INSTITUTE OF TECHNOLOGY
BLANCHARDSTOWN

Institute of Technology
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Baile Bhlainséir

HIGHER CERTIFICATE IN SCIENCE IN COMPUTING IN INFORMATION TECHNOLOGY
BN002 – YEAR 2

BACHELOR OF SCIENCE IN COMPUTING IN INFORMATION TECHNOLOGY
BN013 – YEAR 2

BACHELOR OF SCIENCE (HONOURS) IN COMPUTING
BN104 – YEAR 2

Information Technology Mathematics

COMP H2026

Semester 1
REPEAT

Internal Examiner(s): Mr. Markus Hofmann

External Examiner(s): Mr. John Dunnion
Dr. Richard Studdert

Tuesday 19th August 2008
1.00pm – 3.00pm

Instructions to candidates:

1) Question One Section A is COMPULSORY. Candidates should attempt Question One and ANY other two questions in Section B
2) This paper is worth 100 marks. Question One is worth 40 marks and all other questions are worth 30 marks each.

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Question 1: This question is compulsory **(40 marks)**
Answer **ALL** eight parts.

a) Evaluate (A + C) B where

$$A=\begin{bmatrix} 2 & -1 & -7 \\ 6 & 0 & 2 \\ -3 & 0 & 4 \end{bmatrix} \quad B=\begin{bmatrix} 1 & -3 & 0 \\ 0 & 4 & -1 \\ -2 & -9 & -1 \end{bmatrix} \quad C=\begin{bmatrix} 2 & 1 & -1 \\ 4 & -4 & -3 \\ -7 & 2 & 2 \end{bmatrix}$$

(5 marks)

b) Using the matrices in part a) answer the following questions:

i) What are the *ranks* of matrices A and B^T?
ii) Write down the *elements* c_{23} and b_{12}.
iii) Write down C^T

(5 marks)

c) Define the terms *Hamiltonian Path* and *Euler Path*. Using **two** examples to outline the differences.

(5 marks)

d) Draw a *complete graph* with five vertices. Further determine the *degrees* of each vertex.

(5 marks)

e) Consider the following sequences of steps:

1. Input a non-negative integer *n*
2. *i=2*
3. While n is even and >10 do
   3.1 *n = n/2*
   3.2 *i=i+1*
4. Output i

i) What happens when 0 is input?
ii) What is the output when 72 is input? Using a table trace the values of each variable for each iteration.
iii) Giving reasons say whether this sequence is an algorithm.

(5 marks)

f) Consider the following data: 12, 54, 85, 22, 10, 56, 10, 55, 44, 23, 44, 44, 13
Compute the following:

i) Variance
ii) Median
iii) Mode

(*5 marks*)

g) If one card is to be drawn from a standard deck of cards (52 cards), find the probability that:

i) The card is *Hearts*
ii) The card is a *King of Spade*
iii) The card is a *5* or *10*

(*5 marks*)

h) Define and outline the differences between *Primary Data* and *Secondary Data*

(*5 marks*)

SECTION B: Answer any TWO questions

Question 2: Statistics & Probability **(30 marks)**

a) Calculate the following probabilities:

i. Most small club lotteries are based around picking 4 correct numbers out of 20. What are the probabilities of winning?

(4 marks)

ii. Consider two events A and B, where A is throwing a three with a fair dice, and B is drawing a Queen from a full deck of cards. Determine the probability of the occurrence of both events.

(4 marks)

iii. A box contains five black balls and twelve red balls. Determine the probability of randomly picking a black ball from the box followed by a red ball.

(4 marks)

b) Consider the following case of 96 football players being surveyed. Their weight in stone was recorded as follows:

Weight	Observations
10-11	1
11-12	19
12-13	45
13-14	23
14-15	4
15-16	3
16-17	1

i) Draw a *Histogram* of the grouped data

(3 marks)

ii) Calculate the *Relative Frequencies* of the above grouped data

(3 marks)

iii) Calculate the *Mean* of the grouped data

(3 marks)

vi) Calculate the *Standard Deviation* of the grouped data

(6 marks)

v) Comment on the *two statistical measures* that were calculated in iii) and vi)

(3 marks)

Question 3: Matrices **(30 marks)**

Use the following matrices:

$$A=\begin{pmatrix}4 & -2 & 1\\ 2 & -2 & 3\\ 7 & -1 & 6\end{pmatrix} \quad B=\begin{pmatrix}-1 & 3 & -6\\ 1 & 2 & 0\\ 0 & 3 & -2\end{pmatrix} \quad C=\begin{bmatrix}3 & -1 & 2\end{bmatrix}$$

a) Calculate BB^T

(4 marks)

b) Calculate C(A-B).

(6 marks)

c) Multiply the following two matrices in the order they are given.

$$D=\begin{pmatrix}-5 & 4\\ -1 & 6\end{pmatrix} \quad E=\begin{pmatrix}-2\\ 4\end{pmatrix}$$

(3 marks)

d) Find the *determinant* and the *inverse* of matrix C below:

$$C=\begin{pmatrix}3 & -5\\ 3 & -4\end{pmatrix}$$

(5 marks)

e) Scale the x-coordinate of the 2D point (9, 5) by a factor of 3.

(4 marks)

f) Show how a 3D object matrix can be scaled to one third of its current size.

(4 marks)

g) Demonstrate how to move a point in a 3D environment by 5 points on the y axis.

(4 marks)

Question 4: Graphs, Trees & Recursion **(30 marks)**

a) Find a recursive and a non-recursive definition for the following sequence of positive even natural numbers:

2, 4, 6, 8, 10, 12

(6 marks)

b) Write a recursive method (e.g. int factorial(int n)) for finding the factorial of a number n, expressed mathematically as:

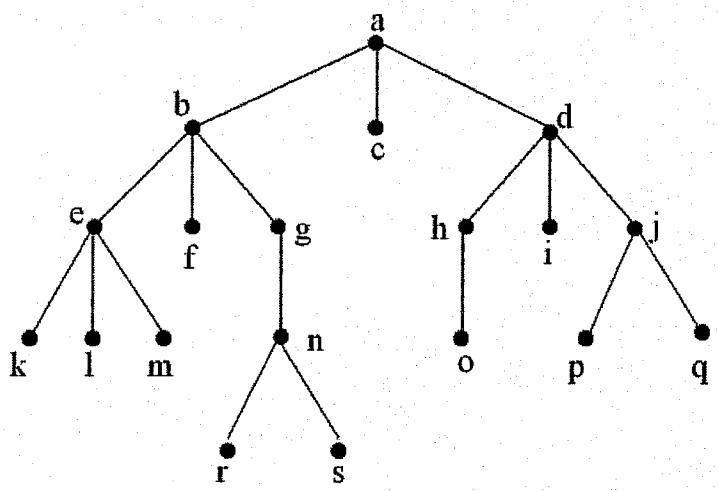
0! = 1
n! = n(n-1)! (n>0)

(5 marks)

c) Draw a directed graph with 3 vertices a, b, c and edges from c to a, c to b and a to b

(2 marks)

d) Answer the following questions about the rooted tree below



i. Which vertex is the root?

(1 marks)

ii. Which vertices are children of d?

(1 marks)

iii. Which vertex is parent of g?

(1 marks)

iv. Draw a subtree of the tree above that is rooted at d.

(2 marks)

v. Perform the in-order traversal of the ordered rooted tree of Question 3a).

(4 marks)

vi. Perform the post-order traversal of the ordered rooted tree of Question 3a).

(4 marks)

vii. Perform the pre-order traversal of the ordered rooted tree of Question 3a).
(4 marks)

Formulae

Quadratic Equation

$$ax^2 + bx + c = 0 \rightarrow x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Determinants

$$\det A = ad - bc$$

$$\det A = a_{11}c_{11} + a_{12}c_{12} + a_{13}c_{13} + \ldots \text{(using row 1)}$$

Inverses

$$A^{-1} = \frac{1}{\det A}\begin{bmatrix} \mathbf{d} & \mathbf{-b} \\ \mathbf{-c} & \mathbf{a} \end{bmatrix}$$

$$A^{-1} = \frac{1}{\det A} C^T$$

Mean

$$\bar{x} = \frac{\sum_{i=1}^{n} x_i}{n} \qquad \bar{x} = \frac{\sum_{i=1}^{M} f_i m_i}{M}$$

Standard Deviation s

$$s^2 = \frac{\sum_{i=1}^{n} \left(x_i - \bar{x}\right)^2}{n-1} \qquad s^2 = \frac{\sum_{i=1}^{M} f_i {m_i}^2 - M\left(\bar{x}\right)^2}{M-1}$$

Binomial Distribution

$$P(r \text{ successes in } n \text{ trials}) = {}^{n}C_r \; p^r \; q^{n-r} = \frac{n!}{(n-r)!\,r!} \; p^r \; q^{n-r}$$

Poisson Distribution

$$P[R = r] = \frac{e^{-\mu} \; \mu^r}{r!}$$

Normal Standard Variable

$$a = \frac{X - \mu}{\sigma}$$