

Inferential analysis

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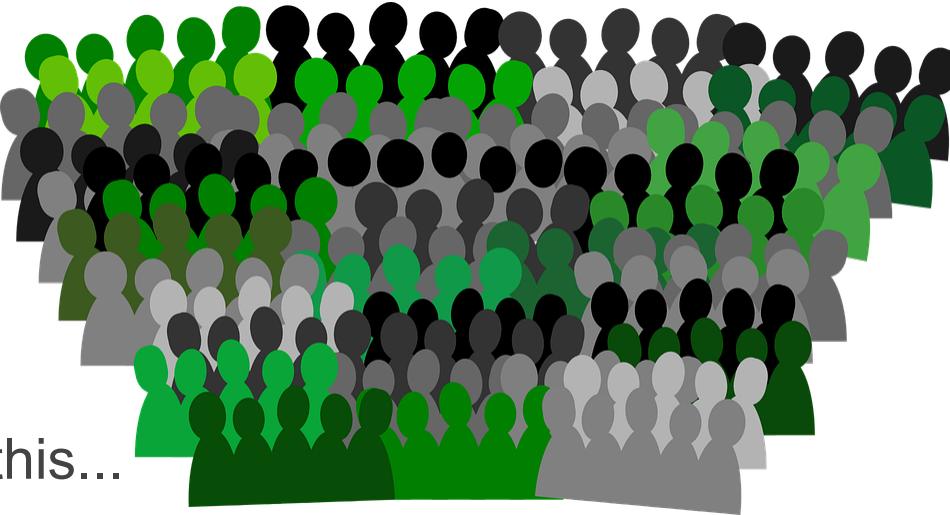


@jasongfleischer

<https://jgfleischer.com>

Population

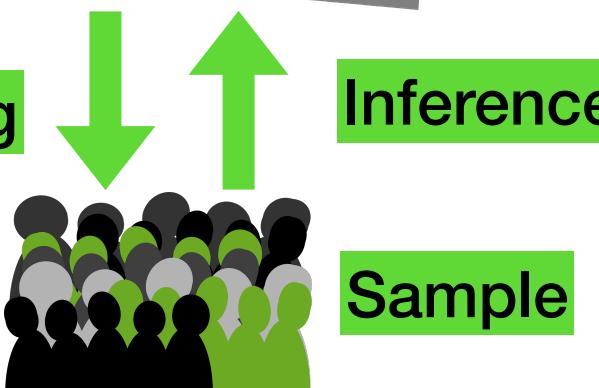
We want to learn
something about this...



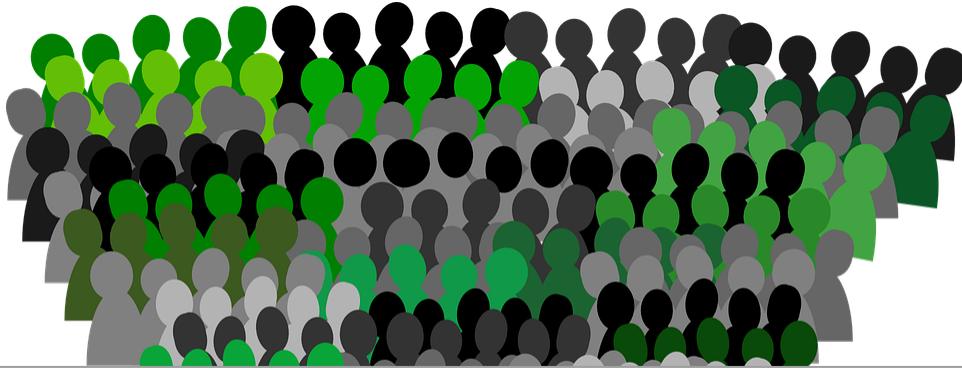
Sampling

Inference

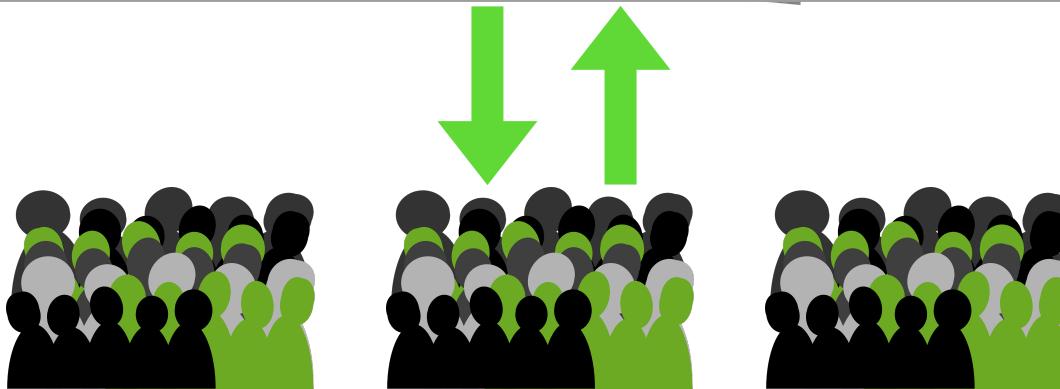
....but we can only *actually*
collect data from this



Sample



Random samples are random!
They differ from each other and the population!





NIH Public Access

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The Effect of Air Pollution Control on Life Expectancy in the United States: An Analysis of 545 US counties for the period 2000 to 2007

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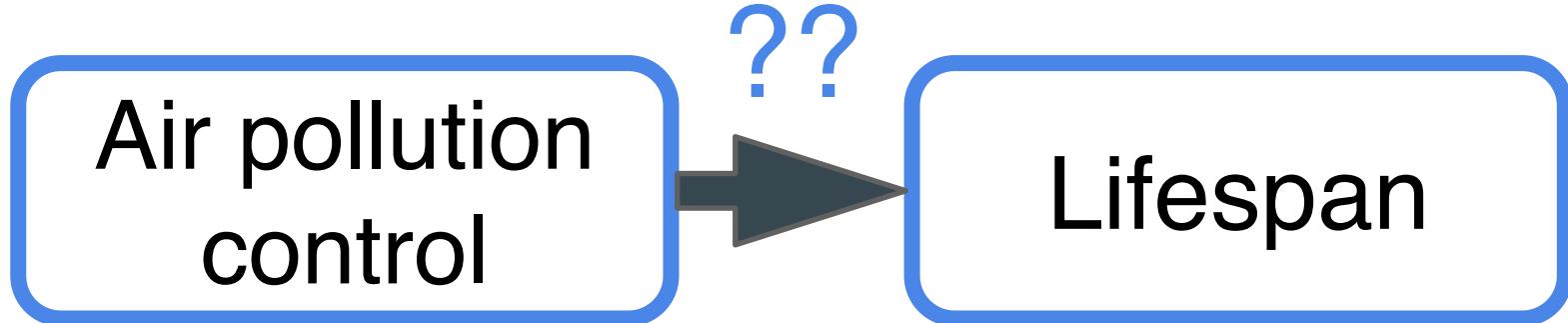
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Majid Ezzati, and

MRC-HPA Centre for Environment and Health and Department of Epidemiology and Biostatistics, Imperial College London, Norfolk Place, St Mary's Campus, London W2 1PG

Francesca Dominici

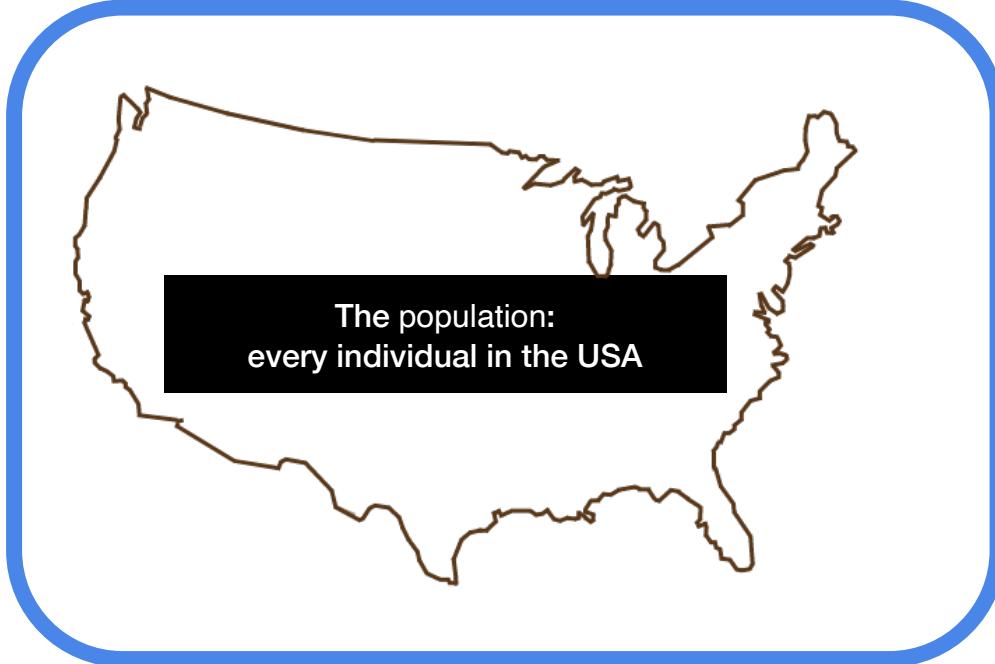
Department of Biostatistics, Harvard School of Public Health, 655 Huntington Avenue, HSPH Building 2, 4th Floor, Boston, MA 02115, fdominic@hsph.harvard.edu, P: (617) 432-1056; F: (617)-739-1781



Is there a relationship between air pollution control and lifespan?

A decrease of 10 $\mu\text{g}/\text{m}^3$ in the concentration of PM2.5 was associated with an increase in mean life expectancy of 0.35 years SD= 0.16 years, $p = 0.033$). This association was stronger in more urban and densely populated counties.

What if we want to know the effect of air pollution on everyone in the United States?





The population:
every individual in the USA

The sample:
545 US counties

#	State	Total Number of Counties	Total Area
1	Texas	254	268,596 mi ²
2	Georgia	159	59,425 mi ²
3	Virginia	133	42,775 mi ²
4	Kentucky	120	40,408 mi ²
5	Missouri	115	69,707 mi ²
6	Kansas	105	82,278 mi ²
7	Illinois	102	57,914 mi ²
8	North Carolina	100	53,819 mi ²
9	Iowa	99	56,273 mi ²
10	Tennessee	95	42,144 mi ²

3143
counties
in the US

CA is #27

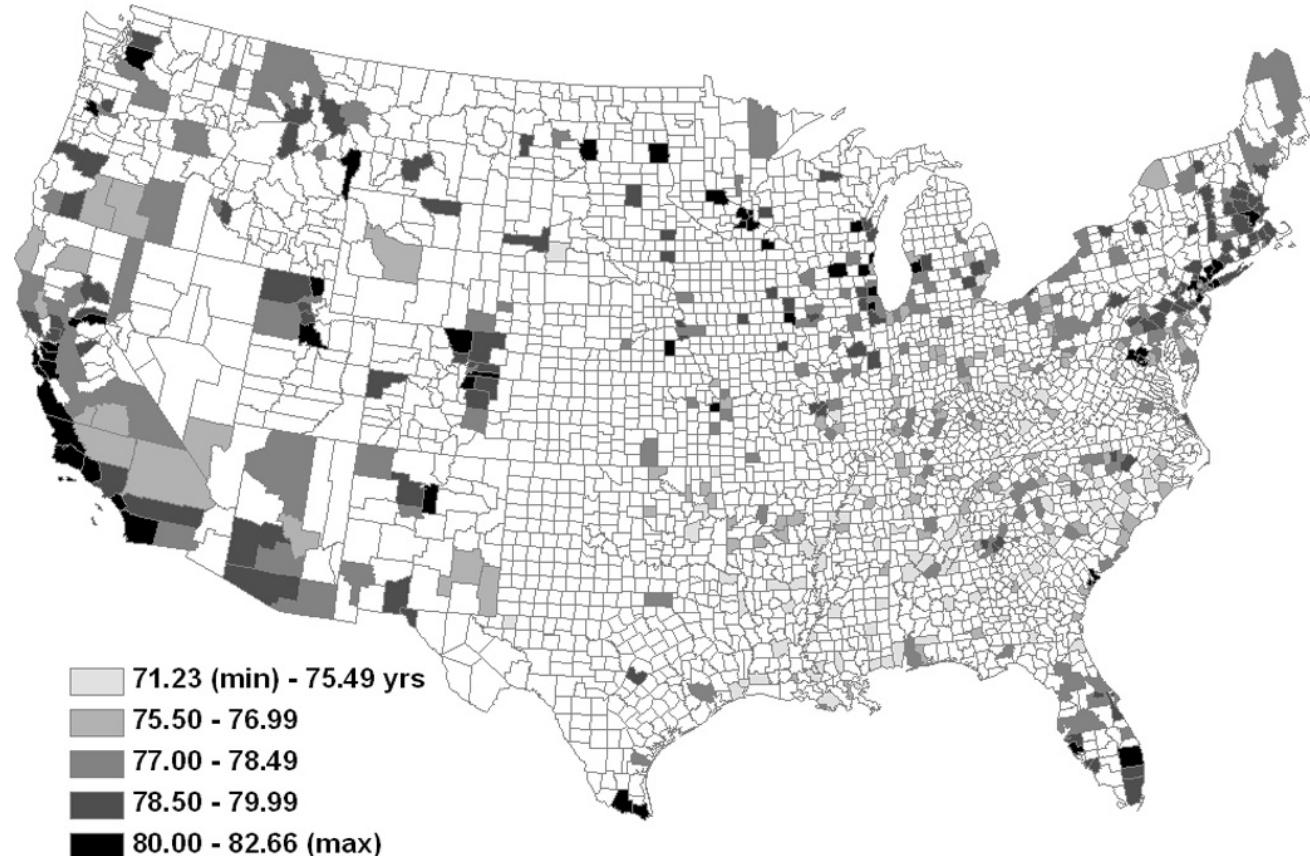
58 counties on 163k mi²

Random Sampler

How would you want to select those 545 counties?

What criteria are ideal?

What do you think they did?



All counties with available matching PM_{2.5} data for 2000 and 2007 from the EPA's Air Quality System. Includes both metropolitan and non-metro counties

Current AQS stations



States typically decide where monitors are placed based on areas of relatively high population and/or areas believed to have relatively higher pollutant concentrations. Each state is responsible for developing its own monitoring plan, which is then reviewed and revised every five years.

Aug 28, 2023



United States Environmental Protection Agency (.gov)

<https://www.epa.gov/outdoor-air-quality-data/who-de...>

⋮

Who decides where monitors get placed? | US EPA

Random Sampler

How might conclusions differ if...

the sample locations were instead completely spatially at random across the USA?

The sample came from the 1980s or the 2020s instead of the 2000s?

Approaches to Inference

CORRELATION

COMPARISON OF MEANS

REGRESSION

NON-PARAMETRIC TESTS

CORRELATION

ASSOCIATION
BETWEEN VARIABLES

i.e. Pearson
Correlation,
Spearman
Correlation, chi-
square test

COMPARISON OF MEANS

DIFFERENCE IN MEANS
BETWEEN VARIABLES

i.e. t-test, ANOVA

REGRESSION

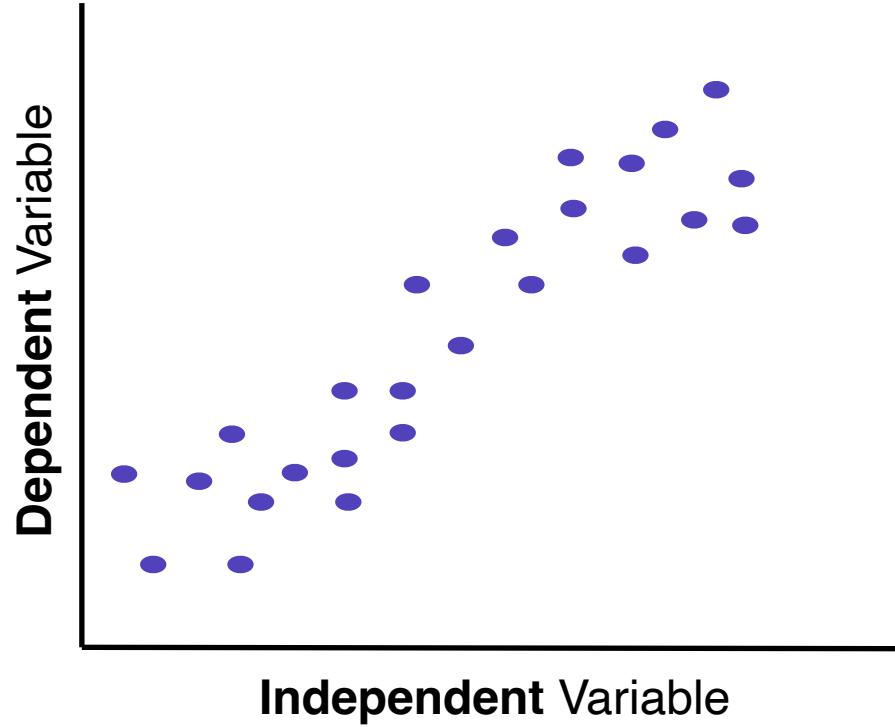
DOES CHANGE IN ONE
VARIABLE MEAN CHANGE
IN ANOTHER?

i.e. simple
regression, multiple
regression

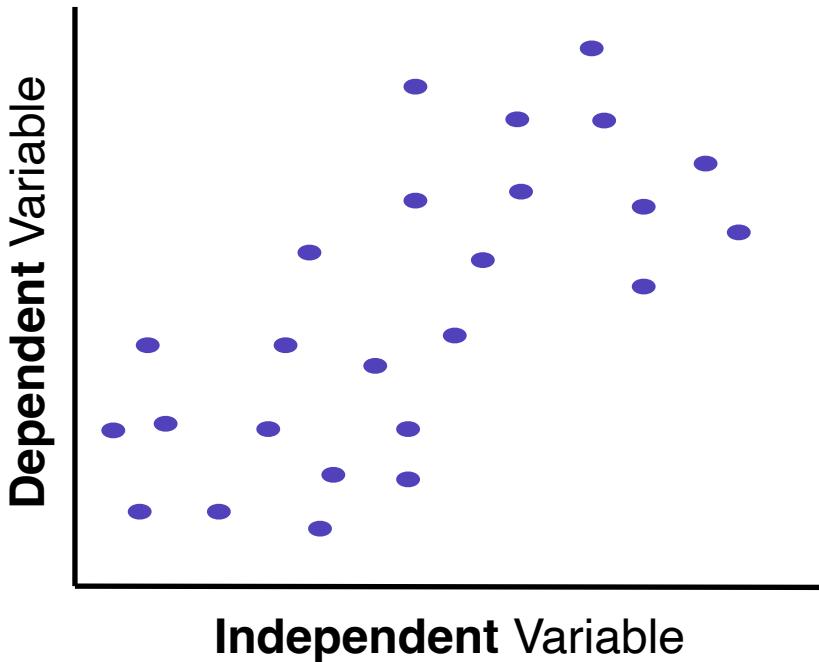
NON-PARAMETRIC TESTS

FOR WHEN ASSUMPTIONS
IN THESE OTHER 3
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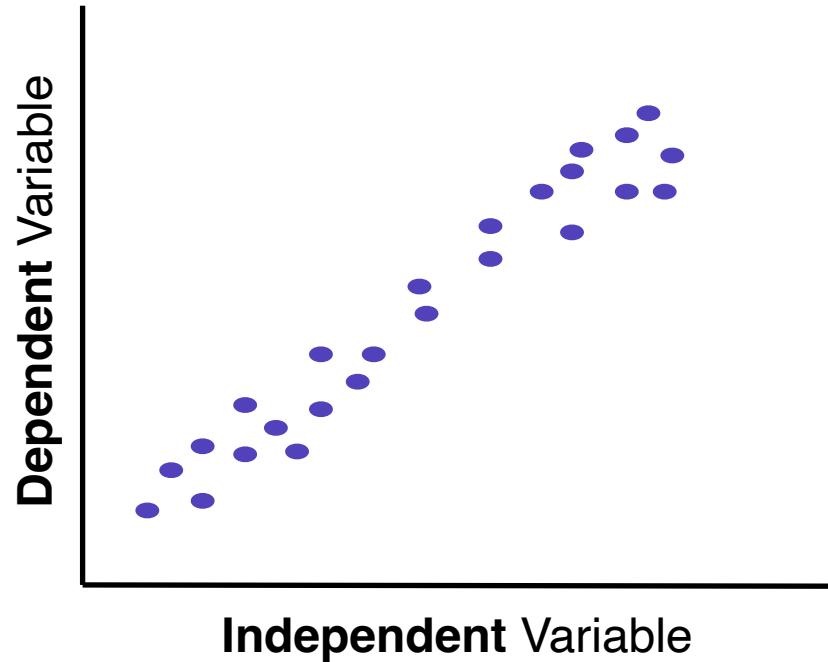
i.e. Wilcoxon rank-
sum test, Wilcoxon
sign-rank test, sign
test



weaker relationship

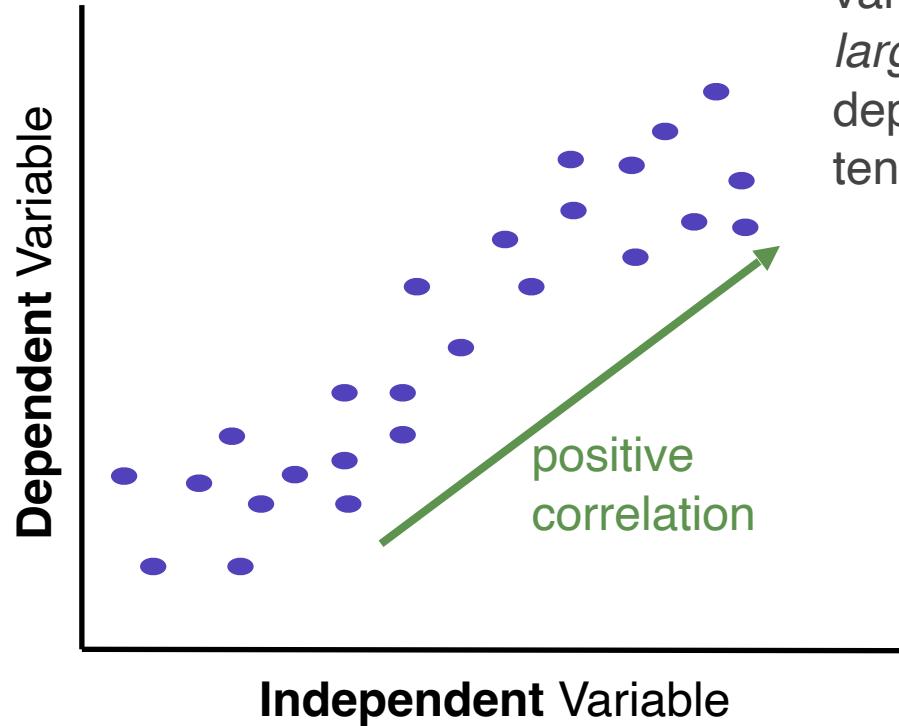


stronger relationship



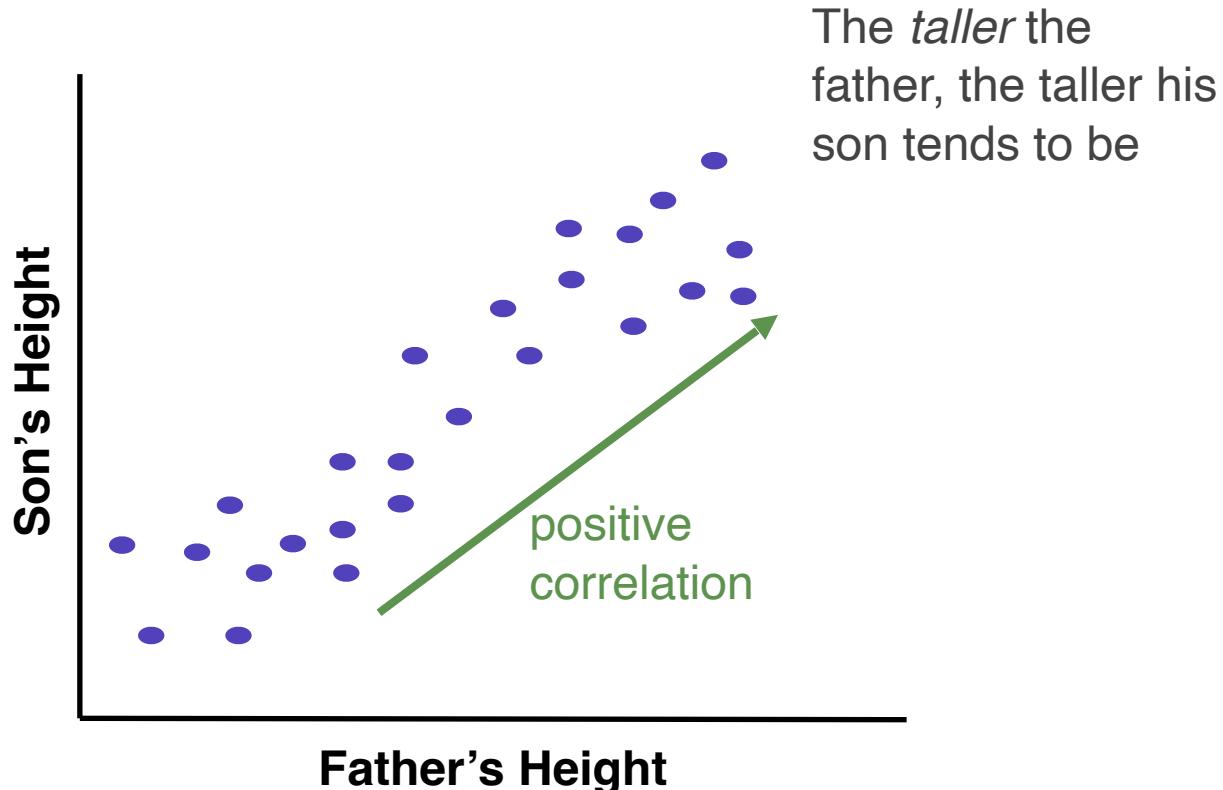
stronger relationship = higher correlation

The *smaller* the independent variable value, the *smaller* the dependent variable tends to be

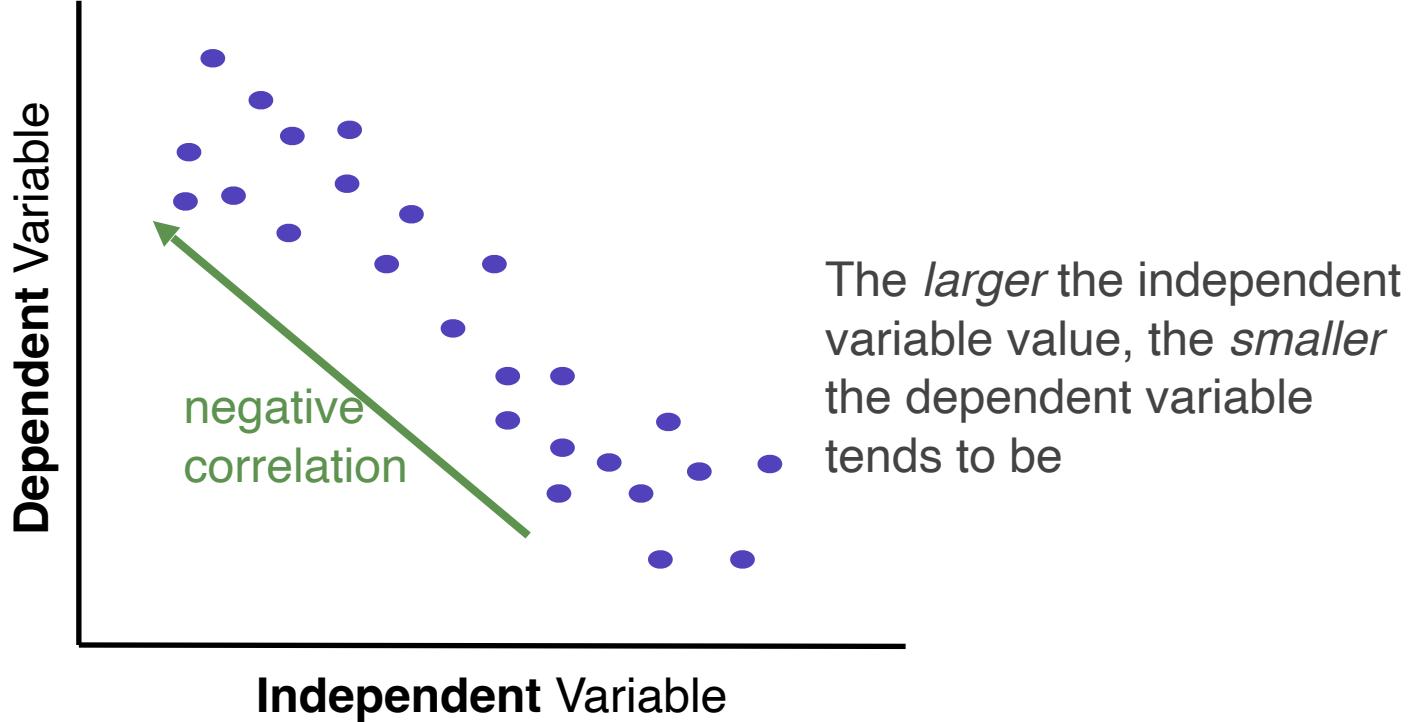


The *larger* the independent variable value, the *larger* the dependent variable tends to be

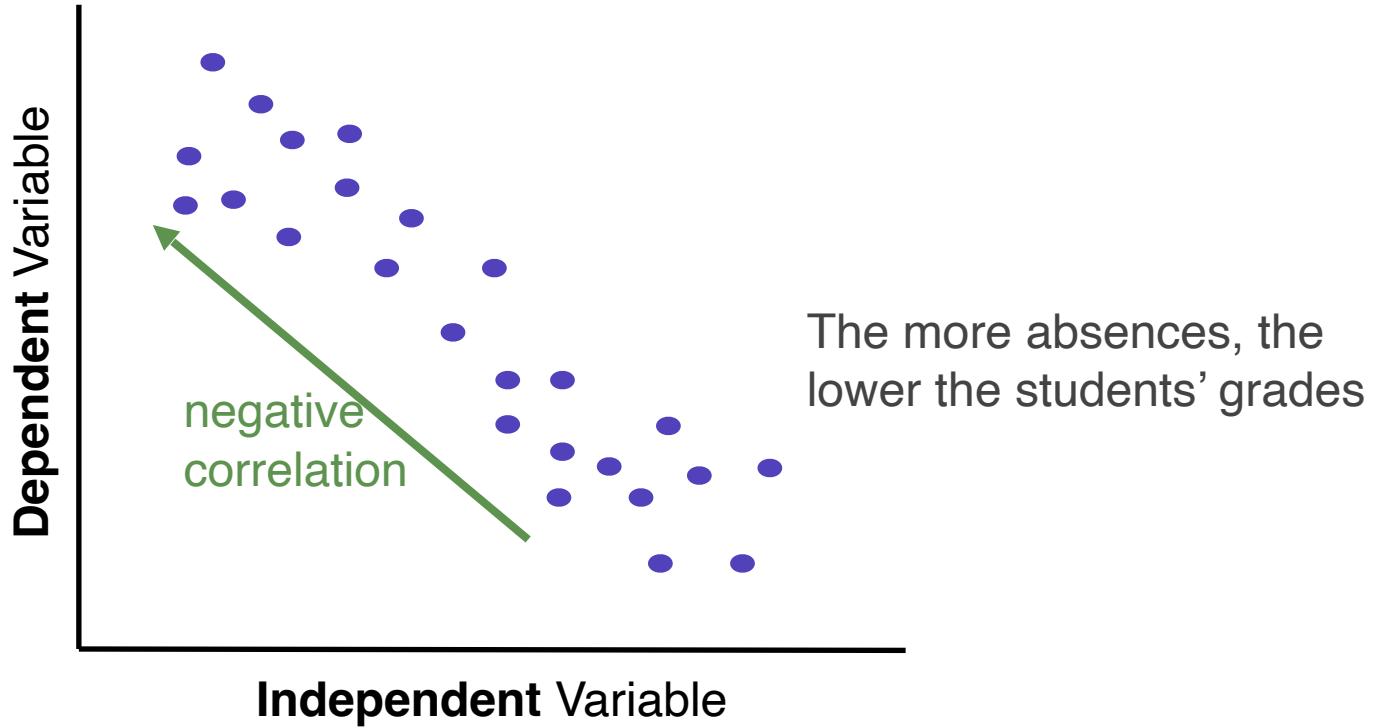
The *shorter* the father, the shorter his son tends to be



The *smaller* the independent variable value, the *larger* the dependent variable tends to be



The *lower* the number of absences, the *higher* the students' grades tend to be



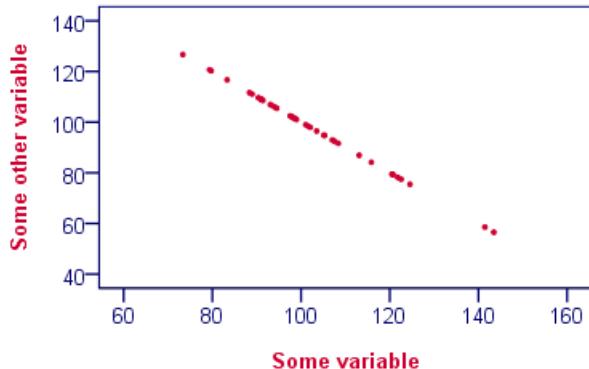
Pearson's *r* :

linear correlation between two variables

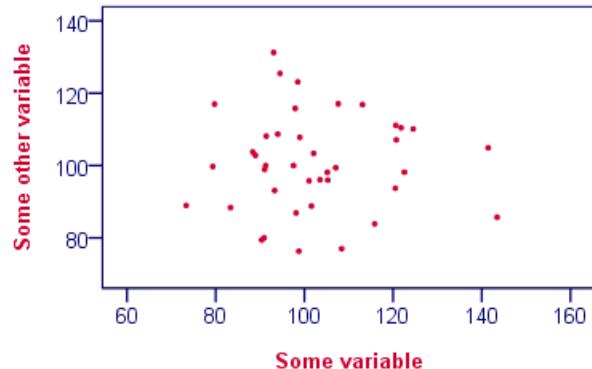
takes values [-1,1]

Correlation is how close the data are to being in a line...
BUT IT HAS NOTHING TO DO WITH THE SLOPE

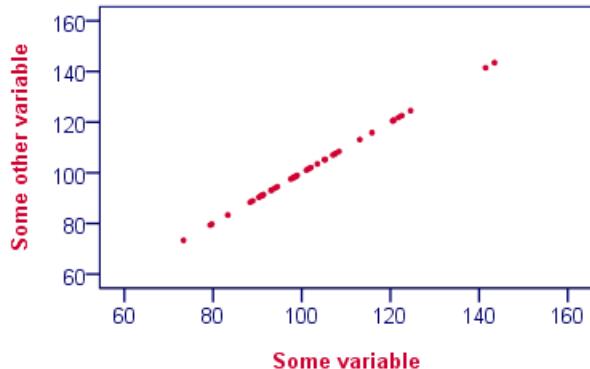
Correlation Coefficient = -1

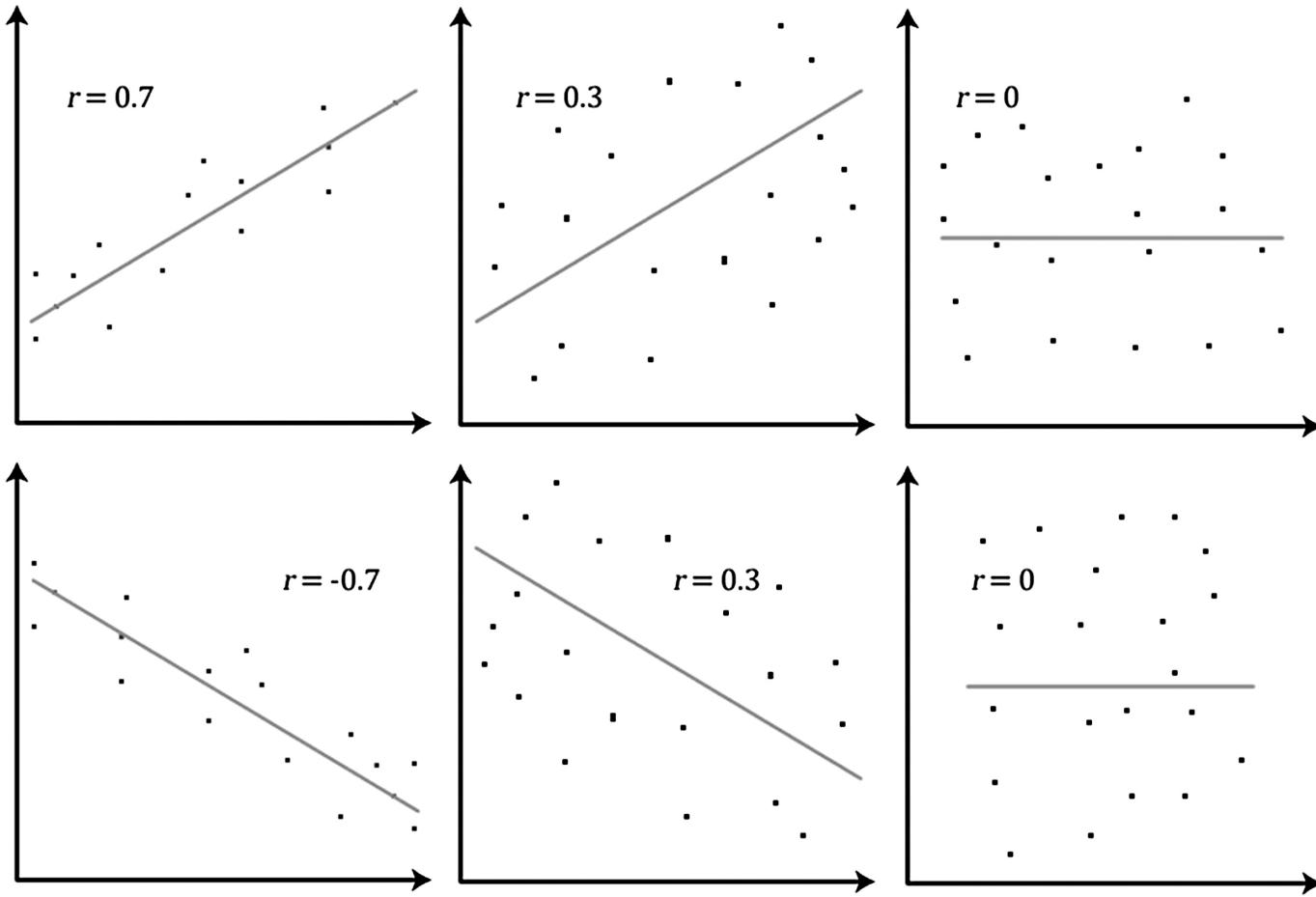


Correlation Coefficient = 0



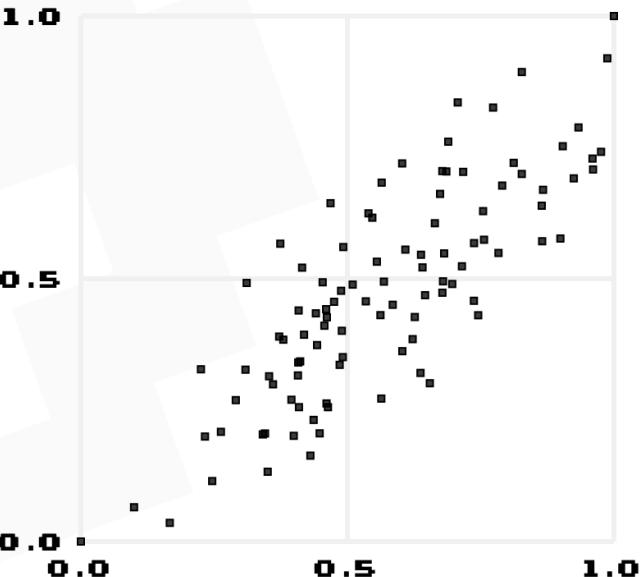
Correlation Coefficient = 1





<https://www.guessthecorrelation.com/>

Which of the following is the Pearson correlation coefficient (r) for this relationship?



<https://forms.gle/M4Q3hFkeoLKCixUX8>

Correlation != Causation

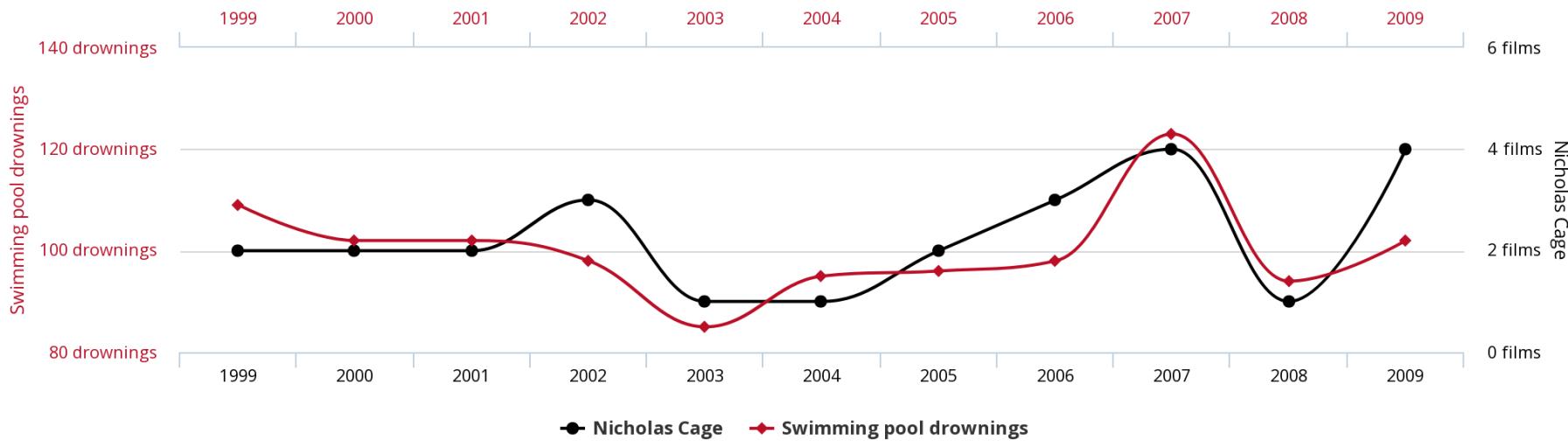
Correlation establishes a relationship.

It does NOT establish causation.

Number of people who drowned by falling into a pool

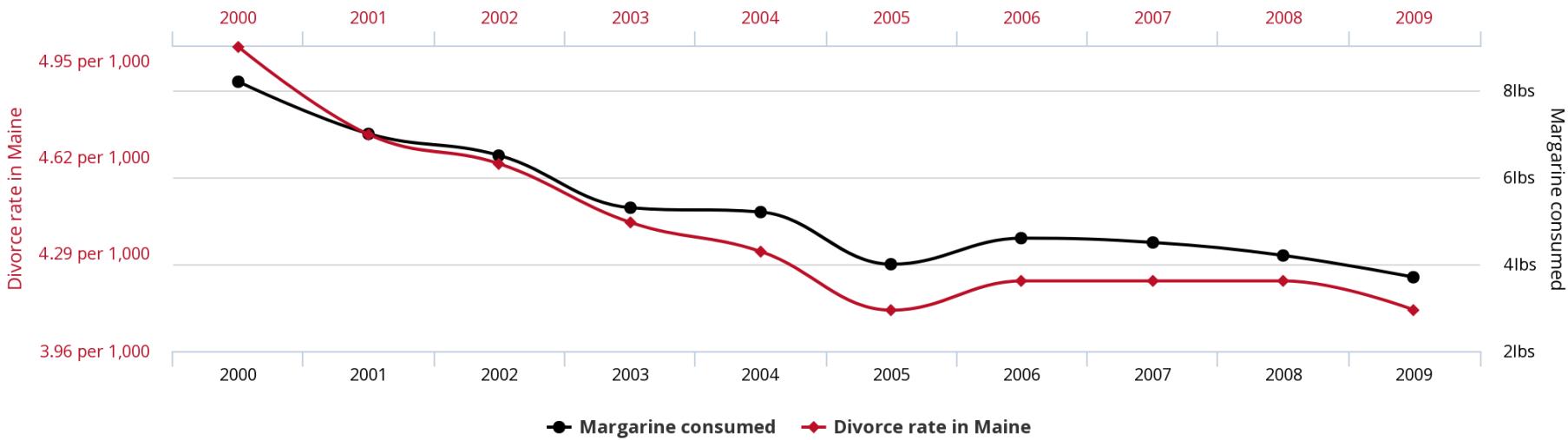
correlates with

Films Nicolas Cage appeared in



tylervigen.com

Divorce rate in Maine correlates with Per capita consumption of margarine



tylervigen.com

What about correlation between categorical variables?

- Make a contingency table ...

Sex \ Handedness	Right-handed	Left-handed	Total
Male	43	9	52
Female	44	4	48
Total	87	13	100

- Then calculate phi (based on chi-squared statistic) ...

[Phi coefficient](#) [edit]

Main article: Phi coefficient

A simple measure, applicable only to the case of 2×2 contingency tables, is the [phi coefficient](#) (ϕ) defined by

$$\phi = \pm \sqrt{\frac{\chi^2}{N}},$$

where χ^2 is computed as in [Pearson's chi-squared test](#), and N is the grand total of observations. ϕ varies from 0 (corresponding to no association between the variables) to 1 or -1 (complete association or complete inverse association), provided it is based on frequency data represented in 2×2 tables. Then its sign equals the sign of the product of the [main diagonal](#) elements of the table minus the product of the off-diagonal elements. ϕ takes on the minimum value -1.0 or the maximum value of +1.0 if and only if every marginal proportion is equal to 0.5 (and two diagonal cells are empty).^[2]

- Yes there are ways to do this in pandas/numpy and other libraries!

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BETWEEN VARIABLES

i.e. Pearson
Correlation,
Spearman
Correlation, chi-
square test

COMPARISON OF MEANS

DIFFERENCE IN MEANS
BETWEEN CONDITIONS

i.e. t-test, ANOVA

REGRESSION

DOES CHANGE IN ONE
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i.e. simple
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IN THESE OTHER 3
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MET

i.e. Wilcoxon rank-
sum test, Wilcoxon
sign-rank test, sign
test



t-test:
tests for difference in means between groups

William Sealy Gosset (13 June 1876 – 16 October 1937) was an English statistician, chemist and brewer who served as **Head Brewer of Guinness** and Head Experimental Brewer of Guinness and was a pioneer of modern statistics. He pioneered small sample experimental design and analysis with an economic approach to the logic of uncertainty. Gosset published under the pen name **Student** and developed most famously **Student's t-distribution** – originally called Student's "z" – and "Student's test of statistical significance".^[1]

Contents [hide]

- 1 Life and career
- 2 See also
- 3 Bibliography
- 4 References
- 5 Further reading
- 6 External links

Life and career [edit]

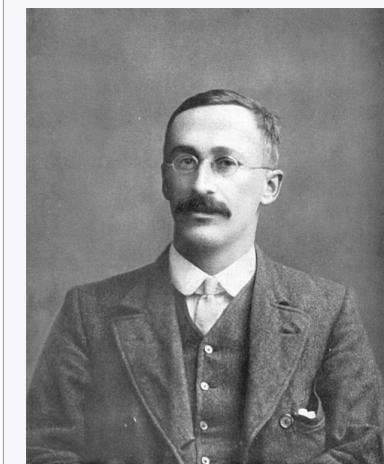
Born in [Canterbury](#), England the eldest son of Agnes Sealy Vidal and Colonel Frederic Gosset, R.E. [Royal Engineers](#), Gosset attended [Winchester College](#) before matriculating as Winchester Scholar in [natural sciences](#) and mathematics at [New College, Oxford](#). Upon graduating in 1899, he joined the brewery of [Arthur Guinness & Son](#) in [Dublin](#), Ireland; he spent the rest of his 38-year career at Guinness.^{[1][2]}

Gosset had three children with [Marjory Gosset](#) (née Phillpotts). Harry Gosset (1907–1965) was a consultant paediatrician; Bertha Marian Gosset (1909–2004) was a geographer and nurse; the youngest, Ruth Gosset (1911–1953) married the Oxford mathematician Douglas Roaf and had five children.

In his job as Head Experimental Brewer at [Guinness](#), the self-trained Gosset developed new statistical methods – both in the brewery and on the farm – now central to the design of experiments, to proper use of significance testing on repeated trials, and to analysis of [economic significance](#) (an early instance of [decision theory](#) interpretation of statistics) and more, such as his small-sample, stratified, and repeated balanced experiments on [barley](#) for proving the best [yielding](#) varieties.^[3] Gosset acquired that knowledge by study, by trial and error, by cooperating with others, and by spending two terms in 1906–1907 in the Biometrics laboratory of [Karl Pearson](#).^[4] Gosset and Pearson had a good relationship.^[4] Pearson helped Gosset with the mathematics of his papers, including the 1908 papers, but had little appreciation of their importance. The papers addressed the brewer's concern with small samples; biometricalians like Pearson, on the other hand, typically had hundreds of observations and saw no urgency in developing small-sample methods.^[2]

Gosset's first publication came in 1907, "On the Error of Counting with a [Haemacytometer](#)," in which – unbeknownst to Gosset aka "Student" – he rediscovered the [Poisson distribution](#).^[3] Another researcher at Guinness had previously published a paper containing trade secrets of the Guinness

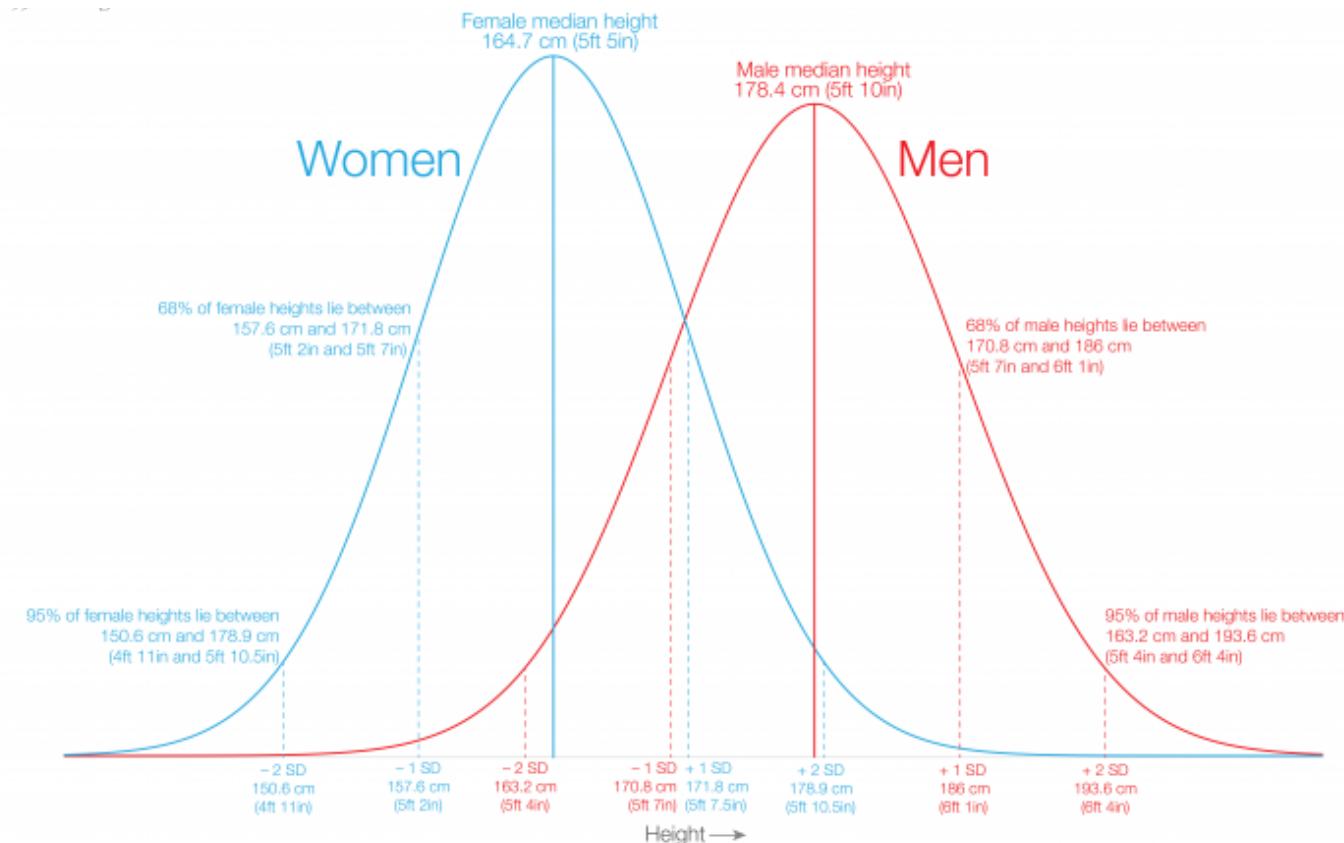
William Sealy Gosset



William Sealy Gosset (aka *Student*) in 1908
(age 32)

Born	13 June 1876 Canterbury , Kent, England
Died	16 October 1937 (aged 61) Beaconsfield , Buckinghamshire, England
Other names	Student
Alma mater	New College, Oxford , Winchester College
Known for	Student's t-distribution, statistical significance, design of experiments, Monte Carlo method, quality control, Modern synthesis, agricultural economics, econometrics
Children	5, including Isaac Henry Gosset
	Scientific career

Do the heights between males and females differ?



t-test Assumptions

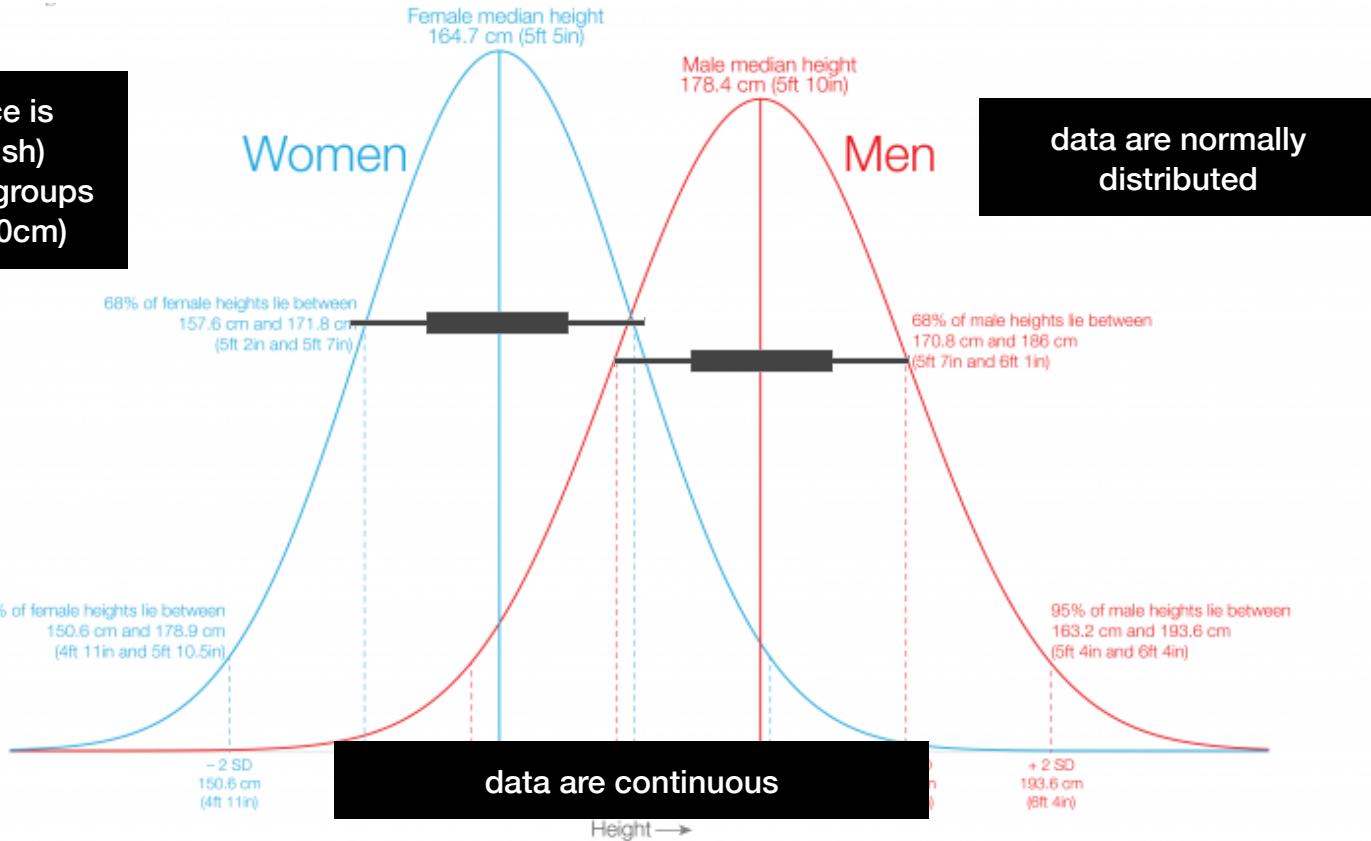
1. Data are continuous
2. Normally distributed
3. Equal variance b/w groups (but can use Welch's test!)
4. Not paired (will talk more about this later)

Do the heights between males and females differ?

variance is equal(ish) between groups (57 vs 50cm)

sample size affects statistic

N=10,000



Do the heights between males and females differ?

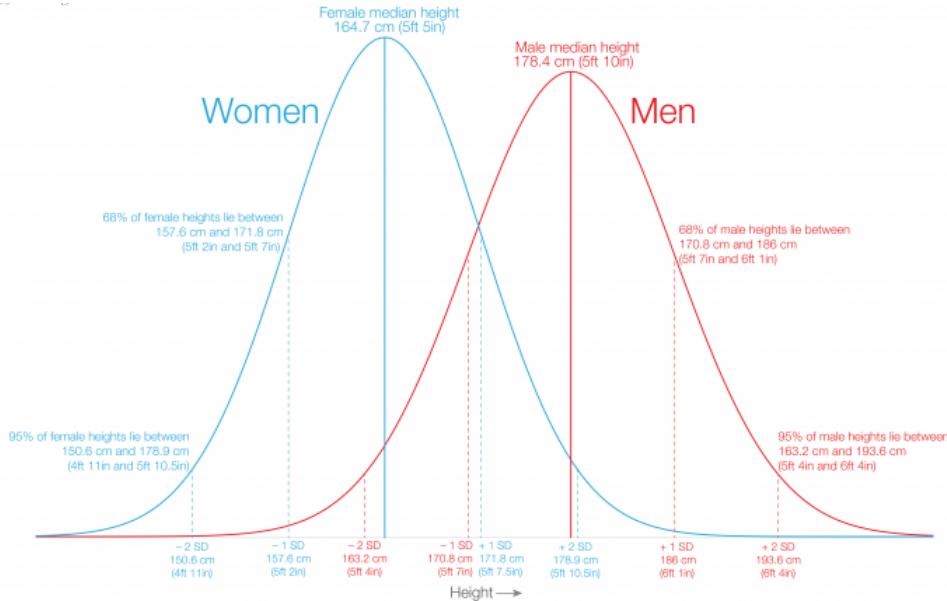
t-statistic: -95.6

p-value << 0.001

95% CI for true difference in means

[-5.43, -5.21]

Yes.

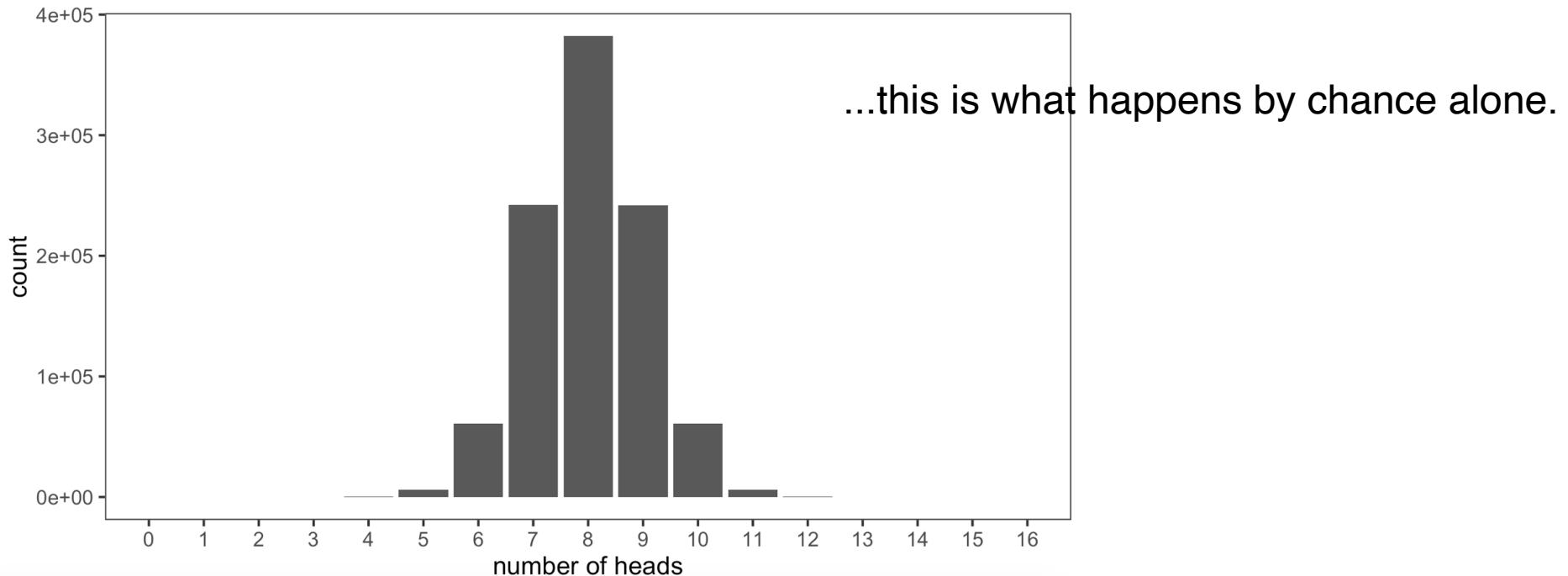


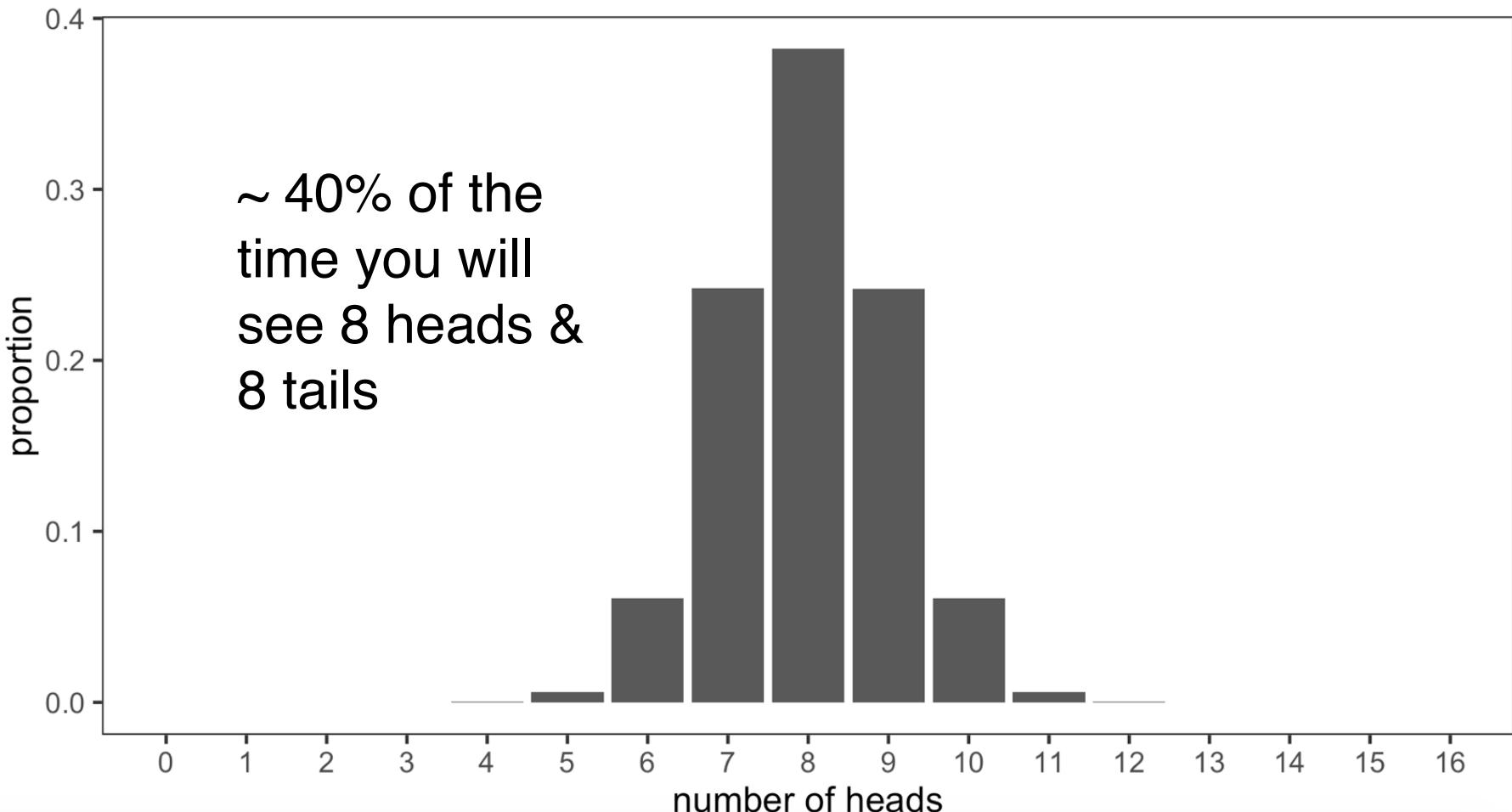
p-value : the probability of getting the observed results (or results more extreme) by chance alone

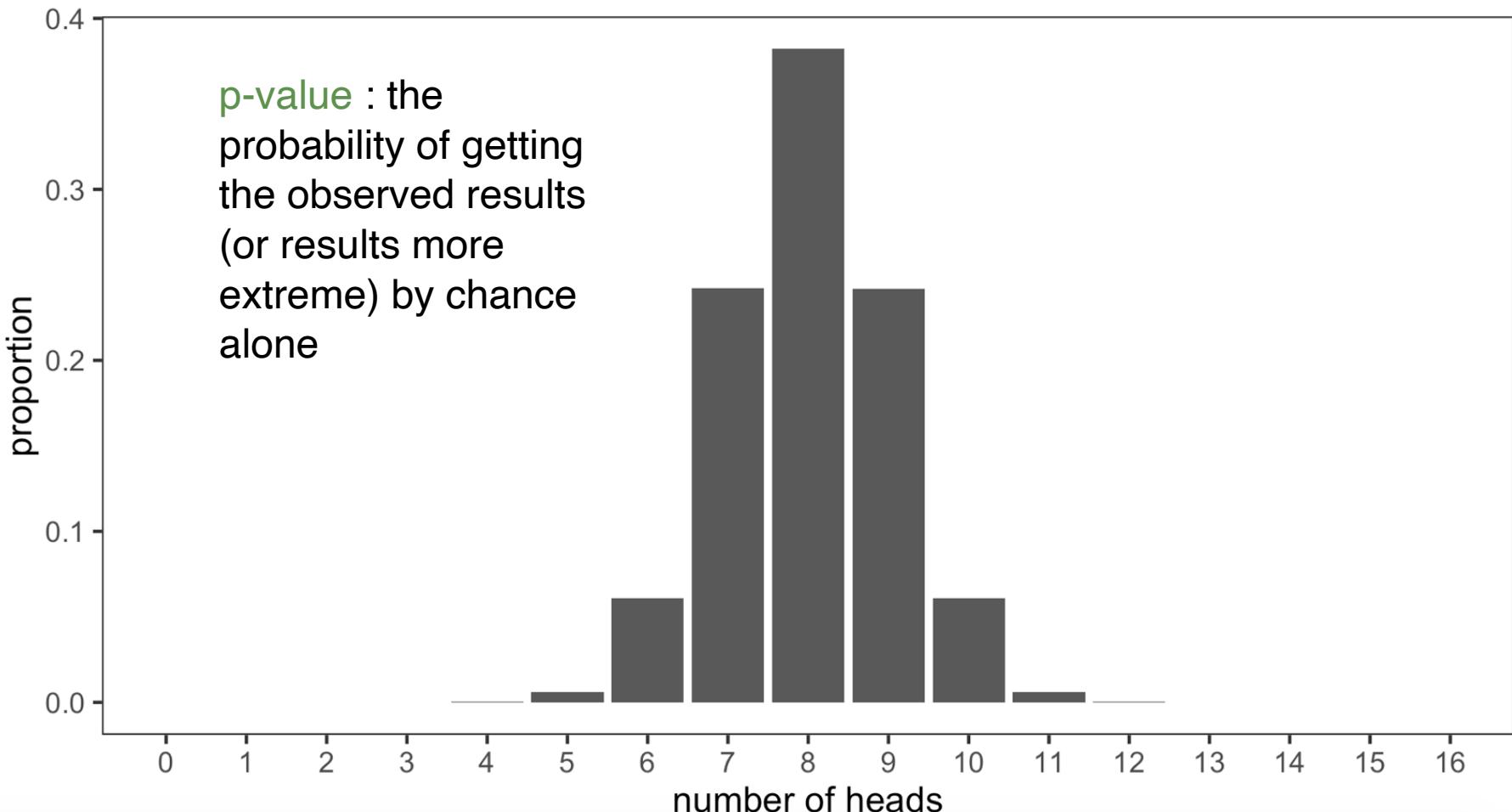


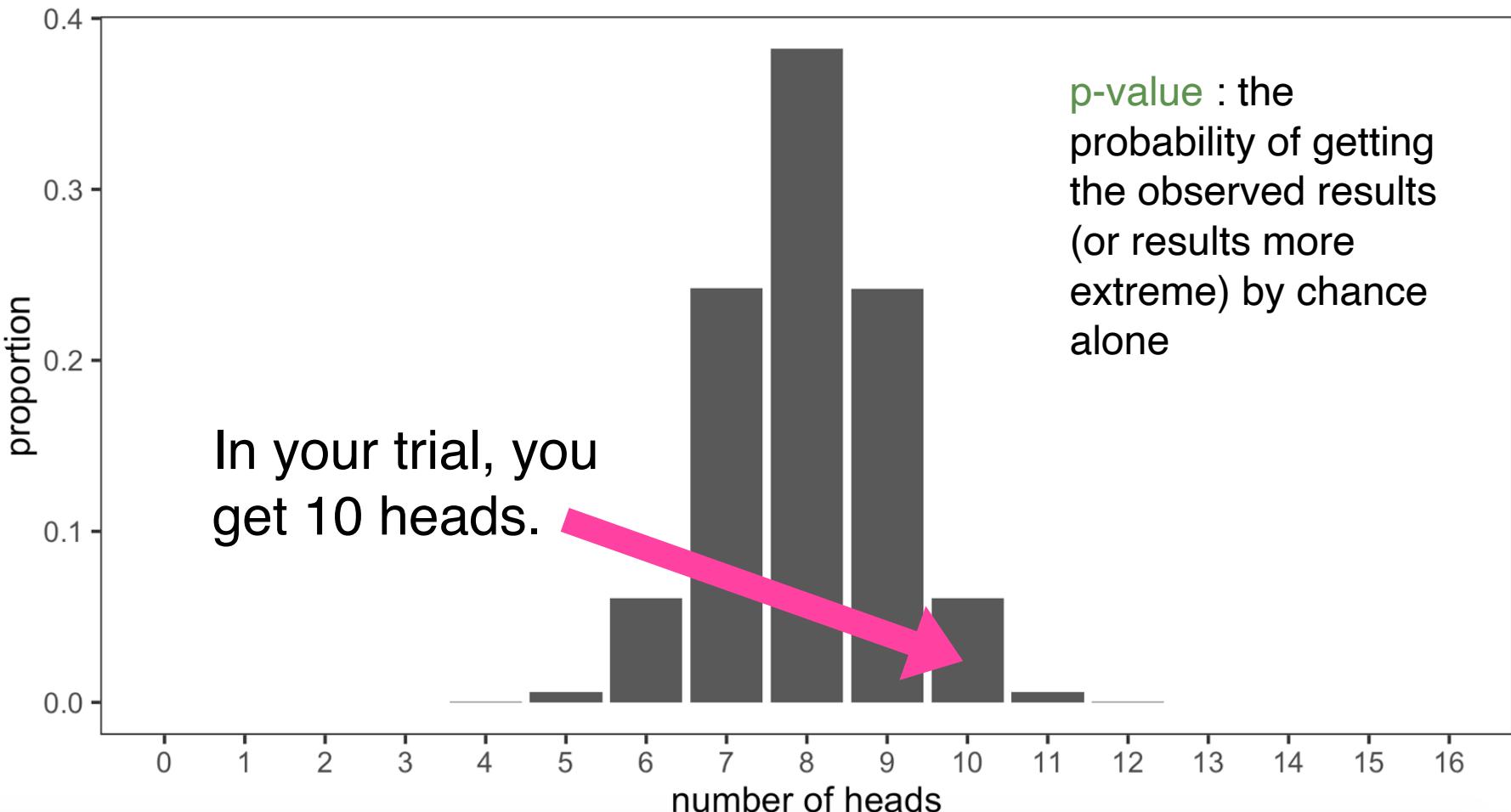
[https://forms.gle/
6MCyp7qFsaHgGKi5A](https://forms.gle/6MCyp7qFsaHgGKi5A)

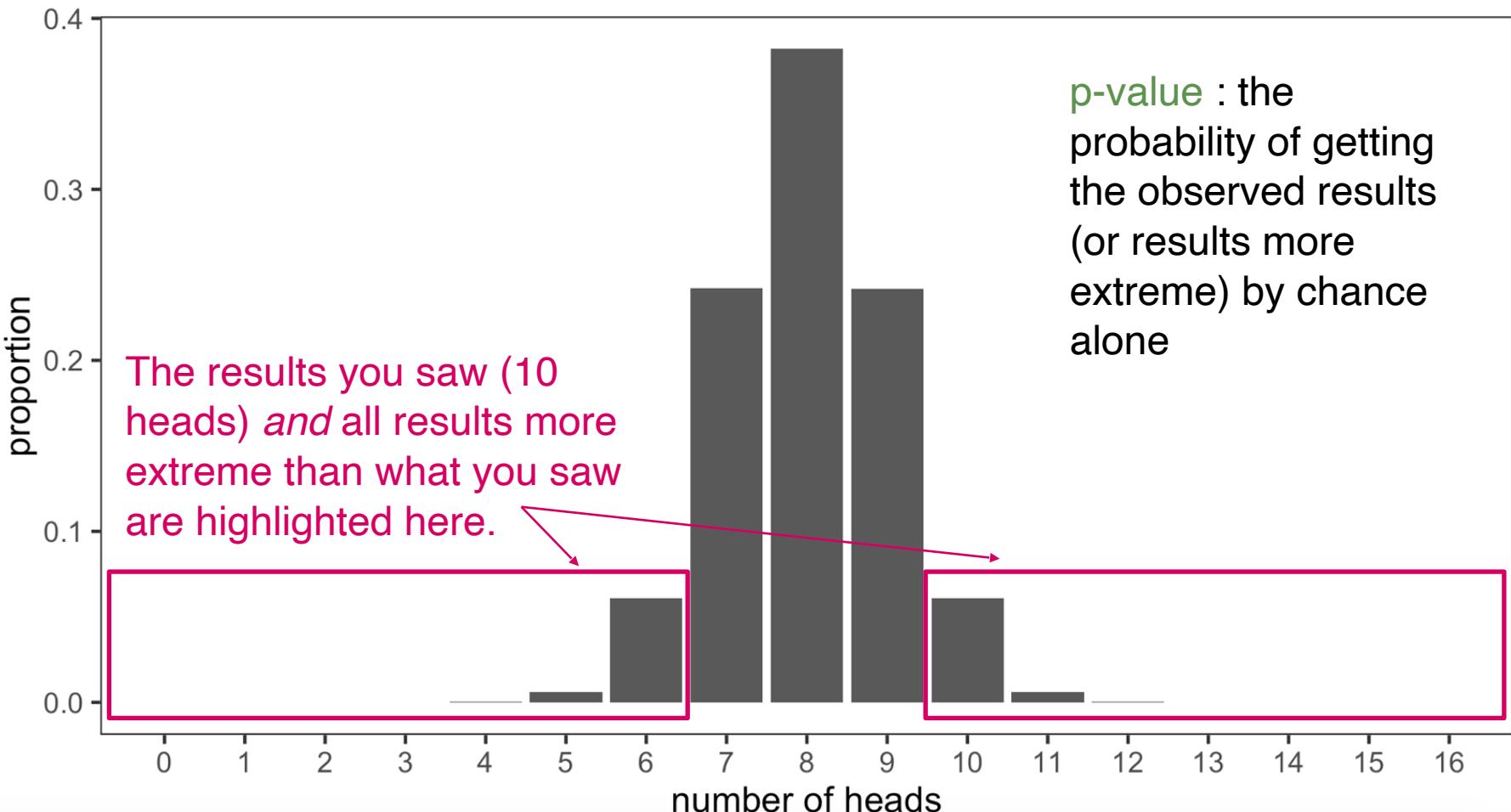
If we flip a coin 16 times and
record the number of heads....
....and then do that 1M times

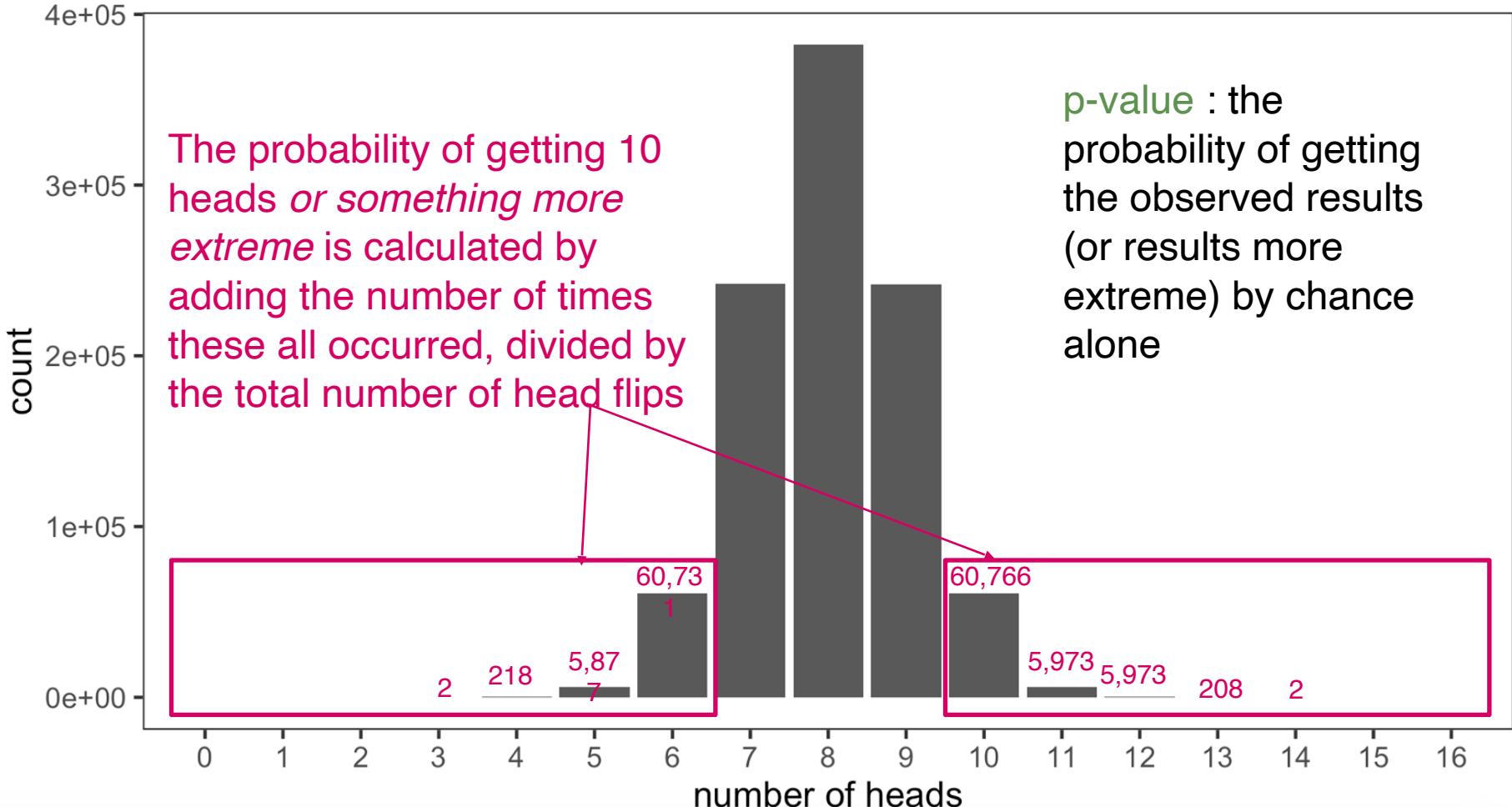


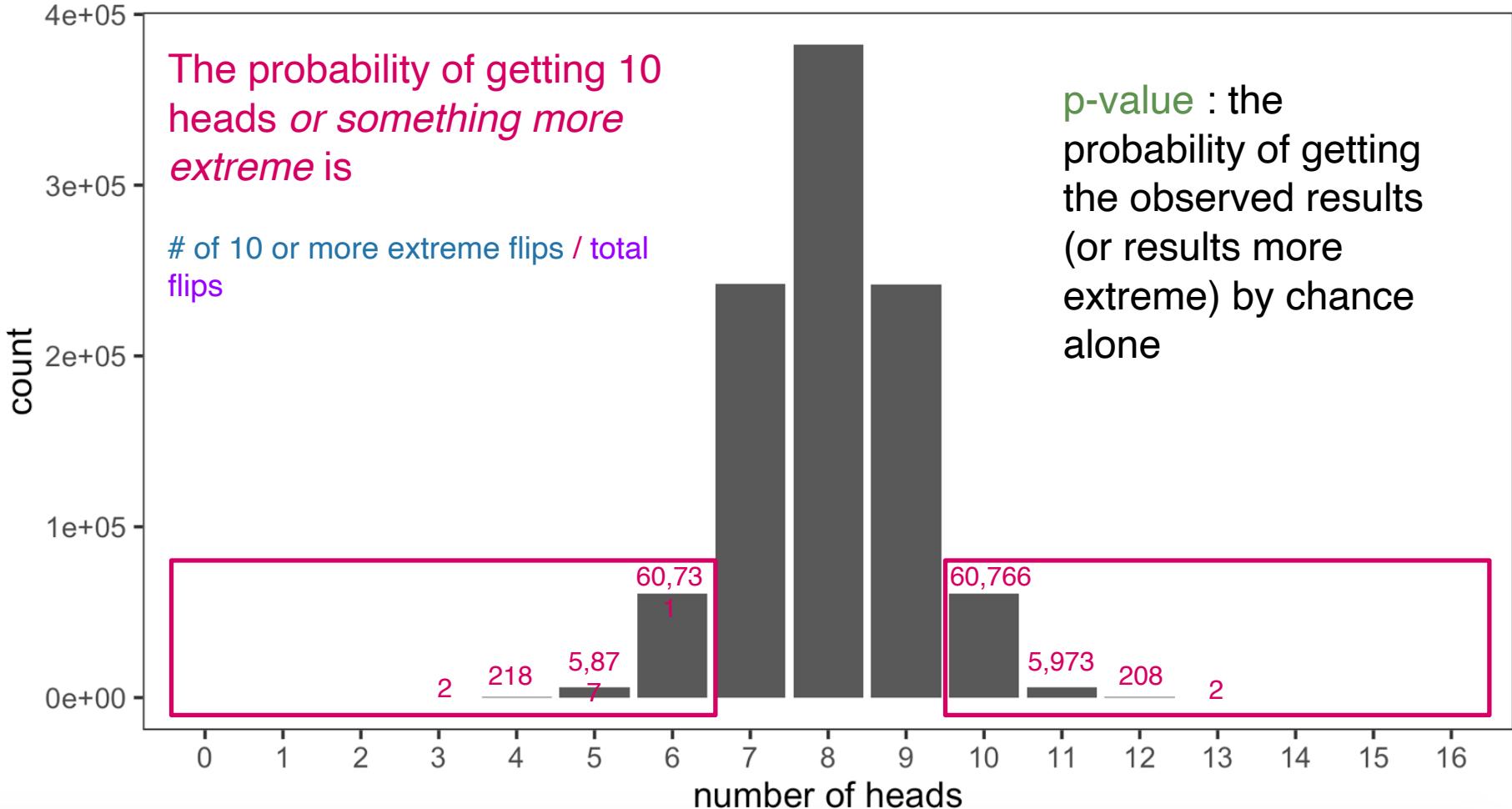


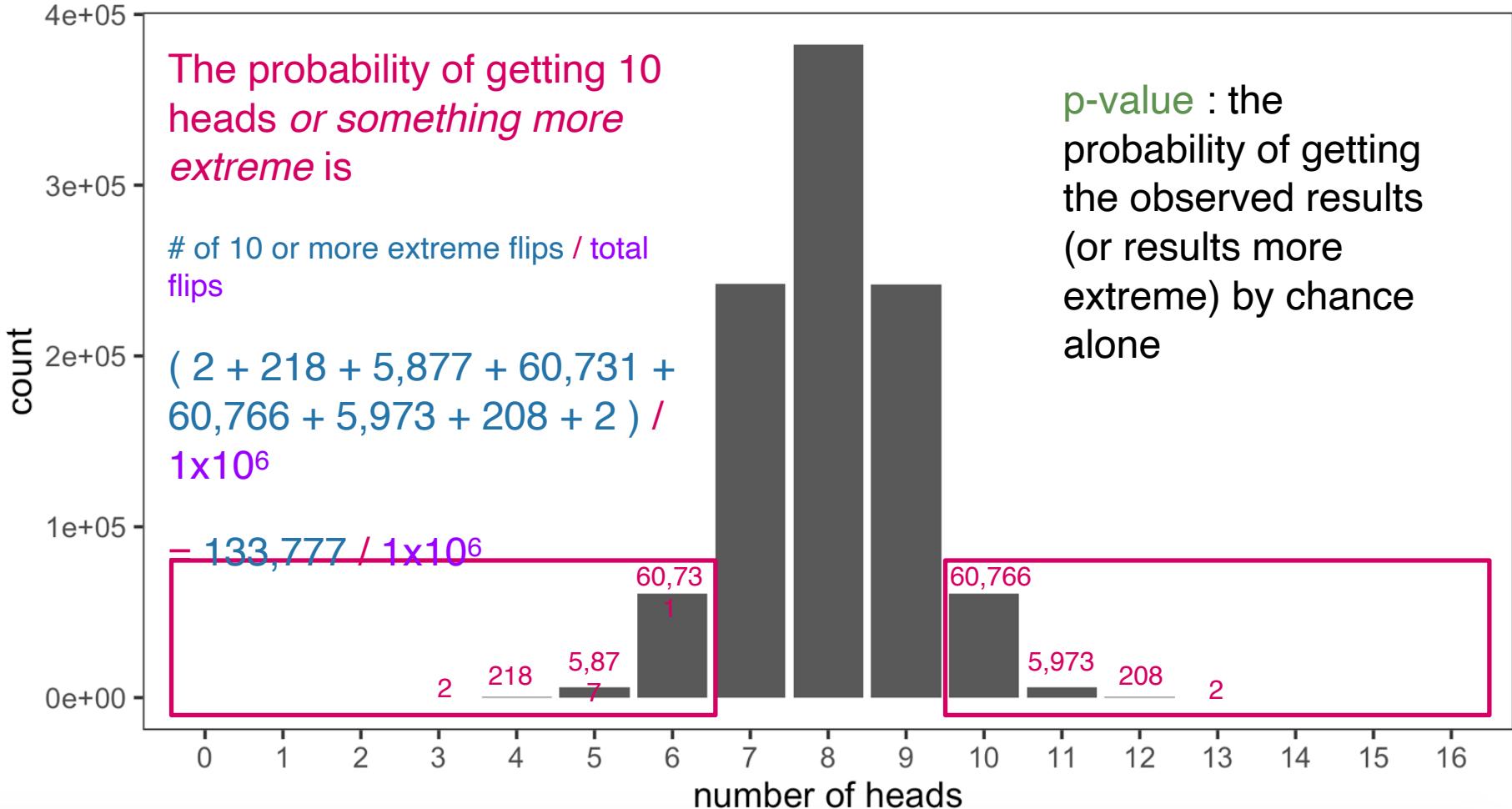


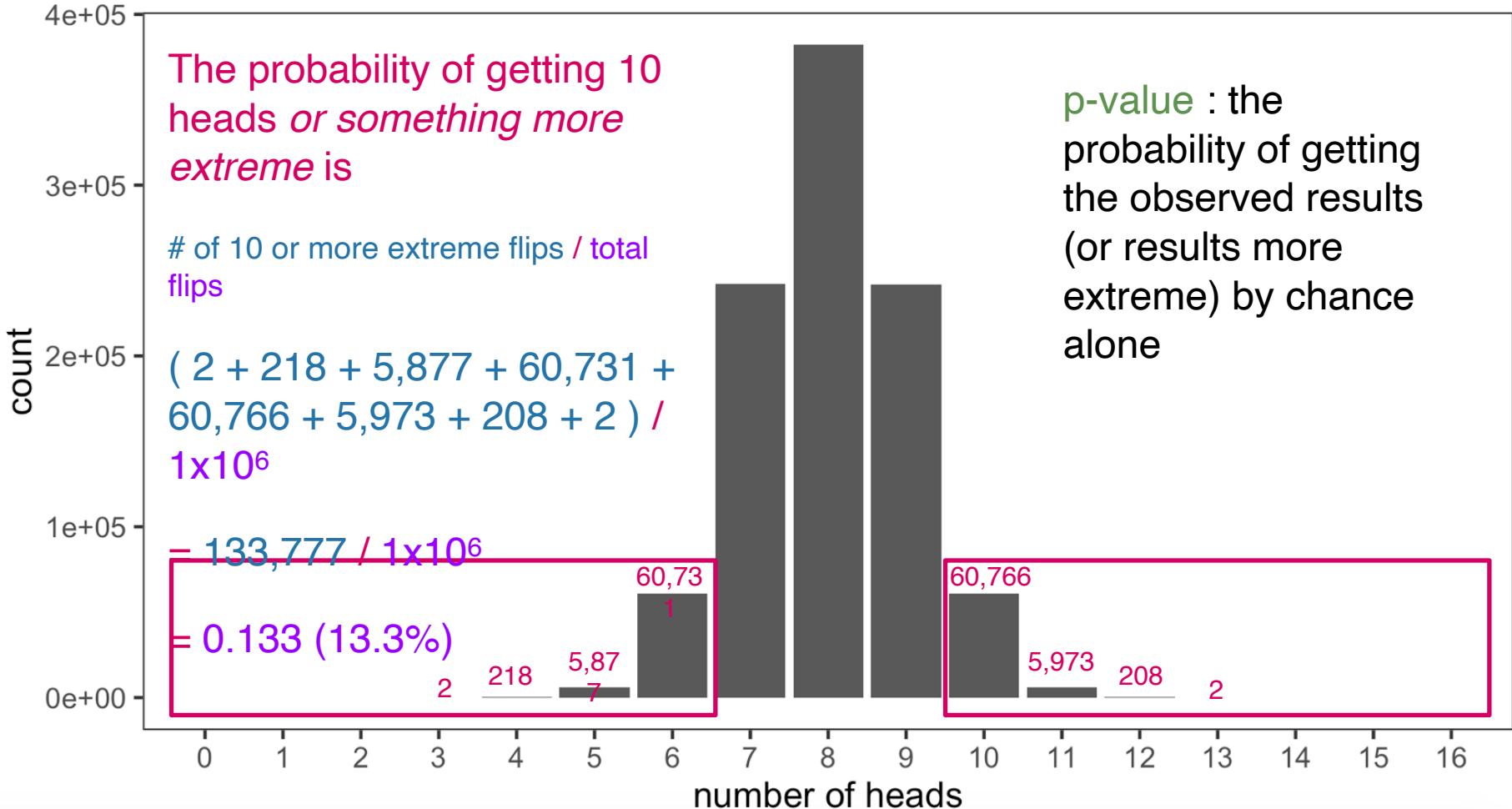


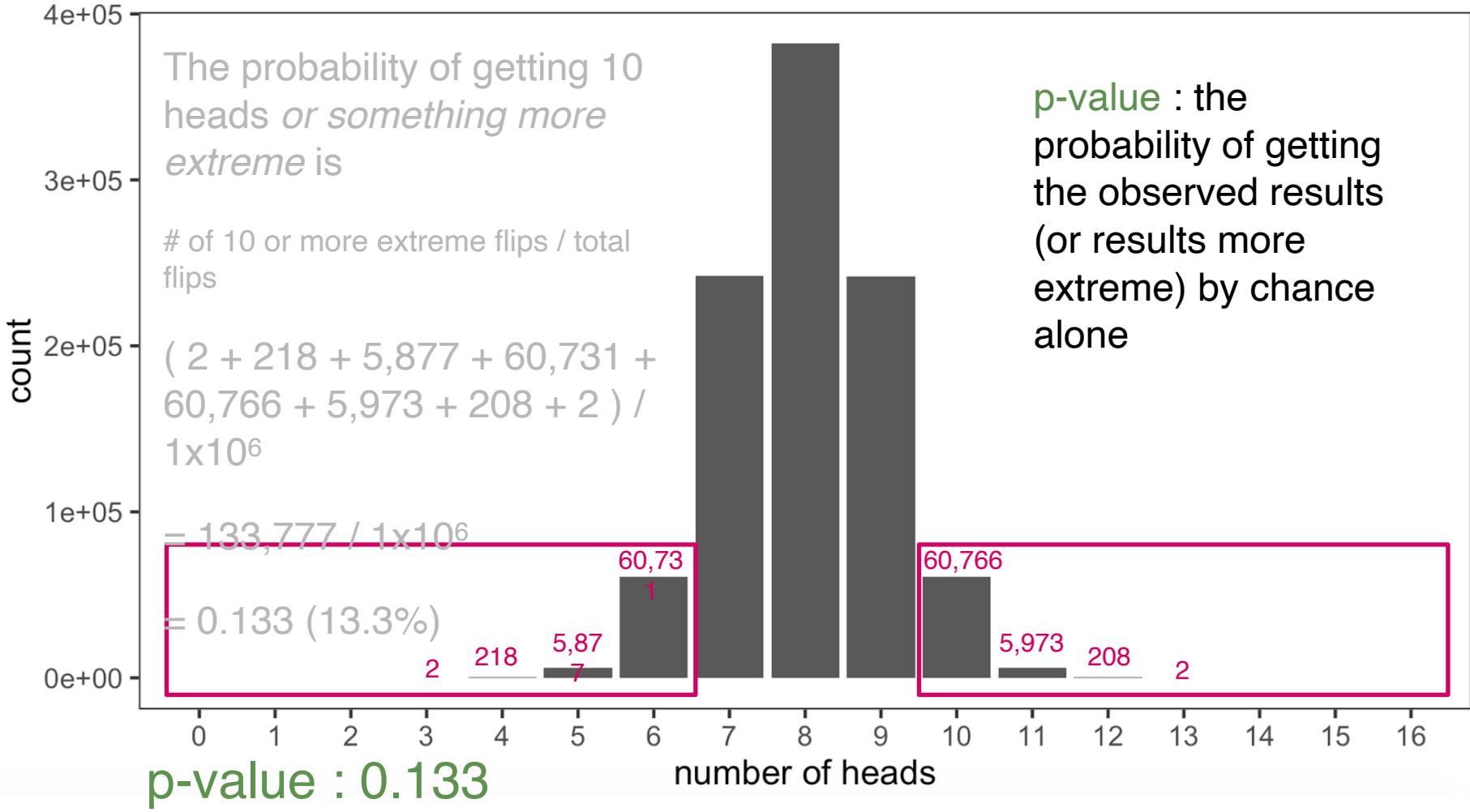


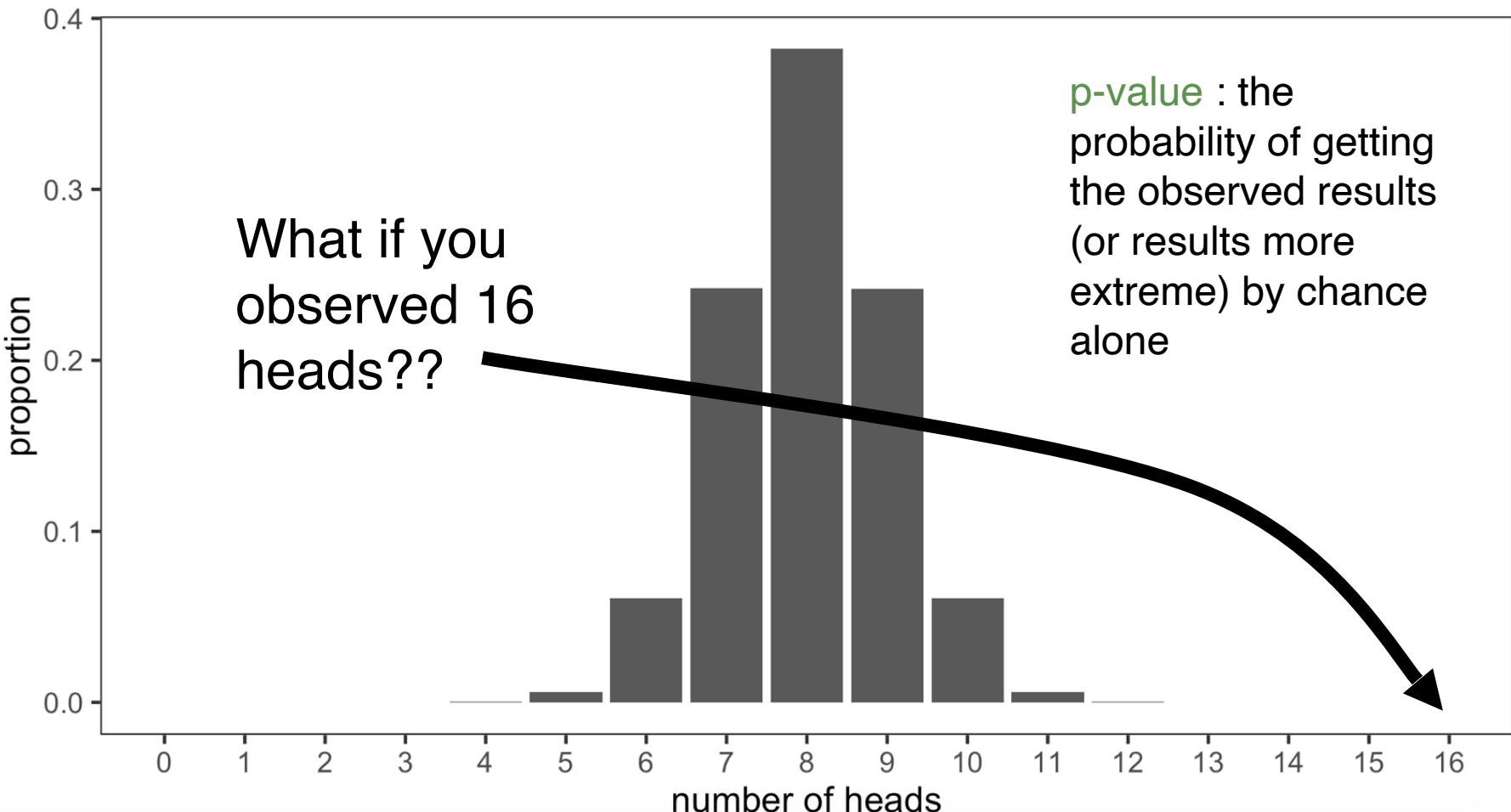


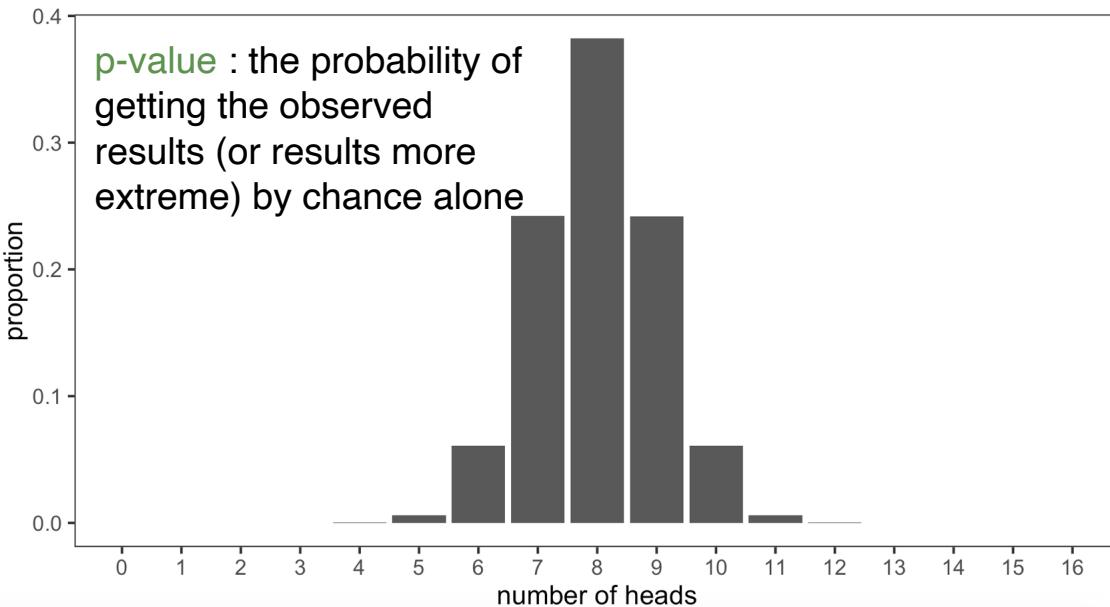












What would be the p-value of you flipping 16 heads?



A
 < 0.13



B
 > 0.13

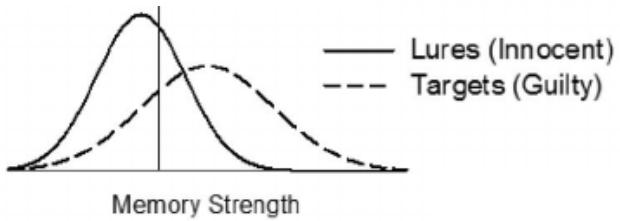


The interpretation of p-values in NHST

- Null hypothesis statistical testing means we assume there is no difference
- P-value represents the evidence against the null
 - What's the likelihood of getting values this weird given an assumption that A and B come from the same distribution?
- P-value is NOT the chance of a Type I error! This is a common mistake!
- When your data are very inconsistent with the null hypothesis, P values can't determine which of the following two possibilities is more probable:
 - The null hypothesis is true, but your sample is unusual due to random sampling error.
 - The null hypothesis is false.



Difference in Means



Why would a t-test *not* be appropriate for these data?

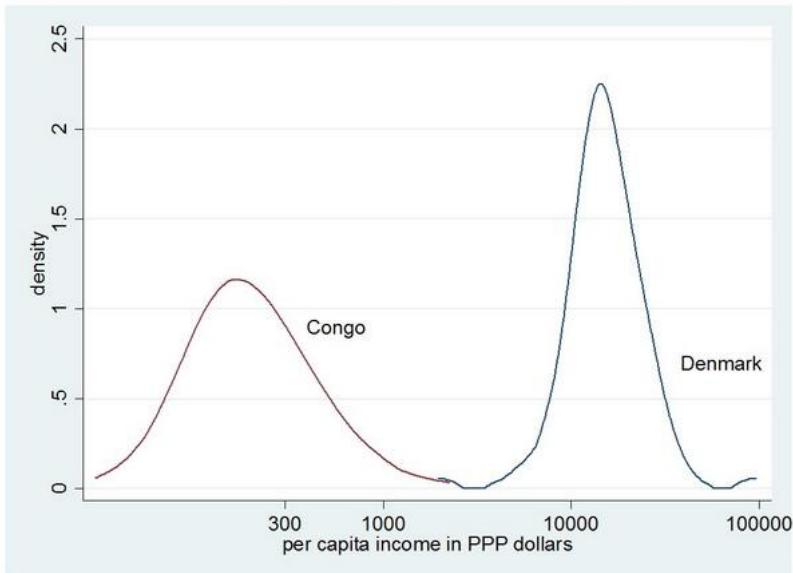


A
Not normally distributed

B
Unequal variances

C
Small sample size

D
Data are not continuous



Would a t-test find a significant difference in means?

A
t-test not appropriate

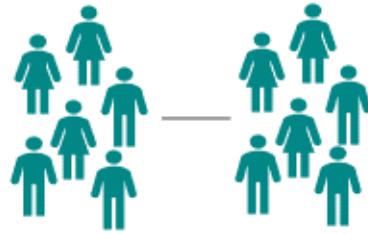
B
Yes

C
No

D
Need more information

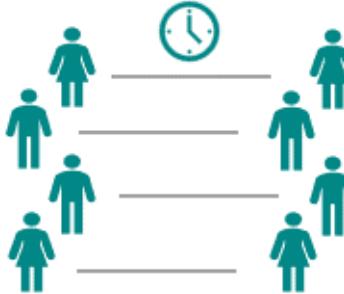
Paired data

Independent samples t-test



Is there a **difference** between
two groups

Paired samples t-test



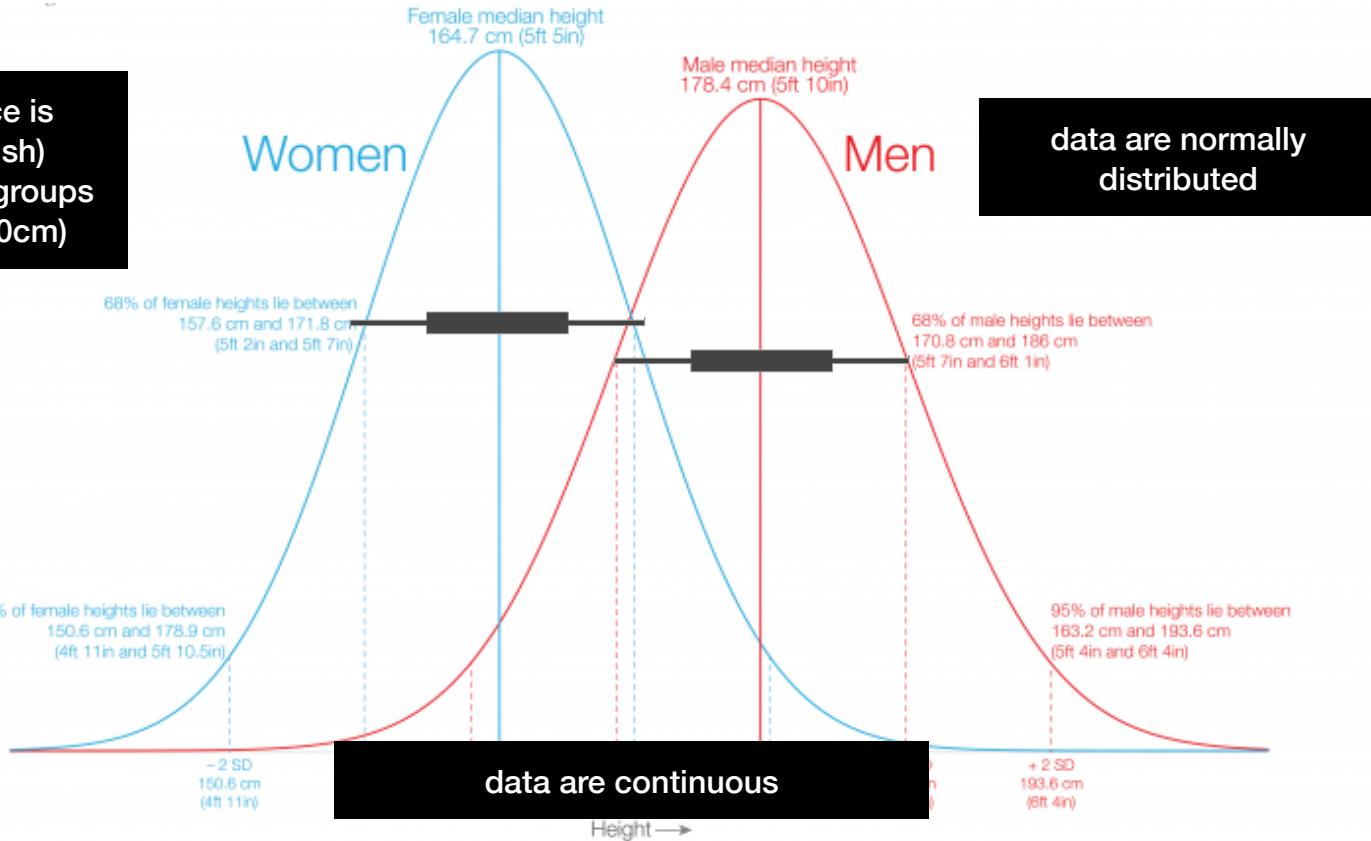
Is there a **difference in a group**
between **two points in time**

Do the heights between males and females differ?

variance is equal(ish) between groups (57 vs 50cm)

sample size affects statistic

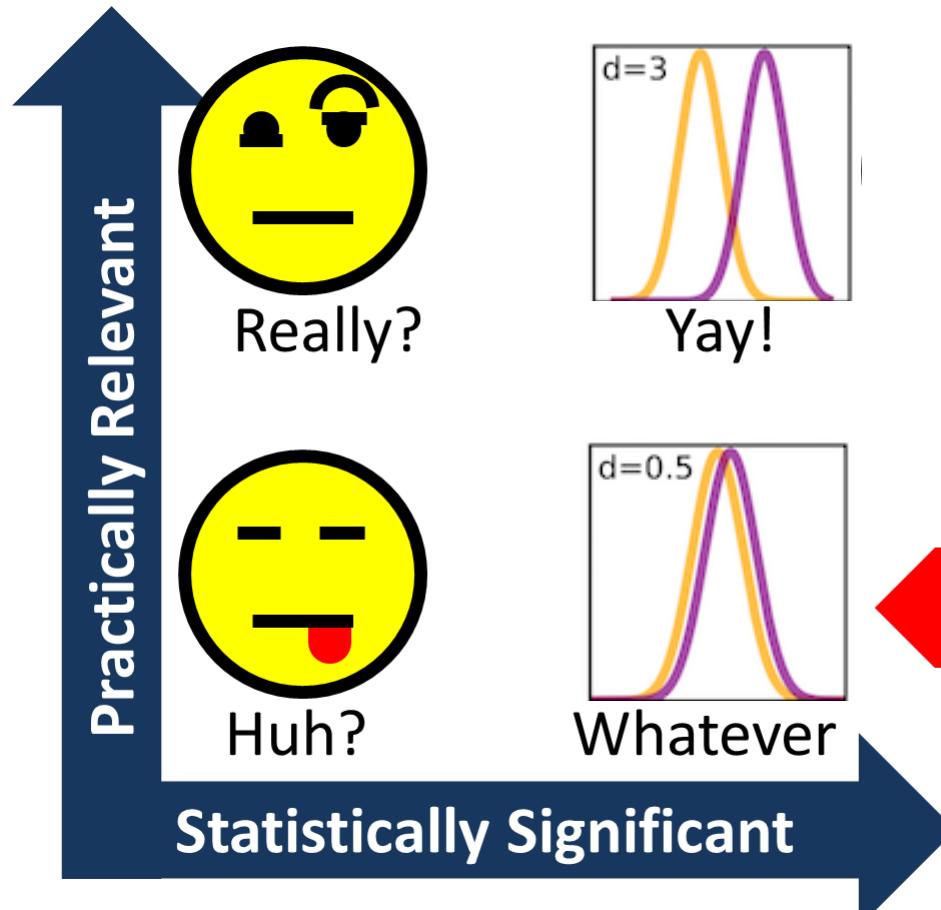
N=10,000



Cohen's d

Cohen's d is defined as the difference between two means divided by a standard deviation for the data

Effect sizes!



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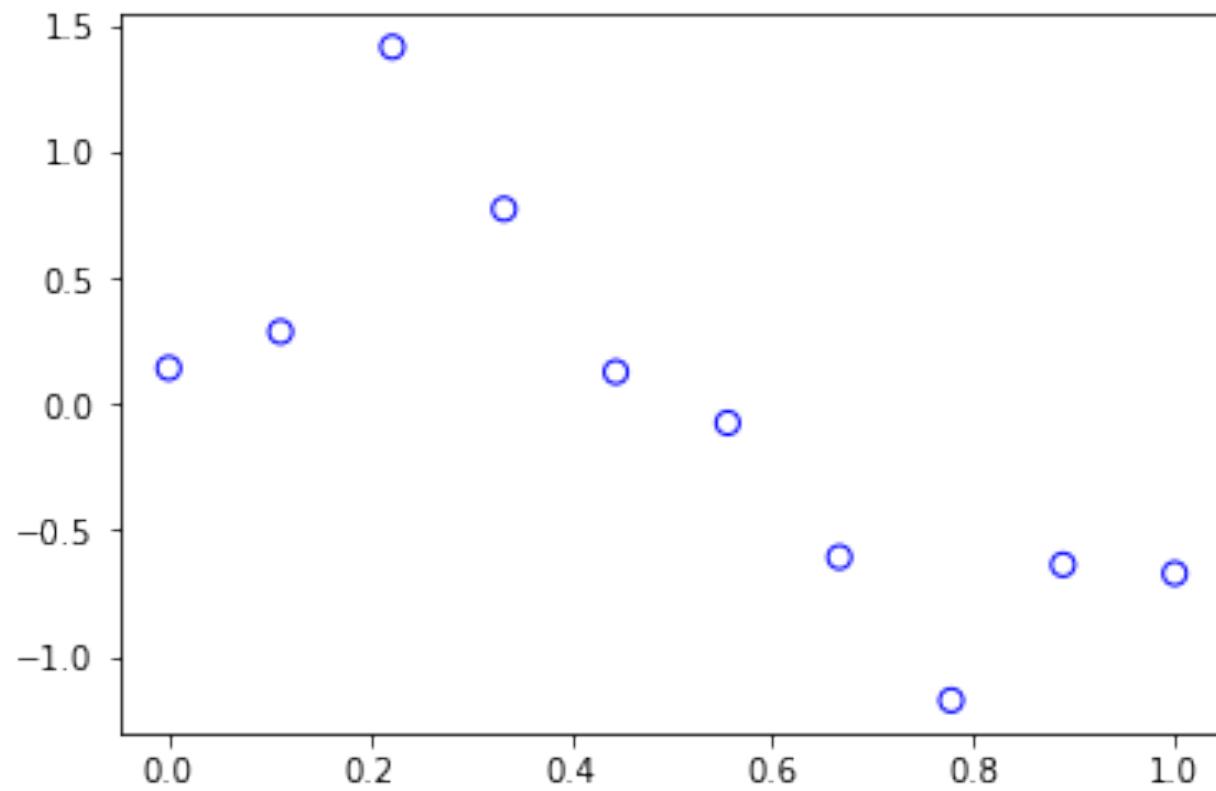
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i.e. Wilcoxon rank-
sum test, Wilcoxon
sign-rank test, sign
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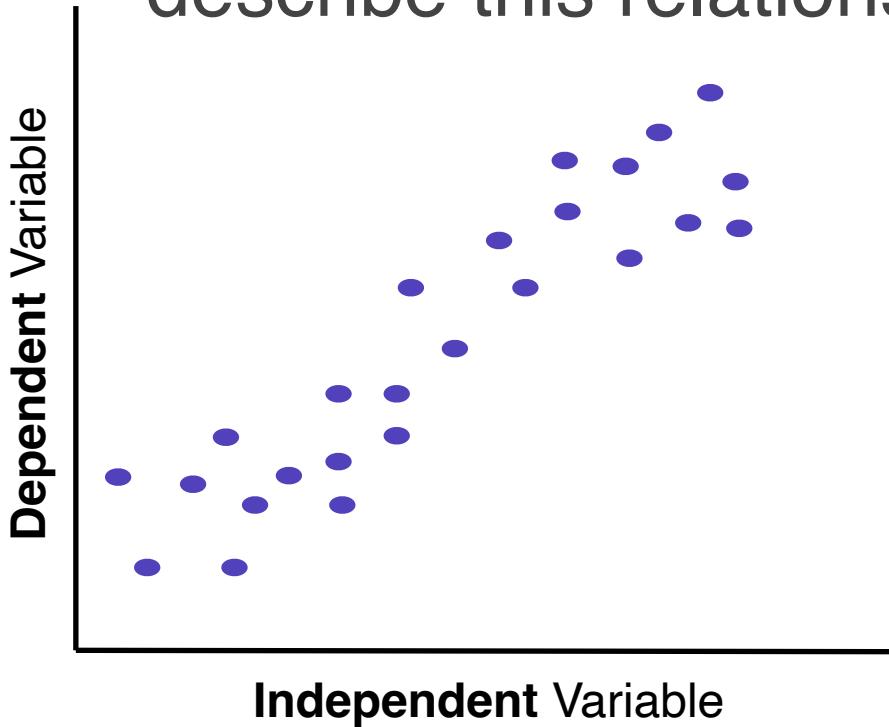
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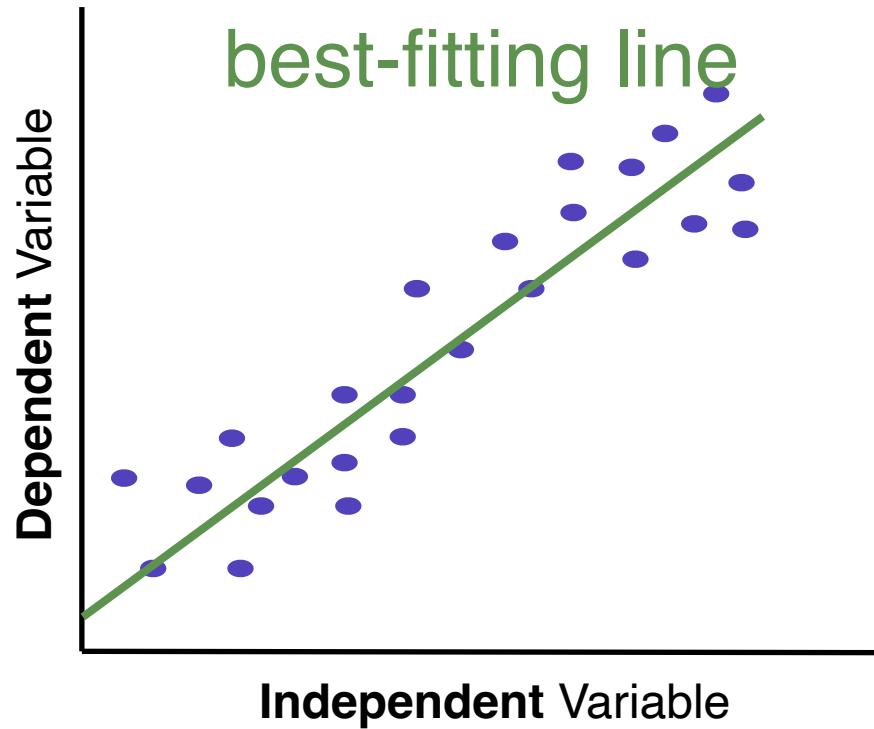
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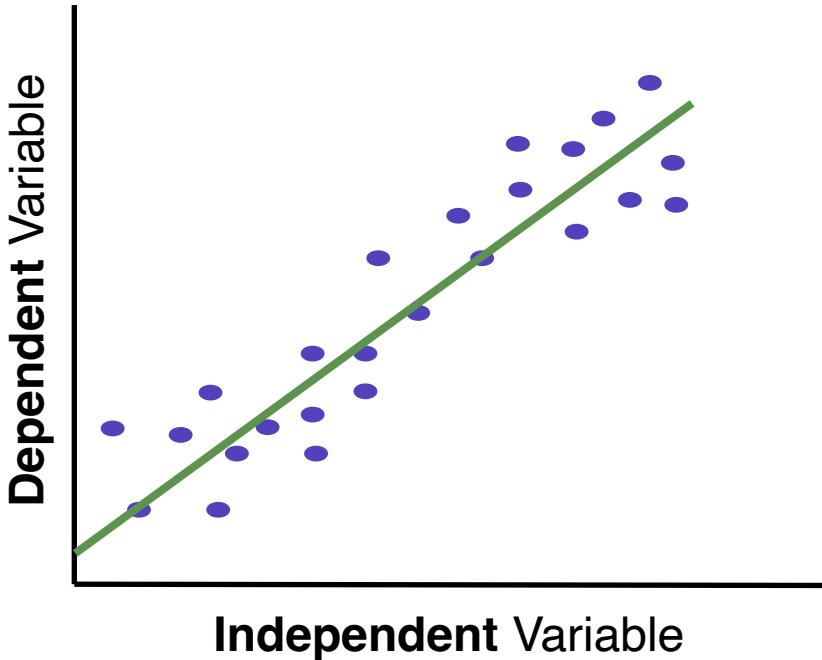
i.e. Wilcoxon rank-
sum test, Wilcoxon
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test

Linear regression can be used to describe this relationship

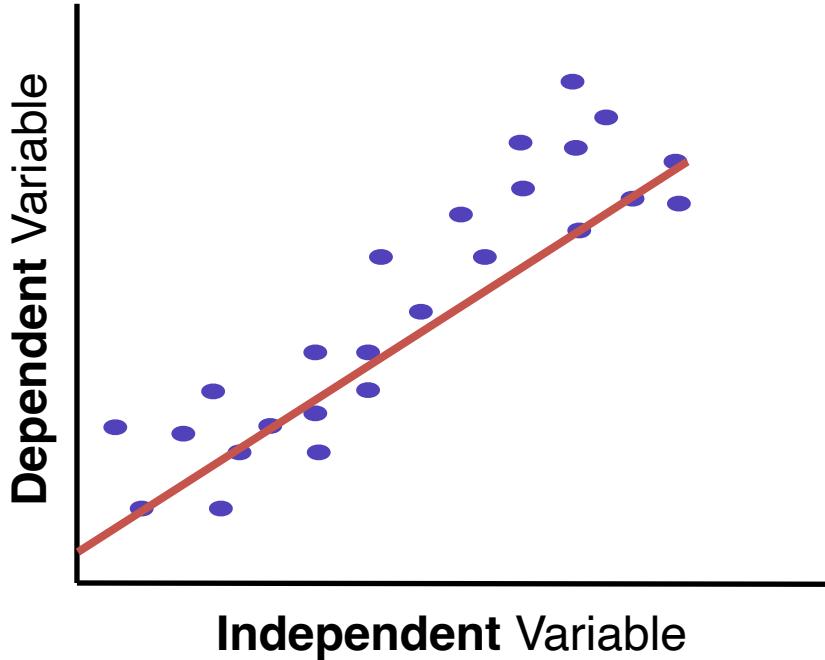


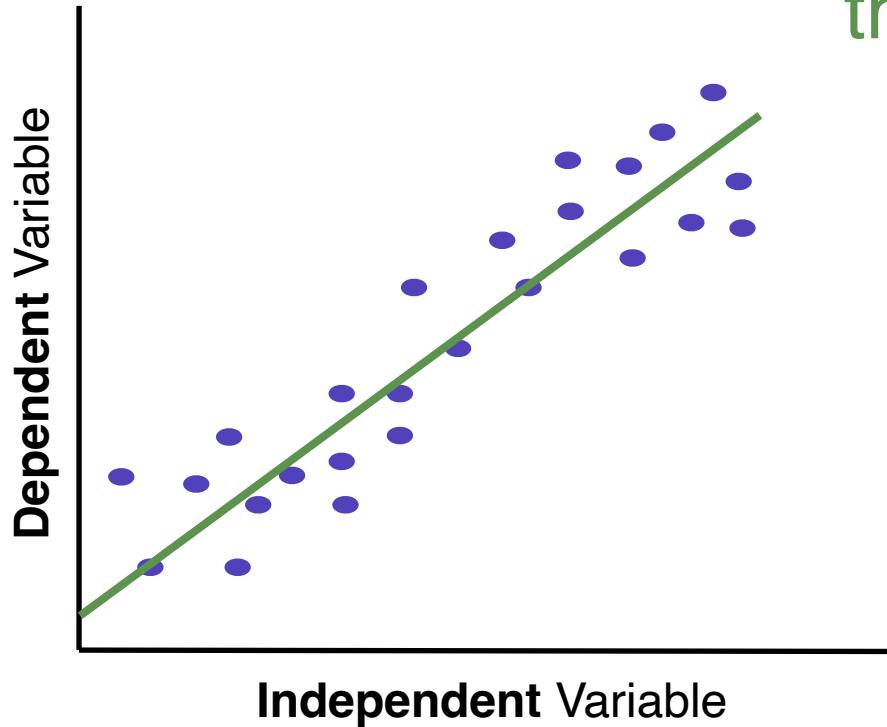


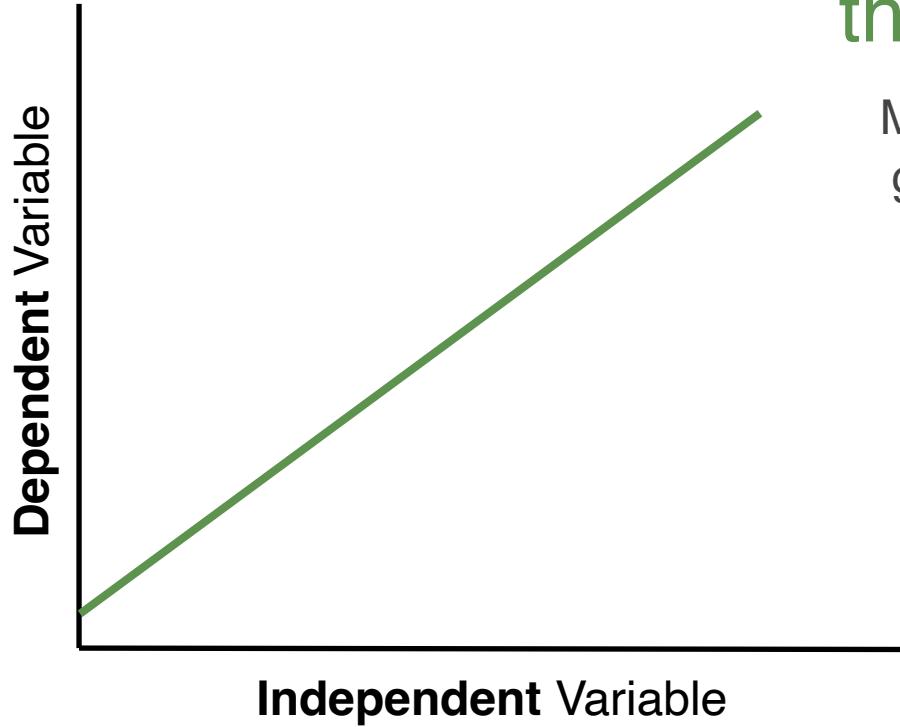
Best-fitting line



NOT a best-fitting line







This line is a model of the data

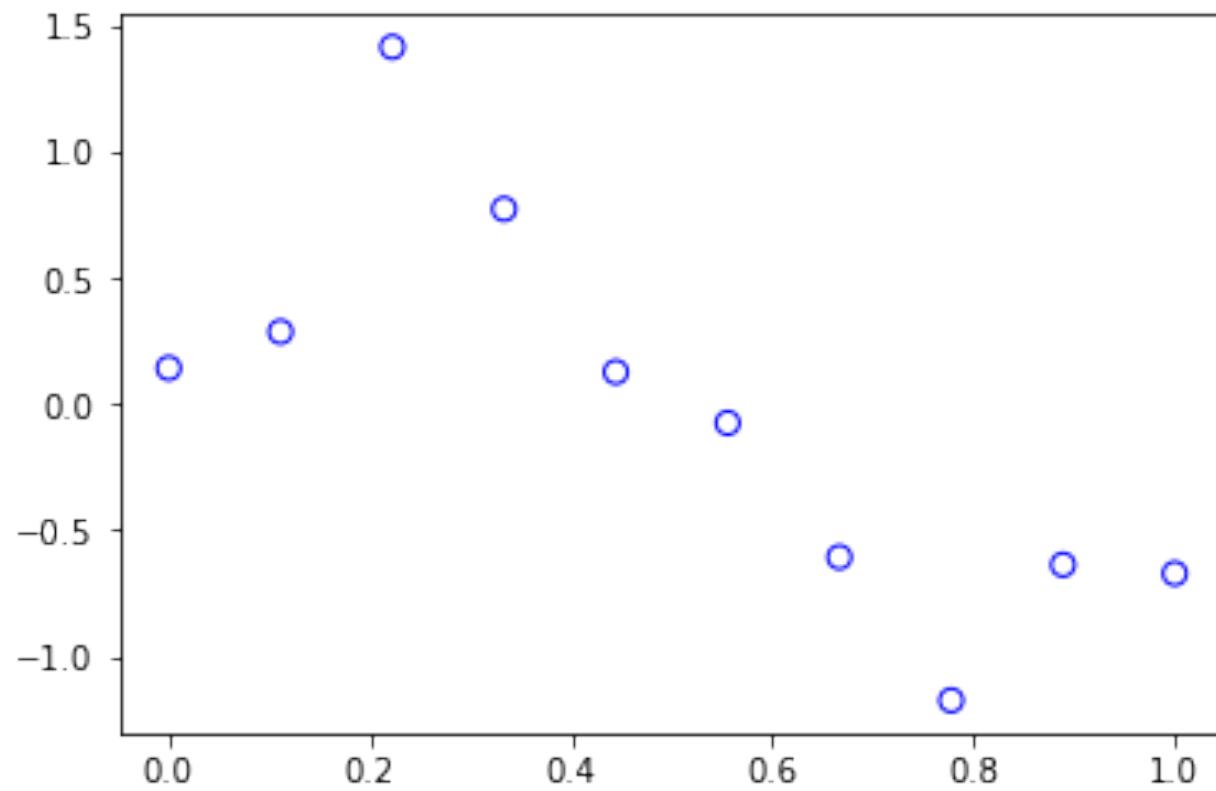
Models are mathematical equations generated to *represent* the real life situation

2.3 Parsimony

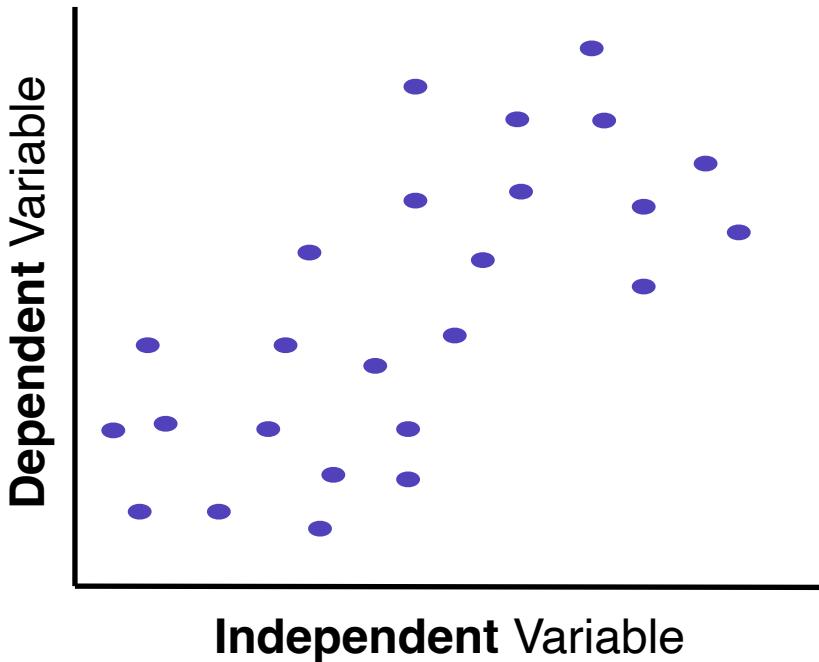
Since all models are wrong the scientist cannot obtain a “correct” one by excessive elaboration. On the contrary following William of Occam he should seek an economical description of natural phenomena. Just as the ability to devise simple but evocative models is the signature of the great scientist so overelaboration and overparameterization is often the mark of mediocrity.

2.4 Worrying Selectively

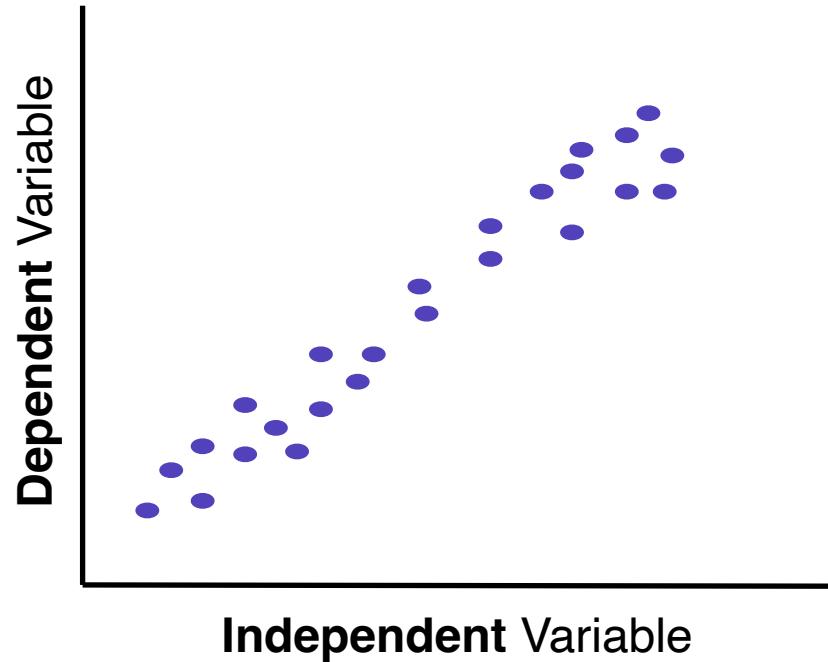
Since all models are wrong the scientist must be alert to what is importantly wrong. It is inappropriate to be concerned about mice when there are tigers abroad.



weaker relationship



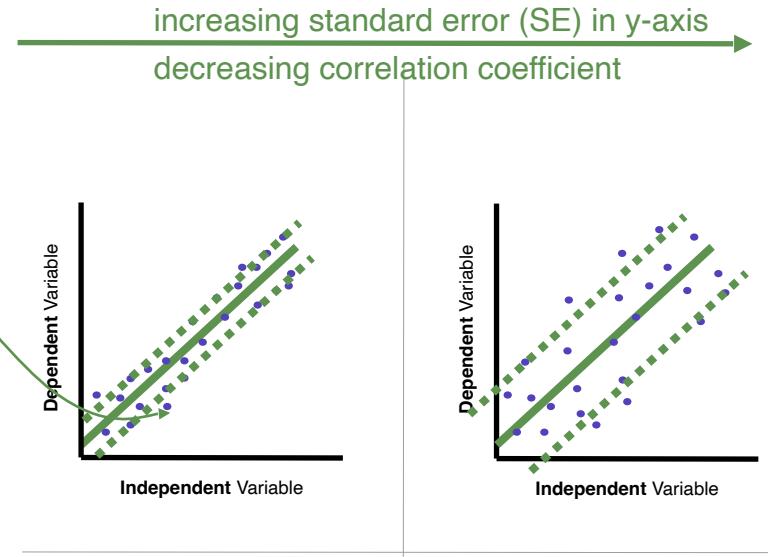
stronger relationship



stronger relationship = higher correlation

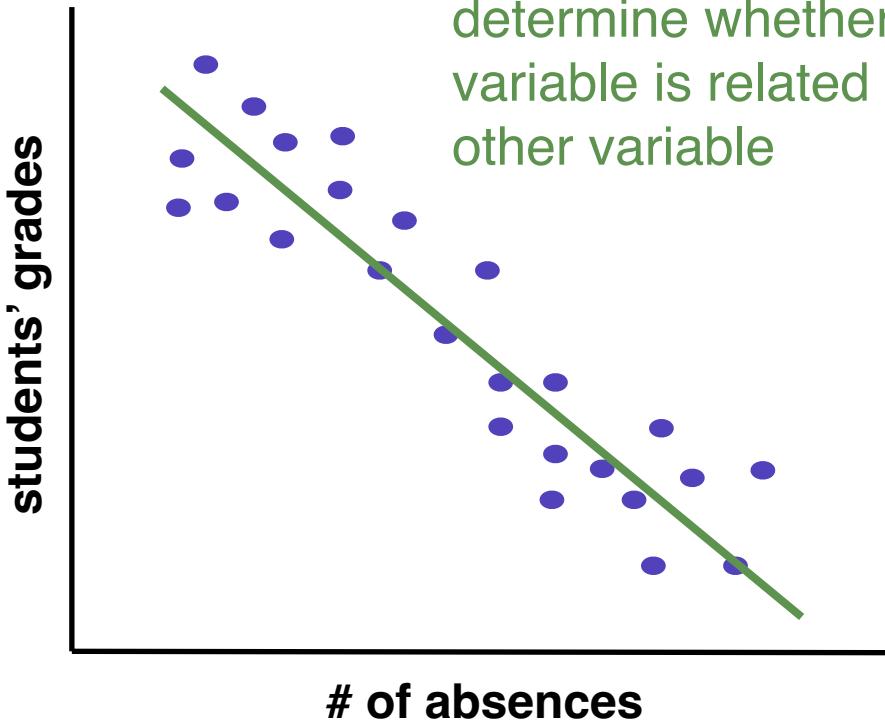
This is a kind of effect size

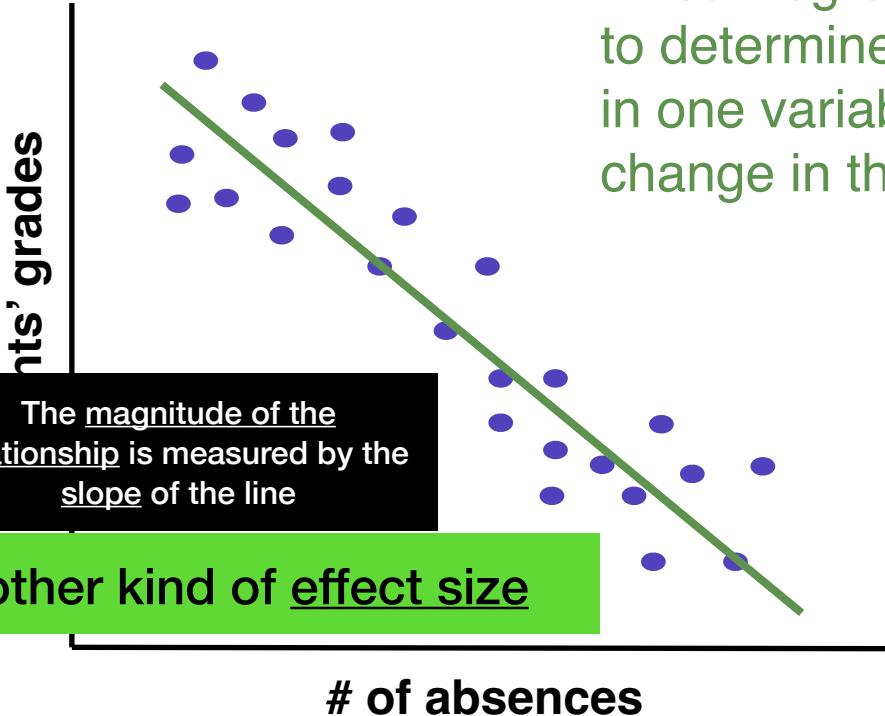
The *closer* the points are to the regression line, the *less uncertain* we are in our estimate



Standard error is standard deviation / \sqrt{n}

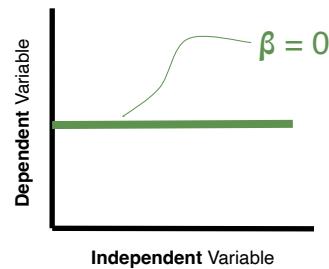
Linear regression can be used to determine whether a change in one variable is related to the change in the other variable



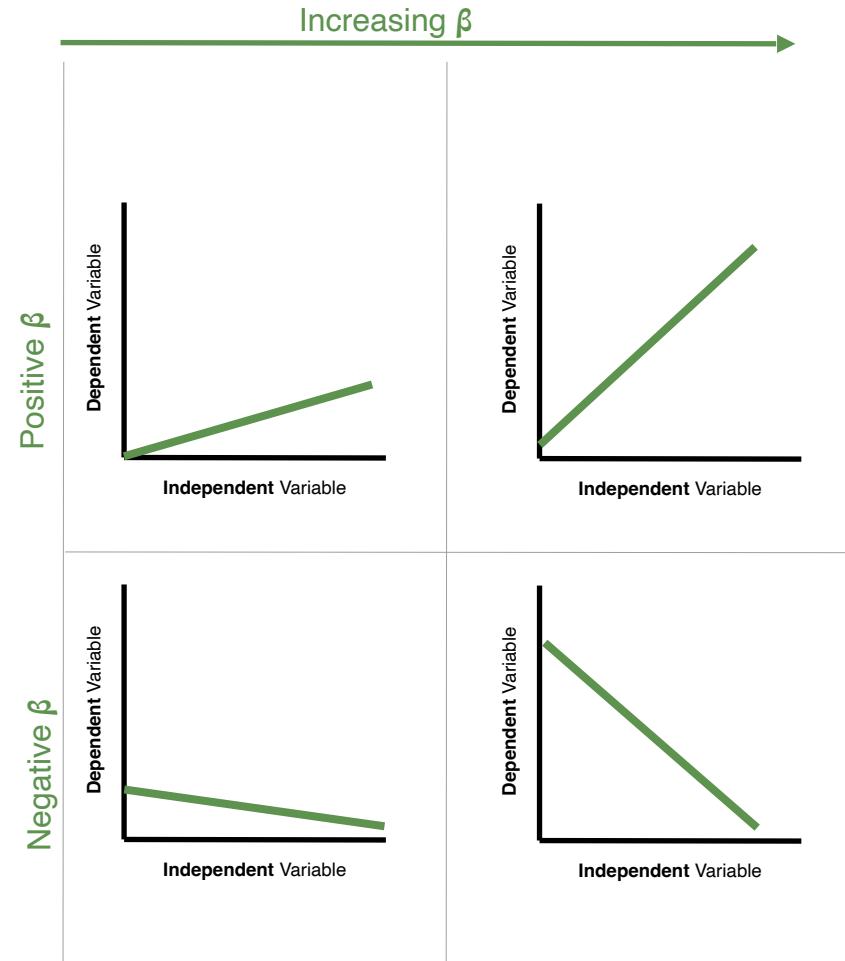
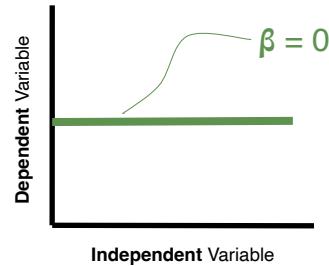


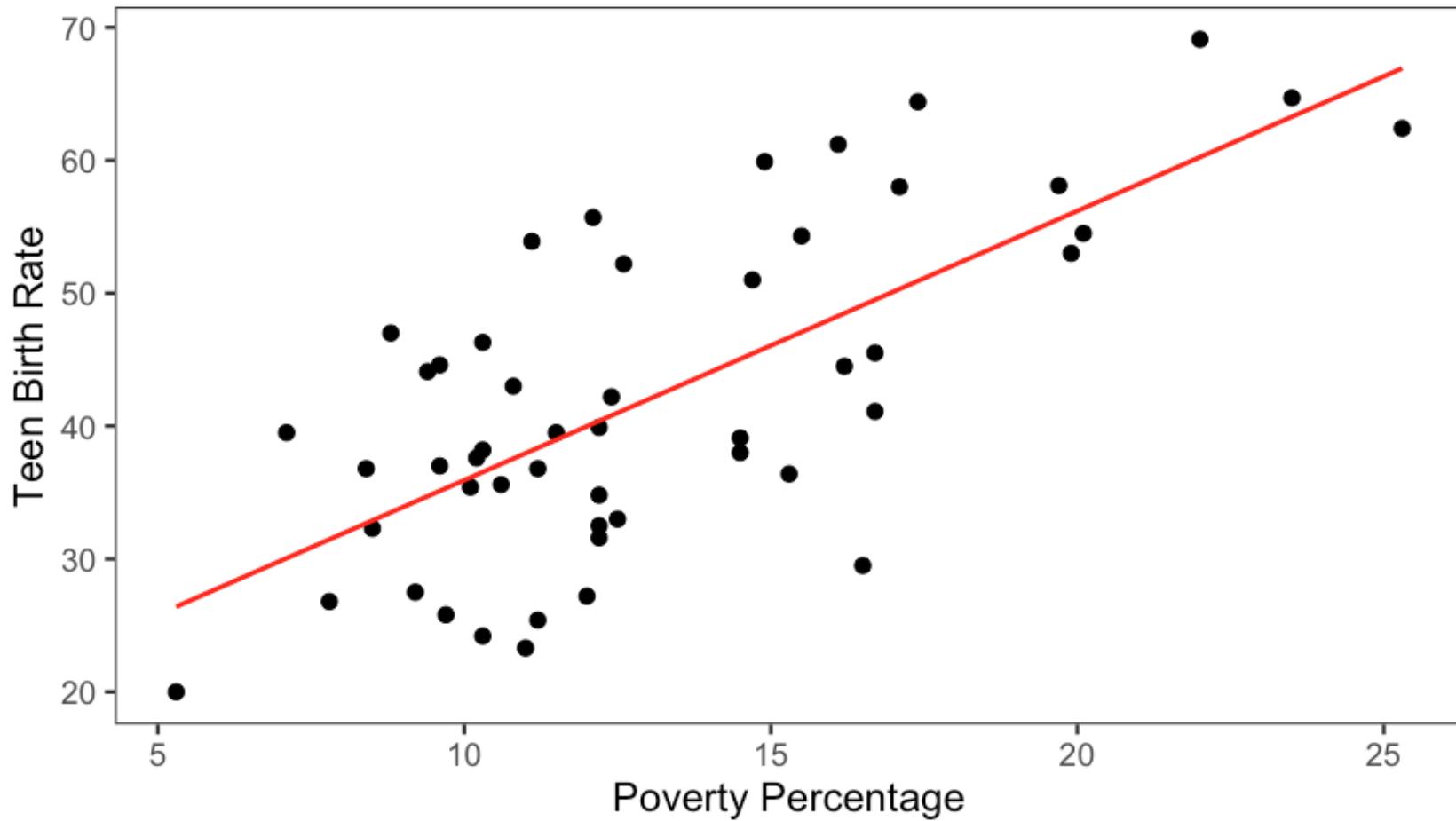
Linear regression can be used to determine whether a change in one variable is related to the change in the other variable

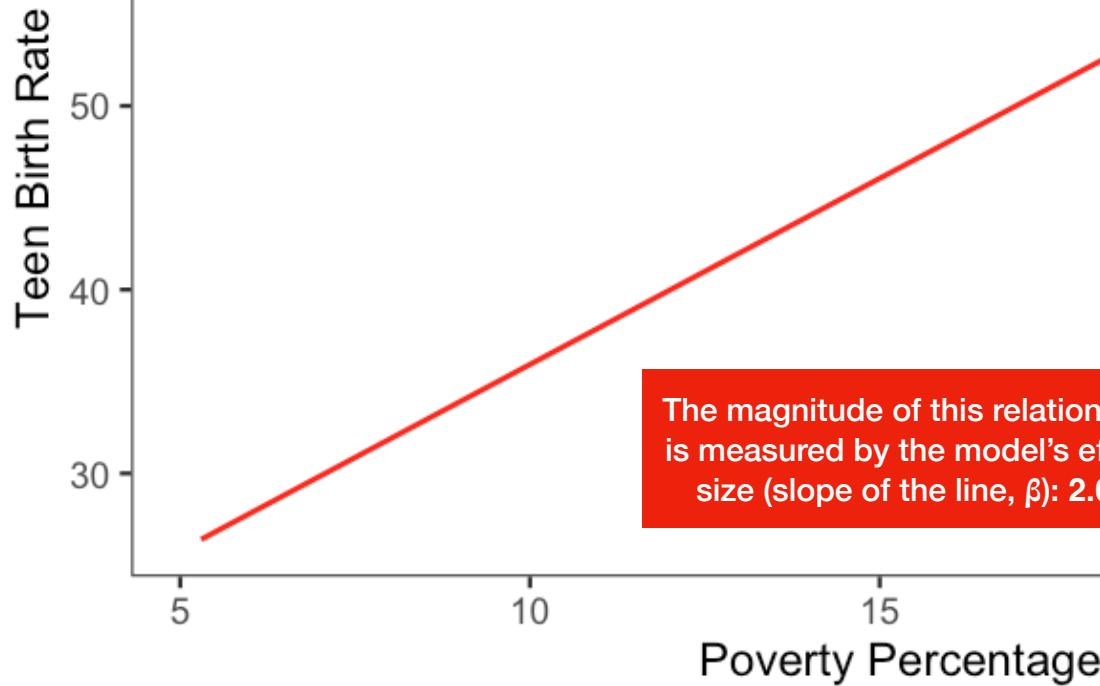
Effect size (β) can
be estimated using
the slope of the line



Effect size (β) can
be estimated using
the slope of the line







The regression line is the model being used to explain the relationship between Poverty Percentage and Birth Rate

The magnitude of this relationship is measured by the model's effect size (slope of the line, β): 2.03

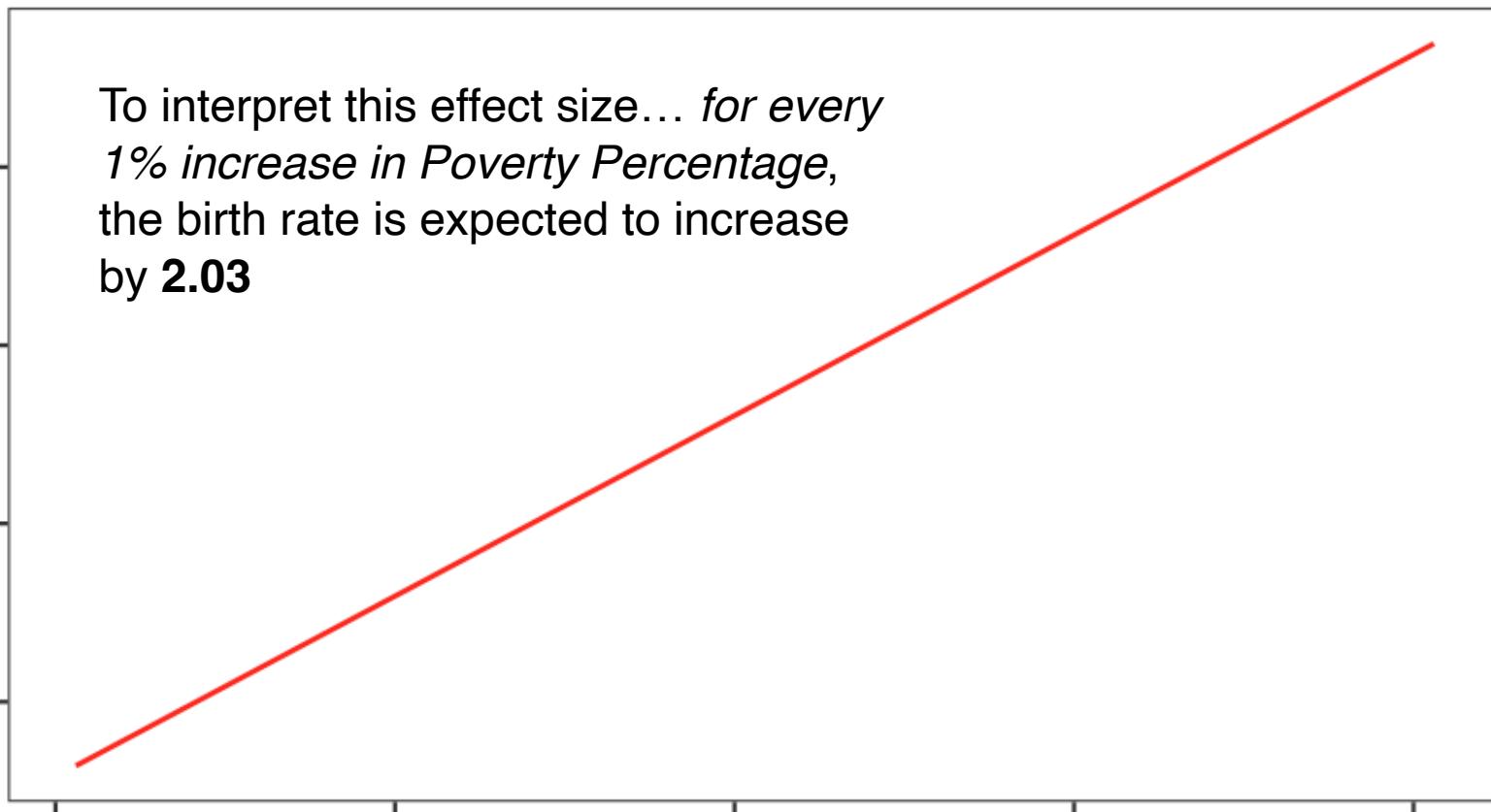
Teen Birth Rate

60
50
40
30

5 10 15 20 25

Poverty Percentage

To interpret this effect size... *for every 1% increase in Poverty Percentage, the birth rate is expected to increase by 2.03*



Teen Birth Rate

60

50

40

30

5

10

15

20

25

Poverty Percentage

...but *how confident* are we in that estimate of the effect size?

For that...we need to look at the standard error (SE) on the estimate of slope

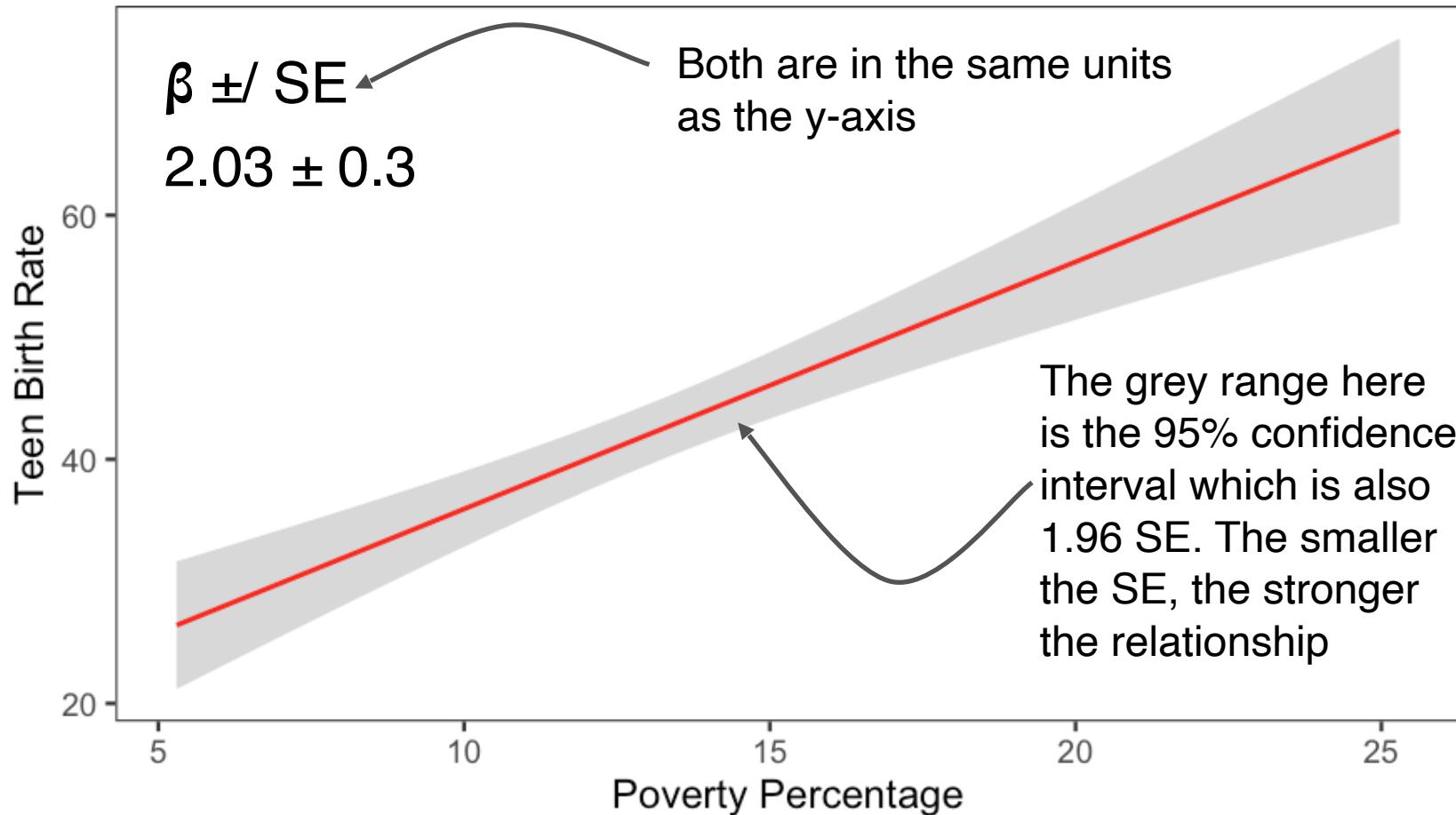
Formula

$$SE = \frac{\sigma}{\sqrt{n}}$$

SE = standard error of the sample

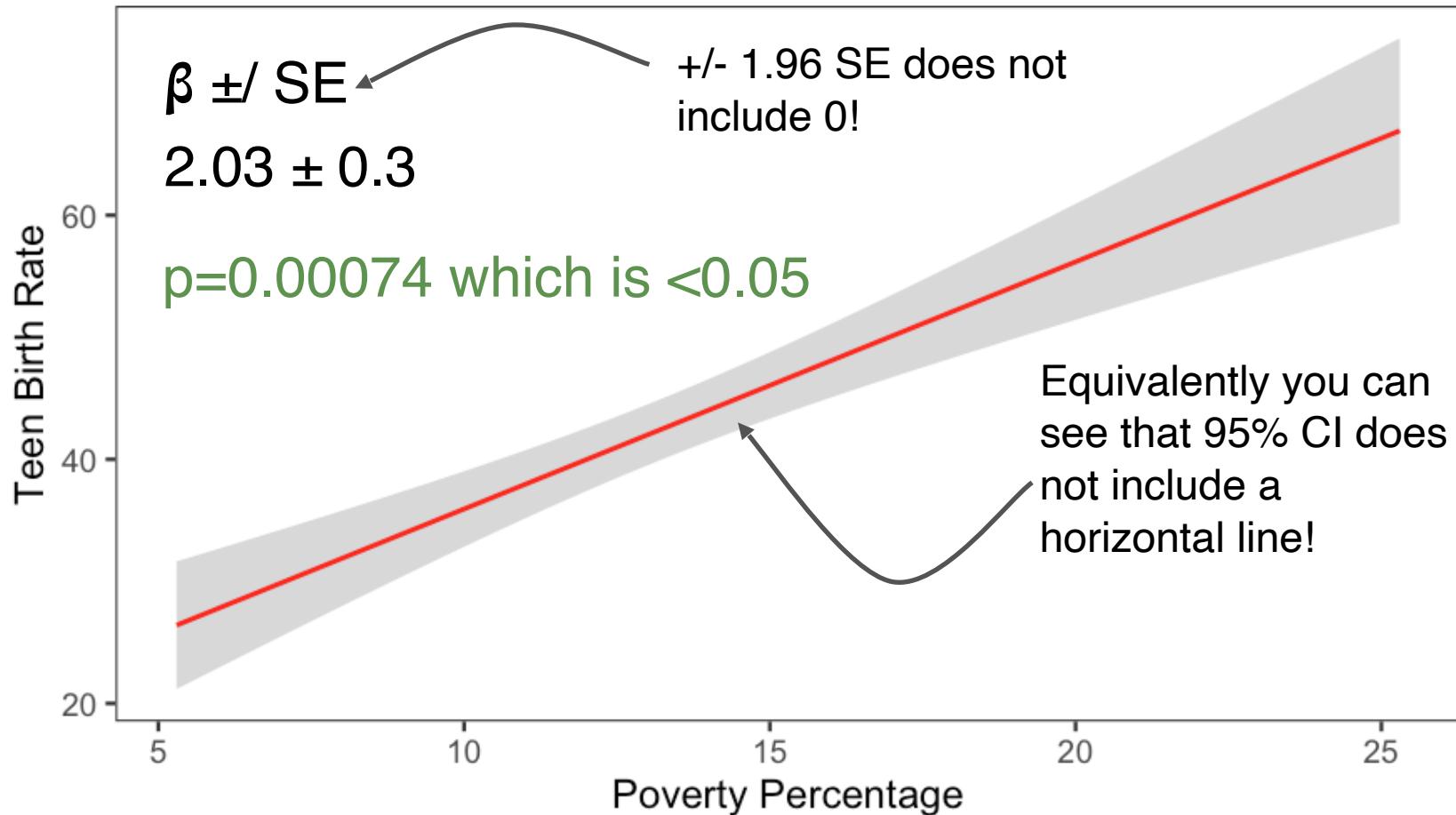
σ = sample standard deviation

n = number of samples



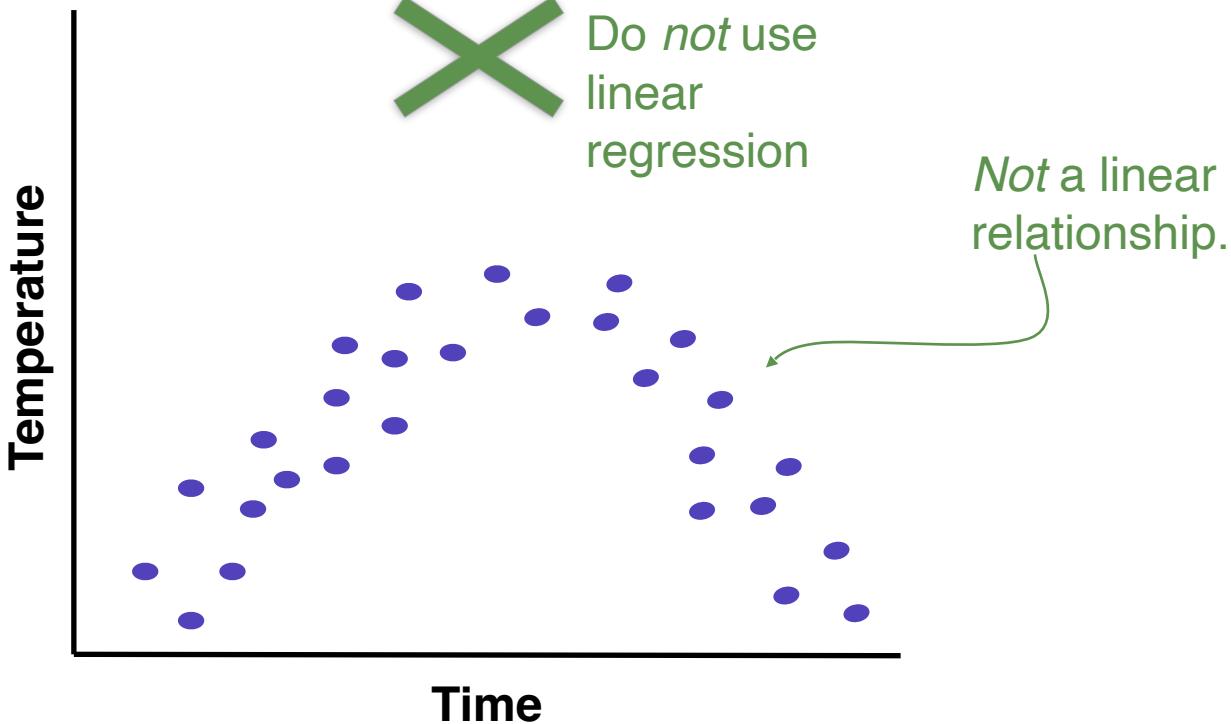
p-value : the probability of getting the observed results (or results more extreme) by chance alone

Takes into account the effect size (β) and the SE

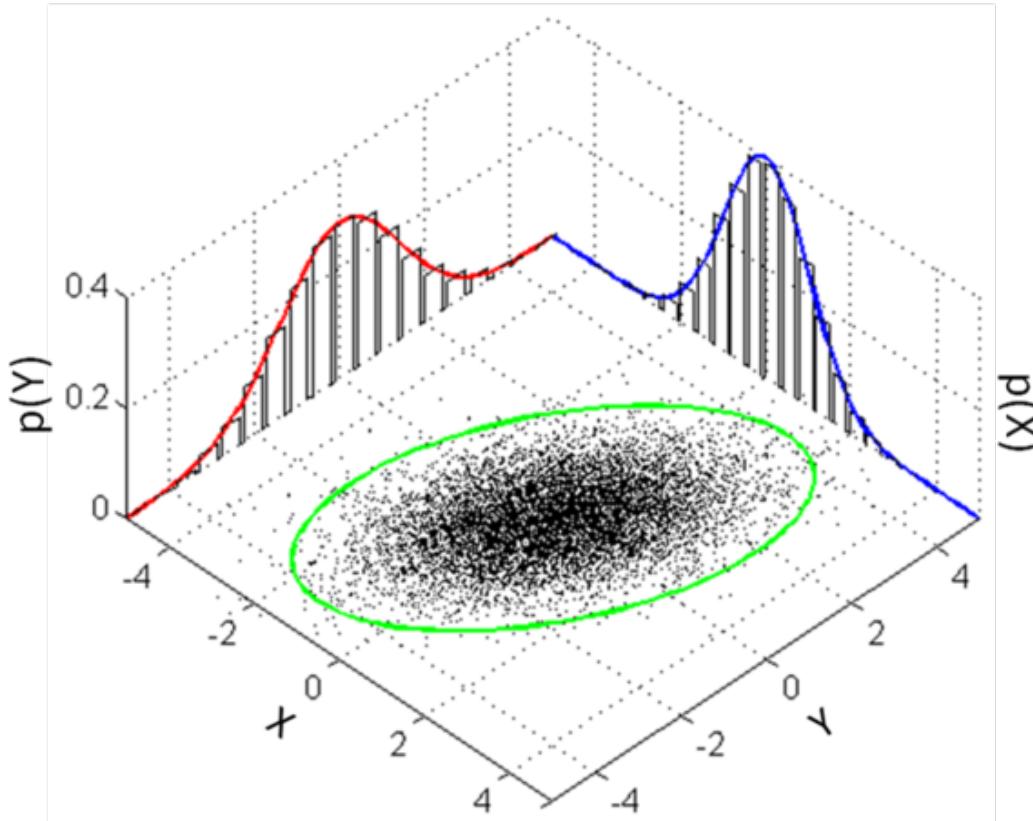


Assumptions of linear regression

1. Linear relationship
2. Multivariate normality
3. No multicollinearity
4. No autocorrelation
5. Homoscedasticity

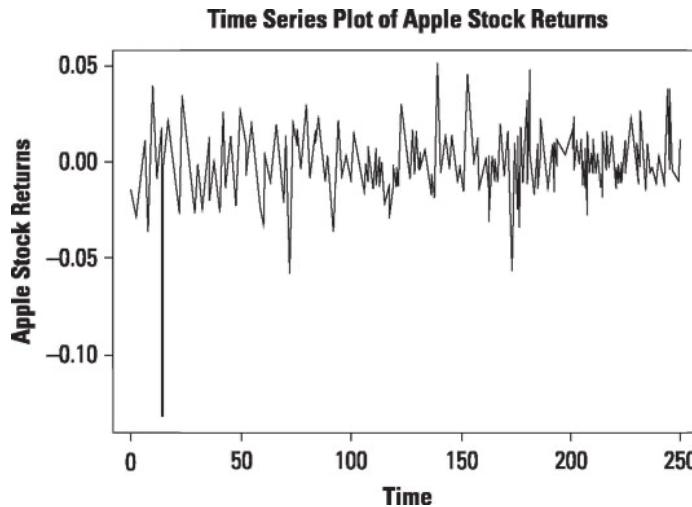
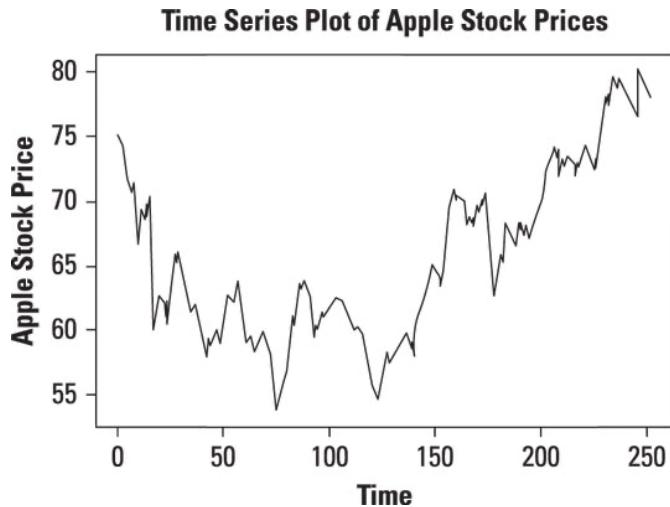


A multivariate
normal probability
distribution (joint
normal)

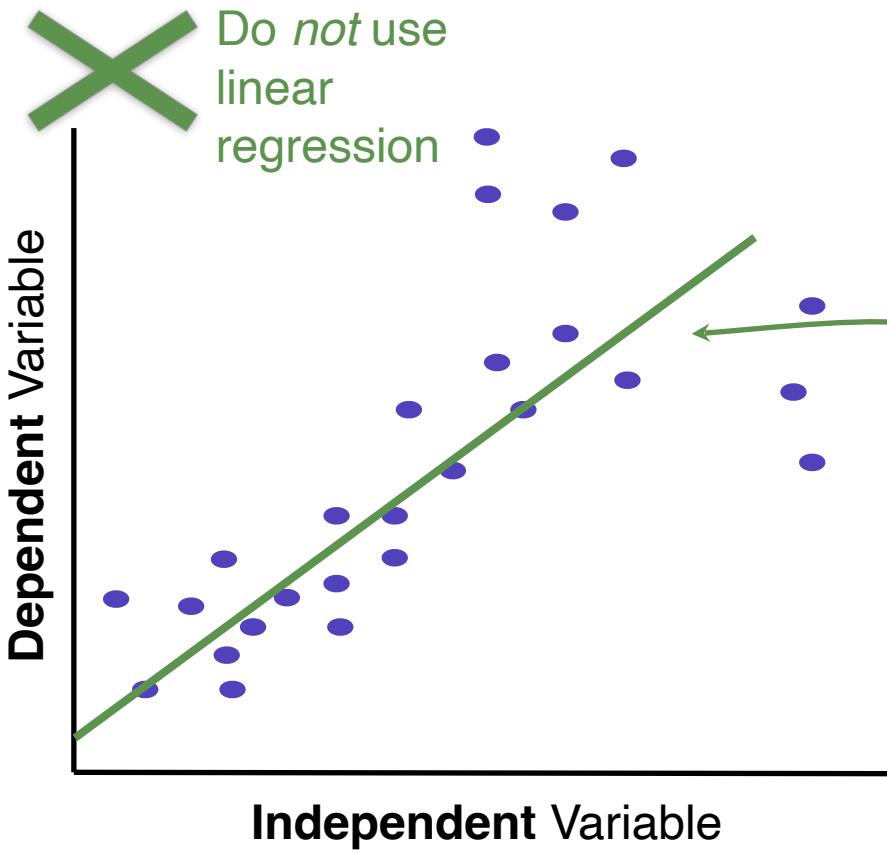


Linear regression assumes no multicollinearity. **Multicollinearity** occurs when the independent variables (in multiple linear regression) are too highly correlated with each other.

Daily returns are
 $\ln(\text{price}_t / \text{price}_{t-1})$



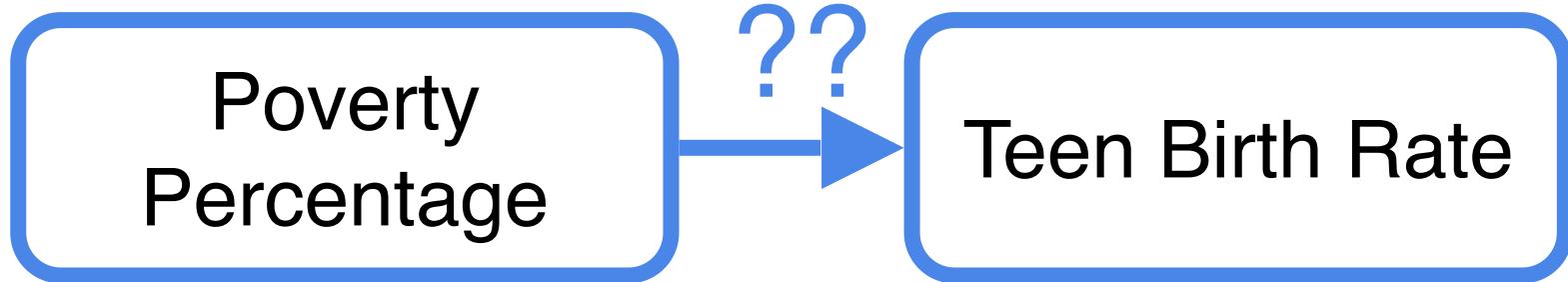
Autocorrelation occurs when the observations are
not independent of one another (i.e. stock prices)



Do *not* use
linear
regression

Not
homoscedastic:
points at this end are much
further from the line than at
the other end

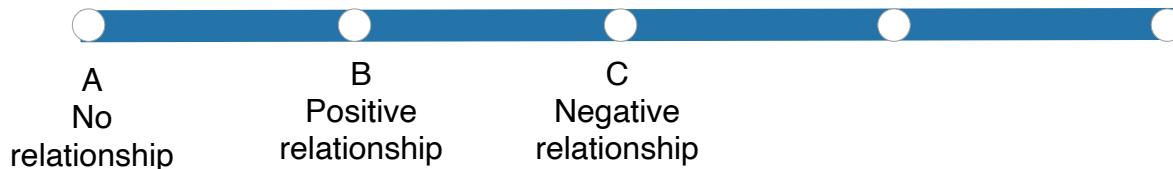
Does Poverty
Percentage affect Teen
Birth Rate?





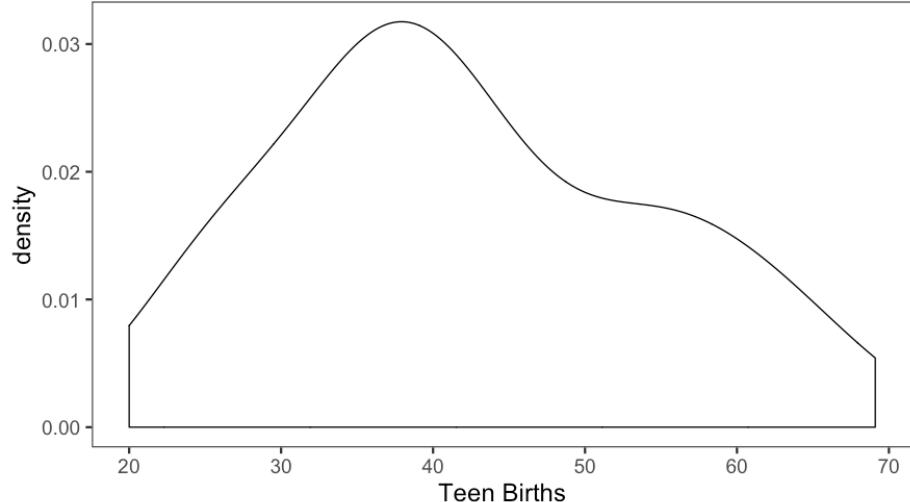
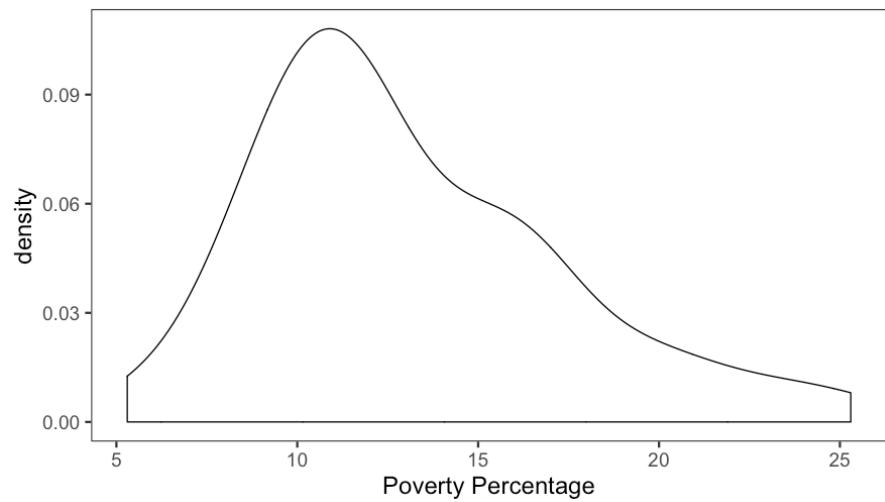
What is the relationship between Poverty Percentage & Teen Birth Rate?

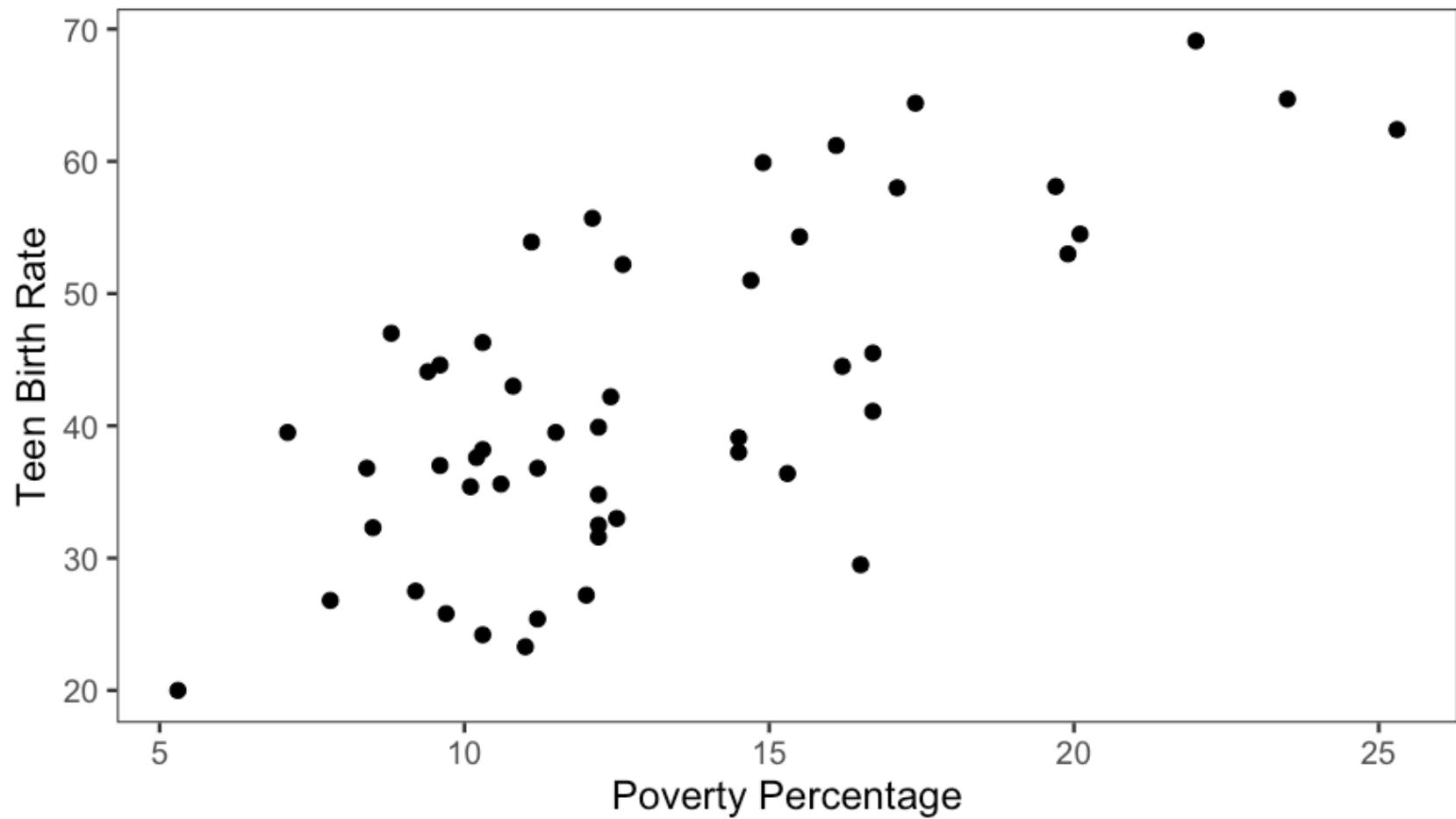
What's your hypothesis?

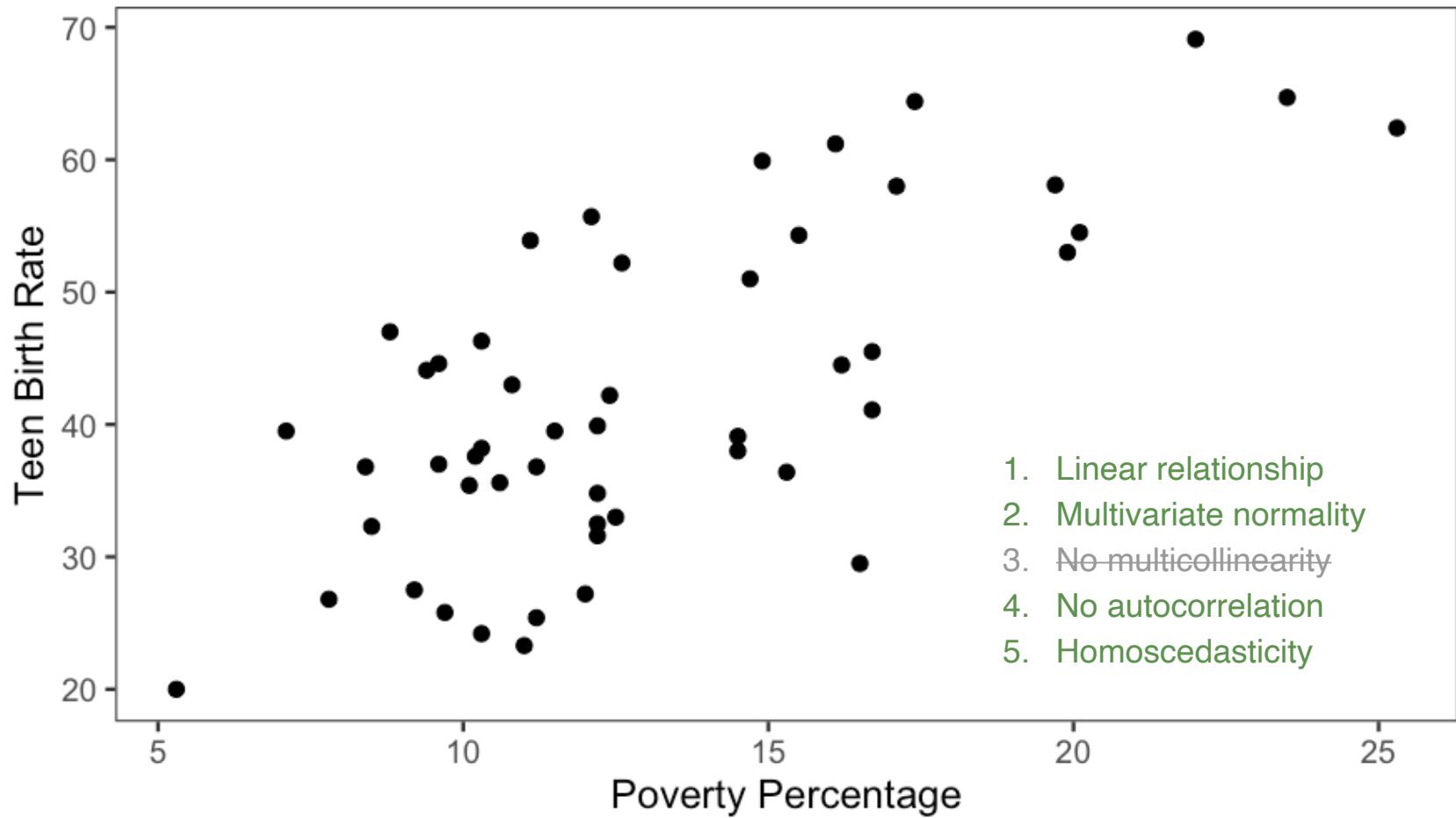


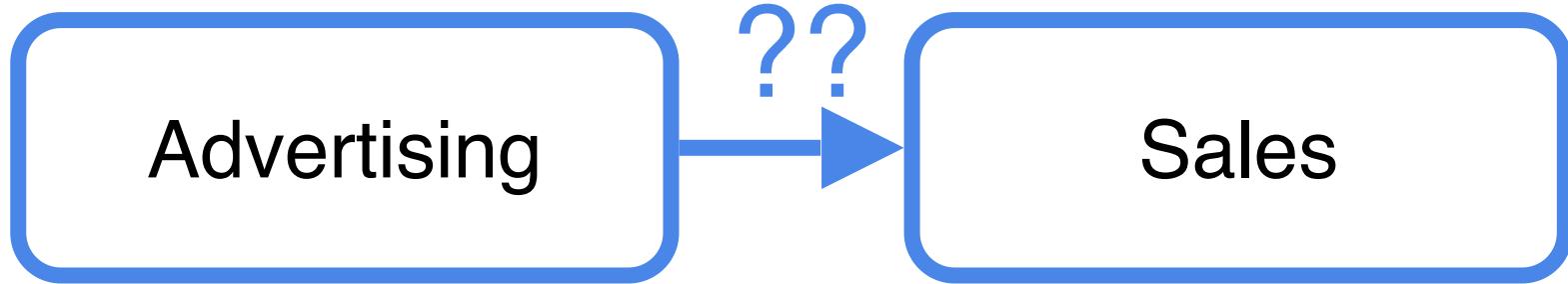
	Location	PovPct	Brth15to17	Brth18to19	ViolCrime	TeenBrth
1	Alabama	20.1	31.5	88.7	11.2	54.5
2	Alaska	7.1	18.9	73.7	9.1	39.5
3	Arizona	16.1	35.0	102.5	10.4	61.2
4	Arkansas	14.9	31.6	101.7	10.4	59.9
5	California	16.7	22.6	69.1	11.2	41.1
6	Colorado	8.8	26.2	79.1	5.8	47.0
7	Connecticut	9.7	14.1	45.1	4.6	25.8
8	Delaware	10.3	24.7	77.8	3.5	46.3
9	District_of_Columbia	22.0	44.8	101.5	65.0	69.1
10	Florida	16.2	23.2	78.4	7.3	44.5
11	Georgia	12.1	31.4	92.8	9.5	55.7
12	Hawaii	10.3	17.7	66.4	4.7	38.2
13	Idaho	14.5	18.4	69.1	4.1	39.1
14	Illinois	12.4	23.4	70.5	10.3	42.2
15	Indiana	9.6	22.6	78.5	8.0	44.6
16	Iowa	12.2	16.4	55.4	1.8	32.5
17	Kansas	10.8	21.4	74.2	6.2	43.0

Normal(ish) distributions





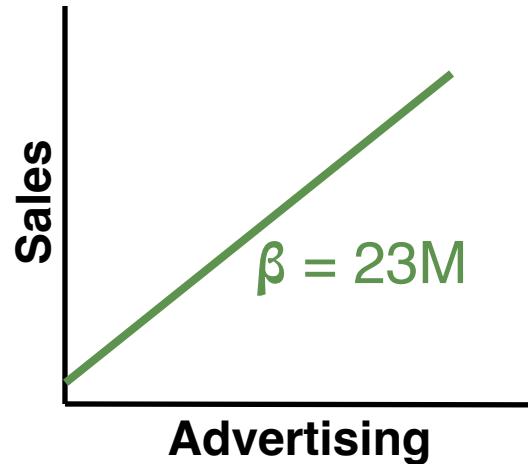






Effect size interpretation

Sales (Million Euro)	Advertising (Million Euro)
651	23
762	26
856	30
1,063	34
1,190	43
1,298	48
1,421	52
1,440	57
1,518	58



The effect size (β) between the advertising and sales is 23M. What does this mean?



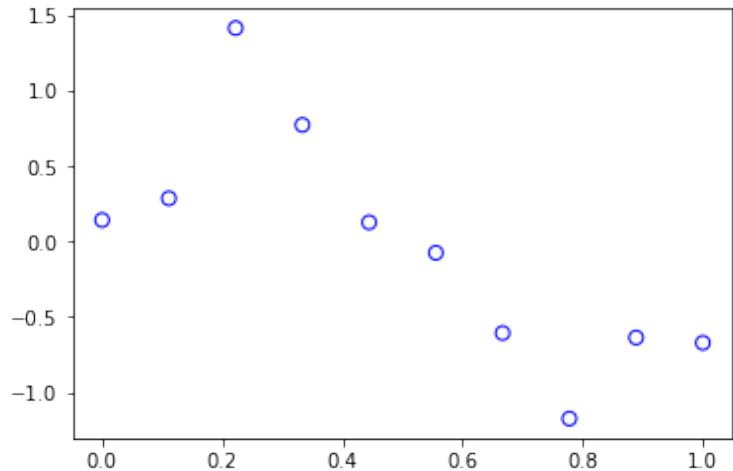
A
For every 1M Euro spent on advertising, the company sees 23M more in sales

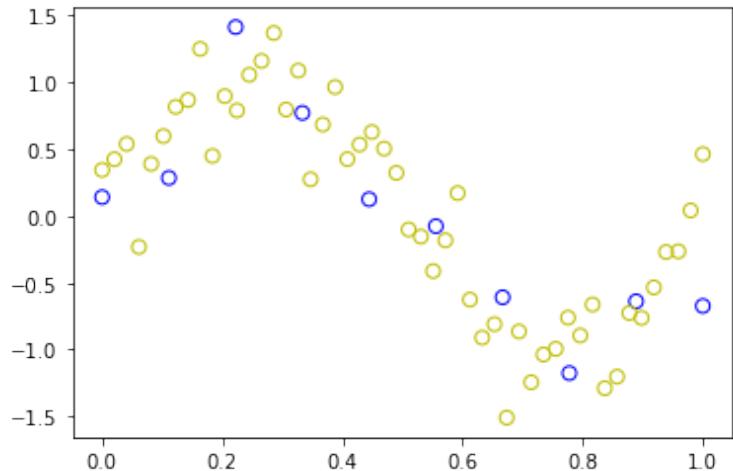
B
For every 1M Euro spent in sales, the company spends 23M more in advertising

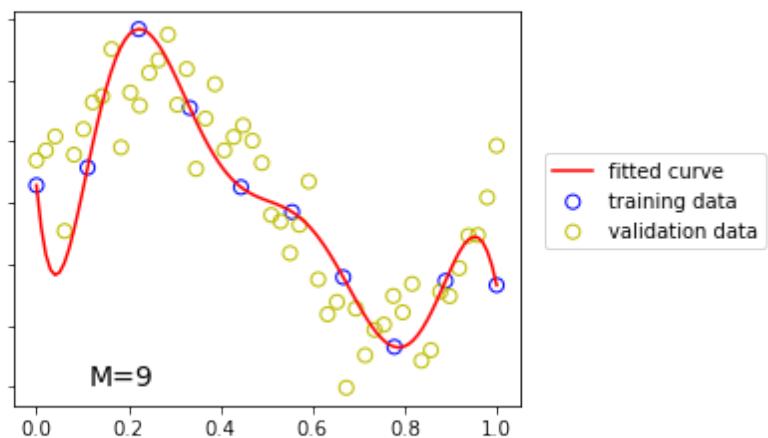
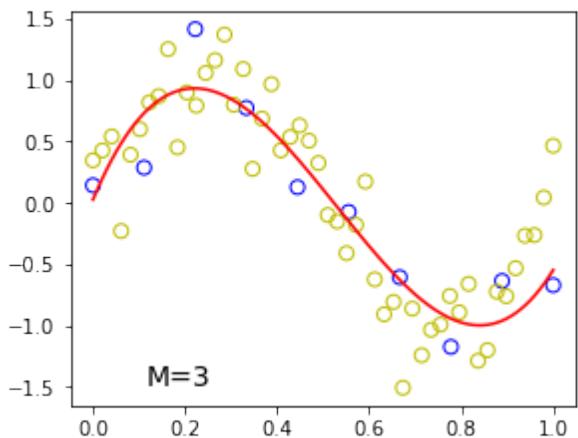
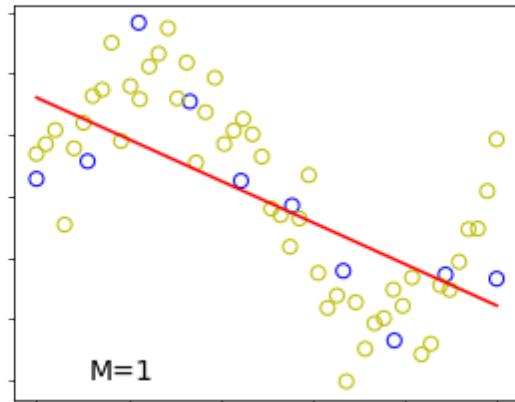
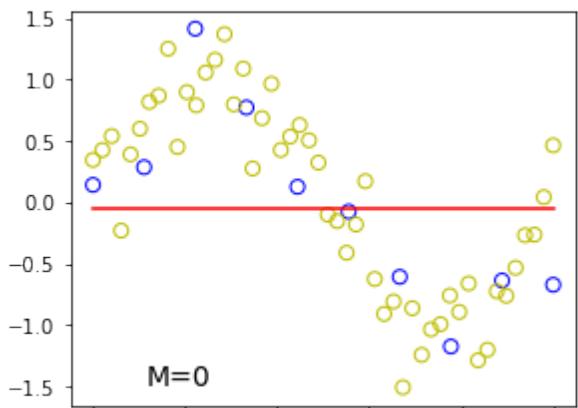
C
For every 1M Euro spent on advertising, the company sees 24M less in sales

D
For every 1M Euro spent in sales, the company spends 23M less in advertising

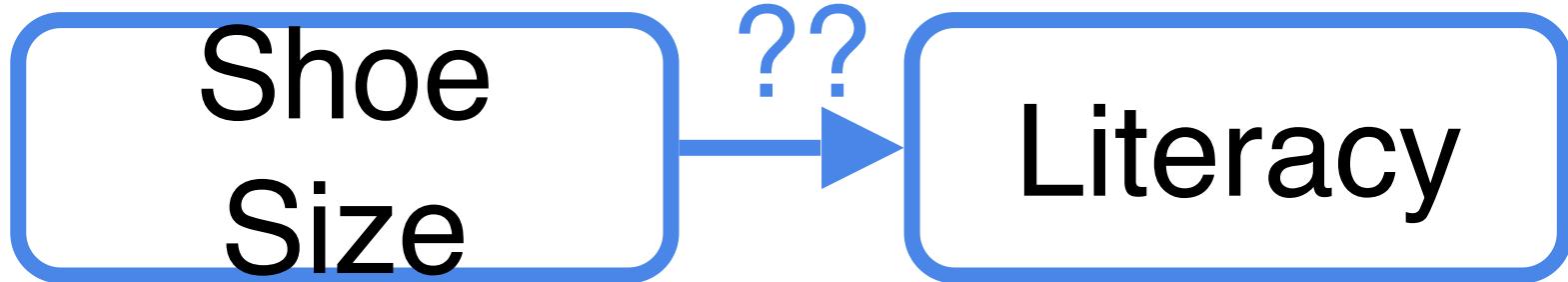
Model selection problems







Confounding



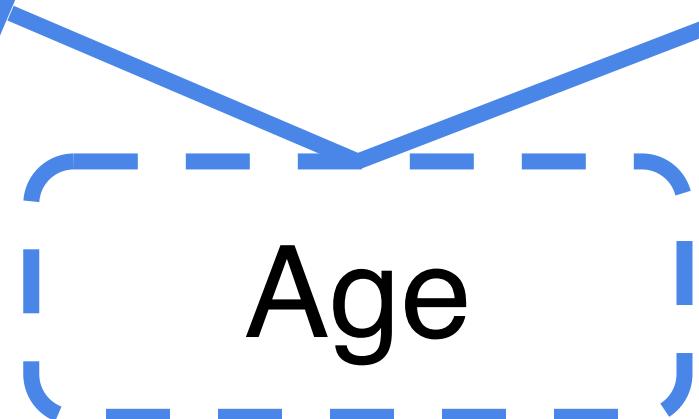


Small shoes
Not literate
Child

Big shoes
Literate
Adult

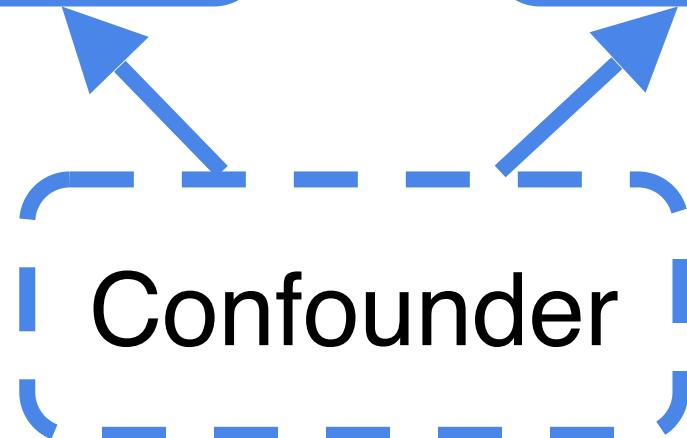
**Shoe
Size**

Literacy



Variable1

Variable2



Confounding



popsicles → crime rate



Your analysis sees an increase in crime rate whenever popsicle sales increase. What could confound this analysis?

- A popsicle preference
- B new gun laws
- C temperature
- D changes in popsicle prices
- E new law enforcement officers

You can plan ahead to avoid confounding and/or include confounders in your models to account for their role on the outcome variable.

Ignoring confounders will lead you
to draw incorrect conclusions

Stratification changes results

Sample: 400 patients with index vertebral fractures

...looks like vertebroplasty was *way* worse for patients!

Vertebroplasty	Conservative care	Relative risk (95% confidence interval)
30/200 (15%)	15/200 (7.5%)	2.0 (1.1–3.6)

subsequent fractures

But wait...at time of initial fracture...

	Vertebroplasty N = 200	Conservative care N = 200
Age, y, mean \pm SD	78.2 ± 4.1	79.0 ± 5.2
Weight, kg, mean \pm SD	54.4 ± 2.3	53.9 ± 2.1
Smoking status, No. (%)	110 (55)	16 (8)

Age and weight are similar between groups. **Smoking Status** differs vastly.

So...let's stratify those results real quick

Smoke			No smoke		
Vertebroplasty	Conservative	RR (95% confidence interval)	Vertebroplasty	Conservative	RR (95% confidence interval)
23/110 (21%)	3/16 (19%)	1.1 (0.4, 3.3)	7/90 (8%)	12/184(7%)	1.2 (0.5, 2.9)

Risk of re-fracture is now similar within group