目錄

Abstract

This practical project will use MATLAB software to simulate, , will adjust random variables to send and receive signals, and to determine the bit error rate and symbol error rate, also discuss binary signals (Orthogonal, Antipodal, On-off), and PAM, 8- PSK and 16QAM, Rayleigh and Rician channel, OSTBC, SIMO, MISO, MIMO. Then analyze the results of SNR error rate under different conditions.

中文摘要

此專題將使用 MATLAB 軟體模擬,將藉由調整隨機變數去傳送及接收訊號，並判斷位元錯誤率及符號錯誤率並探討二元訊號(Orthogonal , Antipodal , On-off)，以及 PAM、8-PSK 及 16QAM， Rayleigh and Rician channel， OSTBC ,SIMO、MISO、MIMO 。進而分析在不同條件的 SNR 錯誤率的結果。

研究動機及目的

目前 4G 壓縮及傳輸資料是利用 256-QAM 及 64-QAM 的調變，而 5G 使用的是 512QAM 及 1024-QAM 這些較多的資料壓縮密度調變/解調變器,所以頻譜效率每 Mbps/100MHz 的利用效率將會更高且增加傳輸速率。另外,5G 使用波束指向配合多輸入多輸出（MIMO）相控陣列天線，MIMO 多輸入多輸出利用電磁波的空間多工和路徑不同多天線系統提高傳輸速率, 5G 的基地台會有數百個天線同時服務使用者(Massive MIMO) 可針對新一代無線資料網路提供多方優勢，以更高的資料傳輸率容納更多使用者，並於加強穩定度之餘降低耗電量。

隨著 5G 競爭逐漸白熱化，世界各國無不開始著手建立創新技術，以期大幅改善網路傳輸率與容量、強化頻譜效率、縮短端對端延時，並提高穩定性,也是我們這次專題要探討的目的。

第一章、Generation of Random Variable

離散隨機變數

- 離散隨機變數的機率分布為機率質量函數(probability mass function, PMF),常見的離散隨機變數有:
- Bernoulli R.V. : $X \in \{0,1\}$
  - PMF : $P(X=1)=p$
    $P(X=0)=1-p$
- Binomial R.V. : $X \in \{0,1,2,3,\ldots, n\}$
  - PMF: $P(X=k)=C_k^n p^k (1-p)^{n-k}, \quad k=0,1,2,\ldots,n$
- Geometric R.V. : $X \in \{0,1,2,3,\ldots..\}$
  - PMF: $P(X=k)=(1-p)^{k-1} \cdot p, \quad k=1,2,3,\ldots.$

隨機變數的產生

- (ex2) 產生Uniform隨機變數 U[a,b]

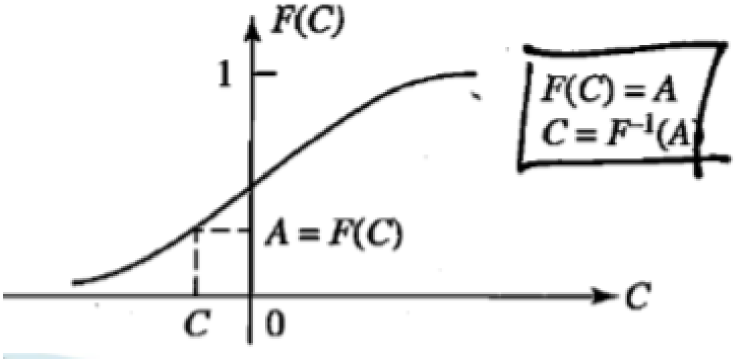
$$F(x)=\begin{cases} 0, & x<a \\ \dfrac{x-a}{b-a}, & a \le x \le b \\ 1, & x>b \end{cases} \Rightarrow F^{-1}(u)=a+(b-a)u$$

1) U=rand(1);
2) X=a+(b-a)U

隨機變數的產生

- 所有的隨機變數都可由Uniform R.V. U~U[0,1]產生
- 若我們要產生一隨機變數X, 其CDF為$F(x)$,步驟為

1) 產生Uniform R.V.U~U[0,1]
   - 在Matlab中,可使用 rand(m,n)來產生m×n個uniform分佈的矩陣
2) 計算CDF的反函數 $F^{-1}(x)$
3) X= F^{-1}(U) 即為我們要產生的隨機變數



大數法則

- Law of Large Number (LLN)
- 若$X_1, X_2, \ldots, X_N$ 為N個互相獨立且相同分佈隨機變數,若這些隨機變數的平均值及變異數均為有限值,則

$$Y_N = \frac{1}{N}\sum_{i=1}^{N} X_i \overset{N\to\infty}{\rightarrow} \mu_X$$

- 根據此定理可用來統計隨機變數的平均值和變異數
- 統計平均值

$$\hat{\mu}_X = \frac{1}{N}\sum_{i=1}^{N} X_i$$

- 統計變異數

$$\hat{\sigma}_X^2 = \frac{1}{N}\sum_{i=1}^{N} (X_i - \hat{\mu}_X)^2$$

隨機變數的轉換

- 若連續隨機變數X的PDF為$p_X(x)$
- 令$Y=g(X)$,其中$g(X)$為一對一的單調(遞增或遞減)函數,那麼Y也是連續隨機變數,PDF為

$$p_Y(y) = p_X\left(g^{-1}(y)\right)\cdot\left|\frac{dg^{-1}(y)}{dy}\right|$$

  - $f(y) = g^{-1}(y)$
- (ex) 假設X~U[0,1],令Y=g(X)=a+(b−a)X

$$g^{-1}(y) = \frac{y-a}{b-a}$$

$$p_Y(y) = p_X\left(\frac{y-a}{b-a}\right)\cdot\frac{1}{b-a} = \begin{cases} \frac{1}{b-a}, & a \le y \le b \\ 0, & otherwise \end{cases}$$

產生相關的多元Gaussian R.Vs

- 產生多元Gaussian $\mathbf{x}=[X_1, X_2, \ldots, X_N]^T \sim N(\mathbf{m}, \mathbf{K})$

1) 產生N個獨立的Gaussian RVs: $W_n \sim N(0,1)$

$$\mathbf{w}=[W_1, W_2, \ldots, W_N]^T$$

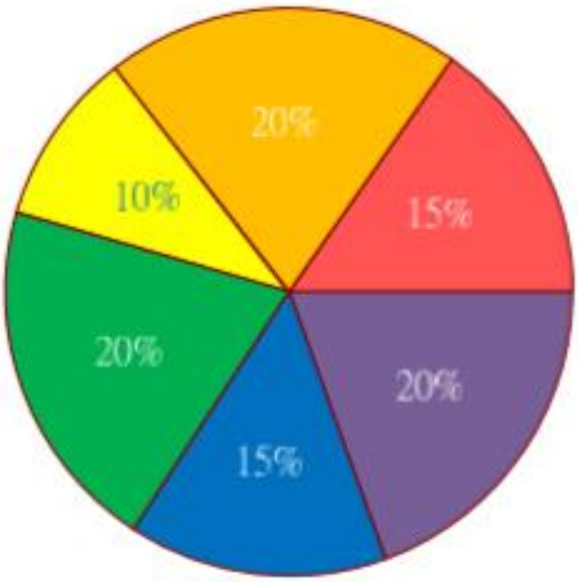
2) 找到矩陣$\mathbf{K}$的分解矩陣$\mathbf{L}$,使得

$$\mathbf{K}=\mathbf{L}\,\mathbf{L}^T$$

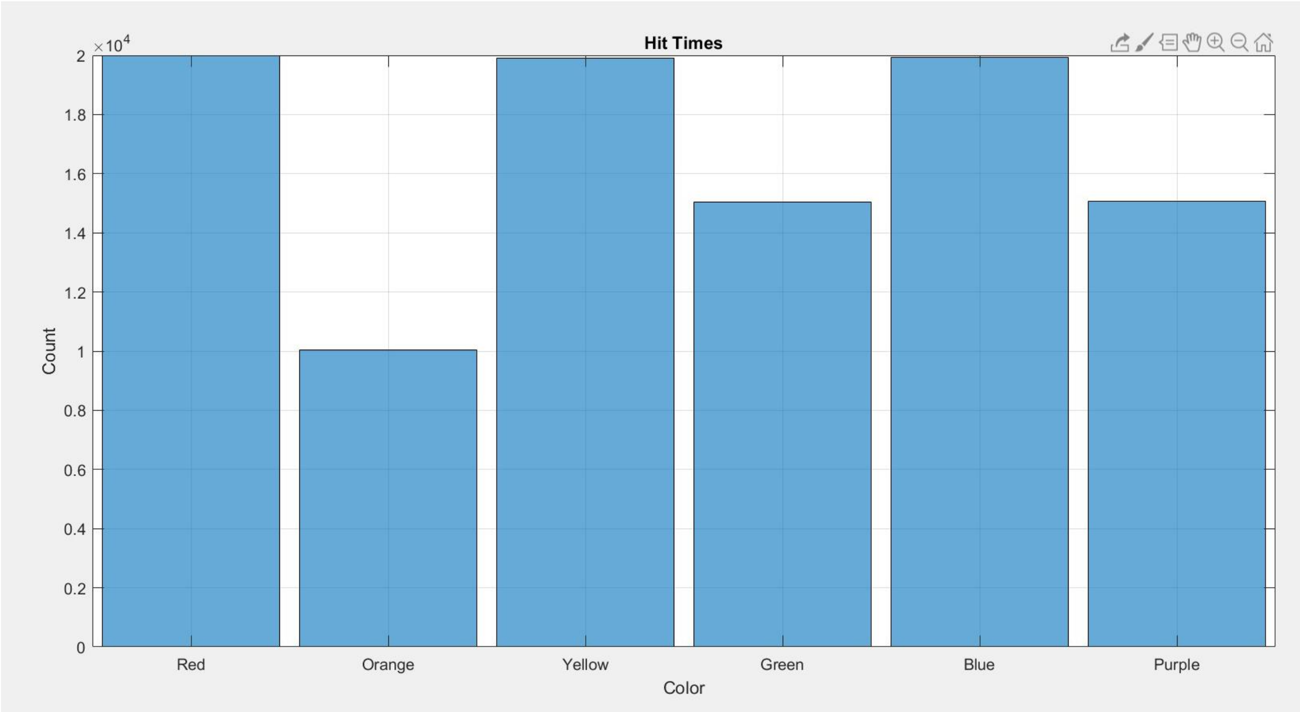
3) $\mathbf{x}=\mathbf{Lw}+\mathbf{m}$ 即為相關性符合矩陣$\mathbf{K}$的隨機向量

結果與展示

5.1 飛鏢投擲 假設我們有個轉盤,轉盤個顏色的面積比例如下圖所示,小明在轉 盤前投擲飛鏢,若飛鏢落在轉盤各處的機率為均勻分布,請寫出程式模擬飛鏢的落 點,並產生 10^5 次落點,以直方圖統計飛鏢落在個顏色區塊的機率



5.1 模擬結果圖(機率分布直方圖)



5.1 程式碼

```
close all;
clc;
clear;

T=10^5;

opt = randsrc(T,1,[1,2,3,4,5,6;0.2,0.1,0.2,0.15,0.2,0.15]);
count = zeros(1,6);

for i=1:100000
        if opt(i) == 1
        count(1) = count(1) + 1;
        elseif opt(i) == 2
            count(2) = count(2) + 1;
        elseif opt(i) == 3
            count(3) = count(3) + 1;
        elseif opt(i) == 4
            count(4) = count(4) + 1;
        elseif opt(i) == 5
            count(5) = count(5) + 1;
        elseif opt(i) == 6
            count(6) = count(6) + 1;
        end
end

figure(1);
histogram('Categories',{'Red','Orange','Yellow','Green','Blue','Purple'},'BinCounts',count);
title('Hit Times');
xlabel('Color');
ylabel('Count');
grid on;
```

5.2 Rayleigh 分佈 Rayleigh 隨機變數 X 的機率密度函數為

$$p(x) = 2x \cdot \exp(-x^2), \qquad x \geq 0.$$

(1) 請計算它 CDF 且利用這個 CDF,寫出產生 Rayleigh 隨機變數的程式

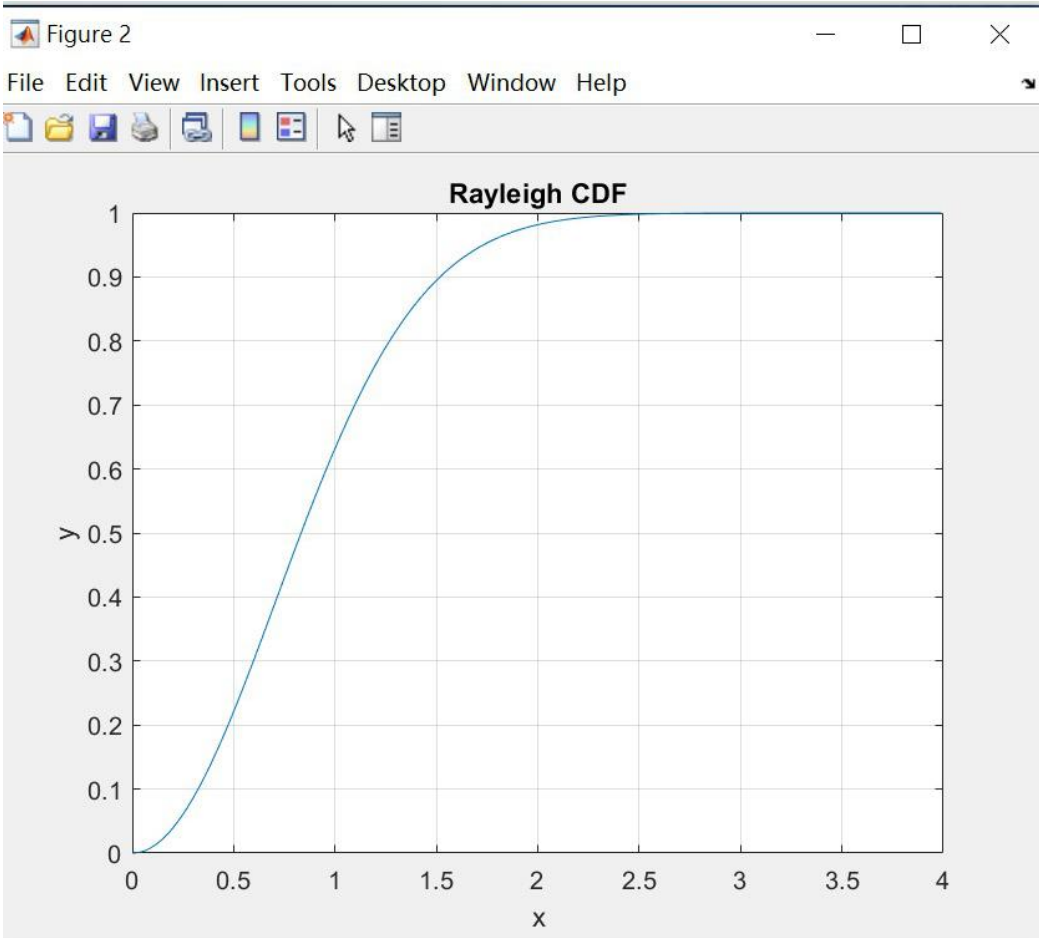
(2) 統計這個隨機變數的平均值及變異數.

(3) 產生 10^6 次隨機變數,畫出這些隨機變數的直方圖,因為 $p(4)$ 的數值已經相當小,因此我們只需畫出 0~4 區間內,間隔為 0.01 的直方圖即可.並將此直方圖轉換成機率密度函數圖,圖上也畫出 p(x)在 0~4 區間的數值做為比對.

(4) 令 $Y=X^2$,請推導 Y 的機率分佈.同時利用你的程式產生 Y,並以(3)的方式畫出 Y 的機率密度函數,驗證看看你推導的結果是否正確.

5.2

(1) 模擬結果圖



(2)我們經過下圖程式可以計算出平均期望值為 0.8861,變異數為

0.2152

(3)模擬結果圖

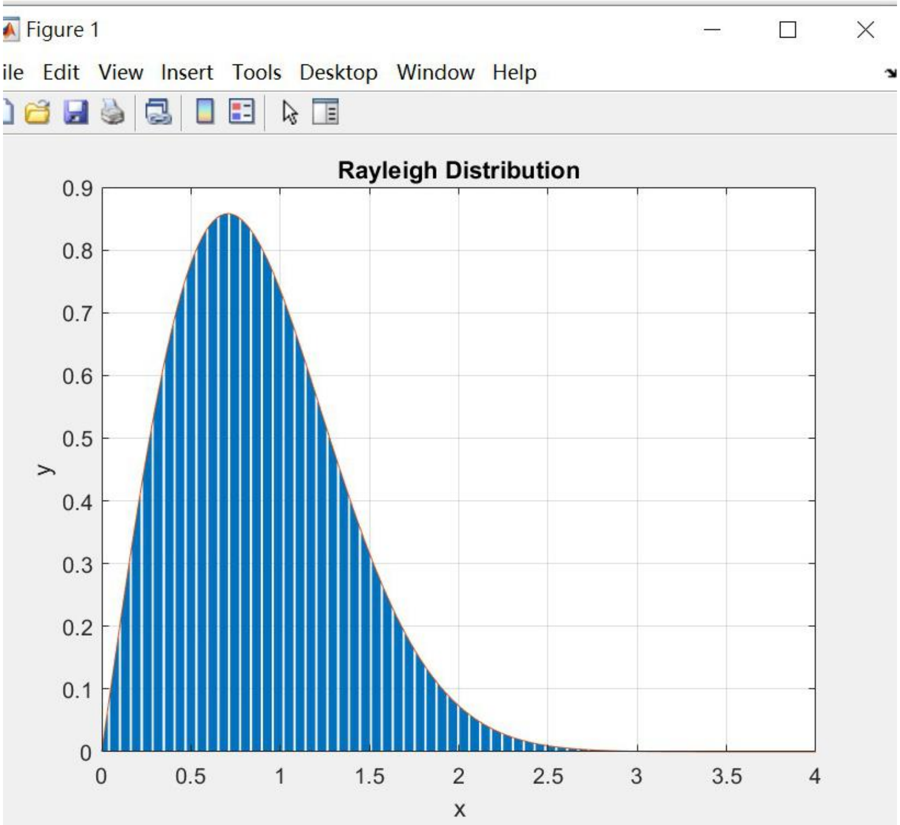


Figure 1
ile Edit View Insert Tools Desktop Window Help
Rayleigh Distribution
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
y
0 0.5 1 1.5 2 2.5 3 3.5 4
x

5.2(3)程式碼

```
close all;
clc;
clear;

v = rand(1);
x = 0:0.000004:3.999996;
B = 1/(2^(0.5));
y = (x./(B.^2)).*exp((-x.^2)./(2*B.^2));
cdf = zeros(1,400);
fun = @(z) 2.*z.*exp(-z.^2);
xx = 0:0.01:3.99;
syms cc;
func = (cc./(B.^2)).*exp((-cc.^2)./(2*B.^2));
funcc = int(func);
pp = diff(funcc)


figure(1);
bar(x,y);
hold on;
plot(x,y);
hold off;
title('Rayleigh Distribution');
xlabel('x');
ylabel('y');
grid on;

mean_of_rayl = mean(y)
var_of_rayl = var(y)

for i=1:400
cdf(i) = integral(fun,0,xx(i));
end

figure(2);
plot(xx,cdf);
title('Rayleigh CDF');
xlabel('x');
ylabel('y');
grid on;
```

5.3 威力彩 威力彩是一種樂透型遊戲，開獎時，開獎單位將從第 1 區 01~38 的 號碼中隨機開出六個號碼，再從第 2 區 01~08 的號碼中隨機開出一個號碼，這一 組六個+一個號碼，就是該期威力彩的中獎號碼.假設每個數字被開出的機率是公 平均等的,請寫出程式產生威力彩的開獎號碼. (請注意,開獎號碼不能重複,也就是 第一個號碼開出時,在第二次開獎時必須移除第一個中獎號碼的可能性)

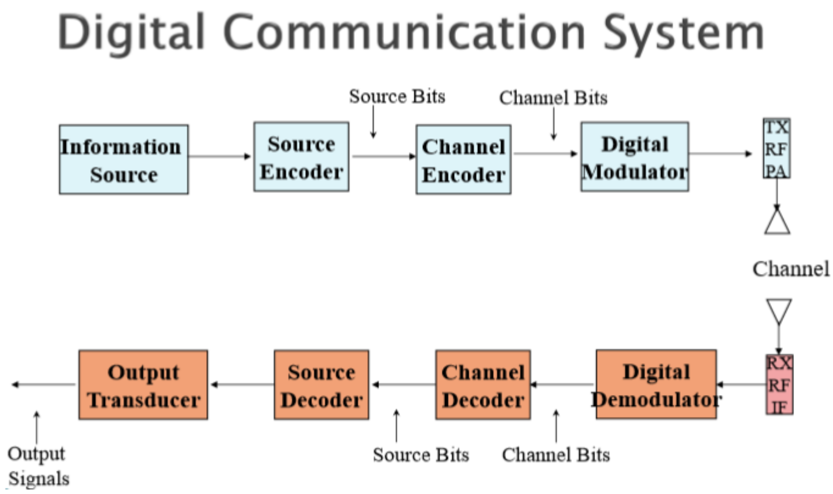
模擬結果:

```
The lottery numbers are
    4    10    13    17    19    24

And the special number is
    6

Congret to the winners!
>>
```

5.3 程式碼

```
close all;
clc;
clear;

ballnum = zeros(1,6);
ballnum = randperm(38,6);
sppool = [1,2,3,4,5,6,7,8];
x = [];

for i = 1:6
    for j = 1:8
        if(ballnum(i) == sppool(j))
            x = [x, j];
        end
    end
end
for k = 1:numel(x)
    sppool(sppool == x(k)) = [];
end

spball_pos = randi(numel(sppool));
special = sppool(spball_pos);
ball = sort(ballnum);

display('The lottery numbers are ');
disp(ball);
display('And the special number is');
disp(special);
display('Congret to the winners!');
```

第二章、Digital Modulation



Digital communication system 即將要傳送的資料先透過通道的編碼和訊號的編碼在經過數位調變,再由天線發射,接下來他端的天線會負責接收,首先先數位解調訊號跟訊號解碼及通道解碼,就是剛剛的步驟的相反流程,而經過上述步驟我們也可以將訊號傳送端與接收端去做比較,而計算出錯誤率,錯誤率比較高就是兩端相差甚遠,反之亦然

Binary Signal Transmission

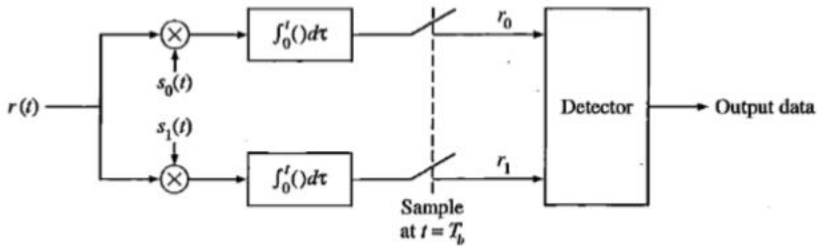
- In baseband, information-bearing signals is transmitted directly through channel without use of sinusoidal carrier
- Digital signals are transferred to as waveforms
- The task of engineers is to design "Rx" to minimize the bit-error-rate (BER)
- In the subsection, we will cover several types of signal transmissions
  - Binary signal transmission
  - Multidimensional signals

Binary Signal Transmission

- Correlator filter
  - Signal correlator:

$$r_0(t) = \int_0^t r(\tau) s_0(\tau) d\tau$$

$$r_1(t) = \int_0^t r(\tau) s_1(\tau) d\tau$$



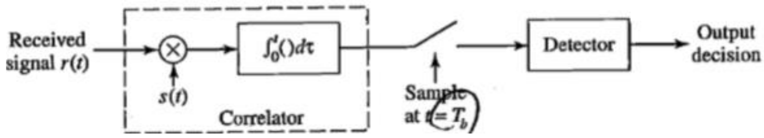
Binary Signal Transmission

Antipodal / on-off binary signal

Antipodal signal:

- Just need a detector
- Received signal waveform:

$$r(t) = \pm s(t) + n(t), \qquad 0 \le t \le T_b$$



Multi-Amplitude Signal Transmission

- Average symbol energy:

$$\mathcal{E}_{av} = \frac{1}{4}\left((-3d)^2 + (-d)^2 + d^2 + (3d)^2\right) = 5d^2$$

- Average energy: per symbol

$$\mathcal{E}_{avb} = \mathcal{E}_{av} / 2$$

- SER in terms of average energy:

$$P_e = \frac{3}{2} Q\left(\sqrt{\frac{2d^2}{N_0}}\right) = \frac{3}{2} Q\left(\sqrt{\frac{2\mathcal{E}_{av}}{5N_0}}\right) = \frac{3}{2} Q\left(\sqrt{\frac{4\mathcal{E}_{avb}}{5N_0}}\right)$$

結果與展示

6.1 Binary Signaling (Orthogonal / Antipodal / On-off)

考慮在 AWGN 通道中傳送二元訊號($s_0(t), s_1(t)$),接收訊號為

$$r(t) = s_i(t) + n(t), \qquad i = 0,1$$

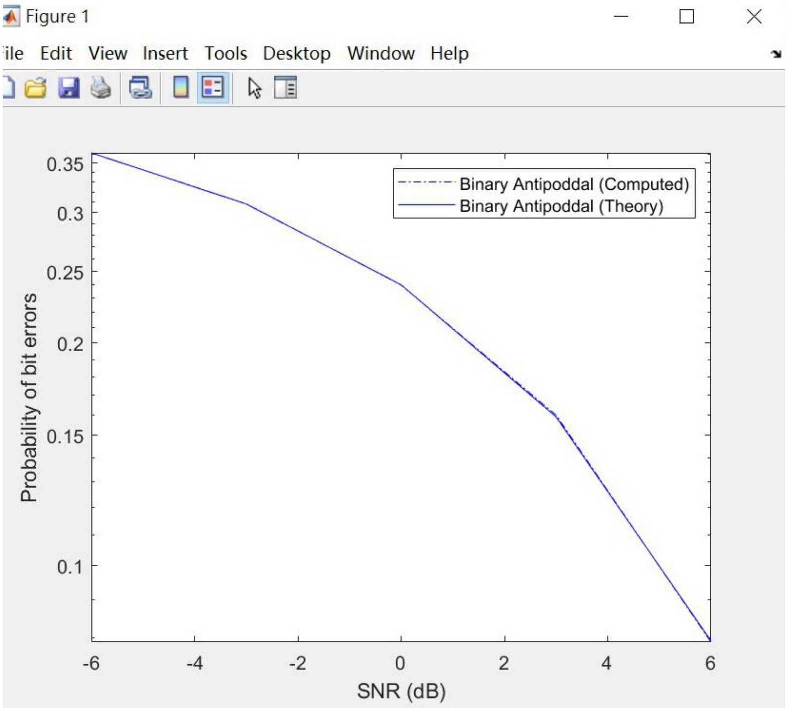
其中 n(t)為高斯白雜訊,功率頻譜密度為 $N_0/2$

(1) 若$s_0(t)$和$s_1(t)$為互相正交的訊號,且能量均為E,在接收段利用 correlator 偵測訊號. 請以 Matlab 程式模擬產生等機率二元隨機訊號,及接收端 correlator 的輸出訊號,偵測訊號後統計錯誤率.在圖上畫出 SNR=E/ N_0 為 -6 dB, -3 dB,0 dB, 3dB, 6dB 的錯誤率圖 (錯誤率請用 log-scale 繪圖). 圖上同時畫出錯誤率的理論值曲線,可評估模擬的準確程度.

(2) 若$s_1(t) = -s_0(t)$,且能量均為E,在接收段利用 correlator 偵測訊號. 請以 Matlab 程式模擬產生等機率二元隨機訊號,及接收端 correlator 的輸出訊號,偵測訊號後統計錯誤率.在圖上畫出 SNR=E/ N_0 為 -6 dB, -3 dB,0 dB, 3dB, 6dB 的錯誤率圖 (錯誤率請用 log-scale 繪圖). 圖上同時畫出錯誤率的理論值曲線,可評估模擬的準確程度.

(3) 若$s_0(t) = 0$,且$s_1(t)$的能量為E,在接收段利用 correlator 偵測訊號. 請以 Matlab 程式模擬產生等機率二元隨機訊號,及接收端 correlator 的輸出訊號,偵測訊號後統計錯誤率.在圖上畫出 SNR=E/ N_0 為 -3 dB,0 dB, 3dB, 6dB, 9dB 的錯誤率圖 (錯誤率請用 log-scale 繪圖). 圖上同時畫出錯誤率的理論值曲線,可評估模擬的準確程度.

6.1(1) 模擬結果圖(Orthogonal Binary Signaling 錯誤率圖)

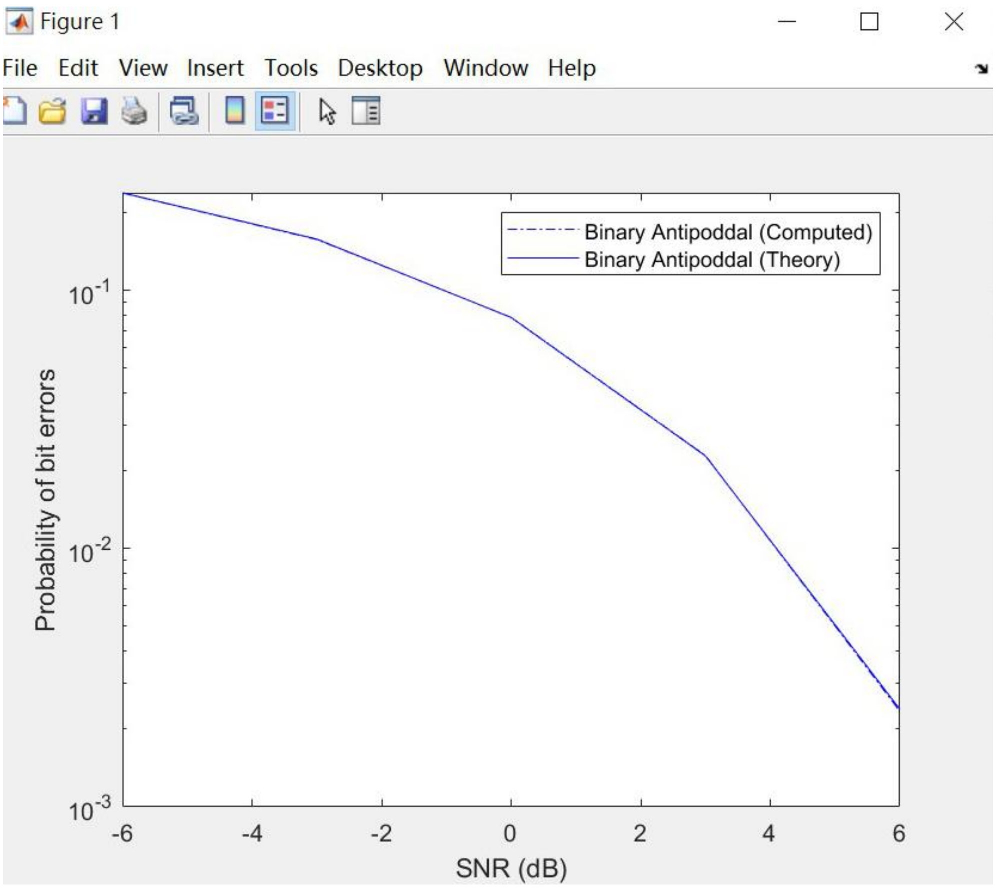


分析:我們可以發現錯誤率圖的理論值與模擬值幾乎相同

6.1(1)程式碼

```
close all;
clc;
clear;

SNR_dB=[-6 -3 0 3 6];
N=10^6;
Count_BER=zeros(1,length(SNR_dB));
TBER = zeros(1,length(SNR_dB));
for n=1:length(SNR_dB)
 SNR=10^(SNR_dB(n)/10);
 N0=1/(SNR);
 TBER(n) = qfunc(sqrt(1/(2*N0)));
 for t=1:N
 %-----------------------------------
 %generate received signal
 b=floor(rand(1)*2); % one-bit data
 w=(randn(1)+j*randn(1))*sqrt(N0/2);
 y=b+w; % 接收訊號
 %-------------------------------------
 %Detection
 x_hat = real(y);
 b_hat=1;
 if x_hat<0.5
 b_hat=0;
 end
 Count_BER(n)=Count_BER(n)+abs(b_hat-b);
 end
end

BER=Count_BER./N;
semilogy(SNR_dB,BER,'b-.');
xlabel('SNR (dB)');
ylabel('Probability of bit errors');
hold on;
semilogy(SNR_dB,TBER,'b-');
legend('Binary Antipoddal (Computed)','Binary Antipoddal (Theory)');
hold off;
```

6.1(2)模擬結果圖(Antipodal binary Binary Signaling 錯誤率圖)

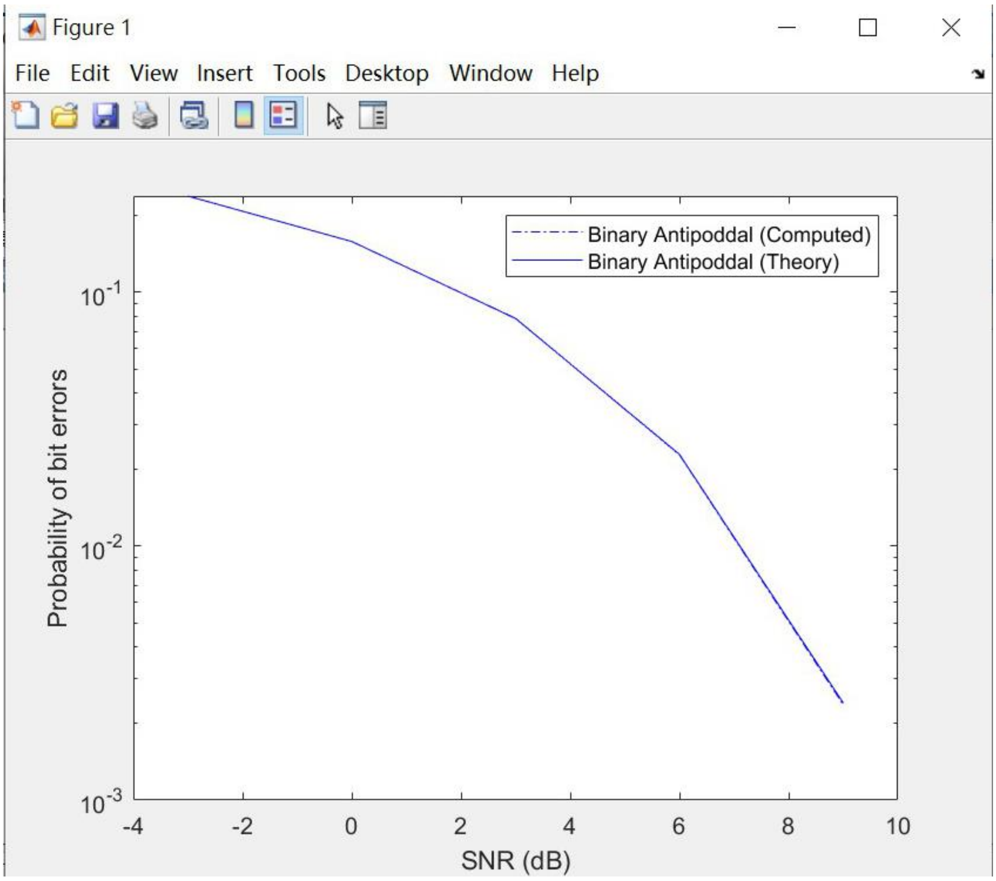


分析:我們可以發現錯誤率圖的理論值與模擬值幾乎相同

6.1(2) 程式碼

```
close all;
clc;
clear;

SNR_dB=[-6 -3 0 3 6];
N=10^6;
Count_BER=zeros(1,length(SNR_dB));
TBER = zeros(1,length(SNR_dB));
for n=1:length(SNR_dB)
 SNR=10^(SNR_dB(n)/10);
 N0=1/SNR;
 TBER(n) = qfunc(sqrt(2/N0));
 for t=1:N
 %----------------------------------
 %generate received signal
 b=floor(rand(1)*2); % one-bit data
 x=(-1)^b;
 w=(randn(1)+j*randn(1))*sqrt(N0/2);
 y=x+w; % 接收訊號
 %------------------------------------
 %Detection
 x_hat=sign(real(y));
 b_hat=0;
 if x_hat<0
 b_hat=1;
 end
 Count_BER(n)=Count_BER(n)+abs(b_hat-b);
 end
end
BER=Count_BER./N;
semilogy(SNR_dB,BER,'b-.');
xlabel('SNR (dB)');
ylabel('Probability of bit errors');
legend('Binary Antipoddal');
hold on;
semilogy(SNR_dB,TBER,'b-');
legend('Binary Antipoddal (Computed)','Binary Antipoddal (Theory)');
hold off;
```

6.1 (3) 模擬結果圖(On-off binary Binary Signaling 錯誤率圖)



分析:我們可以發現錯誤率圖的理論值與模擬值幾乎相同

6.1(3)程式碼

```
close all;
clc;
clear;

SNR_dB=[-3 0 3 6 9];
N=10^6;
Count_BER=zeros(1,length(SNR_dB));
TBER = zeros(1,length(SNR_dB));
for n=1:length(SNR_dB)
 SNR=10^(SNR_dB(n)/10);
 N0=1/(2*SNR);
 TBER(n) = qfunc(sqrt(1/(2*N0)));
 for t=1:N
 %----------------------------------
 %generate received signal
 b=floor(rand(1)*2); % one-bit data
 w=(randn(1)+j*randn(1))*sqrt(N0/2);
 y=b+w; % 接收訊號
 %------------------------------------
 %Detection
 x_hat = real(y);
 b_hat=1;
 if x_hat<0.5
 b_hat=0;
 end
 Count_BER(n)=Count_BER(n)+abs(b_hat-b);
 end
end
BER=Count_BER./N;
semilogy(SNR_dB,BER,'b-.');
xlabel('SNR (dB)');
ylabel('Probability of bit errors');
legend('Binary Antipoddal');
hold on;
semilogy(SNR_dB,TBER,'b-');
legend('Binary Antipoddal (Computed)','Binary Antipoddal (Theory)');
hold off;
```

6.2 PAM Signaling

考慮在 AWGN 通道中傳送 M 元訊號$s_0(t), s_1(t), \ldots, s_{M-1}(t)$,接收訊號為

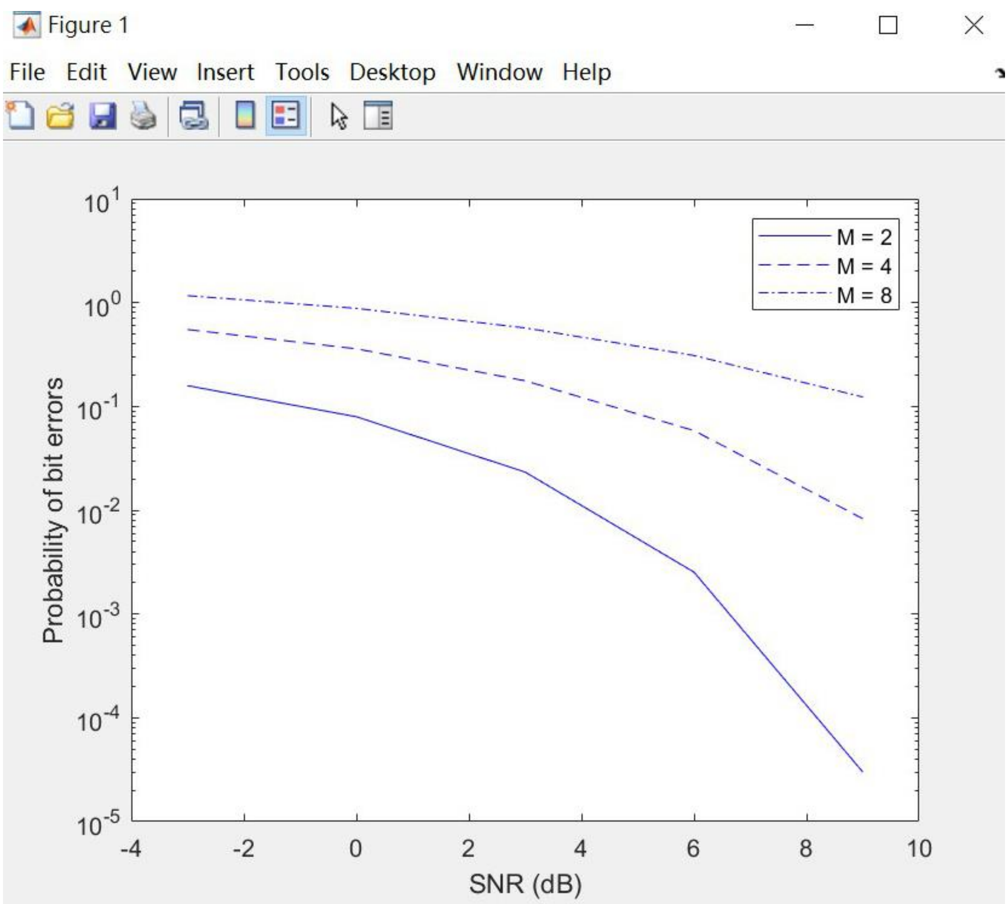
$$r(t) = s_i(t) + n(t) = A_i g(t) + n(t), \qquad i = 0,1,\ldots,M-1$$

其中 n(t)為高斯白雜訊,功率頻譜密度為 $N_0/2$.

(1) 若 M=2,且訊號的位元平均能量為 E_{avb}, 請以 Matlab 程式模擬產生等機率二元隨機訊號,及接收端 correlator 的輸出訊號,偵測訊號後統計錯誤率. 在圖上畫出 $SNR_b=E_{avb}/N_0$ 為 -3 dB,0 dB, 3dB, 6dB, 9dB 的錯誤率圖 (錯誤率請用 log-scale 繪圖).

(2) 同(1),請在圖上加上陸續加上 M=4, M=8, 及 M=16 錯誤率對 SNR_b 的圖

6.2(1) 模擬結果圖(PAM Signaling 錯誤率圖)

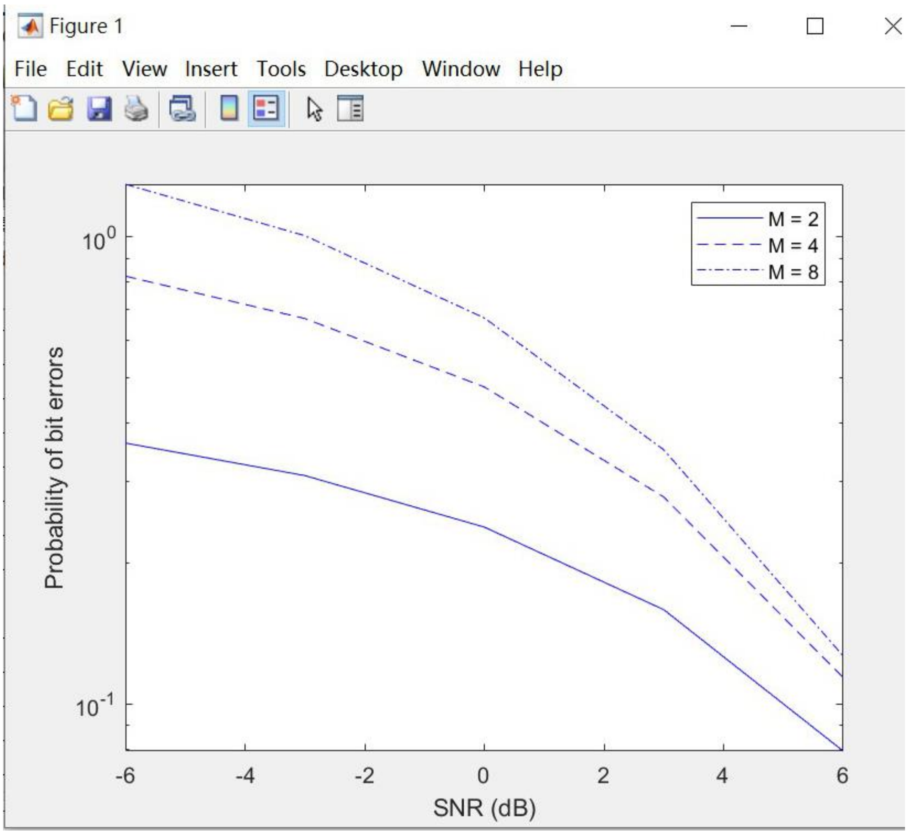


6.2(1)程式碼

```
close all;
clc;
clear;

num1 = [-7 -5 -3 -1 1 3 5 7];
num2 = [-3 -1 1 3];
num3 = [-1 1];
SNR_dB=[-3 0 3 6 9];
N=10^5;
Count_BER=zeros(1,length(SNR_dB));
for n=1:length(SNR_dB)
 SNR=10^(SNR_dB(n)/10);
 N0=1/(SNR);
 for t=1:N
 %-----------------------------------
 %generate received signal
 bn = randi([0,1],1,1); % one-bit data
 b = num3(bn+1);
 w=(randn(1)+j*randn(1))*sqrt(N0/2);
 y=b+w; % 接收訊號
 %-------------------------------------
 %Detection
 x_hat = real(y);
 b_hat=1;
 if x_hat<0
 b_hat=0;
 end
 if b_hat == bn
     error=0;
 else
     error=1;
 end
 Count_BER(n)=Count_BER(n)+error;
 end
end
BER=Count_BER./N;
semilogy(SNR_dB,BER,'b-');
xlabel('SNR (dB)');
ylabel('Probability of bit errors');

for n=1:length(SNR_dB)
 SNR=10^(SNR_dB(n)/10);
 N0=2.5/(SNR);
 for t=1:N
 %-----------------------------------
 %generate received signal
 bn = randi([0,3],1,1); % one-bit data
 b = num2(bn+1);
 w=(randn(1)+j*randn(1))*sqrt(N0/2);
 y=b+w; % 接收訊號
 %-------------------------------------
```

```
 %Detection
 x_hat = real(y);
 b_hat=7;
 if x_hat<-6
 b_hat=0;
 elseif x_hat<-4
     b_hat=1;
     elseif x_hat<-2
     b_hat=2;
     elseif x_hat<0
     b_hat=3;
     elseif x_hat<2
     b_hat=4;
     elseif x_hat<4
     b_hat=5;
     elseif x_hat<6
     b_hat=6;
 end
 if b_hat == bn
     error=0;
 else
     error=1;
 end
 Count_BER(n)=Count_BER(n)+error;
 end
end

hold on;
BER=Count_BER./N;
semilogy(SNR_dB,BER,'b-.');
legend('M = 2','M = 4','M = 8');
hold off;
```

(2) 模擬結果圖(PAM Signaling 錯誤率圖)



6.2(2)程式碼

```
close all;
clc;
clear;

SNR_dB=[-6 -3 0 3 6];
N=10^6;
Count_BER=zeros(1,length(SNR_dB));
for n=1:length(SNR_dB)
 SNR=10^(SNR_dB(n)/10);
 N0=1/SNR;
 for t=1:N
 %----------------------------------
 %generate received signal
 b=floor(rand(1)*2); % one-bit data
 w=(randn(1)+j*randn(1))*sqrt(N0/2);
 y=b+w; % 接收訊號
 %------------------------------------
 %Detection
 x_hat = real(y);
 b_hat=1;
 if x_hat<0.5
 b_hat=0;
 end
 if b_hat == b
     error=0;
 else
     error=1;
 end
 Count_BER(n)=Count_BER(n)+error;
 end
end
BER=Count_BER./N;
semilogy(SNR_dB,BER,'b-');
xlabel('SNR (dB)');
ylabel('Probability of bit errors');

for n=1:length(SNR_dB)
 SNR=10^(SNR_dB(n)/10);
 N0=1/(2*SNR);
 for t=1:N
 %----------------------------------
 %generate received signal
 b=floor(rand(1)*4); % one-bit data
 w=(randn(1)+j*randn(1))*sqrt(N0/2);
 y=b+w; % 接收訊號
 %------------------------------------
```

```
 ..
 %Detection
 x_hat = real(y);
 b_hat=7;
 if x_hat<0.5
 b_hat=0;
 elseif x_hat<1.5
     b_hat=1;
     elseif x_hat<2.5
     b_hat=2;
     elseif x_hat<3.5
     b_hat=3;
     elseif x_hat<4.5
     b_hat=4;
     elseif x_hat<5.5
     b_hat=5;
     elseif x_hat<6.5
     b_hat=6;
 end
 if b_hat == b
     error=0;
 else
     error=1;
 end
 Count_BER(n)=Count_BER(n)+error;
 end
end

hold on;
BER=Count_BER./N;
semilogy(SNR_dB,BER,'b-.');
legend('M = 2','M = 4','M = 8');
hold off;
```

第三章、Digital Demodulation

Two-Dimensional Signals

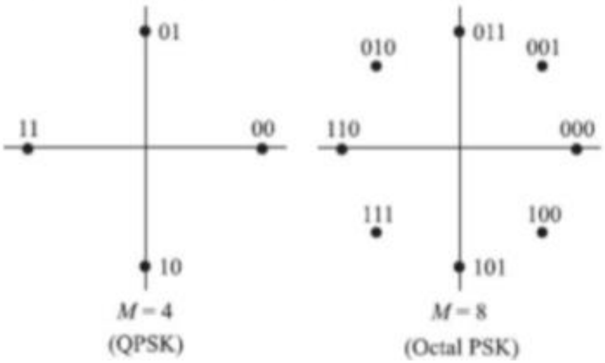
- Phase Shift Keying (PSK)
- Quadrature Amplitude Modulation (QAM)

$$s_m(t) = A_m \underbrace{g(t)\cos(2\pi f_0 t)}_{\phi_0(t)} - B_m \underbrace{g(t)\sin(2\pi f_0 t)}_{\phi_1(t)}$$

  - (A_m, B_m) can be used to represent the signal $s_m(t)$

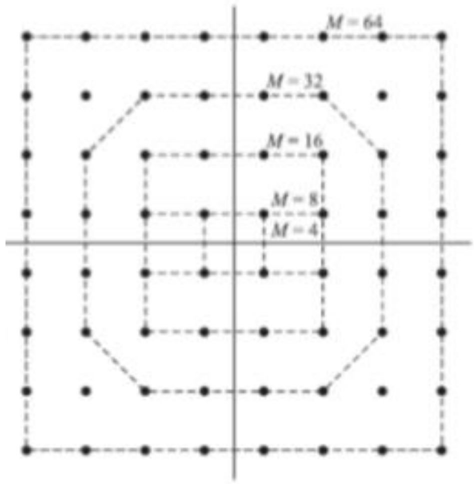
PSK

Two-dimensional signals

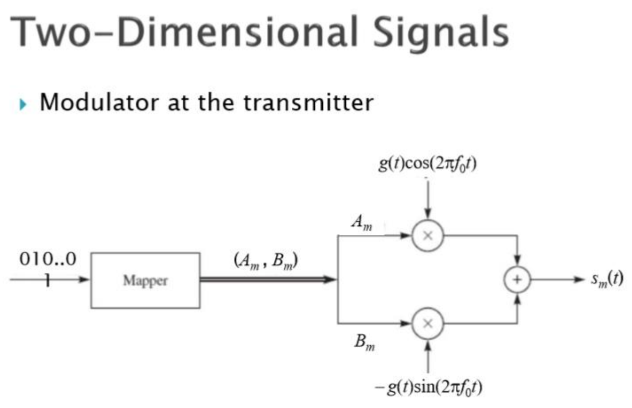


傳送 X 個複數訊號，再把這些複數訊號的實數軸和虛數軸形成的平面切成 Y 等分，組成 Y 個位元的複數等能量訊號，利用 PHASE SHIFT 讓更多的資料被傳送

QAM

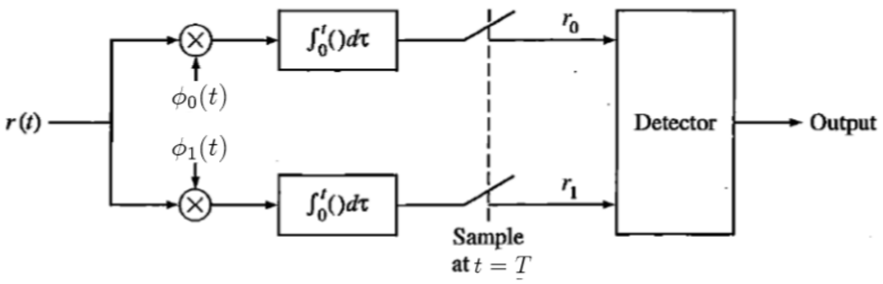


傳送 X 個複數訊號，再把這些複數訊號的實數軸和虛數軸形成的平面切成 Y 等分，組成 Y 個位元的複數等能量訊號同時改變 Amplitude 和 Phase，藉以完成調變的,再利用相位差為 90 度的兩載波,經過參數的變化讓更多的資料被傳送

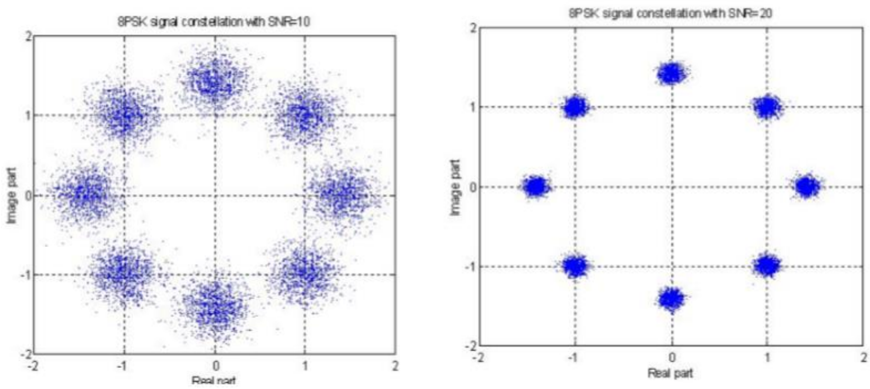


Phase-Shift-Keying (PSK)

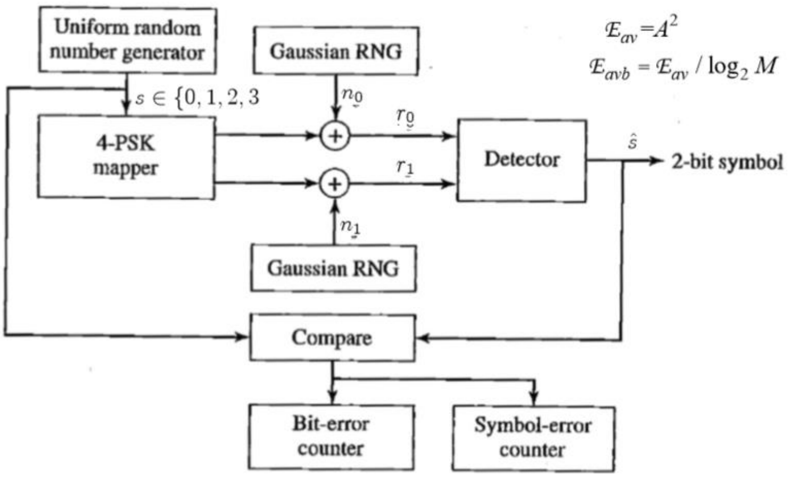
- Let $\phi_0(t) = g(t)\cos(2\pi f_0 t)$
  $\phi_1(t) = -g(t)\sin(2\pi f_0 t)$
- Assume $\phi_i(t)$ has unit energy
  $$\int_0^T |\phi_i(t)|^2 \, dt = 1, \quad i = 0, 1$$
- Receiver structure



Phase-Shift-Keying (PSK)



Phase-Shift-Keying (PSK)



Quadrature Amplitude Modulation (QAM)

- Information is conveyed in amplitude or phase

$$\begin{aligned} s_m(t) &= r_m g(t)\cos(2\pi f_0 t + \theta_m)\,, \qquad m = 1, 2, \cdots, M \\ &= A_{mi} g(t)\cos(2\pi f_0 t) - A_{mq} g(t)\sin(2\pi f_0 t) \end{aligned}$$

  - $r_m = \sqrt{A_{mi}^2 + A_{mq}^2}$
  - $\theta_m = \tan^{-1}\left(A_{mi} / A_{mq}\right)$
  - Combined digital-amplitude and digital-phase modulation
  - Can be represented by

$$s_m = [A_{mi} \quad A_{mq}] \quad \text{or} \quad s_m = A_{mi} + jA_{mq}$$

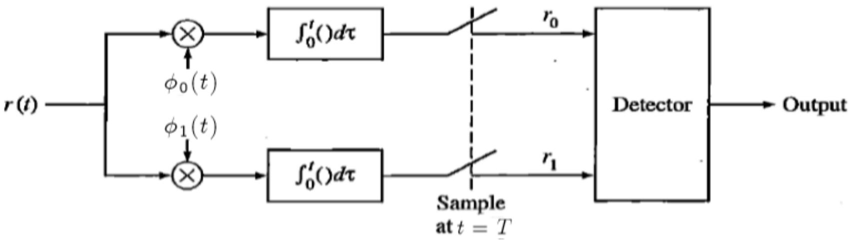
Quadrature Amplitude Modulation (QAM)

- The correlators of QAM is the same as PAM
  - Correlator outputs

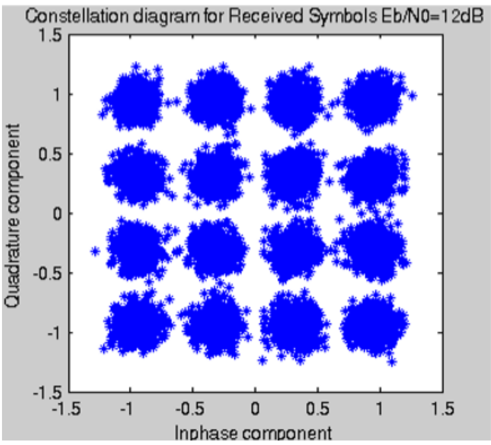
$$r_0 = \int_0^T r(t)\phi_0(t)dt = A_{mi} + n_0$$

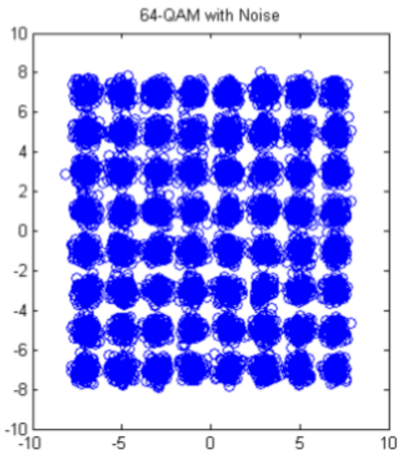
$$r_1 = \int_0^T r(t)\phi_1(t)dt = A_{mq} + n_1$$

  - Noise at the outputs (n_0, n_1) are i.i.d. Gaussian $N(0, N_0/2)$



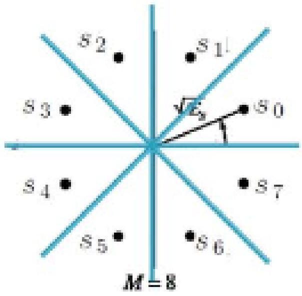
Quadrature Amplitude Modulation (QAM)





結果與展示

Exercise 7



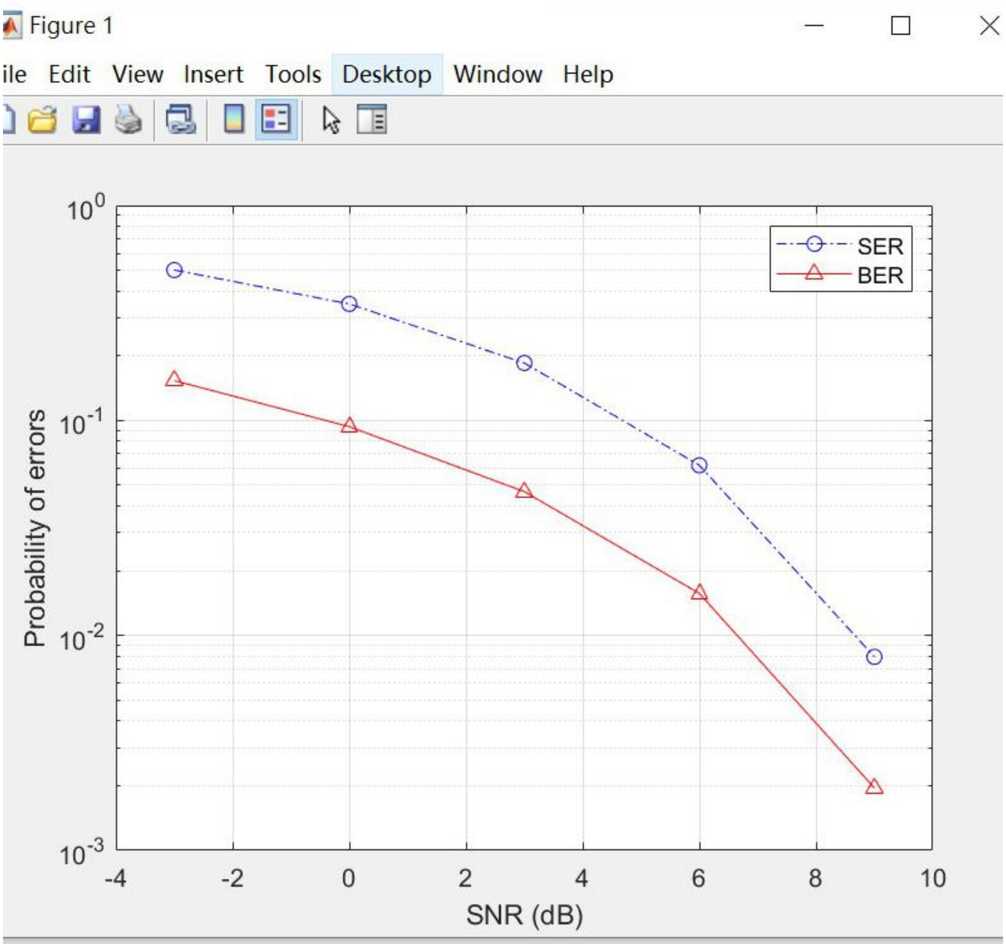
7.1 8-PSK

考慮在 AWGN 通道中傳送 8-PSK 訊號$s_m(t)$,接收訊號為

$$r(t) = s_m(t) + n(t), \qquad m = 0,1,\ldots,7$$

其中 n(t)為高斯白雜訊,功率頻譜密度為 $N_0/2$. 訊號的位元平均能量為 E_{avb}, 請以 Matlab 程式模擬產生等機率隨機訊號,及接收端 correlator 的輸出訊號,偵測訊號後統計錯誤率(SER). 在圖上畫出 $SNR_b = E_{avb}/N_0$ 為 -3 dB,0 dB, 3dB, 6dB, 9dB 的錯誤率圖 (錯誤率請用 log-scale 繪圖).

7.1 模擬結果圖(8-PSK 錯誤率圖)



7.1 程式碼

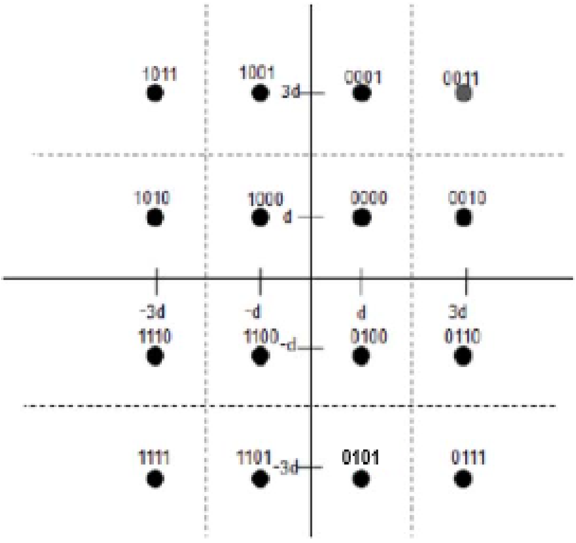
```
close all;
clc;
clear all;

SNR_dB=[-3 0 3 6 9];
N=10^5;
Count_SER=zeros(1,length(SNR_dB));
Count_BER=zeros(1,length(SNR_dB));
Evab = 1/3;
for n=1:length(SNR_dB)
    SNR=10^(SNR_dB(n)/10);
    N0=Evab/SNR;
    for t=1:N
  %---------------------------------
  %generate received signal
  s=floor(rand(1)*8);
        phi=(pi/8)+s*(pi/4);
        x=exp(j*phi);
        b=zeros(1,2); % two-bit data
  b(1)=(imag(x)<0);
        b(2)=(real(x)<0);
        w=(randn(1)+j*randn(1))*sqrt(N0/2);
        y=x+w; % 接收訊號
        %-----------------------------------
        %Detection
  if angle(y)>=0
   s_hat=0;
             b_hat=zeros(1,2);
             if pi/2 >= angle(y) && angle(y) > pi/4
    s_hat=1;
             end
             if 3*pi/4 >= angle(y) && angle(y) > pi/2
                 s_hat=2;
                 b_hat(2)=1;
             end
             if pi >= angle(y) && angle(y) > 3*pi/4
                 s_hat=3;
                 b_hat(2)=1;
             end
  else
             s_hat=4;
             b_hat=ones(1,2);
             if -pi/2 >= angle(y) && angle(y) > -3*pi/4
    s_hat=5;
             end
             if -pi/4 >= angle(y) && angle(y) > -pi/2
                 s_hat=6;
                 b_hat(2)=0;
             end
             if angle(y) > -pi/4
                 s_hat=7;
                 b_hat(2)=0;
             end
  end
  Count_SER(n)=Count_SER(n)+sign(abs(s_hat-s));
  Count_BER(n)=Count_BER(n)+sum(abs(b_hat-b));
 end
end
SER=Count_SER./N;
BER=Count_BER./(2*N);
semilogy(SNR_dB,SER,'b-.o',SNR_dB,BER,'r-^');
xlabel('SNR (dB)');
ylabel('Probability of errors');
legend('SER','BER');
grid on;
```

7.2 16-QAM

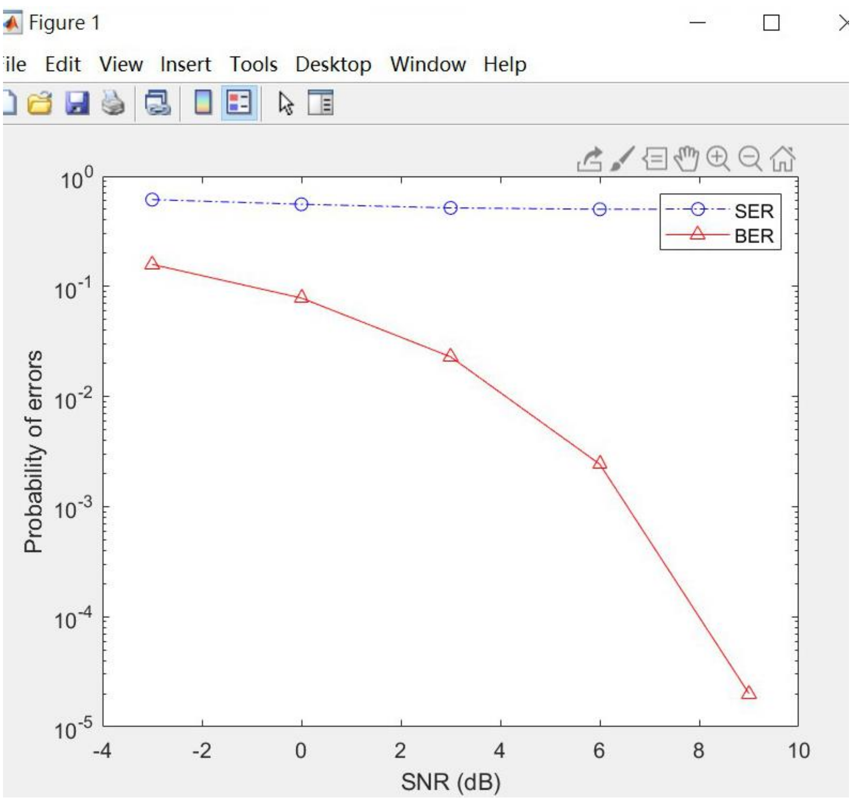
考慮在 AWGN 通道中傳送 16-QAM 訊號$s_m(t)$,接收訊號為

$$r(t) = s_m(t) + n(t), \qquad m = 0,1,\ldots,15$$

其中 n(t)為高斯白雜訊,功率頻譜密度為 $N_0/2$. 訊號的位元平均能量為 E_{avb}, 請以 Matlab 程式模擬產生等機率隨機訊號,及接收端 correlator 的輸出訊號,偵測訊號後統計錯誤率(SER). 在圖上畫出 SNR$_b$=E_{avb}/N_0 為 -3 dB,0 dB, 3dB, 6dB, 9dB 的錯誤率圖 (錯誤率請用 log-scale 繪圖).



7.2 模擬結果圖(16-QAM 錯誤率圖)



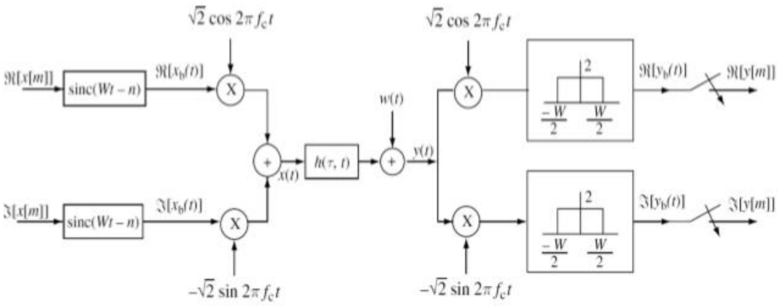
7.2 程式碼

```
close all;
clc;
clear all;

SNR_dB=[-3 0 3 6 9];
N=10^5;
Count_SER=zeros(1,length(SNR_dB));
Eav = 10;
Evab = Eav/4;
for n=1:length(SNR_dB)
    SNR=10^(SNR_dB(n)/10);
    N0=Evab/(SNR);
    for t=1:N
  %----------------------------------
  %generate received signal
        ai = floor(rand(1)*4);
        bi = floor(rand(1)*4);
  s=ai*4+bi;
        x = (2 * ai -3) + j * (2 * bi -3);
        w=(randn(1)+j*randn(1))*sqrt(N0/2);
        y=x+w; % 接收訊號
        %------------------------------------
        %Detection

  if real(y) >= 2
            if imag(y) >= 2
                s_hat=15;
```

```
            end
            if 2 > imag(y) && imag(y) >= 0
                s_hat=14;
            end
            if 0 > imag(y) && imag(y) >= -2
                s_hat=13;
            end
            if -2 > imag(y)
                s_hat=12;
            end
        end
        if 2 > real(y) && real(y) >= 0
            if imag(y) >= 2
                s_hat=11;
            end
            if 2 > imag(y) && imag(y) >= 0
                s_hat=10;
            end
            if 0 > imag(y) && imag(y) >= -2
                s_hat=9;
            end
            if -2 > imag(y)
                s_hat=8;
            end
        end
        if 0 > real(y) && real(y) >= -2
            if imag(y) >= 2
                s_hat=7;
            end
            if 2 > imag(y) && imag(y) >= 0
                s_hat=6;
            end
            if 0 > imag(y) && imag(y) >= -2
                s_hat=5;
            end
            if -2 > imag(y)
                s_hat=4;
            end
        end
        if -2 > real(y)
            if imag(y) >= 2
                s_hat=3;
            end
            if 2 > imag(y) && imag(y) >= 0
                s_hat=2;
            end
            if 0 > imag(y) && imag(y) >= -2
                s_hat=1;
            end
            if -2 > imag(y)
                s_hat=0;
            end
        end
  Count_SER(n)=Count_SER(n)+sign(abs(s_hat-s));
 end
end
SER=Count_SER./N;
semilogy(SNR_dB,SER,'b-.o');
xlabel('SNR (dB)');
ylabel('Probability of errors');
```

第四章、Wireless Fading Channel

Discrete-time baseband model



Discrete-time baseband model

- *Discrete-time baseband equivalent model*

$$y[m] = \sum_{\ell} x[m-\ell] h_{\ell}[m] + w[m]$$

  - $\{\Re(w[m]), \Im(w[m])\}$ are i.i.d. Gaussian noise $N(0, N_0/2)$
- Discrete-time equivalent channel coefficients

$$h_{\ell}[m] = \sum_{i=1}^{N(mT_s)} a_i(mT_s) e^{-j2\pi f_c \tau_i(mT_s)} \mathrm{sinc}(\ell - W\tau_i(mT_s))$$

  - $T_s = 1/W$ (Nyquist rate)
- If the channel is stationary,

$$h_{\ell} = \sum_{i=1}^{N} a_i e^{-j2\pi f_c \tau_i} \mathrm{sinc}(\ell - W\tau_i)$$

$$y[m] = \sum_{\ell} x[m-\ell] h_{\ell} + w[m]$$

Rayleigh fading channel model

- Statistical characterization of $\{h_l[m]\}$

$$h_\ell[m] = \sum_{i=1}^{N(mT_s)} a_i(mT_s)e^{-j2\pi f_c \tau_i(mT_s)}\mathrm{sinc}(\ell - W\tau_i(mT_s))$$

- Assumptions:
  1) There are a large number of independent reflected and scattered paths with random amplitude
  2) None Line of sight (LOS, 直視路徑)
- The phase of the i–th path is $2\pi f_c \tau_i$ modulo 2π, uniformly distributed in $[0, 2\pi]$
- $h_l[m]$ is a sum of many small indep circular symmetric r. vs.
- $\Re(h_l[m])$ & $\Im(h_l[m])$ are i.i.d. zero–mean Gaussian r.vs.
- $|h_l[m]|$ is a Rayleigh r.v. $\Rightarrow$ *Rayleigh fading*

傳輸環境中，假使很多的反散射路徑的直視路徑被遮擋住，那麼接收的訊號的振幅變化會呈現雷利分佈，稱 Rayleigh Fading Channel。

Rayleigh fading channel model

- Frequency–selective fading channel with length L :

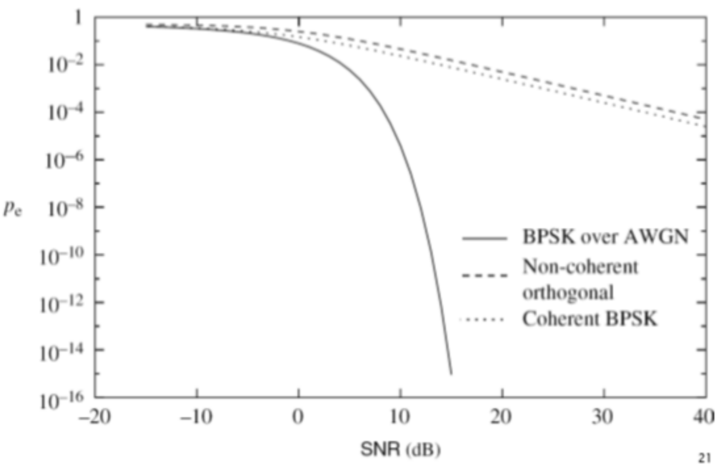
$$h_\ell[m] = \sqrt{Ld^{-\gamma}X_\sigma} \cdot g_\ell[m], \qquad \ell = 0, 1, \cdots, L-1$$

  - $g_\ell[m] \sim CN(0, \sigma_\ell^2)$
  - $\sigma_\ell^2 = \frac{1-e^{-1}}{1-e^{-L}} \cdot e^{-\ell}$
  - $L = G_T G_R \left(\frac{c}{4\pi f}\right)^2$
  - d : distance between transmitter and receiver
  - γ : path–loss exponent, usually $2 \le \gamma \le 6$
  - $X_\sigma = 10^{\phi/10}$: log–normal random variable, $\phi \sim N(0, \sigma^2)$ (slowly variyng with time)

Detection in flat fading channel

- In flat fading channel
$$y[m] = h[m]x[m] + w[m]$$
  - Rayleigh fading $h[m] \sim CN(0,1)$
  - AWGN $w[m] \sim CN(0,N_0)$
- Assume h is known at rx $\Rightarrow$ coherent detection
$$r[m] = \frac{y[m]}{h[m]} = x[m] + \frac{w[m]}{h[m]}$$
  - Given $h[m]$, the noise $w[m]/h[m] \sim CN(0,N_0/|h[m]|^2)$
- With BPSK signaling, the coherent detection is
$$\hat{x}[m] = \begin{cases} \sqrt{E_s}, & \Re\{r[m]\} \geq 0 \\ -\sqrt{E_s}, & \Re\{r[m]\} < 0 \end{cases}$$
- Similar method to detect PAM, PSK and QAM signals

Average BER over fading channel



Rician fading channel model：與 Rayleigh 的差別是有 LOSS

Outage probability for fading channels

- In flat fading channel

$$y[m] = h[m]x[m] + w[m]$$

- Given the channel h[m], the receive SNR is random,

$$\rho = \frac{E_s|h[m]|^2}{N_0}$$

- *Outage probability* (P_{out})
  - P_{out}=Pr{ Channel is in deep fade }
  - P_{out}=Pr{ SNR is dropped below a threshold ρ_{th}}

$$P_{out} = \Pr\{\rho < \rho_{th}\}$$

結果與展示

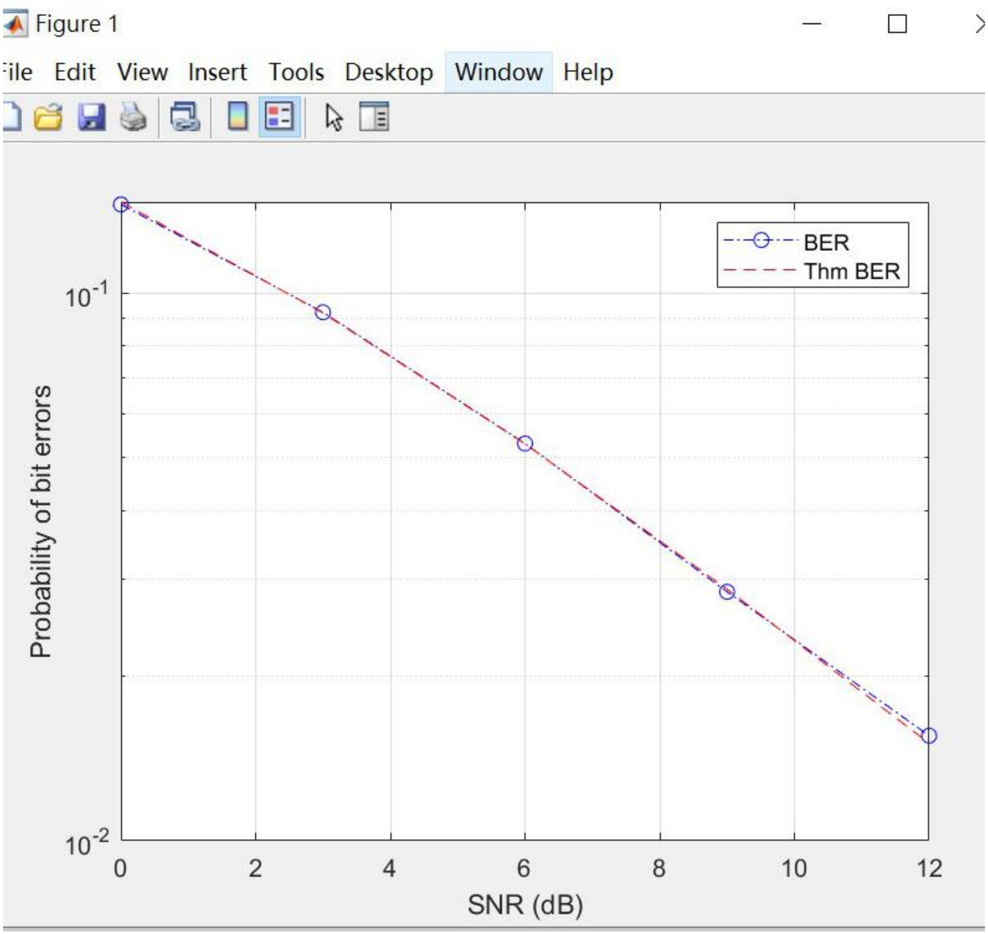
9.1 BER over Rayleigh flat-fading channel

考慮在 Rayleigh flat-fading 通道中傳送 QPSK 訊號,接收訊號為

$$y[m] = h[m]x[m] + n[m]$$

其中$x[m]$是複數表示的 QPSK 訊號，且位元平均能量為 $E_{avb}= \mathbf{E}[|x[m]|^2]/2$, $n[m]$為複數高斯白雜訊,實部及虛部為獨立的高斯隨機變數 $N(0, N_0/2)$，$h[m]$是 Rayleigh fading 通道係數,其機率分佈為$h[m]\sim CN(0,1)$，且假設$h[m]$隨著時間獨立變化.請以 Matlab 程式模擬產生等機率隨機位元訊號,及接收端的輸出訊號,偵測訊號後統計位元錯誤率(BER). 在圖上畫出 $SNR_b=E_{avb}/ N_0$ 為 0 dB, 3dB, 6dB, 9dB, 12dB 的位元錯誤率圖（錯誤率請用 log-scale 繪圖).

9.1 模擬結果圖(QPSK 在 Rayleigh flat-fading 錯誤率圖)



9.1 程式碼

```
close all;
clc;
clear all;

SNR_dB=[0 3 6 9 12];
Nt=10^5;
Count_BER=zeros(1,length(SNR_dB));
P =zeros(1,5);
for n=1:length(SNR_dB)
SNR=10^(SNR_dB(n)/10);
N0=1/(2*SNR);
P(n) = (1/2)*(1-sqrt(SNR/(1+SNR)));
   for t=1:Nt
   %-----------------------------------
   %generate received signal
   s=floor(rand(1)*4); % 4-bit data 0,1,2,3
   phi=(pi/4)+s*(pi/2);
   x=exp(j*phi);
   b=zeros(1,2); % two-bit data
   b(1)=(imag(x)<0);
   b(2)=(real(x)<0);
   h=(randn(1)+j*randn(1))*sqrt(1/2); % Rayleigh fading channel
   w=(randn(1)+j*randn(1))*sqrt(N0/2); % AWGN r=[n1, n2, n3, n4]
   y=h*x+w;
   %------------------------------------
   %Detection
   r=y/h;
   if angle(r)>=0
      b_hat=zeros(1,2);
      if angle(r)>pi/2
         b_hat(2)=1;
      end
   else
      b_hat=ones(1,2);
      if angle(r)>-pi/2
         b_hat(2)=0;
      end
   end
   Count_BER(n)=Count_BER(n)+sum(abs(b_hat-b));
   end
end
BER=Count_BER./(2*Nt);
semilogy(SNR_dB,BER,'b-.o');
hold on;
semilogy(SNR_dB,P,'r--');
hold off;
legend('BER','Thm BER');
xlabel('SNR (dB)');
ylabel('Probability of bit errors');
grid on;
```

9.2 BER over Rician flat-fading channel

考慮在 Rician flat-fading 通道中傳送 QPSK 訊號,接收訊號為

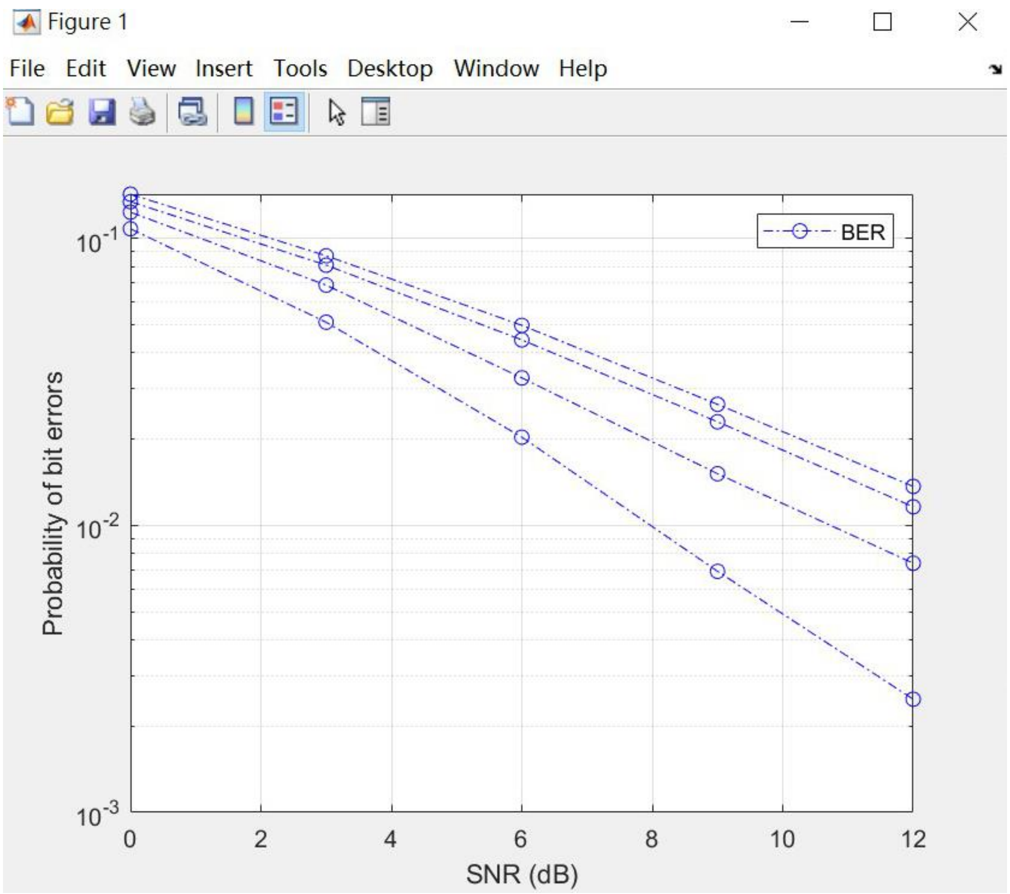
$$y[m] = h[m]x[m] + n[m]$$

其中$x[m]$是複數表示的 QPSK 訊號，且位元平均能量為 $E_{avb}= \mathbf{E}[|x[m]|^2]/2$, $n[m]$為複數高斯白雜訊,實部及虛部為獨立的高斯隨機變數 $N(0, N_0/2)$，$h[m]$是 Rician fading 通道係數,

$$h[m] = \sqrt{\tfrac{\kappa}{\kappa+1}}\, e^{j\pi/4} + \sqrt{\tfrac{1}{\kappa+1}}\, CN(0,1)$$

其中κ為 K factor.假設$h[m]$隨著時間獨立變化.請以 Matlab 程式模擬產生等機率隨機位元訊號, K factor 數值分別為κ =0.5, 1, 2, 4 的通道係數,及接收端的輸出訊號,偵測訊號後統計位元錯誤率(BER). 在圖上畫出 SNR$_b$=E_{avb}/ N_0 為 0 dB, 3dB, 6dB, 9dB, 12dB 的位元錯誤率圖 (請將 **12.1** 及 **12.2** 的位元錯誤率畫在同一張圖上).

9.2 模擬結果圖(QPSK 在 Rician flat-fading 錯誤率圖)



9.2 程式碼

```
close all;
clc;
clear all;

SNR_dB=[0 3 6 9 12];
Nt=10^5;
Count_BER=zeros(1,length(SNR_dB));
k = [0.5 1 2 4];
for ii = 1:4
Count_BER=zeros(1,length(SNR_dB));
for n=1:length(SNR_dB)
SNR=10^(SNR_dB(n)/10);
N0=1/(2*SNR);
  for t=1:Nt
  %----------------------------------
  %generate received signal
  s=floor(rand(1)*4); % 4-bit data 0,1,2,3
  phi=(pi/4)+s*(pi/2);
  x=exp(j*phi);
  b=zeros(1,2); % two-bit data
  b(1)=(imag(x)<0);
  b(2)=(real(x)<0);
  h=sqrt(k(ii)/(k(ii)+1))*exp((j*pi)/4)+sqrt(1/(k(ii)+1))*(randn(1)+j*randn(1))*sqrt(1/2);
  w=(randn(1)+j*randn(1))*sqrt(N0/2); % AWGN r=[n1, n2, n3, n4]
  y=h*x+w;
  %------------------------------------
  %Detection
  r=y/h;
  if angle(r)>=0
    b_hat=zeros(1,2);
    if angle(r)>pi/2
      b_hat(2)=1;
    end
  else
    b_hat=ones(1,2);
    if angle(r)>-pi/2
      b_hat(2)=0;
    end
  end
  Count_BER(n)=Count_BER(n)+sum(abs(b_hat-b));
  end
end
BER=Count_BER./(2*Nt);
semilogy(SNR_dB,BER,'b-.o');
hold on;
end
hold off;
legend('BER');
xlabel('SNR (dB)');
ylabel('Probability of bit errors');
grid on;
```

9.3 Outage probability over Rayleigh or Rician fading channels

考慮在 flat-fading 通道中傳送 QPSK 訊號,接收訊號為

$$y[m] = h[m]x[m] + n[m]$$

其中訊號$x[m]$的位元平均能量為 E_{avb}=$\mathbf{E}[|x[m]|^2]/2$, $n[m]$為複數高斯雜訊 $CN(0, N_0)$，$h[m]$是 Rician fading 通道係數,

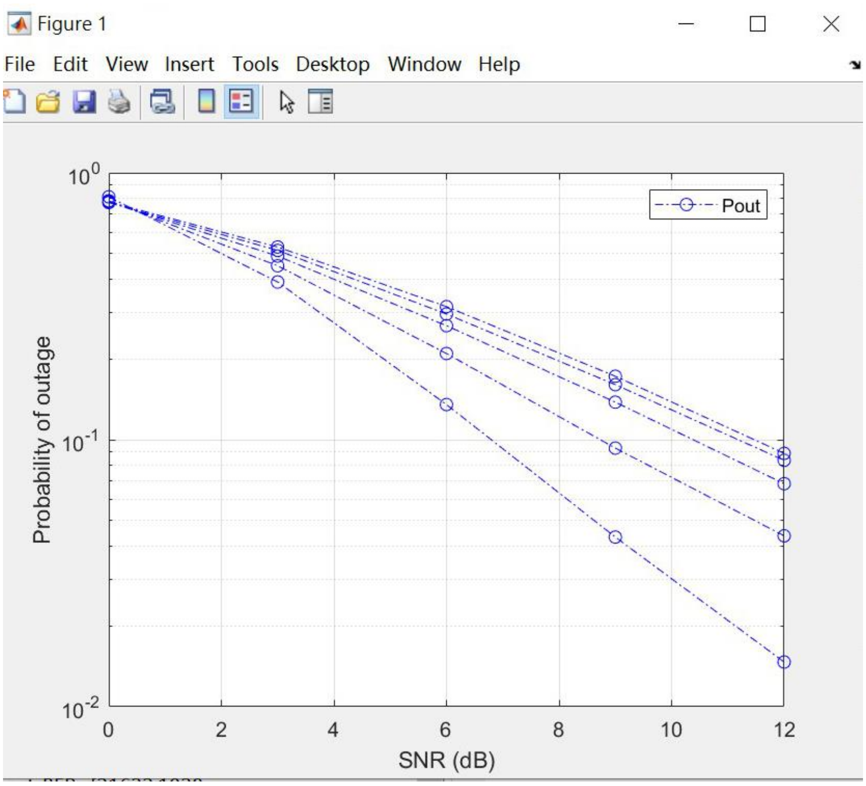
$$h[m] = \sqrt{\frac{\kappa}{\kappa+1}}\, e^{j\pi/4} + \sqrt{\frac{1}{\kappa+1}}\, CN(0,1)$$

其中κ為 K factor,假設$h[m]$隨著時間獨立變化.此接收訊號的訊號雜訊比為

$$\rho = \frac{\mathbf{E}[|x[m]|^2]|h[m]|^2}{N_0}$$

請以 Matlab 程式模擬隨機產生 K factor 數值分別為κ=0, 0. 5, 1, 2, 4 的通道係數, 計算訊號雜訊比ρ之後,統計小於ρ_{th}= 3 中斷機率. 在圖上畫出 SNR$_b$=E_{avb}/ N_0 為 0 dB, 3dB, 6dB, 9dB, 12dB 的中斷機率圖. [註: κ=0 即是 Rayleigh fading 通道]

9.3 模擬結果圖(Rayleigh or Rician fading 通道的錯誤率圖)



分析:我們可以發現當 K 值變大錯誤率將會變小

9.3 程式碼

```
close all;
clc;
clear all;

SNR_dB=[0 3 6 9 12];
Nt=10^5;
Count_BER=zeros(1,length(SNR_dB));
k = [0 0.5 1 2 4];
for ii = 1:5
P = zeros(1,5);
P_count = zeros(1,5);
Count_BER=zeros(1,length(SNR_dB));
for n=1:length(SNR_dB)
SNR=10^(SNR_dB(n)/10);
N0=1/(2*SNR);
    for t=1:Nt
    %-----------------------------------
    %generate received signal
    s=floor(rand(1)*4); % 4-bit data 0,1,2,3
    phi=(pi/4)+s*(pi/2);
    x=exp(j*phi);
    b=zeros(1,2); % two-bit data
    b(1)=(imag(x)<0);
    b(2)=(real(x)<0);
    h=sqrt(k(ii)/(k(ii)+1))*exp((j*pi)/4)+sqrt(1/(k(ii)+1))*(randn(1)+j*randn(1))*sqrt(1/2);
    w=(randn(1)+j*randn(1))*sqrt(N0/2); % AWGN r=[n1, n2, n3, n4]
    y=h*x+w;
    if(2*SNR*abs(h)^2 < 3)
        P_count(n) = P_count(n) +1;
    end
    %-------------------------------------
    %Detection
    r=y/h;
    if angle(r)>=0
        b_hat=zeros(1,2);
        if angle(r)>pi/2
            b_hat(2)=1;
        end
    else
        b_hat=ones(1,2);
        if angle(r)>-pi/2
            b_hat(2)=0;
        end
    end
    Count_BER(n)=Count_BER(n)+sum(abs(b_hat-b));
    end
end
BER=Count_BER./(2*Nt);
P = P_count./(Nt);
semilogy(SNR_dB,P,'b-.o');
hold on;
end
hold off;
legend('Pout');
xlabel('SNR (dB)');
ylabel('Probability of outage');
grid on;
```

第五章、Diversity Combining

Summary of fading channel model

- *Flat fading* model

$$y[m] = h[m]x[m] + w[m]$$

  - $w[m] \sim CN(0, N_0)$
  - For Rayleigh fading, $h[m] \sim CN(0, \sigma_h^2)$
- *Frequency selective fading* model

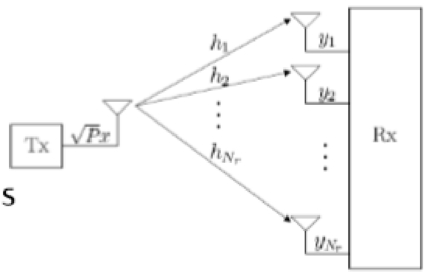
$$y[m] = \sum_{\ell=0}^{L} h_\ell[m]x[m-\ell] + w[m]$$

  - For Rayleigh fading, $h_l[m] \sim CN(0, \sigma_{h,l}^2)$

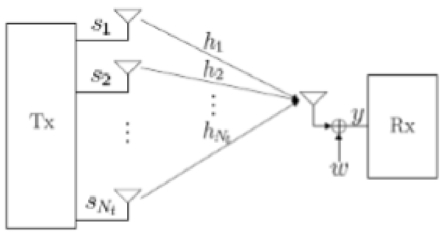
spatial diversity：利用發射或接收端的多根天線所提供的多重傳輸途徑發送相同的資料，以增強資料的傳輸品質。

Spatial Diversity

- Receiver Diversity
  - SIMO channel
  - Linearly combine received symbols to enhance system performance:

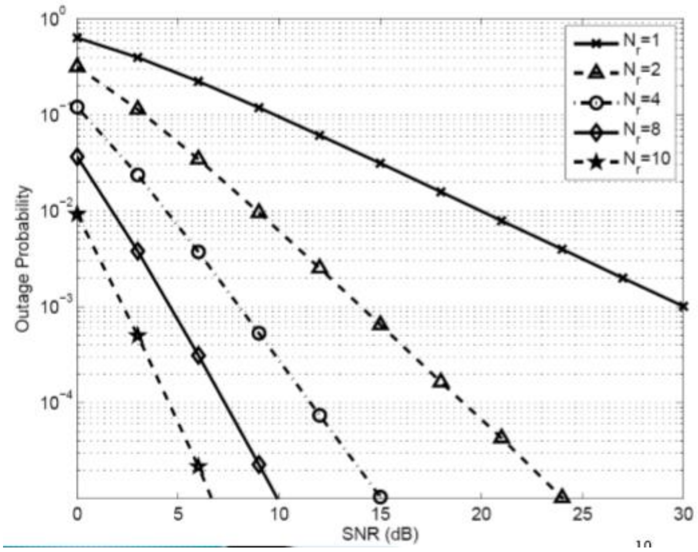


- Transmit Diversity
  - MISO Channel
  - Channel information is not necessary known at Tx
  - Transmission techniques based on different CSI (Channel State Information)

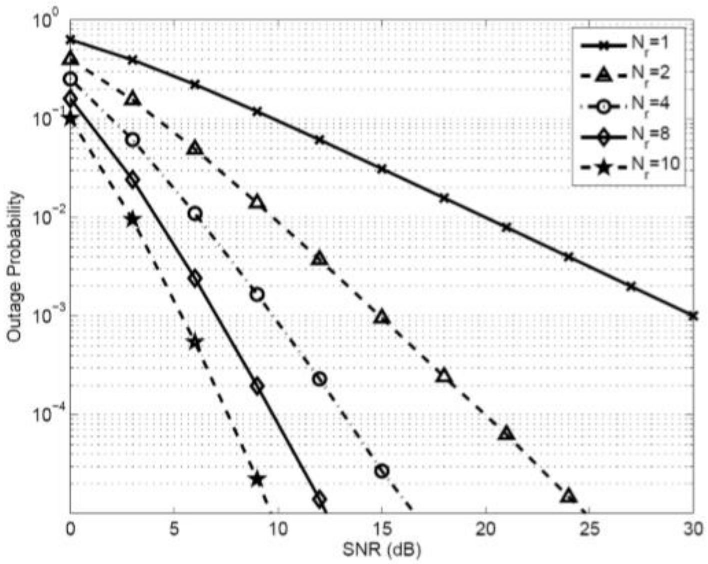


Diversity technologies has: Time Diversity、Frequency Diversity and Spatial Diversity

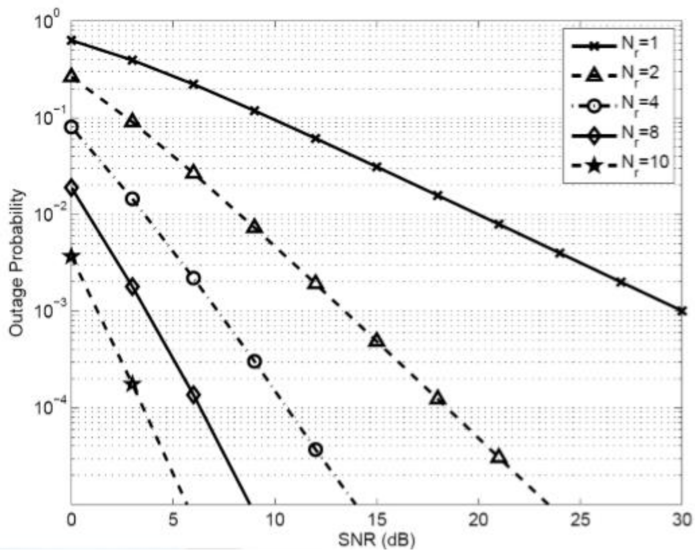
Outage Probability of Equal Gain Combining



Outage Probability of Selection Combining



Outage Probability of MRC



分析:我們可以發現 MRC 是較好的方法,其次是 Equal-Gain-Combining,最後則是 Selection Combining

Equal-Gain Transmission

- Weighting coefficients for equal-gain transmission

$$\beta_k = \sqrt{\frac{E_S}{N_t}} e^{-j\phi_k} = \sqrt{\frac{E_S}{N_t}} \frac{h_k^*}{|h_k|}$$

- Received signal

$$y[n] = \sqrt{\frac{E_S}{N_t}} \left(\sum_{k=1}^{N_r} \beta_k h_k \right) x[n] + w[n]$$

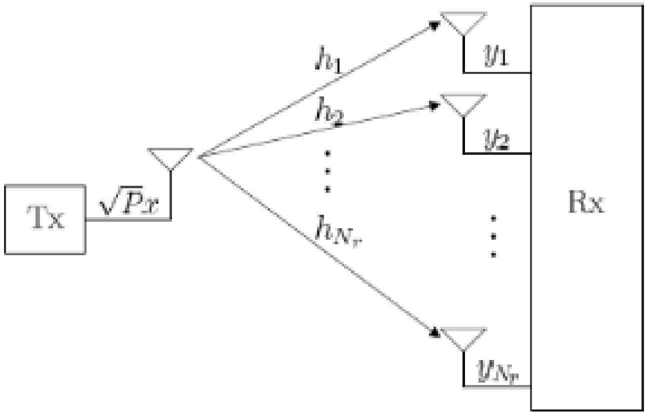
$$= \sqrt{\frac{E_S}{N_t}} \left(\sum_{k=1}^{N_r} |h_k| \right) x[n] + w[n]$$

- Received SNR is

$$\gamma_{EGT} = \frac{E_S \left(\sum_{k=1}^{N_t} |h_k| \right)^2}{N_t N_0}$$

結果與展示

10.1 BER over SIMO Rayleigh fading channel



考慮多接收天線(SIMO)系統,接收端的天線數為 N_r，傳送端送出 QPSK 訊號 $x[m]$之後，第 k 個天線之接收訊號為

$$y_k[m] = h_k[m]x[m] + n_k[m], \qquad k = 1, 2, \ldots, N_r$$

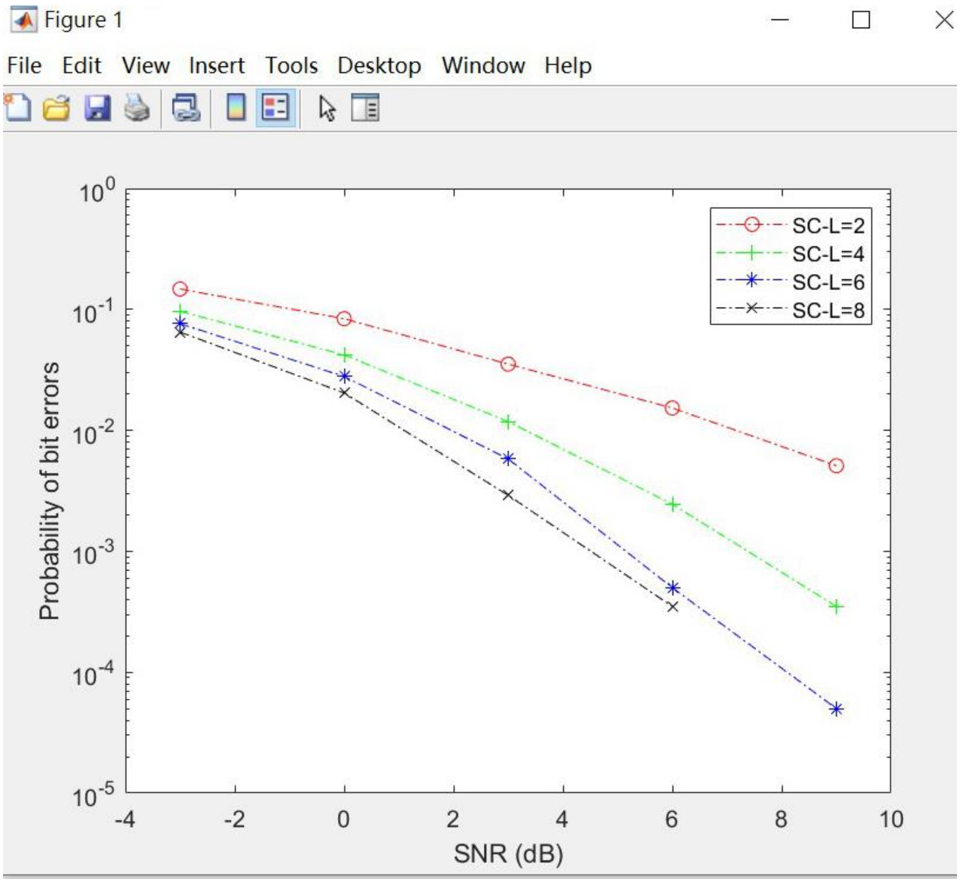
其中 QPSK 訊號$x[m]$的位元平均能量為 E_{avb}= $\mathbf{E}[|x[m]|^2]/2$, $n_k[m]$為複數高斯白雜訊,實部及虛部為獨立的高斯隨機變數 $N(0, N_0/2)$，$h_k[m]$是 Rayleigh fading 通道係數,其機率分佈為$h_k[m] \sim CN(0,1)$，且假設$h_k[m]$隨著時間獨立變化.請以 Matlab 程式模擬產生等機率隨機位元訊號,及 N_r 個天線之接收訊號.

(a) 接收端採用 Selection Combining 後偵測傳送訊號,統計位元錯誤率(BER). 在圖上畫出橫軸為 SNR_b=E_{avb}/N_0 (SNR_b= -3 dB, 0 dB, 3dB, 6dB, 9dB),縱軸為位元錯誤率的圖 (錯誤率請用 log-scale 繪圖),圖上包含 4 條曲線,分別為 N_r =2,4,6,8 的錯誤率.

(b) 接收端採用 Equal-Gain Combining 後偵測傳送訊號,統計位元錯誤率(BER). 在圖上畫出橫軸為 SNR_b=E_{avb}/N_0 (SNR_b= -3 dB, 0 dB, 3dB, 6dB, 9dB),縱軸為位元錯誤率的圖 (錯誤率請用 log-scale 繪圖),圖上包含 4 條曲線,分別為 N_r =2,4,6,8 的錯誤率.

(c) 接收端採用 Maximum Ratio Combining 後偵測傳送訊號,統計位元錯誤率(BER). 在圖上畫出橫軸為 SNR_b=E_{avb}/N_0 (SNR_b= -3 dB, 0 dB, 3dB, 6dB, 9dB),縱軸為位元錯誤率的圖 (錯誤率請用 log-scale 繪圖),圖上包含 4 條曲線,分別為 N_r =2,4,6,8 的錯誤率.

10.1(a)模擬結果圖(Selection Combining 錯誤率圖)



分析: 我們可以發現天線數 Nr 大者的錯誤率比較小

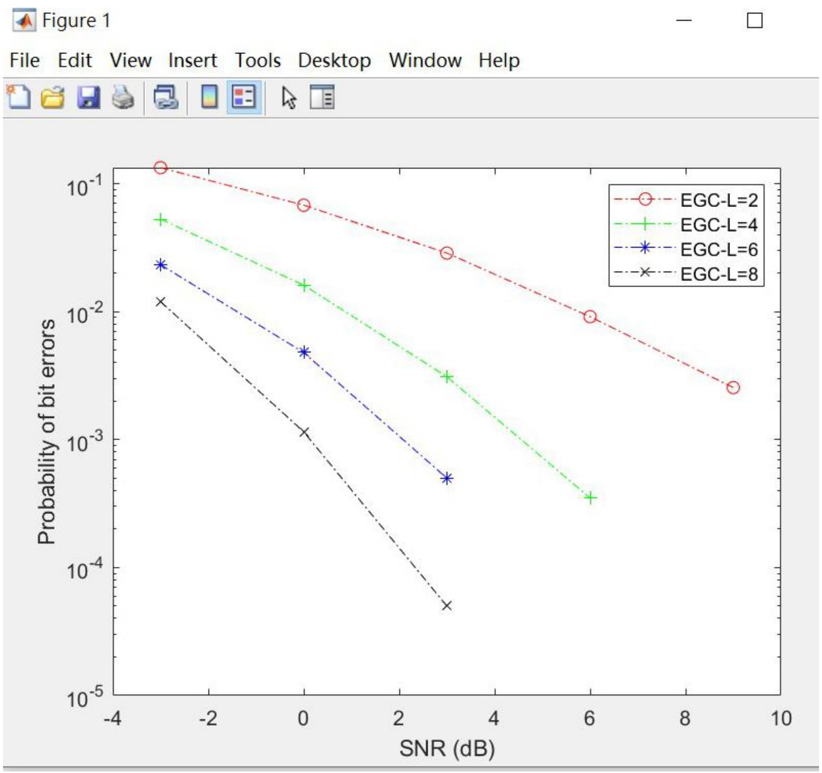
10.1(a)程式碼

```
close all;
clc;
clear all;

SNR_dB=[-3 0 3 6 9];
N=10^4;
Nr=[2 4 6 8];
Count_BER=zeros(1,length(SNR_dB));
for ii = 1:4
Count_BER=zeros(1,length(SNR_dB));
for n=1:length(SNR_dB)
    SNR=10^(SNR_dB(n)/10);
    N0=1/(2*SNR);
    for t=1:N
        %-----------------------------------
        %generate received signal
        s=floor(rand(1)*4); % 4-bit data 0,1,2,3
        phi=(pi/4)+s*(pi/2);
        x=exp(j*phi);
        b=zeros(1,2); % two-bit data
        b(1)=(imag(x)<0);
        b(2)=(real(x)<0);
        h=(randn(Nr(ii),1)+j*randn(Nr(ii),1))*sqrt(1/2); % SIMO channel, Nrx 1
        w=(randn(Nr(ii),1)+j*randn(Nr(ii),1))*sqrt(N0/2); % AWGN r=[n1, n2,n3, n4]
        y=x.*h+w; % Nr天線接收訊號
        %-------------------------------------
```

10.1(a)程式碼

```
        %generate received signal
        s=floor(rand(1)*4); % 4-bit data 0,1,2,3
        phi=(pi/4)+s*(pi/2);
        x=exp(j*phi);
        b=zeros(1,2); % two-bit data
        b(1)=(imag(x)<0);
        b(2)=(real(x)<0);
        h=(randn(Nr(ii),1)+j*randn(Nr(ii),1))*sqrt(1/2); % SIMO channel, Nrx 1
        w=(randn(Nr(ii),1)+j*randn(Nr(ii),1))*sqrt(N0/2); % AWGN r=[n1, n2,n3, n4]
        y=x.*h+w; % Nr天線接收訊號
        %------------------------------------
        %Detection
        %For EGC
        %alpha_egc=exp(-j.*phase(h));
        %z_egc=sum(alpha_egc.*y);
        %h_egc=sum(alpha_egc.*h);
        %r=z_egc/h_egc;

        %For SC
        [m ind]=max((abs(h)));
        gamma_sc=zeros(1, Nr(ii));
        gamma_sc(ind)=1;
        h_sc=gamma_sc *h;
        r=y(ind)/h_sc;

        if angle(r)>=0
          b_hat=zeros(1,2);
          if angle(r)>pi/2
            b_hat(2)=1;
          end
        else
          b_hat=ones(1,2);
          if angle(r)>-pi/2
            b_hat(2)=0;
          end
        end
        Count_BER(n)=Count_BER(n)+sum(abs(b_hat-b));
    end
  end
  BER=Count_BER./(2*N);
  if(ii == 1)
    semilogy(SNR_dB,BER,'r-.o');
  elseif(ii == 2)
    semilogy(SNR_dB,BER,'g-.+');
  elseif(ii == 3)
    semilogy(SNR_dB,BER,'b-.*');
  else
    semilogy(SNR_dB,BER,'k-.x');
  end
  hold on;
  end
  hold off;
  legend('SC-L=2','SC-L=4','SC-L=6','SC-L=8');
  xlabel('SNR (dB)');
  ylabel('Probability of bit errors');
```

10.1(b)模擬結果圖(Equal-Gain Combining 錯誤率圖)

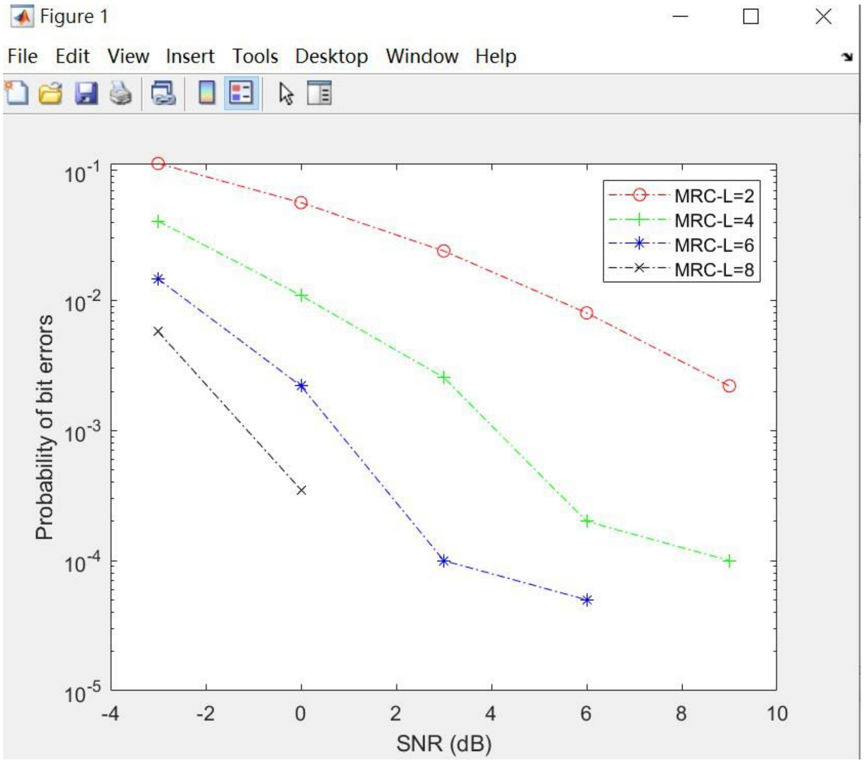


分析: 我們可以發現天線數 Nr 大者的錯誤率比較小

10.1(b)程式碼

```
close all;
clc;
clear all;

SNR_dB=[-3 0 3 6 9];
N=10^4;
Nr=[2 4 6 8];
Count_BER=zeros(1,length(SNR_dB));
for ii = 1:4
Count_BER=zeros(1,length(SNR_dB));
for n=1:length(SNR_dB)
    SNR=10^(SNR_dB(n)/10);
    N0=1/(2*SNR);
    for t=1:N
        %----------------------------------
        %generate received signal
        s=floor(rand(1)*4); % 4-bit data 0,1,2,3
        phi=(pi/4)+s*(pi/2);
        x=exp(j*phi);
        b=zeros(1,2); % two-bit data
        b(1)=(imag(x)<0);
        b(2)=(real(x)<0);
        h=(randn(Nr(ii),1)+j*randn(Nr(ii),1))*sqrt(1/2); % SIMO channel, Nrx 1
        w=(randn(Nr(ii),1)+j*randn(Nr(ii),1))*sqrt(N0/2); % AWGN r=[n1, n2,n3, n4]
        y=x.*h+w; % Nr天線接收訊號
        %------------------------------------
        %Detection
        alpha_egc=exp(-j.*phase(h));
        z_egc=sum(alpha_egc.*y);
        h_egc=sum(alpha_egc.*h);
        r=z_egc/h_egc;
        if angle(r)>=0
            b_hat=zeros(1,2);
            if angle(r)>pi/2
                b_hat(2)=1;
            end
        else
            b_hat=ones(1,2);
            if angle(r)>-pi/2
                b_hat(2)=0;
            end
        end
        Count_BER(n)=Count_BER(n)+sum(abs(b_hat-b));
    end
end
BER=Count_BER./(2*N);
if(ii == 1)
    semilogy(SNR_dB,BER,'r-.o');
elseif(ii == 2)
    semilogy(SNR_dB,BER,'g-.+');
elseif(ii == 3)
    semilogy(SNR_dB,BER,'b-.*');
else
    semilogy(SNR_dB,BER,'k-.x');
end
hold on;
end
hold off;
legend('EGC-L=2','EGC-L=4','EGC-L=6','EGC-L=8');
xlabel('SNR (dB)');
ylabel('Probability of bit errors');
```

10.1(C)模擬結果圖(Maximum Ratio Combining 錯誤率圖)

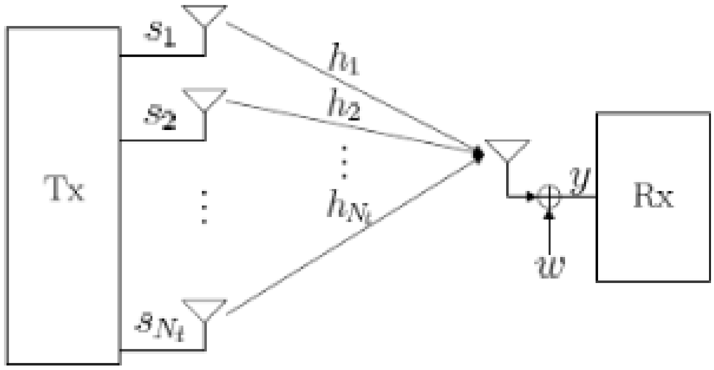


分析: 我們可以發現天線數 Nr 大者的錯誤率比較小

10.1(b)程式碼

```
close all;
clc;
clear all;

SNR_dB=[-3 0 3 6 9];
N=10^4;
Nr=[2 4 6 8];
Count_BER=zeros(1,length(SNR_dB));
for ii = 1:4
Count_BER=zeros(1,length(SNR_dB));
for n=1:length(SNR_dB)
    SNR=10^(SNR_dB(n)/10);
    N0=1/(2*SNR);
    for t=1:N
        %-----------------------------------
        %generate received signal
        s=floor(rand(1)*4); % 4-bit data 0,1,2,3
        phi=(pi/4)+s*(pi/2);
        x=exp(j*phi);
        b=zeros(1,2); % two-bit data
        b(1)=(imag(x)<0);
        b(2)=(real(x)<0);
        h=(randn(Nr(ii),1)+j*randn(Nr(ii),1))*sqrt(1/2); % SIMO channel, Nrx 1
        w=(randn(Nr(ii),1)+j*randn(Nr(ii),1))*sqrt(N0/2); % AWGN r=[n1, n2,n3, n4]
        y=x.*h+w; % Nr天線接收訊號
        %-------------------------------------
        %Detection
        %For EGC
        %alpha_egc=exp(-j.*phase(h));
        %z_egc=sum(alpha_egc.*y);
        %h_egc=sum(alpha_egc.*h);
        %r=z_egc/h_egc;

        %For MRC
        rr = sum(conj(h).*y);
        r = rr / sum(abs(h).^2);

        if angle(r)>=0
            b_hat=zeros(1,2);
            if angle(r)>pi/2
                b_hat(2)=1;
            end
        else
            b_hat=ones(1,2);
            if angle(r)>-pi/2
                b_hat(2)=0;
            end
        end
        Count_BER(n)=Count_BER(n)+sum(abs(b_hat-b));
    end
end
BER=Count_BER./(2*N);
if(ii == 1)
    semilogy(SNR_dB,BER,'r-.o');
elseif(ii == 2)
    semilogy(SNR_dB,BER,'g-.+');
elseif(ii == 3)
    semilogy(SNR_dB,BER,'b-.*');
else
    semilogy(SNR_dB,BER,'k-.x');
end
hold on;
end
hold off;
legend('MRC-L=2','MRC-L=4','MRC-L=6','MRC-L=8');
xlabel('SNR (dB)');
ylabel('Probability of bit errors');
```

10.2 BER over MISO Rayleigh fading channel



考慮多傳送天線(MSIO)系統,接收端的天線數為 N_r，傳送端送出 QPSK 訊號 $x[m]$之後，第 k 個天線之傳送訊號為$s_k[m] = \beta_k x[m]$，$x[m] \in \{(\pm 1 \pm j)/\sqrt{2}\}$是能量為一的 QPSK 訊號,係數$\beta_k$須滿足傳送功率限制,i.e.,$|\beta_1|^2 + |\beta_2|^2 + \cdots + |\beta_{N_t}|^2 = \mathrm{E_{av}}$，位元平均能量為 $E_{avb}= E_{av}/2$，接收端收到的訊號為

$$y[m] = \sum_{k=1}^{N_t} h_k[m] s_k[m] + n[m],$$

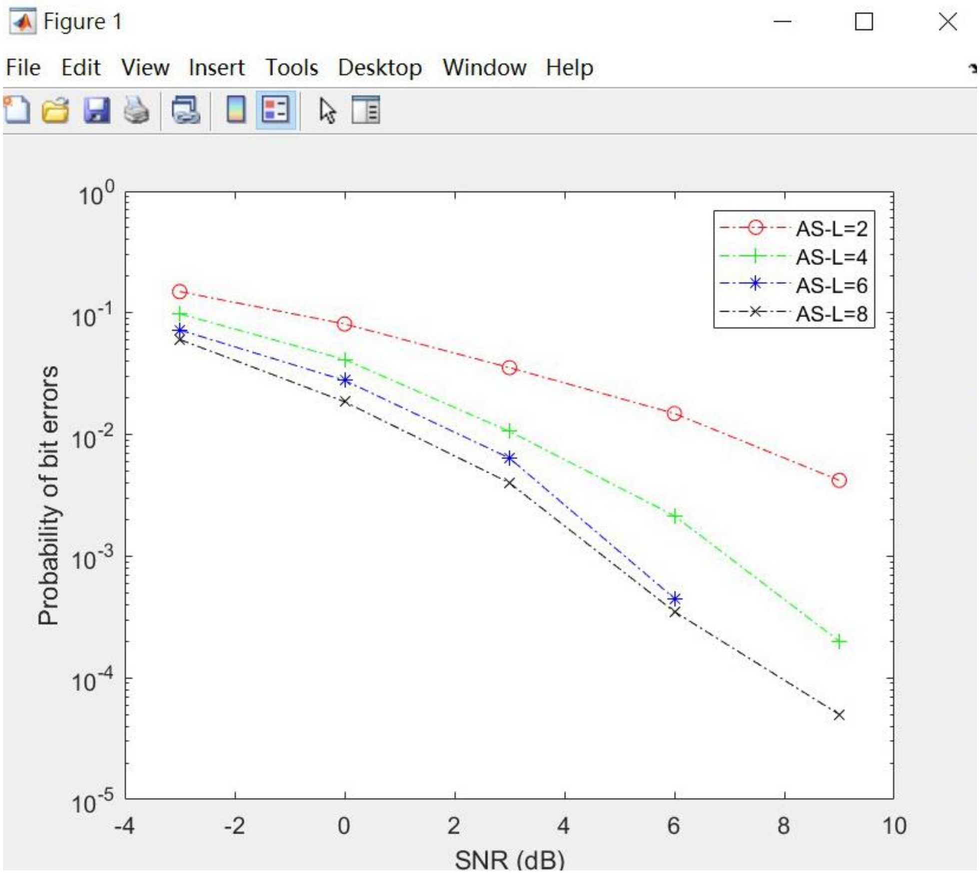
其中 $n[m]$為複數高斯白雜訊,實部及虛部為獨立的高斯隨機變數 $N(0, N_0/2)$，$h_k[m]$是 Rayleigh fading 通道係數,其機率分佈為$h_k[m] \sim CN(0,1)$，且假設$h_k[m]$隨著時間獨立變化.

(a) 請以 Matlab 程式模擬產生等機率隨機位元訊號及 QPSK 符元,若傳送端採用 Antenna selection,請產生 N_t個天線之傳送訊號及接收訊號,偵測 QPSK 符元後,統計位元錯誤率(BER). 在圖上畫出橫軸為 $\mathrm{SNR}_b=E_{avb}/N_0$ (SNR_b= -3 dB, 0 dB, 3dB, 6dB, 9dB),縱軸為位元錯誤率的圖 (錯誤率請用 log-scale 繪圖),圖上包含 4 條曲線,分別為 N_t=2,4,6,8 的錯誤率.

(b) 請以 Matlab 程式模擬產生等機率隨機位元訊號及 QPSK 符元,若傳送端採用 Transmit Beamforming,請產生 N_t個天線之傳送訊號及接收訊號,偵測 QPSK 符元後,統計位元錯誤率(BER). 在圖上畫出橫軸為 $\mathrm{SNR}_b=E_{avb}/N_0$ (SNR_b= -3 dB, 0 dB, 3dB, 6dB, 9dB),縱軸為位元錯誤率的圖 (錯誤率請用 log-scale 繪圖),圖上包含 4 條曲線,分別為 N_t=2,4,6,8 的錯誤率.

(c) 請以 Matlab 程式模擬產生等機率隨機位元訊號及 QPSK 符元,若傳送端採用 Equal-Gain Transmission,請產生 N_t個天線之傳送訊號及接收訊號,偵測 QPSK 符元後,統計位元錯誤率(BER). 在圖上畫出橫軸為 $\mathrm{SNR}_b=E_{avb}/N_0$ (SNR_b= -3 dB, 0 dB, 3dB, 6dB, 9dB),縱軸為位元錯誤率的圖 (錯誤率請用 log-scale 繪圖),圖上包含 4 條曲線,分別為 N_t=2,4,6,8 的錯誤率.

10.2(a)模擬結果圖(Antenna selection 錯誤率圖)

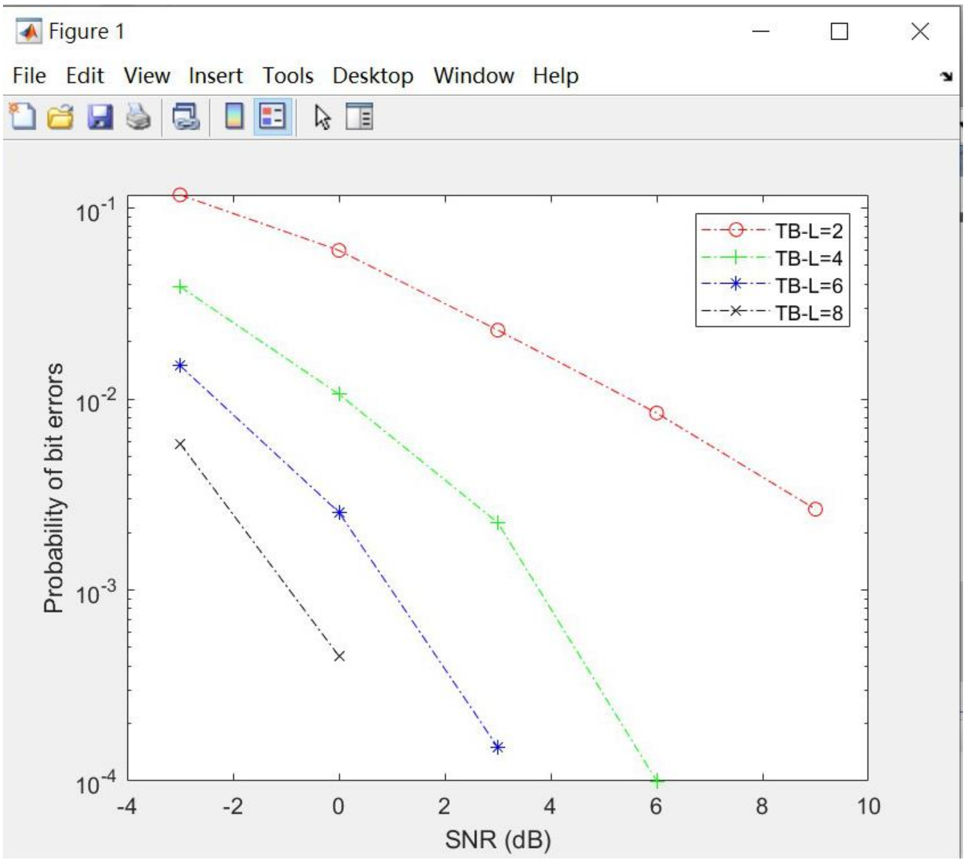


分析: 我們可以發現天線數 Nr 大者的錯誤率比較小

10.2(a)程式碼

```
close all;
clc;
clear all;

SNR_dB=[-3 0 3 6 9];
N=10^4;
Nt=[2 4 6 8];
Count_BER=zeros(1,length(SNR_dB));
for ii = 1:4
Count_BER=zeros(1,length(SNR_dB));
for n=1:length(SNR_dB)
    SNR=10^(SNR_dB(n)/10);
    N0=1/(SNR);
    for t=1:N
        %-----------------------------------
        %generate received signal
        s=floor(rand(1)*4); % 4-bit data 0,1,2,3
        phi=(pi/4)+s*(pi/2);
        x=exp(j*phi);
        b=zeros(1,2); % two-bit data
        b(1)=(imag(x)<0);
        b(2)=(real(x)<0);
        h=(randn(1,Nt(ii))+j*randn(1,Nt(ii)))*sqrt(1/2); % MISO channel,1xNt
        [m ind]=max(abs(h));
        beta_as=zeros(Nt(ii),1);
        beta_as(ind)=1;
        s=x.*beta_as; %Nt天線傳送訊號
        w=(randn(1)+j*randn(1))*sqrt(N0/2); % AWGN r=[n1, n2, n3,n4]
        y=sqrt(2)*h*s+w; % 接收訊號
        %------------------------------------
        %Detection
        h_as=h*beta_as;
        r=y/h_as;
        if angle(r)>=0
            b_hat=zeros(1,2);
            if angle(r)>pi/2
                b_hat(2)=1;
            end
        else
            b_hat=ones(1,2);
            if angle(r)>-pi/2
                b_hat(2)=0;
            end
        end
        Count_BER(n)=Count_BER(n)+sum(abs(b_hat-b));
    end
end
BER=Count_BER./(2*N);
if(ii == 1)
    semilogy(SNR_dB,BER,'r-.o');
elseif(ii == 2)
    semilogy(SNR_dB,BER,'g-.+');
elseif(ii == 3)
    semilogy(SNR_dB,BER,'b-.*');
else
    semilogy(SNR_dB,BER,'k-.x');
end
hold on;
end
hold off;
```

10.2(b)模擬結果圖(Transmit Beamforming 錯誤率圖)



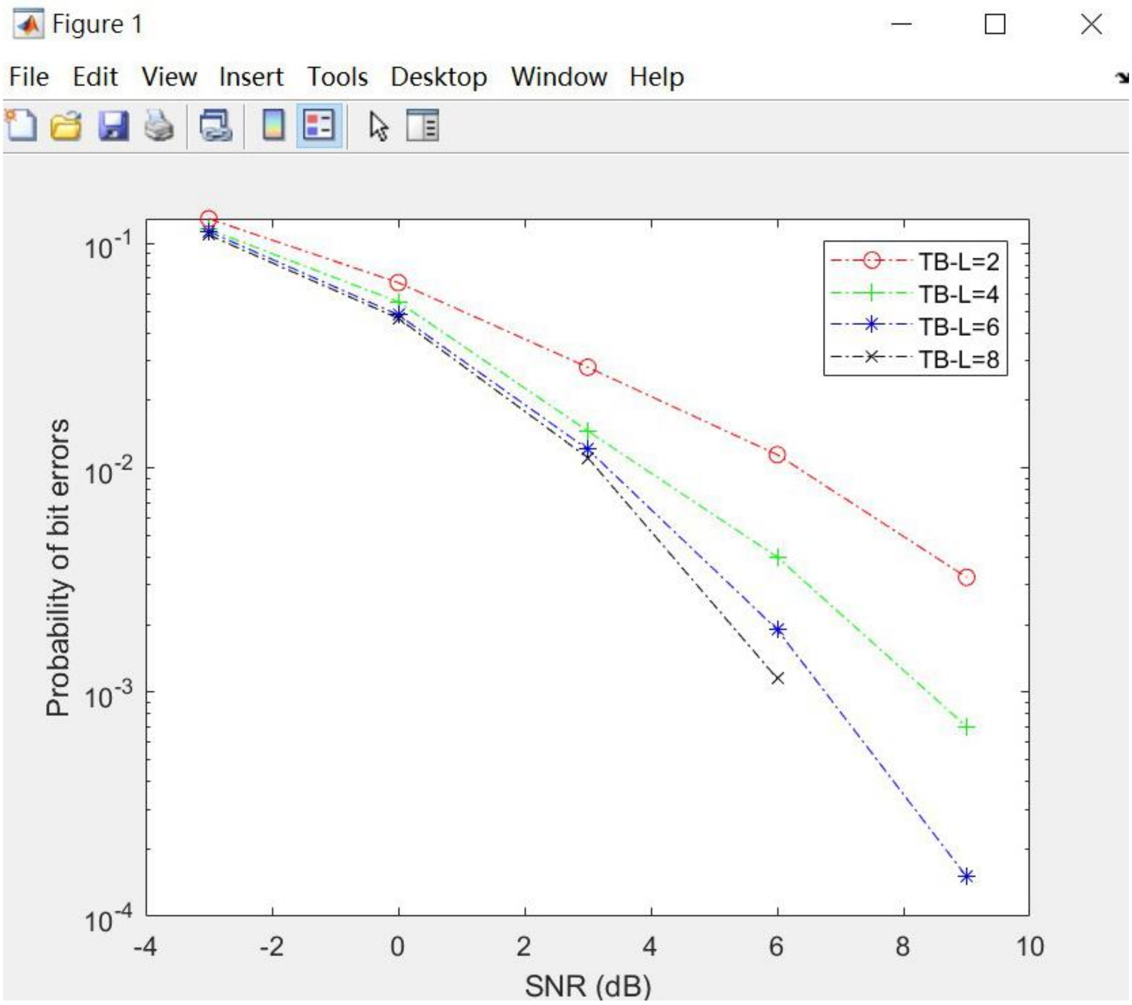
分析: 我們可以發現天線數 Nr 大者的錯誤率比較小,

beamforming：藉由多根天線產生一個具有指向性的波束，將能量集中在欲傳輸的方向，增加訊號品質，並減少與其他用戶間的干擾。

10.2(b)程式碼

```
clc;
clear all;

SNR_dB=[-3 0 3 6 9];
N=10^4;
Nt=[2 4 6 8];
Count_BER=zeros(1,length(SNR_dB));
for ii = 1:4
Count_BER=zeros(1,length(SNR_dB));
for n=1:length(SNR_dB)
    SNR=10^(SNR_dB(n)/10);
    N0=1/(SNR);
    for t=1:N
        %-----------------------------------
        %generate received signal
        s=floor(rand(1)*4); % 4-bit data 0,1,2,3
        phi=(pi/4)+s*(pi/2);
        x=exp(j*phi);
        b=zeros(1,2); % two-bit data
        b(1)=(imag(x)<0);
        b(2)=(real(x)<0);
        h=(randn(1,Nt(ii))+j*randn(1,Nt(ii)))*sqrt(1/2); % MISO channel,1xNt
        beta_tb=sqrt(2/(sum(abs(h).^2)))*conj(h);
        s=x.*beta_tb; %Nt天線傳送訊號
        w=(randn(1)+j*randn(1))*sqrt(N0/2); % AWGN r=[n1, n2, n3,n4]
        y=sum(h.*s)+w; % 接收訊號
        %------------------------------------
        %Detection
        r=y./(h.*beta_tb);
        if angle(r)>=0
            b_hat=zeros(1,2);
            if angle(r)>pi/2
                b_hat(2)=1;
            end
        else
            b_hat=ones(1,2);
            if angle(r)>-pi/2
                b_hat(2)=0;
            end
        end
        Count_BER(n)=Count_BER(n)+sum(abs(b_hat-b));
    end
end
BER=Count_BER./(2*N);
if(ii == 1)
    semilogy(SNR_dB,BER,'r-.o');
elseif(ii == 2)
    semilogy(SNR_dB,BER,'g-.+');
elseif(ii == 3)
    semilogy(SNR_dB,BER,'b-.*');
else
    semilogy(SNR_dB,BER,'k-.x');
end
hold on;
end
hold off;
legend('TB-L=2','TB-L=4','TB-L=6','TB-L=8');
xlabel('SNR (dB)');
ylabel('Probability of bit errors');
```

10.2(C)模擬結果圖(Equal-Gain Transmission 錯誤率圖)



分析:

我們可以發現天線數 Nr 大者的錯誤率比較小

藉由模擬結果錯誤率圖可以發現在 Nr=2 時三種方法錯誤率幾乎相同,但 當 Nr 變大之後 錯誤率將有明顯不同，同時我們可以發現 MRC 是較好的方法，因為相同資料的訊號會在通道中做 MRC，所以錯誤率就會比沒有任何相同訊號的模擬之錯誤率還低。,其次是 Equal-Gain-Combining,最後則是 Selection Combining，但是 selection Combining 成本最低,所以仍值得應用

10.2(c)程式碼

```
close all;
clc;
clear all;

SNR_dB=[-3 0 3 6 9];
N=10^4;
Nt=[2 4 6 8];
Count_BER=zeros(1,length(SNR_dB));
for ii = 1:4
Count_BER=zeros(1,length(SNR_dB));
for n=1:length(SNR_dB)
    SNR=10^(SNR_dB(n)/10);
    N0=1/(SNR);
    for t=1:N
        %-----------------------------------
        %generate received signal
        s=floor(rand(1)*4); % 4-bit data 0,1,2,3
        phi=(pi/4)+s*(pi/2);
        x=exp(j*phi);
        b=zeros(1,2); % two-bit data
        b(1)=(imag(x)<0);
        b(2)=(real(x)<0);
        h=(randn(1,Nt(ii))+j*randn(1,Nt(ii)))*sqrt(1/2); % MISO channel,1xNt
        beta_egt=sqrt(2/Nt(ii)).*(conj(h)./abs(h));
        %Nt天線傳送訊號
        w=(randn(1)+j*randn(1))*sqrt(N0/2); % AWGN r=[n1, n2, n3,n4]
        y=sqrt(2/Nt(ii)).*(sum(beta_egt.*h)).*x+w; % 接收訊號
        %-------------------------------------
        %Detection
        r=y/(sum(abs(h))*sqrt(2/Nt(ii)));
        if angle(r)>=0
            b_hat=zeros(1,2);
            if angle(r)>pi/2
                b_hat(2)=1;
            end
        else
            b_hat=ones(1,2);
            if angle(r)>-pi/2
                b_hat(2)=0;
            end
        end
        Count_BER(n)=Count_BER(n)+sum(abs(b_hat-b));
    end
end
BER=Count_BER./(2*N);
if(ii == 1)
    semilogy(SNR_dB,BER,'r-.o');
elseif(ii == 2)
    semilogy(SNR_dB,BER,'g-.+');
elseif(ii == 3)
    semilogy(SNR_dB,BER,'b-.*');
else
    semilogy(SNR_dB,BER,'k-.x');
end
hold on;
end
hold off;
legend('TB-L=2','TB-L=4','TB-L=6','TB-L=8');
xlabel('SNR (dB)');
ylabel('Probability of bit errors');
```

MIMO technology

MIMO Channel Model

- Received signal in a MIMO Channel

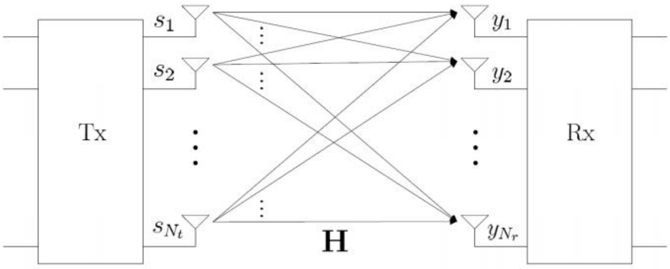
$$\mathbf{y} = \mathbf{H}\mathbf{s} + \mathbf{w}$$

  - Transmit diversity, receiver diversity, or both
  - Have N_t *N_r paths between tx and rx
  - We assume that all paths are independent,

$$M = \text{Rank}(\mathbf{H}) = min(N_t, N_r)$$

  - Power constraint:

$$\mathbf{E}[|s_1|^2] + \mathbf{E}[|s_2|^2] + \cdots + \mathbf{E}[|s_{N_t}|^2] = E$$

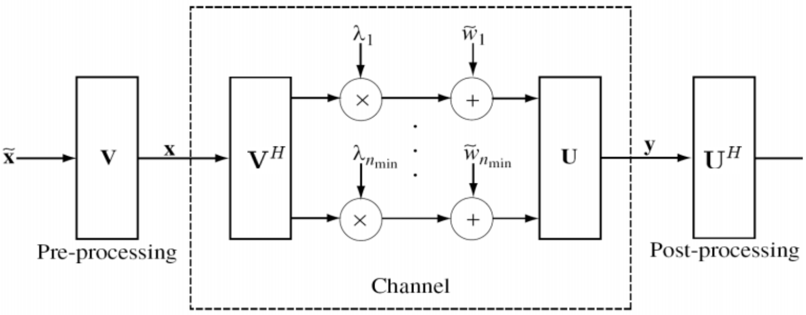


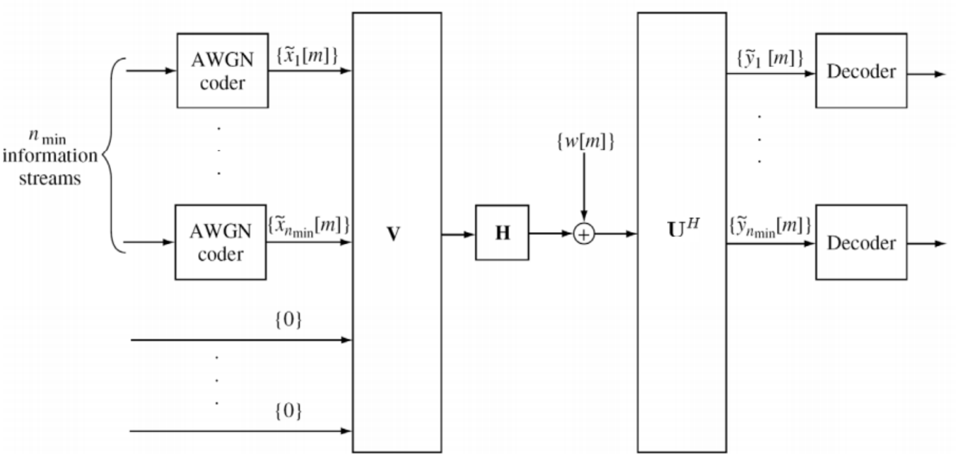
SVD Of MIMO Channel

- $\mathbf{z} = \Sigma\mathbf{x} + \tilde{\mathbf{w}}$
  - $\mathbf{x} = \mathbf{V}^H\mathbf{s} \Longrightarrow \|\mathbf{x}\|^2 = \|\mathbf{s}\|^2$
  - $\mathbf{E}[|x_i|^2] = E_i$, $\sum_i E_i = E$
  - $\tilde{\mathbf{w}} = \mathbf{U}^H\mathbf{w} \Longrightarrow \mathbf{E}[\tilde{\mathbf{w}}\tilde{\mathbf{w}}^H] = N_0\mathbf{I}_{M_r}$
- Via SVD, we have

$$z_i = \lambda_i x_i + \tilde{w}_i, \quad i = 1, 2, \cdots, M$$

  - Each μ_i corresponds to an *eigen-channel*





- Space-Time Block Code(STBC) :
  - A simple and effective method to achieve transmit diversity for quasi-static fading channel
  - Can be generalized to the case with multiple receive antennas to obtain additional receive diversity
- Transmit diversity with two antennas:
  $\Rightarrow$ The Alamouti scheme
- Transmission of Alamouti scheme:

Let

$$\begin{array}{cc} & \xrightarrow{\textit{antennas}} \\ \mathbf{X} = & \left[\begin{array}{cc} x[1] & x[2] \\ -x[2]^* & x[1]^* \end{array}\right] \downarrow \textit{time} \end{array}$$

$$\mathbf{E}[|x[1]|^2] = \mathbf{E}[|\,x[2]|^2] = E/2$$

Optimal Receive for the Alamouti scheme

- The received signals are

$$y[1] = h_1 x[1] + h_2 x[2] + w[1]$$
$$y[2] = -h_1 x[2]^* + h_2 x[1]^* + w[2]$$

$$\mathbf{y} = \begin{bmatrix} y[1] \\ y[2]^* \end{bmatrix} = \begin{bmatrix} h_1 & h_2 \\ h_2^* & -h_1^* \end{bmatrix} \begin{bmatrix} x[1] \\ x[2] \end{bmatrix} + \begin{bmatrix} w[1] \\ w[2]^* \end{bmatrix}$$
$$= \mathbf{H}\mathbf{x} + \mathbf{w}$$

$$\mathbf{H} = \begin{bmatrix} h_1 & h_2 \\ h_2^* & -h_1^* \end{bmatrix}$$

$$\boxed{\mathbf{H}^H\mathbf{H} = (|h_1|^2 + |h_2|^2)\mathbf{I}}$$

$$\mathrm{E}[\mathbf{w}] = \mathbf{0},\ \mathbf{E}[\mathbf{w}\mathbf{w}^H] = N_0\mathbf{I}$$

$$\bar{\mathbf{y}} = \mathbf{H}^H\mathbf{y} = (|h_1|^2 + |h_2|^2)\mathbf{x} + \mathbf{H}^H\mathbf{w}$$

- $\bar{y}[1] = h_1^* y[1] + h_2 y[2]^*$
  $= (|h_1|^2 + |h_2|^2)x[1] + h_1^* w[1] + h_2 w[2]^*$
- $\bar{y}[2] = h_2^* y[1] - h_1 y[2]^*$
  $= (|h_1|^2 + |h_2|^2)x[2] + h_2^* w[1] - h_1 w[2]^*$

$(x[1], x[2])$ can be detected from $(\bar{y}[1], \bar{y}[2])$

- If BPSK is adopted

$$\hat{x}[1] = \text{sign}\left(\Re\left\{\bar{y}[1]\right\}\right)$$
$$\hat{x}[2] = \text{sign}\left(\Re\left\{\bar{y}[2]\right\}\right)$$

OSTBC --Decoding

- (ex) For Alamouti scheme,

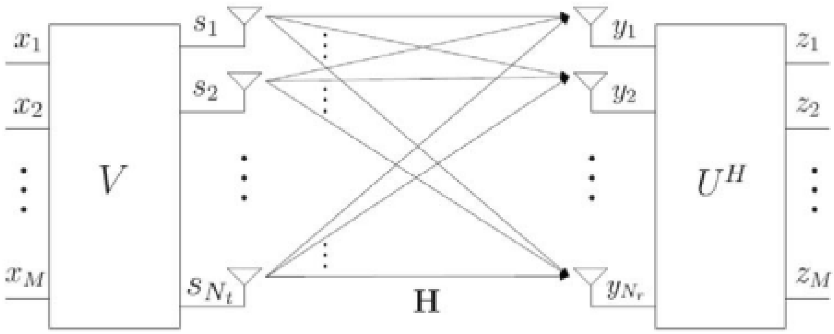
$$\mathbf{A}_1 = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}, \mathbf{A}_2 = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}, \mathbf{B}_1 = \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}, \mathbf{B}_2 = \begin{bmatrix} 0 & 0 \\ -1 & 0 \end{bmatrix}$$

  - $u_1 = \sum_{j=1}^{N_r} \mathbf{h}_j^H \mathbf{A}_1^H \mathbf{y}_j + \mathbf{y}_j^H \mathbf{B}_1 \mathbf{h}_j = \sum_{j=1}^{N_r} h_{1j}^* y_j[1] + h_{2j} y_j[2]^*$
  - $u_2 = \sum_{j=1}^{N_r} \mathbf{h}_j^H \mathbf{A}_2^H \mathbf{y}_j + \mathbf{y}_j^H \mathbf{B}_2 \mathbf{h}_j = \sum_{j=1}^{N_r} h_{2j}^* y_j[1] + h_{1j} y_j[2]^*$
  - $v_1^2 = v_2^2 = \sum_{j=1}^{N_r}\sum_{i=1}^{N_t} |h_{ij}|^2$
  - If BPSK is adopted

$$\hat{x}_m = \text{sign}(\Re\{u_m\})$$

結果與展示

13.1 BER over MIMO Rayleigh fading channel



考慮多輸入多輸出(MIMO)系統,傳送端的天線數為 N_t,接收端的天線數為 N_r, 傳送天線送出訊號 $s_1[m], s_2[m],\ldots, s_{Nt}[m]$ 後,接收訊號為

$$\begin{bmatrix} y_1[m] \\ y_2[m] \\ \vdots \\ y_{Nr}[m] \end{bmatrix} = \begin{bmatrix} h_{11}[m] & h_{12}[m] & \cdots & h_{1N_t}[m] \\ h_{21}[m] & h_{22}[m] & \cdots & h_{2N_t}[m] \\ \vdots & \vdots & \ddots & \vdots \\ h_{Nr1}[m] & h_{Nr2}[m] & \cdots & h_{NrNt}[m] \end{bmatrix} \begin{bmatrix} s_1[m] \\ s_2[m] \\ \vdots \\ s_{Nt}[m] \end{bmatrix} + \begin{bmatrix} w_1[m] \\ w_2[m] \\ \vdots \\ w_{Nr}[m] \end{bmatrix}$$

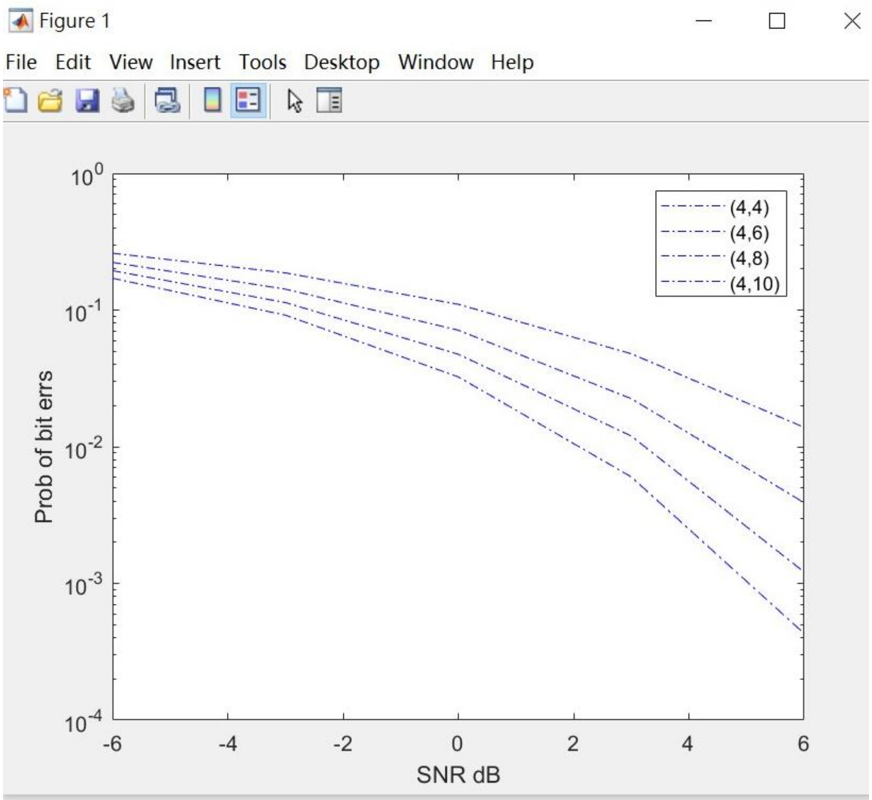
其中傳送訊號的總能量為 E, $w_k[m]$為複數高斯白雜訊 $CN(0, N_0)$， $h_{ij}[m]$是通道係數,其機率分佈為$h_{ij}[m]\sim CN(0,1)$，且假設$h_{ij}[m]$隨著時間獨立變化.令通道係數矩陣的 SVD 分解為$\boldsymbol{H} = \boldsymbol{U\Sigma V}^H$，而訊號 $s_1[m], s_2[m],\ldots, s_{Nt}[m]$是由 $x_1[m], x_2[m],\ldots, x_M[m]$訊號經過$\boldsymbol{V}$做前置處理,而接收段則將接收訊號通過$\boldsymbol{U}^H$將訊號分解出獨立 eigen-channel 的輸出.

請以 Matlab 程式模擬產生 4 個等機率隨機位元訊號,及對應的 2 個 QPSK 訊號,在四個 eigen-channel 傳送兩次 2 個 QPSK 訊號,經過傳送端及接收端的線性處理之後,將 eigen-channel 的輸出訊號以 MRC 結合,偵測 QPSK 訊號，並統計位元錯誤率(BER). 在圖上畫出橫軸為 SNR=E/N_0 (SNR$_b$=0 dB, 3dB, 6dB,

13.1 程式碼

```
clear all;
SNR_dB=[-6 -3 0 3 6];
N=10^4;
Nt=4;
Nr=[4 6 8 10];
Count_BER_2bits=zeros(length(Nr),length(SNR_dB));
% Count_BER_1bit=zeros(1,length(SNR_dB));
for nNr=1:4
for n=1:length(SNR_dB)
SNR=10^(SNR_dB(n)/10);
N0=8/SNR;
for t=1:N

%generate channel
H=(randn(Nr(nNr),Nt)+1j*randn(Nr(nNr),Nt))*sqrt(1/2); % MIMO channel,Nr x Nt
[U S V]=svd(H);

b_2bits=floor(rand(8,1)*2); % one-bit data
x_2bits=zeros(4,1);
for nn=1:1:4
x_2bits(nn,1)=(-1)^b_2bits((2*nn-1),1)+1j*(-1)^b_2bits((2*nn),1); % QPSK
end
s_2bits=V*x_2bits;
w_2bits=(randn(Nr(nNr),1)+1j*randn(Nr(nNr),1))*sqrt(N0/2); % AWGN
y_2bits=H*s_2bits+w_2bits; %Nr天線接收訊號

z_2bits=U'*y_2bits;
sigmasig=diag(S);
z_2bits=z_2bits(1:4,:);
r_2bits=z_2bits./sigmasig;
b_hat_2bits=zeros(8,1);
for nnr=1:1:4
x_hat=sign(real(r_2bits(nnr)))+1j.*sign(imag(r_2bits(nnr)));
 if imag(x_hat)<0
 b_hat_2bits((2*nnr))=1;
 end
 if real(x_hat)<0
 b_hat_2bits((2*nnr-1))=1;
 end
end

Count_BER_2bits(nNr,n)=Count_BER_2bits(nNr,n)+sum(abs(b_hat_2bits-b_2bits));
end

end
end
BER_2bits=Count_BER_2bits./(2*N)/4;
semilogy(SNR_dB,BER_2bits);
xlabel('SNR (dB)');
ylabel('Probability of bit errors');
legend('(4,4)','(4,6)','(4,8)','(4,10)');
```

模擬結果圖 13.1(2 different QPSK over 4eigen-channel 錯誤率圖)



分析: 主要我們要模擬送端有四個，但是接收端有 4,6,8,10 個，然而我們的平行通道也只有四個，我們送出 4 個 qpsk 的資料

經通道後解調得 Z1,Z2,Z3,Z4，再去比較 X1,X2,X3,X4 的錯誤率

SNR=E/N0

接下來我們把平行通道合併 EX 前兩個送一樣的資料 X1,X1,X2,X2

經通道後得 Z1,Z2,Z3,Z4，再去

Z1BAR=Lambda1*Z1+Lambda2*Z2,Z3BAR=Lambda3*Z3+Lambda4*Z4 再用正負下去解調就好。

13.2 OSTBC

考慮多輸入多輸出(MIMO)系統,傳送端的天線數為 N_t,接收端的天線數為 N_r, 傳送天線送出訊號 $s_1[m], s_2[m],\ldots, s_{Nt}[m]$ 後,接收訊號為

$$\begin{bmatrix} y_1[m] \\ y_2[m] \\ \vdots \\ y_{Nr}[m] \end{bmatrix} = \begin{bmatrix} h_{11}[m] & h_{12}[m] & \cdots & h_{1N_t}[m] \\ h_{21}[m] & h_{22}[m] & \cdots & h_{2N_t}[m] \\ \vdots & \vdots & \ddots & \vdots \\ h_{Nr1}[m] & h_{Nr2}[m] & \cdots & h_{NrN_t}[m] \end{bmatrix} \begin{bmatrix} s_1[m] \\ s_2[m] \\ \vdots \\ s_{N_t}[m] \end{bmatrix} + \begin{bmatrix} w_1[m] \\ w_2[m] \\ \vdots \\ w_{Nr}[m] \end{bmatrix}$$

其中傳送訊號的總能量為 E, $w_k[m]$為複數高斯白雜訊 $CN(0, N_0)$, $h_{ij}[m]$是通道係數,其機率分佈為$h_{ij}[m]\sim CN(0,1)$, 且假設$h_{ij}[m]$隨著時間獨立變化.

(a) 假設$N_t = 2$,請以 Matlab 程式模擬產生 4 個等機率隨機位元訊號,及對應的 2 個 QPSK 訊號,以下列方式產生空時編碼的碼字並傳送

$$\mathbf{X} = \begin{bmatrix} x[1] & x[2] \\ -x[2]^* & x[1]^* \end{bmatrix}$$

接收端線性處理接收訊號後, 偵測 QPSK 符元統計位元錯誤率(BER).在圖上畫出橫軸為 SNR=E/N_0 (SNR_b=0 dB, 3dB, 6dB, 9dB, 12dB),縱軸為位元錯誤率的圖 (錯誤率請用 log-scale 繪圖),圖上包含 4 條曲線,分別為 N_r=2,4,6,8 的錯誤率.

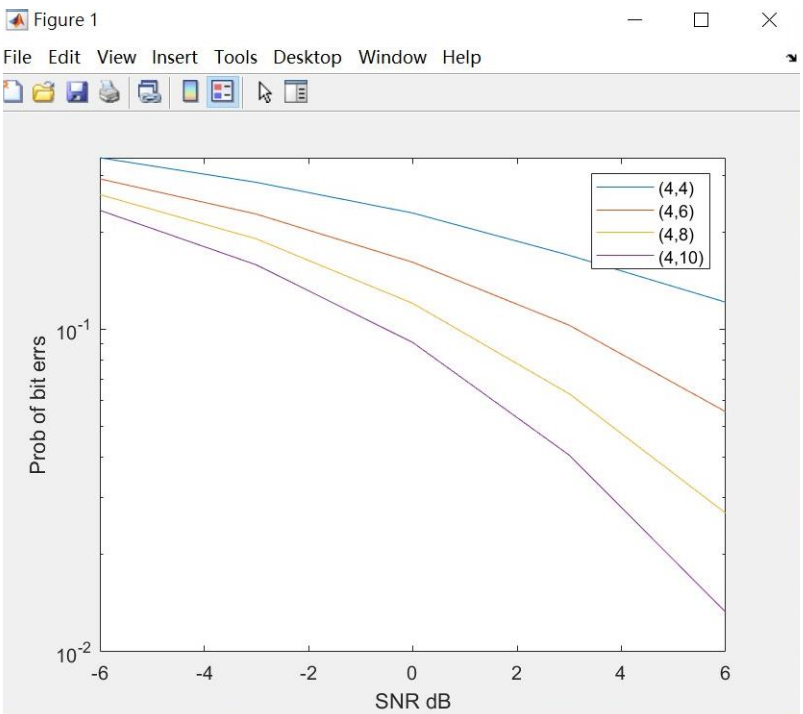
13.2 程式碼

```
% BER_MIMO_SVD.m 1 of 3
%BER for 4 x 2 MIMO systems (precoding thru SVD)
clear all;
SNR_dB=[-6 -3 0 3 6];
N=10^5;
Nt=4;
Nr=[4 6 8 10];
Count_BER_2bits=zeros(length(Nr),length(SNR_dB));
Count_BER_1bit=zeros(1,length(SNR_dB));
for nnr=1:length(Nr)
for n=1:length(SNR_dB)
SNR=10^(SNR_dB(n)/10);
N0=8/SNR;
for t=1:N
%------------------------------------------
%generate channel
H=(randn(Nr(nnr),Nt)+1j*randn(Nr(nnr),Nt))*sqrt(1/2); % MIMO channel,Nr x Nt
[U S V]=svd(H);
%-----------------------------------
%generate tx & rx signal -- 2 bits
b_2bits=floor(rand(2,1)*2); % one-bit data
x_2bits=(-1).^b_2bits; % {0 1}-->{1 -1}
s_2bits=V*[x_2bits;0;0];
w_2bits=(randn(Nr(nnr),1)+1j*randn(Nr(nnr),1))*sqrt(N0/2); % AWGN
y_2bits=H*s_2bits+w_2bits; % Nr天線接收訊號
%-----------------------------------
%generate tx & rx signal -- 1 bits
b_1bit=floor(rand(1)*2); % one-bit data
x_1bit=(-1).^b_1bit; % {0 1}-->{1 -1}
s_1bit=V*[x_1bit;x_1bit;0;0];
w_1bit=(randn(Nr(nnr),1)+1j*randn(Nr(nnr),1))*sqrt(N0/2); % AWGN
% BER_MIMO_SVD.m 2 of 3
y_1bit=H*s_1bit+w_1bit; % Nr天線接收訊號
%-------------------------------------
%Detection -2bits
z_2bits=U'*y_2bits;
sigmasig=diag(S);
z_2bits=z_2bits(1:4,:);
r_2bits=z_2bits./sigmasig;
x_hat_2bits=sign(real(r_2bits));
b_hat_2bits=zeros(2,1);
for nb=1:2
if x_hat_2bits(nb)<0
b_hat_2bits(nb)=1;
end
end
%-------------------------------------
```

分析: 我們假設空時編碼有三根天線，四個時間的，我們可以從碼字

```
%Detection -1bit
% z_1bit=U'*y_1bit;
% z_mrc_1bit=sigmasig'*z_1bit;
% h_eff_1bit=sigmasig'*sigmasig;
% r_1bit=z_mrc_1bit/h_eff_1bit;
% x_hat_1bit=sign(real(r_1bit));
% b_hat_1bit=0;
% if x_hat_1bit<0
% b_hat_1bit=1;
% end
%-----------------------------------------
Count_BER_2bits(nnr,n)=Count_BER_2bits(nnr,n)+sum(abs(b_hat_2bits-b_2bits));
end
end
end
% BER_MIMO_SVD.m 3 of 3
BER_2bits=Count_BER_2bits./(2*N);
BER_1bit=Count_BER_1bit./(N);
semilogy(SNR_dB,BER_2bits,'b-.',SNR_dB,BER_1bit,'r-');
xlabel('SNR (dB)');
ylabel('Probability of bit errors');
legend('(4,4)','(4,6)','(4,8)','(4,10)');
```

模擬結果圖 13.2(OSTBC 錯誤率圖)



分析: 我們假設空時編碼有三根天線，四個時間的，我們可以從碼字

看出 SYMBOL 的關係

至於 Dm 我們可以用 tran(A1)*A1+A1*tran(A1)以此類推

若 X=1+j,則第二小題之能量不平均要除以 4

結論

5G 所使用的是多輸入多輸出系統（Multi-input Multi-output；MIMO）是一種用來描述多天線無線通訊系統的抽象數學模型，能利用發射端的多個天線各自獨立發送訊號，同時在接收端用多個天線接收並恢復原資訊 MIMO 此類多天線技術尚包含 SIMO 和 MIS,而這次的專題則是 MIMO 高端技術的基礎,MIMO 的技術可以利用多天線來抑制頻道衰落，也能增加系統的資料吞吐量及傳送距離，也能利用多根發射天線與多根接收天線所提供之空間自由度來有效提升無線通訊系統之頻譜效率，以提升傳輸速率並改善通訊品質,將對人類文明帶來很深遠的影響

致謝

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文獻資料

- 1.黃婉甄教授專題講義
- 2.Massive MIMO
- https://www.ni.com/zh-tw/innovations/white-papers/14/5g-massive-mimo-testbed--from-theory-to-reality--.html#section-1929677046
- 3.5G
- https://zh.wikipedia.org/zh-tw/5G
- 4.matlab MIMO
- https://ww2.mathworks.cn/help/comm/examples/introduction-to-mimo-systems.html
- 5.**MIMO-OFDM**
- http://wshnt.kuas.edu.tw/network/s1/MIMO-OFDM.htm