INSTITUTE OF TECHNOLOGY
BLANCHARDSTOWN

Year	Year 2
Semester	Semester 1
Date of Examination	Tuesday 17 January 2012
Time of Examination	3.30pm – 5.30pm

Prog Code	BN002	**Prog Title**	Higher Certificate in Science in Computing in Information Technology	**Module Code**	Comp H2026
Prog Code	BN013	**Prog Title**	Bachelor of Science in Computing in Information Technology	**Module Code**	Comp H2026
Prog Code	BN104	**Prog Title**	Bachelor of Science (Honours) in Computing in Information Technology	**Module Code**	Comp H2026

Module Title	Information Technology Mathematics

Internal Examiner(s): *Laura Keyes*
External Examiner(s): *Dr Richard Studdert,*

Instructions to candidates:

1) To ensure that you take the correct examination, please check that the module and programme which you are following is listed in the tables above.
2) Question One Section A is COMPULSORY. Candidates should attempt Question One and ANY other <u>two</u> questions in Section B.
3) This paper is worth 100 marks. Question One is worth 40 marks and all other questions are worth 30 marks each.

DO NOT TURN OVER THIS PAGE UNTIL YOU ARE TOLD TO DO SO

SECTION A: COMPULSORY QUESTION

Question 1: **(40 marks)**

Attempt **ALL** eight parts.

a) Given the following matrices: $A = \begin{bmatrix} 3 \\ 2 \\ -5 \end{bmatrix}$ and $B = \begin{bmatrix} 5 & 6 & 0 \\ -3 & 1 & 8 \end{bmatrix}$

i. What is the *rank* of the two matrices *A* and *B*?
ii. Write down the value of the following elements: a_{21}, and b_{23}
iii. Write down B^T

(5 marks)

b) Find a ***recursive*** and ***non-recursive*** definition for the following sequence of positive natural numbers.

2, 5, 8, 11, 14, 17

(5 marks)

c) Answer the following questions about the ***rooted*** tree in Figure 1.1 below.

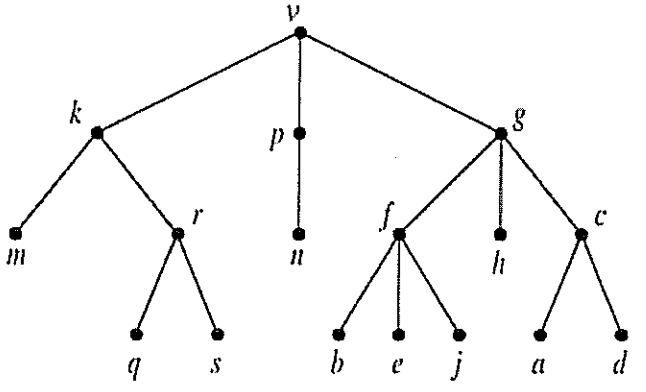

Figure 1.1 rooted tree

i. Which vertex is the *root*?
ii. Which vertices are *children* of *g*?
iii. Which vertex is the *parent* of *n*?
iv. Which vertices are the *leaves* of the tree?
v. Draw the *subtree* of the given tree that is rooted at *k*.

(5 marks)

d) Draw the graph *G* represented by the following adjacency matrix and write down the ***degree*** of each vertex in *G*.

$$\begin{pmatrix} 0 & 1 & 0 & 0 \\ 1 & 1 & 2 & 1 \\ 0 & 2 & 0 & 2 \\ 0 & 1 & 2 & 0 \end{pmatrix}$$

(5 marks)

e) Using the data 18, 10, 27, 6, 28, 11, 2, 26 compute the
   i. Mean
   ii. Standard deviation

(5 marks)

f) A small club lottery is based around picking 4 correct numbers out of 20. What are the probabilities of winning?

(5 marks)

g) If two variables *x* and *y* are related by the equation $y = 3x^2 - x$ calculate the average rate of change of *y* with respect to *x* as *x* varies from 2 to 4.

(5 marks)

h) Find the <u>*area*</u> under the graph $y = x + x^2$ for *x* between 0 and 2.

(5 marks)

SECTION B: Answer any TWO questions

Question 2: Statistics and Probability **(30 marks)**

a) Evaluate the following probabilities:
   i. If a fair dice is rolled, what is the probability of getting a 4?
   ii. If two fair dice are rolled, what is the probability of getting a pair?
   iii. If two fair dice are rolled, what is the probability that the sum of the numbers is below 5?

(4 marks)

b) A student taking a degree course in computing has to choose four from a possible six subjects: *Computer Networks*, *Network Design and Management*, *Web Mining*, *Intelligent Computing*, *Games Development*, *Applied Language Engineering.* How many different combinations of four subjects are there if

   i. the order of selection does not matter
   ii. the order of selection matters

(5 marks)

c) Describe the difference between:

   i. *Observational Data* and *Experimental Data* *(2 marks)*
   ii. *Random Sampling* and *Stratified Random Sampling* *(2 marks)*

d) The following is the distribution of the number of months a group of 22 students have owned their current mobile phone.

Table 2.1

Number of Months	Frequency (No. of Students)
$0 \leq x < 4$	1
$4 \leq x < 8$	2
$8 \leq x < 12$	1
$12 \leq x < 16$	3
$16 \leq x < 20$	4
$20 \leq x < 24$	4
$24 \leq x < 28$	3
$28 \leq x < 32$	1
$32 \leq x < 36$	2
$36 \leq x < 40$	1

   i. Draw a suitable *diagram* of the data given in Table 2.1 above *(4 marks)*
   ii. Calculate the *relative frequencies* of the above grouped data *(3 marks)*
   iii. Calculate the *mean* of the grouped data *(3 marks)*
   iv. Calculate the *standard deviation* s of the grouped data *(5 marks)*
   v. Comment on the *symmetry* or *skewness* of the distribution *(2 marks)*

Question 3: Matrices, Integration & Differentiation **(30 marks)**

a) Write down the *identity matrix* for a 2x2 and 3x3 matrix respectively.

(3 marks)

b) Evaluate $(B + 2A^T)C$ where

$$A = \begin{bmatrix} 1 & 3 & 2 \\ 4 & 5 & 3 \\ 2 & 2 & 1 \end{bmatrix}, \quad B = \begin{bmatrix} 3 & -1 & 4 \\ -2 & 3 & 1 \\ 2 & 4 & -1 \end{bmatrix}, \quad C = \begin{bmatrix} 2 \\ 4 \\ 3 \end{bmatrix}$$

(6 marks)

c) Find the *determinant* and *inverse* of the matrix $D = \begin{pmatrix} 2 & -4 \\ 1 & 4 \end{pmatrix}$

(5 marks)

d) Find the *determinant* of the 3x3 matrix $E = \begin{bmatrix} 4 & -2 & 1 \\ 2 & -2 & 3 \\ 7 & -1 & 6 \end{bmatrix}$

(6 marks)

e) Demonstrate how to *translate* and *scale* a 2D object by a factor of 3 in the *x* and *y*-axis.

(5 marks)

f) Show that the *tangent* to the curve $y = x^2 + 2x - 3$ at the point (-2, -3) is *parallel* to the line given by $2y + 4x - 3 = 0$.

(5 marks)

Question 4: Graphs and Trees **(30 marks)**

a) Explain with the aid of an example how computers in a local area network (LAN) might be modeled using graph theory.

(4 marks)

b) For the weighted graph in Figure 4.1 below, use *Prim's algorithm*, beginning with the indicated vertex *v*, to find a *Minimal Spanning Tree* and give its *weight*.

(8 marks)

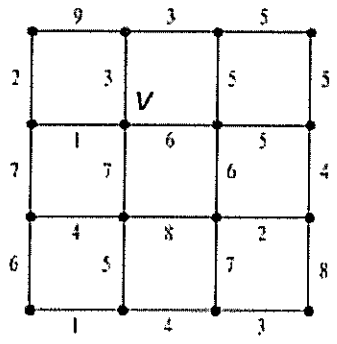

Figure 4.1

c) Are the following two graphs in Figure 4.2 below *isomorphic*? Justify your answer either by finding an *isomorphism* between them or by showing that one has a graph theoretic property which the other does not have.

(8 marks)

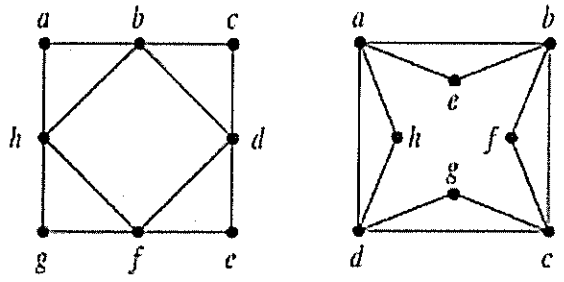

Figure 4.2

d) Perform the *pre-order* and *post-order* traversals of the binary tree in Figure 4.3 below outlining the algorithm used for each type of traversal.

(10 marks)

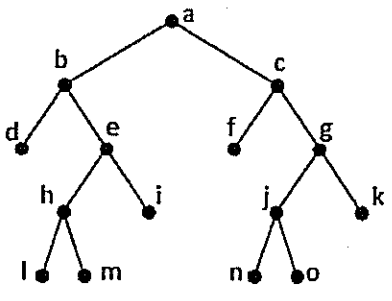

Figure 4.3 Binary Tree

Formulae

Determinants

$\det A = ad - bc$

$\det A = a_{11}c_{11} + a_{12}c_{12} + a_{13}c_{13} + \ldots(\text{using row } 1)$

Inverses

$$A^{-1} = \frac{1}{\det A}\begin{bmatrix} \mathbf{d} & \mathbf{-b} \\ \mathbf{-c} & \mathbf{a} \end{bmatrix}$$

Mean: $\bar{x} = \dfrac{\sum_{i=1}^{n} x_i}{n}$

Standard Deviation: $s^2 = \dfrac{\sum_{i=1}^{n}(x_i - \bar{x})^2}{n-1}$ or $s^2 = \dfrac{\sum_{i=1}^{n} x_i^{\,2} - n(\bar{x})^2}{n-1}$

Grouped Data

Mean: $\bar{x} = \dfrac{\sum_i f_i m_i}{\sum_i f_i}$

Standard Deviation: $s^2 = \dfrac{\sum_i f_i (m_i - \bar{x})^2}{\sum_i f_i - 1}$ or $s^2 = \dfrac{\sum_{i=1}^{M} f_i m_i^{\,2} - M(\bar{x})^2}{M-1}$

Derivation:
First Principles: $\dfrac{dy}{dx} = \lim_{h \to 0} \dfrac{f(x+h) - f(x)}{h}$ **Power Rule:** $\dfrac{dy}{dx} = nx^{n-1}$

Integral:

$$\int ax^n dx = \frac{ax^{n+1}}{n+1} + c, \quad n \neq -1$$