

Extras

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Note for future reference:

$$\text{Pooled Treatment Effect} = \sum_b \frac{n_b}{N} * \frac{\text{Var}(T_{i,b})}{\text{Var}(T_i)} * \hat{\Delta}_b.$$

Question: Does $\sum_b \frac{n_b}{N} * \frac{\text{Var}(T_{i,b})}{\text{Var}(T_i)} = 1$?

Given: $\sum_b n_b = N$

$$\begin{aligned} \sum_b \frac{n_b}{N} * \frac{\text{Var}(T_{i,b})}{\text{Var}(T_i)} &= \sum_b \frac{n_b}{N} * \frac{\text{Var}(T_{i,b})}{\text{Var}(T_i)} \\ &= \sum_b \frac{n_b}{N} * \frac{p_b * (1 - p_b)}{p * (1 - p)} \end{aligned}$$

Assume $n_b = N/b$. (All the blocks have equal size)

$$\begin{aligned} \sum_b \frac{n_b}{N} * \frac{p_b * (1 - p_b)}{p * (1 - p)} &= \sum_b \frac{N/b}{N} * \frac{p_b * (1 - p_b)}{p * (1 - p)} \\ &= \sum_b \frac{1}{b} * \frac{p_b * (1 - p_b)}{p * (1 - p)} \end{aligned}$$

Let $b = 2$, $p_1 = 0.8$ and $p_2 = 0.4$

Additionally $p = 0.6$. This is because you have a 50 percent chance of landing in group 1 or a 50 percent chance in landing in group 2. This implies probability of treatment equals $0.5 * 0.8 + 0.5 * 0.4 = 0.6$

$$\sum_b \frac{1}{b} * \frac{p_b * (1 - p_b)}{p * (1 - p)} = \frac{1}{2} * \frac{0.8 * (1 - 0.8)}{0.6 * (1 - 0.6)} + \frac{1}{2} * \frac{0.4 * (1 - 0.4)}{0.6 * (1 - 0.6)}$$

Top of Fraction

$$0.5 * 0.8 * (1 - 0.8) + 0.5 * 0.4 * (1 - 0.4) = 0.08 + 0.12 = 0.2$$

Bottom of Fraction

$$0.6 * (1 - 0.6) = 0.24$$

This means the weights sum to be

$$\frac{0.2}{0.24} = 0.833$$