Ziptie: Learning Useful Features

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A robot learning to navigate the world finds that the combination of certain sensors gives more information than either of them alone. The combination of x- and y-position tells more about when to watch for obstacles than either variable on its own. The speed and position of arm tells more about what shoulder torque needs to be generated than either variable on its own. These cases of sensor interaction can be hard coded manually. Human designers often exploit their knowledge of the system to do this via feature engineering, but in cases where the robot system is too complex to intuit these useful interactions, they can be learned. Automatically creating these predictive features is the goal of the Ziptie algorithm.

Ziptie makes a non-traditional assumption that all sensor signals, α_i , (otherwise known as **features**) are **Fuzzy Cat**egorical variables with $\alpha_i \in [0,1]$. It also assumes that a fixed number of features, n, are received at discrete time intervals in a vector A.

$$\mathbf{A} = (\alpha_1, \alpha_2, \alpha_3, ..., \alpha_n) \tag{1}$$

The n features of the sensor data can be imagined as n separate **cables**, ϕ_i , each carrying a single signal (as they often are in robots). The challenge of clustering these cables into informative combinations can then be imagined as creating **bundles** of cables, ϕ_{ij} , as with a ziptie.

A new bundle is created when the **agglomeration energy**, γ_{ij} between two cables exceeds a threshold, C_{γ} . Agglomeration energy is the accumulated **coactivation**, κ_{ij} , of the cable pair, where the coactivation at each time step is given by the product of their two activities.

$$\kappa_{ij} = \alpha_i \alpha_j \tag{2}$$

Once bundle ϕ_{ij} is created, it gets first dibs at representing any amount of signal carried on both cables ϕ_i and ϕ_i .

$$\alpha_{ij} = \min(\alpha_i, \alpha_j) \tag{3}$$

The member cables of ϕ_{ij} retain only the residual signal.

$$\alpha_{\hat{i}} = \alpha_i - \alpha_{ij} \tag{4}$$

$$\alpha_{\hat{i}} = \alpha_i - \alpha_{ij}$$

$$\alpha_{\hat{j}} = \alpha_j - \alpha_{ij}$$
(4)

This approach constrains bundles' activities to remain on [0, 1] as well. Bundles can be coactive with cables and with other bundles. Any cable-cable, cable-bundle, or bundlebundle pair whose agglomeration energy exceeds C_{γ} will nucleate a new bundle.

As Ziptie continues to operate on the stream of cable activities, the total number of bundles will continue to grow. As bundles are bundled again together, the number of cables in the largest bundles will grow too. These can be limited from growing too large by introducing a stopping condition, such as a maximum number of bundles or fixed number of time steps.

The rest of this paper attempts to answer the questions you might have:

- What's all this about?
- What is Ziptie good for?
- How does the **Ziptie** algorithm work in practice?
- · What are Fuzzy Categorical variables and why do they matter?
- How is Ziptie related to what **biological brains** do?
- How can I use **Ziptie code**?

1. Concepts and Related Work

1.1. Agglomerative Clustering

This is a method of grouping observations or data points in which the most similar are grouped together right away, then the slightly less similar are added to those clusters. As clusters grow they can also glom on to each other. This process of similar observations and clusters repeatly combining to form larger clusters is agglomerative clustering. It's also called hierarchical clustering because when you trace the lineage observations and mini-clusters combining, it forms a tree showing the hierarchy of similarity between them.

1.2. Multi-membership Clustering

Ziptie is an unusual variant of agglomerative clustering where an item can belong to multiple clusters. Here the ziptie analogy of Figure 1 can be helpful. In a set of wires, imagine pulling out five of them and wrapping them with

one ziptie. Then imagine taking just two of those five, selecting another two loose wires, and wrapping those four with another ziptie. Two of the wires are included in both zipties. Those represent elements with multiple cluster membership.



Figure 1. Clustering with multiple membership.

1.3. Feature Learning

Feature engineering is the practice of cleverly combining several features to get at information that none of them could provide on their own. For example, the x- and y-velocities of an object can be combined to give its overall speed, or specific patterns in a 3×3 collection of pixels can be used to detect edges. Feature learning, also known as **automated feature engineering**, is when features are generated through heuristics or the result of algorithms. There are a collection of **open source tools** for this, which largely focus on time series data sets.

Feature learning is also referred to as **representation learning**. **Principal components analysis** (PCA) is the poster child for unsupervised representation learning. PCA resembles Ziptie in that it finds combinations of features that tend to co-occur. PCA is focused on *dimensionality reduction* in that its goal is to reduce the total number of features used to a small set that distills out most of the informtion in the data.

1.4. Feature Agglomeration

On its surface, scikit-learn's feature agglomeration ¹ is similar to Ziptie. It uses agglomerative clustering to group features into larger groups of features. However, it is different in some important ways, which are illustrated in Section 3.1.

1.5. Sparse Coding

Ziptie is actually an example of **sparse coding** or *sparse dictionary learning*, a family of methods for learning con-

cise ways to represent data.

The goal of sparse coding is to represent using as few features as possible. To do this, sparse coding methods often learn an *overcomplete dictionary of basis functions*. Basis functions are combinations of features, the elements of a dictionary that the sparse coding method uses to re-express each observation. Overcomplete means that the method learns several ways to express the same observation. This allows it the sparsest one, the one with the fewest non-zero elements.

A culinary example of sparse coding is "Moose Tracks" flavor ice cream. It could just as accurately be described as "vanilla with small peanut butter cups and fudge ripples". The name "Moose Tracks" is superfluous; it means exactly the same thing. Its existence shows overcompleteness in the dictionary of ice cream flavor names. But it allows for concise representation, a sparse coding of the

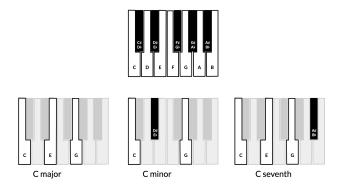


Figure 2. Piano chords as a sparse coding

1.6. ℓ^0 vs ℓ^1 Norms

L0 sparse coding

Computer vision, anomaly detection https://arxiv.org/abs/2104.04289

L0-KSVD https://hal.science/hal-01965904/document

Optimization Mixed integer quadratic programming (computationally expensive) Vs Greedy

1.7. Continual Learning

Continual learning is a particular case of machine learning where the algorithm never stops evolving in response to its inputs, ² also called incremental learning or lifelong learning. Ziptie is a specific flavor of continual learning called

¹Pedregosa, F., Varoquaux, G., Gramfort, A., Michel, V., Thirion, B., Grisel, O., Blondel, M., Prettenhofer, P., Weiss, R., Dubourg, V., Vanderplas, J., Passos, A., Cournapeau, D., Brucher, M., Perrot, M., Duchesnay, E. (2011) Scikit-learn: Machine Learning in Python. Feature agglomeration. *JMLR*, 12, 2825–2830.

²Wang, L., Zhang, X., Su, H., and Zhu, J. (2015) A Comprehensive Survey of Continual Learning: Theory, Method and Application. arXiv. https://arxiv.org/abs/2302.00487

online learning where the algorithm does a small update after every new data point is collected. Ziptie also falls into the niche category of *unsupervised* continual learning like ³ and ⁴ because it isn't learning how to perform a specific task, but instead is learning how to organize and represent its data.

Taken all together, Ziptie sits at the intersection of several families of methods, as in Figure 3.

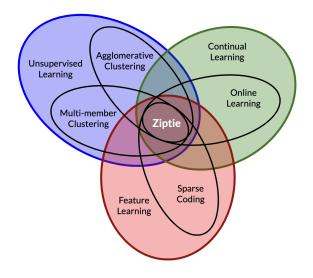


Figure 3. Ziptie at the crossroads.

- Agglomerative Clustering
- Multi-member Clustering
- · Online Learning
- Sparse Coding

2. Applications

It washes the windows and does your taxes.

3. How the Ziptie Algorithm Works

How it works, step-by-step.

3.1. Why coactivation matters

Feature agglomeration was mentioned in Section /refsub-sec:featureagg. Feature agglomeration groups features based on how often they have similar values. It tries to group nearly identical features first. Ziptie on the other hand groups features based on how often they are co-active. (When they are both zero, that doesn't increase their similarity.) This creates groups of features that are simultaneously active, things that happen at the sime time. While this will capture features that are identical, it will also capture unrelated features whose co-occurence gives valuable information.

4. Fuzzy Categorical Variables

symbolic-connectionist divide

5. Biological Motivation

This is biologically motivated.

6. The Ziptie Python Package

Instructions for using the Ziptie library.

7. Versioning and History

8. Licensing

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³Ashfahani, A. and Pratama, M. (2021). Unsupervised Continual Learning in Streaming Environments. arXiv. https://arxiv.org/pdf/2109.09282.pdf

⁴Rao, D., Visin, F., Rusu, A. A., Teh, Y. W., Pascanu, R., and Hadsell, R. (2019) Continual Unsupervised Representation Learning. Paper presented at 33rd Conference on Neural Information Processing Systems (NeurIPS 2019), Vancouver, Canada. https://proceedings.neurips.cc/paper_files/paper/2019/file/861578d797aeb0634f77aff3f488cca2-Paper.pdf