that accepts L

(b) The maximum size of a fooling set F

(c) The number of equivalence classes $\equiv_L$

Proving Non-Regularity

1.) Fooling Sets

2.) DFA analysis - suppose we have $a^n b^n \mid n \geq 0$, suppose $a^n b^n$ takes us to state q_1 and $a^m b^m$ takes us to state q_2 because $\delta^*(q_1 b^n) \neq \delta^*(q_2 b^m)$, means $q_1 \neq q_2$ QED b/c we have ∞ a^m, means we need infinite states QED NOT Regular!

$$\delta^*(p, w) := \begin{cases} \varepsilon\text{-reach}(p) & w = \varepsilon \\ \displaystyle\bigcup_{r \in \varepsilon\text{-reach}(p)} \ \bigcup_{q \in \delta(r,a)} \delta^*(q, x) & \end{cases}$$

- δ^* function for NFA's

Brock Boehler BVB2
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