LEC 22

Evaporation time

We need the # stars w/ $v > v_{esc}$.
Assume a relaxed cluster → Maxwellian distribution.

The # of stars in velocity interval $(v, v+dv)$ per unit vol is

$$dn = n \frac{\eta^2}{\pi^{3/2}} \exp(-\eta^2 v^2) d^3v$$

where $\eta^{-2} = \frac{2}{3}\langle v^2 \rangle = \frac{2}{3}\left(\frac{2T}{M}\right)$

Fraction of stars q w/ $v > \langle v_{esc}^2 \rangle^{1/2}$ is

$$q = \frac{N(v > \langle v_{esc}^2 \rangle^{1/2})}{N} = \frac{4\eta^3}{\pi^{1/2}} \int_{\langle v_{esc}^2 \rangle^{1/2}}^{\infty} e^{-\eta^2 v^2} v^2 dv$$

But $\langle v_{esc}^2 \rangle = 4\langle v^2 \rangle = \frac{6}{\eta^2}$

Thus $q = \frac{4}{\pi^{1/2}} \int_{\sqrt{6}}^{\infty} dx\, x^2 e^{-x^2}$, $x^2 = \eta^2 v^2$

$$\boxed{q = 0.007}$$

Not all these stars will necessarily escape, depending on their encounters on their way out of the cluster. So, typically this is taken as an upper limit on the fraction

$$\left|\frac{\Delta N}{N}\right| \leq \left|-0.007 \frac{\Delta t}{t_{rel}}\right|.$$

this is because at $\Delta t \simeq t_{rel}$, $\frac{\Delta N}{N} = 0.007$ and

$$t_{evap.} = -\frac{N}{dN/dt} \geq \frac{t_{rel}}{0.007} = 1.35 \cdot 10^2 \, t_{rel}$$

The rate of loss of stars is

$$\left|\frac{dN}{dt}\right| \leq \left|-0.007 \frac{N}{t_{rel}}\right|$$

Escape of stars w/ different masses:

There are a few competing effects when we consider the escape of low mass vs high mass stars: low m stars acquire larger velocities but $t_{rel} \sim m^{-1/2}$ is longer for these stars.

- As a result stars w/ $m \simeq 0.4 \langle m \rangle$ have the maximum rate of escape.
- Cluster retains virtually all stars with

- The least massive stars probably are retained as they are away from equilibrium
- A larger proportion of stars w/ $m \approx 0.4 M_\odot$ are lost.

Solar System Dynamics

The solar system: Sun: $M = 2 \cdot 10^{33}$ g
Jupiter: $M = 2 \cdot 10^{30}$ g, 5.2 A.U.
Saturn: $M = 5 \cdot 10^{29}$ g, 9.6 A.U.
Neptune: $M = 10^{29}$ g, 30 A.U.

- Asteroid belt between Mars & Jupiter
- The Kuiper belt is a disk-shaped region past the orbit of Neptune, 30-50 AU
  - many small icy bodies - source of short period comets. Orbits of Kuiper belt objects susceptible to ~~disruption~~ perturbations by the giant planets, and can have close encounters with Neptune.
- Oort cloud: 1950 Jan Oort noticed that comet orbits originate from ~50,000 AU w/o preferential direction ⇒ indirect evidence for a cloud of 10^{12} small objects

The asteroid belt is an interesting location in which dynamical chaos can be seen at work

① It is thought that a planet never formed between Mars & Jupiter or was destroyed shortly after formation due to grav. effects of J

② Kirkwood gaps: In 1867 Kirkwood observed gaps in the spatial distribution of asteroids (see histogram)

The gaps occur at resonances between the orbits of the asteroid and J. E.g. the 3:1 Kirkwood gap is located, where the ratio of the asteroid's orbital period to P_J is 1/3 — 3 orbits of the asteroid for 1 orbit of J.

Kepler's law $P^2 = a^3$, so

$$\left(\frac{P_J}{P_A}\right)^2 = \left(\frac{a_J}{a_A}\right)^3 \Rightarrow \quad 9 = \left(\frac{a_J}{a_A}\right)^3 \Rightarrow \quad a_A = 2.4 \text{ AU} \sim 2.5 \text{ AU}$$

→ The location of the first gap

Asteroid - Sun - Jup can be modeled as a restricted 3-body problem →

→ m_3 (m_A) is negligible compared to m_1 (m_S) and m_2 (m_J). Do not take into account gravitational effects of m_3.

Non-linear systems (which gravitational N-body systems are examples of) can exhibit chaotic behavior. What is chaos?

- two trajectories infinitesimally close at one time, diverge exponentially at a later time
- bifurcations can emerge - solution of a deterministic system jumps between two or more points in phase space
- dense periodic orbits in phase space

Consider a harmonic oscillator w/ forcing:

a) Driven undamped oscillator

$$\ddot{x} + \omega^2 x = \varepsilon \cos \Omega t$$

solution $x(t) = a \sin(\omega t + \delta) + \underbrace{\frac{\varepsilon}{\omega^2 - \Omega^2}}_{\text{resonance}} \cos \Omega t$

b) Damped driven osc.

$$\ddot{x} + \lambda\dot{x} + \omega^2 x = \varepsilon \cos \Omega t$$

$$x(t) = a e^{-\frac{\lambda}{2}t} \sin(\Omega t + \delta') + \frac{\varepsilon \cos(\Omega t + \delta')}{\sqrt{(\omega^2 - \Omega^2) - \lambda^2 \omega^2 \Omega^2}}$$

transient solution

↑ resonance offset by damping

c) Non-linear osc (with or w/o damping)

$$\ddot{x} + \lambda\dot{x} + \omega^2 x + \beta x^3 = \varepsilon \cos(\Omega t)$$

1) Small ε ordered motion (Limit cycle) in phase space

2) Large ε, bifurcation: 2 period cycle

3) Larger ε, 4 period cycle

Motion with n×Period is nonlinear & chaotic

3-body restricted problem in Kirkwood gaps is very similar to the forced NL osc

$V(r) = \frac{1}{r}$, or $\frac{1}{r^2}$ + Ext. nonlinear periodic forcing by Jupiter

→ "Chaotic behaviour" – bifurcation occurs leading to eccentricity jumps

Changes in eccentricity put asteroids in orbits going close to orbits of Earth & Mars → get kicked out with close encounters. $e > 0.3$ are Mars crossers.

In resonance areas orbits become unstable to increased eccentricity orbits → transition to nearby stable orbits or are kicked out by close encounters with planets.