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Small t elastic scattering and the ρ parameter

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ABSTRACT

A simple application of Regge theory, with 9 free parameters, provides a good fit to elastic scattering data at small t from 13.76 GeV to 13 TeV. It yields a value for ρ, the ratio of the real part of the hadronic contribution to the forward amplitude to its imaginary part, close to 0.14 at 13 TeV. Although the exact value obtained for ρ is sensitive to what functional form is chosen for the fit, there is no strong case for the presence of an odderon contribution to forward scattering.

1. The fit

There are two approaches to the extraction of the phase of the forward elastic scattering amplitude from the data. That favoured by experimentalists is to fit the differential cross section at just one energy beyond the Coulomb region and extrapolate it into the Coulomb region. This then gives the hadronic amplitude up to an unknown phase, which is then determined by the data because the Coulomb peak at very small t is sensitive to the interference between the hadronic and Coulomb terms.

However, this approach ignores information linking the phase of the amplitude to its variation with energy, so instead we assume that the hadronic amplitude is described by Regge theory and determine the unknown parameters in it from data beyond the Coulomb peak at a wide range of energies. The data in the Coulomb peak do not then play a part in determining the phase of the amplitude, but we do check that they are well described when the Coulomb contribution is added to the amplitude.

Regge theory has long been known to give an excellent description of soft hadronic processes [1,2]. This paper applies it in its simplest form to small-t data from 13.76 GeV to 13 TeV for total cross sections and elastic scattering at small t, namely $|t| \leq 0.1$ GeV^2, by including in the amplitude the exchange of the soft pomeron $\mathbb{P}$, of the reggeons ρ, ω, f_2, a_2 and of two pomerons $\mathbb{P}\mathbb{P}$. The fit reveals no need [3] for any odderon contribution at small t.

For the reggeon exchanges Regge theory introduces a trajectory $\alpha(t)$ associated with Chew-Frautschi plots of the squares of masses of particles with the same quantum numbers but different spins. According to figure 2.13 of reference [2] the trajectories are found to be exchange-degenerate, $\alpha_+(t)$ for f_2, a_2 and $\alpha_-(t)$ for ρ, ω, with to a good approximation

$$\alpha_\pm(t) = 1 + \epsilon_\pm + \alpha'_\pm t \tag{1a}$$

where

$$\epsilon_+ = -0.3 \quad \alpha'_+ = 0.8 \text{ GeV}^{-2} \qquad \epsilon_- = -0.56 \quad \alpha'_- = 0.92 \text{ GeV}^{-2} \tag{1b}$$

The trajectory for pomeron exchange is assumed similarly to be linear in t, with intercept $1 + \epsilon_{\mathbb{P}}$ and slope $\alpha'_{\mathbb{P}}$ determined by the fit.

Each of these exchanges contributes

$$X_i F(t) (2\nu\alpha'_i)^{\alpha_i(t)} \xi(t) \quad i = \mathbb{P}, \pm \tag{1c}$$

to the elastic amplitude, where

$$2\nu = \tfrac{1}{2}(s-u) = s - 2m^2 + \tfrac{1}{2}t \tag{1d}$$

and the signature factor

$$\xi(t) = -e^{-\frac{1}{2}i\pi\alpha} \quad \text{or} \quad -ie^{-\frac{1}{2}i\pi\alpha} \tag{1e}$$

according to whether the C-parity of the exchange is even or odd. The signature factor determines the complex phase of the contribution. For each exchange there is a real factor $X_i F(t)$ which is not determined by the theory. For simplicity we assume the same function $F(t)$ for each

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$$F(t) = Ae^{a_1 t} + (1 - A)e^{a_2 t} \tag{1f}$$

Regge theory has had many successes over more than half a century, with the above exchanges giving the main contributions to a wide variety of reactions. But it is necessary also to take account of the double-exchange contributions $\mathbb{PP}, \mathbb{P}\rho, \mathbb{P}\omega, \ldots$ to the amplitude, and perhaps triple or more. For simplicity we include only the first of these $A_{\mathbb{PP}}(s,t)$. The trajectory for this exchange is known [2]:

$$\alpha_{\mathbb{PP}}(t) = 1 + 2\epsilon_{\mathbb{P}} + \tfrac{1}{2}\alpha'_{\mathbb{P}} t \tag{2a}$$

However, while this determines the power of s at large s, and the complex phase of the term, that is all that is known about it. To construct a simple model, we introduce the eikonal function (see for example equation (2.49) of reference [2])

$$\chi(s,b) = -\log\left(1 + \frac{i}{8\pi^2 s}\int d^2q e^{i\mathbf{q}.\mathbf{b}} A(s, -\mathbf{q}^2)\right) \tag{2b}$$

so that

$$A(s, -\mathbf{q}^2) = 2is\int d^2b\, e^{-i\mathbf{q}.\mathbf{b}}\,(\chi - \tfrac{1}{2}\chi^2 + \ldots) \tag{2c}$$

A model that is sometimes used to calculate $A_{\mathbb{PP}}(s,t)$ is to take $\chi(s,b)$ to include only the single-exchange contribution, which would give, if we omitted the term $\frac{1}{2}t$ in (1d) (which is justified because we are working only at small t)

$$\begin{aligned}\tilde{\chi}_{\mathbb{P}}(s,b) &= \frac{1}{8i\pi^2 s}\int d^2q e^{i\mathbf{q}.\mathbf{b}} A_{\mathbb{P}}(s, -\mathbf{q}^2)\\ &= \sum_{i=1,2}\frac{iZ_i}{8\pi s D_i}\exp\left(\alpha_{\mathbb{P}}(0)L - \mathbf{b}^2/(4D_i)\right)\end{aligned} \tag{2d}$$

with

$$\begin{aligned}&Z_1 = X_{\mathbb{P}}A \qquad Z_2 = X_{\mathbb{P}}(1-A)\\ &L = \log(2\nu\alpha'_{\mathbb{P}}) - \tfrac{1}{2}i\pi \qquad D_i = a_i + \alpha'_{\mathbb{P}}L\end{aligned} \tag{2e}$$

Then the χ^2 term in (2c) would be the double-exchange contribution to the amplitude,

$$\begin{aligned}A_{\mathbb{PP}}(s,t) = i\sum_{i,j=1,2}&\frac{Z_i Z_j}{16\pi s(D_i + Dj)}\\ &\exp\left(2\alpha_{\mathbb{P}}(0)L + tD_i D_j/(D_i + D_j)\right)\end{aligned} \tag{2f}$$

However, if instead we were to choose $\chi(s,b)$ to include also a double-exchange contribution, when it is inserted into (2c) this would multiply (2f) by some constant C. Our fit finds that C should be close to $\frac{1}{2}$.

So we have 8 free parameters in addition to C, which we determine from the data beyond the Coulomb peak. We then add to the amplitude the photon-exchange term

$$8\pi\alpha_{\mathrm{EM}}G(t)/t \tag{2g}$$

Here $G(t)$ is a squared form factor, equal to 1 at $t = 0$. Choosing the square of either the Dirac or the Pauli form factor, or a combination of them, gives almost the same result, because the term is negligibly small except at extremely small t.

The fits shown in Figs. 1 to 5 are with the choices

$$\begin{aligned}&\epsilon_{\mathbb{P}} = 0.108 \quad X_{\mathbb{P}} = 166.3 \quad X_+ = 201.2\\ &X_- = 119.8 \quad \alpha'_{\mathbb{P}} = 0.321\ \mathrm{GeV}^{-2}\\ &C = 0.5 \quad A = 0.561 \quad a_1 = 0.321\ \mathrm{GeV}^{-2} \quad a_2 = 7.674\ \mathrm{GeV}^{-2}\end{aligned}$$

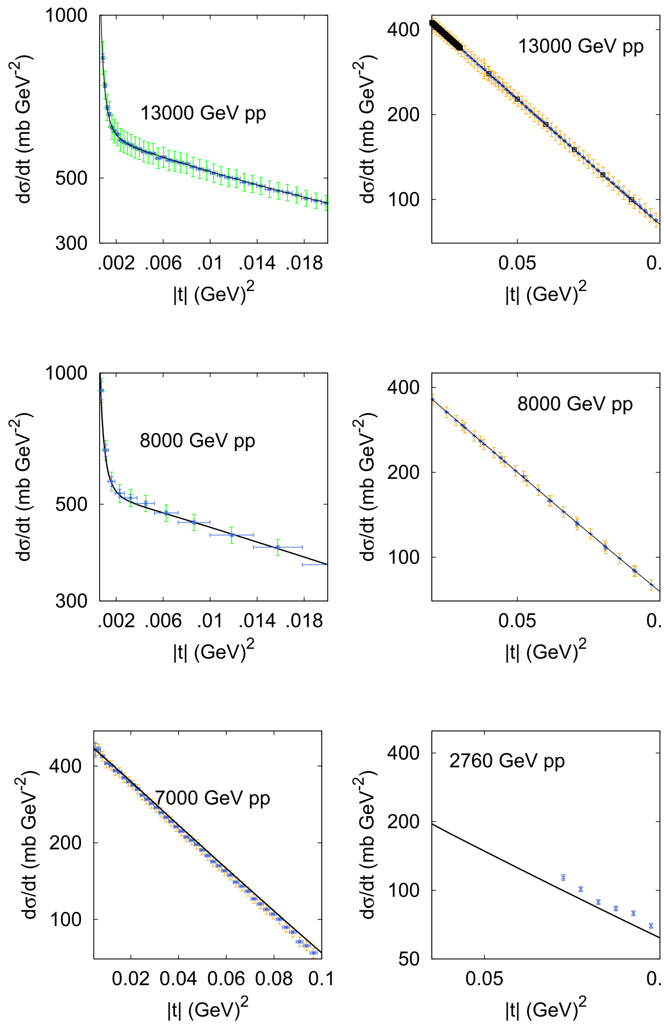

Fig. 1. Fits to TOTEM data [4].

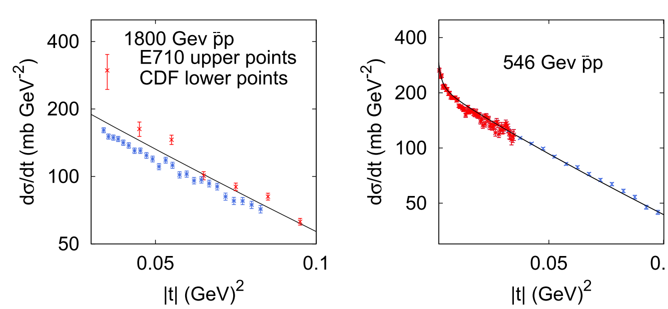

Fig. 2. Fits to CERN and Fermilab $p\bar{p}$ data (which are referenced in reference [10]).

(3)

As is seen in Fig. 5, the value 0.14 obtained for ρ at 13 TeV is rather different from that of at most 0.1 concluded by TOTEM from their data [4]. The reason why the curves rise to a maximum and then fall again as the energy increases is that the $\mathbb{PP}$ term becomes progressively more important.

2. Comments

1 With just 9 adjustable parameters, Regge theory provides a fit to data over a range of energies differing by a factor of 1000. The fit is extremely good, though less than perfect in some cases. There are

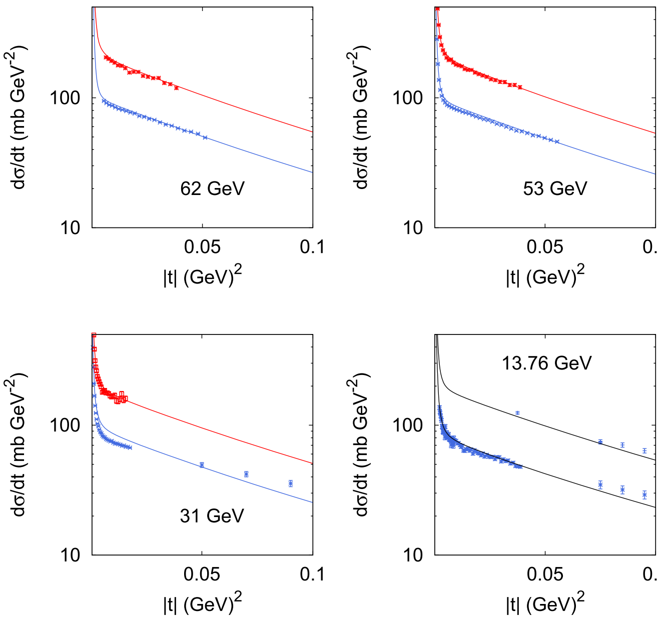

Fig. 3. Fits to fixed target and CERN ISR data (which are referenced in reference [10]). The lower points are pp scattering, the upper points $p\bar{p}$ multiplied by 2.

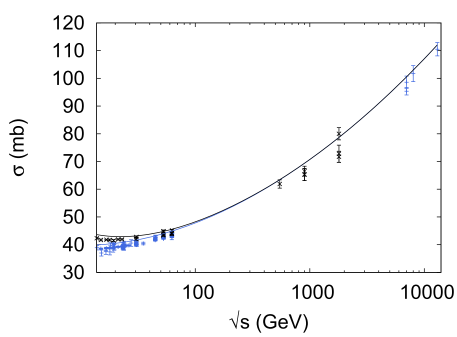

Fig. 4. Fits to the pp and $p\bar{p}$ total cross sections.

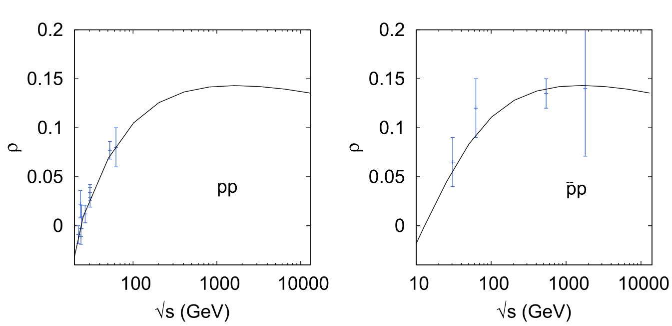

Fig. 5. Outputs for ρ.

also some anomalies in the data. An example is shown in Fig. 6: the data at 7, 8 and 13 TeV agree well with a single exponential in t, but the slope for the 7 TeV data lies between that for 8 and 13 TeV, which is surely anomalous.

2 In their extraction of ρ from their 13 TeV data, TOTEM assume [4] that the ratio of the real to the imaginary part of the hadronic amplitude is independent of t from 0 to -0.1 GeV^2 or more. Fig. 7 shows how it varies with t for the fit described here.

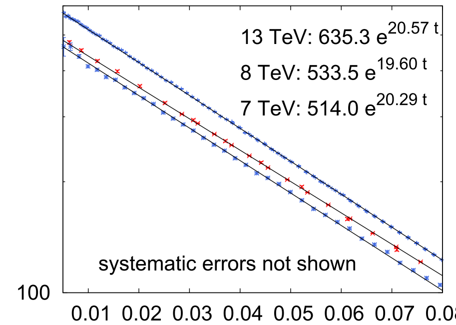

Fig. 6. Exponential fits to TOTEM data.

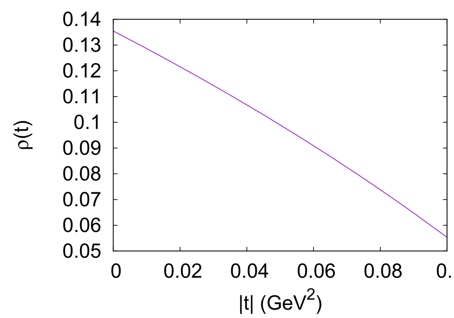

Fig. 7. The calculated ratio $\rho(t)$ of the real to the imaginary part of the hadronic amplitude at 13 TeV.

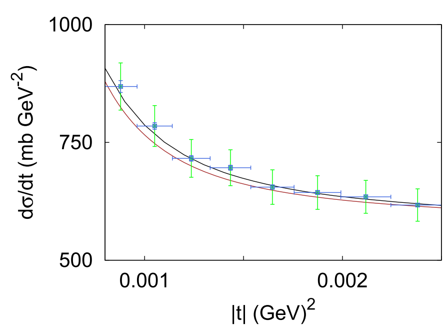

Fig. 8. 13 TeV data without using West-Yennie phase (upper curve) and with (lower curve).

3 TOTEM also extract ρ by using the data only at the one energy 13 TeV. If we perform our fit only at 13 TeV and include the West-Yennie phase [5] in the Coulomb term (2g) we obtain $\rho = 0.1$, in agreement with TOTEM. However, it has been shown [6] that this use of the West-Yennie phase is incorrect because of the variation of the amplitude's phase with t shown in Fig. 7.

4 Fitting the data at only one energy ignores information linking the phase of the amplitude to its variation with s: see the signature factors (1e) for example. If we include in our fit even only the 13 TeV and 8 TeV data, the output for ρ is 0.14 whether or not the West-Yennie phase is included though, as Fig. 8 shows, the fit is slightly better without it.

5 It should be recognised that the value of ρ extracted from data inevitably depends on just what functional form is used to fit the data. This is illustrated in Fig. 9, which shows that over a wide range of values of $\sqrt{s}$ the real part of $\log(-s)$ agrees very well with a power of s, but the corresponding imaginary parts are somewhat different.

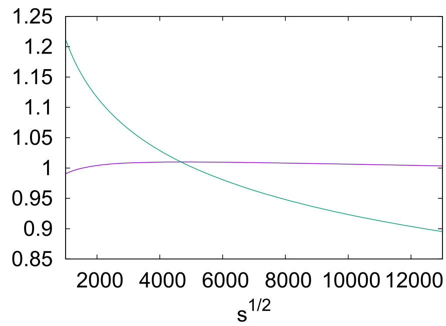

Fig. 9. Ratios of real and imaginary parts of $\log(-s)$ to those of $Cs^{\epsilon}e^{-i\pi\epsilon/2}$ with $C=6.2$ and $\epsilon=0.059$, chosen so that the real parts almost agree.

6 Those who fit total cross section and elastic scattering data often replace powers of s by log factors. In Regge theory this is unnatural: it would correspond to more than just a simple pole in the complex angular momentum plane. The excuse for including a log, or the square of a log, in the amplitude is often said to be to saturate the Froissart-Lukaszuk-Martin bound [7]. However, the bound is about 20 barns at LHC energies and so is irrelevant.

3. Concluding remarks

The value 0.14 obtained for ρ does not encourage the belief [3] that there is an odderon contribution at $t=0$. However, there is good reason to believe that there is an odderon contribution at large t and that it is identified with triple-gluon exchange. Indeed, this led us to predict [8] that pp and $\bar{p}p$ scattering would be different, as was confirmed [9] at the CERN ISR. We included such a term in a previous fit [10]. Note, however, that to lowest order in the strong coupling triple-gluon exchange's contribution to the pp amplitude is real positive, while of the TOTEM odderon would have negative real part at $t=0$.

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