$$u(x,t) = \frac{1}{2}\sum_{+,-}\sqrt{\frac{c_0}{c(X^{\pm}(t))}}\, V\left(\frac{c_0\cdot(x-X^{\pm}(t))}{c(X^{\pm}(t))\cdot\mu}\right) + O(\mu|\log\mu|)$$

$\delta u(x,t) = ?$ (невязка)

$c(x) \in C^{\infty}(\mathbb{R})$

$0 \le c_1 \le c(x) \le c_2 \quad (*)$

$\dot{X} = c(X) \qquad \tau(X) = \int_a^X \frac{d\xi}{c(\xi)}$

□ 1^0. $\frac{1}{\mu^2}$:

1) $$\frac{\partial}{\partial t}u(x,t) \sim \sqrt{\frac{c_0}{c(X^{\pm}(t))}}\, V'\cdot\left\{\frac{c_0}{\mu}\cdot\left(-\frac{(x-X^{\pm})c'(X)c(X)}{c^2(X)} - \frac{c(X)}{c(X)}\right)\right\} =$$

$$= \sqrt{\frac{c_0}{c(X^{\pm}(t))}}\, V'\left\{-\frac{c_0}{\mu}\left(\frac{(x-X)c'(X)}{c(X)} + 1\right)\right\}$$

2) $$\frac{\partial^2}{\partial t^2}u \sim \sqrt{\frac{c_0}{c(X)}}\, V''\,\frac{c_0^2}{\mu^2}\cdot\left(\frac{c'(X)(x-X)}{c(X)} + 1\right)^2$$

3) $$\frac{\partial}{\partial x}u \sim \sqrt{\frac{c_0}{c(X)}}\, V'\cdot\frac{c_0}{\mu}\frac{1}{c(X)}$$

$$\frac{\partial}{\partial x}c^2(x)\frac{\partial}{\partial x}u = 2c(x)c'(x)\sqrt{\frac{c_0}{c(X)}}\, V'\left(\frac{c_0}{\mu}\frac{1}{c(X)}\right) + c^2(x)\sqrt{\frac{c_0}{c(X)}}\, V''\left(\frac{c_0}{\mu}\right)^2\left(\frac{1}{c(X)}\right)^2 =$$

$$\Rightarrow \frac{1}{\mu^2}\frac{\partial}{\partial x}c^2(x)\frac{\partial}{\partial x}u \sim \frac{c^2(x)}{c^2(X)}\sqrt{\frac{c_0}{c(X)}}\, V''\left(\frac{c_0}{\mu}\right)^2$$

4) $$\underbrace{\left(\frac{\partial^2}{\partial t^2} - \frac{\partial}{\partial x}c^2(x)\frac{\partial}{\partial x}\right)u}_{\delta u} = \left(\frac{c_0}{\mu}\right)^2\sqrt{\frac{c_0}{c(X)}}\left(\left(\frac{c'(X)(x-X)}{c(X)} + 1\right)^2 - \frac{c^2(x)}{c^2(X)}\right)V'' =$$

$$= \frac{1}{2}\sum_{\pm}\left(\frac{c_0}{\mu}\right)^2\sqrt{\frac{c_0}{c(X)}}\, V''\left(\frac{c_0(x-X)}{c(X)\mu}\right)\left\{\left(\frac{c'(X)(x+X)}{c(X)}+1\right)^2 - \left(\frac{c(x)}{c(X)}\right)^2\right\} = O(\mu^2)$$

Как тут разложить?

$$= \dots \sim \left(\frac{c'(X)(x-X) + c(X)}{c(X)} - \frac{c(x)}{c(X)}\right)\Big(\quad\Big) =$$

$$c(x) = c(X) + c'(X)(x-X) + O(\mu^2)$$

ограничено

$$= O(\mu^2)\,(\quad)$$

огр.

$$\Rightarrow \delta u = O(\mu^2)$$

$2^\circ)\ \frac{1}{\mu}$

1) $\frac{\partial}{x} c^2(x) \frac{\partial}{\partial(x)} u = \frac{1}{2} \sum_{\pm} \sqrt{\frac{c_0}{c(X)}} \cdot 2\, c(x) c'(x)\, V'\left(\frac{c_0(x-X)}{c(X)\mu}\right) \frac{c_0}{\mu\, c(X)} =$

$$= \sum_{+,-} \frac{c_0\, c(x)\, c'(x)}{\mu\, c(X)} V'\left(\frac{c_0(x-X)}{c(X)\mu}\right) \sqrt{\frac{c_0}{c(X)}}$$

2) $\frac{\partial u}{\partial t} \sim -\frac{1}{2}\sqrt{\frac{c_0}{c(X)}} \cdot c'(X) \cdot V(\ ) + \sqrt{\frac{c_0}{c(X)}}\, V' \cdot \left\{ -\frac{c_0\, c'(X)(x-X)}{c(X)\mu} - \frac{c_0}{\mu} \right\}$

$$= -\sqrt{\frac{c_0}{c(X)}} \left\{ \frac{1}{2} c'(X)\, V + \frac{c_0}{\mu}\left(\frac{c'(x-X)}{c(X)} + 1 \right) V' \right\}$$

$\cancel{\frac{\partial^2 u}{\partial t^2} \sim 2 \left(\sqrt{\frac{c_0}{c(X)}}\right)' V'_t =}$

$\cancel{\left(\sqrt{\frac{c_0}{c(X)}}\right)'_t = -\frac{1}{2}\sqrt{\frac{c_0}{c(X)}}\, c'(X)} =$

$\cancel{V'_t = V' \cdot \left(\frac{c_0}{\mu} \left\{ -\frac{c'(X)(x-X)}{c(X)} - 1 \right\} \right)}$

$$\frac{\partial u}{\partial t} = -\frac{1}{2}\sqrt{\frac{c_0}{c(X)}}\, c'(X)\, V + \frac{c_0}{\mu}\sqrt{\frac{c_0}{c(X)}}\, V' \left\{ \frac{c'(X)(x-X)}{c(X)} + 1 \right\}$$

$$\frac{\partial^2 u}{\partial t^2} = +\frac{1}{2}\sqrt{\frac{c_0}{c(X)}}\, c'(X)\, \frac{c_0}{\mu}\, V' \cdot \left\{ \frac{c'(X)(x-X)}{c(X)} + 1 \right\}$$

$$+ \frac{1}{2}\sqrt{\frac{c_0}{c(X)}}\, c'(X)\, \frac{c_0}{\mu}\, V' \left\{ \frac{c'(X)(x-X)}{c(X)} + 1 \right\}$$

$$- \frac{c_0}{\mu}\sqrt{\frac{c_0}{c(X)}}\, \cancel{c'(X)}\, V' \left\{ c''(x-X) - \frac{c'^2(X)(x-X)}{c(X)} - c'(X) \right\}$$

$$= \sqrt{\frac{c_0}{c(X)}}\, \frac{c_0}{\mu} \left\{ c'(X)\, V' \left[\frac{c'(X)(x-X)}{c(X)} + 1 \right] - \cancel{c'(X)}\, V' \left[c''(x-X) - \frac{c'^2(X)(x-X)}{c(X)} - c'(X) \right] \right\}$$

$$\cancel{= \frac{c_0}{\mu}\sqrt{\frac{c_0}{c(X)}}\, V' \left\{ -c''(X)(x-X) \right\} =}$$

$$= \frac{c_0}{\mu}\sqrt{\frac{c_0}{c(X)}}\, V' \left\{ \frac{2c'^2(X)(x-X)}{c(X)} + 2c'(X) - c''(X)(x-X) \right\}$$

$$\frac{\partial u^2}{\partial t^2} = \sum_{+,-} \frac{c_0}{\mu}\sqrt{\frac{c_0}{c(X)}}\, V' \left\{ \frac{c'^2(X)(x-X)}{c(X)} + c'(X) - \frac{1}{2}c''(X)(x-X) \right\}$$

$$3)\left(\frac{\partial^2}{\partial t^2}-\frac{\partial}{\partial x}c^2(x)\frac{\partial}{\partial x}\right)u=\sum_{+,-}\frac{c_0}{\mu}\sqrt{\frac{c_0}{c(X)}}V'\cdot\{\ \}$$

$$\{\ \}=\frac{c'^2(X)(x-X)}{c(X)}+c'(X)-\frac{1}{2}c''(X)(x-X)-\frac{c(x)c'(x)}{c(X)}\ =$$

$$c(x)=C(X)+C'(X)(x-X)+O(\mu^2)$$
$$c'(x)=C'(X)+C''(X)(x-X)+O(\mu^2)$$

$$\Rightarrow\quad c(x)c'(x)=c(X)\left(C'(X)+C''(X)(x-X)+O(\mu^2)\right)$$
$$+\ c'^2(X)(x-X)+c'(X)C''(X)(x-X)^2$$
$$+\left(c'(X)+C''(X)\right)(x-X)\cdot O(\mu^2)+O(\mu^2)$$
$$+\ c'(X)\cdot O(\mu^2)$$

$$=\ \cancel{c'(X)}+\frac{1}{2}c''(X)(x-X)-\cancel{c'(X)}-c''(X)(x-X)\underbrace{\frac{c'(X)c''(X)(x-X)^2}{c(X)}}_{\to O(\mu^2)}$$
$$-\underbrace{\left(c'(X)+c''(X)\right)(x-X)\cdot O(\mu^2)}_{\to O(\mu^2)}+O(\mu^2)=$$

$$=-\frac{3}{2}c''(x-X)+O(\mu^2)$$

$$\to\frac{1}{\mu}\square_c u=\sum_{+,-}\left\{-\frac{3}{2}c''(X)(x-X)+O(\mu^2)\right\}\cdot\frac{c_0}{\mu}\sqrt{\frac{c_0}{c(X)}}V'$$

$3^0)$ $\frac{1}{\mu^0}=1$:

$$1)\ \frac{\partial}{\partial x}u\sim\cancel{\sqrt{\frac{c_0}{c(X)}}\cdot c'(X)V}\quad\frac{\partial}{\partial x}c^2(x)\frac{\partial}{\partial x}u=0.$$

$$2)\ \frac{\partial^2}{\partial t^2}u=\left(-\frac{1}{2}\sqrt{\frac{c_0}{c(X)}}c'(X)V\right)'_t=\left(\frac{1}{4}\sqrt{\frac{c_0}{c(X)}}c'^2(X)-\frac{1}{2}\sqrt{\frac{c_0}{c(X)}}c''(X)c(X)\right)V$$

$$\frac{1}{\mu^2}\delta u=\sum_{+,-}\frac{1}{4}\sqrt{\frac{c_0}{c(X)}}V(x,t)\left\{c'^2(X)-2c''(X)c(X)\right\}$$
$$+\frac{3}{2}\frac{c_0}{\mu}\sqrt{\frac{c_0}{c(X)}}V'(x,t)c''(X)\underbrace{(x-X)}_{O(\mu)}$$

$$\delta u=\mu^2\sum_{+,-}\left\{\frac{1}{4}V(x,t)\left\{c'^2(X)-2c''(X)c(X)\right\}\sqrt{\frac{c_0}{c(X)}}-\underbrace{c_0\mu\sqrt{\frac{c_0}{c(X)}}V'(x,t)}_{O(\mu)}\right\}+^{?}O(\mu^2)$$

Тогда компоненты невязки:

$$\delta u = \mu^2 \sum_{+,-} \left\{ \sqrt{\frac{c_0}{c(X)}} \left(\frac{1}{4} V(x,t) \left[c'^2(X) - 2c''(X)\, c(X) \right] + \frac{3}{2} c_0 V'(x,t)\, c''(X) \right) \right\}$$