Propositional Logic - Problem Sheet 1

Q1 : If A = B = **true** and x = y = **false** evaluate the truth or falsity of each of the following

(i) $\neg (A \vee X)$

(ii) $\neg A \vee \neg X$

(iii) $A \vee (X \wedge Y)$

(iv) $\neg (A \vee X) \wedge \neg (A \vee Y)$

(v) $(A \Rightarrow B) \Rightarrow Y$

(vi) $(A \Rightarrow X) \Rightarrow [(\neg A) \wedge X]$

(vii) $Y \Rightarrow B \Rightarrow [\neg Y \vee B]$

(viii) $(X \Rightarrow Y) \equiv \neg (X \vee Y)$

(ix) $(A \Rightarrow B) \Rightarrow (\neg A \Rightarrow \neg B)$

(x) $(X \Rightarrow A) \Rightarrow (\neg X \Rightarrow \neg A)$

(xi) $[(A \wedge X) \Rightarrow Y] \Rightarrow (A \Rightarrow Y)$

(xii) $[(X \wedge Y) \Rightarrow A] \Rightarrow [X \Rightarrow (Y \Rightarrow A)]$

(xiii) $[(X \Rightarrow Y) \Rightarrow X] \Rightarrow X$

Q2: Use truth tables to prove the following tautologies

(i) $p \wedge q \Rightarrow p$

(ii) $(\neg p \Rightarrow (p \wedge q)) \equiv p$

(iii) $p \wedge (p \Rightarrow q) \Rightarrow q$

(iv) $(p \wedge q) \vee (\neg p \wedge r) \Rightarrow q \vee r$

(v) $(p \vee q) \wedge (\neg p \vee r) \Rightarrow q \vee r$

(vi) $p \wedge (p \vee q) \Rightarrow p \vee q$

Q3: Use truth tables to characterize the following statement forms as tautologies, contradictory, or contingent

(i) $p \Rightarrow \neg p$

(ii) $(p \Rightarrow \neg p) \wedge (\neg p \Rightarrow p)$

(iii) $(p \Rightarrow p) \Rightarrow p$

(iv) $(p \wedge q) \Rightarrow p$

(v) $(\neg p \wedge q) \wedge (q \Rightarrow p)$

(vi) $[p \wedge q \Rightarrow r] \equiv [(\neg p \wedge \neg q) \vee r]$

Note : A contingent proposition is one which is neither a tautology or a contradiction, e.g. p, $p \wedge q$, $p \vee q$, etc... .

Q4 : Use truth tables to prove the validity or invalidity of each of the following arguments

(i) if $x = 0$ then either $y > 0$ or $z < 0$. Therefore, if $y > 0$ then $x = 0$

(X: $x = 0$, Y: $y > 0$, Z: $z < 0$)

(ii) if $x = 0$ then if $y > 0$ then $z < 0$.$y > 0$.Therefore either $x = 0$ or $z < 0$.

(Use same abbreviations as for (i))

(iii) if the initialisation is correct and if the loop terminates then the required postcondition is guaranteed. The required postcondition is guaranteed. Therefore, if the initialisation is correct the loop terminates (I : the initialisation is correct, T : the loop terminates , P : the required postcondition is guaranteed .)