Lec 11

Synchrotron Self Absorption

For a thermal distr. of e^-, we have Kirchoff's law

$$(\alpha_\nu)_{thermal} = \frac{\eta_\nu}{B_\nu}$$

In the RJ regime $h\nu \ll E$ of electrons we obtain $(\alpha_\nu)_{th} = \frac{\eta_\nu c^2}{2\nu^2 KT}$

For power-law distribution

$N(E) \sim C E^{-p}$ assuming relativistic electrons

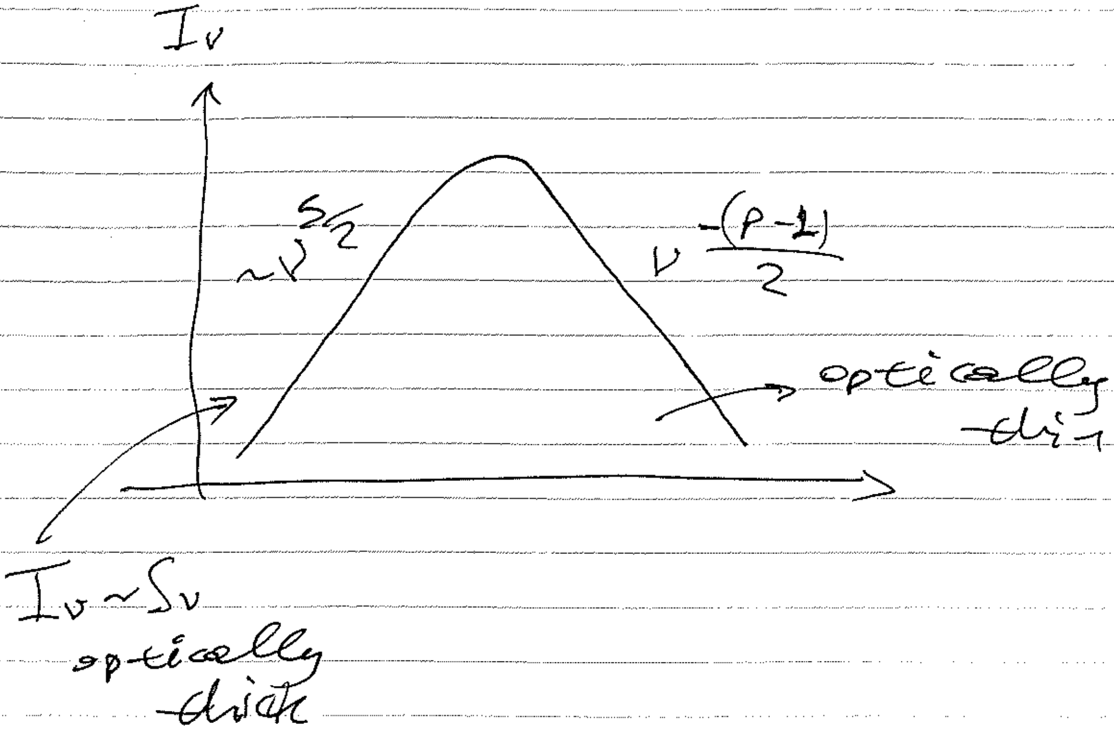
$$\alpha_\nu = \frac{\sqrt{3} e^3}{8\pi m}\left(\frac{3e}{2\pi m^3 c^5}\right)^{p/2} C \,(B\sin\alpha)^{\frac{p+2}{2}}$$

$$\times\, \Gamma\left(\frac{3p+2}{12}\right)\Gamma\left(\frac{3p+22}{12}\right)\nu^{-\frac{(p+4)}{2}}$$

So that the source function

$$S_\nu = \frac{\eta_\nu}{\alpha_\nu} \propto \frac{\nu^{-(p-1)/2}}{\nu^{-(p+4)/2}} \sim \nu^{5/2}$$

Thus typical spectrum



Final remarks on RT

Radiative Equil.

If no energy is generated or absorbed (net) by the gas through interactions with radiation or presence of sources, we can write detailed balance:

$$\underbrace{\iint \alpha_\nu I_\nu \, d\Omega \, d\nu}_{E \text{ removed}} = \underbrace{\iint \eta_\nu \, d\Omega \, d\nu}_{E \text{ emitted}}$$

Coherent (or elastic) scattering doesn't change E, and doesn't produce photons So, we can ignore scattering in ~~de~~ detailed balance. (

The above equation is the condition for Radiative Equilibrium in its general form, but we can simplify it

If α and ~~η~~ η are angle-independent

$$\int d\nu\, \alpha_\nu \int I_\nu\, d\Omega = \int d\nu\, \alpha_\nu\, 4\pi J_\nu =$$

~~[illegible]~~

$$= \int d\nu\, \eta_\nu\, 4\pi$$

For thermal emission ~~[illegible]~~

~~&~~ $\eta_\nu = \alpha_\nu B_\nu$ Kirchoff's law

Thus ~~$\int d\nu\, \alpha_\nu [J_\nu \ldots]$~~ $\Rightarrow$

$$\int d\nu\, \cancel{4}\, \alpha_\nu [J_\nu - B_\nu] = 0$$

IF α_ν is ν-independent then

$$\boxed{J_\nu = B_\nu}$$

In other words RTE becomes LTE-like

More remarks

Usually energy is generated in one region (e.g. "center" of star) and then transported

3 modes of energy (heat) transport

i) Radiation
ii) Convection (boiling water)
iii) Conduction

i) & ii) dominate usually, but iii) important in coronae, clusters and accretion disks.

Radiative equil. implies all heat transport is taking place through radiation in a given region, and that no additional E generation means passive transport is in equilibrium.

Let's take the 0th-moment of the time-dependent RTE to show that radiative equilibrium means flux is constant

$$\frac{1}{c}\frac{\partial I_\nu}{\partial t} + \mu \frac{\partial I_\nu}{\partial z} = \eta_\nu - \chi_\nu I_\nu \quad \int \cdot \, d\Omega$$

$$\frac{4\pi}{c}\frac{\partial J_\nu}{\partial t} + \frac{\partial F_\nu}{\partial z} = \int (\eta_\nu - \chi_\nu I_\nu)\, d\Omega$$

Integrate over ν (assuming coherent scattering)

$$\frac{\partial E}{\partial t} + \frac{\partial F}{\partial z} = \int d\nu \int d\Omega (\eta - \chi I)$$

But, in equilib. we showed that the RHS $= 0$, and without sources or sinks $\frac{dE}{dt} = 0$

$$\Rightarrow \frac{\partial F}{\partial z} = 0 \quad \Rightarrow$$

$$F(z) = \int I_\nu(\mu)\,\mu\,d\mu\,d\nu = \text{const.}$$

$\uparrow F$

no heat/E src/sink

$\uparrow F = \text{const}$

This Flux can be used as the parameter to describe the atmosphere.

It's customary to define an effective temperature, T_{eff}, corresponding to this constant flux

$$F = \sigma T_{eff}^4$$

Flux is set by B.Cs

In rad. equilib. the temperature in the slab $T(z)$ adjusts to ensure constant flux.

In higher dimensions easy to show

$$\nabla \cdot \bar{F} = 0$$

We can use T_{eff} even if the spectrum is not Planckian. We do so s.t. T of a B.B. matches that flux, i.e.

$$\sigma T_{eff}^4 = \int_0^\infty F_\nu \, d\nu = 4\pi \int_0^\infty H_\nu \, d\nu =$$

$$= \underbrace{\frac{L}{4\pi R^2}}$$

For a spherical object of radius R (e.g. a star)

$$\frac{dK}{dz} = -\frac{1}{4\pi}\chi F \Rightarrow \frac{dK}{d\tau} = H = \text{const.}$$

$$d\tau = -\chi\, dz$$

Thus $K(\tau) = \frac{1}{4\pi}F\tau + C = H\tau + C$

~~[illegible]~~

In LTE $J = S = B = \frac{\sigma T^4}{\pi}$

So we need a relation between J & K to solve for $T(z)$

At large τ we had shown, representing the specific intensity as $I = a + b\mu$ produces a non-zero flux and

$$K = \frac{J}{3} \quad \text{~~with } K \to \tfrac{1}{4}F\tau\text{~~}$$

Thus, $\frac{J}{3} = \frac{1}{4\pi}\sigma T_{eff}^4[\tau + c]$

to generalize we allow c to be $q(\tau)$ called a Hopf function (so it is also valid at small τ)

$$J(\tau) = \frac{3}{4\pi}\sigma T_{eff}^4 \left[\tau + q(\tau)\right]$$

Eddington approx. $J = 3K$ everywhere even at $\tau \simeq 0$, then requiring

$F_{rad} = 2\pi \int_0^1 \mu I \, d\mu$ to match $F = const$

yields $\quad C = q(\tau) = \frac{2}{3}$

And with $\quad J = \frac{\sigma T^4}{\pi} \quad$ we get

$$T^4 = \frac{3}{4} T_{eff}^4 \left(\tau + \frac{2}{3}\right)$$

so predicts $T = T_{eff}$ at $\tau = \frac{2}{3}$

Effective depth