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Physics for Astronomy (ASTR 589)

Homework 6

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Worked with Noah, Brian, Ningyuan, Junyu, Genevieve, and Aaron

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1. **Complete the following table for the dynamical timescales for open clusters, globular clusters, and elliptical galaxies. The times are expressed in years. Please consult the literature to find the most recent values of the masses and radii in each of the systems. Choose typical values for M and R in each case.**

We know that the mixing time for a dynamical system is given as

$$\tau_{\rm mix} = \left(\frac{6}{G\pi\rho}\right)^{1/2} = \left(\frac{8\bar{R}^3}{GM}\right)^{1/2} \tag{1}$$

Similarly, the relaxation timescale is given by

$$\tau_{\rm rel} = \frac{1}{3(32G)^{1/2}}\frac{1}{\Lambda}\left(\frac{N\bar{R}^3}{m_2}\right)^{1/2} = \frac{1}{3(32G)^{1/2}}\frac{1}{\ln(N^{2/3}/2)}\left(\frac{N\bar{R}^3}{m_2}\right)^{1/2} \tag{2}$$

assuming that m_2 represents the average stellar mass in the system, $m_2 = M/N$, this is equivalent to

$$\tau_{\rm rel} = \frac{1}{3(32G)^{1/2}}\frac{1}{\ln(N^{2/3}/2)}\left(\frac{N^2\bar{R}^3}{M}\right)^{1/2} \tag{3}$$

Finally, for a relaxed cluster with a Maxwell-Boltzmann velocity distribution, the evaporation timescale is simply

$$\tau_{\rm evap} \approx 135\ \tau_{\rm rel} \tag{4}$$

Plugging the below values into these equations gives the following dynamical timescales for the systems.

System	M	R	N	$\tau_{\rm mix}$	$\tau_{\rm rel}$	$\tau_{\rm evap}$
Open Clusters	1000 $M_\odot$[1]	3 pc[1]	1000[1, 2]	6.93 Myr	36.90 Myr	4.981 Gyr
Globular Clusters	10^5 $M_\odot$[3]	10 pc[3]	10^5[4]	4.22 Myr	1.26 Gyr	169.9 Gyr
Elliptical Galaxies	10^{11} $M_\odot$[5, 6]	10^4 pc[5, 6]	10^9[5, 7]	133.4 Myr	211,717 Gyr	28,581,926 Gyr

2. **Infrared observations of the stellar orbits in the vicinity of the radio source Sgr A* in the center of the Milky Way show that a non-luminous object with mass $4.15 \times 10^6\ M_\odot$ is confined to a volume of at most 10^{-6} pc^3. Argue that the possibility of a cluster of compact objects with this total mass, confined to the same volume, is not a plausible alternative to the black hole hypothesis. Assume that each compact object has a mass larger than $0.1\ M_\odot$.**

Assuming a perfect sphere, a compact object confined to a volume of 10^{-6} pc^3 has a characteristic radius of approximately

$$R = \left(\frac{3V}{4\pi}\right)^{1/3} = \left(\frac{3 \cdot 10^{-6}\ \mathrm{pc}^3}{4\pi}\right)^{1/3} = 0.0062\ \mathrm{pc} \tag{5}$$

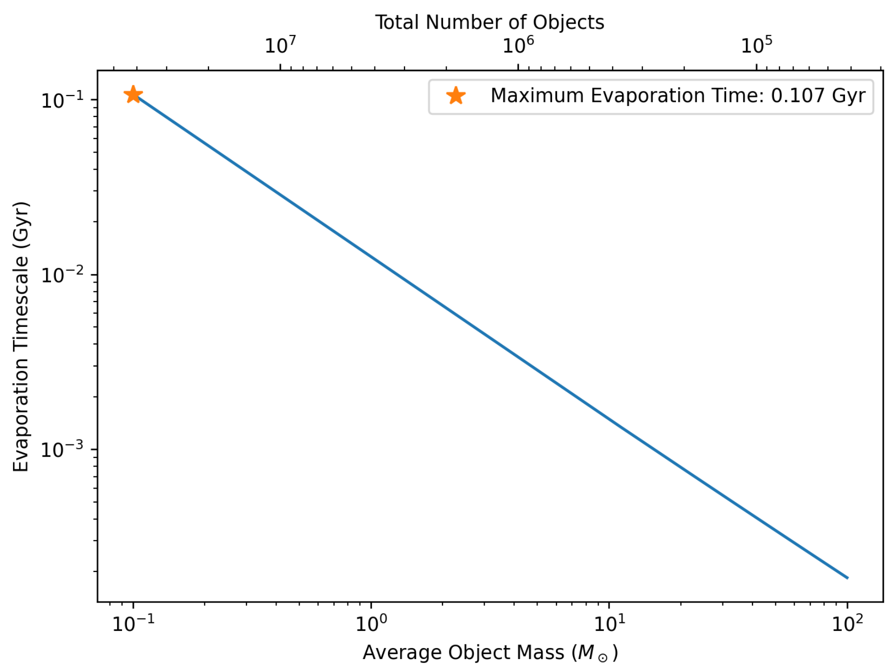
If this source was made up of a cluster of compact objects, the amount of time the system would be stable, i.e. how long until all of the sources were ejected, is approximately the relaxation timescale,

$$\tau_{\mathrm{evap}} = \frac{135}{3(32G)^{1/2}} \frac{1}{\ln(N^{2/3}/2)} \left(\frac{N\bar{R}^3}{m_2}\right)^{1/2} \tag{6}$$

Assuming that each compact object has the same mass, $m_1 = m_2 = \ldots = m_i = m$, the total number of objects is $N = \frac{M}{m}$, making the evaporation timescale

$$\tau_{\mathrm{evap}} = \frac{135}{3(32G)^{1/2}} \frac{1}{\ln\left(\left({}^{M}/\sqrt{8}m\right)^{2/3}\right)} \left(\frac{M\bar{R}^3}{m^2}\right)^{1/2} \tag{7}$$

Plotting this timescale for varying compact object masses, or equivalently different total number of compact objects,



We see that the maximum possible evaporation timescale, assuming all the objects have masses greater than $0.1\ M_\odot$, is only $\boxed{0.107\ \mathrm{Gyr}}$. Assuming that the galactic center formed with the rest of the Milky Way around 10 Gyr ago, there is no possible way for this cluster of compact objects to still exist. Moreover, even if it formed later on in the life of the Milky Way, this number represents the absolute maximum time it could have existed before evaporating, let alone ejecting a majority of the objects. Thus, it is impossible for the compact object Sgr A* in the center of the Milky Way to be a cluster of compact objects.

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