Chapter 4

1.

$$n_i = \sqrt{N_c N_v}\, \exp\left(-\frac{E_g}{2kT}\right)$$

for $T = 300^K$: $kT = 25.8^{meV}$

Si : $\left| \begin{array}{l} E_g = 1.12^{eV}, \; N_c = 2.8\times10^{19}\, cm^{-3}, \; N_v = 1.04\times10^{19}\, cm^{-3} \\ m_n^* = 1.08 m_o, \; m_p^* = 0.56 m_o \end{array} \right.$

$GaAs$: $\left| \begin{array}{l} E_g = 1.42^{eV}, \; N_c = 4.7\times10^{17}\, cm^{-3}, \; N_v = 7.0\times10^{18}\, cm^{-3} \\ m_n^* = 0.067 m_o, \; m_p^* = 0.48 m_o \end{array} \right.$

$$N_c = 2\left(\frac{2\pi m_n^* kT}{h^2}\right)^{3/2}, \quad N_v = 2\left(\frac{2\pi m_p^* kT}{h^2}\right)^{3/2}$$

In these calculations we assume E_g to be constant between $T=300^K$ & $T=500^K$ and electron & holes masses to be T invarient. However, $\sqrt{N_c N_v} \propto T^{3/2}$

pay attention that electron & hole masses reported above are taken from density of state row of Table B.4. Although we haven't used them in calculations it is important to notice that these are different from the effective masses reported earlier in this table. Those values are used for transport.

Si : $\left| \begin{array}{l} T = 300^K \rightarrow n_i \simeq 6.4\times10^{9}\, cm^{-3} \\ T = 500^K \rightarrow n_i \simeq 8.1\times10^{13}\, cm^{-3} \end{array} \right.$

this is smaller than the number we use for $T = 300$ (i.e. $n_i = 1.45\times10^{10}\, cm^{-3}$) because of descrepancy in reported N_c & N_v

$GaAs$: $\left| \begin{array}{l} T = 300^K \rightarrow n_i \simeq 2.0\times10^{6}\, cm^{-3} \\ T = 500^K \rightarrow n_i \simeq 2.63\times10^{11}\, cm^{-3} \end{array} \right.$

2.

$$N_d = 10^{13}\ cm^{-3}, \quad N_a = 2.5 \times 10^{13}\ cm^{-3} \quad @\ T = 300^K$$

It's safe to assume that @ $T = 300^K$ full ionization of dopants have been achieved and all dopants are activated.

We know that the semiconductor would stay neutral:

$$\underbrace{n_0}_{\text{electron Concentration}} + \underbrace{N_a}_{\text{Concentration of negatively charged ionized impurities}} = \underbrace{p_0}_{\text{hole concentration}} + \underbrace{N_d}_{\text{Concentration of positively charged impurities}}$$

Mass action law: $p_0 n_0 = n_i^2$

$$\Rightarrow \quad n_0 = \frac{N_d - N_a}{2} + \sqrt{\left(\frac{N_d - N_a}{2}\right)^2 + n_i^2}$$

or

$$p_0 = \frac{N_a - N_d}{2} + \sqrt{\left(\frac{N_a - N_d}{2}\right)^2 + n_i^2}$$

Table B.4 ($T = 300^K$):

Si: $n_i = 1.5 \times 10^{10}\ cm^{-3}$; Ge: $n_i = 2.4 \times 10^{13}\ cm^{-3}$; GaAs: $n_i = 1.8 \times 10^{6}\ cm^{-3}$

$N_a - N_d = 5 \times 10^{12}\ cm^{-3}$

for Si & GaAs: $\frac{N_a - N_d}{2} \gg n_i \Rightarrow p_0 \approx N_a - N_d \Rightarrow$

Si: $p_0 \approx 5 \times 10^{12}\ cm^{-3}$ & $n_0 = \frac{n_i^2}{p_0} \simeq 4.5 \times 10^{7}\ cm^{-3}$

GaAs: $p_0 \approx 5 \times 10^{12}\ cm^{-3}$ & $n_0 = \frac{n_i^2}{p_0} \simeq 0.648\ cm^{-3}$ (negligibly small)

Ge: $p_0 = \frac{5 \times 10^{12}}{2} + \sqrt{\left(\frac{5 \times 10^{12}}{2}\right)^2 + (2.4 \times 10^{13})^2} \approx 2.7 \times 10^{13} \Rightarrow n_0 = \frac{n_i^2}{p_0} \approx 2.1 \times 10^{13}\ cm^{-3}$

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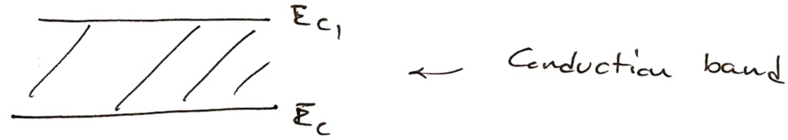
a) $g_c(E) = K$



$$n_o = \int_{E_c}^{E_{c_1}} g_c(E)\, f_M(E)\, dE = \int_{E_c}^{E_{c_1}} K \exp\left(-\frac{E - E_f}{K_B T}\right) dE$$

Based on the knowledge of electron occupancy f_M would be very small a few $K_B T$ above E_c so we can easily change the upper limit of integration to ∞ without losing accuracy:

$$n_o \simeq K \int_{E_c}^{\infty} \exp\left(-\frac{E - E_f}{K_B T}\right) dE = K \cdot kT \exp\left(-\frac{E_c - E_f}{K_B T}\right)$$

b) $T = 200^K$, GaAs : $n_o = 5 p_o$

Mass action law : $n_o \cdot p_o = \frac{n_o^2}{5} = n_i^2$

$$n_i^2 = N_C N_V \exp\left(-\frac{E_g}{KT}\right)$$

Table B.4 : $n_i^2 = 7 \times 10^{18} \times 4.7 \times 10^{17} \times \underbrace{\left(\frac{200}{300}\right)^3}_{\text{like example 1}} \exp\left(-\frac{1.42}{25.8^m \times \frac{200}{300}}\right)$

$n_i^2 \simeq 1.36 \Rightarrow n_o \simeq 2\ cm^{-3}$!!

apparently $T = 200^K$ is too low, you can see this trend from Fig 4.2 aswell

if it was $T = 400^k \rightarrow n_i^2 = \ldots \Rightarrow n_i \simeq 3 \times 10^9\ cm^{-3}$

$\Rightarrow n_o \simeq 6.8 \times 10^9\ cm^{-3}$; $\Rightarrow p_o = 1.36 \times 10^9\ cm^{-3}$; no N_a : neutrality : $N_d = n_o - p_o = 5.44 \times 10^9\ cm^{-3}$

4.

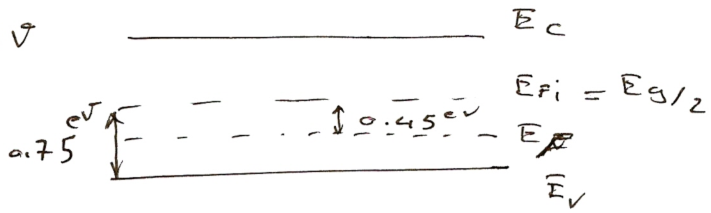
$E_g = 1.5\,eV$, $m_p^* = 10\, m_n^*$, $T = 300K$, $n_i = 10^5\, cm^{-3}$

a)

$$E_{f_i} - \frac{E_g}{2} = \frac{3}{4} \underbrace{K_B T}_{25.8\, meV\ (T=300)} \ln\left(\frac{m_p^*}{m_n^*}\right) \approx 44.5\, meV$$

b)

$$\frac{E_g}{2} - E_f = 0.45\, eV$$



$$N_c = 2\left(\frac{2\pi m_n^* K T}{h^2}\right)^{3/2}, \quad N_v = 2\left(\frac{2\pi m_p^* K T}{h^2}\right)^{3/2} \Rightarrow N_v = 10^{1.5} N_c$$

$$n_i^2 = N_c N_v \exp\left(\frac{-E_g}{KT}\right) \Rightarrow N_c \simeq 74.96 \times 10^{15}\, cm^{-3} \Rightarrow N_v = 2.37 \times 10^{18}\, cm^{-3}$$

assuming that only acceptors are added (no partial compensation)

$\Rightarrow$ ~~m^*~~ and assuming that $N_a \gg n_i$ (we have to return to calculation if this doesn't hold at the end)

$\Downarrow$

$p = N_a$

$$p = N_v \exp\left(\frac{\overbrace{E_V - E_F}^{-0.3}}{KT}\right) \approx 21.1 \times 10^{12}\, cm^{-3} = N_a \gg \underbrace{10^5 = n_i}$$

So the assumption was fine

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$T = 300\,K$ → perfect ionization, $N_a = 7 \times 10^{15}\ cm^{-3}$

a) $p = 7 \times 10^{15} = N_v \exp\left(\frac{E_v - E_F}{KT}\right) \Rightarrow E_F - E_v \approx 0.188\ eV$

$= N_a$

N_v: Table B.4 1.04×10^{19}

valid as $N_a >> n_i$ and full activation of dopants happened already at $300\,K$

b) $E_F - E_v = 0.188\ eV - \underbrace{25.8\ meV}_{KT} \approx 0.162\ eV$

$\Rightarrow p \approx N_v \exp\left(-\frac{0.162}{0.0258}\right) \approx 19.3 \times 10^{15}\ cm^{-3}$

addition of acceptors: $19.3 \times 10^{15} - 7 \times 10^{15} \approx 12.3 \times 10^{15}\ cm^{-3}$

6

$T = 300\,K$, Si, $N_d = 10^{16}\ cm^{-3} >> n_i \Rightarrow n = N_d$

i) $n = N_d = N_c \exp\left(\frac{E_F - E_c}{KT}\right) = N_c \exp\left(\frac{E_F - E_i + E_i - E_c}{KT}\right)$

$\Rightarrow n = \underbrace{\left[N_c \exp \frac{E_i - E_c}{KT}\right]}_{n_i} \exp \frac{E_F - E_i}{KT}$

$\Rightarrow \boxed{n = n_i \exp\left(\frac{E_F - E_i}{KT}\right)}$ ← another important relationship

$\Rightarrow E_F - E_i = \underbrace{KT}_{25.8\,meV} \ln \frac{n}{n_i} = 0.347\ eV$

$n_i \approx 1.45 \times 10^{10}\ cm^{-3}$

ii) $N_a = 10^{17}$ is added $\frac{N_a - N_d}{2} >> n_i$

$E_i - E_F \approx 0.4\ eV$

$\Rightarrow$ neutrality: $p_0 = \frac{N_a - N_d}{2} + \sqrt{\left(\frac{N_a - N_d}{2}\right)^2 + n_i^2} \approx N_a - N_d = 9 \times 10^{16} = n_i \exp\left(\frac{E_i - E_F}{KT}\right)$