Derivation of Algorithms Lab2 Solutions

Problem sheet 2

Question 1

a)
$X \vee (Y \vee X) \vee \neg Y$
$\equiv$ {Associativity}
$X \vee Y \vee X \vee \neg Y$
$\equiv$ {Commutativity}
$X \vee X \vee Y \vee \neg Y$
$\equiv$ {Excluded middle}
$X \vee X \vee TRUE$
$\equiv$ {Constants}
TRUE

b)
$(X \vee Y) \wedge (X \vee \neg Y)$
$\equiv$ {Distribution}
$X \vee (Y \wedge \neg Y)$
$\equiv$ {Contradiction}
$X \vee FALSE$
$\equiv$ {Constants}
X

c)
$X \vee Y \vee \neg X$
$\equiv$ {Associativity}
$X \vee \neg X \vee Y$
$\equiv$ {Excluded middle}
$TRUE \vee Y$
$\equiv$ {Constants}
TRUE

d)
$(X \vee Y) \wedge (X \vee \neg Y) \wedge (\neg X \vee Y) \wedge (\neg X \vee \neg Y)$
$\equiv$ {Distribution}
$X \vee (Y \wedge \neg Y) \wedge (\neg X \vee Y) \wedge (\neg X \vee \neg Y)$
$\equiv$ {Distribution}
$X \vee (Y \wedge \neg Y) \wedge \neg X \vee (Y \vee \neg Y)$
$\equiv$ {Contradiction}
$X \vee FALSE \wedge \neg X \vee FALSE$
$\equiv$ {Constants}
$X \wedge \neg X$
$\equiv$ {Contradiction}
FALSE

e) $(X \wedge Y) \vee (X \wedge \neg Y) \wedge (\neg X \vee Y) \wedge (\neg X \vee \neg Y)$
$\equiv$ {Distribution x 2}
$X \wedge (Y \vee \neg Y) \wedge \neg X \vee (Y \wedge \neg Y)$
$\equiv$ {Excluded middle + contradiction}
$X \wedge TRUE \wedge \neg X \vee FALSE$
$\equiv$ {Constants}
$X \wedge \neg X$
$\equiv$ {Contradiction}
FALSE

f) $(\neg X \wedge Y) \vee X$
$\equiv$ {Commutativity, not really necessary}
$X \vee (\neg X \wedge Y)$
$\equiv$ {Distribution}
$(X \vee \neg X) \wedge (X \vee Y)$
$\equiv$ {Excluded middle}
$TRUE \wedge (X \vee Y)$
$\equiv$ {Constants}
X ∨ Y

g) $\neg X \Rightarrow (X \wedge Y)$
$\equiv$ {$\Rightarrow$}
$\neg \neg X \vee (X \wedge Y)$
$\equiv$ {$\neg \neg$}
$X \vee (X \wedge Y)$
$\equiv$ {Absorption}
X

h) $TRUE \Rightarrow (\neg X \Rightarrow X)$
$\equiv$ {$\Rightarrow$}
$\neg TRUE \vee (\neg X \Rightarrow X)$
$\equiv$ {Constants}
$FALSE \vee (\neg X \Rightarrow X)$
$\equiv$ {$\Rightarrow$}
$FALSE \vee \neg \neg X \vee X$
$\equiv$ {$\neg \neg$}
$FALSE \vee X \vee X$
$\equiv$ {Constants}
X

i) $X \Rightarrow (Y \Rightarrow (X \wedge Y))$
$\equiv$ {$\Rightarrow$ X 2}

$\neg X \vee (\neg Y \vee (X \wedge Y))$
$\equiv$ {Distribution}
$\neg X \vee ((\neg Y \vee X) \wedge (\neg Y \vee Y))$
$\equiv$ {Excluded middle}
$\neg X \vee ((\neg Y \vee X) \wedge TRUE)$
$\equiv$ {Constants}
$\neg X \vee (\neg Y \vee X)$
$\equiv$ {Associativity + Commutativity}
$\neg X \vee X \vee \neg Y$
$\equiv$ {Excluded middle}
$TRUE \vee \neg Y$
$\equiv$ {Constants}
TRUE

j) $\neg X \Rightarrow (\neg X \Rightarrow (\neg X \wedge Y))$
$\equiv$ {$\Rightarrow$ X 2}
$\neg\neg X \vee (\neg\neg X \vee (\neg X \wedge Y))$
$\equiv$ {$\neg\neg$}
$X \vee (X \vee (\neg X \wedge Y))$
$\equiv$ {Distribution}
$X \vee ((X \vee \neg X) \wedge (X \vee Y))$
$\equiv$ {Excluded middle}
$X \vee (TRUE \wedge (X \vee Y))$
$\equiv$ {Constants}
$X \vee (X \vee Y)$
$\equiv$ {Associativity}
$X \vee X \vee Y$
$\equiv$ {Constants}
X ∨ Y

k) $\neg X \Rightarrow Y$
$\equiv$ {$\Rightarrow$}
$\neg\neg X \vee Y$
$\equiv$ {$\neg\neg$}
X ∨ Y

l) $\neg Y \Rightarrow \neg Y$
$\equiv$ {$\Rightarrow$}
$\neg\neg Y \vee Y$
$\equiv$ {$\neg\neg$}
$Y \vee Y$
$\equiv$ {Constants}
Y

Question 3

a) $P \Rightarrow P \wedge P$
$\equiv \{\Rightarrow\}$
$\neg P \vee P \wedge P$
$\equiv$ {Constants}
$\neg P \vee P$
$\equiv$ {Excluded middle}
TRUE

b) $[P \wedge (P \Rightarrow Q)] \Rightarrow Q$
$\equiv \{\Rightarrow\}$
$[P \wedge (\neg P \vee Q)] \Rightarrow Q$
$\equiv$ {Distribution}
$[(P \wedge \neg P) \vee (P \wedge Q)] \Rightarrow Q$
$\equiv$ {Contradiction}
$[FALSE \vee (P \wedge Q)] \Rightarrow Q$
$\equiv$ {Constants}
$(P \wedge Q) \Rightarrow Q$
$\equiv \{\Rightarrow\}$
$\neg(P \wedge Q) \vee Q$
$\equiv$ {De-morgan}
$\neg P \vee \neg Q \vee Q$
$\equiv$ {Excluded middle}
$\neg P \vee TRUE$
$\equiv$ {Constants}
TRUE

c) $[P \wedge (P \wedge Q)] \Rightarrow P \vee Q$
$\equiv \{\Rightarrow\}$
$\neg[P \wedge (P \wedge Q)] \vee P \vee Q$
$\equiv$ {De-morgan}
$\neg P \vee \neg(P \wedge Q) \vee P \vee Q$
$\equiv$ {Commutativity}
$\neg P \vee P \vee \neg(P \wedge Q) \vee Q$
$\equiv$ {Excluded middle}
$TRUE \vee \neg(P \wedge Q) \vee Q$
$\equiv$ {Constants}
TRUE

d) $[(P \Rightarrow Q) \wedge \neg Q] \Rightarrow \neg P$
$\equiv \{\Rightarrow\}$
$[(\neg P \vee Q) \wedge \neg Q] \Rightarrow \neg P$
$\equiv$ {Distribution}

$[(\neg Q \wedge \neg P) \vee (\neg Q \wedge Q)] \Rightarrow \neg P$
$\equiv$ {Contradiction}
$[(\neg Q \wedge \neg P) \vee FALSE] \Rightarrow \neg P$
$\equiv$ {Constants}
$(\neg Q \wedge \neg P) \Rightarrow \neg P$
$\equiv$ {$\Rightarrow$}
$\neg(\neg Q \wedge \neg P) \vee \neg P$
$\equiv$ {De-morgan}
$Q \vee P \vee \neg P$
$\equiv$ {Excluded middle}
$Q \vee TRUE$
$\equiv$ {Constants}
TRUE

e) NOT GIVEN

f) $[(P \Rightarrow Q) \Rightarrow Q] \Rightarrow P$
$\equiv$ {$\Rightarrow$ X 2}
$[\neg(\neg P \vee Q) \vee Q] \Rightarrow P$
$\equiv$ {De-morgan}
$[P \wedge \neg Q \vee Q] \Rightarrow P$
$\equiv$ {Excluded middle}
$(P \wedge TRUE) \Rightarrow P$
$\equiv$ {Constants}
$P \Rightarrow P$
$\equiv$ {$\Rightarrow$}
$\neg P \vee P$
$\equiv$ {Excluded middle}
TRUE

g) $[(P \Rightarrow Q) \wedge (Q \Rightarrow R)] \Rightarrow (P \Rightarrow R)$
$\equiv$ {$\Rightarrow$ X 4}
$\neg[(\neg P \vee Q) \wedge (\neg Q \vee R)] \vee \neg P \vee R$
$\equiv$ { De-morgan}
$\neg(\neg P \vee Q) \vee \neg(\neg Q \vee R) \vee \neg P \vee R$
$\equiv$ {Commutativity}
$\neg P \vee \neg(\neg P \vee Q) \vee \neg(\neg Q \vee R) \vee R$
$\equiv$ {De-morgan x 2}
$\neg P \vee P \wedge \neg Q \vee Q \wedge \neg R \vee R$
$\equiv$ {Excluded middle x 3}
$TRUE \wedge TRUE \wedge TRUE$
$\equiv$ {Constants}
TRUE

h) $[\neg(P \Rightarrow Q) \wedge \neg(\neg P \Rightarrow (Q \vee R))] \Rightarrow (\neg Q \Rightarrow R)$
$\equiv$ {$\Rightarrow$ X 4}
$\neg[\neg(\neg P \vee Q) \wedge \neg(P \vee (Q \vee R))] \vee (Q \vee R)$
$\equiv$ {De-morgan}
$\neg\neg(\neg P \vee Q) \vee \neg\neg(P \vee (Q \vee R) \vee (Q \vee R)$
$\equiv$ {$\neg\neg$}
$(\neg P \vee Q) \vee (P \vee (Q \vee R) \vee (Q \vee R)$
$\equiv$ {Associativity + Commutativity}
$\neg P \vee P \vee Q \vee Q \vee Q \vee R \vee R$
$\equiv$ {Excluded middle + Constants}
TRUE $\vee$ Q $\vee$ R
$\equiv$ {Constants}
TRUE

Question 5

a) NOT GIVEN

b) $(P \vee Q) \wedge Q \equiv Q$
$\equiv$ {Absorption}
Q

c) $[(P \wedge Q) \vee (\neg P \wedge Q) \vee (P \wedge \neg Q)] \equiv P \vee Q$
$\equiv$ {Associativity}
$[(P \wedge Q) \vee (P \wedge \neg Q) \vee (\neg P \wedge Q)]$
$\equiv$ {Distribution}
$P \wedge (Q \vee \neg Q) \vee (\neg P \wedge Q)$
$\equiv$ {Excluded middle}
$P \wedge$ TRUE $\vee (\neg P \wedge Q)$
$\equiv$ {Constants}
$P \vee (\neg P \wedge Q)$
$\equiv$ {Distribution}
$(P \vee \neg P) \wedge (P \vee Q)$
$\equiv$ {Excluded middle}
TRUE $\wedge (P \vee Q)$
$\equiv$ {Constants}
P $\vee$ Q

d) NOT GIVEN
e) NOT GIVEN
f) NOT GIVEN

g) $P \wedge Q \Rightarrow R \equiv P \Rightarrow (\neg Q \vee R)$
$\equiv \{\Rightarrow\}$
$\neg(P \wedge Q) \vee R$
$\equiv$ {De-morgan}
$\neg P \vee \neg Q \vee R$
$\equiv \{\Rightarrow\}$
$P \Rightarrow (\neg Q \vee R)$

h) $P \Rightarrow (Q \vee R) \equiv \neg Q \Rightarrow (\neg P \vee R)$
$\equiv \{\Rightarrow\}$
$\neg P \vee (Q \vee R)$
$\equiv$ {Commutativity/Associativity}
$Q \vee (\neg P \vee R)$
$\equiv \{\Rightarrow\}$
$\neg Q \Rightarrow (\neg P \vee R)$