ECEN 250 Lab10 – Impulse and Step Responses

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Purposes:

- Derive transfer function of RLC circuit and predict impulse response
- Derive step response equations of RLC circuit
- Use lab equipment to evaluate the step response of an RLC circuit.
- Compare the derived equations with simulation results (both impulse and step responses)
- Compare the derived equation for the step response with the actual oscilloscope measurements.

Procedure:

Part 1 - Impulse Response of an RLC Circuit

Part 1a - Calculate the transfer function

Complete the transfer function calculation, $H(s)$, of the following circuit:

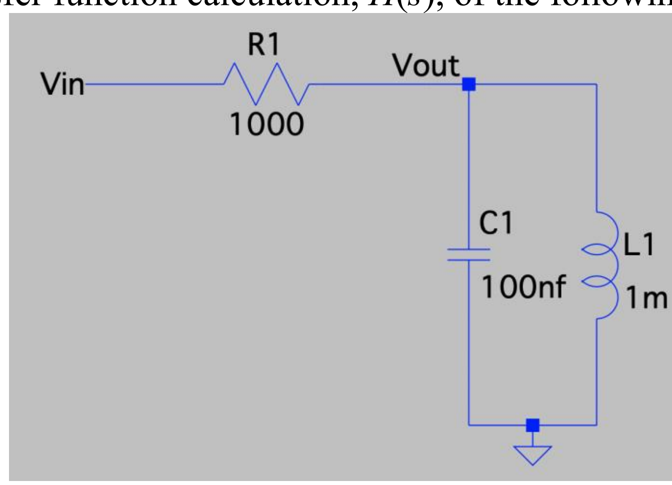


Figure 1a - RLC circuit

$$H(s) = \frac{\dfrac{\frac{1}{sC} sL}{\frac{1}{sC} + sL}}{R + \dfrac{\frac{1}{sC} sL}{\frac{1}{sC} + sL}} = \frac{\frac{1}{RC} s}{s^2 + \frac{1}{RC} s + \frac{1}{LC}}$$

$$H(s) = \frac{10\text{^}4 s}{s^2 + 10\text{^}4 s + 10\text{^}10}$$

If this were used as a filter, what type of filter would it be (chapter 14.4)?
Band-pass filter

$$H(s) = \frac{\beta s}{s^2 + \beta s + \omega_o^2}.$$

Equation 14.18 from the textbook

What is the bandwidth in Hz? 1591
What is the Center frequency in Hz? 15915

Part 1b - Perform partial fraction decomposition of *H*(*s*)

$$H(s) = \frac{K}{s + 5000 - j99875} + \frac{K^*}{s + 5000 + j99875}$$

$$H(s) = \frac{2503 < -90}{s + 5000 - j99875} + \frac{2503 < 90}{s + 5000 + j99875}$$

Part 1c - Find the inverse Laplace Transform of *H*(*s*) (refer to Table 12.3 in the textbook)

TABLE 12.3	Four Useful Transform Pairs		
Pair Number	**Nature of Roots**	$F(s)$	$f(t)$
1	Distinct real	$\frac{K}{s+a}$	$Ke^{-at}u(t)$
2	Repeated real	$\frac{K}{(s+a)^2}$	$Kte^{-at}u(t)$
3	Distinct complex	$\frac{K}{s+\alpha-j\beta}+\frac{K^*}{s+\alpha+j\beta}$	$2\lvert K\rvert e^{-\alpha t}\cos(\beta t+\theta)u(t)$
4	Repeated complex	$\frac{K}{(s+\alpha-j\beta)^2}+\frac{K^*}{(s+\alpha+j\beta)^2}$	$2t\lvert K\rvert e^{-\alpha t}\cos(\beta t+\theta)u(t)$

Note: In pairs 1 and 2, K is a real quantity, whereas in pairs 3 and 4, K is the complex quantity $|K|\angle\theta$.

$$h(t) = 2|K|e^{-\alpha t}cos(\beta t + \theta)u(t)$$

$$h(t) = 2(2503)e^{-5000t}cos(99875t - 90)u(t)$$

This $h(t)$ represents the impulse response. Simulate the impulse response (approximate impulse):

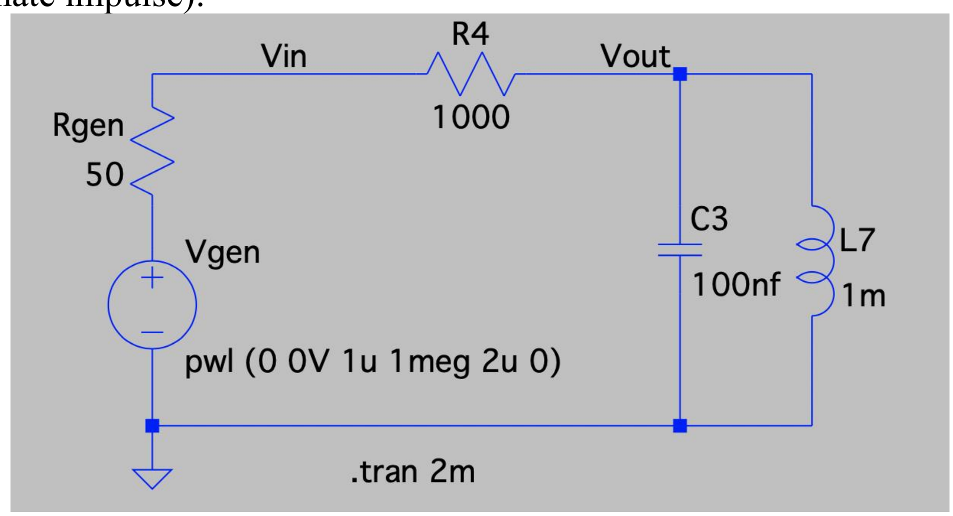
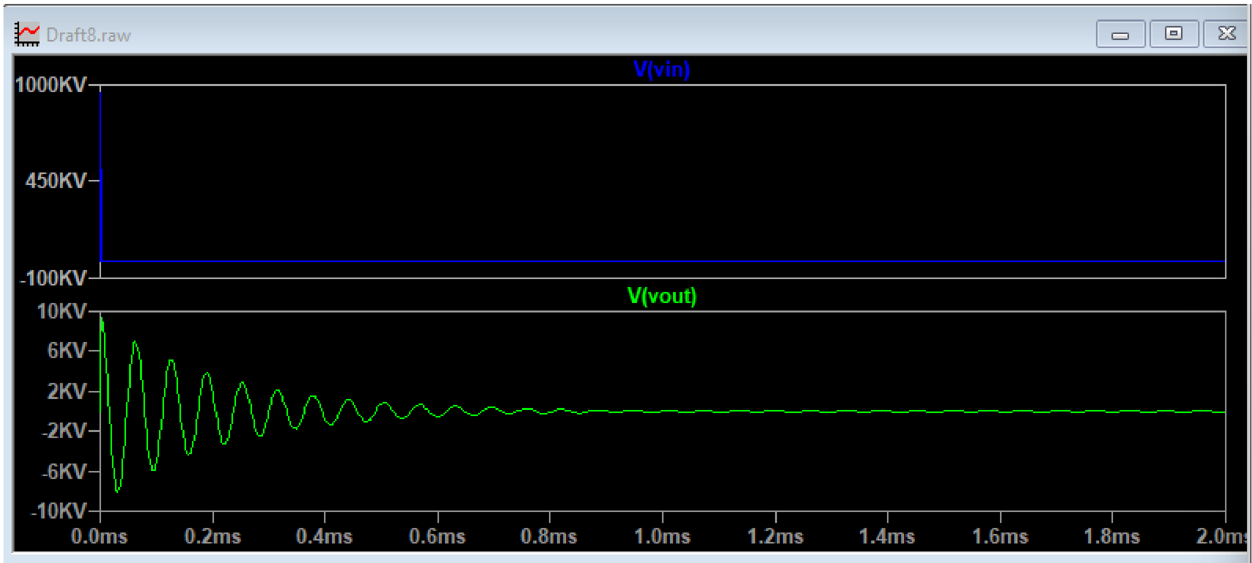


Figure 1b - Impulse response simulation of an RLC circuit

Insert a screenshot of your simulation (Vout must not be in the same plot pane as Vin or you will not see Vout!):



Draft8.raw
V(vin)
1000KV
450KV
-100KV
V(vout)
10KV
6KV
2KV
-2KV
-6KV
-10KV
0.0ms
0.2ms
0.4ms
0.6ms
0.8ms
1.0ms
1.2ms
1.4ms
1.6ms
1.8ms

Part 2 - Modify the Transfer Function with a Step Function Input

Part 2a - Modify $H(s)$ with a step function (divide the $H(s)$ of *part 1a* by s):

$$F(s) = \frac{10\text{^}4}{s^2 + 10\text{^}4s + 10\text{^}10}$$

Part 2b - Derive the new value of K for the partial fraction decomposition:

$$F(s) = \frac{K}{s + 5000 - j99875} + \frac{K^*}{s + 5000 + j99875}$$

$$F(s) = \frac{0.5 < 90}{s + 5000 - j99875} + \frac{0.5 < -90}{s + 5000 + j99875}$$

Part 2c - Derive the Inverse Laplace Transform of $F(s)$ (refer to Table 12.3 in the textbook)

TABLE 12.3 Four Useful Transform Pairs

Pair Number	Nature of Roots	$F(s)$	$f(t)$
1	Distinct real	$\frac{K}{s+a}$	$Ke^{-at}u(t)$
2	Repeated real	$\frac{K}{(s+a)^2}$	$Kte^{-at}u(t)$
3	Distinct complex	$\frac{K}{s+\alpha-j\beta} + \frac{K^*}{s+\alpha+j\beta}$	$2\lvert K\rvert e^{-\alpha t}\cos(\beta t+\theta)u(t)$
4	Repeated complex	$\frac{K}{(s+\alpha-j\beta)^2} + \frac{K^*}{(s+\alpha+j\beta)^2}$	$2t\lvert K\rvert e^{-\alpha t}\cos(\beta t+\theta)u(t)$

Note: In pairs 1 and 2, K is a real quantity, whereas in pairs 3 and 4, K is the complex quantity $|K|\angle\theta$.

$$f(t) = 2|K|e^{-\alpha t}cos(\beta t + \theta)u(t)$$

$$f(t) = 2(0.5)e^{-5000t}\cos(99875t + 90)u(t)$$

Part 2d - Run a SPICE simulation of an RLC Circuit with the step input

Simulate the following circuit in LTspice:

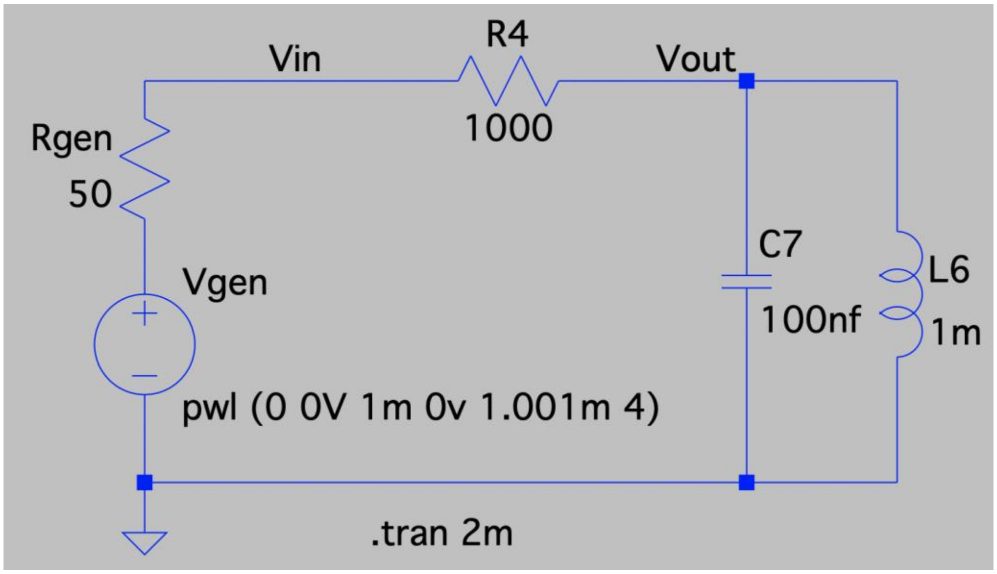
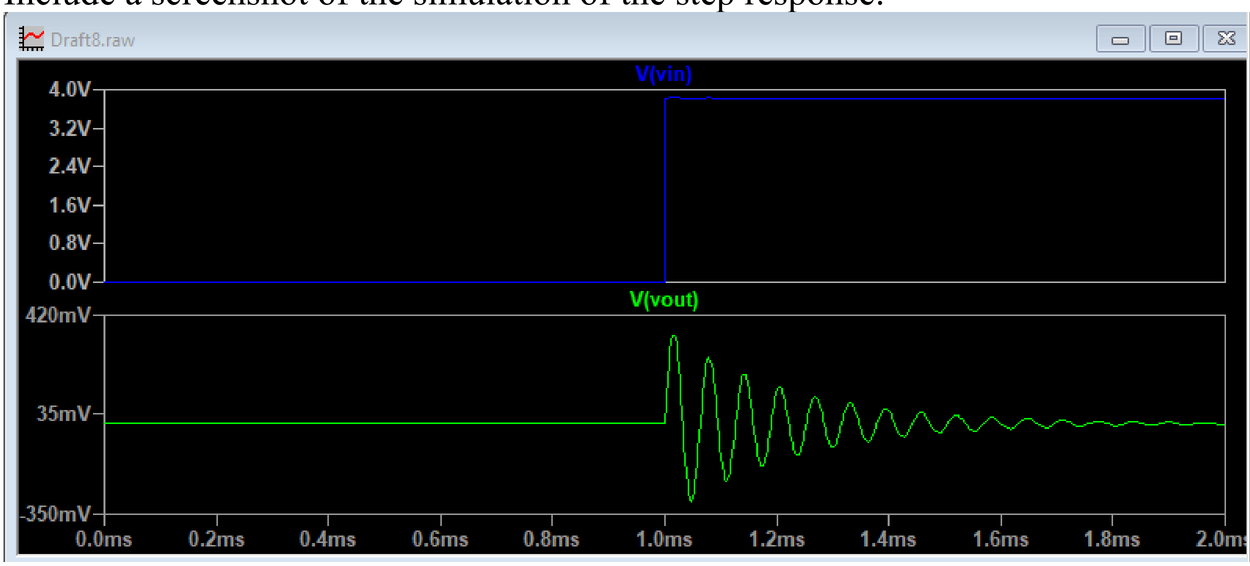


Figure 2 - Simulating a series RLC Circuit with the Step Input (4*u(t-1ms))

Include a screenshot of the simulation of the step response:

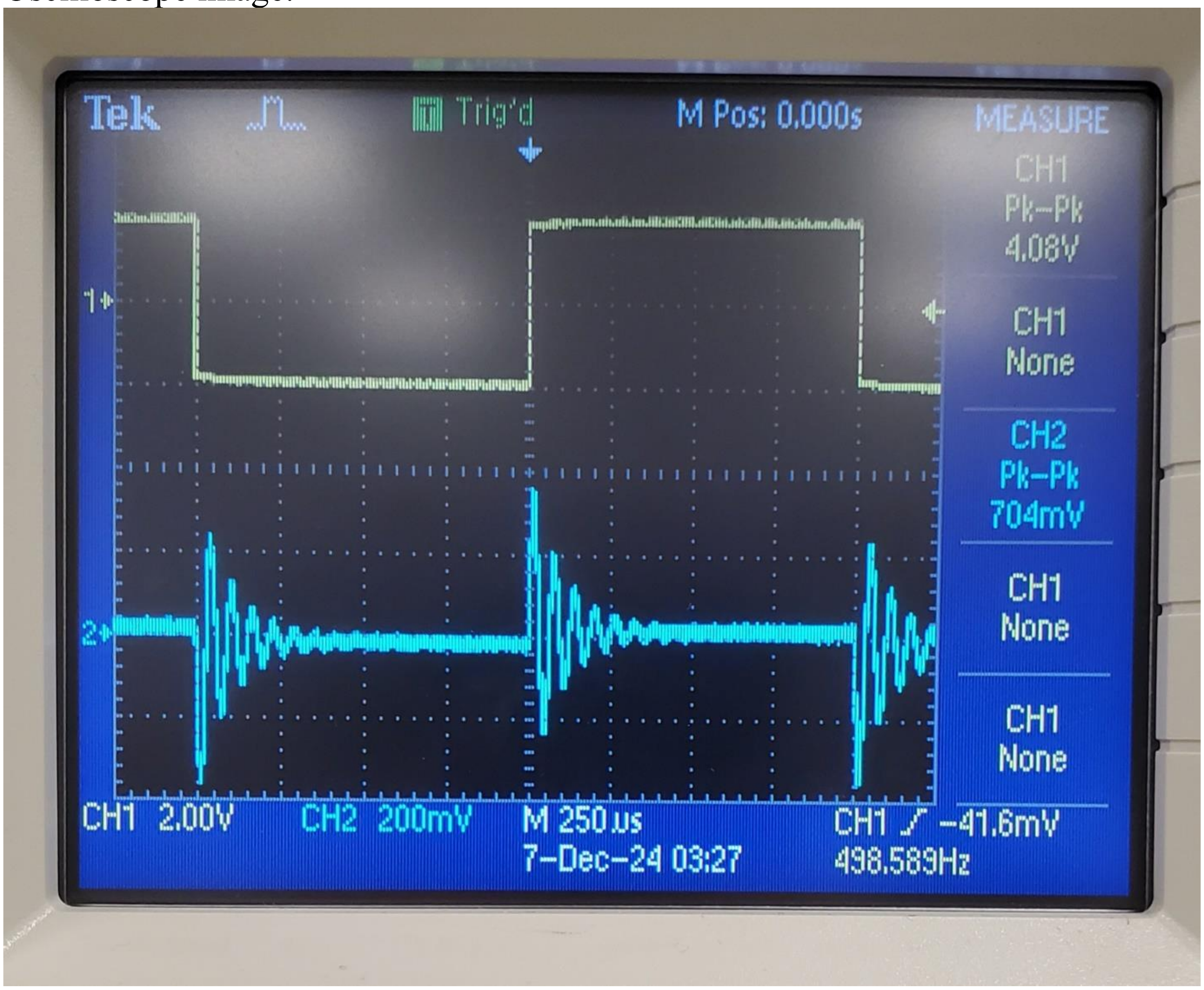


Part 3 - Build the RLC Circuit

Part 3a - Build the circuit of part 2.

Approximate a step response by testing the circuit using a 500Hz square wave with an amplitude of 4Vp-p.

Oscilloscope image:



Conclusions (write a conclusion statement that discusses each of the purposes of the lab):

In this lab, we saw how the transfer function, simulation, and physical circuit all represented the same thing going on and the math actually works. We derived the transfer and step function of the RLC circuit and calculated using partial fractions the response. We then simulated it and compared the equations with the simulation, finding that after the impulse or step there was a jump up in Vout and then had a damped exponential waveform going back down. We then built the circuit and found the same thing using the function generator and oscilloscope measurements.