BULLETIN OF THE ASTRONOMICAL INSTITUTES OF THE NETHERLANDS

1946 November 27 Volume X No. 376

COMMUNICATION FROM THE OBSERVATORY AT LEIDEN

Gas and smoke in interstellar space, by *J. H. Oort* and *H. C. van de Hulst* [1]).

The simultaneous occurrence of the solid and gaseous phase in interstellar space in the galactic and other systems suggests that a process must be acting by which gas is released from the solid particles. Evaporation explains the presence of gaseous hydrogen and helium, but not of the other gases. Expulsion of surface atoms of these latter elements by the action of photons seems likewise insufficient. The present investigation is concerned with a third process, namely that of volatilization of solid particles by mutual encounters. Because of this volatilization particles with radii larger than the wave-length of light must be practically absent. Calculations further indicate that it is *possible* that the observed gas density is maintained in this way. The motions and forms of interstellar clouds are briefly discussed in this connection. It is probable that, in large features, the clouds of gas and the clouds of solid particles are identical. The process of melting together of colliding particles has been briefly considered.

In the sixth section the distribution function of the radii of the particles has been studied which would result if an equilibrium between gas and solid particles were maintained by the processes of sublimation and of evaporation through collisions. The distribution is characterized by a steep drop, beginning at a radius between one and one-tenth time the wave-length of light (Figure 4 and Table 2).

In the last section mean absorption coefficients a_{pg} and effective mean radii of the scattering particles have been calculated from the theory developed in this paper, and with the aid of new data derived by one of the authors for the scattering by dielectric particles. No astronomical absorption data were used in this calculation. The effective mean radii are found to be of the order of 10^{-5} cm. The values of a_{pg} resulting from the theory are likewise of the order of those observed. The uncertainty of the computed values is caused mainly by lack of precise information on (a) the motions and forms of interstellar clouds, (b) the average gas density, and (c) the velocity of formation of new solid particles. Inversely, the observed "absorption" and its variation with wave-length may be used to fix the distribution curve of the radii of the interstellar particles. We thus find that the unit of length in Figure 4 and Table 2 is $0{\cdot}134\ \mu$, and that $n(0)$, the frequency of particles with very small radii, is 5.10^{-9}. The latter value indicates that the number of new solid particles formed per cm^3 per second is of the order of 6.10^{-29}, which is smaller than previous estimates. The dependence of scattering on wave-length following from this distribution of radii is shown in Figure 8. Figure 9 shows how much particles of different radii contribute to the total photographic scattering. The maximum amount of light is scattered by particles of $0{\cdot}31\ \mu$ radius. The total number of solid particles contained in 1 cm^3 would be $1{\cdot}3.10^{-13}$, their average radius $0{\cdot}15\ \mu$, and their total mass $0{\cdot}43.10^{-26}$ g, or about 1/1000th of the total gas density. The average life of a particle would be about 50 million years.

1. *Introduction.*

In 1935 LINDBLAD [2]) has opened up a fruitful line of attack on interstellar problems by investigating the hypothesis that the small solid particles in the galactic system had originated by condensation of the interstellar gas. From a rough estimate he concluded that the size to which the particles would have grown during the time the galactic system has existed was in agreement with the size attributed to these particles by SCHALÉN on the basis of his investigations on the absorption of light by small particles. The subject has been further investigated by one of us in an unpublished prize essay [3]). It was shown, among other things, that hydrogen and helium can only condense to a limited extent, and that the density of the other gases might be of the right order to explain the present sizes of the solid particles, though the data had to be stressed somewhat to avoid too large dimensions.

Upon closer inspection the picture of the one-sided process of gradual sublimation of the gas on to the solid particles without the occurrence of an inverse process meets, however, with a difficulty, which will be considered in the present article. The nature of the difficulty is this. If the "smoke" particles have originated by condensation in the interstellar gas the speed of their formation must increase with at least the second power of the gas density, and probably with a rather higher power. The number of solid particles present after a certain time will therefore be strongly dependent upon the original gas density ρ_0. If for a certain value of ρ_0 the present densities of smoke and of condensable gas (that is all gases except hydrogen and helium) would be of the same order, a relatively small increase in ρ_0 would have caused a practically complete disappearance of all condensable gases, while for smaller ρ_0 we would soon have come to a point where no appreciable amounts of solid particles could have been formed. In most places we would thus expect either no solid particles to have formed at all, or else the condensation to have practically exhausted the condensable gases. The intermediate phase, where the gases and the solid particles would exist side by side, should be of rare occurrence.

[1]) The manuscript for this article had been finished in February 1945. Its publication having been delayed by subsequent events, we have deliberately delayed it further, for making some minor alterations suggested by the literature that became available to us after the liberation.

[2]) *Nature*, **135**, 133, 1935.

[3]) C.f. also *Nederlandsch Tijdschrift voor Natuurkunde*, **10**, 251, 1943.

We would certainly expect large differences in the relative densities of gas and smoke between various parts of a stellar system. Now, we have only scant information concerning the density of gas and smoke in different regions of the galactic system, but what evidence there is, indicates that the densities of smoke and gases other than hydrogen and helium are quite comparable. There are certainly no signs of exhaustion of the condensable gases relative to hydrogen in any part of our system; nor does this seem to occur in other well-observable stellar systems (c.f. pp. 189–190).

We seem thus to be pressed to the conclusion that there must be at work a process which prevents the total sublimation of the gases capable of being condensed. There are two types of such inverse processes which may be considered, viz. photodissociation of molecules on the surfaces of the smoke particles and consequent escapes of atoms, and volatilization of smoke particles by collisions with other smoke particles. In the following both processes will be further discussed.

2. *The proportion between gas and smoke if evaporation can be neglected.*

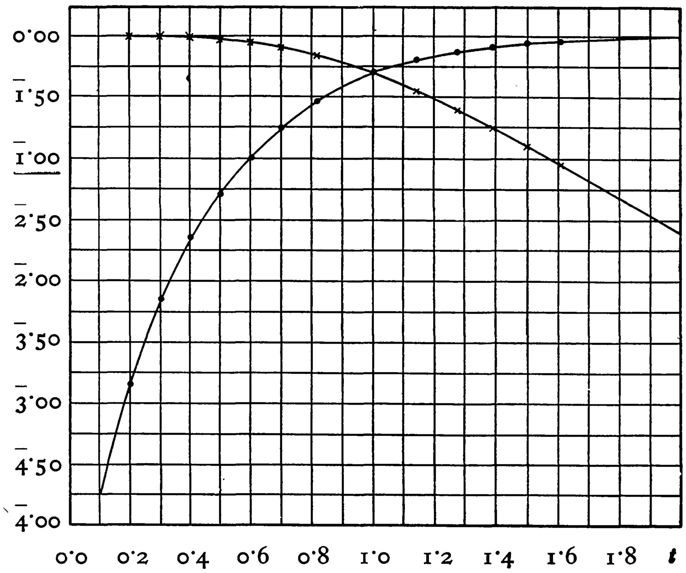
From the discussions by TER HAAR[1]) and by KRAMERS and TER HAAR[2]) it follows that the frequency of the formation of smoke particles is proportional to at least the third power of the gas density, ρ. As these authors have shown, the density of CH and CH^+ molecules is probably determined by an equilibrium between processes of radiation capture, ionization and photodissociation. The density of these molecules is proportional to ρ^2. If photodissociation were unimportant for the larger molecules, and if the probabilities for addition of further atoms were such that a semi-stationary state had been reached, with a constant stream of growing particles (such as considered, for instance, by BECKER and DÖRING[3])), the velocity of this stream would be proportional to ρ^3.

It seems likely, however, that in at least one of the following steps the capture probability is so low that the quasi-equilibrium has not yet been reached; moreover, it is possible that photodissociation is a determining factor not only for the numbers of CH and CH^+ molecules, but likewise for one or more higher molecules. In both cases the expression for the velocity of formation of the solid particles should be multiplied by as many factors of ρ as the number of steps in which the holding-up process is effective. The velocity may therefore be estimated to be at least proportional to ρ^4 (c.f. pp. 198–199). In the following calculations the exponent has been assumed to be 4.

Let us define ρ_g as the density of all gases other than H and He, of which latter gases, as first clearly stated by VAN DE HULST, only a minor fraction can condense on to the smoke particles. Let us further denote by ρ_s the density of the smoke in g/cm³, and by ρ_o that of smoke plus gas *inclusive H and He*. The increase of the smoke density with the time for a homogeneous mass of gas has been studied in detail by one of us[1]), by considering the general case that the velocity of smoke formation is proportional to $\rho_g{}^n$. If we adopt a certain value of n, and if we assume that the smoke particles do not lose atoms in any way, the variations of ρ_g and ρ_s with the time can be easily calculated numerically. The curves in Figure 1 have been so computed for the case $n=4$. The details of this computation have been omitted as they are irrelevant to our present purpose. The unit of the abscissae in this figure is the time $t_{1/2}$ needed to transform half of the non-hydrogen or -helium gas into smoke. This time is related to ρ_o by the following formula

$$t_{1/2} = c\,\rho_o{}^{-3/2}\,. \qquad (1)$$

FIGURE 1



Crosses show the change of log ρ_g with time, dots that of log ρ_s, ρ_g being the density of gases other than H and He, ρ_s that of smoke (unit $\rho_g + \rho_s$; the unit of time is that needed to transform half of ρ_g into smoke). The probability of formation of solid particles was supposed to be proportional to $\rho_g{}^4$.

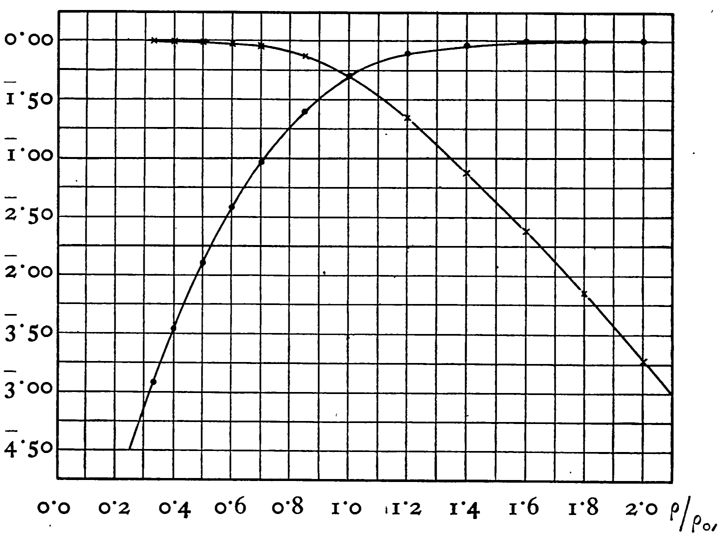
It is easy to see that this formula must be valid in the first stages of the process of condensation. For, as long as the exhaustion of the gas is negligible the number of particles formed is $c_1\,\rho_o{}^4\,t$, the maximum radius is $c_2\,\rho_o\,t$, and the mean mass $c_3\,\rho_o{}^3\,t^3$, c_1, c_2, c_3 being constants. The total mass of the smoke particles

1) *B.A.N.* **10**, 1, 1943.
2) *B.A.N.* **10**, 137, 1946.
3) *Ann. d. Phys.* (V) **24**, 719, 1935; c.f. also TER HAAR, *l.c.*

1) Still unpublished.

after a time t is therefore $c_1 c_3 \rho_0{}^7 t^4$, and the time needed to transform a small fraction c_4 of the gas into smoke is equal to $\left(\frac{c_4}{c_1 c_3}\right)^{1/4} \rho_0{}^{-3/2}$, or proportional to $\rho_0{}^{-3/2}$. It can be proved that in later stages, when the exhaustion can no longer be neglected, the time t needed to transform a given fraction of the gas into smoke is proportional to the same power of ρ_0.

FIGURE 2



Dependence of present values of log ρ_g and log ρ_s upon original gas density (unit of ρ_g and ρ_s is $\rho_g + \rho_s$).
$\rho_0{}'$ is the total original density for which present-time values of ρ_g and ρ_s become equal.

Let us denote by $\rho_0{}'$ an original total density such that, after a time equal to the age of the galactic system, say 3.10^9 years, ρ_s would have become equal to ρ_g. The unit of time in Figure 1 is then 3.10^9 years. In the galactic system ρ_g and ρ_s are at present of about the same order. It follows that $\rho_0{}'$ should be close to the total gas density in this system, or to 3.10^{-24}. We now want to know what the present gas and smoke density would have become if the original density had been ρ_0 instead of $\rho_0{}'$. According to formula (1) the time needed to reach the phase $\rho_g = \rho_s$ would then be $(\rho_0{}'/\rho_0)^{3/2}$ times larger. In order to use Figure 1 for calculating ρ_g and ρ_s in this case, the unit of time has therefore to be taken $(\rho_0{}'/\rho_0)^{3/2}$ times larger, so that the period of 3.10^9 years corresponds to $(\rho_0/\rho_0{}')^{3/}$ units. In order to find ρ_g and ρ_s at this epoch in case of an original density ρ_0 we must therefore enter Figure 1 with $t = (\rho_0/\rho_0{}')^{3/2}$. In this way we easily derive the relation, at the present epoch, of ρ_g and ρ_s to ρ_0. It is shown in Figure 2. Evidently, the present values of ρ_g and ρ_s depend very strongly upon the original density. In order that the present ratio ρ_s/ρ_g lie between 0·1 and 10, ρ_0 should be between $0{\cdot}70\,\rho_0{}'$ and $1{\cdot}37\,\rho_0{}'$. These limits are so close together that the simultaneous occurrence of absorbing clouds and diffuse nebulae emitting oxygen lines should be a relatively rare phenomenon. For instance, for $\rho_0 = 2\rho_0{}'$ only one five-hundredth part of the original amount of ρ_g would have remained uncondensed.

Information on interstellar gas from observations of emission nebulosities is available for the galactic system, M33, the Andromeda Nebula and the Magellanic Clouds. The latter give no certain evidence of absorption, and will therefore be omitted from the discussion.

In the galactic system there is abundant evidence of the occurrence of solid particles as well as of gaseous oxygen. It is even possible to obtain estimates of the average values of ρ_s and ρ_g. The absorption of light near the galactic plane, coupled with the fact that this "absorption" varies with the wave-length, enables us to estimate the smoke density. As all gases except hydrogen and helium are supposed to condense, and as oxygen is probably more abundant than iron, the particles are likely to be non-conductive. The particles responsible for the greater part of the "absorption" should then have dimensions of the order of the wave-length of light. If we assume a mean radius of 10^{-5} cm, and optical constants as adopted by GREENSTEIN for "ice and frozen gases", for instance[1]), we find from his data that a density of 2.10^{-26} g/cm³ would be required to give an absorption of 1 mag/kps. As regards ρ_g no trustworthy direct data relating to oxygen, nitrogen, etc. have yet been derived, but the total gas density including hydrogen and helium may be fairly reliably estimated at 3.10^{-24}. The relative abundance of elements heavier than H and He can only be guessed. According to the latest estimates the number of these atoms is about 1/500th of the total[2]), which would give $\rho_g \backsim 10^{-25}$.

It is thus seen that ρ_s and ρ_g are of the same order. It may be remarked that in many places strong emission nebulae displaying oxygen lines are found connected with dark regions where the smoke density may exceed the average density by a factor of the order of 100. Such cases cannot, however, be adduced as independent evidence that the gas cannot be exhausted by condensation, for there is ample evidence that the interstellar clouds possess considerable relative motions, so that fresh gas may continually be carried into the smoke clouds.

Out of the great number of nebulous knots in M33, 25 have been observed by MAYALL and ALLER[3]). All have emission spectra, in which the forbidden line

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1) *Harv. Circ.* No. 422, Table X.
2) C. f. footnote 1, p. 198.
3) *Ap. J.* **95**, 5, 1942; *Lick Contr.* Ser. II, No. 1.

λ3727 of OII is present in great strength, together with the Balmer lines. The nebula is full of absorbing clouds [1]), and these as well as the emission patches are distributed practically over its entire surface.

In the Andromeda nebula H. W. BABCOCK [2]) has observed five emission nebulae. Three of these lie outside the main body of the nebula, the two others are situated in or near regions of considerable absorption. The emission lines shown are again those of oxygen and hydrogen.

In addition to these observations of individual emission patches there is evidence, in many spiral systems, of general emission throughout the bodies of these systems. MAYALL [3]), who has collected the largest material, even reports that the great majority of later-type spirals show general emission of the oxygen line λ 3727. Though in some nebulae the great width of the emission lines would seem to suggest a different origin, there is no reason to doubt that in most cases the emission is due to interstellar gas of the type we observe in the galactic system. In some cases it seems evident that the emission lines cut right across regions of distinct absorption.

Elliptical and So nebulae show neither absorption nor emission. The absence of solid particles in these objects can easily be understood on the basis of the conclusions reached above. We have only to assume that the gas density in these systems is less than, say, $0{\cdot}5\ \rho_0'$.

We have seen that in systems showing absorption, whenever the interstellar gas can be observed, the oxygen lines show up with relative strengths of the same order; nowhere does the oxygen appear to be sensibly depleted. As it is not plausible that in all these systems the total gas density would have been within the narrow limits $0{\cdot}70\rho_0'$ and $1{\cdot}37\rho_0'$, between which according to the above considerations solid particles could have been formed without the oxygen having been used up, we are led to conclude that our premise of one-sided condensation has been wrong, and that a process must be taking place by which atoms are released from the solid particles.

This conclusion tends also to clarify another problem, the data on which have been found somewhat difficult to reconcile. In a former article [4]) one of us has investigated the sizes which particles could reach in 10^9 years if there had been no loss of atoms from their surfaces. It was found that only by choosing rather extreme values for all relevant data could the results be reconciled with the observed absorption. In particular the density of the condensable gas had to be taken considerably lower than the most recent estimates of the proportion between C, N, O and Ne atoms and those of hydrogen seem to indicate, or than would be needed to explain the abundance of CH and CN molecules in interstellar space. The difficulty disappears of course if we drop the hypothesis of exclusive condensation.

Doubt has sometimes been expressed as to whether interstellar particles can be formed at all from the interstellar gas. The existing particles would then have to be of meteoric origin. The considerations given in the following sections are not favourable for a meteoric origin of scattering particles, in as much as it is difficult to see how even the maximum number of meteors that might be admitted could yield dust at a rate sufficient to explain the interstellar scattering.

1) C. f. for instance, HUBBLE, *The Realm of the Nebulae*, Pl. X.
2) *Lick Bull.* **19**, 41, 1939.
3) *Lick Bull.* **19**, 33, 1939.
4) *Nederlandsch Tijdschrift voor Natuurkunde*, **10**, 251, 1943. See also TER HAAR, *B.A.N.* **10**, 1, 1943 and KRAMERS and TER HAAR, *B.A.N.* **10**, 137, 1946.

3. *Processes by which atoms can be detached from the particles.*

There are three possible ways in which atoms that have condensed on to the solid particles may return to the interstellar gas: (a) by gradual evaporation from the surface, (b) by photodissociation from the surface, or (c) by evaporation of an entire particle upon being hit by another particle of comparable size.

(a) VAN DE HULST has shown that, while at the temperatures pertaining to interstellar space particles consisting of hydrogen or helium would evaporate rapidly, particles formed of solid oxygen for instance would not loose a single atom in 10^9 years by the process of evaporation [1]). Evaporation is therefore ineffective except in limiting the hydrogen and helium content.

(b) This process consists either in photodissociation of a molecule near the surface and escape of one of the components, or, for the heteropolar crystals, in the release of an atom from the crystal lattice by means of a light-quantum. Our knowledge of these processes, and of the way an interstellar particle is built, is as yet too scant to permit an adequate discussion, but from what we know of the interstellar molecules CH and CN [2]) it does not seem probable

1) *Nederlandsch Tijdschrift voor Natuurkunde*, **10**, 253, 1943; c.f. also *Thesis*, Utrecht, p. 78, 1946.
2) C.f. KRAMERS and TER HAAR, *B.A.N.* **10**, 137, 1946. For CH^+ a value of 10^{-15} is indicated in their Table 1 for the probability of photodissociation of an ion per second. This means that if the surface layers of the particles consisted of CH^+ ions, about one atom would be released per cm² per sec. The authors give good reasons to believe that the probability would be still rather lower for CN. Now if ρ_g represents the interstellar density of gases other than H and He, expressed in number of atoms per cm³, and if two H atoms are supposed to condense for each heavier atom, the number of atoms condensing on each cm² of the surface would be $3.10^5 \rho_g$. With $\rho_g = 0{\cdot}002$ cm^{-3} this becomes 600. Compared to this the effect of photodissociation would therefore be negligible.

that the rôle of the photodissociation would be predominant, at least not under the conditions prevailing in the galactic system. The dissociation will be enhanced in the vicinity of early-type stars, but it seems doubtful whether the increase would be sufficient to explain the considerable quantities of gas observed. It might only be of importance if condensations of interstellar matter were generally accompanied by concentrations of hot stars. There may be such an association in a few cases, but these seem to be exception rather than rule. It may well be, however, that photodissociation influences the composition of the particles, in so far as only those molecules or crystal lattices can be formed which are sufficiently resistant against the action of the photons.

(c) It is well known that the clouds of interstellar gas have considerable relative velocities, such that they will unavoidably run into each other in the course of time. At such an impact the solid particles in each of the two colliding clouds will have a certain chance to hit a solid particle in the other cloud. If the kinetic energy lost in the collision is large enough the two colliding particles will evaporate; otherwise they will either melt together or part intact after an elastic collision.

Although the data available for numerical calculation are very fragmentary we shall try to estimate the importance of this evaporation. It will be shown in section 6 that, once accepting this process to be the determining factor in the relation between gas and smoke, important conclusions can be drawn concerning the distribution of the diameters of the smoke particles, and that these conclusions depend only to a small extent upon the exact data assumed.

4. *Forms and motions of interstellar clouds.*

We start with a model based upon the following suppositions:

(I) The distribution of gas and smoke is generally identical.

Of the various arguments favouring this hypothesis the strongest is that, as first clearly pointed out by SPITZER [1]), the small solid particles must follow the motions of the gas. So, when the moving clouds of gas which we now observe were formed, solid particles will have been carried along practically just as though they were gas particles. The clouds and streams of gas shown by the spectrographs must therefore be made up of both gas and smoke. Another argument proving the close connection between condensations of gas and smoke may be based upon HUBBLE's discovery that the nebulae surrounding O- en Bo-type stars always show an emission spectrum, while in general those illuminated by later-type stars are reflection nebulae, in which the light is presumably scattered by clouds of solid particles. If we should suppose that these latter clouds consisted exclusively of solid particles, without a corresponding concentration of gas, it would be difficult to understand why such clouds are never illuminated by O and Bo stars, and give a pure reflection nebula around a hot star. Yet another argument in favour of a close connection between gas and smoke is provided by the luminous emission rims often found bordering dark nebulae.

On the other hand it has been pointed out that relatively near-by stars which happen to lie behind a dense obscuring cloud show but faint interstellar absorption lines, by no means corresponding to their large colour-excesses [1]). However, it appears to us that this does not prove that the absorbing clouds contain no corresponding concentrations of gas. It seems likely from other data that circumstances are such that the interstellar lines formed in the clouds are super-saturated. Indeed, observations of the intensity ratio of the D-lines of sodium show that the intensity is far from proportional to the number of atoms. O. C. WILSON and MERRILL have pointed out [2]) that the observed ratio of this doublet is in general about 1·2, instead of 2, as it would have to be in case of simple proportionality. Under these circumstances a sensible strengthening of lines by a dense interstellar gas cloud is not to be expected.

We have previously thought that the galactic concentration of the gas was different from that of the smoke, but in a recent investigation based on a much extended material VAN RHIJN has shown that this result was probably fortuitous, and that the actual concentrations of the two types of matter are sensibly the same [3]).

It should be remarked that, although it is thus very probable that the distribution of gas and smoke is closely related *in general*, there is unambiguous evidence of difference in *detail*, for example in the nebula around σ Scorpii [4]).

(II) A large fraction of gas and smoke is concentrated into more or less discrete clouds or streams having considerable relative velocities.

If only a small fraction of the gas were contained in the moving clouds, while the major part were distributed more or less evenly, and followed the general galactic rotation, without large internal motions, it would be difficult to understand how the average line intensity could increase proportionally

[1]) *Ap. J.* **93**, 369, 1941.

[1]) In general this is shown by the smallness of the correlation between colour-excesses and interstellar line intensities, which has often been commented upon. Striking special cases have been indicated by GREENSTEIN and STRUVE in *Ap. J.* **90**, 625, 1939, and by W. W. MORGAN in *Ap. J.* **90**, 632, 1939.

[2]) *Ap. J.* **86**, 44, 1937; *Mt Wilson Contr.* No. 570.

[3]) *Groningen Publ.* No. 50, p. 6 a.f., 1946.

[4]) STRUVE, *Ap. J.* **86**, 94, 1937.

with the average distance, and how these line intensities could fail to show the effects of differential galactic rotation[1]). A still more convincing and much more direct argument in support of statement (II) is furnished by the results recently reported by ADAMS, indicating that 80% of the stars investigated on Mount Wilson with the large dispersion of the Coudé spectrograph show multiple interstellar lines[2]). It seems clear from this paper that the amount of smoothly distributed matter cannot greatly surpass that contained in the distinct, moving clouds.

As is well known, the evidence of colour-excesses and of investigations on absorption wholly confirms that the distribution of interstellar matter is highly uneven, much of the absorption taking place in fairly concentrated clouds. Again it is difficult to obtain a numerical estimate of the proportion between the amount of matter concentrated in the clouds to that in between. It would seem that the latter cannot be of a larger order than the former; it may be considerably less.

(III) In order to compute the probability of a collision between two interstellar clouds we must know the number of separate clouds met by a line of given length. As practically all early-type stars at a distance of more than 500 parsecs show interstellar lines (usually consisting of more than one component) it seems a safe estimate that a line of 1000 parsecs length will on the average cross at least five independent cloud systems.

This number is all we need for the present investigation. It is of interest, however, to inquire to what average dimensions and frequency of cloud systems this result corresponds. If it be assumed that the distribution of interstellar matter and that of early-type stars are independent of each other (which is by no means certain) we find that about 14% of the space near the galactic plane would be covered by interstellar cloud complexes[3]). If N is the number of cloud systems per kps^3 and r their average radius we would then have $\frac{4}{3}\pi r^3 N = 0{\cdot}14$. The condition that a line of sight of 1 kps would cross 5 clouds gives $\pi r^2 N = 5$. We thus find $r = 0{\cdot}021$ kps, $N = 3600$. The result for the average radius agrees well enough with what we know about the two conspicuous cloud systems in our vicinity, in Taurus-Auriga and in Ophiuchus-Scorpius, respectively; the largest diameters of these have been estimated at 50 ps, while the denser nuclei might measure about 30 ps.

1) C.f. O. C. WILSON and MERRILL, *Ap. J.* **86**, 44, 1937; *Mt Wilson Contr.* No. 570.

2) *Ap. J.* **97**, 105, 1943; *Mt Wilson Contr.* No. 673.

3) For, among the roughly 160 stars with spectra between O and B2, brighter than $7^m{\cdot}5$ and north of $-15°$ declination, 23 occur in HUBBLE's list of stars connected with diffuse nebulae, showing that they are situated in parts of space containing dense concentrations of interstellar material (*Ap. J.* **56**, 400, 1923; *Mt Wilson Contr.* No. 250, Table 1).

Naturally, these numbers remain very uncertain. The picture of more or less regular clouds is, moreover, certainly an inadequate representation of the real clouds, with their highly irregular, whispy, and often stratified, structures.

The above estimates might certainly be improved by a systematic study of all available material; this, however, lies beyond the scope of the present article, which only intends to give a provisional orientation.

(IV) The average peculiar velocity of the clouds in radial direction has, somewhat arbitrarily, been assumed to be 13 km/sec. Peculiar motions of this order are required to explain the lack of correlation between the intensity of interstellar lines and the effects of galactic rotation on the radial velocities, as well as to understand why the intensities increase roughly proportionally with the distance. The data on multiple lines, in so far as these have come to our knowledge, confirm that the velocities are of this order. We further suppose that the average velocity in the direction perpendicular to the galactic plane is 7·5 km/sec, roughly corresponding to the spread in this direction of gas and absorbing material. The average space velocity is then found to be 22·5 km/sec. As the velocity in the z-direction is only half that in the other co-ordinates the frequency function of the space velocities v may approximately be written as

$$f(v) = 2\,h^2 v\, e^{-h^2 v^2}, \text{ with } \frac{\sqrt{\pi}}{2h} = 22{\cdot}5. \qquad (2)$$

(V) The clouds will continually sweep gas and smoke lying between them. The smoke particles entering a cloud will be braked by the gas atoms in the cloud, and come to rest after having penetrated a certain distance D_0 into it. This distance will depend upon the radius of the particle, but in general it will be considerably smaller than the diameter of the cloud. Before having lost its velocity the particle will have a certain chance of hitting another smoke particle of suitable size, with a speed sufficient to cause evaporation of both particles. The same accident may occur to particles at the edges of two colliding clouds. At such a collision particles from both clouds will run into the denser layer formed between the two colliding clouds[1]).

Real conditions will deviate in several respects from the schematical picture sketched. Near the interface between two colliding clouds there will be pressure and velocity gradients, and these will likewise occur near the front surface of a cloud moving through a smooth interstellar medium. It is possible, though

1) The clouds may partly become luminous as a consequence of such a collision. These effects will be discussed in a subsequent article. C.f. also J. M. BURGERS, *Proc. Koninklijke Nederlandsche Akademie*, **49**, 589, 1946, and J. H. OORT, *George Darwin Lecture*, *M.N.* **106**, 1946 (in print).

not probable, that these gradients would have some influence on the probability of collision of the solid particles [1]).

(VI) Prof. MINNAERT has drawn our attention to the fact that this picture of clouds sweeping up the interposed gas, and of their mutual encounters, may not be adequate in case the motions are of a turbulent nature. In ordinary turbulent fluids and gases the motions of the whirls are such that they interfere with each other to a much smaller extent than would happen in the case of purely random motions. It is of importance, therefore, to inquire whether the motion of the interstellar medium is turbulent in this sense.

On one hand there is indeed reason to believe that the interstellar motions would be analogous to the turbulent motions observed in ordinary gases. As was first pointed out by ZWICKY [2]) Reynolds' number is quite large for interstellar media (for our region of the galactic system we estimate it to be of the order of 10^9), so that turbulent motion is to be expected. Prof. BURGERS, to whom we are much indebted for a discussion on this subject, has indicated that it is not unlikely that the units in these turbulent motions would be of sizes comparable with the distances between interstellar clouds. If this is so, the observed peculiar velocities of these clouds may have originated from something like enormous "whirls", continually formed in the interstellar gas as a consequence of the differential rotation of the galactic system.

On the other hand, though the velocities of interstellar clouds might *originate* through processes analogous to those which under terrestrial conditions give rise to turbulent motions, it seems at least questionable whether the analogy could also be extended to the subsequent orbits. If interstellar matter moved in whirls, like the particles in ordinary turbulent matter, we should expect to find evidence of these whirls in the arrangement of interstellar formations. In reality neither dark clouds nor extended luminous nebulae show any distinct traces of such an arrangement [3]). Nor has any evidence of whirls been found in the dark matter observed in near-by extra-galactic systems. It should be noted that the interstellar medium appears to differ also in another important point from ordinary matter in turbulent motion, namely in the outspoken concentration of the matter in often sharply bounded clouds with densities considerably exceeding those in the intervening space.

1) Through recent discussions with Professor J. M. BURGERS our picture of the phenomena occurring at an encounter between interstellar clouds has been much clarified. Professor BURGERS has calculated the density of the intermediate layer between the colliding clouds, and the way in which the thickness of this layer increases with the time. He has also pointed out that the transitions between the clouds and the intermediate layer are quite abrupt, so that the cloud particles must indeed enter with full speed into this layer (J. M. BURGERS, *l.c.*).

2) C.f. *Ap. J.* **93**, 411, 1941.

3) At least not on the scale with which we are concerned. Whirl-like formations of small size are evident in several places, the finest example being presented by the nebula around Merope, which consists of numerous roughly parallel streaks, perhaps 15′ long, and about 8″ wide. The thickness of these whirls is thus of the order of 1000 astronomical units, or about 10000 times smaller than the large-scale interstellar irregularities considered above.

That the clouds must have undergone many collisions, and cannot have subsisted as more or less the same units throughout the greater part of the life of the galactic system is also indicated by the notable similarity of the smoke particles in- and outside the clouds. The two groups of particles have grown to practically the same dimensions, for they generally show the same law of scattering [1]). This would be difficult to understand if the particles had not continually shifted from dense clouds into more tenuous regions, and vice versa.

In the following we have provisionally treated the observed interstellar motions as being really directed at random; it is well to keep in mind that this assumption is still uncertain.

5. *Provisional estimate of the importance of collisional evaporation.*

We may now proceed to make a rough estimate of the amount of evaporation to be expected from the collisions.

In order to find the energy required for evaporation we have assumed the following "average" values for specific heats etc.: specific heat of solid phase 0·2, of the fluid phase 0·3, melting heat 30, evaporation heat 130. The exact values of the temperatures at which melting and evaporation take place are not of much consequence; we have taken 0° and 100°, respectively. With these constants the heat required to evaporate an interstellar particle is roughly 250 calories, or $1{\cdot}05.10^{10}$ erg per gram. If two particles of mass m and with relative velocity v km/sec collide centrally the kinetic energy lost is $\frac{1}{4}mv^2.10^{10}$. Equating this to the evaporation heat, $2m.1{\cdot}05.10^{10}$, we obtain for the minimum relative velocity required to vaporize the particles

$$v_{min} = \sqrt{8.1{\cdot}05} = 2{\cdot}90 \text{ km/sec}. \qquad (3)$$

According to our suppositions the average velocity with which a smoke particle enters a cloud when swept up by it is 22·5 km/sec. In computing the distance D down to which the particle will penetrate into the

1) This argument has obtained fresh support from an investigation by STEBBINS and WHITFORD (*Ap. J.* **98**, 20, 1943; *Mt Wilson Contr.* No. 680). They show how accurately the law of scattering is the same in all the regions studied, and conclude that "some sort of equilibrium must have been reached".

cloud before its relative velocity has decreased to 2·90 km/sec, it was assumed that the gas atoms hitting the particle transfer their momentum to it, and, being mostly H and He atoms, evaporate again in an arbitrary direction. Following B. STRÖMGREN's estimate that in about 90% of the galactic stratum H (and He) are non-ionized we have neglected the effects of the passages of charged nuclei, though in a rigorous calculation these should certainly be taken into account. We thus obtain the following equation

$$m\,dv = -\pi r^2 \rho v^2\, dt, \tag{4}$$

in which m is the mass of the particle, assumed to be constant, r its radius and ρ the total gas density in the cloud. Solving the equation we obtain

$$v = v_0\left(1 + \frac{k\,v_0}{r}t\right)^{-1}, \tag{5}$$

t being counted from the moment of entrance into the cloud; v_0 is the velocity at that moment;

$$k = 3\rho/4s = 2{\cdot}25.10^{-24}\,\Delta, \tag{6}$$

in which s, the specific weight of the particle, was taken 1; Δ is the factor by which the density in the cloud exceeds the average interstellar gas density, which was assumed to be 3.10^{-24} g/cm³. It follows that

$$D = \int_0^t v\,dt = \frac{r}{k}(\ln v_0 - \ln v_{min}) =$$

$$= \frac{4{\cdot}44.10^{23}\,r\,(\ln v_0 - \ln v_{min})}{\Delta}, \tag{7}$$

ln denoting the natural logarithm. For $r = 10^{-5}$ cm and $\Delta = 10$, D would be $0{\cdot}91.10^{18}$ cm or 0·30 parsec. It will be noted that the distance to which the particles penetrate is quite small compared to the probable dimensions of the cloud.

The average number of particles of radius r hit by a particle of the same radius while penetrating to a depth D into the cloud is $\pi\,(2r)^2 D n \Delta$, if n is the average number of particles, all supposed to be of the same size, in 1 cm³ of interstellar space. A rough estimate of n may be obtained from the observed absorption. Including the dark nebulae the average absorption may be estimated as about 1 mag/kps. If, as seems probable, the particles are non-metallic the absorption must largely be due to particles with radii of the order of 10^{-5} cm, say between $0{\cdot}5.10^{-5}$ and $1{\cdot}5.10^{-5}$. Now, with ice particles for instance, we would need $7{\cdot}6.10^{-12}$ particles of radius 10^{-5} per cm³ to get an absorption of 1 mag/kps[1]). We may, accordingly, estimate n to be about $7{\cdot}6.10^{-12}$. The number of collisions leading to evaporation is

$$4\pi.4{\cdot}44.10^{23}\,r^3\,n\,(\ln v_0 - \ln v). \tag{8}$$

Inserting the numerical values given above, and $r = 10^{-5}$, we find 0·087. The chance of evaporation upon meeting a cloud is therefore by no means negligible. In this calculation all collisions were treated as central, while in reality in the non-central collisions part of the energy goes into the rotation and is not transformed into heat. For the present rough survey this was neglected. The circumstance that in case of unequal particles somewhat higher velocities would be required was also neglected, but will be taken into account in the next section.

In order to judge the importance of collisional evaporation we must still estimate the number of encounters with clouds which an average particle will undergo during its lifetime. According to the data given under **4** (III) and (IV) the total space swept by the clouds contained in 1 kps³ in 10^6 years is 0·11 kps³. The probability that a particle meets a cloud is thus 0·11 per million years, the probability of evaporation is 0·11.0·087 or 0·010. Consideration of mutual encounters between clouds leads to about the same result. The average lifetime of a particle would thus not exceed 100 million years, or, say, one thirtieth of the age of the galactic system. During its life a particle would collide a dozen times with a cloud. The clouds should be conceived as being continually formed and re-formed, so that a given particle can be swept up more than once.

It follows from these estimates that evaporation through collision may indeed provide a satisfactory explanation of the observed fact that the average radius of solid particles in the galactic system has not grown beyond a few thousand Angströms at most; particles larger than those considered above are likely to be practically absent, as the probability of evaporation increases strongly with the radius (c.f. formula (8)). The result also shows how the condensable gases can permanently exist in all systems containing dark material. The process will be worked out in more detail in the next section. It will be shown there that this evaporation can account also quantitatively for the amount of gas in the interstellar space in the galactic system.

6. *The distribution function of the radii.*

In this section we shall investigate the state which would result after conditions in interstellar space have stabilized; that is, when smoke particles have grown in such numbers and to such sizes that further birth and growth are just balanced by evaporation through collision. It is perhaps well to emphasize that we do not know whether such a state has already been reached in the galactic system; but it seems of interest to investigate what the equilibrium stage will look

[1]) This number may be directly obtained from the results computed by GREENSTEIN (*Harvard Circ.* No. 422, Table X).

like, and to confront it with what has been observed.

If $n(r)dr$ represents the number of particles with radii between r and $r + dr$ the condition for a steady state is

$$\dot{r}\, n(r) = \dot{r}\, n(r + dr) + P(r)\, n(r)\, dr. \qquad (9)$$

Here $\dot{r}$ is the growth of the radius per unit of time (a million years). $P(r)$ is the probability that a particle of radius r evaporates in a unit of time; this probability depends implicitly on $n(r)$. The left-hand member gives the number of particles growing into the interval r to $r + dr$, the right-hand member shows the number leaving it, either by growth or through evaporation. For the sake of simplicity no terms have been included to take account of the melting together of two particles meeting with moderate velocities; it will be shown in the next section that their inclusion would not have had much influence on our conclusions. The quantity $\dot{r}$ may be considered as a constant parameter, depending only upon the density and temperature of the gas (and to some extent upon the motions of the smoke particles) but not upon the radius; for the number of atoms condensing per unit of time upon each surface element is independent of the radius.

In order to determine $P(r)$ we shall consider the case of loose particles swept up by a cloud [1]). If we should have supposed that the number of "loose" particles is small compared to that contained in the clouds consideration of the mutual encounters of the clouds would have led to approximately the same result.

Let r be the radius of the particle swept up, r' that of the particle with which it collides in the cloud, and let r'/r be denoted by x. Let, further, v_0 denote the velocity with which the particle enters the cloud, and v the minimum velocity required to vaporize the two particles at an average collision. If m is the mass of the particle with radius r and velocity v, the velocity V of the combined mass is given by

$$m\,(1 + x^3)\, V = m v,$$

its kinetic energy is

$$\tfrac{1}{2} m\,(1 + x^3)\, V^2 = \tfrac{1}{2} m v^2 \frac{1}{1 + x^3}.$$

The kinetic energy released in the form of heat in the case of a head-on collision is therefore

$$\tfrac{1}{2} m v^2 - \tfrac{1}{2} m v^2 \frac{1}{1 + x^3} = \tfrac{1}{2} m v^2 \frac{x^3}{1 + x^3}. \qquad (10)$$

Using again a value of $1{\cdot}05 . 10^{10}$ erg for the evaporation energy per gram, the minimum velocity v, in km/sec, required for evaporation in case of a central collision is found to be

$$v = 1{\cdot}45\, (x^{3/2} + x^{-3/2})\ \text{km/sec}. \qquad (11)$$

It will be noted that v is a pure function of x. Values of v for various values of x are given in Table 1. In case the collision is not central, part of the energy (10) is absorbed by the rotation of the combined mass. Considering the uncertainty of other parts of the calculations we did not deem it worth while to make a rigorous computation of the effect of this rotation, which would have required estimates of the flattening of the combined mass formed as a result of the collision. In the following we have simply assumed that the velocities given by (11) would suffice for evaporation at all encounters. It may be roughly estimated that for $x = 1$ the value of the function $w(x)$ (see below) has become about 50% too high by this procedure; the error becomes negligible for $x \leq 0{\cdot}5$ and $x \geq 2$. The effect upon the result for $n(r)$ is not very serious.

Once v has been found, the greatest depth D to which a particle with radius r can penetrate with sufficient velocity to volatilize upon hitting a particle with radius r' may be computed by means of (7), inserting v for v_{min}. The probability for such a particle of evaporating through collision with a particle of radius between r' and $r' + dr'$, when swept up by a cloud, is then

$$\pi\, (r + r')^2\, D \Delta\, n(r')\, dr', \qquad (12)$$

or, using (7) and introducing $x = r'/r$,

$$4{\cdot}44 . 10^{23}\, \pi\, r^4\, (1 + x)^2 \{\ln v_0 - \ln v(x)\}\, n(xr)\, dx. \qquad (13)$$

In order to obtain, for a certain value of r and x, the total probability of evaporation, (13) should be multiplied by the probability that in a million years the particle is swept up by a cloud with velocity between v_0 and $v_0 + dv_0$, and the resulting expression should be integrated over all values of v_0 greater than v. Using the result found in the last paragraph of the preceding section the probability of encountering a cloud with velocity v_0 may be taken $0{\cdot}11 f(v_0)$ [1]), $f(v_0)$

[1]) The primitive picture of particles being swept up by a cloud seems sufficient for our present purpose, although, in reality, the phenomena are more complicated. As BURGERS has pointed out, a compression wave in the interstellar gas will precede a moving cloud, while another compression region will proceed into the cloud itself. It is easily seen, however, that these complications will have no influence on our calculations, because the resulting values for the probability of evaporation depend upon the relative velocities, and not upon the densities of the encountering layers.

[1]) After the calculations had been finished it was found that a factor $v_0/22{\cdot}5$ had inadvertently been omitted, the correct expression for the probability of encountering a cloud with a velocity v_0 being $0{\cdot}11\, v_0 f(v_0)/22{\cdot}5$. The same factor $v_0/22{\cdot}5$ should be applied to the integrands of (14) and (15). In view of the uncertainty of all relevant data, the influence of which greatly exceeds that of the omitted factor, we did not deem it worth while to repeat all computations with the corrected formulae. For particles effective in interstellar scattering the correct expressions would have made the probability of evaporation about 20% higher. This error counterbalances about half of the error which was introduced by the treatment of the non-central collisions.

being given by (2). A factor 2 should still be added in order to take account of the fact that at each evaporation of a particle entering the cloud a cloud particle is likewise vaporized. The evaporated cloud particle will in the average be smaller than the particle entering the cloud, so that, strictly, a more refined reasoning would be required. The complication arises from the more or less artificial distinction made between cloud particles and non-cloud particles. In section 7 an attempt has been made to remove this imperfection of the theory. It is indicated that the factor 2 just mentioned is approximately correct.

We thus obtain the following expression for the total chance of evaporation as a function of r and x

$$9{\cdot}8.10^{22}\,\pi r^4\,(1+x)^2\,n(xr)\,dx\int_v^\infty \{\ln v_0 - \ln v(x)\}\,f(v_0)\,dv_0{}^1). \qquad (14)$$

$P(r)$ is obtained by integrating (14) over all values of x. Introducing

$$w(x) = 0{\cdot}134\,(1+x)^2\int_v^\infty \{\ln v_0 - \ln v(x)\}\,f(v_0)\,dv_0{}^1), \qquad (15)$$

$w(x)$ thus containing the factors which depend on x only[2]), we get

$$P(r) = c\,r^4\int_0^\infty w(x)\,n(xr)\,dx, \qquad (16)$$

where

$$c = 2{\cdot}3.10^{24}. \qquad (17)$$

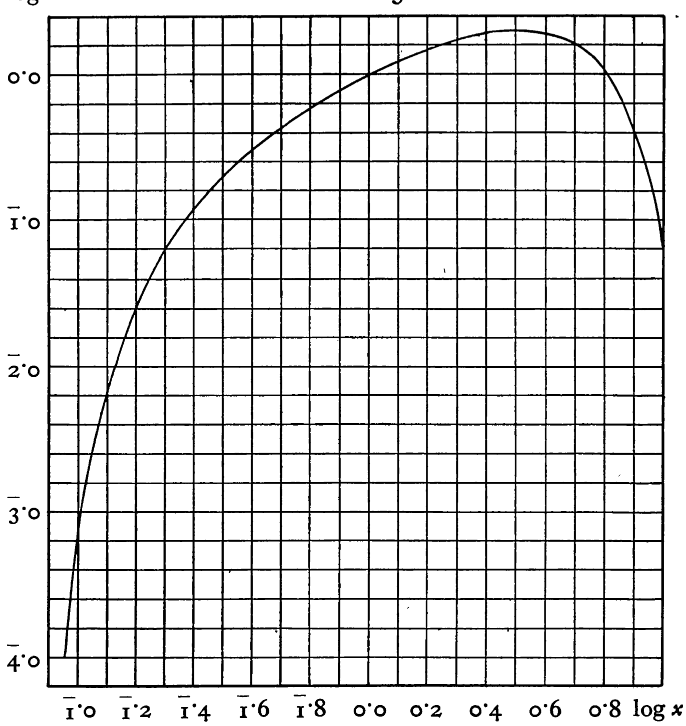
Table 1 and Figure 3 show the function w as used in the following computations. It was obtained from a somewhat rough numerical integration, and was assumed to be zero for $x < 0{\cdot}08$, which is equivalent with assuming that there exist no clouds with velocities exceeding 63 km/sec. The results in the table have been checked by means of the formula given in footnote 2.

TABLE 1

Values of $w(x)$ and of the minimum velocities required for evaporation

$x = r'/r$	v	$w(x)$	$x = r'/r$	v	$w(x)$
	km/sec			km/sec	
0·09	53·7	0·0001	0·75	3·17	0·737
·10	45·8	·0007	1·00	2·90	1·000
·12	34·9	·0048	1·33	3·17	1·307
·14	27·8	·0135	1·67	3·78	1·554
·17	20·8	·0342	2·50	6·09	1·918
·20	16·3	·0666	3·33	9·08	1·973
·25	11·8	·120	4·00	11·8	1·918
·30	9·08	·178	5·00	16·3	1·665
·40	6·09	·307	7·14	27·8	0·702
0·60	3·78	0·560	10·00	45·8	0·071

FIGURE 3



Equation (9) can now be written

$$\frac{d\ln n(r)}{d\ln r} = -\frac{c}{\dot{r}}\,r^5\int_0^\infty w(x)\,n(xr)\,dx. \qquad (18)$$

In order to eliminate the dimensions in (18) we introduce the dimensionless quantities

$$\mathbf{n} = n(r)/n(0),\ \mathbf{r} = r/l, \qquad (19)$$

l being the unit of length in cm. We thus obtain

$$\frac{d\ln \mathbf{n}(\mathbf{r})}{d\ln \mathbf{r}} = -q\,\mathbf{r}^5\int_0^\infty w(x)\,\mathbf{n}(x\mathbf{r})\,dx, \qquad (20)$$

in which

$$q = \frac{c\,n(0)}{\dot{r}}\,l^5. \qquad (21)$$

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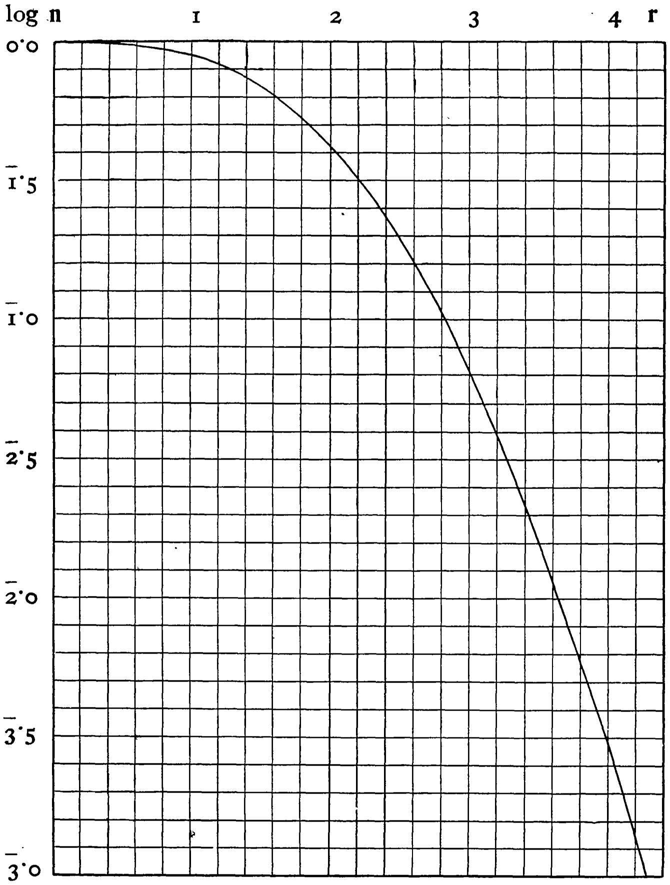
1) See footnote p. 195, second column.

2) The constant 0·134 was chosen so that $w(1) = 1$. Using (2), and inserting $v = 1{\cdot}45\,(x^{3/2} + x^{-3/2})$ from (11), we can write $w(x) = 0{\cdot}134\,(1+x)^2 \times \frac{1}{2}K(y)$, in which

$$y = \left\{1{\cdot}45\,h\,(x^{3/2} + x^{-3/2})\right\}^2, \text{ and } K(y) = \int_y^\infty e^{-x}\,\frac{dx}{x}.$$

The latter function may be obtained from existing tables.

FIGURE 4



The distribution function of the radii

The value of q can be changed at will by the choice of the unit of length, l. Therefore, the equation need only be solved for one particular value of q. We have taken

$$q = 0{\cdot}1725, \tag{22}$$

and therefore

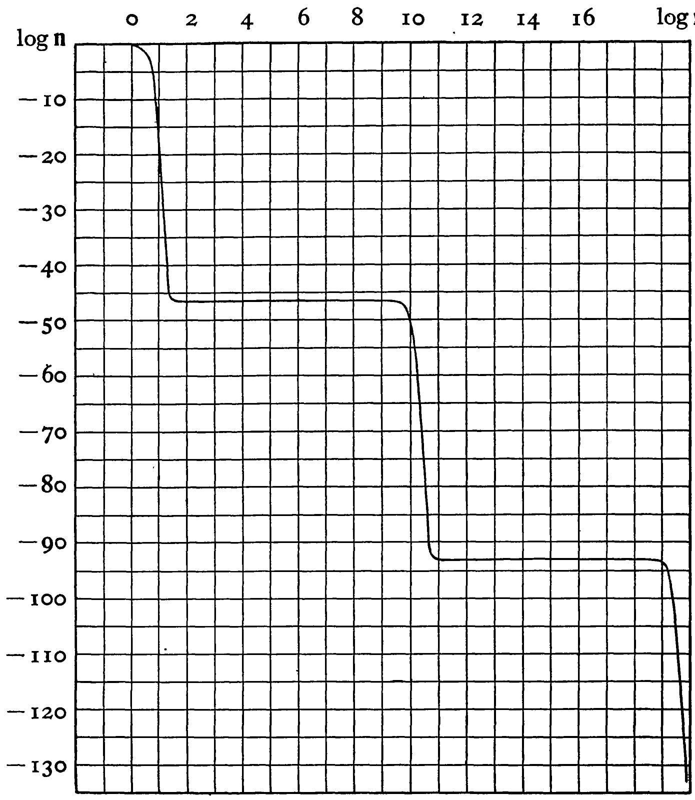
$$l = \left(\frac{\dot{r}}{kn(0)}\right)^{1/5}, \text{ with } k = \frac{c}{0{\cdot}1725}. \tag{23}$$

For the numerical integrations (20) was used in the form

$$\mathbf{n}(\mathbf{r}) = e^{-q\int\limits_0^{\mathbf{r}}\left\{\mathbf{r}^4\int\limits_0^{\infty} w(x)\,\mathbf{n}(x\mathbf{r})\,dx\right\}d\mathbf{r}} \quad . \tag{24}$$

Inserting provisional estimates of $\mathbf{n}(x\mathbf{r})$ in the exponent, new values of $\mathbf{n}(\mathbf{r})$ were computed by means of (24), and in this way a solution of the differential equation was found without too much trouble after a few successive approximations. The solution is given in Table 2 and Figures 4 and 5. It has, of course, no practical significance beyond about $\mathbf{r} = 5$, where the frequency becomes vanishingly small. When proceeding to still larger values of $\mathbf{r}$ the density finally drops so low that the chance of collision with a particle of comparable size, and therefore, also

FIGURE 5



The distribution function of the radii for large values of $\mathbf{r}$

the chance of evaporation, becomes negligible. This happens near $\mathbf{r} = 40$. The frequency then becomes constant. A new drop follows eventually, but not before $\mathbf{r}$ has grown to about 10^{10}, when the integrand between the brackets in (24) becomes again appreciable. A little reflexion shows that the drops and constant parts will recur in exactly the same way after intervals of 9·286 in log r. As seen from Table 2 the level at which $n(r)$ becomes constant for the second time lies near log n = log $n(0) - 46{\cdot}43$. Let us denote this value of n by $n_2(0)$. Now, the exponent in (24) is proportional with r^5 and n, so that in the second horizontal part of the curve this exponent will become equal to that in the first part for values of r^5 which

TABLE 2
The distribution function of the radii

$\mathbf{r}$	log $\mathbf{n}(\mathbf{r})$	$\mathbf{r}$	log $\mathbf{n}(\mathbf{r})$
0	0·00	12	5·47 — 30
1	9·95 — 10	14	0·11 — 30
2	9·63 — 10	16	5·62 — 40
3	8·83 — 10	18	2·09 — 40
4	7·49 — 10	20	9·45 — 50
5	5·62 — 10	24	6·17 — 50
6	3·29 — 10	28	4·63 — 50
7	0·61 — 10	34	3·81 — 50
8	7·67 — 20	40	3·61 — 50
9	4·57 — 20	50	3·57 — 50
10	1·45 — 20	60	3·57 — 50

are $n(0)/n_2(0)$ times larger. If we denote the thus corresponding values of the radius by r_1 and r_2, r_1 lying on the first part, r_2 on the second part of the curve, the shape of the curve $n(r_2)$ will be the same as that of $n(r_1)$. For, in the exponent of (24) we can write $n(xr) = \frac{n(xr_2)}{n(0)}$ as $\frac{n(xr_2)}{n_2(0)} \frac{n_2(0)}{n(0)}$. The second factor is compensated by the factor $(r_2/r_1)^5$, so that equation (24) in r_2 becomes identical with the original equation in r_1. The solution satisfying the latter will therefore also satisfy the former, hence $n(r_2)/n_2(0) = n(r_1)/n(0)$. The second part of the curve of $\log n$ against $\log r$ will thus be identical with the first part shifted over $\log n_2(0) - \log n(0)$ in $\log n$ and over $\frac{1}{5} \{ \log n_2(0) - \log n(0) \}$ or 9·286 in $\log r$. This property has been used in constructing Figure 5, a figure which may be of mathematical interest, but which has no physical significance. For it should be noted that particles of large radius will no longer be stopped or carried along by the clouds, and conditions will therefore become different from those on which the present theory is based.

The interval between successive drops and the height of the steps depend on the function $w(x)$. If the evaporation heat should be larger, or the average velocity of the clouds lower, so that $w(x)$ gets a narrower maximum (which means that particles can only evaporate when meeting particles of more nearly their own size), the steps may become much smaller. In the limiting case, when evaporation would take place only when two identical particles collide, the steps disappear altogether, and we obtain $n(r) = Cr^{-5}$.

There are no signs of a more or less continuous transition between these small interstellar particles and the meteors, as the latter are much more frequent than the interstellar particles of these sizes would be according to Table 2. It should be emphasized, however, that it is still doubtful whether any of the meteors are hyperbolic, and thus interstellar. In any case the true number of hyperbolic meteors may well be of an entirely lower order than that indicated in recent researches.

In order to obtain the actual distribution function of the radii we still have to determine the unit of length from equation (23). Each of the three parameters entering into l has a large uncertainty.

The value of $\dot{r}$ depends on the interstellar density, ρ_g, of the gases other than hydrogen and helium, upon their temperature, and upon the composition of the solid particles. If it is assumed that all heavier atoms hitting a particle stick to it, and that for each such atom two hydrogen atoms can be absorbed, and if the influence of the electrical charge of the particles is neglected, we find easily that

$$\dot{r} = 3\rho_g v / 4s. \tag{25}$$

In this formula v is the average velocity of the heavier atoms; we shall adopt a value of $3 \cdot 6 . 10^5$ cm/sec or $1 \cdot 14 . 10^{19}$ cm/million years, corresponding to the average velocity of oxygen atoms at $T = 10000°$. The specific weight s of the particles will be assumed to be 1. Concerning ρ_g there is considerable uncertainty. The density of interstellar hydrogen can be fairly well estimated; with B. STRÖMGREN we shall assume an average of two atoms per cm^3. Nothing reliable is however known about the relative concentration of the more frequent heavier atoms like oxygen. If, by analogy to what has been found in stellar atmospheres and planetary nebulae [1]), it is assumed that there is one "heavy" atom to thousand hydrogen atoms, so that $\rho_g = 5 \cdot 3 . 10^{-26}$, $\dot{r}$ is found to be $4 \cdot 5 . 10^{-7}$ cm/million years. Because of the uncertainty concerning the interstellar density of O, Ne, N, C, etc. and concerning other factors, we have taken as extreme possible limits $4 . 10^{-8}$ and $4 . 10^{-6}$, while $4 . 10^{-7}$ cm/million years has been adopted as the most probable value.

The parameter $n(0)$ is determined by the rapidity with which new solid particles are being formed. Let the number of particles formed per unit of time be denoted by N. Because near $r = 0$ the evaporation through encounters is negligible, we have

$$N = \dot{r}\, n(0). \tag{26}$$

An extensive discussion of the formation of solid particles has been given by TER HAAR [2]), and by KRAMERS and TER HAAR [3]). In the latter article it was found that the frequency of the formation of tri-atomic molecules was at most $7 . 10^{-17}$ $\rho_{CH}\, \rho_H$ per second, ρ_{CH} and ρ_H standing for the numbers of CH molecules and H atoms per cm^3. In the case of a quasi-equilibrium of the sort considered by BECKER and DÖRING (c.f. also p. 188 above), where the numbers of the different molecules do not vary any longer with the time, this expression would also give the number of solid particles formed per second. KRAMERS and TER HAAR have shown, however, that in the range of molecules along which the solid particles are built up, there is probably at least one step for which the stationary state has not yet been reached, namely that in which the second non-hydrogen atom is added.

[1]) UNSÖLD, *Zs. f. Ap.* **21**, 22, 1941; RUSSELL, cited in *Astr. News Letter* No. 3. For the planetary nebulae we may now refer to an article by ALLER and MENZEL in *Ap. J.* **102**, 239, 1945, which has appeared since the above article was written. According to WYSE (*Ap. J.* **95**, 385, 1942) there are no indications of important differences of chemical composition between the planetaries and the diffuse nebulae, so that it is not unlikely that the relative abundance cited applies also to the interstellar gas.

[2]) *B.A.N.* **10**, 1, 1943.

[3]) *B.A.N.* **10**, 137, 1946.

At such a step the stream of growing molecules is slowed down by a factor $\gamma\tau$, where γ is the probability of capture at an average collision and τ the number of collisions in 3.10^9 years[1]). In the present case $\gamma\tau$ may be estimated as perhaps $1/10$. The number of solid particles formed per second would then be of the order of $10^{-17}\,\rho_{CH}\,\rho_H$. With $\rho_{CH} \sim 10^{-6}$, $\rho_H \sim 1$ this becomes 10^{-23} per second, or $N = 3.10^{-10}$ per 10^6 years. With $\dot{r} = 4.10^{-7}$ we then obtain $n(0) = 10^{-3}$ cm^{-4}. The value of N must be considered as an upper limit, for, in the first place, the factor 7.10^{-17} was an upper limit, in the second place the numbers of triatomic and higher molecules might be cut down by photodissociation, while finally it is quite possible that there are several more steps in which the equilibrium has not been reached. Indeed the calculations of section 8 make it appear probable that some five more factors of $1/10$ should be added. We have accordingly considered a large range of possible values of $n(0)$, extending from 10^{-3} to 10^{-9} cm^{-4}.

As regards the third constant in (23), k, measuring the probability of evaporation, a provisional estimate has led to the value $c = 2{\cdot}3.10^{24}$ given in (17), corresponding with $k = 1{\cdot}3.10^{25}$. Apart from the possibility of whirl-like motions, which has been discussed on p. 193, and which did not seem very probable, the true value of k is likely to be somewhat higher. For the value quoted was based on the assumption that a line of sight of 1000 ps would cross five independent clouds, which is certainly a minimum value. Moreover, considerable internal motions have been observed inside the Orion nebula; such internal motions within the clouds themselves may evidently enhance the chance of collisions. The following calculations have been carried through for three values of k, viz. $1{\cdot}3.10^{24}$, $1{\cdot}3.10^{25}$ and $1{\cdot}3.10^{26}$. The true value is more likely to be in between the last two values.

Table 3 on p. 202 shows the data from which the distribution of the radii can be inferred, for these three values of k, for $\dot{r} = 4.10^{-8}, 4.10^{-7}$ and 4.10^{-6}, and for four different values of $n(0)$. The results corresponding to the direct estimates for $\dot{r}$ and k given above have been printed in italics, the other numbers are intended to show the extreme possible range corresponding to the uncertainty of the data. For each combination of the parameters, l shows the factor by which the abscissae in Figure 4 and the values of $\mathbf{r}$ in Table 2 should be multiplied in order to be reduced to cm. The quantities $r_{1/2}$ in the second division indicate the radii for which the frequency has become $\frac{1}{2}n(0)$. They are meant to be rough measures of the radius where the steep drop in the frequency curve starts; at twice this radius the frequency has already fallen to $\frac{1}{125}\,n(0)$. From Figure 4 we find that $r_{1/2}$ is $1{\cdot}86\,l$.

1) Kramers and ter Haar, *l.c.* p. 140.

It will be observed that for most of the range covered by the parameters the radius $r_{1/2}$ lies between one and one-tenth times the wave-length of light, and therefore generally in the region where the scattering is selective but not yet according to Rayleigh's law. This is a very satisfactory result, indicating that the theory of evaporation through collision may well be capable of explaining the observed law of scattering. More concerning this will be said in the last section. With previous theories, based on continual growth of the particles, this point formed a notable difficulty, in as much as it could only be considered as a pure accident that the velocity of growth and the age of the galactic system were just such that the frequency function of the radii broke off somewhere near the wave-length of light. In the case now considered variation of the parameters has much less effect upon l and $r_{1/2}$ because of the 5th root occurring in (23).

We may inquire to what radius the particles will on the average grow. Once the function $\mathbf{n}(\mathbf{r})$ is known this average maximum radius can be computed with the aid of (16). If we take also roughly account of the possibility of melting together (as discussed in section 7) we find $\mathbf{r}_{max} = 1{\cdot}6$. This is about $1{\cdot}5$ times the average radius of all particles.

With the constants adopted in section 8 this average maximum radius would correspond to $0{\cdot}22$ microns, and the average life of a particle would be 54 million years. It disappears, either by evaporation through collision with another particle, or by fusing together with the colliding particle, the probability of the latter process being about half that of the former.

7. *The error made by neglecting fusion.*

In the foregoing calculations the effect of collisions resulting in a fusion of two particles has been neglected. In this section we shall show by a rough computation that the effect of these fusion processes is relatively unimportant.

We assume, somewhat arbitrarily, that two particles will fuse together as soon as the kinetic energy set free at a collision would suffice to heat up the total mass to its melting point and to liquify one half of it. With the values previously assumed for the melting point and melting heat (p. 193) an energy of 70 cal or $0{\cdot}29.10^{10}$ erg would be needed for a total mass of 1 g. The minimum velocity in km/sec required for fusion would then be

$$v' = 0{\cdot}76\,(x^{3/2} + x^{-3/2}), \tag{27}$$

which is analogous to formula (11).

From (14) we see that the number of collisions in which particles hit each other with velocities higher than v is proportional to

$$M(v) = \int_v^\infty (\ln v_0 - \ln v)\, f(v_0)\, dv_0 \tag{28}$$

and that the number of such collisions between a particle of radius r to $r+dr$ and a particle of radius r' to $r'+dr'$ is

$$K r (r+r')^2 \, n(r) dr \, n(r') \, dr' \, M(v) \qquad (29)$$

per unit volume per unit time. The value of the constant is

$$K = 9{\cdot}8\pi \, 10^{22} \, (10^6 \, \text{year})^{-1}. \qquad (30)$$

The number of collisions giving evaporation is read from (29) by substituting into it the value v defined by (11). On substituting the value v' defined by (27), however, we obtain the number of collisions resulting in either evaporation or fusion of the particles. The difference between these numbers is the net number of fusion processes.

Before we proceed to calculate these numbers, a slight inconsistency of the formulae used in the preceding section must be removed. At first sight it seems queer that (29) involves r and r' in a different way. This however is inherent to the picture we made of the clouds sweeping up the particles they meet. The cloud particle has the radius r' and the entering particle the radius r. As these particles are not in the same manner involved in the events described in section 5, the formula giving the number of collisions is asymmetrical. Therefore (29) is correct, though inevitably it is based on a somewhat ill-defined model.

The equation of equilibrium used in section 6 may be written in a slightly different form, which is convenient for our present discussion:

$$-\frac{d\mathbf{n}(\mathbf{r})}{d\mathbf{r}} \cdot \frac{1}{Q} = \mathbf{n}(\mathbf{r}) \, \mathbf{r} \int_0^\infty (\mathbf{r}+\mathbf{r}')^2 \, \mathbf{n}(\mathbf{r}') \, M\{v(x)\} \, d\mathbf{r}'; \qquad (31)$$

it is identical with (20) if $Q = 0{\cdot}134 \, q$. Like (29) this equation is asymmetrical with respect to $\mathbf{r}$ and $\mathbf{r}'$.

If independent solutions were to be made for the distributions of "entering particles" and "cloud particles", we should obtain two slightly different functions. In assuming the same function $n(r)$ throughout the interstellar medium, however, we follow the suggestion, made in section 4, that the clouds might be formed continually. Some reasoning shows that the introduction of a factor 2 into (14), doubling the values of q and Q in all further equations up to (31), is not strictly correct. Instead we must replace the factor $\mathbf{r}$ in (29) and (31) by $\mathbf{r}+\mathbf{r}'$, so that we obtain

$$-\frac{d\mathbf{n}(\mathbf{r})}{d\mathbf{r}} \cdot \frac{1}{Q} = \mathbf{n}(\mathbf{r}) \, \mathbf{r}^4 \int_0^\infty (1+x)^3 \, \mathbf{n}(\mathbf{r}x) \, M\{v(x)\} \, dx. \qquad (32)$$

The terms of this equation denote increase and decrease of the number of particles of a specified radius r. We can take the fusion into account by adding two more terms to the right-hand member of this equation. One gives the number of particles fusing together with another particle, and the other gives the number of particles formed by fusion. The first two terms of the right-hand member can then be taken together by merely replacing $v(x)$ by $v'(x)$. The last term, however, requires a more detailed computation. Therefore, we write our new equation:

$$-\frac{d\mathbf{n}(\mathbf{r})}{d\mathbf{r}} \cdot \frac{1}{Q} = \mathbf{n}(\mathbf{r}) \, \mathbf{r}^4 \int_0^\infty (1+x)^3 \, \mathbf{n}(\mathbf{r}x) \, M\{v'(x)\} \, dx + \text{third term}. \qquad (33)$$

A provisional estimate of the fusion effect can be made without computing the third term, if we integrate (33) from $r = 0$ to $r = \infty$. Let the suffix e denote evaporation and f denote fusion. The integrated equation, which gives the balance of the *total number* of particles, then takes the form

$$\frac{1}{Q} = I_e + I_f - \tfrac{1}{2} I_f. \qquad (34)$$

We have written the last term in this form, since *two* particles fusing together yield *one* new particle of a different diameter.

As a first approximation to the solution of (33) we use the function $\mathbf{n}(\mathbf{r})$ given by the solid line in Figure 6, which is approximately the same function as given in Table 2. By substituting it into the right-hand member of (32) and integrating this, we find $I_e = 70$; this means that

$$\frac{1}{Q} = 70 \qquad (35)$$

is the value of $1/Q$ for which $\mathbf{n}(\mathbf{r})$ is the solution of (32). In the same way the integrated right-hand member of (31) has the value 38, which is nearly half the above value, as we had already expected. Owing to the asymmetry of (31) the proportion is not exactly $\frac{1}{2}$. On integrating the first term in the right-hand member of (33) we finally find $I_e + I_f = 104$. Therefore $I_f = 34$, and

$$\frac{1}{Q} = 70 + 34 - 17 = 87 \qquad (36)$$

is the value of $1/Q$ for which $n(r)$ *may* be the solution of (33). Since Q is proportional to q and, therefore, by (21), to l^5, the changes in the scale factor l are readily obtained. According to (35) and (36) the fusion processes could be taken into account by multiplying l and $r_{1/2}$ by a factor $(70/87)^{1/5} = 0{\cdot}96$. Evidently this is of no consequence for the conclusions stated in section 6. Therefore, though in an exact calculation these factors should be taken into account, we have left the results in Table 2 unchanged.

It is not a priori evident that the shape of the distribution function remains the same, while the scale of r is slightly changed. This assumption, which is the basis of our provisional estimate, cannot be checked by the integrated equation (34). We must ascertain that (33) is satisfied for all values of r. To do so, we must yet compute the third term.

If in (29) we replace $M(v)$ by $M(v')-M(v)$, this expression denotes the number of fusion processes between particles of radii r and r'. Let this be shortly written

$$N(r, r')\, dr\, dr'.$$

The radius r'' of the new particle, which for convenience is assumed to be spherical, is given by $r''^3 = r^3 + r'^3$. Let $y = r^3/r''^3$ be the fraction of the mass contributed by the first particle and $1-y = r'^3/r''^3$ the other part. We change then to new variables r and y and find that $x = y^{1/3}/(1-y)^{1/3}$, and, after some reductions, that the above equation is transformed into

$$N\{y^{1/3}r'', (1-y)^{1/3}r''\}\, \tfrac{1}{3}\, y^{-2/3}(1-y)^{-2/3}\, dy\, r''dr''. \quad (37)$$

The number of particles that have radii between r'' and $r''+dr''$, formed by fusion per unit time and unit volume, is found by integrating (37) from $y = 0$ to $y = 1$. The value of this integral remains the same if we replace the asymmetrical factor r in the integrand by $r+r'$ and add a factor $\frac{1}{2}$. Finally, after omission of the constant K and the differential dr'', we obtain

$$\text{third term} = \tfrac{1}{2} r''^4 \int_0^1 n(r)\, n(r')\, \{M(v') - M(v)\}\, (r+r')^3 \frac{r''^4}{3 r^2 r'^2}\, dy \quad (38)$$

as the final form of the third term of (33). Here r and r' should be regarded as functions of r'' and y as shown in (37).

FIGURES 6 AND 7

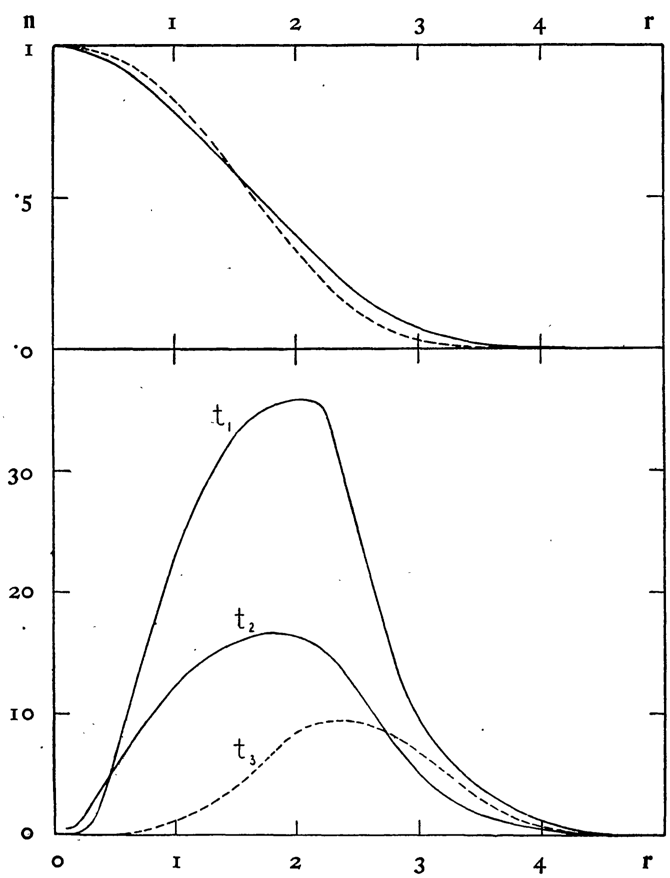


Fig. 6 (above): Influence of fusion on the distribution function of the radii. The full-drawn curve shows the distribution corresponding to Figure 4 and Table 2, the dotted curve shows the distribution roughly corrected for the effects of fusion.

Fig. 7 (below): Numbers of particles disappearing by collisional evaporation (t_1), and by fusing together with a colliding particle (t_2). The number of new particles formed as a consequence of the fusion is shown by the dotted curve t_3.

In order to make the decisive check we calculated graphically the right- and left-hand members of (33). There appeared to be only a small difference: the fusion processes tend to make the right-hand curve somewhat higher and narrower than it was supposed to be. A second approximation to the correct solution of (33) might now be made. If the result of the right-hand member is interpreted in terms of dn/dr, the function $n(r)$ obtains a somewhat steeper slope than previously assumed, as shown by the dotted line in Figure 6. But obviously there is no reason for making such a new approximation, since no change in our conclusions would result.

Interesting informations can be read from the graphs of the separate terms of equation (33) shown in Figure 7. The term t_1 is the number of particles expiring by evaporation. About seventy per cent of the evaporating particles have radii between $r = 1$ and 2·5. The next term t_2 showing the number of particles expiring by fusion shows a similar curve. Instead of these, new particles appear in numbers shown by the curve of t_3. The net result of the fusion processes is a loss of particles with radii 0 to 2·7 and a gain of bigger particles. Total areas under each curve have been mentioned above. A check on the calculation of t_3 is that the area it encloses is indeed 17, which is half the area enclosed by t_2.

8. *The extinction by interstellar particles.*

We wish to compute the interstellar extinction and scattering which will occur if the frequency function of the radii is such as derived in the preceding sections. We assume rather arbitrarily that the particles have the refractive index 1·33 like ice, and that they are perfectly spherical. Other refractive indices, ranging from 1·2 to 1·5, or somewhat irregular forms, would give much the same results. In view of the probable mode of formation we consider it improbable that the

TABLE 3

Constants of the frequency function of the radii and photographic absorption coefficients for various combinations of the parameters $n(0)$, k and $\dot{r}$

	$\dot{r}$ →	$4 \cdot 10^{-8}$	$4 \cdot 10^{-7}$	$4 \cdot 10^{-6}$	$4 \cdot 10^{-8}$	$4 \cdot 10^{-7}$	$4 \cdot 10^{-6}$	$4 \cdot 10^{-8}$	$4 \cdot 10^{-7}$	$4 \cdot 10^{-6}$
$n(0)$	k	log l			$r_{1/2}$ (microns)			log a_{pg}		
10^{-9}	$1·3 \cdot 10^{24}$	−4·50	−4·30	−4·10	·59	·94	1·48	0·46	1·04	1·59
	$1·3 \cdot 10^{25}$	−4·70	−4·50	−4·30	·37	·59	·94	−0·10	0·46	1·04
	$1·3 \cdot 10^{26}$	−4·90	−4·70	−4·50	·24	·37	·59	−0·84	−0·10	0·46
10^{-7}	$1·3 \cdot 10^{24}$	−4·90	−4·70	−4·50	·24	·37	·59	1·16	1·90	2·46
	$1·3 \cdot 10^{25}$	−5·10	−4·90	−4·70	·15	·24	·37	0·17	1·16	1·90
	$1·3 \cdot 10^{26}$	−5·30	−5·10	−4·90	·09	·15	·24	−0·88	0·17	1·16
10^{-5}	$1·3 \cdot 10^{24}$	−5·30	−5·10	−4·90	·09	·15	·24	1·12	2·17	3·16
	$1·3 \cdot 10^{25}$	−5·50	−5·30	−5·10	·06	·09	·15	−0·08	1·12	2·17
	$1·3 \cdot 10^{26}$	−5·70	−5·50	−5·30	·04	·06	·09	−1·37	−0·08	1·12
10^{-3}	$1·3 \cdot 10^{24}$	−5·70	−5·50	−5·30	·04	·06	·09	0·63	1·92	3·12
	$1·3 \cdot 10^{25}$	−5·90	−5·70	−5·50	·024	·04	·06	−0·75	0·63	1·92
	$1·3 \cdot 10^{26}$	−6·10	−5·90	−5·70	·015	·024	·04	−2·15	−0·75	0·63

particles would be metallic, as has been supposed by some investigators.

Data on the extinction of light in the photographic region by spheres of different dimensions have been compiled by GREENSTEIN[1]). More complete data on the extinction, ranging from a very small to a very large proportion of radius to wave-length, were published by one of us[2]).

In computing the law of selective extinction for the distribution function of the radii and the value of the refractive index assumed, we used the curves given in this thesis. GREENSTEIN's data are in essential agreement with ours. Detailed results of the computations for various other distribution functions and other values of the refractive index will be published elsewhere[3]).

Let E be the "efficiency factor" for extinction, i.e. the amount of light actually removed from the incident wave, divided by the amount which would be removed if only the geometrically obstructed light were scattered. For a single particle E is a known function of $x = \frac{2\pi r}{\lambda}$ [4]).

In exactly the same way we define the mean efficiency factor $\bar{E}$ for a given mixture of particles of various sizes. If $n(r)$ is the distribution function of the radii, then

$$\bar{E} = \frac{\int E(x)\, n(r)\, r^2\, dr}{\int n(r)\, r^2\, dr} \quad . \tag{39}$$

1) *Harvard Circ.* No. 422, 1937 (Table X).
2) H. C. VAN DE HULST, *Thesis*, Utrecht, 1946 = *Rech. Astr. de l'Obs. d'Utrecht*, XI, part I.
3) Ibidem, Vol XI, part 2.
4) *Thesis l.c.* Figure 21.

If the unit of length is 1 cm the total absorption expressed in mag/kpc is

$$a = 10^{22 \cdot 02}\, \bar{E} \int n(r)\, r^2\, dr. \tag{40}$$

The distribution function $n(r)$ derived in the preceding sections has a well-defined form, but the scales both of ordinates and abscissae are subject to some uncertainty. Therefore we introduce

$$u = \frac{r}{r_{1/2}} = \frac{\mathbf{r}}{\mathbf{r}_{1/2}}, \quad \mathbf{n}(u) = \frac{n(r)}{n(0)}; \tag{41}$$

we then find by (39) that $\bar{E}$ is a function of

$$x_{1/2} = \frac{2\pi r_{1/2}}{\lambda} \tag{42}$$

that can be computed by numerical integration, and finally we find from (40) that the absorption coefficient is

$$a = 10^{22 \cdot 02}\, \bar{E}(x_{1/2})\, n(0) r_{1/2}^3 \int \mathbf{n}(u)\, u^2\, du. \tag{43}$$

All integrals extend from 0 to ∞. That in (43) can be computed from the function $\mathbf{n}(\mathbf{r})$ as given in Figure 4. Its value is 0·500.

By graphical integration according to (39) we constructed the curve showing $\bar{E}$ as a function of $x_{1/2}$. Since $x_{1/2}$ is proportional to $1/\lambda$, this curve gives the correct shape of the curve showing the interstellar extinction as a function of $1/\lambda$. Only the scales of both co-ordinates should still be fixed, by fixing the values of $r_{1/2}$ and $n(0)$.

We can now draw the extinction curve on the correct scale for each combination of parameters contained in Table 3. The values of a_{pg} read from these curves are given in the last section of the table. It is very satisfactory to see that the observed value,

$\log a_{pg} = 0{\cdot}0$ to $0{\cdot}3$, lies well within the possible range [1]). In a similar way it is satisfactory to find that $r_{1/2}$ is of the order of the wave-length, such as is needed to explain why the wave-length dependence lies between a λ^{-4}-law and a λ^{0}-law. These considerations give us a preliminary confidence in the result of the preceding sections. But a more accurate check may be obtained by proceeding from the observed law of interstellar extinction, and by trying to fix the parameters $r_{1/2}$ and $n(0)$ in (43) in such a way that theory and observations agree.

First we determine the correct scale of the abscissae. As is well known, the interstellar extinction, when plotted against $1/\lambda$, shows a nearly linear relation, with a slight curvature in the ultra-violet. The correct zero, which would be reached for very long wave-lengths, is not accurately known from the observations. In former discussions this datum was only used to infer that the particles must be of the order of the wave-length. In the present more accurate comparison we observe that the nearly linear increase of a in the observed interval $1{\cdot}0$ to $3{\cdot}0\ \mu^{-1}$ must coincide with the nearly linear increase of $\bar{E}$ in the interval $1{\cdot}57$ to $4{\cdot}70$ of $x_{1/2}$. According to (42) this means

$$r_{1/2} = 0{\cdot}25\,\mu, \qquad l = 1{\cdot}34.10^{-5},\ \log l = -4{\cdot}87. \tag{44}$$

If $r_{1/2}$ would differ as much as 20 per cent from this value the agreement would become much worse.

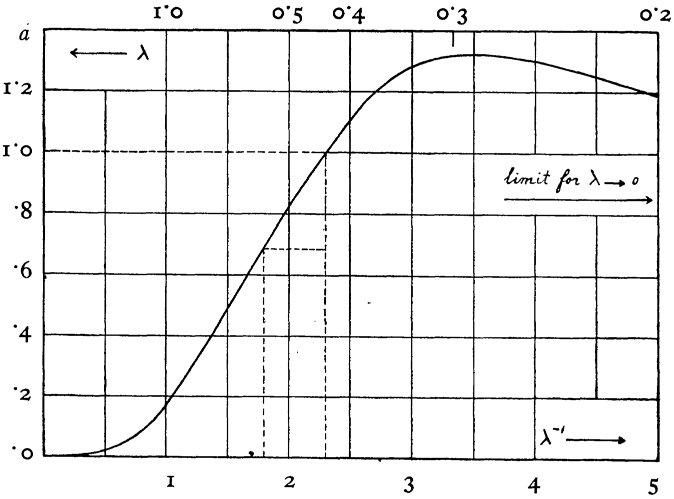
Next we determine the scale of the ordinates. For the photographic wave-length $\lambda = 0{\cdot}440\ \mu$ and the value of $r_{1/2}$ just found, we have $x_{1/2} = 3{\cdot}58$; the corresponding value of $\bar{E}$ is $2{\cdot}33$. Assuming a photographic extinction $a = 1$ mag/kps we can now compute $n(0)$ from (43). We thus obtain

$$n(0) = 5.10^{-9}\,\mathrm{cm}^{-4}. \tag{45}$$

This value may be wrong by a factor 2, like the absorption coefficient itself.

Having thus fixed the probable values of $r_{1/2}$ and $n(0)$, we constructed by (43) the complete curve of the extinction as a function of the inverse wave-length. It is shown in Figure 8. We note that in the observable range it has exactly the form that is revealed by the observations: a nearly linear graph with a less pronounced increase in the ultra-violet region. The value of the ratio of photographic to selective

FIGURE 8

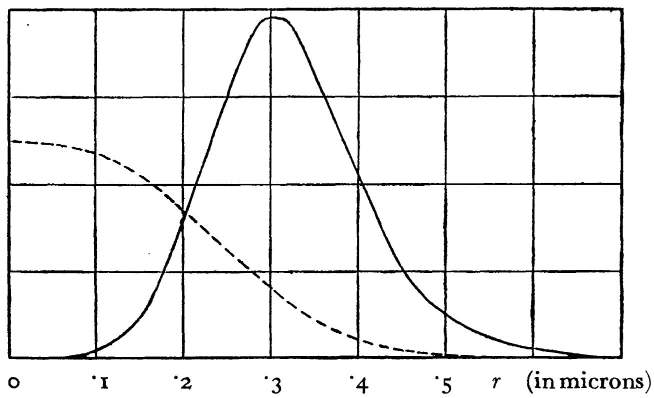


Computed interstellar extinction coefficient a, in mag/kps, as a function of $1/\lambda$, if the scales of the co-ordinates are fixed according to (44) and (45). The dotted lines indicate the visual and photographic absorptions.

extinction in the $0{\cdot}550\ \mu - 0{\cdot}440\ \mu$ system is found to be $3{\cdot}2$, in approximate agreement with most modern values.

As a further illustration we constructed Figure 9, in which the dotted curve shows the final distribution function of the radii, and the full curve the relative contributions of the particles of various sizes to the photographic absorption coefficient. The strongest contribution comes from the particles with radii $0{\cdot}31\ \mu$. Therefore, the particles with radii $1{\cdot}2$ times $r_{1/2}$ are most effective. To a fair approximation this holds for all wave-lengths. For the photographic wave-length it corresponds to $x_{\mathrm{eff}} = 4{\cdot}4$. The scattering by particles of these sizes has a strong preference for forward directions. The value of the asymmetry factor may be $0{\cdot}7$ to $0{\cdot}8$ [1]). This result is in good agreement with the observations by HENYEY and

FIGURE 9



Relative contribution of particles of various sizes to the total photographic scattering (full curve). The dotted curve shows the distribution function of the radii.

[1]) We may note, in passing, how little the absorption coefficient depends upon $n(0)$. It is far from proportional to this number: for $\dot{r} = 4.10^{-7}$, $k = 1{\cdot}3.10^{25}$, for instance, the million-fold range in $n(0)$ corresponds with but a five-fold range in a_{pg}; between $n(0) = 10^{-7}$ and 10^{-3} an *increase* in $n(0)$ appears even to entail a *decrease* in the absorption. This arises from the fact that when there are many particles they do not get a chance of growing, as the probability of evaporation through collision increases proportionally. The possible bearing of this on the occurrence and localisation of absorption in extra-galactic systems will be discussed in a subsequent article (c.f. also J.H. OORT, *George Darwin Lecture*, *M.N.* **106**, 1946, in print.)

[1]) VAN DE HULST, *Thesis*, Figure 23.

GREENSTEIN [1]) on diffuse radiation in the galaxy, which also require a large asymmetry factor.

Finally we compute the space density of interstellar smoke. It is

$$\rho_s = \tfrac{4}{3}\pi\sigma \int n(r)\, r^3\, dr,$$

where σ is the density of a particle. Using (41) we find

$$\rho_s = \tfrac{4}{3}\pi\sigma\, n(0)\, r_{1/2}^{\,4} \int \mathbf{n}(u)\, u^3\, du. \qquad (46)$$

Here the integral has the value 0·53. Estimating σ = 1 g/cm^3, and substituting the values (44) and (45) we then find

$$\rho_s = 0{\cdot}43 . 10^{-26}\ \mathrm{g/cm^3}.$$

This is about one-tenth of the density, ρ_g, of the "condensable gases" as estimated in section 6, and about one-thousandth of the total gas density. The density of the smoke for other values of the parameters may be computed by means of (46).

[1]) *Ap. J.* **93**, 70, 1941; *Ann. d'Astroph.* **3**, 117, 1940.

Summarizing the results of this section we state that the observational data on interstellar scattering are in rather good agreement with the theoretical calculations, if $r_{1/2}$ and $n(0)$ are fitted to the values (44) and (45). These values lie well within the range of the a priori estimates made in section 6, and compiled in Table 3. Only $n(0)$ is somewhat smaller than we expected, but this difference is easily explained by assuming that in the formation of condensation nuclei there are still other steps that are seriously hampered. We should *not* conclude from this agreement that the distribution law of the radii shown by Figure 9 is the correct one. There are various other forms that may give similar extinction curves, so that an observational determination of the distribution law is not very well possible.

We like to thank Profs. J. M. BURGERS, KRAMERS, KRONIG and MINNAERT, whose helpful and pleasant discussions on various points have contributed to our investigation.