INSTITUTE OF TECHNOLOGY BLANCHARDSTOWN

Year	Year 2
Semester	Semester 1
Date of Examination	Friday 16th January 2009
Time of Examination	9.30am – 11.30am

Prog Code	BN002	**Prog Title**	Higher Certificate in Science in Computing in Information Technology	**Module Code**	COMP H2026
Prog Code	BN013	**Prog Title**	Bachelor of Science in Computing in Information Technology	**Module Code**	COMP H2026
Prog Code	BN104	**Prog Title**	Bachelor of Science (Honours) in Computing	**Module Code**	COMP H2026

Module Title	Information Technology Mathematics

Internal Examiner(s): *Laura Keyes*
External Examiner(s): *Mr John Dunnion*
Dr Richard Studdert

Instructions to candidates:

1) **To ensure that you take the correct examination, please check that the module and programme which you are following is listed in the tables above.**
2) **Question One Section A is COMPULSORY. Candidates should attempt Question One and any two questions in Section B**
3) **This paper is worth 100 marks. Question One is worth 40 marks and all other questions are worth 30 marks each.**

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SECTION A: COMPULSORY QUESTION

Question 1: This question is compulsory
Answer all eight parts. Each part is worth 5 marks.

a) Given the matrices $A = \begin{bmatrix} 1 \\ 3 \\ -4 \end{bmatrix}$ and $B = \begin{bmatrix} 4 & 6 & 0 \\ -3 & 1 & 10 \end{bmatrix}$ answer the following:

i) What is the rank of matrix A, A^T and B
ii) Write down the elements a_{21} and b_{22}

b) Evaluate the determinant of $A = \begin{bmatrix} 1 & -1 & 2 \\ 3 & 1 & -5 \\ 1 & 1 & -2 \end{bmatrix}$ by expanding across the first row.

c) Outline the difference between Random and Stratified Random Sampling.

d) A sample of the temperatures for 7 days in June is as follows:

30 21 26 27 25 32 29

Compute the (show your work):
i) Mean
ii) Standard deviation
iii) Median

e) Evaluate the following:
i) If a dice is rolled, what is the probability of getting a 3?
ii) If two dice are rolled, what is the probability of getting a pair?
iii) If two dice are rolled, what is the probability that the sum of the numbers is below 5

f) Evaluate $t(2)$, $t(3)$, and $t(4)$ for the following recursively defined sequences:

$t(1) = 0$
$t(n) = 2t(n-1) + 1 \quad (n>1)$

g) Draw a directed graph with 3 vertices *a, b, c* and edges from *c* to *a, c* to *b* and a to *b*

h) What is a *Tree* structure? In your answer provide an example of a Tree representation to explain the following concepts: *root node*; *parent node*; *child node*; *leaf node.*

(40 marks)

SECTION B: ANSWER ANY TWO QUESTIONS

Question 2: Statistics and Probability

a) The following table represents a comparison of car sales in 2007 and 2008 for the months April to July. Construct a ***time series*** plot for data given. Comment on rise and fall of sales in this period between 2007 and 2008.

Month	*Car Sales2007*	*Car Sales2008*
April	21,427	17,587
May	14,251	6,512
June	15,399	7,907
July	13,150	14,210

(5 marks)

b) A survey of 298 college students was carried out to gather information on the attendance of students at the college social events in an academic year. The findings are represented in the table below. Using this data:

Attendance	Frequency (No. of Students)
< 10%	31
≥ 10% and < 20%	16
≥ 20% and < 30%	13
≥ 30% and < 40%	23
≥ 40% and < 50%	29
≥ 50% and < 60%	39
≥ 60% and < 70%	49
≥ 70% and < 80%	44
≥ 80% and < 90%	33
≥ 90%	21

i) Calculate the ***Relative Frequencies*** of the above *grouped* data

(2 marks)

ii) Calculate the ***Mean*** of the *grouped* data

(3 marks)

iii) Calculate the ***Standard Deviation*** of the *grouped* data

(5 marks)

iv) Draw a ***Histogram*** of the *grouped* data

(4 marks)

v) Comment on the symmetry or skewness of the distribution of the histogram from part iv)

(2 marks)

c) 5 people are asked to choose a number from 1 to 50. What is the probability that two or more people pick the same number?

(9 marks)

Question 3: Matrices

a) Write down the *identity* matrix for a 2x2 and a 3x3 matrix respectively.
(2 marks)

b) Given the matrices $A=\begin{bmatrix}2 & 1 & -2\\ 1 & 2 & 2\end{bmatrix}$, $B=\begin{bmatrix}0 & -1\\ 1 & 3\end{bmatrix}$, $C=\begin{bmatrix}-2 & 3\\ 1 & 1\end{bmatrix}$ and $D=\begin{bmatrix}1\\ -2\\ 3\end{bmatrix}$

find the following sum and products, or where appropriate, state that they are undefined.

i) $A+C$ (2 marks)
ii) AD (4 marks)
iii) $A^T(B+C)$ (5 marks)

c) Let $D=\begin{bmatrix}1 & 1 & 1\\ 2 & 3 & 1\\ 4 & 9 & 1\end{bmatrix}$. Calculate the *determinant* and *inverse* of the matrix D.
(9 marks)

d) Perform the following transformations:

i) Translate the *x*-coordinate of the 2D point (2, 5) by a factor of 2.
(4 marks)

ii) Produce a matrix that would scale a 3D object to *half* its current size.
(4 marks)

Question 4 Graphs, Trees and Recursion

a) Consider the following sequence of steps:

```
1. Input a positive integer n
2. answer = n
3. While n > 1 do
        3.1 n = n-1
        3.2 answer = answer * n
4. Output answer
```

i) What happens when 4 is input? (2 marks)

ii) Is this sequence of steps an algorithm? Give a reason for your answer. (3 marks)

b) Explain how computers in a Local Area Network (LAN) might be modelled using graph theory. (5 marks)

c) Describe **pre-order** traversal of a binary search tree. What is the output of a pre-order traversal of the following binary search tree? (8 marks)

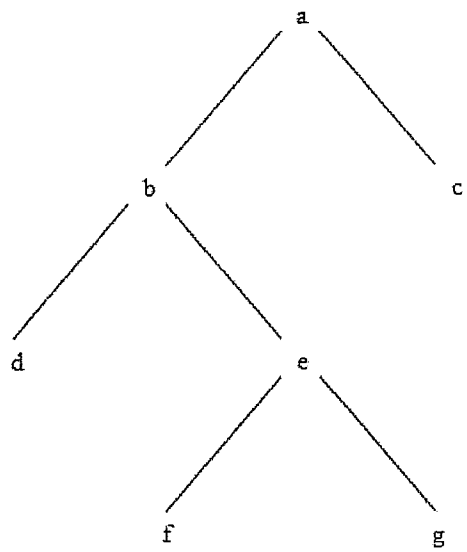

Figure 4.1 Binary Tree

d) Consider the graph G depicted below in figure 4.2: (12 marks)

i) Draw the corresponding adjacency matrix of G

ii) Verify that the sum of the degrees of the vertices equals twice the number of edges.

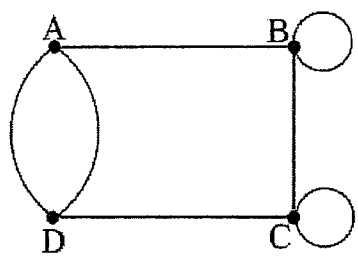

Figure 4.2 Graph *G*

Formulae

Determinants

$$\det A = ad - bc$$

$$\det A = a_{11}c_{11} + a_{12}c_{12} + a_{13}c_{13} + ...(\text{using row } 1)$$

Inverses

$$A^{-1} = \frac{1}{\det A}\begin{bmatrix} \mathbf{d} & \mathbf{-b} \\ \mathbf{-c} & \mathbf{a} \end{bmatrix}$$

$$A^{-1} = \frac{1}{\det A} C^T$$

Mean

$$\bar{x} = \frac{\sum_{i=1}^{n} x_i}{n} \qquad \bar{x} = \frac{\sum_{i=1}^{M} f_i m_i}{M}$$

Standard Deviation *s*

$$s^2 = \frac{\sum_{i=1}^{n} x_i^{\,2} - n(\bar{x})^2}{n-1} \quad \text{or} \quad s^2 = \frac{\sum_{i=1}^{n} (x_i - \bar{x})^2}{n-1}$$

$$s^2 = \frac{\sum_{i=1}^{M} f_i m_i^{\,2} - M(\bar{x})^2}{M-1}$$

Binomial Distribution

$$P(r \text{ successes in } n \text{ trials}) = {}^{n}C_r \; p^r \; q^{n-r} = \frac{n!}{(n-r)!\,r!} \; p^r \; q^{n-r}$$

Poisson Distribution

$$P[R = r] = \frac{e^{-\mu} \; \mu^r}{r!}$$

Normal Standard Variable

$$a = \frac{X - \mu}{\sigma}$$