

Derivation of Algorithms Lab1 Solutions

Problem sheet 1

Question 1

$A = B = \text{true}$ and $X = Y = \text{false}$

i)

$\neg (A \vee X)$

$\equiv \{\text{substitution}\}$

$\neg (\text{true} \vee \text{false})$

$\equiv \{\vee\}$

$\neg \text{true}$

$\equiv \{\neg\}$

false

iv)

$\neg(A \vee X) \wedge \neg(A \vee Y)$

$\equiv \{\text{substitution}\}$

$\neg(\text{true} \vee \text{false}) \wedge \neg(\text{true} \vee \text{false})$

$\equiv \{\vee\}$

$\neg \text{true} \wedge \neg \text{true}$

$\equiv \{\neg\}$

$\text{false} \wedge \text{false}$

$\equiv \{\text{Constants}\}$

false

vii)

$Y \Rightarrow B \Rightarrow [\neg Y \vee B]$

$\equiv \{\text{substitution}\}$

$\text{false} \Rightarrow \text{true} \Rightarrow [\neg \text{false} \vee \text{true}]$

$\equiv \{\neg\}$

$\text{false} \Rightarrow \text{true} \Rightarrow [\text{true} \vee \text{true}]$

$\equiv \{\vee\}$

$\text{false} \Rightarrow \text{true} \Rightarrow \text{true}$

$\equiv \{\Rightarrow\}$

$\text{true} \Rightarrow \text{true}$

$\equiv \{\Rightarrow\}$

true

xii)

$$[(X \wedge Y) \Rightarrow A] \Rightarrow [X \Rightarrow (Y \Rightarrow A)]$$

$\equiv \{\text{Substitution}\}$

$$[(\text{false} \wedge \text{false}) \Rightarrow \text{true}] \Rightarrow [\text{false} \Rightarrow (\text{false} \Rightarrow \text{true})]$$

$\equiv \{\wedge\}$

$$[\text{false} \Rightarrow \text{true}] \Rightarrow [\text{false} \Rightarrow (\text{false} \Rightarrow \text{true})]$$

$\equiv \{\Rightarrow\}$

$$[\text{false} \Rightarrow \text{true}] \Rightarrow [\text{false} \Rightarrow \text{true}]$$

$\equiv \{\Rightarrow \text{ twice}\}$

$$\text{true} \Rightarrow \text{true}$$

$\equiv \{\Rightarrow\}$

true

Question 2

Use truth table to prove the following tautologies

ii)

$$(\neg P \Rightarrow (P \wedge Q)) \equiv P$$

P	Q	$\neg P$	$(P \wedge Q)$	$\neg P \Rightarrow (P \wedge Q)$	$(\neg P \Rightarrow (P \wedge Q)) \equiv P$
T	T	F	T	T	T
T	F	F	F	T	T
F	T	T	F	F	T
F	F	T	F	F	T

v)

$$(P \vee Q) \wedge (\neg P \vee R) \Rightarrow Q \vee R$$

P	Q	R	$\neg P$	$(P \vee Q)$	$(\neg P \vee R)$	$(P \vee Q) \wedge (\neg P \vee R)$	$Q \vee R$	$(P \vee Q) \wedge (\neg P \vee R) \Rightarrow Q \vee R$
T	T	T	F	T	T	T	T	T
T	T	F	F	T	F	F	T	T
T	F	F	F	T	F	F	F	T
F	F	F	T	F	T	F	F	T
F	F	T	T	F	T	F	T	T
F	T	T	T	T	T	T	T	T
F	T	F	T	T	T	T	T	T
T	F	T	F	T	T	T	T	T

Question 3

Use truth table to characterise the following statement forms as tautologies, contradictions, or contingents.

i) $P \Rightarrow \neg P$

P	$\neg P$	$P \Rightarrow \neg P$
T	F	F
F	T	T

Therefore this is a contingent statement

v) $(\neg P \wedge Q) \wedge (Q \Rightarrow P)$

P	Q	$\neg P$	$(\neg P \wedge Q)$	$(Q \Rightarrow P)$	$(\neg P \wedge Q) \wedge (Q \Rightarrow P)$
T	T	F	F	T	F
T	F	F	F	T	F
F	T	T	T	F	F
F	F	T	F	T	F

Therefore this is a contradictory statement