Lec. 8

Strömgren Spheres

If $T_* \geq 20{,}000\,K \rightarrow \gamma$ w/ $\lambda \leq 912\,\mathring{A} \rightarrow$ ionization of H

(If $\lambda < 584\,\mathring{A}$ or $< 304\,\mathring{A}$, $\longrightarrow$ single or double ionization of He)

The ionized region formed is called an HII region.
The size of the ionized region is called a Strömgren radius

Let a pure H cloud w/ uniform density n. When it's partially ionized it will contain HI (neutral H) and p (protons - ionized H)

$n = n_H + n_p = n_H + n_e$ (b/c H produces 1e when ionized)

We define ionization fraction x

$$x = \frac{n_p}{n} = \frac{n_e}{n} = \frac{n_p}{n_H + n_p} \quad \text{and} \quad 1 - x = \frac{n_H}{n}$$

The ionizing radiation has

$$n_\nu = \frac{\#\ \text{photons}}{\text{vol}}$$

From bound-free trans. (Last lecture)

$$\sigma_\nu = \sigma_0 \left(\frac{\nu_0}{\nu}\right)^3, \quad \text{For } H: \quad \sigma_0 = 6.3 \cdot 10^{-18}\ \text{cm}^2$$

$$\nu_0 = \frac{13.598\ \text{eV}}{h} = 3.29 \cdot 10^{15}\ \text{Hz}$$

Focusing on ionizations from ground state

$$\frac{\#\ \text{ionizations}}{\text{sec} \cdot \text{vol}} = \int_{\nu_0}^{\infty} n_H\, \sigma_\nu \cdot c \cdot n_\nu \, d\nu$$

($n \sigma v$ is number of interactions per unit time for 1 ~~particle~~ photon) × (number density of photons)

But n_ν falls with the distance from the star r.

a) flux $\downarrow$ $\frac{1}{r^2}$

b) γ are absorbed by neutral atoms

↓ time ergs^{-1}

Thus

$$n_\nu(r) = \frac{L_\nu}{4\pi r^2 \, c h\nu} e^{-\tau_\nu(r)}$$

$$d\tau_\nu(r) = \sigma_\nu \, n_H(r) \cdot dr$$

$$\tau_\nu(r) = \sigma_\nu \int_0^r n_H(r)\,dr = \left(\frac{\nu_0}{\nu}\right)^3 \underbrace{\int_0^r \sigma_0 \, n_H(r)\,dr}_{\text{call this } \tau_0(r)}$$

Typically the spectrum of a hot star falls off rapidly (exponentially) above ν_0. The cross section is also falling off ν^{-3}.

So, we approximate the majority of the effect by setting

$\sigma_\nu = \sigma_0$, and call the ionizing flux

$$S_{uv} = \int_{\nu_0}^{\infty} \frac{L_\nu}{h\nu}\,d\nu$$

→ think of it from the 2nd mean value theorem

More specifically:

$$\int_a^b G(x) f(x)\, dx = G(a^+) \int_a^{\cancel{y}} f(x)\, dx$$

with $y \in [a,b]$

$$G(a^+) = \lim_{x \to a^+} G(x)$$, with G a monotonically decreasing function

So,

$$\frac{\#\text{ ionizations}}{\text{time} \cdot \text{vol}} = \int_{\nu_0}^{\infty} c\, n_H\, \sigma_\nu\, n_\nu\, d\nu =$$

$$= \frac{c\, n_H}{4\pi r^2 c} \int_{\nu_0}^{\infty} \sigma_\nu e^{-\tau_\nu(r)} \frac{L_\nu}{h\nu}\, d\nu =$$

$$= \frac{\cancel{c}\, n_H}{4\pi r^2 \cancel{c}}\, \sigma_0 e^{-\tau_0(r)} \int_{\nu_0}^{y} \frac{L_\nu}{h\nu}\, d\nu$$

but since L_ν drops exponentially with $\nu > \nu_0$ we can take $y \to \infty$

So,

$$\frac{\#\text{ ionizations}}{\text{time} \cdot \text{vol.}} = n_H \underbrace{\frac{\sigma_0 S_{uv} e^{-\tau_0(r)}}{4\pi r^2}}_{= \zeta} = \zeta\, n_H$$

In equilibrium, this must equal the rate of recomb./vol $= \xi n_e n_p$

$\xi = \langle U \sigma_{fb} \rangle$ = recombination coef.

$$= \left[\frac{cm^3}{s} \right]$$

Note: recombinations can take place to any level, so

$$\xi = \sum_{n=1}^{\infty} \xi_n$$

but only recombinations to ground level produce to the ground level 100% produce ionizing γ. This would produce a diffuse flux of γ, which would not have the r^{-2} dependence, we took in our treatment.

To get around this problem, assume that any ionizing γ produced by recombs. would be re-absorbed locally so should be excluded from our global ionization balance then recomb. coef.

$$\xi^{(2)} = \sum_{n=2}^{\infty} \xi_n \quad \text{and}$$

$$J n_H = \xi^{(2)} n_e n_p = \sigma_0 n_H \frac{S_{uv} e^{-\tau_0(r)}}{4\pi r^2}$$

Integrate from $r=0$ to $r=\infty$ remembering that $d\tau = \sigma n_H \cdot dr$

$$\int_0^{\infty} \underline{\sigma_0 n_H} S_{uv} e^{-\tau_0(r)} \underline{dr} = \int_0^{\infty} \xi^{(2)} n_e n_p 4\pi r^2 dr$$

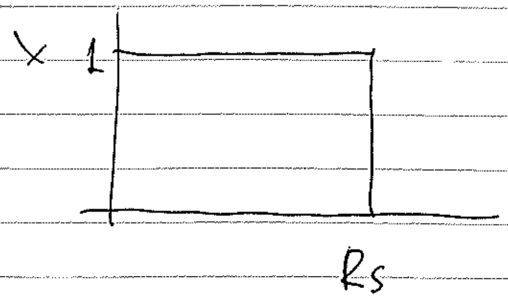
$$\int_0^{\infty} S_{uv} e^{-\tau_0(r)} d\tau = \int_0^{\infty} \xi^{(2)} n_e n_p 4\pi r^2 dr$$

Since $\tau = \int (x) \cdot n \sigma_0 dr$, it's difficult to solve for x exactly.

Let's assume that the transition from ionized to neutral happens like a step function (turns out to be a decent approx.)

$x \simeq 1$ or $n_p = n_e = n$ $\quad 0 < r < R_s$

$x = 0 \quad , r > R_s$



Then $\int_0^{\tau_s} e^{-\tau} d\tau = 1 - e^{-\tau_s}$

so

$$S_{uv} = \frac{4}{3}\pi R_s^3 \xi^{(2)} n^2 \quad \text{or}$$

$$R_s = \left(\frac{3 S_{uv}}{4\pi \xi^{(2)} n^2} \right)^{1/3}$$

Take $n = 5 \cdot 10^3 \ cm^{-3}$, $S_{uv} = 10^{48} \ s^{-1}$

$$\xi^{(2)} = 2.5 \cdot 10^{-13} \left(\frac{T}{10^4 K} \right)^{1/2} cm^3 \cdot s^{-1}$$

$$R_s \simeq 1 pc$$

* When working on your HW, you may want to write the eq. in terms of x and n, and take x to be constant when calculating τ, so that it's not an integral

$$\frac{1-x}{x^2} = \frac{4\pi r^2 \xi^{(2)} n}{\sigma_0 e^{-\tau} S_{uv}}$$

Thomson Scattering

Simplest form of interaction of photons w/ charged particles. The E of the photon does not change (elastic scat.)

Assume EM wave accelerates the e^-, which radiates in response

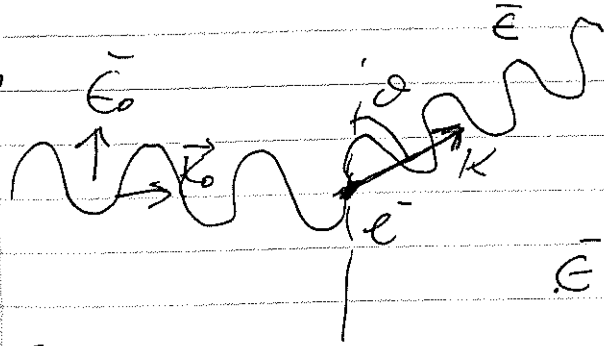
Power radiated into polarization state $\bar{\epsilon}$ is given by

$$\frac{dP}{d\Omega} = \frac{e^2}{4\pi c^3} |\bar{\epsilon} \cdot \bar{a}|^2, \quad \bar{a} \text{ is accel.}$$

← power per solid angle

nor

nela



$$\bar{E}(x,t) = \bar{\epsilon}_0 E_0 e^{i(\bar{k}_0 \cdot \bar{r} - \omega t)}$$

$$\bar{\epsilon} \cdot \bar{a} = \bar{\epsilon} \cdot \bar{\epsilon}_0 \cdot \frac{e}{m} E_0 e^{i(\bar{k} \cdot \bar{r} - \omega t)}$$

Thus
$$\left\langle \frac{dP}{d\Omega} \right\rangle = \underbrace{\frac{c}{8\pi} |E_0|^2}_{\text{time averaged Poynting vector}} \underbrace{\left(\frac{e^2}{mc^2}\right)^2 |\bar{\epsilon} \cdot \vec{\epsilon}_0|^2}_{\frac{d\sigma}{d\Omega}}$$

incident radiation

$$|\bar{\epsilon}\cdot\bar{\epsilon}_0|^2 = \frac{1}{2}(1+\cos^2\theta)$$

scattering is not isotropic

Thus $\frac{d\sigma}{d\Omega} = \left(\frac{e^2}{mc^2}\right)^2 \frac{1}{2}(1+\cos^2\theta)$

Thomson formula for scattering of rad. by free charge.

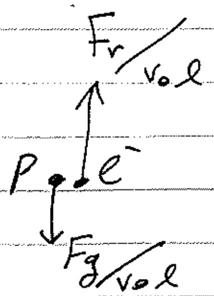
Total cross section $\sigma = \int \frac{d\sigma}{d\Omega} d\Omega$

$$\sigma_T = \frac{8\pi}{3}\left(\frac{e^2}{mc^2}\right)^2 \quad \text{frequency-ind.}$$

So; the opacity due to scatt.

$$\chi = \sigma_T \cdot n_e$$

Simple, and frequently used application of Thompson scat. is the Eddington Limit or Eddington Luminosity. used as the max. luminosity a star/BH can have to maintain balance between gravity and radiation pressure.



p feels gravity

e^- rad. force but

$n_e = n_p$ and no charge separation

So

$$\begin{cases} \dfrac{F_g}{\text{Vol}} = \dfrac{GM\rho}{r^2} = \dfrac{GMm_p n_p}{r^2} \\[2ex] \dfrac{F_r}{\text{vol}} = n_e \sigma_T \dfrac{F}{c} = n_e \sigma_T \dfrac{\frac{L}{4\pi r^2}}{c} \end{cases}$$

energy flux

$$\frac{\text{energy flux}}{c} = \text{momentum flux} = \frac{\text{force}}{\text{area}}$$

σ_T

fraction of force felt

equate and take $n_e = n_p$ w/ $L = L_{Edd}$

$$\boxed{L_{Edd} = \frac{4\pi G M m_p c}{\sigma_T}}$$

depends only on mass