

Problem Sheet 3 : Predicate Calculus

Q1 : Let $U = \{2,4,6\}$ and let $P(x) \equiv x \bmod 2 = 0$
Evaluate $\forall x P(x)$, $\exists x P(x)$.

Q2 : Let $U = \{5,6,7,11\}$, $P(x) \equiv x < 10$
Evaluate $\forall x P(x)$, $\neg \exists x P(x)$

Q3 : Specify a Universe of Discourse for which the following propositions are true.
(Try to choose the Universe to be as large a subset of the integers as possible.)

- i) $\forall x [x > 0]$,
- ii) $\forall x [x = 3]$
- iii) $\exists y \forall x [x + y < 0]$

Q6 : Write down quantified predicates for

- i) x is a multiple of k
- iv) x is prime number

Q8 : Prove

- ii) $\neg \exists x \neg P(x) \equiv \forall x P(x)$

Q9: Let $A[0..N]$, $N \geq 1$ be an integer array.

Write down formal assertions for each of the following:

- i) All elements of A are in the range $1..100$
- iii) $A[j..k]$ contains an even integer value
- v) Max is the largest value in A
- viii) j = index of smallest element in $A[i..N]$