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Comparison of pp and $p\bar{p}$ differential elastic cross sections and observation of the exchange of a colorless C-odd gluonic compound

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We describe an analysis comparing the $p\bar{p}$ elastic cross section as measured by the D0 Collaboration at a center-of-mass energy of 1.96 TeV to that in pp collisions as measured by the TOTEM Collaboration at 2.76, 7, 8, and 13 TeV using a model-independent approach. The TOTEM cross sections extrapolated to a center-of-mass energy of $\sqrt{s} = 1.96$ TeV are compared with the D0 measurement in the region of the diffractive minimum and the second maximum of the pp cross section. The two data sets disagree at the 3.4σ level and thus provide evidence for the t-channel exchange of a colorless, C-odd gluonic compound, also known as the odderon. We combine these results with a TOTEM analysis of the same C-odd exchange based on the total cross section and the ratio of the real to imaginary parts of the forward elastic scattering amplitude in pp scattering. The combined significance of these results is larger than 5σ and is interpreted as the first observation of the exchange of a colorless, C-odd gluonic compound.

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At high energies, the scattering amplitudes of both proton-proton (pp) and proton-antiproton ($p\bar{p}$) elastic collisions are dominated by t-channel exchanges that carry the quantum numbers of the vacuum. A charge (C) even and a subdominant C-odd combination can be formed from the pp and $p\bar{p}$ elastic scattering amplitude [1–5]. The differences between these cross sections are due to processes that involve the exchange of the odderon [1, 2], which couples differently to particles and their charge conjugates. In the quantum theory of strong interactions, quantum chromodynamics, this C-odd exchange is described by the t-channel exchange of a colorless three-gluon compound at leading order [6–8], where the binding strength among the gluons is greater than the strength of their interaction with other particles. At TeV energies the effects of t-channel virtual meson exchanges may be neglected [9].

The TOTEM Collaboration reported strong evidence that the measurements [10] of the ρ parameter, the ratio of the real to imaginary part of the forward elastic scattering amplitude, and the total cross section in pp scattering are inconsistent with the hypothesis of purely C-even exchanges for a range of models [11]. In this paper, we present independent evidence for C-odd exchanges based on a comparison of the pp and $p\bar{p}$ elastic scattering cross sections as a function of momentum transfer t, and combine the two measurements.

The D0 experiment [12] collected elastic $p\bar{p}$ data [13] at a center-of-mass energy $\sqrt{s}$ of 1.96 TeV using special Tevatron optics and beams containing only one proton and one anti-proton bunch. About two million elastic events were collected in two separate data collection periods. The scattered p and $\bar{p}$ were measured in the forward proton detectors (FPD). The FPD consisted of sets of scintillating fiber detectors downstream of the interaction point (IP) along the proton and anti-proton beam lines. The momenta of the scattered p and $\bar{p}$ were measured in two stations of fibers located at distances of 23 and 31 m from the IP. The resolution for the measurement of the squared transverse momentum $|t|$ varied from 0.02 GeV^2 at $|t| \approx 0.25$ GeV^2 value to 0.04 GeV^2 at $|t| \approx 1.2$ GeV^2. The systematic uncertainties in the D0 results included the effects of the beam divergence, the uncertainty in the FPD positions, the choice of the Monte Carlo (MC) ansatz function used to calculate the acceptance, and the efficiencies of the scintillating fibers [13].

The TOTEM Collaboration at the CERN Large Hadron Collider (LHC) measured the differential elastic pp cross sections at $\sqrt{s} = 2.76$ [14], 7 [15], 8 [16] and 13 [17] TeV. The TOTEM experiment [18] utilizes sets of Roman Pot detectors (RPs) to detect elastically and diffractively scattered protons at very small angles. The RP system is composed of two arms placed symmetrically about interaction point 5 of the LHC. Each arm contains several RP stations between 213 m and 220 m from the IP. The detectors and their configurations were changed for the measurements at the different energies, but each measurement was based on a pair of RPs with a lever arm between 5 and 7 m. There are three RPs in each station, one approaching the beam from the top, one from the bottom, and one horizontally. Each RP is equipped with a stack of 10 silicon strip detectors designed with the objective of reducing the insensitive area at the edge facing the beam to only a few tens of micrometers. In each detector, the 512 strips with 66 μm pitch are oriented at an angle of $+45^\circ$ (five planes) and -45° (five planes) with respect to the detector edge facing the beam.

Figure 1 shows the D0 and TOTEM differential cross sections used in this study as functions of $|t|$ [19]. All pp cross sections show a common pattern of a diffractive minimum ("dip") followed by a secondary maximum ("bump") in $d\sigma/dt$. Fig. 2 shows the ratio R of the differ-

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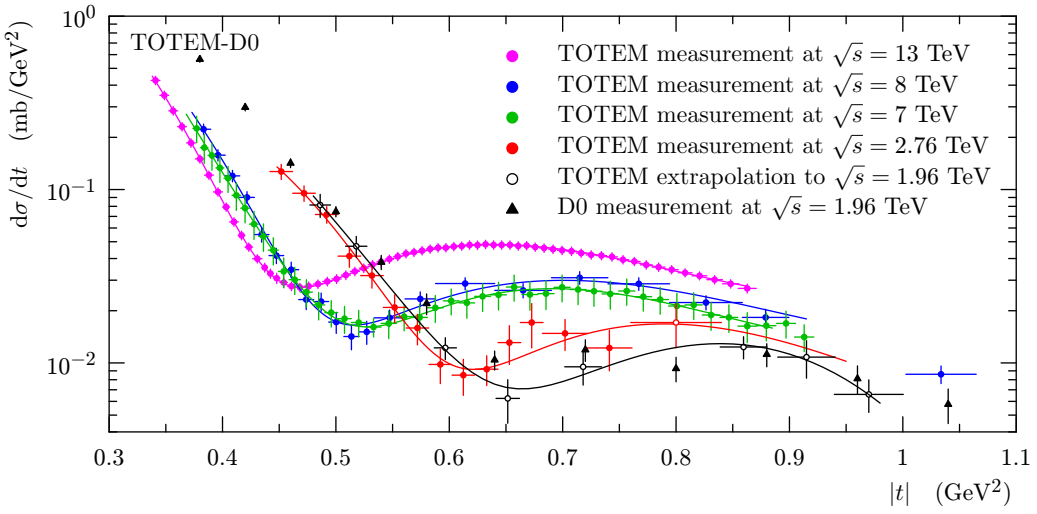

FIG. 1: The measured pp elastic cross sections as functions of $|t|$ at 2.76, 7, 8, and 13 TeV (full circles), and the extrapolation (discussed in the text) to 1.96 TeV (empty circles). The lines show the double exponential fits to the TOTEM data (see text). The $p\bar{p}$ measurement by the D0 Collaboration at 1.96 TeV is also shown in full triangles.

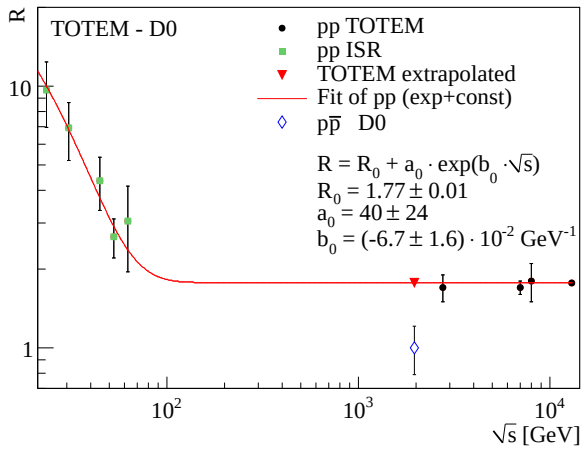

FIG. 2: The ratio, R, of the cross sections at the bump and dip as a function of $\sqrt{s}$. The points for $\sqrt{s} < 100$ GeV are from the ISR measurements. The pp data are fitted to the function noted in the legend. The ratio for the D0 $p\bar{p}$ results is shown at $\sqrt{s} = 1.96$ TeV.

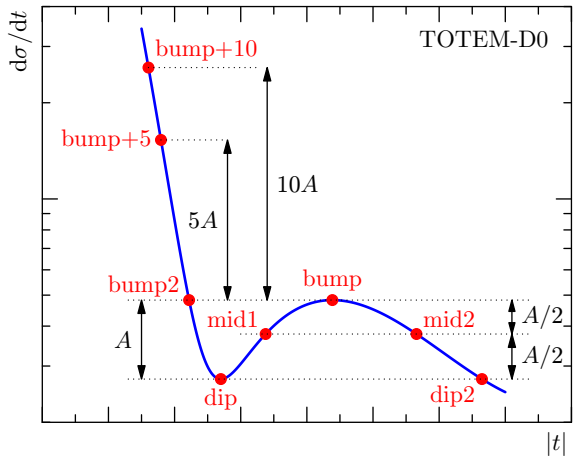

FIG. 3: Schematic definition of the characteristic points in the TOTEM differential cross section data. A represents the vertical distance between bump and dip.

ential cross sections measured at the bump and dip locations as a function of $\sqrt{s}$ for ISR [20] and TOTEM [14–17] pp elastic cross section data. The pp data are fitted using the formula $R = R_0 + a_0 \exp(b_0\sqrt{s})$. We note that R decreases as a function of $\sqrt{s}$ in the ISR regime and flattens out at LHC energies. Since there is no discernible dip or bump in the $p\bar{p}$ cross section, we estimate R by taking the maximum ratio of the measured $d\sigma/dt$ values over the three neighboring bins centered on the bump and dip locations predicted by the TOTEM measurements. The $p\bar{p}$ value, R =1.0 $\pm$ 0.2, differs from the pp ratio by more than 3σ assuming that the flat R behavior of the pp cross section ratio at the LHC continues down to 2 TeV.

Motivated by the features of the pp elastic $d\sigma/dt$ measurements, we define a set of eight characteristic points. as shown in Fig. 3. For each characteristic point, we identify the values of $|t|$ and $d\sigma/dt$ at the closest measured points to the characteristic point, thus avoiding the use of model-dependent fits. In cases where two adjacent points are of about equal value, the data bins are merged. This leads to a distribution of $|t|$ and $d\sigma/dt$ values as a function of $\sqrt{s}$ for all characteristic points as shown in Fig. 4. The uncertainties correspond to half the bin size in $|t|$ (comparable to the $|t|$-resolution) and to the published uncertainties on the cross sections.

The values of $|t|$ and $d\sigma/dt$ as functions of $\sqrt{s}$ for each

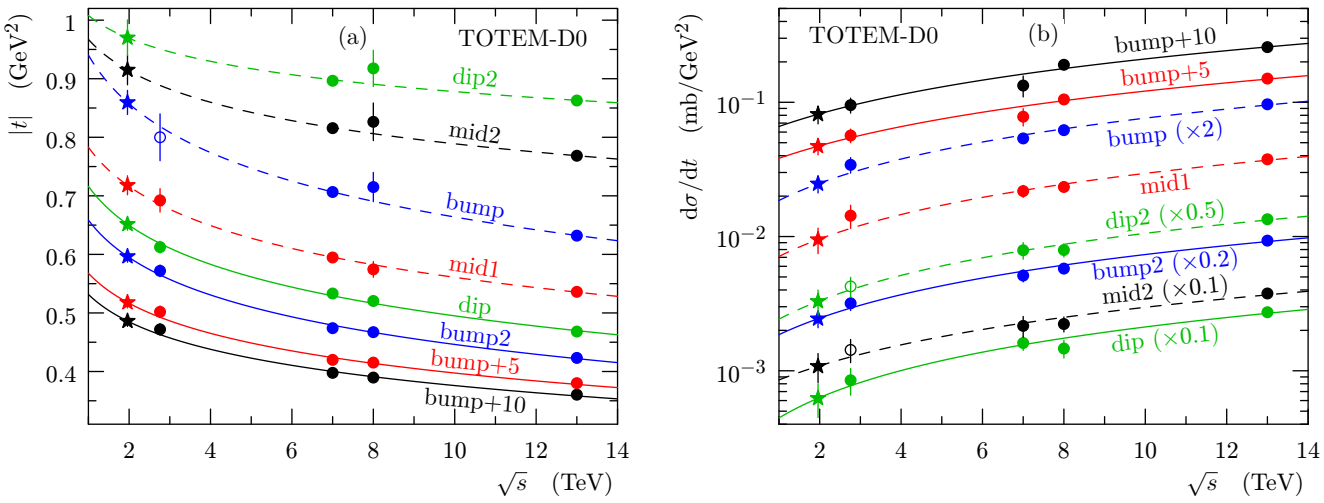

FIG. 4: Characteristic points in (a) $|t|$ and (b) $d\sigma/dt$ from TOTEM measurements at 2.76, 7, 8, and 13 TeV (circles) as a function of $\sqrt{s}$ extrapolated to Tevatron center-of-mass energy (stars) using (a) Eq. 1 and (b) Eq. 2. On (b), a multiplication factor indicated in parenthesis is applied in order to distinguish the different fits. Filled symbols are from measured points; open symbols are from extrapolations or definitions of the characteristic points.

characteristic point are fitted using the functional forms

$$|t| = a\log(\sqrt{s}) + b \quad (1)$$

$$(d\sigma/dt) = c\sqrt{s} + d. \quad (2)$$

The parameter values are determined for each characteristic point separately and the same functional form describes the dependence for all characteristic points. The fact that the same form can be used for all points is not obvious and might be related to general properties of elastic scattering [21]. The χ^2 values for the majority of fits are close to 1 per degree of freedom (dof). Alternative forms that give adequate fits yield extrapolated values that are the same within uncertainties, hence the approach used is essentially model-independent.

The $|t|$ and $d\sigma/dt$ values extrapolated for the characteristic points for pp interactions at 1.96 TeV are displayed as open black circles in Fig. 1. The uncertainties on the extrapolated $|t|$ and $d\sigma/dt$ values are computed using a full treatment of the fit uncertainties, taking into account the fact that the systematic uncertainties of the different characteristic points are not correlated because they correspond to different data sets and running conditions.

To compare the extrapolated pp elastic cross sections with the $p\bar{p}$ measurements, we compute the pp cross sections at the same $|t|$-values as in the D0 measurements (in the interval $0.50 \leq |t| \leq 0.96$ GeV^2). We fit the pp extrapolated data at 1.96 TeV with the function

$$h(t) = a_1 e^{-a_2|t|^2 - a_3|t|} + a_4 e^{-a_5|t|^3 - a_6|t|^2 - a_7|t|}. \quad (3)$$

The fit gives a χ^2 of 0.63 per dof [22]. The first exponential in Eq. (3) describes the cross section up to the location of the dip, where it falls below the second exponential that describes the asymmetric bump and subsequent falloff. This functional form (with non-zero a_7) also provides a good fit for the measured pp cross sections at all energies as shown by the fitted functions in Fig. 1.

We evaluate the pp extrapolation uncertainty from MC simulation in which the cross section values of the eight characteristic points are varied within their Gaussian uncertainties and the fits given by Eq. 3 are performed. Fits without a dip and bump position matching the extrapolated values within their uncertainties are rejected, and slope and intercept constraints are used to discard unphysical fits [23]. The MC simulation ensemble provides a Gaussian-distributed pp cross section at each t-value, allowing a 1σ uncertainty band to be defined. However, the dip and bump matching constraints cause the median of the band to deviate from the best-fit cross sections. For the χ^2 comparison with the D0 measurements below we choose to use the center of the band.

We scale the pp extrapolated cross section so that the optical point (OP), $d\sigma/dt(t = 0)$, is the same as that for $p\bar{p}$. The cross sections at the OP are expected to be equal if there are only C-even exchanges. Possible C-odd effects [21] are taken into account below as systematic uncertainties. Rescaling the OP for the extrapolated pp cross section would not itself constrain the behavior away from $t = 0$. However, as demonstrated in Refs. [24, 25] the ratio of the pp and $p\bar{p}$ integrated elastic cross sections becomes one in the limit $\sqrt{s} \to \infty$. The parts of the elastic cross sections in the low $|t|$ Coulomb-nuclear interference region and in the high $|t|$ region above the exponentially falling diffractive cone that do differ for pp and $p\bar{p}$ scattering contribute negligibly to the total elastic cross sections. Thus, to excellent approximation, the integrated pp and $p\bar{p}$ elastic cross sections in the exponential diffractive region should be the same, implying that the logarithmic slopes should be the same. As this is the case within uncertainty for the pp and $p\bar{p}$ cross sections

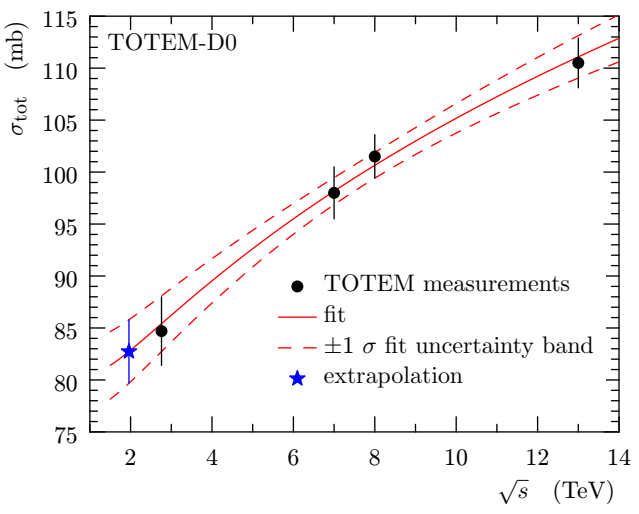

FIG. 5: Total cross section measurements by TOTEM at 2.76, 7, 8, and 13 TeV (black circles) extrapolated to 1.96 TeV (blue star). The dashed lines represent the 1σ uncertainty band.

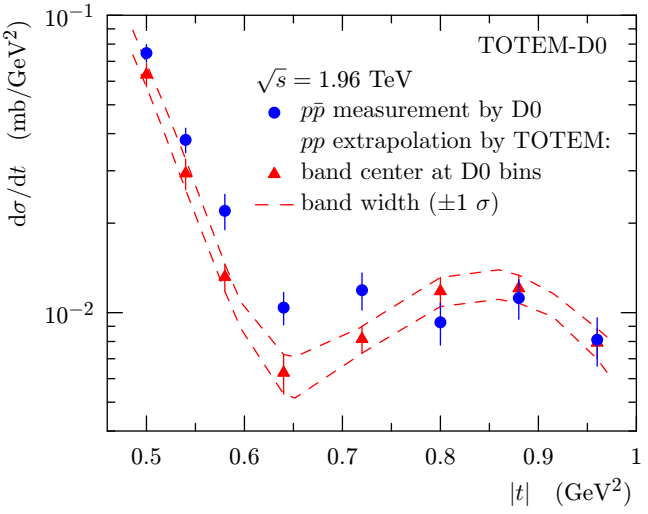

FIG. 6: Comparison between the D0 $p\bar{p}$ measurement at 1.96 TeV and the extrapolated TOTEM pp cross section, rescaled to match the OP of the D0 measurement. The dashed lines show the 1σ uncertainty band.

before the OP normalization, we choose to constrain the scaling to preserve the measured logarithmic slopes. We assume that no t-dependent scaling at t values beyond the diffractive cone ($|t| \geq 0.55$) is necessary.

To obtain the OP for pp at 1.96 TeV, we compute the total cross section by extrapolating the measurements by the TOTEM Collaboration at 2.76, 7, 8, and 13 TeV as illustrated in Fig. 5. A two-parameter fit of σ_{tot} is performed using

$$\sigma_{tot} = b_1 \log^2(\sqrt{s}/1\text{TeV}) + b_2 \quad (4)$$

with a $\chi^2 = 0.27$ for 2 dof, $b_1 = 4.63 \pm 0.72$ mb, and $b_2 = 80.64 \pm 3.36$ mb, leading to an estimate of the total cross section at the Tevatron energy of $\sigma_{tot} = 82.7 \pm 3.1$ mb. The extrapolated cross section is converted to a differential cross section $d\sigma/dt = 357 \pm 26$ mb/GeV^2 at $t = 0$ using the optical theorem

$$\sigma_{tot}^2 = \frac{16\pi(\hbar c)^2}{1+\rho^2}\left(\frac{d\sigma}{dt}(t=0)\right). \quad (5)$$

We assume $\rho = 0.145$ based on the COMPETE extrapolation [11]. The D0 Collaboration published an exponential fit of $d\sigma/dt$ in the range $0.26 < |t| < 0.6$ GeV^2 [13], which is extrapolated to $t = 0$ to give the OP cross section of 341 ± 48 mb/GeV^2. Thus the TOTEM OP and extrapolated $d\sigma/dt$ values are rescaled by 0.954 ± 0.071 (consistent with the OP uncertainties), where the uncertainty is due to that on the TOTEM extrapolated OP. We note that we do not claim that we have performed a measurement of $d\sigma/dt$ at the OP at $t = 0$ since this would require additional measurements of the elastic cross section closer to $t = 0$, but we require equal OPs simply to obtain a common and somewhat arbitrary normalization for the two data sets.

The assumption of the equality of the pp and $p\bar{p}$ elastic cross sections at the OP could be modified if an odderon exists [1, 2]. A reduction of the significance of a difference between pp and $p\bar{p}$ cross sections would only occur if the pp total cross section were larger than the $p\bar{p}$ total cross section at 1.96 TeV. This is the case only in maximal odderon scenarios [21], in which a 1.19 mb difference of the pp and $p\bar{p}$ total cross sections at 1.96 TeV would correspond to a 2.9% effect for the OP. This is taken as an additional systematic uncertainty and added in quadrature to the quoted OP uncertainty estimated from the TOTEM total cross section fit. The effect of additional (Reggeon) exchanges [9, 26, 27], different methods for extrapolation to the OP, and potential differences in ρ for pp and $p\bar{p}$ scattering are negligible compared with the uncertainties in the experimental normalization. The comparison between the extrapolated and rescaled TOTEM pp cross section at 1.96 TeV and the D0 $p\bar{p}$ measurement is shown in Fig. 6 over the interval $0.50 \leq |t| \leq 0.96$ GeV^2.

We perform a χ^2 test to examine the probability for the D0 and TOTEM differential elastic cross sections to agree. The test uses the difference of the integrated cross section in the examined $|t|$-range with its fully correlated uncertainty, and the experimental and extrapolated points with their covariance matrices. The correlations for the D0 measurements at different t-values are small, but the correlations between the eight TOTEM extrapolated data points are large due to the fit using Eq. 3, particularly for neighboring points. Given the constraints on the OP normalization and logarithmic slopes of the elastic cross sections, the χ^2 test with six degrees of freedom yields the p-value of 0.00061, corresponding to a significance of 3.4σ.

We make a cross check of this result using an adaptation of the Kolmorogov-Smirnov test in which correlations in uncertainties are taken into account using simu-

lated data sets [28, 29]. This cross check, including the effect of the difference in the integrated cross section in the examined $|t|$-range via the Stouffer method [30], gives a p-value for the agreement of the pp and $p\bar{p}$ cross sections that is equivalent to the χ^2 test.

We interpret this difference in the pp and $p\bar{p}$ elastic differential cross sections as evidence that two scattering amplitudes are present and that their relative sign differs for pp and $p\bar{p}$ scattering. These two processes are even and odd under crossing (or C-parity) respectively and are identified as pomeron and odderon exchanges [1, 2]. The dip in the elastic cross section is generally associated with the t-value where the pomeron-dominated imaginary part of the amplitude vanishes. Therefore the odderon, believed to constitute a significant fraction of the real part of the amplitude, is expected to play a large role at the dip. In agreement with predictions [21, 31], the pp cross section exhibits a deeper dip and stays below the $p\bar{p}$ cross section at least until the bump region.

We combine the present evidence using the Stouffer method with the independent evidence of the odderon found by the TOTEM Collaboration using the measurements of the ρ parameter and total cross section [10] in a completely different $|t|$ domain. For the model preferred by COMPETE [11], the TOTEM ρ measurement at 13 TeV provided a 4.6σ significance [32], leading to a total significance of 5.7σ for the t-channel exchange of a colorless C-odd gluonic compound when combined with the present result. The combined significance ranges from 5.2 to 5.7σ depending on the model [11, 31] when also including the model uncertainties.

In conclusion, we have compared the D0 $p\bar{p}$ elastic cross sections at 1.96 TeV and the TOTEM pp cross sections extrapolated to 1.96 TeV from measurements at 2.76, 7, 8, and 13 TeV using a model independent method [33]. The pp and $p\bar{p}$ cross sections differ with a significance of 3.4σ, and this stand-alone comparison provides evidence that a t-channel exchange of a colorless C-odd gluonic compound, i.e. an odderon, is needed to describe elastic scattering at high energies [21]. When combined with the result of Ref. [10], the significance is in the range 5.2 to 5.7σ and thus constitutes the first experimental observation of the odderon.

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