INSTITUTE OF TECHNOLOGY

BLANCHARDSTOWN

Year	Year 2
Semester	Semester 1
Date of Examination	Thurs 17th Jan 2013
Time of Examination	12.30pm – 2.30pm

Prog Code	BN002	**Prog Title**	Higher Certificate in Science in Computing in Information Technology	**Module Code**	Comp H2026
Prog Code	BN013	**Prog Title**	Bachelor of Science in Computing in Information Technology	**Module Code**	Comp H2026
Prog Code	BN104	**Prog Title**	Bachelor of Science (Honours) in Computing	**Module Code**	Comp H2026

Module Title	Information Technology Mathematics

Internal Examiner(s): *Laura Keyes*
External Examiner(s): *Mr Michael Barrett, Dr Tom Lunney*

nstructions to candidates:

) To ensure that you take the correct examination, please check that the module and programme which you are following is listed in the tables above.
) Question One is COMPULSORY. Candidates should attempt Question One and ANY other <u>two</u> questions.
) This paper is worth 100 marks. Question One is worth 40 marks and all other questions are worth 30 marks each.

DO NOT TURN OVER THIS PAGE UNTIL YOU ARE TOLD TO DO SO

Question 1: **(40 marks)**

Attempt **ALL** eight parts.

a) Given the following matrices: $A = \begin{bmatrix} 1 & 2 \\ 4 & -5 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & -4 & 0 \\ 2 & 1 & 2 \end{bmatrix}$

i. What is the *rank* of the two matrices *A* and *B*?
ii. Write down the value of the following elements: a_{22}, and b_{13}
iii. Write down the *transpose* of B.

(5 marks)

b) *Translate* the 2D point $P = \begin{bmatrix} 2 \\ 4 \\ 1 \end{bmatrix}$ by a factor of 5 in the *x*-axis.

(5 marks)

c) What is a *Tree* structure? In your answer provide an example of a Tree representation to explain the following concepts: *root node*; *parent node*; *child node*; *leaf node*.

(5 marks)

d) Draw the graph with the following adjacency matrix: $\begin{bmatrix} 0 & 0 & 1 & 2 & 1 \\ 0 & 1 & 1 & 1 & 0 \\ 1 & 1 & 0 & 0 & 1 \\ 2 & 1 & 0 & 0 & 1 \\ 1 & 0 & 1 & 1 & 2 \end{bmatrix}$

(5 marks)

e) Using the data 15, 12, 11, 14, 15, 10 compute the
i. Mean
ii. Standard deviation

(5 marks)

f) If two dice are thrown, what is the probability that the **sum** is greater than 6?

(5 marks)

g) If two variables *x* and *y* are related by the equation $y = 4 - 6x$ calculate the average rate of change of *y* with respect to *x* as *x* varies from 2.5 to 3.

(5 marks)

h) Solve the following indefinite integrals:

i. $\int 2x^2 dx$

ii. $\int 6(x - x^2)dx$

(5 marks)

Question 2: Statistics and Probability **(30 marks)**

a) Evaluate the following probabilities:
   i. If a fair dice is rolled, what is the probability of getting a 5?
   ii. If two fair dice are rolled, what is the probability of getting a pair?

(4 marks)

b) There are six counties in a list. In how many ways can four counties be chosen:

   i. if the **order matters**?
   ii. if the **order doesn't matter**?

(6 marks)

c) Given that a valid user-code for a certain computer system consists of exactly 6 characters of the form *XY123A*. (Assume that there is no distinction made between lower and uppercase letters). In how many of these user-codes does the digit 0 occur at least once?

(6 marks)

d) Using the following frequency distribution:

Table 2.1

Number of Months	Frequency (No. of Students)
14 to 16	3
16 to 18	7
18 to 20	10
20 to 22	8

i. Draw a suitable *diagram* of the data given in Table 2.1 above *(4 marks)*
ii. Calculate the *mean* of the grouped data *(3 marks)*
iii. Calculate the *standard deviation* of the grouped data *(5 marks)*
iv. Comment on the *symmetry* or *skewness* of the distribution *(2 marks)*

Question 3: Matrices, Integration & Differentiation **(30 marks)**

a) Given the following matrices:

$$A = \begin{bmatrix} 1 & 3 & 2 \\ 4 & 5 & 3 \\ 2 & 2 & 1 \end{bmatrix} \quad B = \begin{bmatrix} 3 & -1 & 4 \\ -2 & 3 & 1 \\ 2 & 4 & -1 \end{bmatrix} \quad C = \begin{bmatrix} 2 \\ 4 \\ 3 \end{bmatrix}$$

$$D = \begin{bmatrix} 2 & 6 \\ 1 & 8 \end{bmatrix}$$

i. Evaluate $(B + 2A^T)C$ *(5 marks)*
ii. Find the *determinant* and *inverse* of the matrix D *(5 marks)*
iii. Find the *determinant* of matrix B *(6 marks)*

b) Use differentiation to:

i. Find the value of the *gradient* to the curve $y = 2x^2 + 4x - 6$ at the point where the x coordinate has a value of -2.
ii. Find any *local maximum* or *minimum* points on the curve.

(9 marks)

c) Find the *area* under the graph $y = 2x + 2x^2$ for x between 0 and 4.

(5 marks)

Question 4: Graphs and Trees **(30 marks)**

a) Find the number of paths of length 3 between *c* and *d* in the graph in Figure 4.1 below.

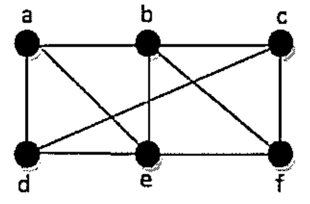

Figure 4.1

(4 marks)

b) For the weighted graph in Figure 4.2 below, use *Prim's algorithm* to find a *Minimal Spanning Tree* and give its *weight*.

(8 marks)

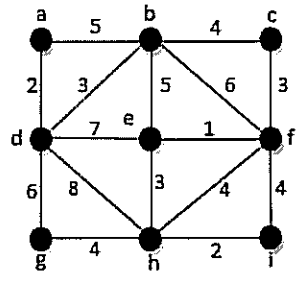

Figure 4.2

c) Identify which if any, of the following graphs below are *isomorphic*. Justify your answer either by finding an *isomorphism* between them or by showing that one has a graph theoretic property which the other does not have.

(8 marks)

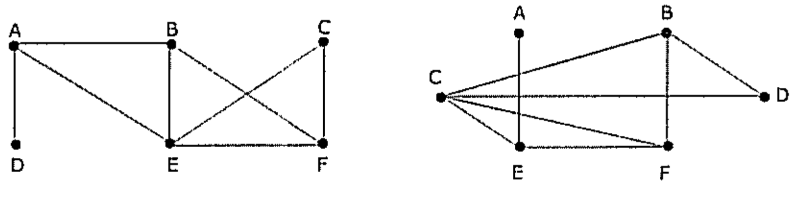

Figure 4.3

d) Perform the *pre-order* and *post-order* traversals of the binary tree in Figure 4.4 below outlining the algorithm used for each type of traversal.

(10 marks)

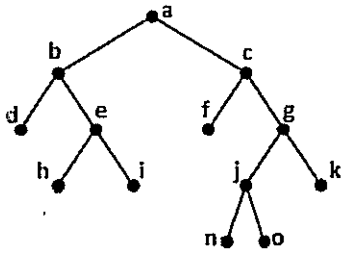

Figure 4.4 Binary Tree

Formulae

Determinants

$\det A = ad - bc$

$\det A = a_{11}c_{11} + a_{12}c_{12} + a_{13}c_{13} + \ldots \text{(using row 1)}$

Inverses

$$A^{-1} = \frac{1}{\det A}\begin{bmatrix} \mathbf{d} & \mathbf{-b} \\ \mathbf{-c} & \mathbf{a} \end{bmatrix}$$

Mean: $\bar{x} = \dfrac{\sum_{i=1}^{n} x_i}{n}$

Standard Deviation: $s^2 = \dfrac{\sum_{i=1}^{n}(x_i - \bar{x})^2}{n-1}$ or $s^2 = \dfrac{\sum_{i=1}^{n} x_i^2 - n(\bar{x})^2}{n-1}$

<u>**Grouped Data**</u>

Mean: $\bar{x} = \dfrac{\sum_i f_i m_i}{\sum_i f_i}$

Standard Deviation: $s^2 = \dfrac{\sum_i f_i (m_i - \bar{x})^2}{\sum_i f_i - 1}$ or $s^2 = \dfrac{\sum_{i=1}^{M} f_i m_i^2 - M(\bar{x})^2}{M-1}$

Derivation:
First Principles: $\dfrac{dy}{dx} = \lim_{h \to 0} \dfrac{f(x+h) - f(x)}{h}$ **Power Rule:** $\dfrac{dy}{dx} = nx^{n-1}$

Integral:

$$\int ax^n dx = \frac{ax^{n+1}}{n+1} + c, \quad n \neq -1$$