

# Extras

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$$\text{Pooled Treatment Effect} = \sum_b \frac{n_b}{N} * \frac{\text{Var}(T_{i,b})}{\text{Var}(T_i)} * \hat{\Delta}_b.$$

$$\text{Question: Does } \sum_b \frac{n_b}{N} * \frac{\text{Var}(T_{i,b})}{\text{Var}(T_i)} = 1?$$

$$\text{Given: } \sum_b n_b = N$$

$$\begin{aligned} \sum_b \frac{n_b}{N} * \frac{\text{Var}(T_{i,b})}{\text{Var}(T_i)} &= \sum_b \frac{n_b}{N} * \frac{\text{Var}(T_{i,b})}{\text{Var}(T_i)} \\ &= \sum_b \frac{n_b}{N} * \frac{p_b * (1 - p_b)}{p * (1 - p)} \end{aligned}$$

Assume  $n_b = N/b$ . (All the blocks have equal size)

$$\begin{aligned} \sum_b \frac{n_b}{N} * \frac{p_b * (1 - p_b)}{p * (1 - p)} &= \sum_b \frac{N/b}{N} * \frac{p_b * (1 - p_b)}{p * (1 - p)} \\ &= \sum_b \frac{1}{b} * \frac{p_b * (1 - p_b)}{p * (1 - p)} \end{aligned}$$

Let  $b = 2$ ,  $p_1 = 0.8$  and  $p_2 = 0.4$

Additionally  $p = 0.6$ . This is because you have a 50 percent chance of landing in group 1 or a 50 percent chance in landing in group 2. This implies probability of treatment equals  $0.5 * 0.8 + 0.5 * 0.4 = 0.6$

$$\sum_b \frac{1}{b} * \frac{p_b * (1 - p_b)}{p * (1 - p)} = \frac{1}{2} * \frac{0.8 * (1 - 0.8)}{0.6 * (1 - 0.6)} + \frac{1}{2} * \frac{0.4 * (1 - 0.4)}{0.6 * (1 - 0.6)}$$

Top of Fraction

$$\begin{aligned} &0.5 * 0.8 * (1 - 0.8) + 0.5 * 0.4 * (1 - 0.4) \\ &0.08 + 0.12 \\ &0.2 \end{aligned}$$

Bottom of Fraction

$$\begin{aligned} &0.6 * (1 - 0.6) \\ &0.24 \end{aligned}$$

This means the weights sum to be

$$\frac{0.2}{0.24} = 0.833$$