A Comparative Study of Dynamic Linear Models and ARIMA & SARIMA Models for Renewable Energy Forecasting

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Objective

Energy Prediction via DLMs

- Choose a Dynamic Linear Model suitable for our data
- Find optimal parameters for the same
- Analyse forecast results
- Comparative study of DLMs with ARIMA and SARIMA

Why use Dynamic Linear Models?

Statistical analysis of time series data is usually faced with the problem that we have only one realization of a process whose properties we might not fully understand.

In linear trend analysis, we assume that there is an underlying change in the background that stays approximately constant over time.

In dynamic regression systems, by explicitly allowing for variability in the regression coefficients we let the system properties change in time. Also, unlike ARMA models, they can be applied to non-stationary data without transformation.

Furthermore, the use of unobservable state variables allows direct modelling of the processes that are driving the observed variability, such as seasonality or external forcing, and we can explicitly allow for some modelling error.

Ref: DLM Tutorial - MJ Laine

How do DLMs work? What is a Kalman Filter?

DLM (Gaussian Linear State Space Model)

State space models consider a time series as the output of a dynamic system perturbed by random disturbances. They allow a natural interpretation of a time series as combination of trend, seasonal or regressive components. In a state space model we assume that there is an unobservable Markov chain (x_t), called the *state process*, and that y_t is an *imprecise measurement* of x_t. A trivial DLM consists of two sets of equations:

- $y_t = F_t x_t + v_t$ (Observation Equation)
- $x_t = G_t x_{t-1} + w_t$ (Model Equation)

Here y_t represents the observation at time t, v_t and w_t are sequences of independent gaussian random errors (*observation error and evolution error*) and x_t corresponds to the unobserved state of the system having a *prior distribution* for $x_0 \sim N(m_0, C_0)$. F_t and G_t are the *observation* and *system matrices*. A linear regression model (with lagged values of observation as regression variable) would look like:

- $y_t = (y_{t-1} x_t) + v_t$ *where* $v_t \sim N(0, V_t)$
- $x_t = G_t x_{t-1} + w_t$ *where* $w_t \sim N(0, W_t)$

V_t and W_t are variance matrices.

Ref: DLM with R, Petris et al.

Kalman Filter

Model building can be a major difficulty: there might be no clear identification of physically interpretable states, or the state space representation could be non unique, or unsuitable choice of parameters could result in an inadequate model.

To estimate the state vector we compute the *conditional densities* $\pi(x_s|y_{1:t})$. We distinguish between problems of *filtering* (when $s = t$), *state prediction* ($s > t$) and *smoothing* ($s < t$).

In a DLM, the Kalman filter provides the formula for updating our current inference on the state vector as new data become available, that is, for passing from the *filtering density* $\pi(x_t|y_{1:t})$ to $\pi(x_{t+1}|y_{1:t+1})$.

It allows us to compute the *predictive* and *filtering distributions* recursively, starting from $x_0 \sim N(m_0, c_0)$ then computing $\pi(x_1|y_1)$, and proceeding recursively as new data becomes available. This is the usual Bayesian sequential updating, in which the *posterior at time t* takes the role of a *prior distribution* for what concerns the observations after time t.

*Posterior = (Likelihood*Prior)/Evidence*
Prior : Prob. of event without observation
Evidence : The observation
Likelihood : Prob. of observation given event
Posterior : Updated belief considering both prior & obs.

Ref: DLM with R, Petris et al.

How does the filter work?

Filtering

Taking the vector of observations $y_{1:t}$, the *filtering distribution* $\pi(x_t|y_{1:t})$ is computed recursively as

1. Start with $x_0 \sim N(m_0, C_0)$
2. One step forecast for the *state*:

$$x_t|y_{1:t-1} \sim N(a_t, R_t) \text{ where } a_t = G_t m_{t-1} \text{ and } R_t = (G_t C_{t-1} G'_t) + W_t$$

3. One step forecast for the *observation*:

$$y_t|y_{1:t-1} \sim N(f_t, Q_t) \text{ where } f_t = F_t a_t \text{ and } Q_t = (F_t R_{t-1} F'_t) + V_t$$

4. Compute the *posterior* at time t:

$$x_t|y_{1:t} \sim N(m_t, C_t) \text{ where } m_t = a_t + R_t f'_t Q_t^{-1}(y_t - f_t) \text{ and } C_t = R_t - (R_t F'_t Q_t^{-1} F_t R_t)$$

Ref: Lalas : KF

Smoothing and Forecasting

Smoothing deals with estimating the state sequence $x_{1:t}$ given data $y_{1:t}$. Backward recursive algorithm can be used to obtain $\pi(x_t|y_{1:T})$ for fixed T and $t = 0:T$

1. Start with $x_T|y_T \sim N(m_T, C_T)$ at time $t = T$
2. For t in T-2 to 0:

$$x_t|y_{1:T} \sim N(s_t, S_t)$$

where $s_t = m_t + C_t G'_{t+1} R^{-1}_{t+1}(s_{t+1} - a_{t+1})$

and

$$S_t = C_t - (C_t G'_{t+1} R^{-1}_{t+1}(R_{t+1} - S_{t+1}) R^{-1}_{t+1} G'_{t+1} C_t)$$

To calculate *forecast distributions*, for $k = 1, 2, \ldots$

1. Start with a sample from $x_T|y_T \sim N(m_T, C_T)$
2. Forecast the *state*:

$$x_{T+k}|y_{1:T} \sim N(a_k^T, R_k^T)$$

where $a_t^k = G_{T+k} a^T_{k-1}$

and $R_k^T = G_{T+k} R^T_{k-1} G'_{T+k} + W_{T+k}$

3. Forecast the *observation*:

$$y_{T+k}|y_{1:T} \sim N(f_k^T, Q_k^T)$$

where $f_t^k = F_{T+k} a^T_{k-1}$

and $Q_k^T = F_{T+k} R^T_{k-1} F'_{T+k} + V_{T+k}$

Ref: Lalas : KF

How do (S)ARIMA models work?

From AR to ARIMA, and SARIMA

- Auto-regressive models take into account previous data (the number of) values depending on the order p.
- Moving-average models express present value as a linear combination of the mean of the series, the present error term and the past error terms(q: order).
- An ARIMA model is the combination of the above two with an additional operation of differencing (d) for non-stationary series allowing it to become stationary.
- Seasonality in a time series is a regular pattern of changes that repeats over T time periods. A seasonal ARIMA model incorporates both seasonal and non-seasonal ARIMA models in a multiplicative fashion.

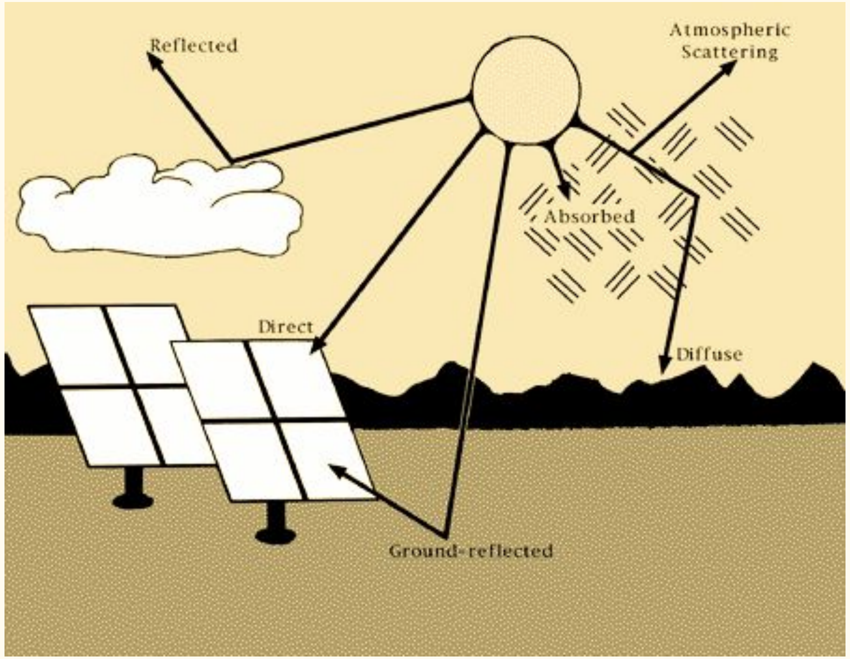
So what are we trying to estimate?

GHI of Bhadla Solar Park *[Clearsky GHI and Wind Speed]*

GHI *(Global Horizontal Irradiance)* refers to the amount of radiant power from sunlight received by a particular surface, perpendicular to the sun's rays.

It is useful when monitoring a solar power plant, finding optimal placement location of solar plants and for assessing solar power plant feasibility.

The higher the GHI value, the more power a system will produce. Its measurement unit is Watts per Sq Meter (W/m^2).



Source: What is GHI - Solar Powered Blog

Wind Speed

Data for multiple locations is considered in various forms – daily, weekly and monthly.

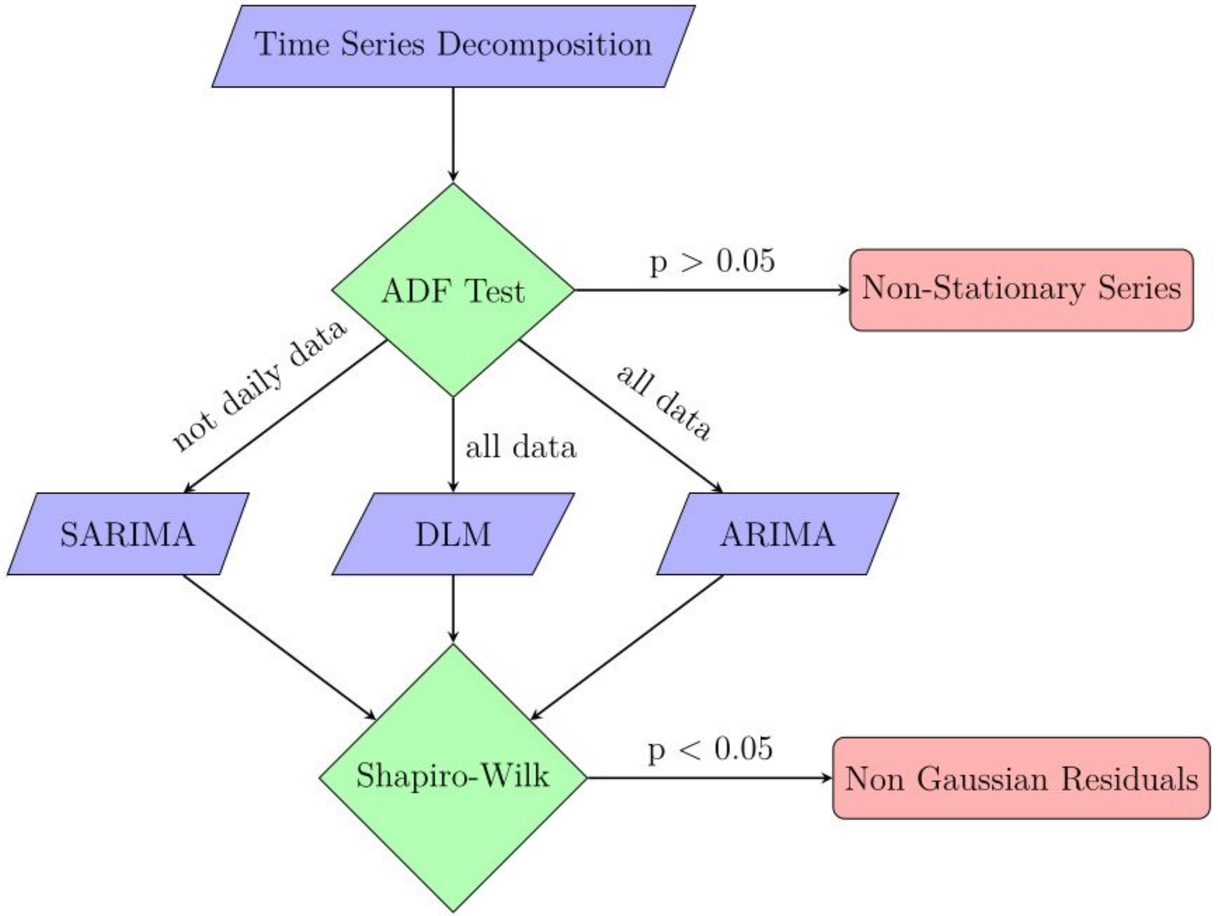
- Tiruvananthpuram
- Bhopal
- Hamirpur
- Jafrabad

The wind speed data for 10 years is used to analyse weekly and monthly data and 6 years for daily.

It is useful when monitoring a wind power plant, finding optimal placement location of windmills and for assessing their feasibility.

Implementation

[*Github : DLM*]



```
### TODO
# 1. Tuning parameters
# 2. Numerical analysis of prediction
# 3. Connect used functions with the math
###

# Read and extract needed data
df = pd.read_csv('Bhadla new.csv')
sort_cols = ["Year", "Month", "Day", "Hour"]
df.sort_values(by=sort_cols, inplace=True)
df = df[(df["Hour"]> 7) & (df["Hour"] < 17)]
df = df[["Year", "Month", "Day", "Hour", "GHI",
"Clearsky GHI"]]
ghi = df[["GHI"]]

# Split data 80 training - 20 testing
train = ghi.head(int(len(ghi)*0.8))
test = ghi.tail(int(len(ghi)*0.2))
regressor = ghi.tail(int(len(ghi)-1))
ghi = ghi.head(int(len(ghi)-1))
```

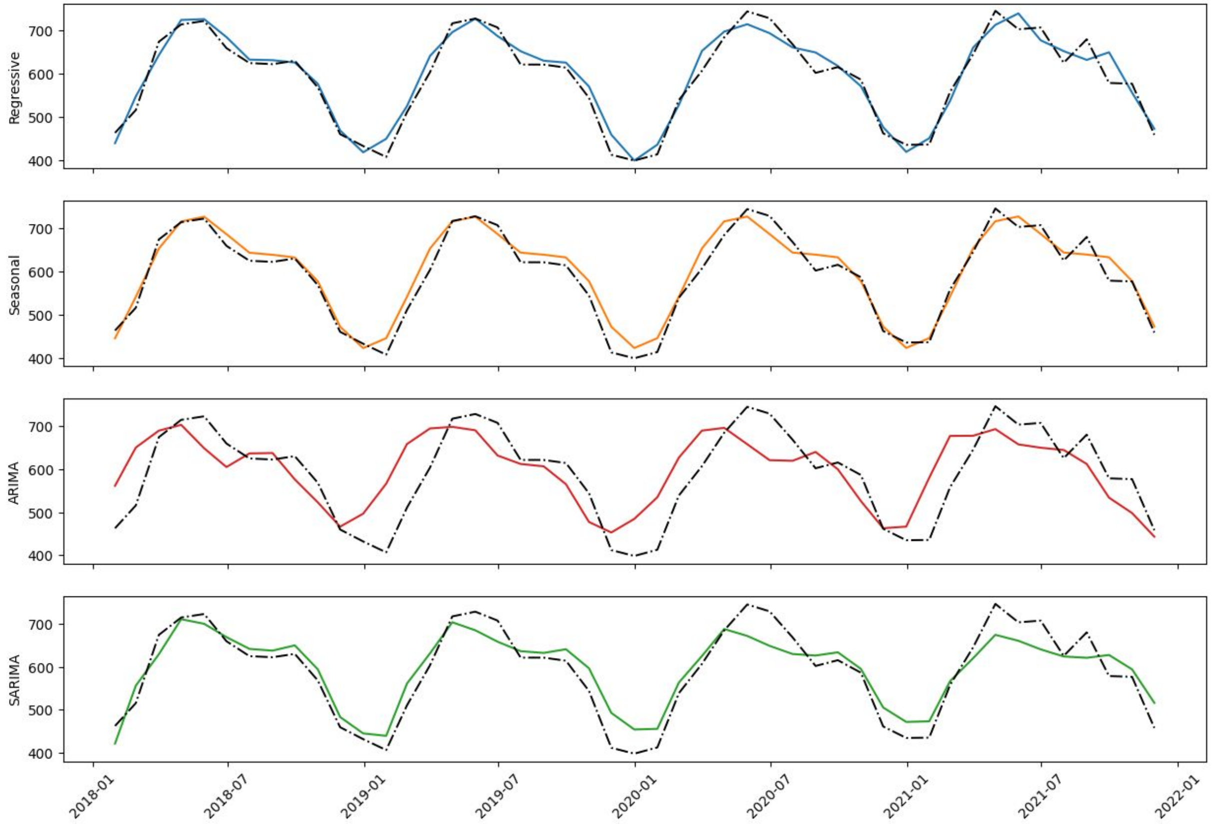
```
# TS Decomposition
from statsmodels.tsa.seasonal import seasonal_decompose
result = seasonal_decompose(ghi, model='additive',
period=12*30*9)
result.plot()
plt.show()

# DLM Design, Filtering and Smoothening
dyn = dynamic(features = regressor, discount = 1, name
= 'b', w=10)
longSeas = longSeason(period=12, data=ghi, name="Yearly
Seasonality") # stay=30*9
ghiDLM = dlm(ghi) + dyn + longSeas
ghiDLM.fit()
ghiDLM.plot()

# Prediction
ghiDLM.plotPredictN(N=len(test)-1, date = len(train)-1)
(predictMean, predictVar) =
ghiDLM.predictN(N=len(test)-1, date=len(train)-1)
ghiDLM.getMSE()
residual = ghiDLM.getResidual(filterType='predict')
plt.plot(residual[-len(test)+1:])
```

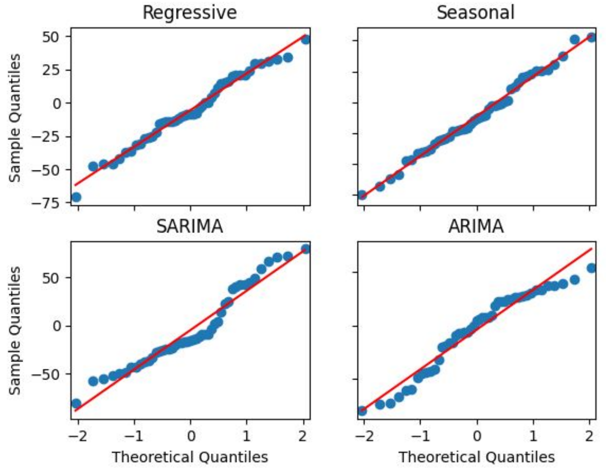
What did we get?

Bhadla - Monthly

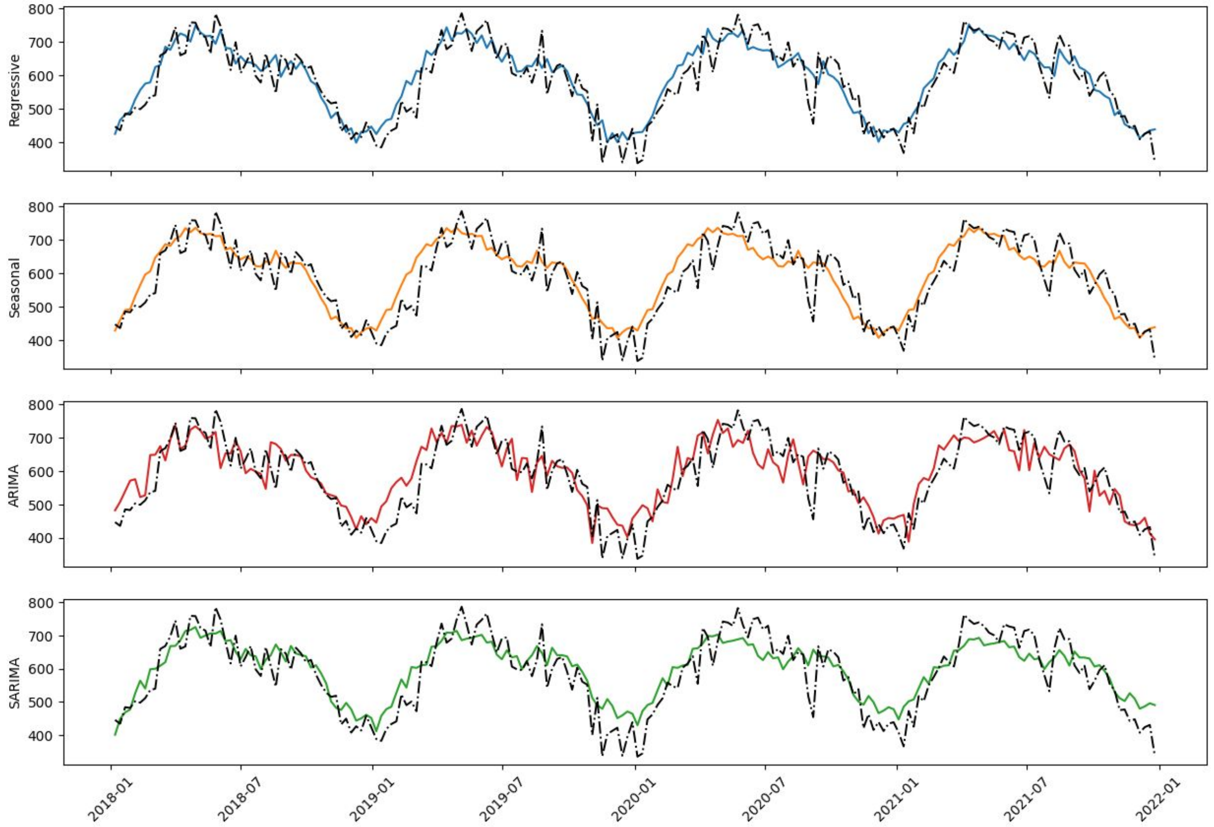


The series is stationary and DLM residuals follow normal distribution but ARIMA and SARIMA residuals don't.

Error	Regressive	**Seasonal**	SARIMA	ARIMA
RMSE	26.4476	25.6720	39.6350	71.3908
MAE	21.8707	21.2476	33.7098	58.5001
MAPE	0.0372	0.0365	0.0587	0.0987

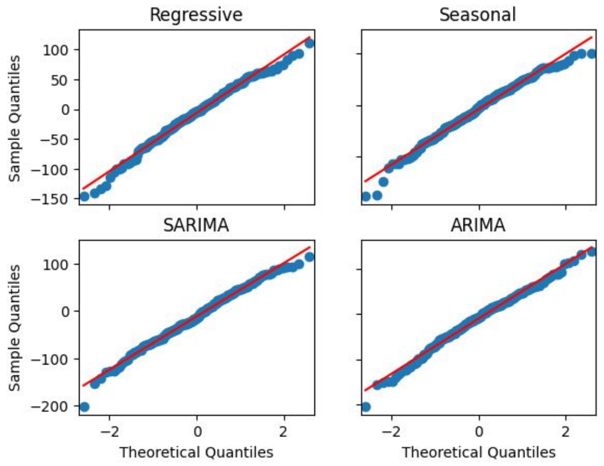


Bhadla - Weekly

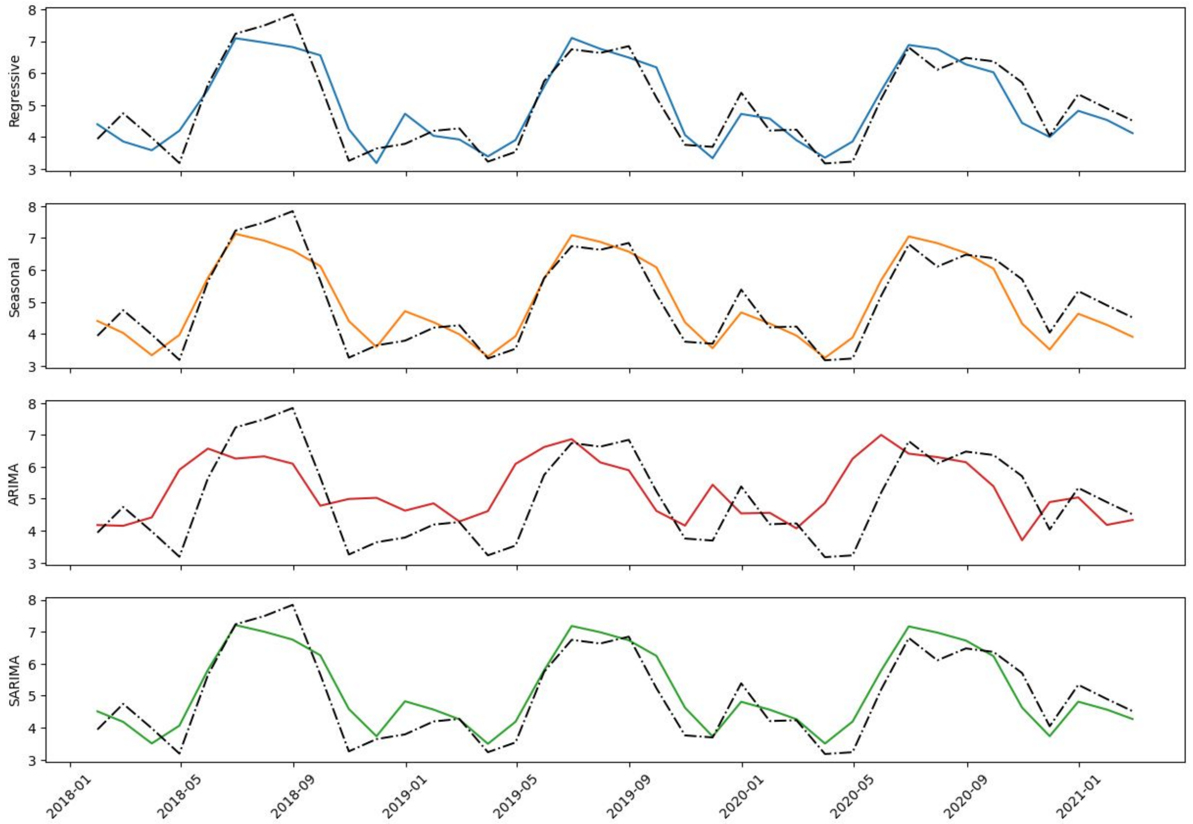


The series is stationary and all the models except the seasonal DLM model show normal characteristics.

Error	**Regressive**	Seasonal	SARIMA	ARIMA
RMSE	48.4967	53.4963	56.3986	61.0935
MAE	38.4183	42.9263	45.7328	47.8263
MAPE	0.0663	0.0735	0.0786	0.0821

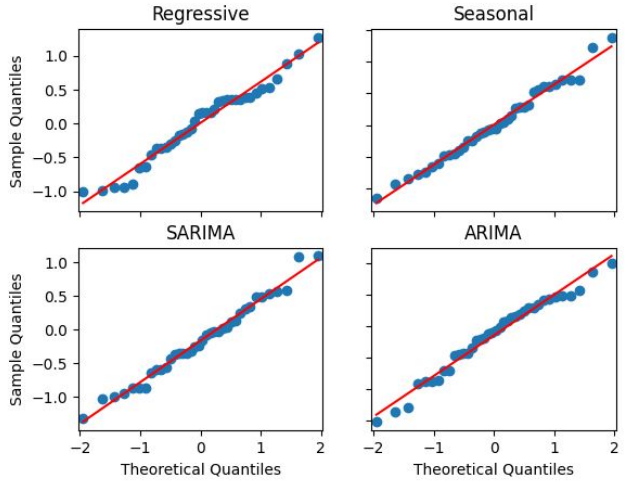


Tiruvananthpuram - Monthly

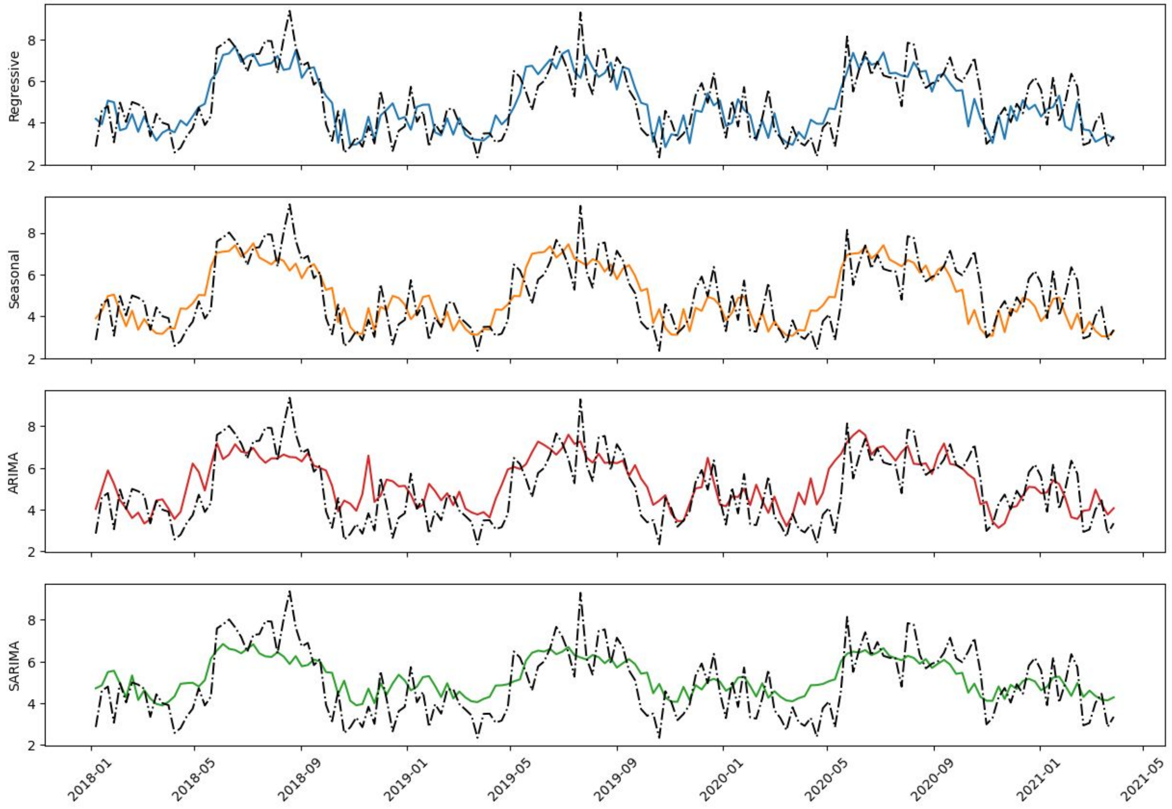


The series is not stationary, nevertheless all four model residuals pass the Shapiro-Wilk test.

Error	**Regressive**	Seasonal	SARIMA	ARIMA
RMSE	0.5636	0.5862	0.5964	1.2335
MAE	0.4688	0.4774	0.4838	0.9814
MAPE	0.0993	0.1015	0.0980	0.1849

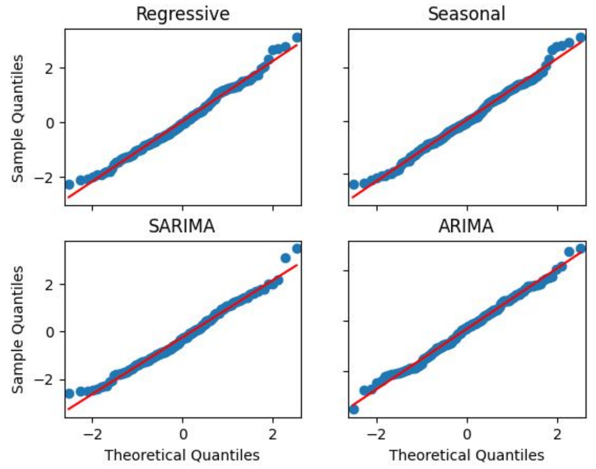


Tiruvananthpuram - Weekly

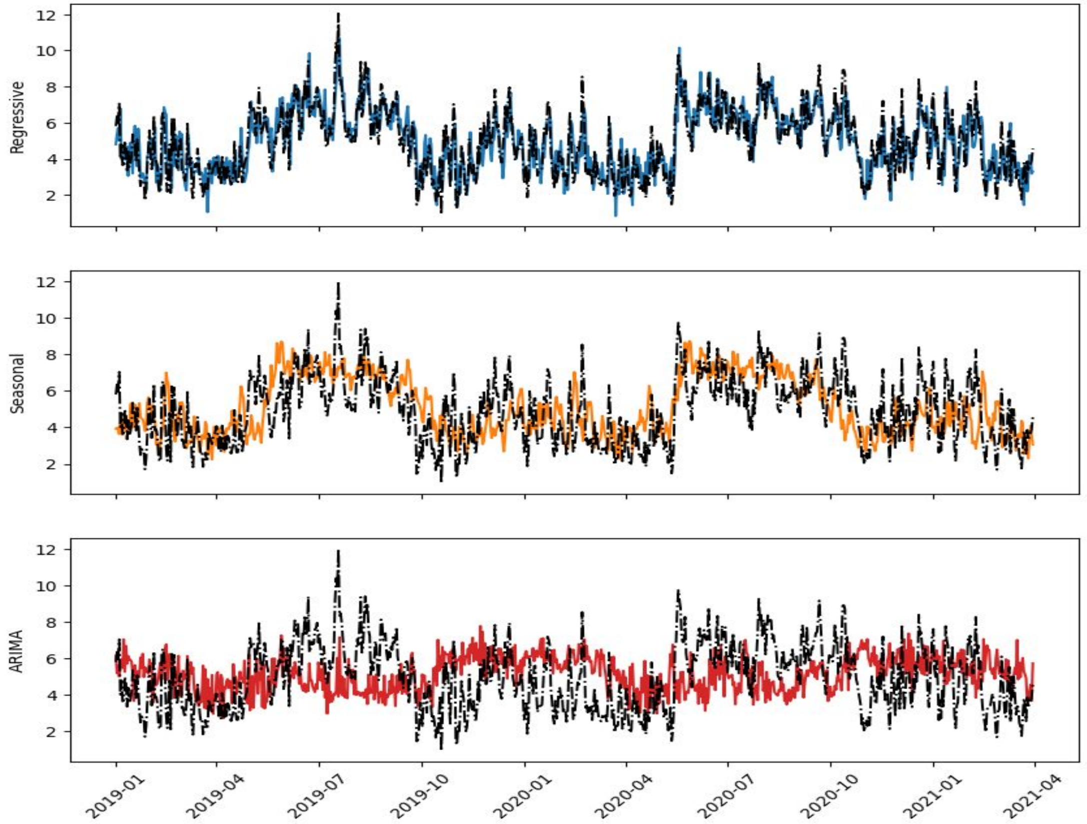


The series is stationary and all four model residuals pass the Shapiro-Wilk normality test.

Error	**Regressive**	Seasonal	SARIMA	ARIMA
RMSE	1.0835	1.1327	1.1986	1.2137
MAE	0.8795	0.9031	0.9905	0.9822
MAPE	0.1892	0.1927	0.1939	0.1909

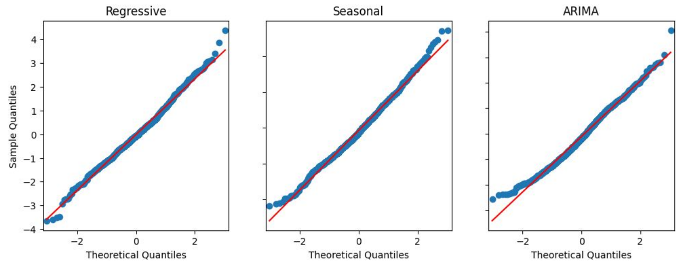


Tiruvananthpuram - Daily

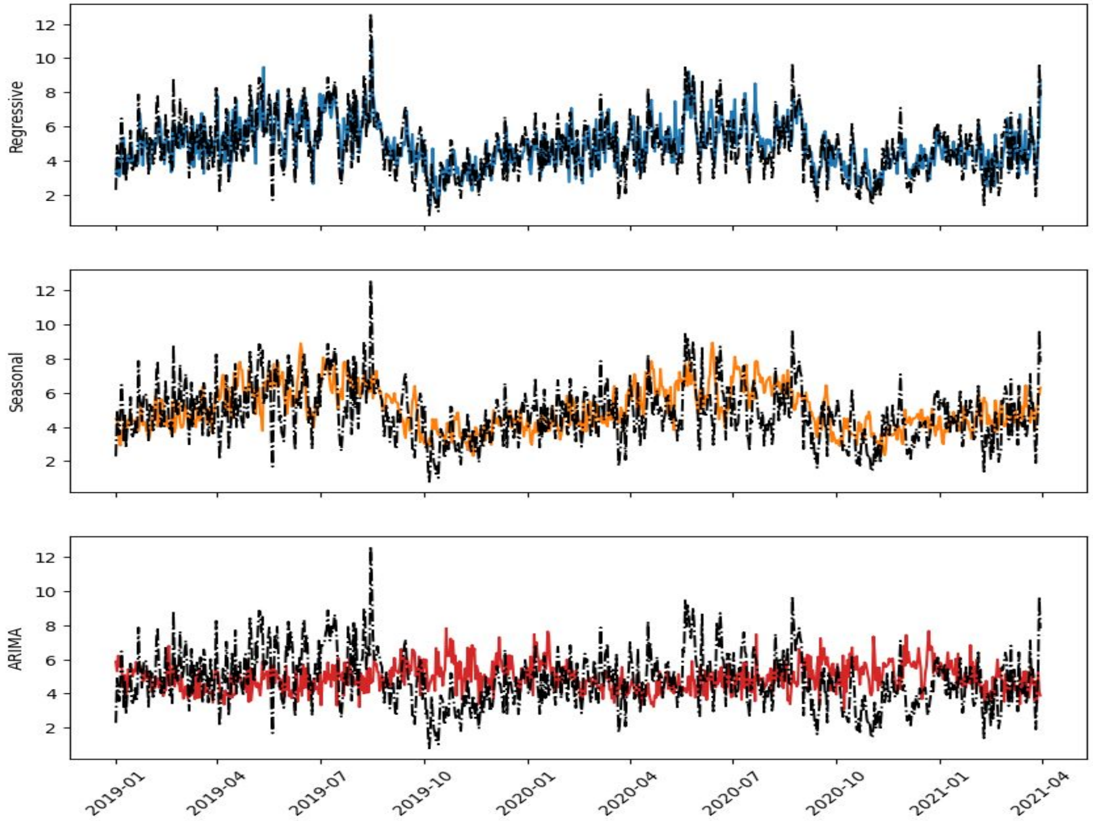


The series is stationary but all four models fail the normality test, probably due to large number data points. Looking at the Q-Q plot, they do exhibit normal characteristics.

Error	**Regressive**	Seasonal	ARIMA
RMSE	1.1719	1.6634	2.1902
MAE	0.9186	1.3385	1.8171
MAPE	0.2103	0.2824	0.3631

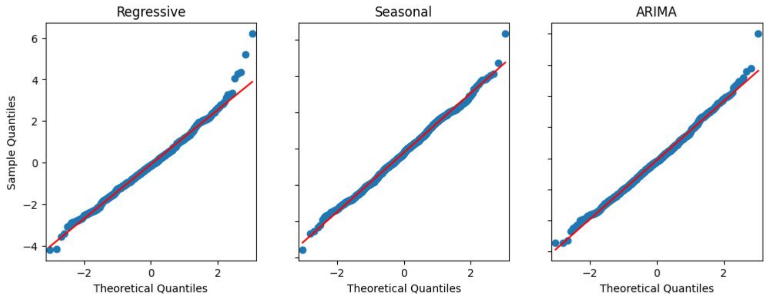


Bhopal - Daily

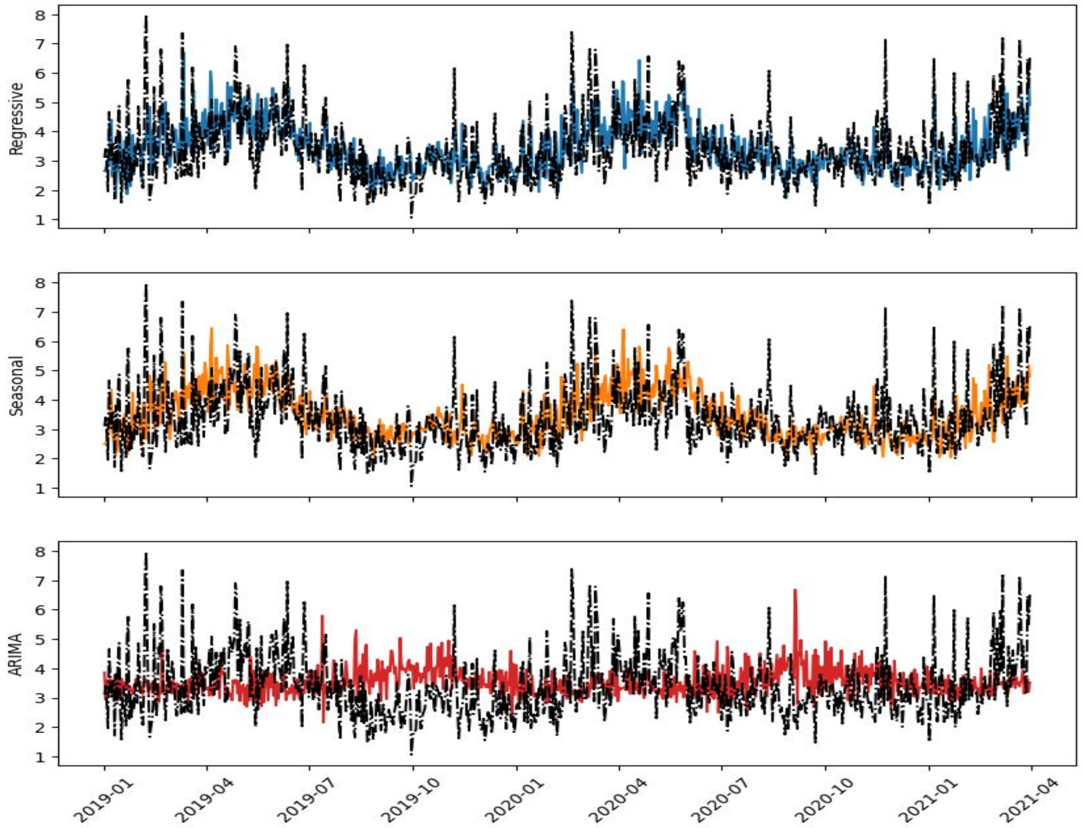


The series is stationary but all the models fail the normality test, except the seasonal DLM model.

Error	**Regressive**	Seasonal	ARIMA
RMSE	1.3081	1.6424	1.8901
MAE	1.0180	1.3210	1.4946
MAPE	0.2192	0.2695	0.3060

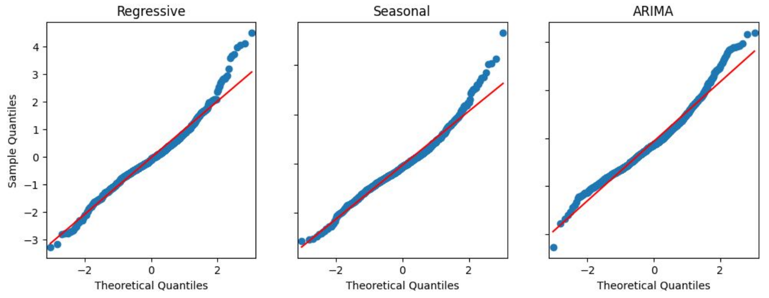


Hamirpur - Daily

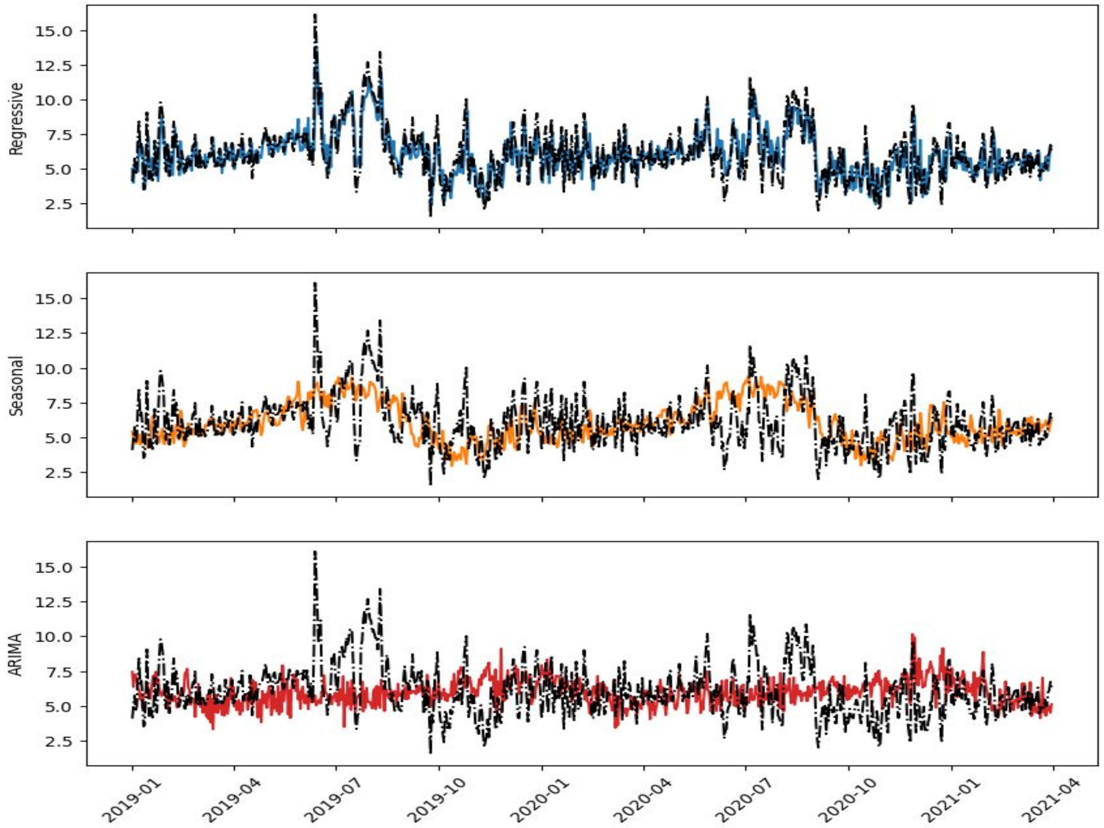


The series is stationary but all the models fail the normality test, even the Q-Q plots show deviation.

Error	**Regressive**	Seasonal	ARIMA
RMSE	1.0356	1.1094	1.2514
MAE	0.7693	0.8315	0.9721
MAPE	0.2246	0.2410	0.2733

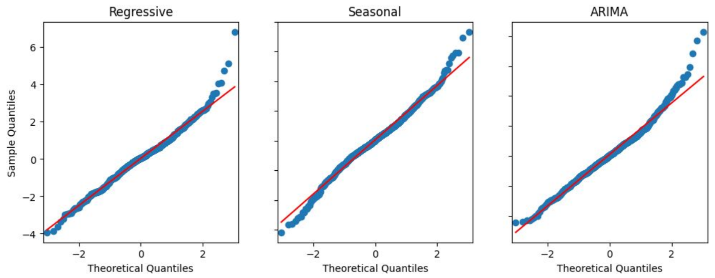


Jafrabad - Daily



The series is stationary but all the models fail the normality test, even the Q-Q plots show deviation.

Error	**Regressive**	Seasonal	ARIMA
RMSE	1.2677	1.8268	2.1575
MAE	0.9601	1.3808	1.6352
MAPE	0.1679	0.2362	0.2786



Results

- Because of the non-stationary nature of monthly tiru. data, ARIMA predictions for it were starkly inaccurate compared to other models.

- On all the other cases including that one, DLM models outshone their (S)ARIMA rivals.

- Regressive DLM model performed well in all cases except the Monthly GHI predictions for Bhadla where seasonal DLM model was a bit better.

- For a lot of the residuals where Shapiro-Wilk test failed, the Q-Q plots still indicated normality.

Challenges and Possible Future Work

- Lack of literature tackling actual implementation
- Switching between R and python
- Long period of no tangible progress due to broken pyDLM library
- Get daily estimates for SARIMA to compare, if possible
- Look into regressive models that have more than one value lag

Thank You