Derivation of Algorithms Problem Sheet 3 Solutions

Question 1

Let U = {2,4,6} and let $P(x) \equiv x \bmod 2 = 0$

Evaluate $\forall x P(x)$ and $\exists x P(x)$

$\forall x P(x)$
$\equiv$ {substitution}
$2 \bmod 2 = 0 \wedge 4 \bmod 2 = 0 \wedge 6 \bmod 2 = 0$
$\equiv$ {arithmetic}
true $\wedge$ true $\wedge$ true
$\equiv$ {constants}
true

$\exists x P(x)$
$\equiv$ {substitution}
$2 \bmod 2 = 0 \vee 4 \bmod 2 = 0 \vee 6 \bmod 2 = 0$
$\equiv$ {arithmetic}
true $\vee$ true $\vee$ true
$\equiv$ {constants}
true

Question 2

Let U = {5,6,7,11} and let $P(x) \equiv x < 10$

Evaluate $\forall x P(x)$ and $\neg \exists x P(x)$

$\forall x P(x)$
$\equiv$ {substitution}
$5 < 10 \wedge 6 < 10 \wedge 7 < 10 \wedge 11 < 10$
$\equiv$ {arithmetic}
true $\wedge$ true $\wedge$ true $\wedge$ false
$\equiv$ {constants}
false

$\neg \exists x P(x)$
$\equiv$ {$\exists$ negation}
$\forall x \neg P(x)$ NOTE : $\neg P(x) \equiv x \geq 10$
$\equiv$ {substitution}
$5 \geq 10 \wedge 6 \geq 10 \wedge 7 \geq 10 \wedge 11 \geq 10$
$\equiv$ {arithmetic}
false $\wedge$ false $\wedge$ false $\wedge$ true

$\equiv$ {constants}
<u>false</u>

Question 3

Specify a universe of discourse for the following propositions

ii) $\forall x[x = 3]$, U = {3}

iii) $\exists y \forall x[x + y < 0]$, U = {-1, -2 , -3, -4, …}

Question 6

i) X is a multiple of k **$\exists x \forall k[x = k * t], t \in N$**

iv) X is a prime. **NOTE:** all numbers are evenly divisible by themselves and 1. Definition of prime number is one that is greater then 1 and has no other divisors other then itself and 1. Therefore:

$$\mathbf{(1 < x) \wedge \forall y[\exists z(y \bullet z = x) \Rightarrow (y = 1 \vee y = x)]}$$

OR

$$\mathbf{(1 < x) \wedge \forall y[(x \bmod y = 0) \Rightarrow (y = 1 \vee y = x)]}$$

Question 8

ii) $\neg\exists x \neg P(x) \equiv \forall x P(x)$
$\equiv$ {$\exists$ negation}
$\forall x \neg\neg P(x)$
$\equiv$ {$\neg\neg$}
$\forall x P(x)$

Question 9

i) $\forall x: 0 \le x < N : A.x \ge 1 \wedge A.x \le 100$

iii) $0 \le j < N \wedge 0 \le k < N \wedge \forall x: j \le x < k: A.x \bmod 2 = 0$

v) $\forall x: 0 \le x < N: A.x \le MAX$

viii) $0 \le i < N \wedge 0 \le j < N \wedge \forall x: i \le x < N: A.x \ge A.j$