On the Validity of Analytic Potential Approximations for Galaxy Disks

ASTR 513, Statistics & Computation, Term Project

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Outline

Why Analytic Potentials ?

Galaxy dynamics is directly impacted by its gravitational potential.
Advantages of an analytic potential:

- Intuitive understanding of internal orbits and resonances
- Tracer particle simulations & precision control
- Satellite dynamics
- Harmonic and an-harmonic expansions
- Understanding of phase space behaviour
- Non-equilibrium processes and perturbations

Coordinate System

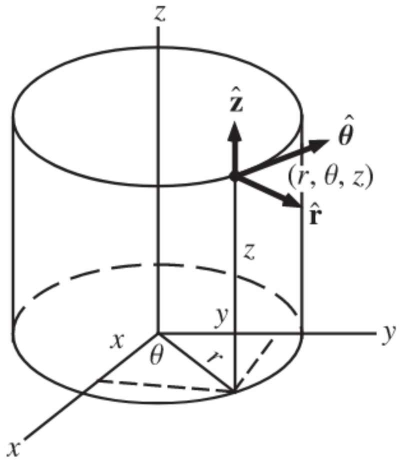


Figure: Cylindrical coordinate system. Source: Wolfram

Potential - Density Pairs

Potential-Density relation:

$$\nabla^2\Phi = 4\pi G\rho \tag{1}$$

From observations of the brightness profile of disks,

$$\rho(r, z) = \frac{M}{4\pi a^2 b} exp\left(-\frac{r}{a}\right) sech^2\left(\frac{z}{b}\right), \tag{2}$$

M is a normalization mass, a is the radial scale length, b is the vertical scale length.

Potential corresponding to this mass distribution is not analytical. Therefore, analytic approximations need to be employed.

Miyamoto-Nagai Approximation

The Miyamoto-Nagai (1975) disk is the most popular analytical approximation to the exponential density profile:

$$\Phi(r,z) = \frac{GM}{\sqrt{r^2 + \left[a + \sqrt{z^2+b^2}\right]^2}}, \tag{3}$$

The corresponding density distribution is:

$$\rho(r,z) = \frac{b^2 M}{4\pi}\left[\frac{ar^2 + \left(a + 3\sqrt{z^2+b^2}\right)\left(a + \sqrt{z^2+b^2}\right)^2}{\left[r^2 + \left(a + \sqrt{z^2+b^2}\right)^2\right]^{\frac{5}{2}}\left(z^2+b^2\right)^{\frac{3}{2}}}\right] \tag{4}$$

Validity of Miyamoto - Nagai Approximation ?

The validity of the Miyamoto - Nagai approximation has not been investigated quantitatively in depth. Questions I ask:

- How well does the Miyamoto - Nagai profile fit the exponential disk ?
- Can the same parameters describing the exponential disk be used for the Miyamoto - Nagai approximation ?
- Where does the Miyamoto - Nagai approximation break down ?
- What can be the physical consequences if the Miyamoto-Nagai approximation is not valid ?

I utilize existing simulation data of the equilibrium disk of the Large Magellanic Cloud for this work

LMC Equilibrium Disk

I extract the phase space information of stars (x, y, z, v_x, v_y, v_z) from Besla et al. 2012 simulations. These simulations contain 10^6 star particles ($M_* = 2500$) in an exponential disk, and 10^5 CDM particles in a Hernquist halo.

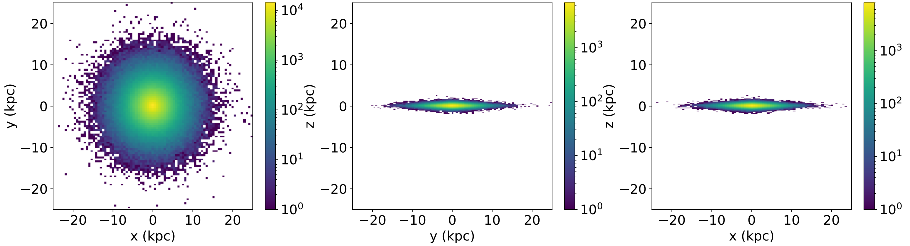


Figure: Stellar density map of the equilibrium LMC disk.

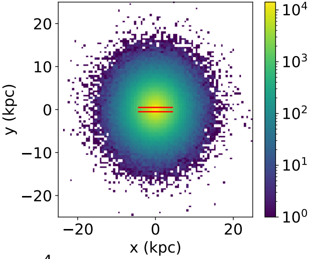
Obtaining Radial & Vertical Densities

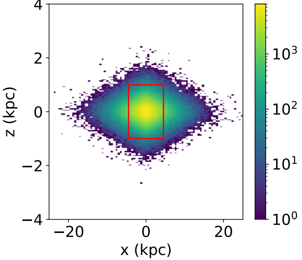
- Extract stars in a cuboidal volume $x \in (-4.5, 4.5)$ kpc, $y \in (-0.5, 0.5)$ kpc, $z \in (-1, 1)$ kpc, and binned them.
- Density:

$$\rho(x, z) = \frac{N(x, z) M_*}{\Delta V}$$

- Error on density:

$$\delta\rho(x, z) = \frac{\sqrt{N(x, z)} M_*}{\Delta V}$$





Sampling Setup

I use parallel MCMC to sample the posterior distribution of the model parameters. The χ^2 is defined as:

$$\chi^2 = \sum_{ij} \left(\frac{\rho_{data}(r_i, z_j) - \rho_{model}(r_i, z_j, a, b, M)}{\sigma_{ij}} \right)^2, \tag{5}$$

where $model \in \{exponential, Miyamoto - Nagai\}$.

The likelihood is defined as:

$$\mathcal{L} = e^{-\frac{\chi^2}{2}} \tag{6}$$

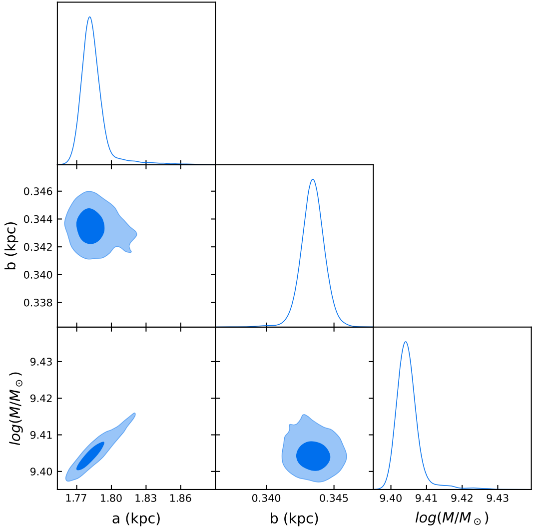
The parameters $a, b, log(M/M_\odot)$ are sampled using uniform priors.

Fitting of Exponential Profile

Priors:

- $a \in (0.1, 5)$ kpc
- $b \in (0.1, 0.5)$ kpc
- $log(M/M_{\odot}) \in (9, 10)$

Chain length $= 200$
No. of // chains $= 200$

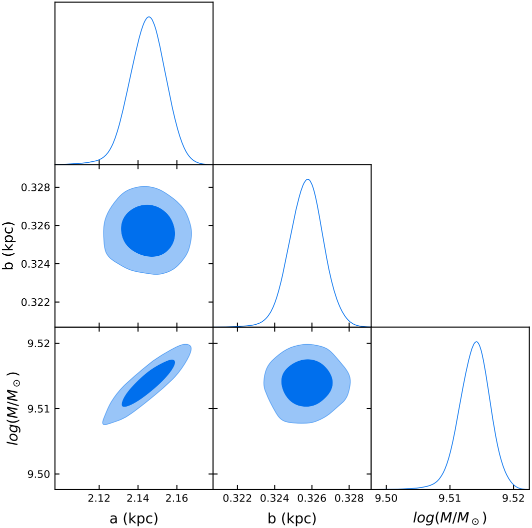


Fitting of Miyamoto-Nagai Profile

Priors:

- $a \in (0.1, 5)$ kpc
- $b \in (0.1, 0.5)$ kpc
- $log(M/M_{\odot}) \in (9, 10)$

Chain length = 200
No. of // chains = 200



Exponential Profile Fit

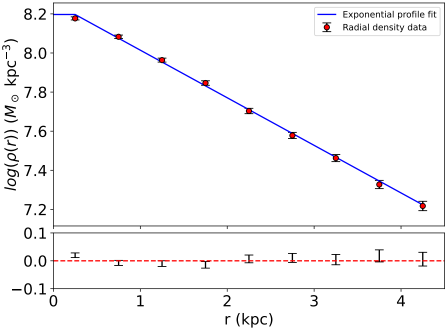


Figure: Radial density profile

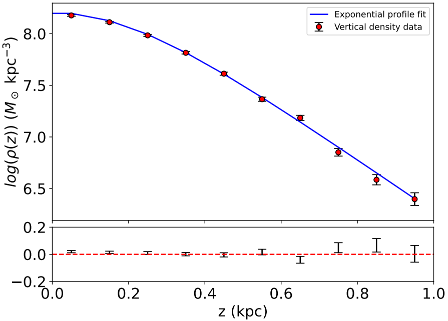


Figure: Vertical density profile

Miyamoto-Nagai Profile Fit

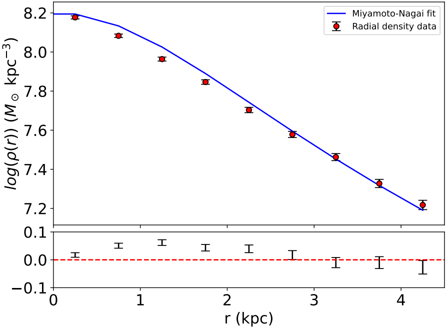


Figure: Radial density profile

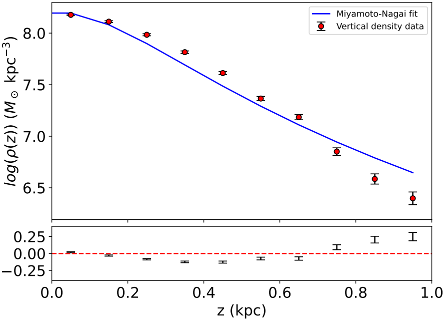


Figure: Vertical density profile

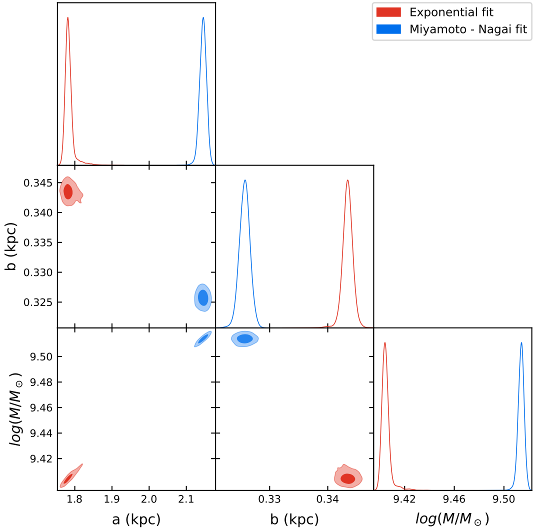
Comparison of Exponential & Miyamoto-Nagai Fits

Exponential fit:

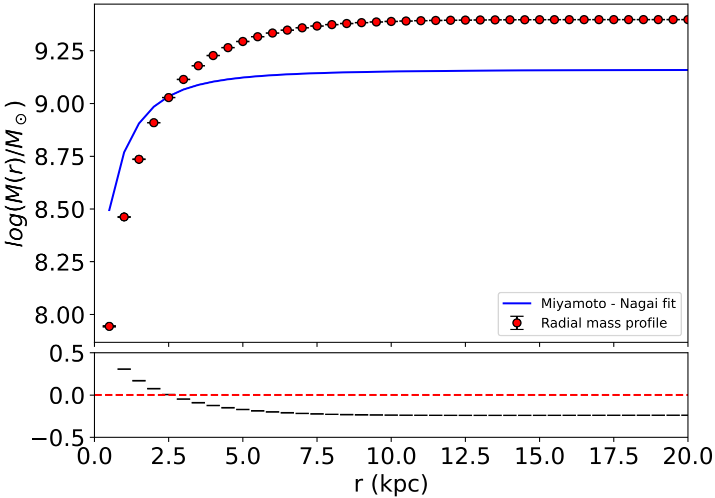
- $a = 1.78 \pm 0.01$ kpc
- $b = 0.343 \pm 0.001$ kpc
- $log(\frac{M}{M_\odot}) = 9.405 \pm 0.004$
- $\chi^2_{red,dof} = 1.03$

Miyamoto-Nagai fit:

- $a = 2.14 \pm 0.01$ kpc
- $b = 0.326 \pm 0.001$ kpc
- $log(\frac{M}{M_\odot}) = 9.514 \pm 0.003$
- $\chi^2_{red,dof} = 13.42$



Mass Profile Comparison



Conclusions

I explored quantitatively how well does the Miyamoto-Nagai analytical approximation describe an exponential galaxy disk:

- Miyamoto-Nagai is a considerably worse fit to the exponential disk, particularly in the vertical direction and disk outskirts
- The parameters of the exponential disk cannot be used to describe the Miyamoto-Nagai approximation
- Miyamoto-Nagai approximation can lead to underestimation of forces by a factor of ~ 2 and orbital speeds by a factor of ~ 1.5 in the disk outskirts

Multiple component Miyamoto-Nagai potential, or a combination of Miyamoto-Nagai and Hernquist potential might be the way to go forward.