ASTR 589 – Physics of Astrophysics
Assignment I on Radiative Transfer and Processes
Due Date: Thursday September 14 on D2L by end of day

1. Analytically or numerically integrate the 1D, time independent radiative transfer equation through a slab (or use the formal solution to the transfer equation) to find $I_\nu(\mu)$ for the following source functions and temperature profiles:
   a. $S_\nu(T) = B_\nu(T)$ and $T(z) =$const.
   b. $S_\nu(T) = B_\nu(T)$ and $T(z) = T_0\tau^2$
   Here, τ is the dimensionless depth variable at $\mu = 1$, $B_\nu(T)$ is the blackbody function, and you may take $\tau_{\max} = 10$ through the entire slab for $\mu = 1$ (i.e., at normal incidence to the slab). The boundary conditions are $I_\nu(\mu) = 0$ for $-1 \le \mu < 0$ specified at $\tau = 0$ and $I_\nu(\mu) = 0$ for $0 \le \mu \le 1$ specified at $\tau = \tau_{\max}$.

2. A distant source emits blackbody radiation at temperature T_s. There is an intervening nebula at temperature $T_n < T_s$. Calculate the specific intensity of the radiation you would observe by looking at the source through the nebula as well as along a line of sight to the nebula that does not intersect the source. You may assume that the frequency of observation is smaller than both kT_s and kT_n so that you may work in the Rayleigh-Jeans limit. Can you use these two observations to infer the optical depth through the nebula at that wavelength?

3. Photoionization is a process in which a photon is absorbed by an atom and an electron is ejected. An energy at least equal to the ionization potential is required. Let this energy be $h\nu_0$ and let σ_ν be the cross section for photoionization. Show that the number of photoionizations per unit volume and per unit time is

   $$4\pi n_a \int_{\nu_0}^{\infty} \frac{\sigma_\nu J_\nu}{h\nu} d\nu,$$

   where n_a is the number density of atoms.

4. We discussed in class that there is a close connection but not a one-to-one relationship between the photon mean-free-path λ_p and the degree of isotropy of the radiation field. Give an example for each case (where l is the characteristic length scale of the medium):

   $$\lambda_p << l$$

   but I_ν is anisotropic; and

   $$\lambda_p > l$$

   but I_ν is isotropic. Note that the second case is more rare.