ELEC321 HW#2

1. Avg energy of electron gas at T.E. $= \frac{3K_BT}{2}$; @RT : $K_BT \simeq 25.8^{meV}$

@ $T = 80^K$: $K_BT \simeq 6.88^{meV}$

@ $T = 4.2^K$: $K_BT \simeq 0.36^{meV}$

a) RT : $E \simeq 38.7^{meV}$, $P = \sqrt{2m_eE^{(5)}} \simeq 106.1 \times 10^{-27}\ kgm/s$

$m_e \simeq 9.1 \times 10^{-31}\ kg$ & $P = \frac{h = 6.62 \times 10^{-34}}{\lambda} \Rightarrow \lambda \simeq 6.24^{nm}$

b) $T = 80^K$: $P \simeq 54.8 \times 10^{-27}\ kgm/s \Rightarrow \lambda \simeq 12.1^{nm}$

c) $T = 4.2^K$: $P \simeq 12.5 \times 10^{-27}\ kgm/s \Rightarrow \lambda \simeq 52.9^{nm}$

2. $E_e = E_{ph}$ (a)

$$E_e = \frac{1}{2}mv^2 = \frac{P^2}{2m_e} = \frac{h^2}{2m_e\lambda_e^2}, \quad m = 9.1 \times 10^{-31}\ kg$$

$$E_{ph} = h\nu = h\frac{c}{\lambda_{ph}}$$

$$\xrightarrow{(a)} \quad \frac{c}{\lambda_{ph}} = \frac{h}{2m_e\lambda_e^2} \quad \& \quad \lambda_{ph} = 10\lambda_e$$

$$\Rightarrow \lambda_{ph} = \frac{50h}{2m_ec} = \frac{50 \times 6.62 \times 10^{-34}}{\cancel{2} \times 9.1 \times 10^{-31} \times 3 \times 10^{8}} \simeq 60.62 \times 10^{-12} \times 2\,m$$

$$\Rightarrow E_{ph} = \frac{hc}{2 \times \lambda_{ph}} \simeq \frac{3.28 \times 10^{-15}}{2}\ J \simeq \frac{20.5}{2}\ keV \simeq 10.25\ keV$$

3. $$\psi(x) = \underbrace{\sqrt{\frac{2}{a_0}}\, \exp\left(-\frac{x}{a_0}\right)}_{\text{real}}$$

a) wave function: $\Psi(x,t) = \psi(x).\varphi(t) = \sqrt{\frac{2}{a_0}} \exp\left(-\frac{x}{a_0}\right).\exp\left(\frac{-jEt}{\hbar}\right)$

b) $$\text{PDF} = |\underbrace{\psi(x)}_{\text{real}}|^2 = \left[\sqrt{\frac{2}{a_0}} \exp\left(-\frac{x}{a_0}\right)\right]^2 = \frac{2}{a_0} \exp\left(-\frac{2x}{a_0}\right)$$

probab. of finding particle between $0 \le x \le \frac{a_0}{4}$ $= \int_0^{a_0/4} |\psi(x)|^2 dx = \int_0^{a_0/4} \frac{2}{a_0} \exp\left(-\frac{2x}{a_0}\right) dx$

$$= -\exp\left(-\frac{2x}{a_0}\right)\Big|_0^{a_0/4} = 1 - \exp(-0.5) = 0.393$$

4. infinite potential well with width $= 12\,\text{Å}$ ← a

$$E_n = \frac{\hbar^2 n^2 \pi^2}{2m_e a^2} \qquad m_e = 9.1\times10^{-31}\,\text{Kg}$$

a) $E_1 \simeq 262\,\text{meV}$ & $E_2 \simeq 1.04\,\text{eV}$

b) $$\underbrace{E_2 - E_1}_{\text{in J}} = h\nu_{ph} = h\frac{C = 3\times10^8\,\text{m/s}}{\lambda_{ph}} \Rightarrow \lambda_{ph} \simeq 1.58\,\mu\text{m}$$

5. $$V(x,y,z) = \begin{cases} 0 & 0<x<a,\ 0<y<a,\ 0<z<a \\ \infty & \text{elsewhere} \end{cases}$$

T.I Schrödinger wave equation in 3D:

$$\nabla^2\psi(x,y,z) + \frac{2m}{\hbar^2}\left[E - V(x,y,z)\right]\psi(x,y,z) = 0$$

In Cartesian coordinates: $\nabla^2\psi = \frac{\partial^2\psi}{\partial x^2} + \frac{\partial^2\psi}{\partial y^2} + \frac{\partial^2\psi}{\partial z^2}$

Assume : $\psi(x,y,z) = \psi_1(x)\,\psi_2(y)\,\psi_3(z)$

replace in the T.I.S.E & multiply both sides by $\frac{1}{\psi(x,y,z)}$

$$\Rightarrow \underbrace{\frac{1}{\psi_1}\frac{\partial^2\psi_1}{\partial x^2}}_{\substack{\text{function of } x\\(a)}} + \underbrace{\frac{1}{\psi_2}\frac{\partial^2\psi_2}{\partial y^2}}_{\substack{\text{function of } y\\(b)}} + \underbrace{\frac{1}{\psi_3}\frac{\partial^2\psi_3}{\partial z^2}}_{\substack{\text{function of } z\\(c)}} + \underbrace{\frac{2m}{\hbar^2}\left[E - V(x,y,z)\right]}_{\text{Constant}} = 0$$

$\Rightarrow$ (a), (b) & (c) are constants $\Rightarrow$

$$(I):\quad \frac{\partial^2\psi_1}{\partial x^2} = -k_1^2\psi_1 \,,\quad \frac{\partial^2\psi_2}{\partial y^2} = -k_2^2\psi_2 \,,\quad \frac{\partial^2\psi_3}{\partial z^2} = -k_3^2\psi_3$$

Within the well : $V(x,y,z) = 0$

$$\Rightarrow -(k_1^2 + k_2^2 + k_3^2) = -\frac{2mE}{\hbar^2} \quad (d)$$

$$\overset{(I)}{\Rightarrow} \left|\begin{array}{l} \psi_1(x) = A_1\cos k_1x + B_1\sin k_1x \\ \psi_2(y) = A_2\cos k_2y + B_2\sin k_2y \\ \psi_3(z) = A_3\cos k_3z + B_3\sin k_3z \end{array}\right.$$

Boundary Conditions of infinite potential well:

$$\psi_1(0^\pm) = \psi_2(0^\pm) = \psi_3(0^\pm) = 0 \Rightarrow \left|\begin{array}{l} A_1 = 0 \\ A_2 = 0 \\ A_3 = 0 \end{array}\right.$$

$$\psi_1(a^\pm) = \psi_2(a^\pm) = \psi_3(a^\pm) = 0 \Rightarrow \left|\begin{array}{l} k_1a = n_x\pi \,,\ n_x = 1,2,3,\dots \\ k_2a = n_y\pi \,,\ n_y = 1,2,3\dots \\ k_3a = n_z\pi \,,\ n_z = 1,2,3\dots \end{array}\right.$$

$$\overset{(d)}{\Rightarrow} \quad E = \frac{\hbar^2\pi^2}{2ma^2}\left(n_x^2 + n_y^2 + n_z^2\right)$$

$m_0 = 9.1\times10^{-31}\,kg$ $\quad a = 1.5^{nm}$, $E = 0.2^{eV}$

6. GaAs: $m = 0.067\, m_0$

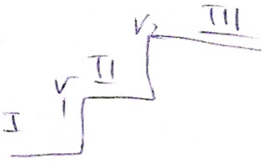
$V_0 = 0.8^{eV}$

$$T \approx 16\,\frac{E}{V_0}\left(1-\frac{E}{V_0}\right)\exp(-2K_2 a)$$

$$K_2 = \sqrt{\frac{2m(V_0-E)}{\hbar^2}} \overset{GaAs}{\Rightarrow} \boxed{T \approx 0.138}$$

Si: $m = 1.08\,m_0 \Rightarrow k_2 \approx 4.12\times10^{9}\ m^{-1} \Rightarrow T \simeq 13\times10^{-6}$

$\Rightarrow$ It's harder for heavier electrons to tunnel!



Problem 2.36

Solution to T.I. schrödinger eq:

Assume electrons coming from $-\infty$

Reg I: $\psi_1(x) = \underbrace{A_1\exp(jK_1x)}_{\text{incident e-wave}} + \underbrace{B_1\exp(-jK_1x)}_{\text{reflected e-wave}}$, $K_1 = \sqrt{\frac{2mE}{\hbar^2}}$

Reg II: $\psi_2(x) = \overbrace{A_2\exp(jK_2x)}^{\text{transmitted e-wave}} + \overbrace{B_2\exp(-jK_2)}^{\text{reflected from the right barrier}}$, $K_2 = \sqrt{\frac{2m(E-V_1)}{\hbar^2}}$

Reg III: $\psi_3(x) = \overbrace{A_3\exp(jK_3x)}^{\text{transmitted e-wave}}$, $K_3 = \sqrt{\frac{2m(E-V_2)}{\hbar^2}}$

there is no barrier to the right so no more reflection

mag. velocity of transmitted wave in Reg III

Def of transmission: $T = \frac{V_3}{V_1}\cdot\frac{A_3A_3^*}{A_1A_1^*}$

magnitude vel. of incident in Reg I

B.C i) $\psi_1(x=0^-) = \psi_2(x=0^+) \Rightarrow A_1 + B_1 = A_2 + B_2$

V is finite ii) $\frac{\partial\psi_1}{\partial x}\Big|_{x=0^-} = \frac{\partial\psi_2}{\partial x}\Big|_{x=0^+} \Rightarrow K_1A_1 - K_1B_1 = K_2A_2 - K_2B_2$

B.C at $x = a$

i) $\psi_2(x = a^-) = \psi_3(x = a^+) \Rightarrow A_2 \exp(jK_2 a) + B_2 \exp(-jK_2 a) =$

$$= A_3 \exp(jK_3 a)$$

ii) $\left.\frac{\partial \psi_2}{\partial x}\right|_{x=a^-} = \left.\frac{\partial \psi_3}{\partial x}\right|_{x=a^+} \Rightarrow K_2 A_2 \exp(jK_2 a) - K_2 B_2 \exp(-jK_2 a)$

$$= K_3 A_3 \exp(jK_3 a)$$

$k_2 a = 2n\pi \Rightarrow \exp(\pm jK_2 a) = 1$

$$\Rightarrow \left| \begin{array}{l} A_1 + B_1 = A_2 + B_2 \\ K_1 A_1 - K_1 B_1 = K_2 A_2 - K_2 B_2 \\ A_2 + B_2 = A_3 \exp(jK_3 a) \\ K_2 A_2 - K_2 B_2 = K_3 A_3 \exp(jK_3 a) \end{array} \right. \Rightarrow A_1 = \frac{1}{2} A_3 \left[1 + \frac{K_3}{K_1}\right] \exp(jK_3 a) \quad (i)$$

Reminder

$$T = \frac{v_3}{v_1} \frac{A_3 \cdot A_3^*}{A_1 \cdot A_1^*}$$

$$K_1 = \sqrt{\frac{2mE}{\hbar^2}} \Rightarrow v_1 = \frac{\hbar K_1}{m}$$

total E, potential energy

$$K_3 = \sqrt{\frac{2m(E - V_2)}{\hbar^2}} \Rightarrow v_3 = \frac{\hbar K_3}{m}$$

Kinetic

$$\Rightarrow T = \frac{K_3}{K_1} \frac{A_3 A_3^*}{A_1 A_1^*}$$

$$(i) \Rightarrow T = \frac{K_3}{K_1} \times 4 \times \frac{\cancel{A_3 A_3^*}}{\cancel{A_3 A_3^*}\left[\left(1 + \left(\frac{K_3}{K_1}\right)\right]^2 \cancel{+ \frac{K_3}{K_1} \times 2\cos K_3 a}\right]} = \frac{4 K_1 K_3}{(K_1 + K_3)^2}$$