Lecture 20

Radiatively inefficient / Thick (Advection Dominated) Accretion disk (flows)

RIAF, ADAF

In the solution of thin disks, when we considered the energy balance, we saw that

$$\underbrace{\Sigma T u_r \frac{dS}{dR}}_{\text{advected entropy}} = \underbrace{\nu \Sigma \left(R \frac{d\Omega}{dR} \right)^2}_{\text{entropy generated by viscosity}} - \underbrace{F_c}_{\text{cooling flux lost to radiation}}$$

We made the assumption that cooling is efficient s.t. radiation takes away locally all the energy that is generated, and advection can be neglected.

When is that valid?

$$t_{cool} \ll t_{accr.} \simeq \frac{R}{u_r} \simeq \frac{2}{3} \frac{R^2}{\nu}$$

but fails when cooling time is

$$t_{cool} \sim \frac{E_{th}}{L} \text{ is long!}$$

This happens for two reasons in two different regimes

At low ṁ → low ρ
emissivities ($\propto n$) are small. Recall

that all radiative processes go as n^2 (e.g. Bremsstrahlung) or n

(eg synchrotron, Compton, line emission). In addition, thermalization timescale is dominated by collisional processes (e.g. Coulomb collisions between charged particles), which go as $\sim n^2$.

As a result, in the low ṁ case F_c is low because the gas is an inefficient emitter

In the opposite, high $\dot{M}$ regime, t_{cool} is long for a different reason

high $\dot{M} \rightarrow$ high $\rho \rightarrow$ high $\eta \rightarrow$ high χ, high τ

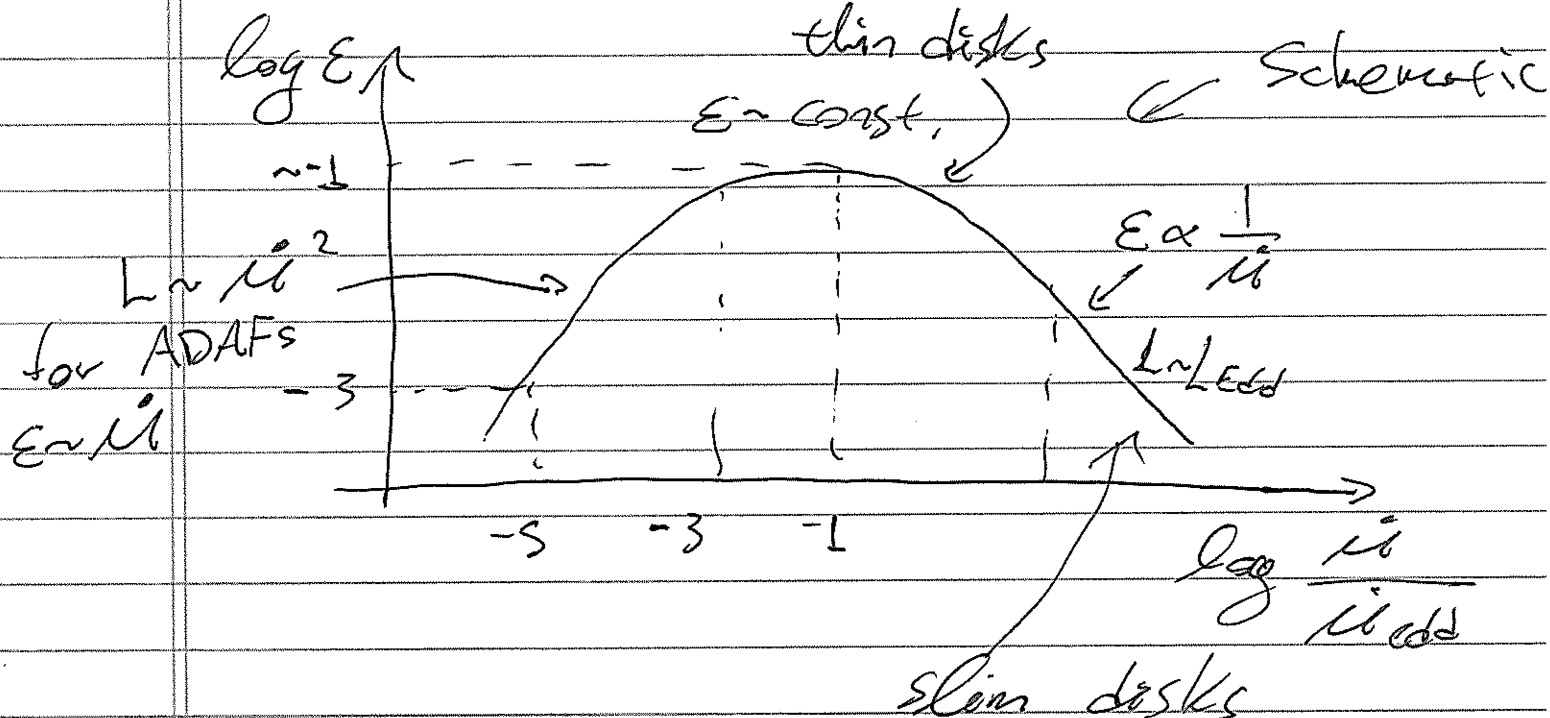
So, the diffusion timescale for radiation gets longer. When $t_{diff} > t_{accr}$, radiation gets trapped in the flow, and is advected with it.

In both cases, in the E eqn, F_c becomes sub-dominant to the other terms, and

$$\Sigma T u_r \frac{ds}{dR} = \nu \Sigma \left(R \frac{d\Omega}{dR} \right)^2$$

implies that much of the viscous entropy is advected (carried inward) with the fluid.

As a result, radiative efficiency has an inverted U shape



In both cases, the scale height of the disk increases $\frac{H}{R} \to 1$, again for different reasons.

For thin disks, we neglected advection and found $H = \frac{c_s}{\Omega} \Rightarrow$

$$\frac{H}{R} = \frac{c_s}{v_\varphi} \ll 1 \quad , \quad c_s = \left(\frac{kT}{m}\right)^{\frac{1}{2}}$$

At low $\dot{M}$, disks can't cool efficiently so $T \uparrow$ and approach the virial temperature, and $c_s \sim \Omega R$

$\Rightarrow$ ~~ADAF~~ ADAF/RIAF are both pressure and angular momentum supported.

Large $T \rightarrow$ large thermal pressure $\rightarrow$ can't neglect the pressure gradient in the radial direction in the momentum eqn (which we did for thin disks).

- The presence of radial pressure support $\rightarrow$ fluid elements are no longer on Keplerian orbits $\rightarrow$ these flows are sub-Keplerian.

- The transition from azimuthal velocities to radial plunging at the ISCO is not as sharp as in thin disks.

~~Lec 24~~

Wave steepening & Shocks

In the small-amplitude perturbation we found solutions of ρ, p, u of the form $f(x \pm c_s t)$. This means that the solutions are of the form $p = p(u)$, $\rho = \rho(u)$ or $p = p(\rho)$ $u = u(\rho)$. For finite amplitude waves this is no longer true.

But, assuming $\rho = \rho(u)$, we can write the ~~continuum~~ momentum eqn. as

$$\frac{\partial u}{\partial t} + \left[u + \frac{1}{\rho} c_s^2 \frac{d\rho}{du} \right] \frac{\partial u}{\partial x} = 0$$

and the continuity eqn. as

$$\frac{\partial u}{\partial t} + \left[u + \rho \frac{du}{d\rho} \right] \frac{\partial u}{\partial x} = 0$$

Comparing the two yields

$$\frac{du}{d\rho} = \pm \frac{c_s}{\rho}$$

~~Lec 21~~

$$\frac{\partial u}{\partial t} + (u \pm C_s)\frac{\partial u}{\partial x} = 0$$

and $$\frac{\partial e}{\partial t} + (u \pm C_s)\frac{\partial e}{\partial x} = 0$$

$\downarrow$

These are non-linear advection eqn's. In particular the higher u or C_s the faster/slower the solution propagates

The advection speed is $u+C_s$ and $u-C_s$

Consider propagation along

$$\frac{dx}{dt} = u \pm C_s \quad \text{then}$$

$$\frac{\partial u}{\partial x} = \frac{\partial u}{\partial t}\frac{\partial t}{\partial x} = \frac{1}{u \pm C_s}\frac{\partial u}{\partial t}$$

so $$\frac{\partial u}{\partial t} + \frac{\partial u}{\partial t} = 0 \Rightarrow u = \text{const.}$$

The curves $\frac{dx}{dt} = u \pm C_s$ are called characteristics

Consider adiabatic EOS

$P = k\rho^{\gamma} \Rightarrow c_s^2 \propto \rho^{\gamma-1} \Rightarrow$

$$(\gamma-1)\frac{d\rho}{\rho} = 2\,\frac{dc_s}{c_s}$$

Since $\frac{du}{d\rho} = \pm\frac{c_s}{\rho} \Rightarrow du = \pm\frac{\cancel{c_s}}{\gamma-1}\,\frac{2\,dc_s}{\cancel{c_s}}$

$$\Rightarrow \boxed{u = \pm\frac{2}{\gamma-1}(c_s - c_0)} \quad \text{or}$$

$$\cancel{c_s = c_0 \pm \frac{1}{2}(\gamma-1)u}$$

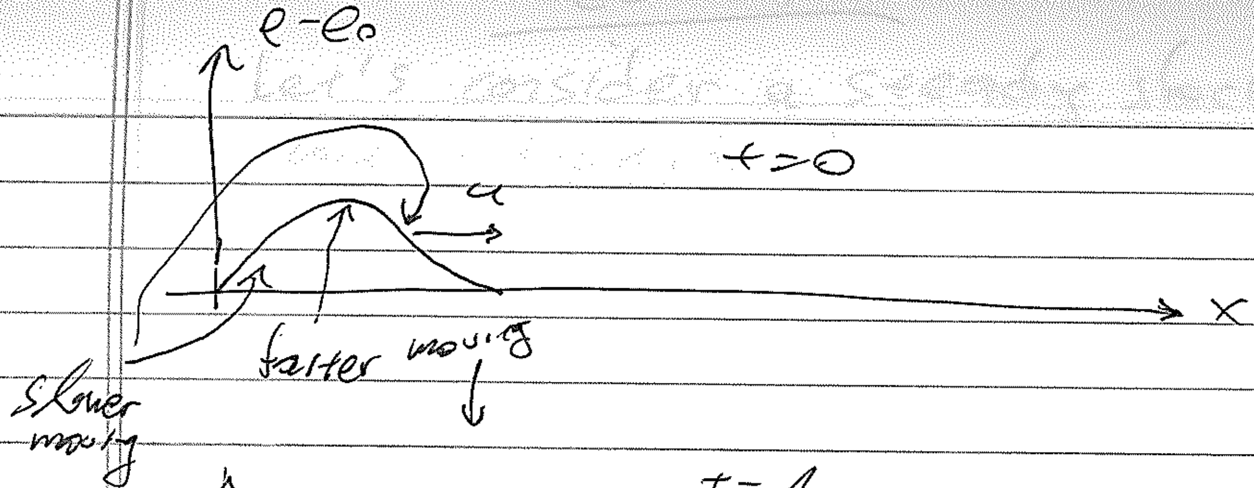
$$\cancel{u + c_s = c_0 \pm \frac{1}{2}(\gamma-1)u + u} \rightarrow c_0 \pm \frac{1}{2}(\gamma+1)u$$

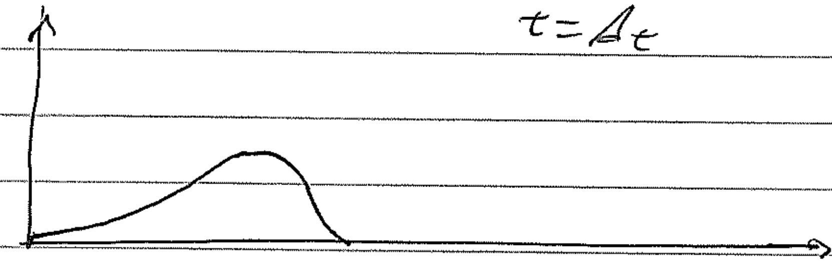
$$\rightarrow \cancel{c_0 \pm \frac{1}{2}(\gamma-2)u}$$

$$\cancel{u - c_s = c_0 \mp \frac{1}{2}(\gamma-1)u - u}$$

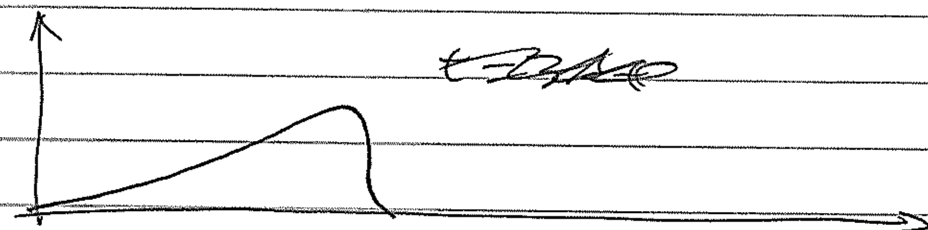
Thus, larger $\rho \Rightarrow$ larger $c_s \Rightarrow$ larger u.

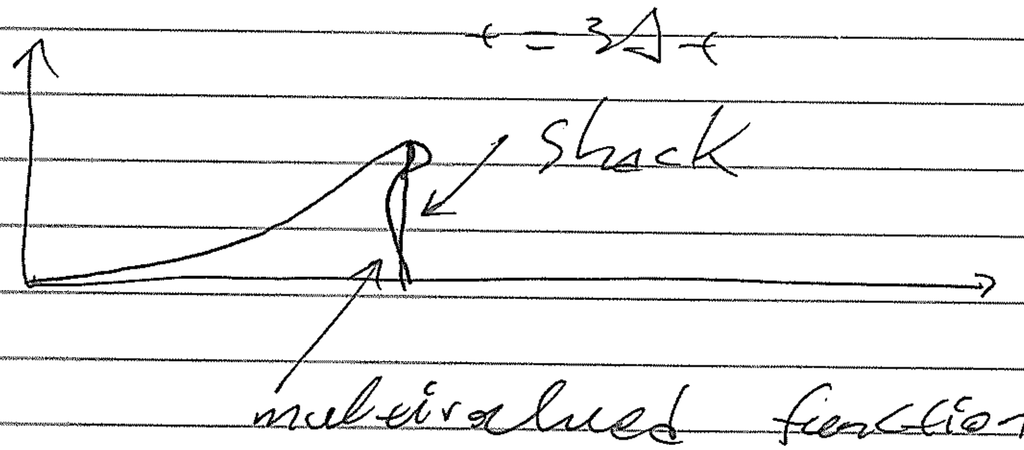
Consider a sound wave moving to the right, i.e. at $u + c_s$


e − e0
t=0
u
x
faster moving
slower moving


t = Δt

t = 2Δt

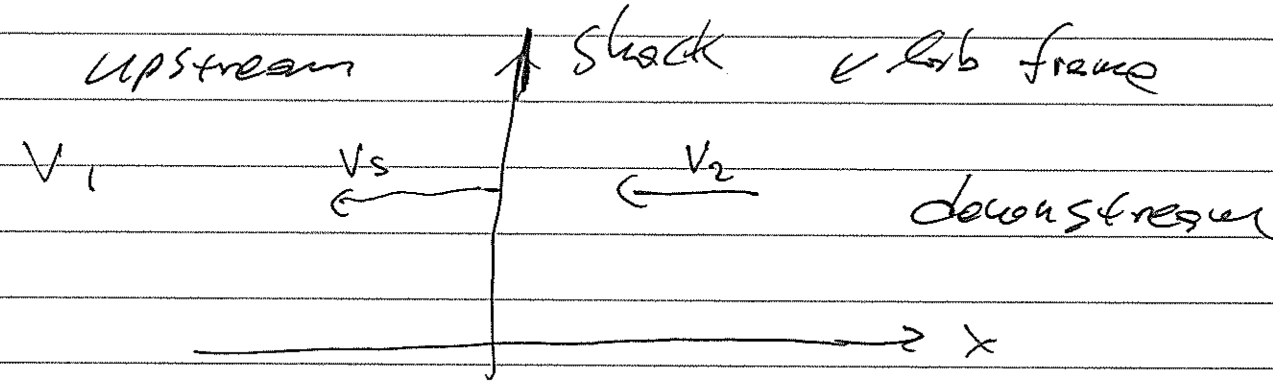


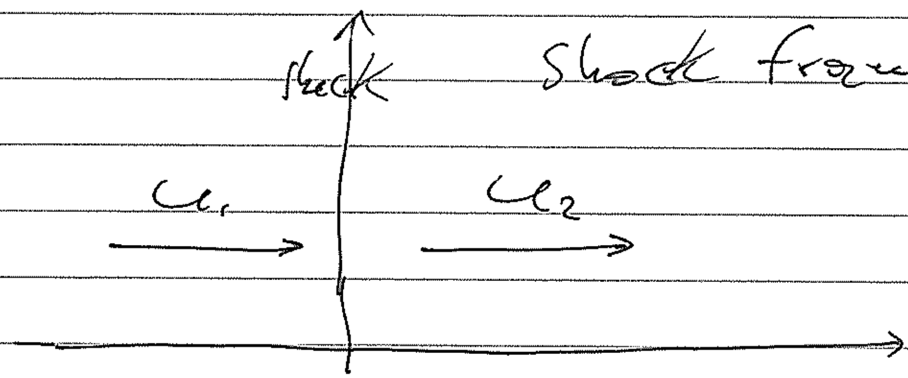

t = 3Δt
Shock
multivalued function

Lec 21

Let's consider a steady shock (time independent)

Let's transform quantities to the shock frame





upstream enters shock at $u_1 = V_1 - V_s$

downstream leaves shock at $u_2 = V_2 - V_s$

Cons. laws

$$\frac{d(\rho u)}{dx} = 0$$

$$\frac{d(\rho u^2 + p)}{dx} = 0$$

$$\frac{d}{dx}\left[\rho u\left(h + \frac{1}{2}u^2\right)\right] =$$

$h \rightarrow \varepsilon + \frac{p}{\rho}$ specific endalpy

Internal energy