

1. $2/n, 37, \sqrt{n}, n, n \log \log n, n \log n, n \log(n^2), n \log^2 n, n^{1.5}, n^2, n^2 \log n, n^3, 2^{\frac{n}{2}}, 2^n$

$$n \log(n^2) = 2n \log n = O(n \log n)$$

$n \log(n^2)$ and $n \log n$ grow at the same rate

2. which grows faster? $n \log n$ or $n^{1+\frac{\epsilon}{\sqrt{\log n}}}$ when $\epsilon > 0$?

assume: $f(x) > g(x)$ and prove by contradiction, where $n \log n = f(x)$ and $n^{1+\frac{\epsilon}{\sqrt{\log n}}} = g(x)$

$$\begin{aligned} n \log n &> n^{1+\frac{\epsilon}{\sqrt{\log n}}} \\ n \cdot \log n &> n \cdot n^{\frac{\epsilon}{\sqrt{\log n}}} \\ \log \log n &> \log n^{\frac{\epsilon}{\sqrt{\log n}}} = \frac{\epsilon}{\sqrt{\log n}} \log n \\ \log \log n &> \frac{\epsilon}{\log n^{\frac{1}{2}}} = \frac{\epsilon}{\log n^{\frac{1}{2}}} \cdot \frac{2 \log n}{2} \\ \log \log n &> \epsilon \sqrt{\log n} \\ \text{let } X &= \log n \text{ and we get} \\ \epsilon \sqrt{X} &< \log X \\ (\epsilon \sqrt{X})^2 &< (\log X)^2 \\ \epsilon^2 L &< \log^2 L \end{aligned}$$

Because ϵ is a constant greater than 0, we can see from the above that $\log^2 X < \epsilon^2 X$, this is then a contradiction to our statement above showing that $n^{1+\frac{\epsilon}{\sqrt{\log n}}}$ grows faster.

3. a. $O(n)$
 b. $O(n^2)$
 c. $O(n^3)$
 d. $O(n^2)$
 e. $O(n^5)$
 f. $O(n^4)$

Figure 1: mergeTime table

N	Time
5000	0.027925491
10000	0.057844639
15000	0.087766171
20000	0.12067771
30000	0.180515289
40000	0.257311821
50000	0.324133396

Figure 2: mergeTime graph

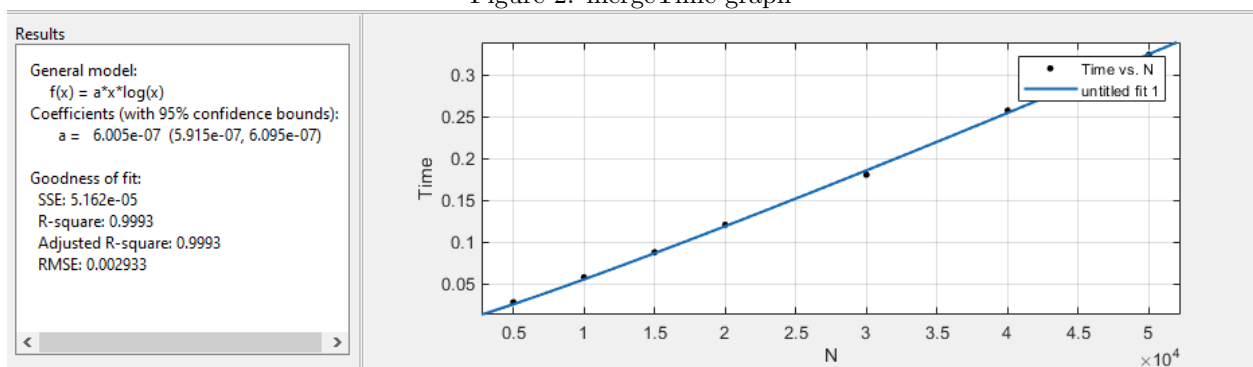


Figure 3: insertTime table

N	Time
5000	1.088094
10000	4.22339344
15000	9.55557251
20000	16.8260305
30000	38.0102339
40000	68.4598172
50000	108.419545

Figure 4: insertTime graph

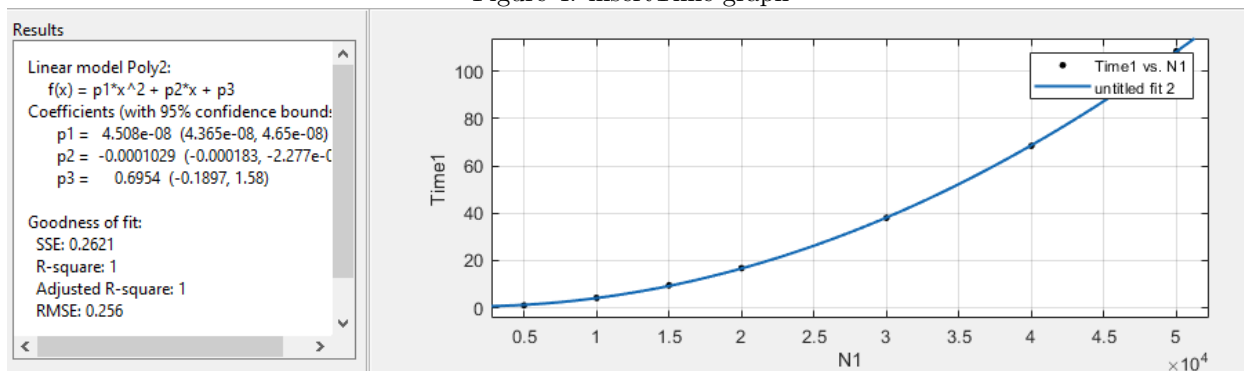
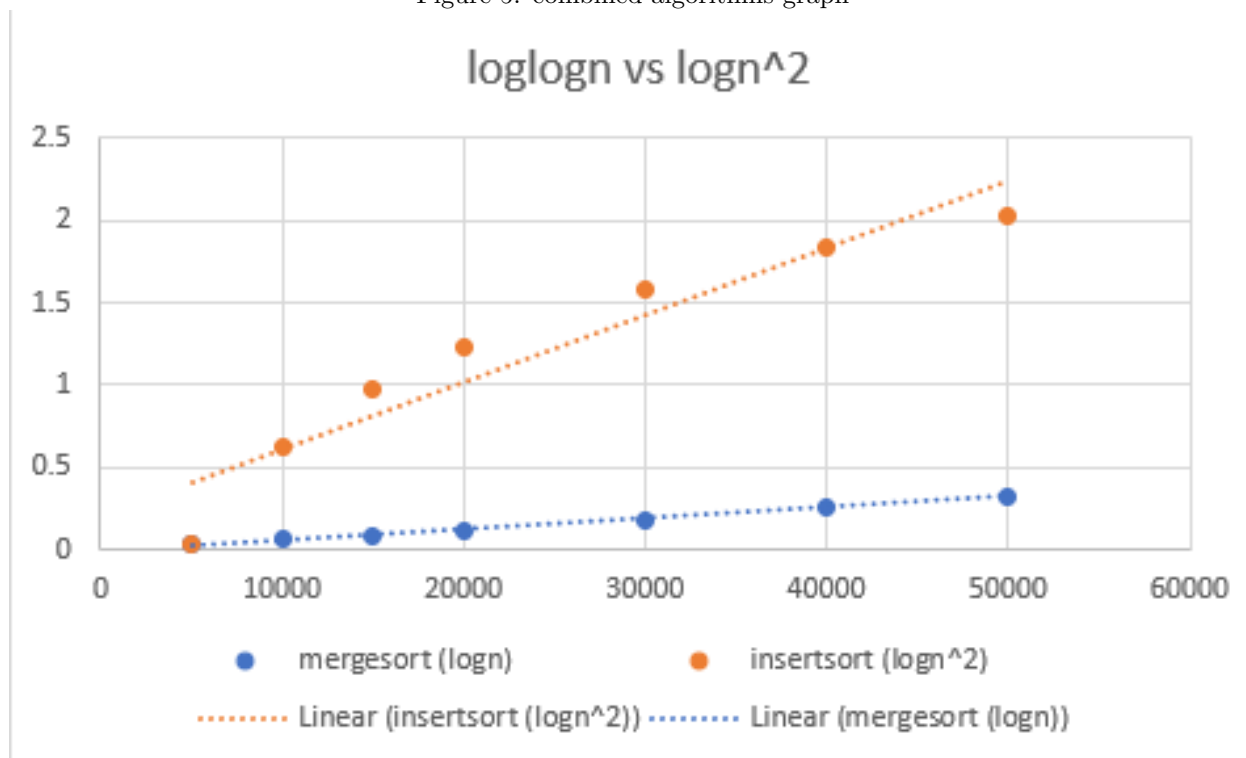


Figure 5: combined algorithms graph



The merge sort equation that best fits the graph curve is an $n \log n$ line, the best fit for the mergesort graph is an n^2 line.

The experimental run times vs the theoretical running times of the algorithms compare closely of $n \log n$ for Merge sort, and n^2 for insertion sort. The merge sort curve looks linear however, this is likely because the times are too close together to display a proper curve. This means that the variation of the run times was too small.