Propositional Logic - Problem sheet 2

Q1: Each proposition below can be simplified to one of the six propositions false, true, x, y, $x \wedge y$ and $x \vee y$. Simplify them using laws (i) - (xi)

(a) $x \vee (y \vee x) \vee \neg y$

(b) $(x \vee y) \wedge (x \vee \neg y)$

(c) $x \vee y \vee \neg x$

(d) $(x \vee y) \wedge (x \vee \neg y) \wedge (\neg x \vee y) \wedge (\neg x \vee \neg y)$

(e) $(x \wedge y) \vee (x \neg y) \wedge (\neg x \vee y) \wedge (\neg x \vee \neg y)$

(f) $(\neg x \wedge y) \vee x$

(g) $\neg x \Rightarrow (x \wedge y)$

(h) $\underline{true} \Rightarrow (\neg x \Rightarrow x)$

(i) $x \Rightarrow (y \Rightarrow (x \wedge y))$

(j) $\neg x \Rightarrow (\neg x \Rightarrow (\neg x \wedge y))$

(k) $\neg x \Rightarrow y$

(l) $\neg y \Rightarrow \neg y$

Q2: Using truth tables prove

(i) laws of associativity

(ii) laws of implication

(iii) laws of distribution

Q3: Using laws (i) - (xi) prove

(a) $p \Rightarrow p \wedge p$

(b) $[\, p \wedge (p \Rightarrow q)] \Rightarrow q$

(c) $[\, p \wedge (p \vee q) \,] \Rightarrow p \vee q$

(d) $[\, (p \Rightarrow q) \wedge \neg\, q \,] \Rightarrow \neg\, p$

(e) $[\, (p \vee q) \wedge \neg\, p \,] \Rightarrow q$

(f) $[\, (p \Rightarrow q) \Rightarrow q \,] \Rightarrow q$

(g) $[\, (p \Rightarrow q) \wedge (q \Rightarrow r)] \Rightarrow (p \Rightarrow r)$

(h) $[\, \neg\, (p \Rightarrow q) \wedge \neg\, (\neg\, p \Rightarrow (q \vee r)) \,] \Rightarrow (\neg\, q \Rightarrow r)$

Q4: Prove that the following are contradictions

(a) $(\neg\, p \wedge\ ) \wedge (q \Rightarrow p)$

(b) $(p \Rightarrow \neg\, p) \wedge (\neg\, p \Rightarrow p)$

Q5: Prove the following equivalences

(a) $p \vee (q \wedge p) \equiv p$

(b) $(p \vee q) \wedge q \equiv q$

(c) $[\, (p \wedge q) \vee (\neg\, p \wedge q) \vee (p \wedge \neg\, q) \,] \equiv p \vee q$

(d) $[\, (p \Rightarrow q) \wedge (p \Rightarrow \neg\, q) \,] \equiv \neg\, p$

(e) $[\, (p \Rightarrow \neg\, q) \wedge (p \Rightarrow \neg\, r) \,] \equiv \neg\, (p \wedge (q \vee r))$

(f) $[\, p \wedge q \equiv p \,] \equiv p \Rightarrow q$

(g) $p \wedge q \Rightarrow r \equiv p \Rightarrow (\neg\, q \vee r)$

(h) $p \Rightarrow (q \vee r) \equiv \neg\, q \Rightarrow (\neg\, p \vee r)$