ASTR 589 – Physics of Astrophysics
Assignment V, on Fluid Dynamics
Due Date: Tuesday, November 23

1. (a) Consider the hydrostatic equilibrium equation in spherical symmetry of a fluid distribution

$$\frac{dP}{dr} = -\frac{Gm(r)\rho}{r^2} \tag{1}$$

Integrate this equation over all volume V to prove the Virial theorem for static equilibrium fluid configurations

$$3\int PdV + W = 0, \tag{2}$$

where W is the gravitational potential energy $W = -\int \frac{Gm(r)\rho(r)}{r} dV$.

(b) Adopting an ideal gas equation of state and a constant density star, use the Virial theorem to estimate the characteristic (Virial) temperature for the Sun assuming its composed of pure hydrogen.

(c) Assuming an ideal gas equation of state, what is the physical interpretation of the $3\int PdV$ term?

2. Taking the inner edge of a thin accretion disk to be at the ISCO around a non-spinning black hole, calculate the maximum possible effective temperature as a function of the mass of the black hole and the mass accretion rate. What do you expect this temperature to be for accretion around a $10M_\odot$ and a $10^9 M_\odot$ black hole at 10% of the critical mass accretion rate $\dot{M}_{\rm Edd}$?

(b) Given your answer to the previous part, at what wavelength would you have designed the Event Horizon Telescope in order to observe/image the inner accretion flow of an AGN? Is this what really happens? And how do you explain AGN observations in X-rays?

3. Steepening of an Acoustic Wave into a Shock Wave: Consider an acoustic wave that propagates in a static, uniform, non-magnetized medium of adiabatic index γ. Estimate the time it takes (in units of the period of the wave) for the acoustic wave to steepen into a shock wave, as a function of the fractional perturbation in density. How far does the sound wave travel before it becomes a shock wave? Evaluate your answer for a 440 Hz sound wave that is propagating in dry air at a speed of about 350 m/s and corresponds to a density perturbation of about 10%. Why don't sounds in everyday life always turn into shock waves?

4. In class, we derived the Rankine-Hugoniot jump conditions by assuming that γ is the same in the pre-shock and post-shock regions. This need not be the case in general. For example, a shock may dissociate hydrogen molecules and/or ionize hydrogen atoms, so that part of the pre-shock kinetic energy goes into dissociation and ionization.

   From elementary thermodynamics, we know that

$$\gamma = 1 + \frac{2}{f}, \tag{3}$$

   where f is the number of internal degrees of freedom.

   (a) Ignoring radiation and magnetic fields, write down the one-dimensional jump conditions in the case $\gamma_1 \neq \gamma_2$.

   (b) A supernova shock wave travels with speed $v_s = 10^3$ km s^{-1} into a purely molecular hydrogen cloud of density $N_{H_2} = 10^4$ cm^{-3} and $T_1 = 10$ K. The molecules get completely dissociated by the shock and the resulting atoms get completely ionized. Given that the dissociation energy per molecule is $E_d = 4.5$ eV and the ionization energy per atom is $E_i = 13.6$ eV, calculate the temperature T_2 and the electron density n_e behind the shock front.

Conceptual Questions and Review. Do not turn in these answers.

Accretion and Shocks

1. How do we treat the discontinuity in the equations of fluid dynamics when describing a shock?

2. Describe the physical setup of the problem that is often referred to as Bondi-Hoyle accretion. Use dimensional arguments to express the accretion rate in this setup as a function of the mass of the central object, the density and sound speed in the ambient medium, and the velocity of the object through the medium.

3. Use dimensional arguments to express the time it takes for a viscous ring orbiting a central object to be accreted as a function of the distance of the ring from the object and the coefficient of kinematic viscosity.

4. Provide at least one specific argument from observations which proves that molecular viscosity cannot be the dominant source of angular momentum transport in an accretion disk.

5. In a steady-state, geometrically thin, optically thick accretion disk, what determines the rate of mass accretion? what determines the rate with which the angular momentum of the central object increases (these are trick questions!)

6. In a steady-state, geometrically thin, optically thick accretion disk, the flux of radiation emerging from a ring at distance $R >> R_{\rm in}$ is

$$F = \frac{3GM\dot{M}}{4\pi R^3} .$$

   How does this flux compare to the potential energy released (per unit surface area) as matter crosses the ring? Where does the extra energy come from?

7. Describe the significance of the innermost stable circular orbit around a black hole in setting the efficiency of emission from the accretion disk, under the standard assumptions.

8. Calculate the luminosity from an accretion disk at which radiation forces (along the vertical radiation) balance gravity and show that it scales as the spherically-symmetric Eddington luminosity.