Lec 4

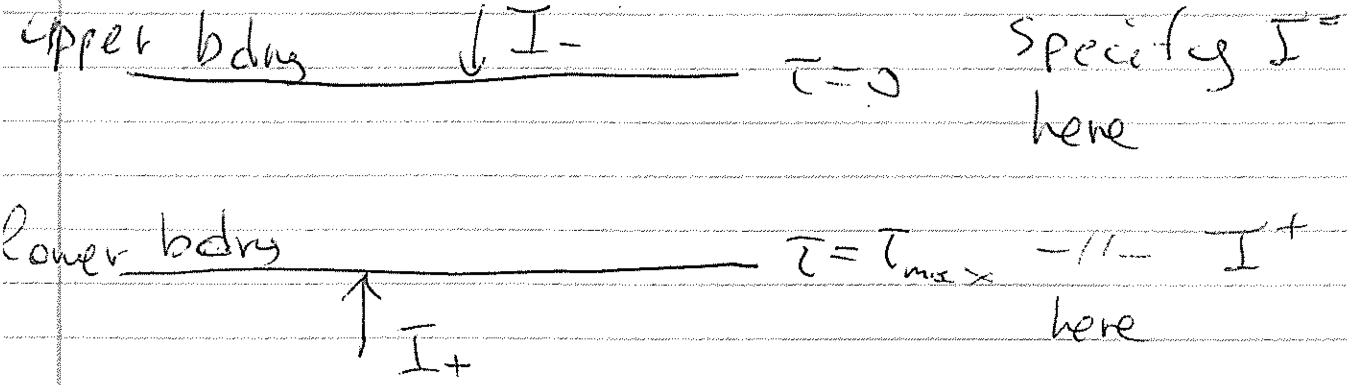
Solving the RTE

Like all PDEs / ODEs, RTE needs boundary conditions.

How many BCs?

1D planar geometry is $\mu \times \nu$ Eqs. which are ODEs. So we need $\mu \times \nu$ BCs

- For a finite slab of specified τ_{max}, we can either specify I at τ_{max}, or $\tau = 0$

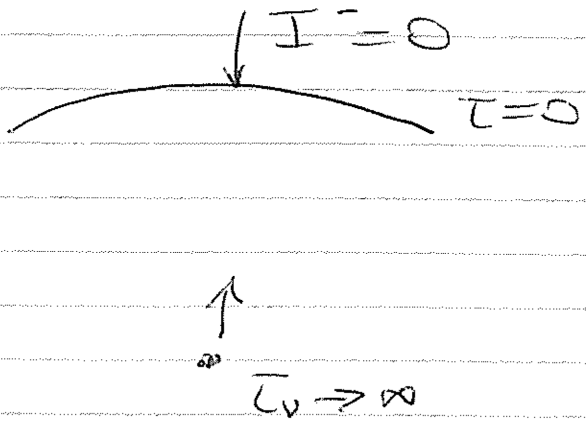


$$I(\tau_\nu = 0, \mu, \nu) = I^-(\mu, \nu) \quad -1 \leq \mu \leq 0$$

$$I(\tau_\nu = \tau_{max}, \mu, \nu) = I^+(\mu, \nu) \quad 0 \leq \mu \leq 1$$

These determine unique sol.

- For semi-infinite case
(e.g. star w/ nearly ∞ optical depth)



$$\lim_{\tau \to \infty} I_\nu \cdot e^{-\frac{\tau_\nu}{\mu}} = 0 \quad \forall \mu.$$

But, we've seen that isotropic I has $F=0$, so we wouldn't solve the RTE for $I(\mu)$, but for its moments.

Simple examples and the Formal solution of the RTE

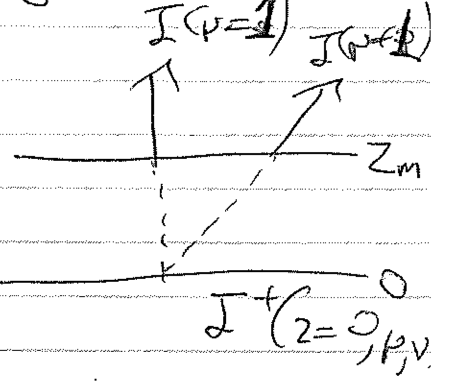
1) No material is present

$$\chi_\nu = \eta_\nu = 0$$

then $$\frac{\partial I_\nu}{\partial z} = 0 \Rightarrow I_\nu = \text{const.}$$

Specific Intensity is constant!

2) Material emits at freq. ν but does not absorb or scatter (decay of metastable levels in low density gas for example)



$$\mu \frac{\partial I_\nu}{\partial z} = \eta_\nu$$

$$\cancel{I_\nu(\mu) = \mu^{-1}\int_0^{z_m} \eta(z,\nu)\,dz + I}$$

$$I_\nu(\mu, z) = I^+(z=0,\mu,\nu) + \mu^{-1}\int_0^{z} \eta(z',\nu)\,dz'$$

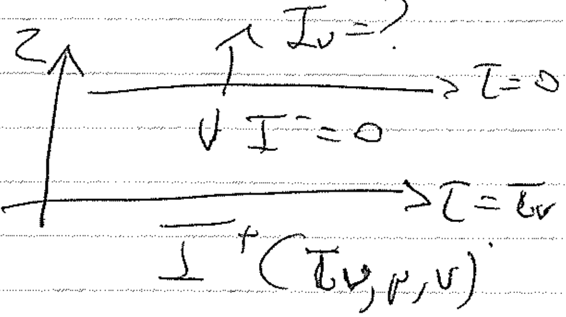
geometrical path-length factor

3) Radiation is absorbed but not emitted (eg. filter at a particular frequency) or scattered

$$\mu \frac{\partial I_\nu}{\partial z} = -\chi_\nu I_\nu = -\alpha_\nu I_\nu$$

$$d\tau_\nu = -\alpha_\nu dz$$

$$I(\tau_\nu = 0, \mu, \nu) = I_\nu^+(\tau_\nu, \mu) e^{-\frac{\tau_\nu}{\mu}}$$



exponentially attenuated.

This ties back to the def. of optical depth as the # of m.f.p. → i.e. survival prob. of a photon goes down as $e^{-\tau}$.

4/ Formal solution

$$\mu \frac{\partial I_\nu}{\partial \tau_\nu} = I_\nu - S_\nu \quad \Rightarrow$$

$$\frac{\partial I_\nu}{\partial \tau_\nu} - \frac{1}{\mu} I_\nu = -\frac{1}{\mu} S_\nu \quad (1)$$

Note $\frac{\partial}{\partial \tau_\nu}\left(I_\nu e^{-\tau_\nu/\mu}\right) =$

$$= \frac{\partial I_\nu}{\partial \tau_\nu} e^{-\frac{\tau_\nu}{\mu}} + I_\nu e^{-\tau_\nu/\mu}\left(-\frac{1}{\mu}\right) =$$

$$= e^{-\tau_\nu/\mu}\left(\frac{\partial I_\nu}{\partial \tau_\nu} - \frac{1}{\mu} I_\nu\right) \quad (2)$$

$(1)+(2) \Rightarrow \quad \frac{\partial}{\partial \tau_\nu}\left(I_\nu e^{-\tau_\nu/\mu}\right) = -\frac{1}{\mu} e^{-\frac{\tau_\nu}{\mu}} S_\nu$

$\Downarrow$

$$I_\nu(\tau,\mu) = I_\nu(\tau_0,\mu)\, e^{-(\tau_0-\tau)/\mu} +$$
$$+ \frac{1}{\mu}\int_{\tau_0}^{\tau} S_\nu(\tau')\, e^{-(\tau'-\tau_0)/\mu}\, d\tau'$$

If S_ν is known, we have a complete sol. of the rad. transfer equ.

So may be coupling different
μ's and ν's, e.g. in scattering
Then the sol.'n of these coupled
eqn's is still quite
difficult (actually they
are coupled integro-differential
eqn's because an integral over
I appears ~~[illegible]~~ on the R.H.S
~~[illegible]~~ in S_ν).