


# Timing the LMC-SMC Interaction Using Phase Space Projections

  
*ASTR513: Statistics & Computation, Term Project Proposal*  
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## Proposal Summary

The availability of high quality Gaia astrometric observations of the stars in the Magellanic clouds makes it possible to observationally validate predictions of N-body simulations of the LMC-SMC system. However, this can only be done if a robust statistical framework is developed that can connect such simulations to observations. In this term project, we propose to develop a proof-of-concept statistical model to address this need. We plan to validate our model using existing simulation data of the LMC-SMC system. We shall use the phase space structure of the simulated LMC stars to identify signatures of a recent interaction with the SMC. We propose a Bayesian framework that can be used to time the most recent LMC-SMC interaction using a simple toy model. We plan to perform the Bayesian analysis with the help of MCMC sampling, and finally obtain the time elapsed since the last LMC-SMC interaction. We shall compare this time to what is predicted in the N-body simulations as a test of our approach.

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## 1 Introduction

We are currently in the era of high precision astrometry of the Milky Way (MW) and its satellites. The Gaia mission has provided us with unprecedented position and velocity information of stars in these galaxies, and more is to be expected in the upcoming data releases. Also, high resolution N-body codes have enabled us to come up with ever powerful simulations of dynamical interactions between these galaxies. However, we currently lack a statistical framework that can connect the kinematic observations with the dynamical simulations. With this motivation, we propose to build such a statistical framework.

The interaction between the Large and Small Magellanic Clouds (LMC & SMC) has been of particular interest over the last two decades, given the myriad of observations about these galaxies that still need to be explained (e.g. [2]). This system represents the closest pair of interacting galaxies to the Milky Way, , thus it can also be used as an excellent laboratory to test theories of galaxy mergers, stellar dynamics, dark matter, star-formation, and ultimately our cosmological models.

In recent times, high precision astrometric and photometric information have been obtained for the LMC and SMC (eg. [3, 5, 6]). Thus, it has become increasingly important to build a statistical model that can be used to verify the predictions of the LMC-SMC N-body simulations using observations. Such a statistical framework would not only help us to explain the diverse observations of this system, but will also enable us to use this framework as a laboratory to test our theories.

In this project, we shall particularly focus on a key prediction of the simulation of [4] (hereafter B12), which is that the LMC and SMC must have had a close encounter  $\sim 100$  Myr ago. In order to verify this prediction using observations, we first need to identify astrometric observables that encode the past history of the LMC-SMC interactions.

Recently, [1] (hereafter A18) unravelled various structures in the phase space projections of the MW stars using the unprecedented Gaia observations. Some of these projections are shown in Fig. 1.

A18 demonstrate using simple toy models that such structures are caused by the mixing of different frequencies in the dynamics of the phase space space of the MW stars. They argue that such mixing is likely triggered by the interactions of the MW with its satellites. [7] (hereafter L19) showed using N-body

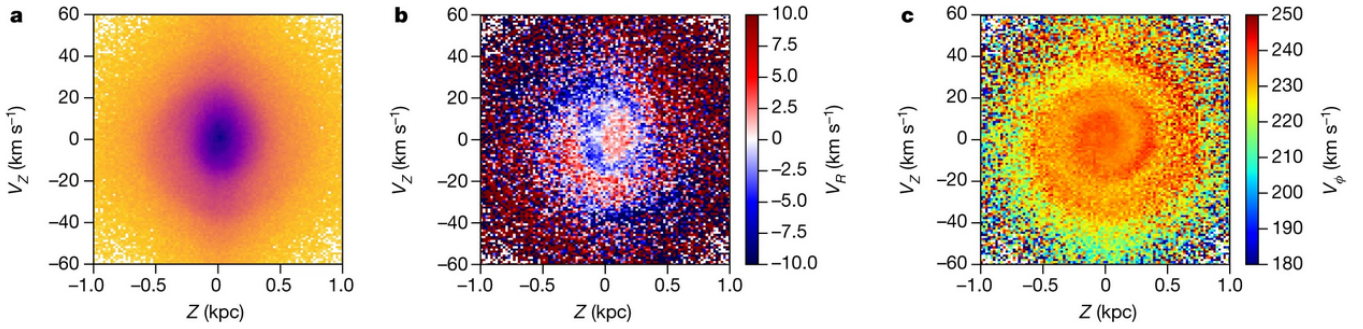


Figure 1: Various phase space projections of the Gaia data for MW stars, in Galactic cylindrical coordinates. (a) shows the 2D histogram of  $Z$  and  $V_Z$ , (b) shows  $V_R(Z, V_Z)$ , and (c) shows  $V_\phi(Z, V_Z)$ . Various interesting structures are observed in these phase space projections. Source: Fig. 1 of A18

simulations that these phase space structures can be accurately explained by an interaction of the MW with the Sagittarius dwarf galaxy (SDG)  $\sim 800$  Myr ago.

**In the near future it will be possible to construct phase space projections of the LMC with Gaia observations. In this term project, we aim to develop a proof-of-concept statistical approach using existing simulation data of B12 to show that the LMC-SMC interaction can be timed with the help of such phase space structures. Such a proof-of-concept solution will be critical to interpret actual observational data in the future.**

## 2 Phase Space Projections of the Simulated LMC

N-body Smoothed Particle Hydrodynamic simulations of B12 represent a significant leap in our understanding of the LMC-SMC system. They used the code GADGET3 [9] initialized with  $10^6$  and  $10^5$  star particles for the LMC and SMC respectively. Each star particle had a mass of  $2500M_\odot$ . B12 were able to successfully explain some of the key observables of the LMC-SMC system, like the trailing component of the Magellanic stream.

Using the simulation data of B12, we construct the phase space projections of LMC for two snapshots: (i) the final snapshot where the LMC and SMC have already interacted, (ii) the initial snapshot where the LMC disk is isolated and is in equilibrium. These are shown in Fig. 2. We notice that the morphology of the structures seen in the LMC phase space projections (Fig. 2, final snapshot) is different from that seen in the MW (Fig. 1).

The morphology of the phase space structures is precisely what encodes the past history of galactic interactions. L19 demonstrate how this morphology changes as a function of time elapsed since the last close encounter between two galaxies. Thus, visually comparing the observed morphology of the MW phase space to the morphology of the simulations at various times allowed them to constrain when in the past the SDG closely interacted with the MW.

We will adopt a similar method for the LMC. However, we shall treat the phase space morphology of the LMC disk stars in a statistical way, as opposed to a visual examination. Performing a full-fledged N-body simulation (like that of L19) of the LMC-SMC interaction is beyond the scope of this term project. Instead, we shall construct a toy model that will be applied to already existing simulation data of B12.

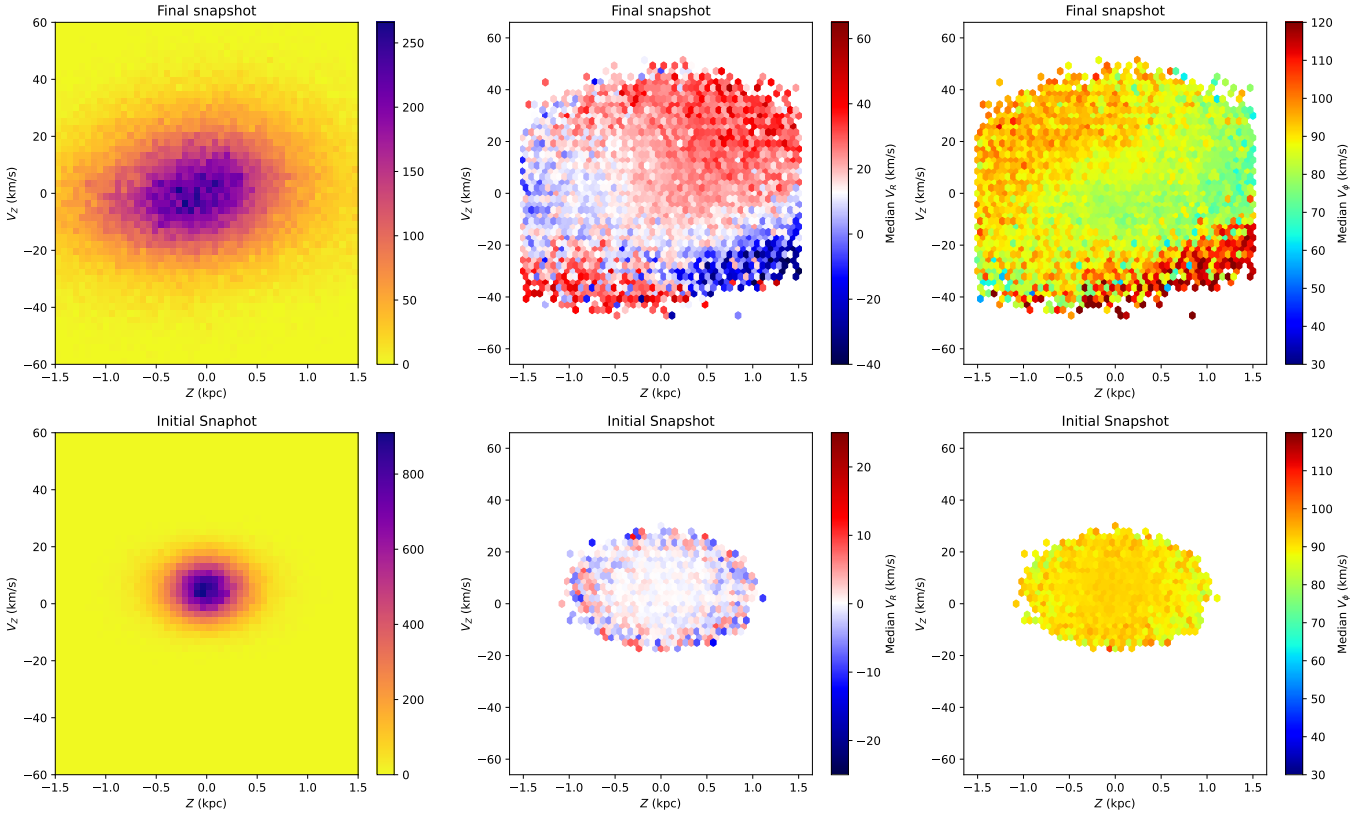


Figure 2: Phase space structures (refer Fig. 1) for star particles in the simulated LMC. Top row is for the final snapshot, where the LMC and SMC have already interacted. Bottom row is for the equilibrium LMC disk, at the start of the simulation. *We propose to test whether the phase space morphology seen in these figures can be used to time the interaction between the LMC and SMC.*

### 3 Motivating a Toy Model

The idea is to construct a toy model that can be used to evolve the phase space of an isolated LMC disk such that we can obtain the required phase space structures. For this purpose, we use the An-harmonic Oscillator Model (AHM) described in A18. This model can only be used for the first phase space projection - the 2D histogram of  $Z$  and  $V_Z$  (Fig. 2, first panel). Hence, from this point onward, we shall primarily focus on this particular projection.

When stars of a galaxy disk are perturbed in the  $Z$  direction, the potential of the disk acts like a restoring force. The resulting motion can be described by the AHM potential of the form (eq. 1):

$$\Phi(Z) \propto -\alpha_0 + \frac{1}{2}\alpha_1 Z^2 - \frac{1}{4}\alpha_2 Z^4, \quad (1)$$

where  $\alpha_0$ ,  $\alpha_1$  and  $\alpha_2$  are the AHM coefficients. These coefficients can be obtained by taking a Taylor series expansion of the disc potential profile along the  $Z$  direction. In the simulations of B12, the disk density distribution is initially modelled as an exponential profile. The potential for an exponential disc is not analytically defined and needs to be approximated. We plan to obtain the disc potential profile by fitting the density distribution with a potential model of a form similar to the Miyamoto - Nagai disk [8]. This would yield both the values and the errors on the AHM coefficients, which we will treat as a prior in the Bayesian analysis we present in the next section.

Under the AHM potential, oscillation frequencies take the form (eq. 2):

$$\nu(A, R) = \sqrt{\alpha_1(R)} \left[ 1 - \frac{3\alpha_2(R)A^2}{8\alpha_1(R)} \right], \quad (2)$$

where  $A$  is the amplitude of oscillation and  $R$  is the radial coordinate of the star in Cylindrical coordinates. Note that an-harmonic oscillations have amplitude dependent frequencies, as opposed to the case in simple harmonic oscillations.

The dynamical equations under the AHM potential for each star are (eq. 3, 4):

$$Z = A \cos [\nu(A, R)t + \phi_0] \quad (3)$$

$$V_Z = -A\nu(A, R) \cos [\nu(A, R)t + \phi_0], \quad (4)$$

where  $\phi_0$  is the initial phase, which we shall obtain from the initial distribution of stars in the LMC disk. We shall determine the amplitude  $A$  using conservation of energy applied to the initial phase.

We assume the onset of these perturbations ( $t = 0$ ) coincides with the last close passage between the LMC and the SMC. We also assume that just before  $t = 0$ , the state of the LMC disk was the same as the initial snapshot of the simulation. Our aim is to evolve the initial snapshot under eqs. (3, 4) and find out at which time  $t = \tau$  the first phase space projection of the LMC, i.e. the  $(Z, V_Z)$  2D histogram, matches with the final snapshot of the simulation.

## 4 Fitting Methodology & Expected Results

In Fig. 3, we show the  $(Z, V_Z)$  2D histogram of the LMC in overdensity. Following L19, we define overdensity as:

$$\delta\rho = \frac{\rho}{\bar{\rho}} - 1, \quad (5)$$

where  $\rho$  depicts the actual counts in each 2-D  $(Z, V_Z)$  bin, and  $\bar{\rho}$  denotes the smoothed counts as obtained by convolving the actual counts with a Gaussian of  $\sigma = 4$  pixels. For removing noise, we further convolve  $\delta\rho$  with a Gaussian of  $\sigma = 1$  pixel.

An open spiral shape is clearly visible in the final snapshot of the simulation. We will test whether this structure is likely the result of the last close encounter between the LMC and the SMC.

We will treat the 2D histogram of  $(Z, V_Z)$  of the final simulation snapshot as an image data vector  $\vec{D}$ . We plan to obtain error on  $\vec{D}$  from the Poisson noise of the 2D histogram. We will then evolve the initial simulation snapshot ( $I$ ) as a function of time  $t$ , under the AHM toy model, and construct the resulting 2D histogram of  $(Z, V_Z)$ . We will treat the latter as an image model vector  $M(\vec{I}, t)$ . We might apply image processing steps on both  $\vec{D}$  and  $M(\vec{I}, t)$ , in order to enhance the strength of the open spiral feature.

The likelihood function that we plan to construct would compare  $\vec{D}$  and  $M(\vec{I}, t)$ . We shall supply suitable priors to  $t$  and the AHM coefficients and obtain the posterior of  $t$  using Markov-Chain-Monte-Carlo sampling. We expect that the posterior of  $t$  should be peaked at  $t = \tau$ , which would correspond to the time since the last close encounter between the Magellanic clouds. This would be the main result of our statistical analysis.

Finally, we will compare the value of  $\tau$  obtained from our fits of the phase space with the value in the simulations of B12. A consistency between the two would be a solid justification of the correctness of our simple statistical approach for treating phase space structures. This would enable us to develop this approach further, and use it to compare actual Gaia observations with predictions of N-body simulations.

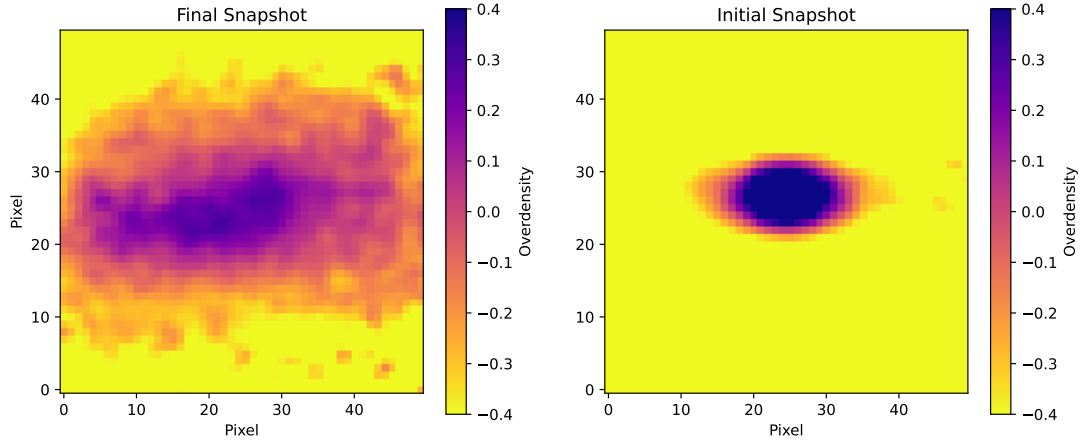


Figure 3: The 2D histogram of  $(Z, V_Z)$  in overdensity (eq. 5). Left panel denotes the final snapshot of the simulation, and right panel denotes the initial snapshot. An open spiral shape is clearly visible in the final snapshot. *We will test whether this structure is likely the result of the last close encounter between the LMC and the SMC.*

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