Lec. 17

Bondi accretion con't

$\cancel{\dot{M} = 4\pi \rho_\infty c_s r_s^2}$

$$\dot{M} = 4\pi \lambda_s \left(\frac{GM}{c_{s,\infty}^2}\right)^2 \rho_\infty c_{s,\infty}$$

λ_s	Γ
1.12	1
$\sqrt{e^3}/4$	4/3
1/4	5/3

So, for ideal gas $\Gamma = 5/3$

$$\dot{M} = \pi \frac{G^2 M^2}{c_{s,\infty}^{\cancel{4}\,3}} \rho_\infty \cancel{c_{s,\infty}}$$

$$= \pi \frac{G^2 M^2}{c_{s,\infty}^3} \rho_\infty$$

If star is moving, i.e, gas velocity at infinity U_∞, then Bondi proposed

Bondi - Hoyle

$$\dot{M} = \frac{2\pi G^2 M^2 \rho_\infty}{(c_{s,\infty}^2 + v_\infty^2)^{3/2}}$$

But, Shima et al '85 suggested

$$\dot{M} = \frac{4\pi G^2 M^2 \rho_\infty}{(c_{s,\infty}^2 + v_\infty^2)^{3/2}}$$

which matches the Hoyle-Lyttleton rate when $\frac{c_{s,\infty}}{v_\infty^2} \ll 1$

So, this is now called the

Bondi-Hoyle-Lyttleton

accretion rate.

Thin Accretion disks

Shakura-Sunyaev 1973

Accretion onto a "star" of mass M, characterized by a rate $\dot{M}$.

Assumptions

Steady state $\frac{\partial}{\partial t} \to 0$

(I) The disk is axisymmetric $\frac{\partial}{\partial \varphi} \to 0$

(II) The disk is geometrically thin $\frac{H}{R} \ll 1$

(III) $U_r \ll U_\varphi$ minor drift in radial direction due to viscosity

(IV) $V_\varphi = \Omega \cdot R = \left(\frac{GM}{R}\right)^{1/2}$ Keplerian profile

(V) Disk self-gravity is negligible

(VI) Radial & Azimuthal pressure gradients are negligible. Vertical pressure serves to support the disk in z-direction.

Under these assumptions, we can z-integrate the hydro eqn.s and solve for the vertically integrated quantities e.g. $\Sigma \equiv \int \rho \, dz$ = surface density

Cont' equation

$$\frac{\partial \rho}{\partial t} + \nabla (\rho u) = 0 \Rightarrow$$ cylindrical coords.

u_r no z dep. $\Rightarrow$

$$\frac{1}{R}\frac{\partial}{\partial R}\left(R \rho u_r\right) = 0$$

$$\frac{1}{R}\frac{\partial}{\partial R} R \int \rho u_r \, dz = 0 \Rightarrow$$

$$R \Sigma(R) u_r = \text{const.} \Rightarrow$$

$$\boxed{-2\pi R \Sigma(R) u_r = \dot{M}} \quad (1)$$

Define vertical scale height H

$$2 \rho_c H \Sigma \Rightarrow \boxed{2H = \frac{\Sigma}{\rho_c}}, \quad \rho_c = \rho(z=0)$$

So $\boxed{\dot{M} = 4\pi R \rho_c u_r H}$ (2)

The ϕ-component of the Euler (momentum eqn)

$$\frac{1}{R}\frac{\partial}{\partial R}\left[R u_r \Sigma R^2 \Omega\right] = \frac{1}{R}\frac{\partial}{\partial R}\left[\nu \Sigma R^3 \frac{d\Omega}{dR}\right]$$

$$\Rightarrow \quad R u_r \Sigma R^2 \Omega - \nu R^3 \Sigma \frac{d\Omega}{dR} = \text{const.}$$

$$\xrightarrow{\times 2\pi}$$

$$\underbrace{2\pi R u_r \Sigma}_{-\dot M} R^2 \Omega - 2\pi \nu R^3 \Sigma \frac{d\Omega}{dR} = \text{const.}$$

$$\xrightarrow{\times u_r}$$

$$\underbrace{-u_r \cdot \dot M R^2 \Omega}_{\text{ang. mom. advected inward}} - \underbrace{2\pi \nu R^3 \frac{d\Omega}{dR} u_r}_{\text{ang. mom. diffusing outward by viscosity}} = \underbrace{\dot J u_r}_{\text{flux of ang. moment}} \qquad (3)$$

$$\dot J = \text{const.}$$

In the case of BH accretion the inner edge of the disk is at the innermost stable circular orbits, and we postulate that there are no viscous torques

$$\left.\frac{d\Omega}{dR}\right|_{R_{in}} = 0 .$$

At the ISCO the eq. 3 $\Rightarrow$

$$\dot{J} = \dot{M} R_{in}^2 \Omega_{R_{in}}$$

So, Eq. (3) becomes

$$-\dot{M}\left[R^2\Omega - R_{in}^2 \Omega_{R_{in}}\right] - 2\pi\nu R^3 \Sigma \frac{d\Omega}{dR} = 0$$

Assuming Keplerian $\Omega = \sqrt{\frac{GM}{R^3}}$

and $\frac{d\Omega}{dR} = -\frac{3}{2}\sqrt{GM}\, R^{-5/2}$

$$\boxed{3\pi\nu\Sigma = \dot{M}\left[1 - \left(\frac{R_{in}}{R}\right)^{\frac{1}{2}}\right]} \quad (4)$$

Important concl. of eq. (4), For a given $\dot{M}$

$$\Sigma \sim \nu^{-1}$$

i.e., larger viscosity $\Rightarrow$ faster rate of radial inflow, and smaller Σ.

$$(1) \xRightarrow{(4)} -2\cancel{\pi} R u_r \frac{\cancel{\dot{M}}\left[1 - \left(\frac{R_{in}}{R}\right)^{\frac{1}{2}}\right]}{3\cancel{\pi}\nu} = \cancel{\dot{M}}$$

So at $R \gg R_{in}$

$$\boxed{u_r \simeq -\frac{3}{2}\frac{\nu}{R}} \quad (5)$$

$\boxed{\text{So indeed } |u_r| \nearrow \text{ as } |\nu| \nearrow}$

$\boxed{\text{Lecture 18}}$

Now, energy eqn

We had written

$$\rho T \frac{DS}{dt} = \frac{\sigma_{ij}\sigma^{ij}}{2\eta}$$

For thin disk, we need to include radiative cooling, because if the visc heat is not radiated the disk will puff up.

In this case

$$\underbrace{\Sigma T u_r \frac{dS}{dR}}_{\text{entropy advected}} = \underbrace{\nu \Sigma \left(R \frac{d\Omega}{dR}\right)^2}_{\substack{\text{entropy generated} \\ \text{by viscosity}}} - \underbrace{F_c}_{\substack{\text{cooling} \\ \text{radiative} \\ \text{flux}}}$$