

Central elastic scattering

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Abstract

We comment on the phase selection of the scattering amplitude, emphasizing the elastic overlap function should have the central impact parameter profile and the role of the reflective scattering mode. Emerging problems with the use of peripheral impact parameter dependence of the elastic overlap function are explicitly indicated. Their solution is an elimination of the phases connected to peripheral form of the elastic overlap function. Contrary, we adhere to a peripheral form of the *inelastic* overlap function having its maximum at nonzero value of the impact parameter. Phenomenologically, such a dynamics of hadron scattering is motivated by a hadron structure a hard central core presence.

Introduction. Role of the scattering amplitude phase

The phase of the elastic scattering amplitude plays an important role in physics interpretation of hadron scattering. It is just as important as the scattering amplitude modulus. Unfortunately, the phase is an essentially unknown quantity, and the experimental data on the differential cross-section of the elastic scattering allow one to reconstruct the amplitude $F(s, t)$ with its phase at $-t = 0$ only with the use of Coulomb–Nuclear Interference (CNI) contribution at very small values of $-t$. Moreover, certain model assumptions are needed even at this reconstruction. In the simplest case of the assumed constant (i.e. t -independent) phase the relation of a ratio of the real to imaginary parts of the forward scattering amplitude ρ with the phase $\arg F(s, t)$ is the following:

$$\arg F(s, t) = \frac{\pi}{2} - \arctan \rho.$$

However, the phase t -dependence can be nontrivial and therefore several various parameterizations of nuclear amplitude phase dependencies on t have been used in [1]. The significant uncertainty allows one to use a wide range of assumptions on the phase behavior and it would be useful to find some arguments limiting the phase selection.

The role of phase in hadron scattering is under active theoretical discussion nowadays, cf. e.g. [2–6], due to its relation to the recent measurements of the real to imaginary part ratio ρ .

The elastic and inelastic overlap functions $h_{l,el}$ and $h_{l,inel}$ introduced by Van Hove [7] are related by unitarity and are strongly interdependent with phase. The important role of the phase has been clearly demonstrated under analysis of the LHC experimental data obtained by the TOTEM Collaboration [1].

To this end, it is instrumental to address the elastic and inelastic overlap functions in order to limit the choice of phases suitable for the further considerations and phenomenological analysis. It is aim of present note.

Thus, we discuss an exclusion of the phase t -dependencies on the ground of unitarity and analyticity of the scattering amplitude. The peripheral distribution of elastic scattering has been suggested by the analogue tunneling picture of strong interactions based on the hypothesis of "maximal importance" of particle production [8]. The cases, subject to elimination, correspond to a peripheral form of the elastic overlap function [9]. Despite the explicit form of the phase dependence after elimination of the peripheral distribution of elastic scattering remains to be unknown and leave us with results of phenomenological models only, it allows one to reduce an arbitrariness under the phase selection in the data analysis [1].

The reasons for elimination of the peripheral option of $h_{l,el}$ at the phase pre-selection stage are discussed in Section 1. Section 2 is devoted to the central

form of the elastic overlap function in the impact parameter representation and its correlation with the reflective scattering mode.

1 Problems with peripheral form of the elastic overlap function

The partial elastic scattering matrix element can always be represented as the complex function:

$$S_l(s) = \kappa_l(s) \exp[2i\delta_l(s)] \quad (1)$$

with the two real functions κ_l, δ_l . Herewith κ_l can vary in the interval $0 \leq \kappa_l \leq 1$ and is known as an absorption factor. The value $\kappa_l = 0$ means a complete absorption of the corresponding initial state.

The function $h_{l,el}(s) \equiv |f_l(s)|^2$ is a contribution of the elastic intermediate states while $h_{l,inel}(s)$ is a contribution of the inelastic intermediate states into the unitarity relation:

$$\text{Im}f_l(s) = h_{l,el}(s) + h_{l,inel}(s). \quad (2)$$

The latter can be expressed through the function $\kappa_l(s)$ by the relation

$$\kappa_l^2(s) = 1 - 4h_{l,inel}(s). \quad (3)$$

The normalization is such that $S_l = 1 + 2if_l$. In Eq. (2) $f_l(s)$ is the partial scattering amplitude, the functions $h_{l,el}(s)$ and $h_{l,inel}(s)$ correspond to the elastic and inelastic overlap functions $h_{el}(s, b)$ and $h_{inel}(s, b)$ which can be related to the probability distributions of the elastic and inelastic interactions over impact parameter $b = 2l/\sqrt{s}$ (cf. [10]).

Evidently, unitarity relation implies that the limiting behavior $\text{Im}f_l \rightarrow 1$ leads to a vanishing real part of the scattering amplitude, i.e. $\text{Re}f_l \rightarrow 0$, cf. [11].

Eq. (2) can be rewritten in the form

$$\text{Im}f_l(s)[1 - \text{Im}f_l(s)] = [\text{Re}f_l(s)]^2 + h_{l,inel}(s). \quad (4)$$

We consider the region of $l \gg 1$, $s \gg 4m_\pi^2$ and large $2l/\sqrt{s}$, i.e. the region of peripheral interactions at high energies. Eq. (4) can be simplified in this region since $\text{Im}f_l(s) \ll 1$. The smallness of $\text{Im}f_l(s)$ is a result of the axiomatic field theory [12] (short-range nature of strong interactions) and the unitarity relation can be approximated by the following one:

$$\text{Im}f_l(s) \simeq [\text{Re}f_l(s)]^2 + h_{l,inel}(s). \quad (5)$$

Thus, we have an evident upper bound for the function $[\text{Re} f_l(s)]^2$ as a consequence of unitarity and short-range nature of strong interactions:

$$[\text{Re} f_l(s)]^2 \leq \text{Im} f_l(s). \quad (6)$$

Any phenomenological model with a nonvanishing real part of the amplitude should test its amplitude against this inequality along with the other constraints.

It will be shown further that unitarity being combined with Mandelstam analyticity allows one to obtain a more stringent constraint for the profile of the elastic overlap function (which is just $[\text{Re} f_l(s)]^2$ in the region under consideration) and, as a consequence, for the scattering amplitude phase since the overlap function strongly depends on it.

Namely, it is known (cf. [13]) that the amplitude $f_l(s)$ (its real and imaginary parts) decrease exponentially with l at large values of l and s according to the Froissart–Gribov formula [14, 15]:

$$f_l(s) \simeq \omega(s) \exp(-\mu \frac{2l}{\sqrt{s}}), \quad (7)$$

where $\omega(s)$ is a complex function of energy and μ is determined by the position of the lowest singularity in the t -channel. Thus, the exponent is the same for the real as well as for the imaginary parts of the amplitude f_l and is determined by the mass of two pions. Eq. (7) originates from Mandelstam representation and is consistent also with the analyticity of the amplitude $F(s, t)$ in the Lehmann–Martin ellipse [12].

We address the problems related to a peripheral option for the elastic scattering in what follows. Peripheral dominant role of $h_{l,el}(s)$ has been discussed in [9, 16–18] and could occur due to a nontrivial contribution of the real part at large values of l , i.e. by the relevant choice of the scattering amplitude phase.

If the elastic scattering is assumed to be dominant in peripheral region and the contribution of the inelastic states can be neglected in Eq. (5), the following approximate relation should be considered as valid at large values of l, s and $2l/\sqrt{s}$:

$$\text{Im} f_l(s) \simeq [\text{Re} f_l(s)]^2. \quad (8)$$

Both Eq. (8) or the elastic unitarity

$$\text{Im} f_l(s) = h_{l,el}(s) \quad (9)$$

are in conflict with Eq. (7) since the latter points to the same exponent of the decrease with l for the real and imaginary parts of the scattering amplitude. Therefore, the peripheral domination of the elastic scattering should be discarded with the corresponding elimination of the respective phases.

Alternative option anticipates a central elastic scattering. Then Eq. (5) can be approximated in the following form

$$\text{Im} f_l(s) \simeq h_{l,inel}(s), \quad (10)$$

confirming a shadow nature of elastic scattering at large values of l . The use of the phases, leading to peripheral dominance of the elastic scattering [1] looks like an artificial version because of shadow nature of the amplitude at large impact parameters takes place (cf. e.g. [19, 20]). Moreover, we can conclude on the following behaviour of $h_{l,inel}(s)$ resulting from Eq. (7)

$$h_{l,inel}(s) \simeq \text{Im} \omega(s) \exp(-\mu \frac{2l}{\sqrt{s}}) \quad (11)$$

in the region of large l and s variation. In addition, the relations

$$[\text{Re} f_l(s)]^2 \ll h_{l,inel}(s) \quad (12)$$

and, consequently,

$$[\text{Re} f_l(s)]^2 \ll \text{Im} f_l(s) \quad (13)$$

are being valid for the large values of l , s and $2l/\sqrt{s}$.

Those relations reflect a shadow nature of elastic scattering, cf. Eq. (10), i.e. they correspond to the central profile of elastic scattering in the impact parameter representation. The large- l tail of $\text{Im} f_l(s)$ at high energies is due to the inelastic collisions.

One should conclude that assumption of approximate elastic unitarity applicability in this region of l and s variation contradicts to analytical properties of the scattering amplitude. Indeed, this is not surprising from the general principles and from viewpoint based on semiclassical scattering picture of hadrons.

2 Central elastic scattering

We proceed with the discussion of physical picture which corresponds to central elastic scattering. It is a common practice to analyse the hadron scattering in the impact parameter representation (cf. [21]) which is a convenient way to use a semiclassical picture of hadron collisions at high energies [10, 22, 23]. Indeed, the impact parameter b is a conserved quantity at high energies and the scattering amplitude is determined by the Fourier–Bessel transformation of the amplitude $F(s, t)$. The elastic and inelastic overlap functions can be interpreted as the differential contributions to the elastic and inelastic cross–sections respectively over the impact parameter b [10].

Large- b behaviour of the scattering amplitude can be obtained from the relation similar to Froissart–Gribov projection formula [22, 24, 25]. Note, that due to high collision energy and the exponential decrease of the amplitude $F(s, t)$ (both of the real and imaginary parts) with $-t$, the effect of finite integration range over $-t$ at finite energy is not significant and has therefore been neglected by the use of the infinite integration limit.

In what follows central profile of the elastic overlap function will be considered. The respective estimates lead to the value of the elastic overlap function equal to 0.31 at $b = 0$ [1]¹ indicating transition to the reflective scattering mode [27] in a complete agreement with the results of [28, 29]. It has been shown that the inelastic overlap function $h_{inel}(s, b)$ is very close to its limiting value $h_{inel}^{max} = 1/4$ in the region of small and moderate impact parameters, $0 \leq b \simeq 0.4$ fm at the LHC energy $\sqrt{s} = 13$ TeV. Deviation of h_{inel} from its maximal value is small and negative in this region of the impact parameters and

$$h_{el}(s, b) > 1/4 > h_{inel}(s, b).$$

In fact, $h_{inel}(s, b)$ has a shallow local minimum at $b = 0$. Unitarity relation at this energy value and the impact parameters range gives

$$(\text{Im}f(s, b) - 1/2)^2 + (\text{Re}f(s, b))^2 \simeq 0. \quad (14)$$

The estimates $\text{Re}f(s, b) \simeq 0$ and $\text{Im}f(s, b) \simeq 1/2$ result from Eq. (14). Thus, the impact parameter picture of the elastic scattering together with unitarity can provide at least a qualitative explanation of the recent result on the unexpectedly small real to imaginary parts ratio of the forward scattering amplitude [30].

In fact, there is no room for a significant real part of the elastic scattering amplitude at $\sqrt{s} = 13$ TeV even at high values of the impact parameters. The real part of the amplitude $f(s, b)$ can be neglected due to its smallness (cf. for the numerical estimations [31], qualitative arguments in favor of such an approximation have been given in Section 1).

So, the replacement $f \rightarrow if$ is used and the function $S(s, b)$ is taken to be real. The $S(s, b)$ can acquire the negative values at $b < r(s)$ at high enough energy [27]. The $r(s)$ increases as $\ln s$ at $s \rightarrow \infty$ and its value at $\sqrt{s} = 13$ TeV is approximately equal to 0.4 fm. At $b < r(s)$ the reflection appears, i.e the function $S(s, b)$ in Eq. (1) crosses zero at $b = r(s)$ and the value of the phase δ changes abruptly from 0 to $\delta = \pi/2$.

Under the reflective scattering regime, $f > 1/2$, an *increase* of the elastic scattering amplitude f correlates with *decrease* of h_{inel} at small values of b due to

¹From another point of view, this value can be used for the estimation of the value of $[\text{Re}f(s, b = 0)]^2$. It is approximately equal to 0.03 since $\text{Im}f(s, b = 0) \simeq 0.53$ [26] at the energy $\sqrt{s} = 7$ TeV.

unitarity relation

$$[f(s, b) - 1/2]^2 = 1/4 - h_{inel}(s, b) = \kappa^2(s, b)/4. \quad (15)$$

The negative $S(s, b)$ correspond to $\delta = \pi/2$. The term “reflective” comes from optics where the phases of incoming and outgoing waves under reflection differ by π .

The physical picture of high energy hadron interaction region in transverse plane corresponds to a reflective disk surrounded by a black ring. It should be emphasized that it is a picture of the interaction region and not the one of the individual protons. Appearance of the reflective ability at hadron interactions can be associated with a soft deconfinement proposed in [32] due to possibility to probe a hard core in hadron structure. The reflective scattering can also be associated with formation of the color conducting medium in the intermediate state of hadron interaction [33]. Such a medium can be associated with the Quarkyonic Matter discussed in [32, 34, 35].

This picture finds its confirmation in the experiments at JLab and the LHC [1, 36]. The results are in favor of the dominance of elastic scattering in the soft deconfined state. The mechanism is energy-dependent and leads to the elastic scattering dominance at $s \rightarrow \infty$ resulting from its increasing decoupling from the inelastic production [37].

Central profile of the elastic overlap function is encoded into the relation

$$\frac{\partial h_{el}}{\partial b} = \left(\frac{1 - S}{S} \right) \frac{\partial h_{inel}}{\partial b}. \quad (16)$$

resulting from [38]. Note, that $(1 - S)/S < 0$ in the reflective scattering region. The respective qualitative dependencies are presented at Fig. 1. The picture of hadron scattering is schematically represented at Fig. 2 [39]. Because of presence of the hard core, the elastic scattering of hadrons has a central profile and looks similar to the collisions of billiard balls. Under central collisions the cores survive in soft processes. Inelasticity is therefore depleted in central collisions due to unitarity.

3 Concluding remarks

The peripheral profile of the elastic overlap function, i.e. its domination over the inelastic one at large values of b , corresponds to assumption of the *elastic* unitarity validity at high energies and is to be attributed to the dominant contribution of the amplitude’s real part. Evidently, at high energies and large values of b such an assumption faces the troubles due to violation of the consequences of the scattering matrix analyticity and unitarity.

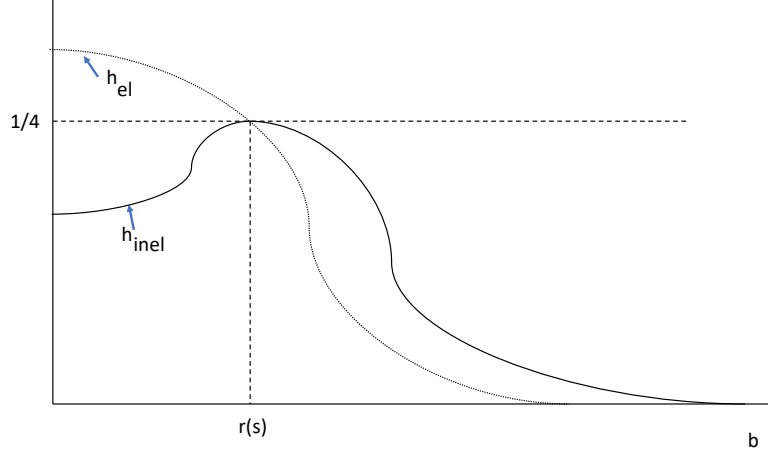


Figure 1: Qualitative b -dependencies of the elastic and inelastic overlap functions h_{el} and h_{inel} .

The inconsistency of the peripheral dominance of the elastic scattering results, in particular, in disregard of the shadow nature of elastic scattering at large impact parameters. The peripheral profile dominance of h_{el} masks an absence of absorption in this region. The proper exclusion of this option corresponds to the respective elimination of the relevant class of the amplitude phase t -dependencies and, thus, can be treated as an emergent constraints for the phase. This exclusion, which means an elimination of the peripheral geometric elastic scattering, provides a consent with the unitarity and analyticity and leads to a clear physical picture of the elastic scattering restoring its shadow nature at large values of the collision impact parameters.

Thus, any choice of phenomenological model or analysis of the experimental data which allows nonvanishing contribution of the amplitude real part should control the predicted profile of the elastic overlap function in the impact parameter representation to guarantee that the inequality

$$[\text{Re}f(s, b)]^2 \ll h_{inel}(s, b)$$

is valid at large values of b . It should be applied at the preselection stage.

The appearance of the constraint for the real part of the scattering amplitude is not unexpected since its real and imaginary parts are related due to dispersion relation. Full implementation of this connection is not realized at the moment, a possible variant of such implementation has been discussed in [40].

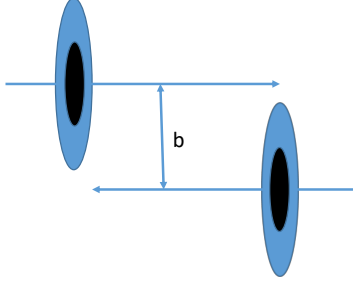


Figure 2: Proton scattering at the impact parameter b .

It is rather reasonable, that a hadron can be seen as a structure with a hard central core coated by a thick but breakable shell. Similar structure of hadrons has been proposed in [41]. The peripheral collisions of hadrons lead to the shells' destruction which is naturally associated with inelastic diffraction processes. This breaking down the shells can therefore be a reason of the peripheral nature of the single or double inelastic diffraction processes.

Diffraction on the outer shell has a shadow nature and it is observed as a diffraction peak with the first dip in $d\sigma/dt$. Diffraction on the core has a geometrical nature and hides the secondary dips and bumps in differential cross-section [39]. The latter is due to the reflective scattering.

The reflective scattering mode can be interpreted as a result of the central core presence in the hadron structure responsible for the appearance of the second diffraction cone in the differential cross-section of the elastic scattering [39]. This mode implies elastic scattering dominance at $s \rightarrow \infty$. Moreover, the feature of this mode is an absence of the noticeable contribution from the real part, i.e. the elastic overlap function always has central profile and the inelastic one has peripheral profile.

Presence of the central proton's core could be associated with a chiral symmetry spontaneous breaking mechanism when the two scales Λ_{QCD} and Λ_χ relevant for the color confinement and spontaneous chiral symmetry breaking are different (recent discussion is in [42]).

Finally, besides the use of CNI for the phase studies, there is a yet another possibility of phase-sensitive experimentation related to the polarization studies.

Those are technically difficult, but results of the polarization measurements are sensitive to the helicity amplitudes phases [43].

References

- [1] G. Antchev et al. (The TOTEM Collaboration), *Eur. Phys. J. C* **26**, 661 (2016).
- [2] I.M. Dremin, *Particles* **2**, 57 (2019).
- [3] V.A. Petrov, arXiv: 2001.04844v2.
- [4] V.A. Khoze, A.D. Martin, M.G. Ryskin, *Phys. Rev. D* **101**, 016018 (2020).
- [5] L. Durand, P. Ha, *Phys. Rev. D* **102**, 036025 (2020).
- [6] E. Ferreira, A.K. Kohara, T. Kodama, arXiv: 2011.13335v1.
- [7] L. Van Hove, *Nuovo Cimento* **28**, 798 (1963).
- [8] B. Schrempp, F. Schrempp, *Nucl. Phys. B* **163**, 397 (1980).
- [9] V. Kandrát, M. Locajíček, D. Krupa, *Proc. of the IX Blois Workshop on Elastic and Diffractive Scattering, Průhonice near Prague, Czech Republic, June 9–15, 2001*, p. 247, Eds. V Kandrát, P. Závada.
- [10] B.R. Webber, *Nucl. Phys. B* **87**, 269 (1975).
- [11] S.M. Troshin, *Phys. Lett. B* **682**, 40 (2009).
- [12] A. Martin, *Phys. Rev.* **129**, 1432 (1963).
- [13] P.D.B. Collins, *An Introduction to Regge Theory and High-Energy Physics*, 460pp, Cambridge University Press, Cambridge 1977.
- [14] M. Froissart, *Phys. Rev.* **123**, 1053 (1961).
- [15] V.N. Gribov, *JETP* **41**, 667 (1961).
- [16] R. Cahn, *Z. Phys. C* **15**, 253 (1982).
- [17] V. Kandrát, M. Locajíček, *Z. Phys. C* **63**, 619 (1994).
- [18] V. Kandrát, M. Locajíček, *Mod. Phys. Lett. A* **11**, 2241 (1996).
- [19] J. Pumplin, G.L. Kane, *Phys. Rev. D* **11**, 1183 (1975).
- [20] G. Cohen–Tannoudji, V.V. Ilyin, L.L. Jenkovszky, *Lett. Nuov. Cim.* **5**, 957 (1972).
- [21] F. Halzen, *Lectures presented at the 1973 Summer Institute on Particle Interactions at Very High Energies, Louvain, Belgium*.

- [22] R. Blankenbecler, M.L. Goldberger, Phys. Rev. **126**, 766 (1962).
- [23] M.L. Goldberger, K.M. Watson, Collision Theory, John Wiley & Sons, New-York–London–Sydney, 1964.
- [24] R. Henzi, Nuovo Cim. A **46**, 370 (1966).
- [25] B. Schrempp, F. Schrempp, Nucl. Phys. B **54**, 525 (1973).
- [26] A. Alkin et al, Phys. Rev. D **89**, 091501(R) (2014).
- [27] S.M. Troshin, N.E. Tyurin, Int. J. Mod. Phys. A **22** 4437 (2007) .
- [28] A. Alkin et al., arXiv: 1807.06471v2.
- [29] T. Csörgő, R. Pasechnik, A. Ster, Acta Phys. Pol. B Proc. Suppl. **12** , 779 (2019).
- [30] G. Antchev et al. (The TOTEM Collaboration), Eur. Phys. J. C **79**, 785 (2019).
- [31] I.M. Dremin, V.A. Nechitailo, S.N. White, Eur. Phys. J. C **77**, 910 (2017).
- [32] K. Fukushima, T. Kojo, W. Weise, Phys. Rev. D **102**, 096017 (2020).
- [33] S.M. Troshin, N.E. Tyurin, J. Phys. G, **46**, 105009 (2019).
- [34] L. McLerran, R.D. Pisarski, Nucl. Phys. A **796**, 83 (2007).
- [35] L. McLerran, Acta Phys. Pol. B **51**, 1067 (2020).
- [36] V.D. Burkert, L. Elouadrhiri, F.X. Girod, Nature **557**, 396 (2018).
- [37] S.M. Troshin, N.E. Tyurin, Phys. Lett. B **707**, 558 (2012).
- [38] S.M. Troshin, N.E. Tyurin, Phys. Rev. D **88**, 077502 (2013).
- [39] S.M. Troshin, N.E. Tyurin, EPL **129**, 31002 (2020).
- [40] S.M. Troshin, N.E. Tyurin, Mod. Phys. Lett. A **32**, 1750028 (2017).
- [41] M.M. Islam, Nucl. Phys. B (Proc. Suppl.) **25**, 104 (1992).
- [42] N. Evans, K.S. Rigatos, arXiv: 2012.00032v1.
- [43] M.G. Echevarria et al., PoS SPIN2018 (2019) 063.