# ASTR 589 – Physics of Astrophysics
# Assignment II, on Radiative Transfer and Processes
# Due Date: Thursday Oct 5th

1. Consider a 1D atmosphere that is heated from below such that the temperature decreases from the bottom toward the surface. Assume that all extinction coefficients are angle-independent (i.e., no magnetic fields, anisotropic scattering etc). Using the formal solution, find out whether limb darkening, limb brightening, or both can occur in this atmosphere.
   Hints:
   *(i)* Express limb darkening/brightening as a condition on $dI_\nu/d\mu$.
   *(ii)* An intermediate step you reach will probably be a condition on some derivative of the source function.
   *(iii)* After that intermediate step, consider the case with no scattering. The general case with absorption plus scattering is more difficult but you may want to tackle it as a bonus.

2. According to the standard model of the sun, the central density is 153 g cm$^{-3}$ and the Rosseland mean opacity at the center is 2.17 cm$^2$ g$^{-1}$.
   (a) Calculate the mean-free-path of a photon at the center of the sun.
   (b) If this mean-free-path remained constant for the photon's journey to the surface, calculate the average time it would take for the photon to escape the sun.

3. **Strömgren spheres.** Consider a pure hydrogen nebula surrounding a hot star of radius $R$. At some distance $r$ from the star, the ionization equilibrium equation

$$n_{H^0} \int_{\nu 0}^{\infty} \frac{4\pi J_\nu}{h\nu} \sigma_\nu d\nu = n_p n_e \alpha(T) \tag{1}$$

becomes

$$\frac{n_{H^0} R^2}{r^2} \int_{\nu 0}^{\infty} \frac{\pi F_\nu(R)}{h\nu} \sigma_\nu e^{-\tau_\nu} d\nu = n_p n_e \alpha(T), \tag{2}$$

where the optical depth is

$$\tau_\nu(r) = \int_0^r n_{H^0}(r') \sigma_\nu dr', \tag{3}$$

and the ionization cross section is approximately given by

$$\sigma_\nu = 6.3 \times 10^{-18} \left(\frac{\nu_0}{\nu}\right)^3 \text{ cm}^2, \tag{4}$$

where $\nu_0$ corresponds to the ionization threshold for hydrogen at 912 Å. We will assume that the star emits like a blackbody. Define the ionization fraction $x$ such that $n_{H^+} = x n_H$, $n_{H0} = (1-x) n_H$.

*(a)* Integrate numerically equation (2) to plot the ionization fraction as a function of the distance from the star in parsecs. Repeat the exercise for these two cases:

i) $T_{\text{eff}} = 45,000$ K, $R/R_{\odot} = 11$

ii) $T_{\text{eff}} = 40,000$ K, $R/R_{\odot} = 20$

Assume that $T_e = 10,000$ K in the nebular gas so that $\alpha = 2.59 \times 10^{-13}$ cm$^3$s$^{-1}$, and $n_H = 10$ cm$^{-3}$ throughout the nebula. You may assume that $a_{\nu} = 6.3 \times 10^{-18}$ cm$^2$ in the calculation of the optical depth.

*(b)* Calculate the number of ionizing photons for the two black bodies using

$$Q(H^0) = \int_{\nu 0}^{\infty} \frac{L_{\nu}}{h\nu} d\nu. \tag{5}$$

*(c)* Calculate the Strömgren radius of both stars using $Q(H^0)$ from part (b).