Lec 6

Radiative Transitions: Bound-bound & bound-free

- light produced during transitions between dif. energy states
- understand: 1) the strength of the radiation
  2) selection rules for ~~radiative~~ radiative transitions

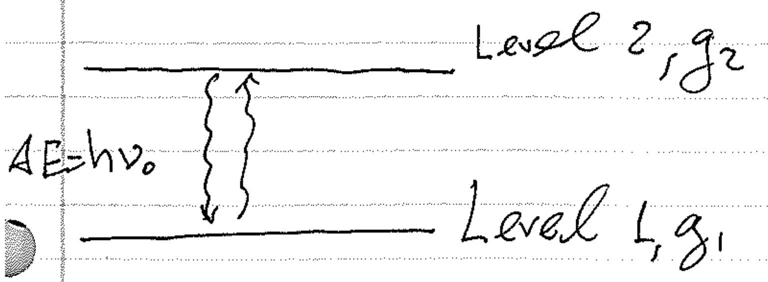
The Einstein Coefficients

At the macroscopic level, Kirchoff's law for a thermal emitter dictates

$$\eta_\nu = \alpha_\nu B_\nu$$

which must imply some relationship between emission and absorption at a microscopic level

Consider a 2 level atomic system and its interaction w/ radiation



1: E, stat. weight g_1

2: $E + h\nu_0$, — " — g_2

There are three processes:

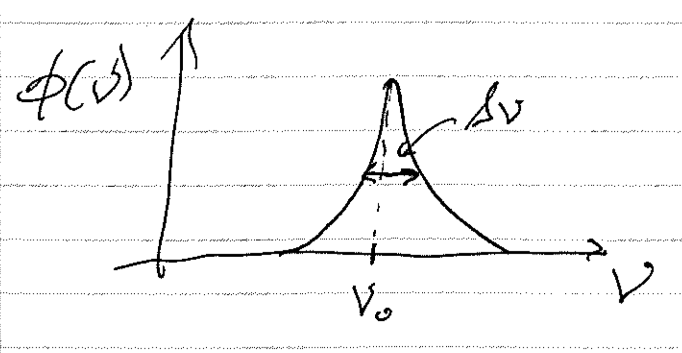
1.- Spontaneous emission: Einstein coef. A

A_{21} = transition prob./time for spont. em. (s^{-1})

2. Absorption : Einstein coef. B

This is proportional to the incoming density of photons at ν_0. But, due to Heisenberg uncertainty principle (and other mechanisms), $\Delta E = h\nu_0$ is <u>not</u> a δ function. The atom can absorb photons of slightly different E (with decreased strength / efficiency)

Line profile function : $\phi(\nu)$



$$\int_0^\infty \phi(\nu)\, d\nu = 1$$

If $\Delta\nu$ small then $\phi(\nu) \sim \delta(\nu)$

$B_{12} \cdot \bar{J}$ = transition prob./time for absorption

$$\bar{J} = \int_0^\infty J_\nu \, \phi(\nu)\, d\nu ,$$

mean intensity

But, in microscopic balance, A_{21} and A_{12} do not obey Kirchoff's law, unless there exists another process also proportional to J but causes emission

(Planck's function)

3. Stimulated emission

$B_{12} \cdot \bar{J}$ = transition prob./time for stimulated emission

In thermodynamic equilib.

$$J_\nu = B_\nu \quad \text{and}$$

$$\frac{\text{transition rate}}{\text{vol}}\ 1 \to 2 = \frac{\text{transition rate}}{\text{vol}}\ 2 \to 1$$

If n_1, n_2 the number densities of atoms in levels 1 & 2, then

$$n_1 B_{12} \bar{J} = n_2 B_{21} \bar{J} + n_2 A_{21} \Rightarrow$$

$$\bar{J} = \frac{A_{21}/B_{21}}{(n_1/n_2)\left(\frac{B_{12}}{B_{21}}\right) - 1}$$

But, in TE

$$\frac{n_1}{n_2} = \frac{g_1 \exp(-E/KT)}{g_2 \exp(-(E + h\nu_0)/KT)} = \frac{g_1}{g_2} \exp\left(\frac{h\nu_0}{KT}\right)$$

$*(E + h\nu_0)/KT$

So

$$\bar{J} = \frac{A_{21}/B_{21}}{(g_1 B_{12}/g_2 B_{21}) \exp\left(\frac{h\nu_0}{kT}\right) - 1} \qquad (1)$$

In TE we know $J_\nu = B_\nu$, but B_ν varies slowly on the scale of $\Delta\nu$, thus $\bar{J} = B_\nu$

For (1) to equal B_ν at all T, we must have

$$\boxed{\begin{aligned} g_1 B_{12} &= g_2 B_{21} \\ A_{21} &= \frac{2h\nu^3}{c^2} B_{21} \end{aligned}}$$

Einstein relations

Notice that there is no T in these eqns. → apply for all T and hold whether in TE or not!!!

If we know one of the 3 Einstein coefficients → we know all!

Einstein relations are example of a detailed balancing involves 2 processes

Einstein relations extend Kirchoff's law to include non-thermal emission, occurring when matter is not in TE.

Absorption & emission in terms of Einstein coef.

To obtain the emission coefficient η_ν we must make an assumption about how the emitted rad. is distributed. Assume line profile $\phi(\nu)$. Then

$$\frac{\text{energy}}{dV\,d\Omega\,d\nu\,dt} = \eta_\nu$$

Since $h\nu_0$ is distributed over 4π sr we can write for the energy emitted

$$\underbrace{\left(\frac{h\nu_0}{4\pi}\right)\phi(\nu)}_{\text{energy emitted in } d\nu \text{ and } d\Omega}\;\underbrace{n_2 A_{21}}_{\#\ \text{of transitions}/\text{time}}\; dV\, d\Omega\, d\nu\, dt$$

Thus $$\eta_\nu = \frac{h\nu_0}{4\pi} n_2 A_{21} \phi(\nu)$$

Absorption coef.

Similarly, total energy absorbed in dt and dV is

$$\int \underbrace{dt\, dV\, n_1 B_{12} \bar{J}\, d\Omega}_{\text{number of absorptions}} \; \frac{h\nu_0}{4\pi}$$

↑ energy absorbed

Thus

$$dV\, dt\, d\Omega\, d\nu\, \frac{h\nu_0}{4\pi}\, n_1 B_{12}\, \phi(\nu)\, I_\nu$$

energy absorbed out of a beam in freq. range $d\nu$, sol. angle $d\Omega$, dt, dV

Since $dV = dl\, dS$

and the energy absorbed in path dl is

$$dI_\nu = -\alpha_\nu I_\nu\, dl$$

$$\alpha_\nu = \frac{h\nu}{4\pi}\, n_1 B_{12}\, \phi(\nu)$$

↑ uncorrected for stimulated emission

We can treat stimulated emission as negative absorption, so similarly we correct

$$\alpha_\nu = \frac{h\nu}{4\pi}\, \phi(\nu)\, (n_1 B_{12} - n_2 B_{21})$$

Then the transfer equ. in terms of the Einstein coeffs.

$$\frac{dI_\nu}{ds} = -\frac{h\nu}{4\pi}\left(n_1 B_{12} - n_2 B_{21}\right)\phi(\nu) I_\nu + \frac{h\nu}{4\pi} n_2 A_{21} \phi(\nu)$$

The source func. $\frac{\eta_\nu}{\alpha_\nu}$

$$S_\nu = \frac{n_2 A_{21}}{n_1 B_{12} - n_2 B_{21}} = \frac{2h\nu^3}{c^2}\left(\frac{g_2 n_1}{g_1 n_2} - 1\right)^{-1}$$

Einstein Relations

generalized Kirchoff's law!

Cases:

1. Thermal emission (LTE)

$$\frac{n_1}{n_2} = \frac{g_1}{g_2} \exp\left(\frac{h\nu}{KT}\right)$$

then $\alpha_\nu = \frac{h\nu}{4\pi} n_1 B_{12} \left[1 - \exp\left(-\frac{h\nu}{KT}\right)\right] \phi(\nu)$

$$S_\nu = B_\nu(T)$$

2. Non-thermal emission

$$\frac{n_1}{n_2} \neq \frac{g_1}{g_2} \exp\left(\frac{h\nu}{kT}\right)$$

e.g. cold plasma or hot non-Maxwellian plasma

3. Inverted populations; masers

In LTE

$$\frac{(n_1/g_1)}{(n_2/g_2)} = \exp\left(\frac{h\nu}{kT}\right) > 1$$

But, it is possible to put atoms in the upper state, so that

$$\frac{n_1}{g_1} < \frac{n_2}{g_2}, \text{ then } \alpha_\nu < 0$$

since $\alpha_\nu \sim (n_1 B_{12} - n_2 B_{21})$

$$\sim \left(n_1 \frac{g_2}{g_1} - n_2\right) \sim$$

Einstein rels.

$$\sim \left(\frac{n_1}{g_1} - \frac{n_2}{g_2}\right)$$

notice also that $\tau_\nu < 0$

more emission than absorption
stimulated