ECEN 250 Lab9 – Impulse Functions

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Purposes:

- Grasp a better understanding of the impulse function and its approximation.
- Learn what is meant by "impulse response" of a circuit
- Learn how to use the Laplace transform pairs to evaluate a simple circuit with an impulse as the input signal.

Procedure:

Part 1 - SPICE transient simulation of an RC circuit with an impulse function

Simulate the following circuit in LTspice:

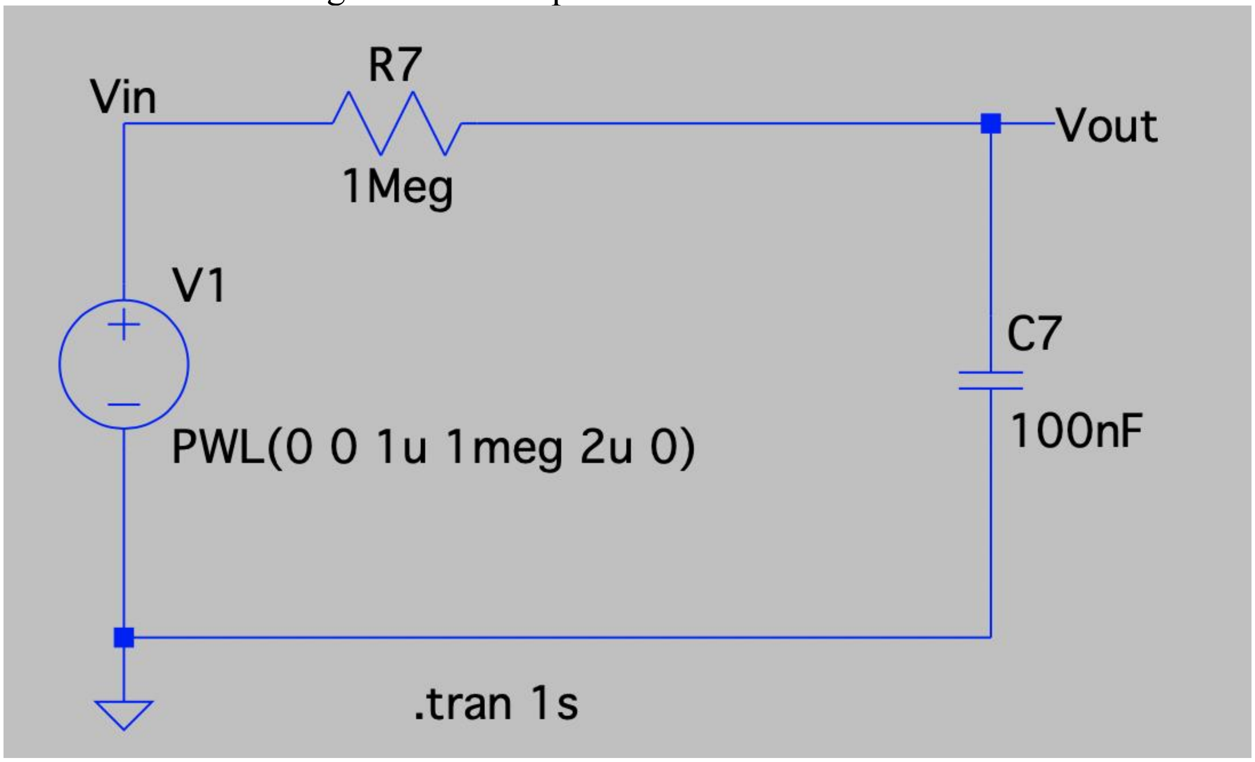
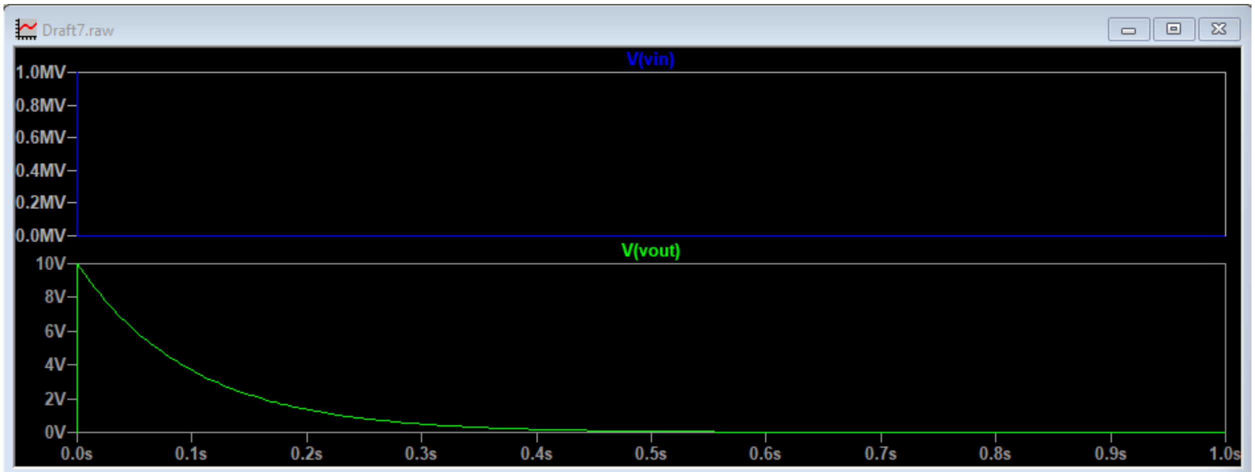


Figure 1 - A SPICE simulation of an RC circuit with a square wave

Place a screenshot of your simulation below (display vin and out in separate plot panes):



The s-domain transfer function of this circuit can be derived from the voltage divider equation using s-domain impedances:

$$H(s) = \frac{\frac{1}{sC}}{R + \frac{1}{sC}} = \frac{\frac{1}{RC}}{s + \frac{1}{RC}} = \frac{1}{RC} \times \frac{1}{s + \frac{1}{RC}}$$

Since $H(s) = Vout(s)/Vin(s)$, the output will differ for each **Type** of input listed in **Table 12.1**. The simplest input type is $F(s) = 1$. The inverse Laplace transform of 1 is $\delta(t)$, the impulse function. The inverse transform, $h(t)$, describes the output signal of a circuit with an impulse as the input signal! We call $h(t)$ the *"impulse response"* of a circuit.

TABLE 12.1	An Abbreviated List of Laplace Transform Pairs	
Type	$f(t)\ (t > 0^-)$	$F(s)$
(impulse)	$\delta(t)$	1
(step)	$u(t)$	$\frac{1}{s}$
(ramp)	t	$\frac{1}{s^2}$
(exponential)	e^{-at}	$\frac{1}{s+a}$
(sine)	$\sin \omega t$	$\frac{\omega}{s^2+\omega^2}$
(cosine)	$\cos \omega t$	$\frac{s}{s^2+\omega^2}$
(damped ramp)	te^{-at}	$\frac{1}{(s+a)^2}$
(damped sine)	$e^{-at}\sin \omega t$	$\frac{\omega}{(s+a)^2+\omega^2}$
(damped cosine)	$e^{-at}\cos \omega t$	$\frac{s+a}{(s+a)^2+\omega^2}$

Finding *h*(*t*) from *H*(*s*)

The $F(s)$ column of the *exponential* row of **Table 12.1** has the same form as our *RC* transfer function:

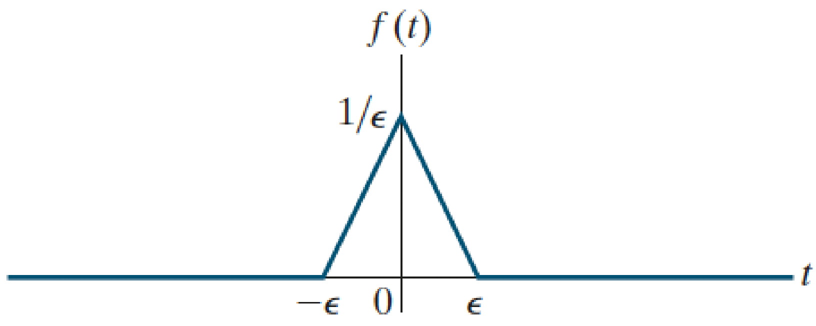
$$H(s) = \frac{1}{RC} \cdot \frac{1}{s + \frac{1}{RC}}$$

since

$$F(s) = \frac{1}{s+a}$$

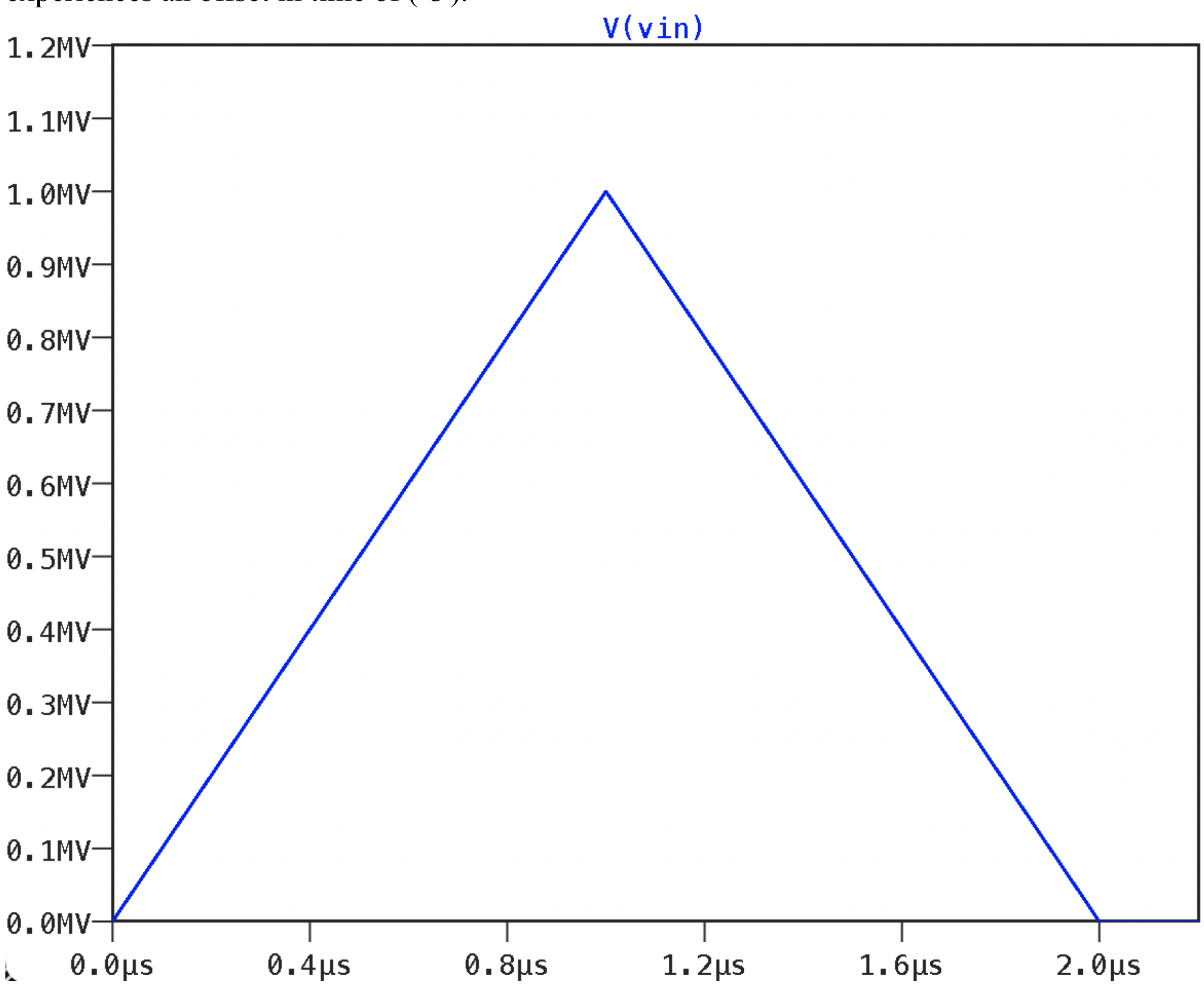
Let $\frac{1}{RC}$ be represented by ***a*** and treat the preceding $\frac{1}{RC}$ like a coefficient or scaler for the Laplace transform pair in **Table 12.1**.

The resulting inverse Laplace transform for the *RC* circuit is $h(t) = 1/RC\ (e^{-t/RC})$. Because $h(t)$ represents the impulse response of a circuit, $vout(t) = h(t)$ when $\delta(t)$ is applied to the input!



Approximating $\delta(t)$ with a small value of ε

How small does ε need to be to accurately represent an impulse? This lab is a simulation-based lab. For a simulated impulse in SPICE, the value of ε is not infinitely small and the start of the "upwards" ramp happens at t = 0. The peak occurs at t = ε. In other words, the impulse experiences an offset in time of (-ε).



vin(t) approximating an impulse function with a time shift: $\delta(t - 1\mu s)$

SPICE command to approximate impulse: PWL(0 0 1u 1meg 2u 0)
SPICE "PWL" parameter summary:

PWL(<Time 1> <Voltage 1> <Time 2> <Voltage 2> <Time 3> <Voltage 3> …)

$$vout(t) = 1/RC\ (e^{-(t-\varepsilon)/RC})$$

Simulated Impulse ε (s)	Impulse peak Voltage (1/ ε)	R (Ω)	C (F)	τ (RC)	Expected *vout* peak (if we had an ideal impulse)	Simulated *vout* peak	Is this a good approximation of an impulse (use 90% of ideal as a threshold)?	τ / ε
10n	100MV	1Meg	100n	0.1s	10V	10V	Yes	10M
100n	10MV	1Meg	100n	0.1s	10V	10V	Yes	1M
1u	1MV	1Meg	100n	0.1s	10V	10V	Yes	100K
10u	100KV	1Meg	100n	0.1s	10V	100V	No	10K
100u	10KV	1Meg	100n	0.1s	10V	1KV	No	1K
1n	1GV	1K	100n	100us	10KV	10KV	Yes	100K
10n	100MV	1K	100n	100us	10KV	100V	No	10K
100n	10MV	1K	100n	100us	10KV	1KV	No	1K
1u	1MV	1K	100n	100us	10KV	10KV	Yes	100
10u	100KV	1K	100n	100us	10KV	90KV	No	10

Is there a τ / ε value that you can use as a target0. to approximate an impulse function?

Yes, there are 4 values. 100, 100K, 1M, and 10M

Part 2 - Construct the circuit of Figure 1 and apply a 1MV impulse (just kidding).

Conclusions (write a conclusion statement that discusses each of the purposes of the lab):

In this lab, we learned a lot about the impulse function and how it behaves. It's hard to physically work with them because of such high voltages, but we simulated them. We approximated the peak voltage in our simulations and compared that to what we expected it to be to see if our T/E value was a good value to approximate the impulse. Our impulse response is just what happens to Vout when we apply the impulse and we learned how to use Laplace transform pairs to figure out that impulse response. This is done by getting out t domain equation into a form we can use, converting it to the s domain, using the table to transform it, then going back to the t domain.