Mihalas Stellar Atmospheres — Office hours 10-12,

Chandrasekhar "Radiative Transfer"

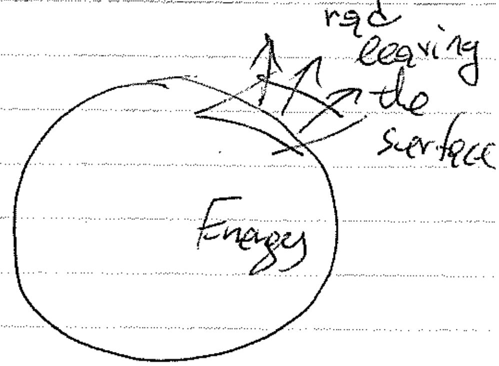
Basic Properties of Radiation

Energy (scalar)

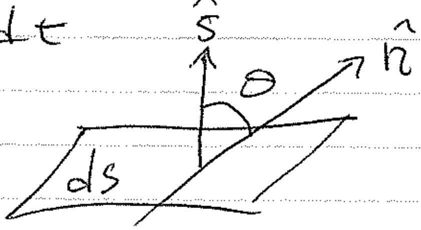
Luminosity $= \dfrac{\text{Energy}}{\text{time}}$, $L = \dfrac{dE}{dt}$ (scalar)

energy flux $= \dfrac{\text{Energy}}{\text{time} \cdot \text{area}}$

(Vector: across a surface)



Amount of E passing through surface element $d\vec{s}$ in direction $\hat{n}$ in time dt



$\hat{n}$ = direction of propagation

$\hat{n}$ and $\hat{s}$ don't have to be aligned.

The projected area is $\hat{n} \cdot \hat{s} \cdot ds = ds \cdot \cos\theta$

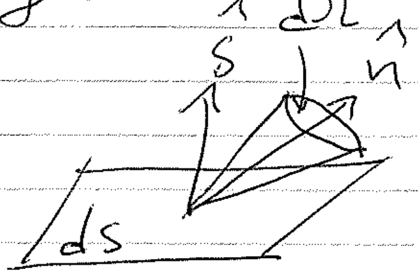
Thus, $\vec{F} = \dfrac{dE}{dt\, ds \cos\theta}$ or

$$dE = \vec{F} \cdot d\vec{s} \cdot dt = |F| \cdot ds \cdot \cos\theta$$

Intensity: energy per unit time per unit area, radiated into solid angle $d\Omega$ along $\hat{n}$.

$$I = \frac{\text{energy}}{\text{time} \cdot \text{area} \cdot \text{solid angle}}$$

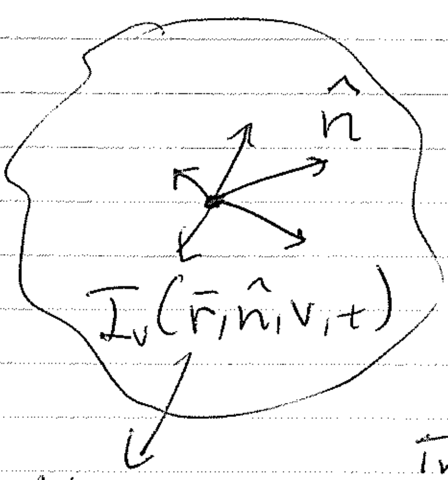
$$= \frac{dE}{dt\, ds \cos\theta\, d\Omega}$$



Specific Intensity: Intensity per unit frequency (ν)
or Brightness

$$I_\nu = \frac{dE}{dt\, ds \cos\theta\, d\Omega\, d\nu}$$

Thus I_ν depends on $\vec{r}$, $\hat{n}$, ν and can be time dependent!



In general, unless you are at the boundary of an object with free space, there's Intensity going into <u>all</u> directions.

<u>But</u> not equal amounts into all directions necessary.

How many variables does I_ν depend on?

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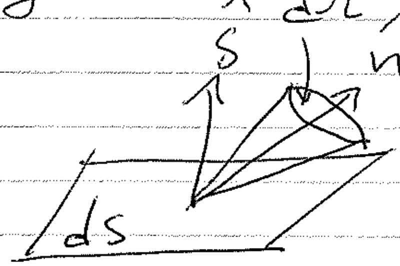

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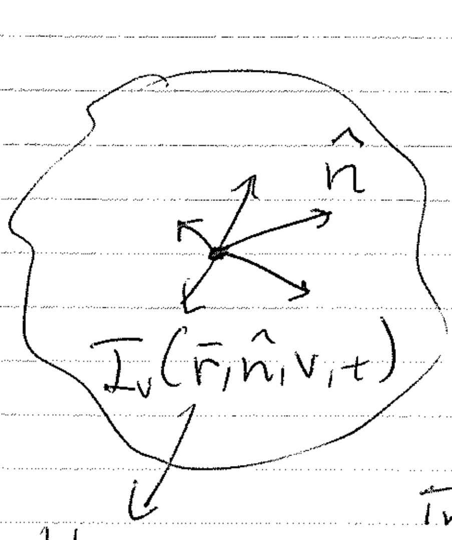

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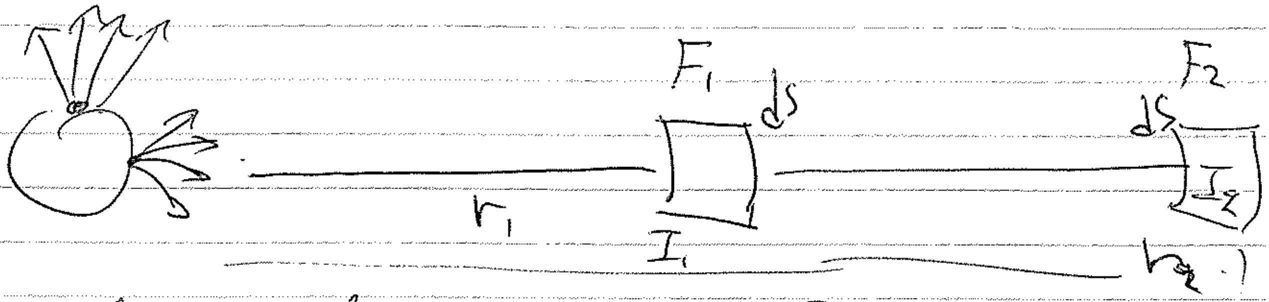
Question: An object has luminosity L and emits isotropically in a steady state fashion as shown below



What is the relation of F_1, F_2, I_1, I_2?

Flux is energy per unit area, but as ~~area~~ r increases area subtends smaller solid angle

Recall $d\Omega = \frac{dS}{r^2}$

Thus, flux is diluted by solid angle as $\frac{1}{r^2}$

Therefore $\frac{F_2}{F_1} = \left(\frac{r_1}{r_2}\right)^2$

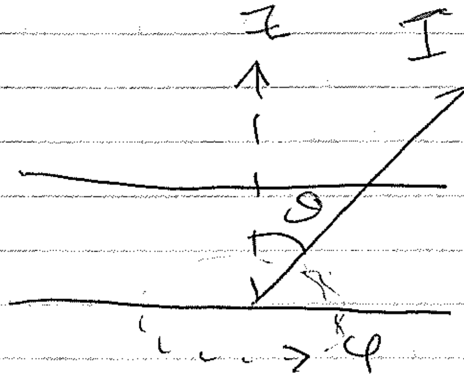
$r_2 = 2r_1 \Rightarrow F_2 = \frac{F_1}{4}$

But, intensity is defined per solid angle, so $I_1 = I_2$

Constancy of intensity along a ray travelling in free space.

Intensity in two common geometries

Planar/Slab



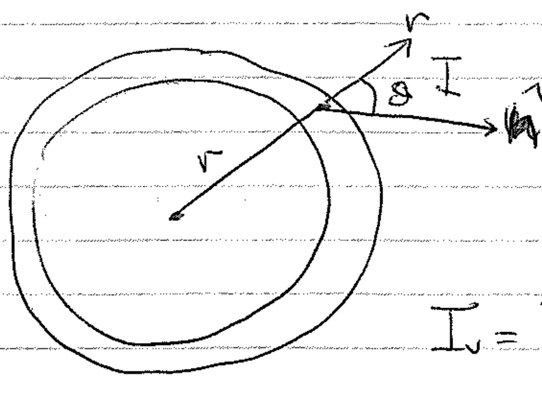
properties independent of x, y

Surface normal $\hat{s} = \hat{z}$, call it $\hat{k}$
no φ dependence in any variable

$$\hat{k} \cdot \hat{n} = \cos\theta = \mu$$

$$I_\nu = I_\nu(z, \theta, \nu, t) \quad \text{or} \quad I_\nu = I_\nu(z, \mu, \nu, t)$$

Spherical symmetry



$\bar{r} = (r, \Theta, \Phi)$
No Θ, Φ dependence

lower case θ angle between $\hat{n}, \hat{r}$

$I_\nu = I_\nu(r, \theta, \nu, t)$ or

$I_\nu = I_\nu(r, \mu, \nu, t)$

Moment of Radiation field

("directional averages")

- Specific intensity of rad. field is made up of photons. But $|v| = c$, so averages over velocity become averages over direction.

Zeroth moment: Mean Intensity

ge

$$\boxed{\bar{J}(r, \nu, t) \equiv \frac{1}{4\pi} \int I(\bar{r}, \hat{n}, \nu, t)\, d\Omega} =$$

$$= \frac{1}{4\pi} \int I \sin\theta \, d\theta \, d\phi =$$

$$= -\frac{1}{4\pi} \int I \, d\mu \, d\varphi$$

$\bar{J} \equiv$ overage specific intensity over all angles

In 1-D plane geometry, no ϕ-dependence

$$\bar{J} = \frac{1}{4\pi} \int_0^{2\pi} d\varphi \int_{-1}^{1} d\mu \, I(z, \mu, \nu, t) =$$

$$= \frac{1}{2} \int_{-1}^{1} I(z, \mu, \nu, t)\, d\mu$$

The total E is

$$E = \frac{1}{c} \int dV \int d\Omega \, d\nu \, I_\nu$$

If we are interested in the energy in specific ν, we drop integral w.r.t. $d\nu$

And for uniform I_ν (independent of $\vec{r}$)

$$E = \frac{1}{c} V \int d\Omega \, I = \frac{4\pi}{c} J \cdot V$$

So, the "monochromatic" energy density is

$$\boxed{\mathcal{E}_R(\bar{r}, \nu, t) = \frac{E}{V} = \frac{4\pi}{c} J(\bar{r}, \nu, t)}$$