Ур-е переноса

$$-i^2\frac{\partial}{\partial t^2}\ \hat{L} = -\left(i\frac{\partial}{\partial t}+\hat{l}\right)\left(i\frac{\partial}{\partial t}-\hat{l}\right)$$

$$\hat{l} = c(x)\frac{\partial}{\partial x}\quad \hat{l}^* = \frac{\partial}{\partial x}\circ c(x)\quad \hat{l}_W := \frac{1}{2}(\hat{l}+\hat{l}^*)$$

связь с ур-м переноса

$$\frac{\partial}{\partial t}\psi - \hat{l}\psi = \frac{\partial}{\partial t} - c(x)\frac{\partial}{\partial x}\psi = 0$$

№10

$$[c(\mu x), \frac{\partial}{\partial x}] = \hat{l} - \hat{l}^* = -\mu c'(\mu x) = -\tilde{c}'(x)$$

Утв

$$\Box_{\tilde{c}} = \left(\frac{\partial}{\partial t} \pm \hat{l}^*\right)\left(\frac{\partial}{\partial t} \mp \hat{l}^*\right) + O(\mu)$$

□ $$\Box_{\tilde{c}} = \frac{\partial^2}{\partial t^2} - \frac{\partial}{\partial x}\tilde{c}(x)^2\frac{\partial}{\partial x}$$

рассмотрим

$$\hat{a} = \cancel{-\hat{l}\hat{l}^*}\left(\frac{\partial}{\partial t}+\hat{l}\right)\left(\frac{\partial}{\partial t}-\hat{l}^*\right) =$$

$$= \frac{\partial^2}{\partial t^2} + \hat{l}\frac{\partial}{\partial t} - \frac{\partial}{\partial t}\hat{l}^* - \hat{l}\hat{l}^*$$

$$\hat{L} = \frac{\partial^2}{\partial t^2} + \frac{\partial}{\partial t}\tilde{c}(x)^2\frac{\partial}{\partial x}$$

рассмотрим ра действие разности операторов на решение $u(x,t)$

$$(\Box_{\tilde{c}} - \hat{a})\,u(x,t) =$$

$$= -\hat{a}\,u(x,t) = \left(\frac{\partial^2}{\partial t^2} + \overbrace{\left(\tilde{c}(x)\frac{\partial}{\partial x}\right)}^{l}\frac{\partial}{\partial t} - \frac{\partial}{\partial t}\overbrace{\frac{\partial}{\partial x}\tilde{c}(x)}^{l^*} - \hat{l}\hat{l}^*\right)u =$$

$$= \left(\frac{\partial^2}{\partial t^2} + \frac{\partial}{\partial t}[\tilde{c}(x), \frac{\partial}{\partial x}] - \tilde{c}(x)\frac{\partial^2}{\partial x^2}\circ\tilde{c}(x)\right)u$$

$\boxed{=}$

$$\hat{l}u = \tilde{c}(x)\frac{\partial}{\partial x}u = \frac{\partial}{\partial x}\tilde{c}(x)u - \tilde{c}'(x)u$$

$$\hat{l}^*(u) = \frac{\partial}{\partial x}c(x)\,u = c'(x)u + u'_x\,c(x) \Rightarrow$$

$$\Rightarrow \left(c(x)\frac{\partial}{\partial x} + c'(x)\right)u)$$

$$\Rightarrow \quad \hat{l} = \frac{\partial}{\partial x}\circ\tilde{c}(x) - \tilde{c}'$$

$$l^* = \tilde{c}(x)\frac{\partial}{\partial x} + \tilde{c}'(x)$$

$$\boxed{=}\left(\frac{\partial^2}{\partial t^2} + \frac{\partial}{\partial t}\left[\tilde{c}(x)\frac{\partial}{\partial x}\right] - \hat{l}\hat{l}^*\right)u =$$

$$= \frac{\partial^2}{\partial t^2}u + \frac{\partial}{\partial t}\left[\tilde{c}(x)\frac{\partial}{\partial x}\right]u - \left(\frac{\partial}{\partial x}\tilde{c}^2(x)\frac{\partial}{\partial x} + \tilde{c}(x)\frac{\partial}{\partial x}\tilde{c}(x) + \tilde{c}'\frac{\partial}{\partial x} - \tilde{c}'^2(x)\right)u =$$

$$= \underbrace{\left(\frac{\partial^2}{\partial t^2} - \frac{\partial}{\partial x}\tilde{c}^2(x)\frac{\partial}{\partial x}\right)}_{=0}u + \frac{\partial}{\partial t}\left[\tilde{c}(x)\frac{\partial}{\partial x}\right]u$$

$$- \tilde{c}'\overbrace{\frac{\partial}{\partial x}\tilde{c}}^{l^*}u + \tilde{c}'\overbrace{\tilde{c}\frac{\partial}{\partial x}}^{l}u + \tilde{c}'^2u =$$

$$= \frac{\partial}{\partial t}\left[\tilde{c}(x), \frac{\partial}{\partial x}\right]u + \tilde{c}'\underbrace{\left[\tilde{c}(x), \frac{\partial}{\partial x}\right]}_{\sim\mu^2}u + \underbrace{\tilde{c}'^2u}_{\sim\mu^2} =$$

$$\cancel{= \left(\frac{\partial}{\partial t} + \tilde{c}'\right)} = \frac{\partial}{\partial t}\mu\tilde{c}(x\mu)u + O(\mu)$$

$c(x\mu)$ ограничена

для $t \in [0, T]$

u - ограничена

$$\frac{\partial}{\partial t}\left[\tilde{c}(x)\frac{\partial}{\partial x}\right] = \left[\tilde{c}, \frac{\partial}{\partial x}\right]\frac{\partial}{\partial t}u \sim \mu.$$

$$\Rightarrow u \in C^2(\mathbb{R}^2)$$

$$\varphi(\xi \pm t) = \frac{1}{2}\exp\left(-\left(\frac{x-a}{\mu}\right)\right) \dots \mu$$

$$\Rightarrow \eta(x,t) =$$

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1)* $c(x) = x^2$

$$\frac{\partial}{\partial t}u \pm \frac{\partial}{\partial x}uc(x) = 0$$

$$\begin{cases} \frac{\partial}{\partial t}u \pm (x^2u' + 2xu) = 0. \\ u(0) = \frac{1}{2}\exp\left(-\left(\frac{x-a}{\mu}\right)^2\right) \end{cases}$$

$$\frac{dt}{1} = \frac{dx}{\pm x^2} = \frac{du}{\mp 2xu}$$

$$dt = \pm\frac{dx}{x^2} \Rightarrow t = \mp\frac{1}{x} + F_1$$

$$\Rightarrow F_1 = \frac{1}{x} \pm t,$$

$$\frac{dx}{\pm x^2} = \frac{du}{\mp 2xu}$$

$$\frac{dx}{\pm x} = \frac{du}{\mp 2u} \Rightarrow \frac{dx}{x} = -\frac{du}{2u}$$

$$\ln|x| = -\frac{1}{2}\ln|u| + F_2$$

$$\Rightarrow \tilde{F}_2 = x\cdot\sqrt{u}$$

$$F_2 = x^2u$$

$$\Rightarrow F_1 = \frac{1}{x} \pm t \qquad F_2 = x^2u.$$

$$u = \frac{1}{x^2}\varphi\left(\frac{1}{x} \pm t\right)$$

$$u(0) = \frac{1}{x^2}\varphi\left(\frac{1}{x}\right) = \frac{1}{2}e^{-\left(\frac{x-a}{\mu}\right)^2} \qquad \varphi = \frac{x^2}{2}e^{-\left(\frac{x-a}{\mu}\right)^2}$$

$$\varphi\left(\frac{1}{x}\right) = x^2 e^{-\left(\frac{x-a}{\mu}\right)^2}$$

$$\cancel{\varphi(y)} \qquad \varphi(y) = \frac{1}{y^2} e^{-\left(\frac{1/y - a}{\mu}\right)^2} \qquad x - a(xt \pm 1)$$

$$\cancel{\varphi\left(\frac{1}{x} \pm t\right) =}$$

$$\varphi(y \pm t) = \frac{1}{(y \pm t)^2} e^{-\left(\frac{\frac{1}{y \pm t} - a}{\mu}\right)^2} =$$

$$\frac{1}{(y \pm t)^2} = \left(\frac{1}{x} \pm t\right)^{-2} = \left(\frac{tx \pm 1}{x}\right)^{-2} = \left(\frac{x}{tx \pm 1}\right)^2$$

$$\cancel{u(x,t) = \frac{1}{(tx+1)^2} e^{-(x-a}}$$

$$\boxed{u^{\pm}(x,t) = \frac{1}{2} \frac{1}{(tx \pm 1)^2} e^{-\frac{1}{\mu^2}\left(\frac{x + a(xt \pm 1)}{1 \pm xt}\right)^2}}$$

$\hat{\ell}$

$$\frac{\partial}{\partial t} u \pm x^2 \frac{\partial}{\partial x} u = 0.$$

$$u_t \pm x^2 u_x = 0$$

$$\frac{dt}{1} = \pm \frac{dx}{x^2} \Rightarrow t = \mp \frac{1}{x} + C$$

$$F = \frac{1}{x} \pm t, = \frac{1 \pm xt}{x},$$

$$u = \varphi\left(\frac{1 \pm xt}{x}\right)$$

$$u(0) = \varphi\left(\frac{1}{x}\right) = \frac{1}{2} e^{-\left(\frac{x-a}{\mu}\right)^2}$$

$$\varphi(y) = \frac{1}{2} e^{-\frac{1}{\mu^2}\left(\frac{1}{y} - a\right)^2} = \frac{1}{2} e^{-\frac{1}{\mu^2}\left(\frac{1-ay}{y}\right)^2}$$

$$\varphi(y \pm t) = \frac{1}{2} \exp\left\{-\frac{1}{\mu^2}\left(\frac{1 - a(y \pm t)}{y \pm t}\right)^2\right\} =$$

$$= \frac{1}{2} \exp\left\{-\frac{1}{\mu^2}\left(\frac{1 - a\frac{1 \pm xt}{x}}{\frac{1 \pm xt}{x}}\right)^2\right\} = \frac{1}{2} \exp\left\{-\frac{1}{\mu^2}\left(\frac{x - a(1 \pm xt)}{1 \pm xt}\right)^2\right\}$$

$$\boxed{\Rightarrow u^{\mp}(x,t) = \frac{1}{2}\exp\left\{-\frac{1}{\mu^2}\left(\frac{x-a(1\pm xt)}{1\pm xt}\right)^2\right\}}$$

$$l_W : \cancel{u} = l_W = \frac{1}{2}\left(l + l^{*}\right) =$$

$$= \frac{1}{2}\left(c(x)\frac{\partial}{\partial x} + \frac{\partial}{\partial x}c(x)\right)$$

$$u_t \pm \frac{1}{2}\left(c(x)u_x + c'(x)u + c(x)u_x\right) = 0.$$

$$u_t \pm \frac{1}{2}\left(2x^2u_x + 2xu\right) = 0. \qquad u_t \pm \left(x^2u_x + \cancel{2}xu\right) = 0.$$

Общ. решение:

$$\frac{dt}{1} = \frac{\cancel{2}dx}{\pm\cancel{2}x^2} = \frac{du}{\pm xu}$$

$$t = \mp\frac{1}{x} + c_1 \to F_1 = \frac{1}{x} \pm t,$$

$$\frac{du}{\pm x} = \frac{dx}{\pm x^2} \Rightarrow \frac{du}{u} = \frac{dx}{x} \to \ln u = \ln x + c_2$$

$$F_2 = \cancel{u - \ln x}\ \frac{u}{x}$$

$$\cancel{\varphi_2}\quad u = \cancel{\ln x} + \varphi\left(\frac{1}{x} \pm t\right)\cdot x$$

$$u(t=0) = \cancel{\ln(x)} + x\varphi\left(\frac{1}{x}\right) = \frac{1}{2}\exp\left\{-\frac{1}{\mu^2}(x-a)^2\right\}$$

$$\varphi\left(\frac{1}{x}\right) = \cancel{\ln\frac{1}{x}}\ \frac{1}{x}\cdot\frac{1}{2}\exp\left\{-\frac{1}{\mu^2}(x-a)^2\right\}$$

$$\varphi(\xi) = \cancel{\ln|\xi|} + \frac{\xi}{2}\exp\left\{-\frac{1}{\mu^2}\left(\frac{1-a\xi}{\xi}\right)^2\right\}$$

$$\varphi(\xi \pm t) = \cancel{\ln|\xi \pm t|} + \frac{\xi \pm t}{2}\exp\left\{-\frac{1}{\mu^2}\left(\frac{1-a(\xi\pm t)}{\xi \pm t}\right)^2\right\} =$$

$$= \cancel{\ln\left|\frac{1}{x} \pm t\right|} + \frac{1}{2}\exp\left\{-\frac{1}{\mu^2}\left(\frac{x-a(1\pm tx)}{1\pm tx}\right)\right\}$$

$$\frac{1}{\xi \pm t} = \frac{1}{\frac{1}{x} \pm t} = \frac{x}{1 \pm xt},$$

Получили

$$\hat{l}^{*}: \quad u^{\pm}(x,t) = \frac{1}{2}\frac{1}{(1\pm tx)^2}\exp\left\{-\frac{1}{\mu^2}\left(\frac{x \mp a(1\pm xt)}{1\pm xt}\right)^2\right\}$$

$$\hat{l}: \quad u^{\pm}(x,t) = \frac{1}{2}\exp\left\{-\frac{1}{\mu^2}\left(\frac{x-a(1\pm xt)}{1\pm xt}\right)^2\right\}$$

$$\hat{l}_w: \quad u^{\pm}(x,t) = \cancel{\ln\left(\frac{1}{x}\pm t\right) a \frac{1}{2}\exp}$$

$$= \frac{1}{2}\frac{1}{1\pm xt}\exp\left\{-\frac{1}{\mu^2}\left(\frac{x-a(1\pm xt)}{1\pm xt}\right)^2\right\}$$

$$t = \int_a^x \frac{d\xi}{\xi^2} = -\frac{1}{\xi}\Big|_a^x \Rightarrow \frac{1}{a}-\frac{1}{x} = t.$$

$$\frac{1}{x} = \frac{1}{a} - t = \frac{1-at}{a}$$

$$x = \frac{a}{1-at}.$$

$$X^{\pm}(t,a) = \frac{a}{1\pm at}$$

$$T^{*}(X^{\pm}(t,a)) = \frac{1}{a} - \frac{1}{x}.$$

№13 $\quad b = \tau_0 \quad \gamma = g = 1.$

$$\begin{cases} N(y,\tau) = \dfrac{1+i\tau}{\left((1+i\tau)^2+4y\right)^{3/2}} \\ U(y,\tau) = \dfrac{2i}{\left((1+i\tau)^2+4y\right)^{3/2}} \end{cases}$$

Подставим:

$$N_0^\tau(y,\tau) = D_0 \cdot N\left(\frac{y}{\mu^2}, \frac{\tau}{\mu}\right)$$

$$U_0^\tau(y,\tau) = \frac{c_0}{\mu} U\left(\frac{y}{\mu^2}, \frac{\tau}{\mu}\right)$$

$$N_0^\tau = D_0 \cdot \frac{1+i\frac{\tau}{\mu}}{\left(\left(1+i\frac{\tau}{\mu}\right)^2+4\frac{y}{\mu^2}\right)^{3/2}}$$

$$U_0^\tau(y,\tau) = \frac{c_0}{\mu} \cdot \frac{2i}{\left(\left(1+i\frac{\tau}{\mu}\right)^2+4\frac{y}{\mu^2}\right)^{3/2}}$$

$$N_0^\tau(y,\tau) = U_0^\tau(\;) \cdot \frac{1+i\frac{\tau}{\mu}}{2i}$$

$$N = U \cdot \frac{1+\tau/\mu}{2}$$

Ур-я: $\begin{cases} N_\tau + (yU)_y = 0 & (1) \\ U_\tau + N_y = 0 & (2) \end{cases}$

(1) - (2)

$$\Rightarrow N_\tau - U_\tau + yU_y + U - N_y = 0.$$

$$\begin{cases} N_\tau = U_\tau \cdot \dfrac{1+i\,\tau/\mu}{2i} + U \cdot \dfrac{1}{2\mu} \\ N_y = \dfrac{1+\tau/\mu}{2} U_y \end{cases}$$

$$U_\delta = \frac{2iC_0}{\mu}\cdot\left(-\frac{3}{2}\right)\cdot\frac{2(1+i\,\delta/\mu)\, i/\mu}{(\quad)^{5/2}}$$

$$U_y = \frac{2iC_0}{\mu}\left(-\frac{3}{2}\right)\frac{4/\mu^2}{(\,..\,)^{5/2}}$$

$$N_\delta = D_0\left(\frac{i}{\mu}\right)\cdot\frac{1}{(\,..\,)^{3/2}} + D_0\left(1+\frac{i}{\mu}\delta\right)\cdot\left(-\frac{3}{2}\right)\frac{2\left(1+\frac{i}{\mu}\delta\right)\cdot i/\mu}{(\,..\,)^{5/2}}$$

$$N_y = D_0\cdot\left(1+\frac{i}{\mu}\delta\right)\left(-\frac{3}{2}\right)\cdot\frac{4/\mu^2}{(\,..\,)}$$

(1) $N_\delta + yN_y + U =$

$$= D_0\left(\frac{i}{\mu}\right)\cdot\frac{1}{(\,..\,)^{3/2}} - \frac{3}{2}\cdot\frac{2\left(1+i/\mu\,\delta\right)^2\frac{i}{\mu}D_0}{(\quad)^{5/2}}$$

$$-\frac{3}{2}\,\frac{2iC_0}{\mu}\,y\,\frac{(4/\mu^2)}{(\quad)^{5/2}} + \frac{C_0}{\mu}\cdot\frac{2i}{(\quad)^{3/2}} =$$

$$= \frac{1}{(\,)^{3/2}}\left(D_0\frac{i}{\mu} - \frac{C_0}{\mu}\cdot 2i\right) - \frac{3}{2}\,\frac{1}{(\,)^{5/2}}\left(2\left(1+\frac{i}{\mu}\delta\right)^2\frac{i}{\mu}D_\delta\right.$$

$$\left.+\frac{2iC_0}{\mu}\cdot y\cdot\left(\frac{4}{\mu^2}\right)\right)$$

(2): $N_\delta - N_y =$

$$= -\frac{3}{2}\,\frac{2iC_0}{\mu}\,\frac{2(1+i\,\delta/\mu)\cdot\frac{i}{\mu}}{(\,)^{5/2}} - \frac{3}{2}\,D_0\left(1+\frac{i}{\mu}\delta\right)\cdot\frac{4/\mu^2}{(\,)^{5/2}}$$

$$= \frac{3}{2}\left(1+\frac{i}{\mu}\delta\right)\left(\frac{2C_0\cdot 2}{\mu^2(\,)^{5/2}} - \frac{D_0}{\mu^2}\cdot\frac{4}{(\,)^{5/2}}\right)$$

$$= \frac{3\cdot 4}{\mu^2}\left(1+\frac{i}{\mu}\delta\right)\left(\frac{C_0 - D_0}{(\,)^{5/2}}\right)$$

$$T_0 = \frac{b}{c_0} = \sqrt{\frac{b}{\gamma g}} \qquad D_0 = \gamma b \qquad c_0 = \sqrt{g D_0}$$

$$D_0 = C_0 = 1$$

$$N_\delta - yN + U = \overset{1}{\overset{\shortparallel}{D_0}}\left(\frac{i}{\mu}\right)\cdot\frac{1}{(\ )^{3/2}} - \frac{3}{2}\,\frac{2\left(1+\frac{i}{\mu^2}\right)^2 \frac{i}{\mu} \overset{1}{\overset{\shortparallel}{D_0}}}{(\ )^{5/2}}$$

$$-\frac{3}{2}\,\frac{2i\overset{1}{\overset{\shortparallel}{C_0}}}{\mu}\,y\,\frac{\left(4/\mu^2\right)}{(\ )^{5/2}} + \frac{\overset{1}{\overset{\shortparallel}{C_0}}}{\mu}\,\frac{2i}{(\ )^{3/2}} =$$

$$= \frac{i}{\mu}\cdot\frac{1}{(\ )^{3/2}} - 3\,\frac{\left(1+\frac{2i}{\mu}\delta - \left(\frac{\delta}{\mu}\right)^2\right)\frac{i}{\mu}}{(\ )^{5/2}}$$

$$-3\,\frac{i}{\mu}\,y\,\frac{\left(4/\mu^2\right)}{(\ )^{5/2}} + \frac{2i}{\mu}\cdot\frac{1}{(\ )^{3/2}} =$$

$$= \frac{3i}{\mu}\,\frac{1}{(\ )^{3/2}} - \frac{3i}{\mu}\,\frac{1}{(\ )^{5/2}} - \frac{3i}{\mu}\left(\frac{2i}{\mu}\delta\right)\cdot\frac{1}{(\ )^{5/2}}$$

$$+\frac{3i}{\mu}\left(\frac{\delta}{\mu}\right)^2\cdot\frac{1}{(\ )^{5/2}} - \frac{3i}{\mu}\,y\,\frac{\left(4/\mu^2\right)}{(\ )^{5/2}} =$$

$$= \frac{3}{\mu}\,\frac{1}{(\ )^{3/2}} - \frac{1}{(\ )^{5/2}}\left(\frac{2i}{\mu}\delta - \left(\frac{\delta}{\mu}\right)^2 + y\cdot\frac{4}{\mu^2}\right) =$$

$$= \frac{1}{(\ )^{3/2}} - \frac{1}{(\ldots)^{5/2}}\left(1 + \frac{2i\delta}{\mu} - \left(\frac{\delta}{\mu}\right)^2 + \frac{4y}{\mu^2}\right) = 0.$$