Laws of the Calculus

Let P, Q, R be propositions

1. Constants

   $P \vee true \equiv true$

   $P \vee false \equiv P$

   $P \wedge true \equiv P$

   $P \wedge false \equiv false$

   $true \Rightarrow P \equiv P$

   $false \Rightarrow P \equiv true$

   $P \Rightarrow true \equiv true$

   $P \Rightarrow false \equiv \neg P$

2. Law of excluded middle : $P \vee \neg P \equiv true$

3. Law of contradiction: $P \wedge \neg P \equiv false$

4 Negation : $\neg\neg P \equiv P$

5. Associativity: $P \vee (Q \vee R) \equiv (P \vee Q) \vee R$

   $P \wedge (Q \wedge R) \equiv (P \wedge Q) \wedge R$

6. Commutativity: $P \vee Q \equiv Q \vee P$

   $P \wedge Q \equiv Q \wedge P$

7. Idempotency: $P \vee P \equiv P$

   $P \wedge P \equiv P$

8. De Morgan's laws : $\neg (P \wedge Q) \equiv \neg P \vee \neg Q$

   $\neg (P \vee Q) \equiv \neg P \wedge \neg Q$

9. Implication $P \Rightarrow Q \equiv \neg P \vee Q$

   $P \Rightarrow Q \equiv \neg Q \Rightarrow \neg P$

   $(P \wedge Q) \Rightarrow R \equiv P \Rightarrow (Q \Rightarrow R)$

10. (If and only if) $\equiv$: $P \equiv Q \equiv (P \Rightarrow Q) \wedge (Q \Rightarrow P)$

11. Absorption: $[P \wedge (P \vee R) \equiv P]$

$[P \vee (P \wedge R) \equiv P]$

12. Laws of distribution: $P \wedge (Q \vee R) \equiv (P \wedge Q) \vee (P \wedge R)$

$$P \vee (Q \wedge R) \equiv (P \vee Q) \wedge (P \vee R)$$

13. **Predicate Calculus**

Universal Quantification

$[(\forall x : P(x)) \wedge (\forall x : Q(x)) \equiv (\forall x : P(x) \wedge Q(x))]$
$[(\forall x: P(x)) \vee (\forall x : Q(x)) \Rightarrow (\forall x: P(x) \vee Q(x))]$
$[Q \vee (\forall x : P(x)) \equiv (\forall x: Q \vee P(x))]$, where x not free in Q
$[Q \wedge (\forall x: P(x)) \equiv (\forall x: Q \wedge P(x))]$, where x not free in Q

Existential Quantification

$[(\exists x: P(x) \wedge Q(x)) \Rightarrow (\exists x : P(x)) \wedge (\exists x : Q(x))]$
$[(\exists x : P(x)) \vee (\exists x: Q(x)) \equiv (\exists x: P(x) \vee Q(x))]$
$[Q \vee (\exists x: P(x)) \equiv (\exists x : Q \vee P(x))]$, where x not free in Q
$[Q \wedge (\exists x: P(x)) \equiv (\exists x : Q \wedge P(x))]$, where x not free in Q
$[(\exists x : P(x)) \equiv \neg(\forall x: \neg P(x))]$
$[(\neg\exists x : P(x)) \equiv (\forall x: \neg P(x))]$

14**. Universal Quantification over Ranges**

$[\forall i : R : P \equiv \forall i : \neg R \vee P]$ Trading
$[\forall i : false : P \equiv true]$
$[\forall i : i = x : P \equiv P(i := x)]$ One-point rule
$[(\forall i : R : P) \wedge (\forall i : R : Q) \equiv (\forall i : R : P \wedge Q)]$
$[(\forall i : R : P) \wedge (\forall i : S : P) \equiv (\forall i : R \vee S : P)]$
$[(\forall i : R : P) \vee (\forall i : R : Q) \Rightarrow (\forall i : R : P \vee Q)]$
$[Q \vee (\forall i : R : P) \equiv (\forall i : R : Q \vee P)]$
$[Q \wedge (\forall i : R : P) \equiv (\forall i : R : Q \wedge P)]$

15. **Existential Quantification over Ranges**

$[\exists i : R : P \equiv \exists i : R \wedge P]$ Trading
$[\exists i : false : P \equiv false]$
$[\exists i : i = x : P \equiv P(i := x)]$ One-point rule
$[(\exists i : R : P \wedge Q) \Rightarrow (\exists i : R : P) \wedge (\exists i : R : Q)]$
$[(\exists i : R : P) \vee (\exists i : R : Q) \equiv (\exists i : R : P \vee Q)]$
$[Q \vee (\exists i : R : P) \equiv (\exists i : R : Q \vee P)]$
$[Q \wedge (\exists i : R : P) \equiv (\exists i : R : Q \wedge P)]$

16. **Argument Forms**

Modus Ponens:	$P \Rightarrow Q, P \therefore Q$
Modus Tollens:	$P \Rightarrow Q, \neg Q \therefore \neg P$
Hypothetical Syllogism:	$P \Rightarrow Q, Q \Rightarrow R \therefore P \Rightarrow R$
Disjunctive Syllogism:	$P \vee Q, \neg P \therefore Q$
Simplification:	$P \wedge Q \therefore P$
Conjunction:	$P, Q \therefore P \wedge Q$
Addition:	$P \therefore P \vee Q$
Constructive Dilemma:	$(P \Rightarrow Q) \wedge (R \Rightarrow S), P \vee R \therefore Q \vee S$
Destructive Dilemma:	$(P \Rightarrow Q) \wedge (R \Rightarrow S), \neg Q \vee \neg S \therefore \neg P \vee \neg R$