Origin of the diffraction cone shrinkage

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Abstract

We comment on the origin of a diffraction cone shrinkage which may not be related to the contribution of the linear Regge trajectories, but can result from the scattering matrix unitarity in the framework of the geometrical models.

Prediction of logarithmic shrinkage of the diffraction cone and its relation with the Regge trajectory parameter $\alpha'(0)$ was one of the most evident achievements of the Regge model extension to the relativistic case of hadron scattering (cf. e.g. [1]). All secondary Regge trajectories have different values of intercept $\alpha_{R_i}(0)$ ($\alpha_{R_i}(0) - 1 < 0$) but approximately the same value of $\alpha'_{R_i}(0) \simeq 0.9$ $(GeV/c)^{-2}$, i.e. the trajectories are linear and almost parallel [2]. The Pomeron trajectory is different, it has been introduced to reconcile the Regge model with the experimental data. Currently it has an intercept $\alpha_P(0) - 1 > 0$ and the slope $\alpha'_P(0) \simeq 0.25\,(GeV/c)^{-2}$ or even less [3], which is significantly lower than $\alpha'_R(0)$.

It appeared that the rate of the diffraction cone parameter

$$B(s) \equiv \frac{d}{dt} \ln \frac{d\sigma}{dt}|_{t=0}, \tag{1}$$

growth becomes higher than the linear logarithmic extrapolation to the LHC energy range [4,5]. In Eq. (1), $d\sigma/dt$ is a differential cross–section of proton–proton elastic scattering. To bring the Regge model closer to the experiment on diffraction cone parameter growth it has been supposed that the slope of the Pomeron trajectory $\alpha'_P(0)$ is an energy–dependent function (cf. [6]). Such a dependence can reflect the negative contribution of absorption correction due to increasing contribution of multi–Pomeron exchanges. It might be due to Odderon presence also while complication of the Regge model in the form of Dipole Pomeron does not help [7].

Regarding the possible form of transition from a linear logarithmic dependence of $B(s) = a + \alpha'_P(0) \ln s$ predicted by the Regge–pole model to the asymptotic $\ln^2 s$ dependence [8] the following observation should be mentioned. Namely, using the function $a + b \ln s + c \ln^2 s$ to fit the data provides a negative value for the factor b and leads therefore to the difficulty with its interpretation as a slope of Pomeron trajectory $\alpha'_P(0)$.

There is another point regarding Regge model in the LHC energy range. Namely, many papers suggest that the secondary Regge trajectories give a negligible contribution to the elastic amplitude $F(s,t)$ and the Pomeron contribution only is significant at the LHC. Moreover, it has been claimed that contribution of the secondary trajectories can be neglected already at $\sqrt{s} \sim 100\ GeV$ [6].

The above statements are based on the analysis of the data for the total cross–sections. However, the Pomeron trajectory has a significantly lower slope $\alpha'_P(0)$ compared to $\alpha'_R(0)$ and conclusion on the vanishing contributions of the secondary Regge trajectories inevitably leads to prediction of slowdown of $B(s)$ increase in the energy range of Pomeron dominance. Thus, the Regge–pole model predicts slowdown of $B(s)$ prior to speeding up of its growth due to account for the unitarity implemented by multi-Pomeron exchanges or due to an effect of the Odderon contribution. Schematically, the predicted energy dependence can be depicted at

Fig. 1. Such slowdown of $B(s)$ which might be treated as a signal of a Pomeron

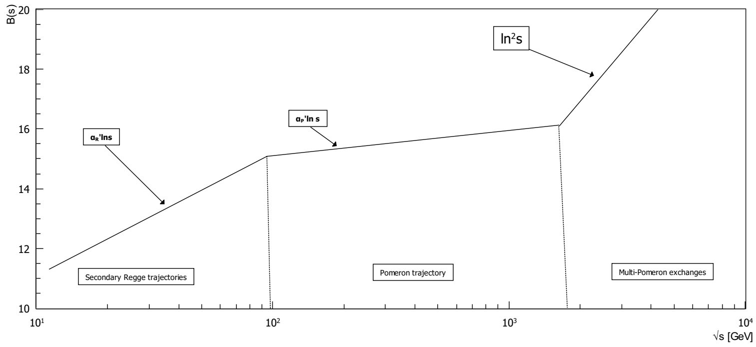

Figure 1: Three regimes of the diffraction cone slope parameter $B(s)$ energy dependence in the Regge-pole model amended with contribution of multi–Pomeron exchanges or Odderon at the LHC energy range.

dominance has not been confirmed experimentally[1].

The resolution of the problem might be a twofold one. First, the energy range between CERN ISR and the LHC should be thoroughly scanned experimentally without an assumption on the approximate equality of pp and $\bar{p}p$ diffraction cone parameters with higher statistical significance. This problem is correlated with the searches for possible signals of the Odderon contributions to the scattering amplitude (cf. [9] and references therein).

Another solution of that qualitative discrepancy is the use of an approach supposing an alternative origin of the diffraction cone parameter growth, namely increase of $B(s)$ at all the energies due to unitarity. To demonstrate this possibility we have considered a wide class of the geometrical models in [10] and [11]. One of the first models of that kind was proposed by Heisenberg [12] for the multiple meson production. A historical remark concerning this model should be placed here. It is often claimed (cf. e.g. [13]) that this model can be considered as a precursor for the saturation of the famous Froissart–Martin bound for the total cross–sections [14, 15]. But, in fact this model has led to the $\ln^2 s$–dependence of the inelastic interactions cross–section only. It has nothing to do with the total cross–sections that include elastic ones. Only after the Froissart–Martin bound for the total cross–sections has been derived, the model [12] can be considered as providing $\ln^2 s$–dependence for the total cross–sections since it is a maximal possible rate of the total cross–sections growth.

[1]This conclusion has been made under assumption of approximate equality of the diffraction cone parameters of pp and $\bar{p}p$ elastic scattering in the energy range between CERN ISR and the LHC energies.

The input amplitude of the geometrical models implies initially an energy–independent diffraction cone parameter behavior, and energy dependence of this parameter arises as a result of subsequent unitarization. This parameter associated with radius of interaction increases at low and moderate energy values, where the total cross–section does not demonstrate any increase yet [16]; it is decreasing or constant one. The growth of the diffraction cone parameter observed experimentally is a reason for unitarization of the input amplitude in geometrical models and this argument is essential even at low and medium energies.

Unitarization adjusts energy dependence of $B(s)$ to the experimental trends at such energies. It has been shown that the resulting energy dependence of the diffraction cone is consistent with the data and can be described up to the LHC energies by a power–like function [10]:

$$B(s) \simeq a + bs^{\lambda}. \tag{2}$$

with value of the exponent $\lambda \simeq 0.1$. This dependence being relevant for the preasymptotic energy region cannot be evidently extended to the asymptotics. The parameter $B(s)$ has the following ultimate asymptotic energy dependence

$$B(s) \sim \ln^2 s \tag{3}$$

replacing in the limit $s \to \infty$ a power–like dependence of Eq. (2). The transition between these two dependencies is automatically realized in the U–matrix form of unitarization [11].

In the U–matrix approach to unitarization (in the pure imaginary case) the relation of the scattering amplitude $f(s,b)$ with the input quantity $u(s,b)$ in the impact parameter representation has a rational form:

$$f(s,b) = u(s,b)/[1 + u(s,b)]. \tag{4}$$

The geometrical models use a factorized form of the input, i.e. the function $u(s,b)$ is taken as a product

$$u(s,b) = g(s)\omega(b), \tag{5}$$

where $g(s) \sim s^{\lambda}$ can be related to an effective rate of the relativistic kinetic energy to mass conversion [17–19]. The function $\omega(b)$ is taken in the form to be consistent with the analyticity of the resulting scattering amplitude in the Lehmann–Martin ellipse (see [20])

$$\omega(b) \sim \exp(-\mu b),$$

which is determined by a convolution of the two matter distributions in the colliding hadrons [21].

The function $u(s,b)$ obtains contributions from multiparticle intermediate states [22] and its monotonic increase with energy can be interpreted as a result of increasing contribution of the newly opened channels of the multiparticle final states. The above form of unitarization, Eq. (4), and the form of the input quantity $u(s, b)$, Eq. (5), lead to appearance of the reflective scattering[2] [23] and to the Eqs. (2) and (3) in the relevant energy ranges, respectfully.

In this way diffraction cone behaviour including its observed changes is a result of the unitarization. Indeed, the diffraction cone slope parameter being calculated with a factorized input function does not depend on energy prior to unitarization.

The recent results of the measurements of $B(s)$ at the LHC where the different regimes observed dependent on the energy intervals and their interpretations make similar measurements in pp–scattering at the higher energies as well as at lower energies very interesting.

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