SIR

September 4, 2024

[4]:
```python
import numpy as np
import matplotlib.pyplot as plt
```

0.0.1 Response To Math Activity

I did the Reading Math Activity using the video on Moodle, and really enjoyed it!

When the problem was shown initially, the first thing that stood out to me was the word "minimized" at the end of the problem description. The rest of the wording in the problem seemed approachable and familiar, but contextualizing it as a "minimization" problem kind of threw me for a loop.

After reading through the problem, I was cautiously hopeful that I could solve it, as the equations looked pretty straightforward. But the word "minimize" and the course number made me feel concerned that I was missing something, or that my approach would be too simple. I was trying to remember back to my other classes to think of where I have seen this problem before, or if I already knew a good algorithm to solve these kinds of problems. Overall, I was a little intimidated even though the equations themselves were not overly complicated.

After the screen changed and the 183 from the first equation on the original page was rewritten as a price I had exactly this reaction:

Suddenly the problem seemed very obvious! I didn't need to go back and look for an equation or strategy to be able to solve this problem, because there was a much simpler way to approach it and a better way to think about it.

After working through the problem using the metaphor of coins, I was mostly thinking about how to apply this problem solving strategy to other problems, and recognizing where I had used this strategy (maybe not purposefully) in the past - some of the calculus I learned in my first degree was much more approachable in the context of thermodynamics or stellar physics than in the pure math.

Going forward, I'm excited to be able to recognize when I am using this strategy, and learn how to apply it to more complicated problems or when I've run into an obstacle!

[46]:
```
def SIR(ti, tf, Si, Ii, Ri):
    dt = 1
    for i in range (0, tf):
        Sprime = -0.00001 * Si * Ii
        Iprime = (0.00001 * Si*Ii) - (Ii/14)
        Rprime = (Ii/14)
        dS = Sprime * dt
        dI = Iprime * dt
        dR = Rprime * dt
        ti = ti + dt
        Si = Si + dS
        Ii = Ii + dI
        Ri = Ri + dR
    print(f"After {ti} days, the number of Susceptible is {Si}, the number of␣
 ↪Infected is {Ii}, and the number of Recovered is {Ri}")
    return (Si, Ii, Ri)

def plotSIR(t, tf, S, I, R):
    tlist = []
    Slist = []
    Ilist = []
    Rlist = []
    dt = 1
    maxDI = 0
    for i in range (t, tf):
        Sprime = -0.00001 * S * I
        Iprime = (0.00001 * S*I) - (I/14)
        Rprime = (I/14)
        dS = Sprime * dt
        dI = Iprime * dt
        if (dI>maxDI):
            maxDI = dI
            maxt = t
        dR = Rprime * dt
        t = t + dt
        tlist.append(t)
        S = S + dS
        Slist.append(S)
        I = I + dI
```

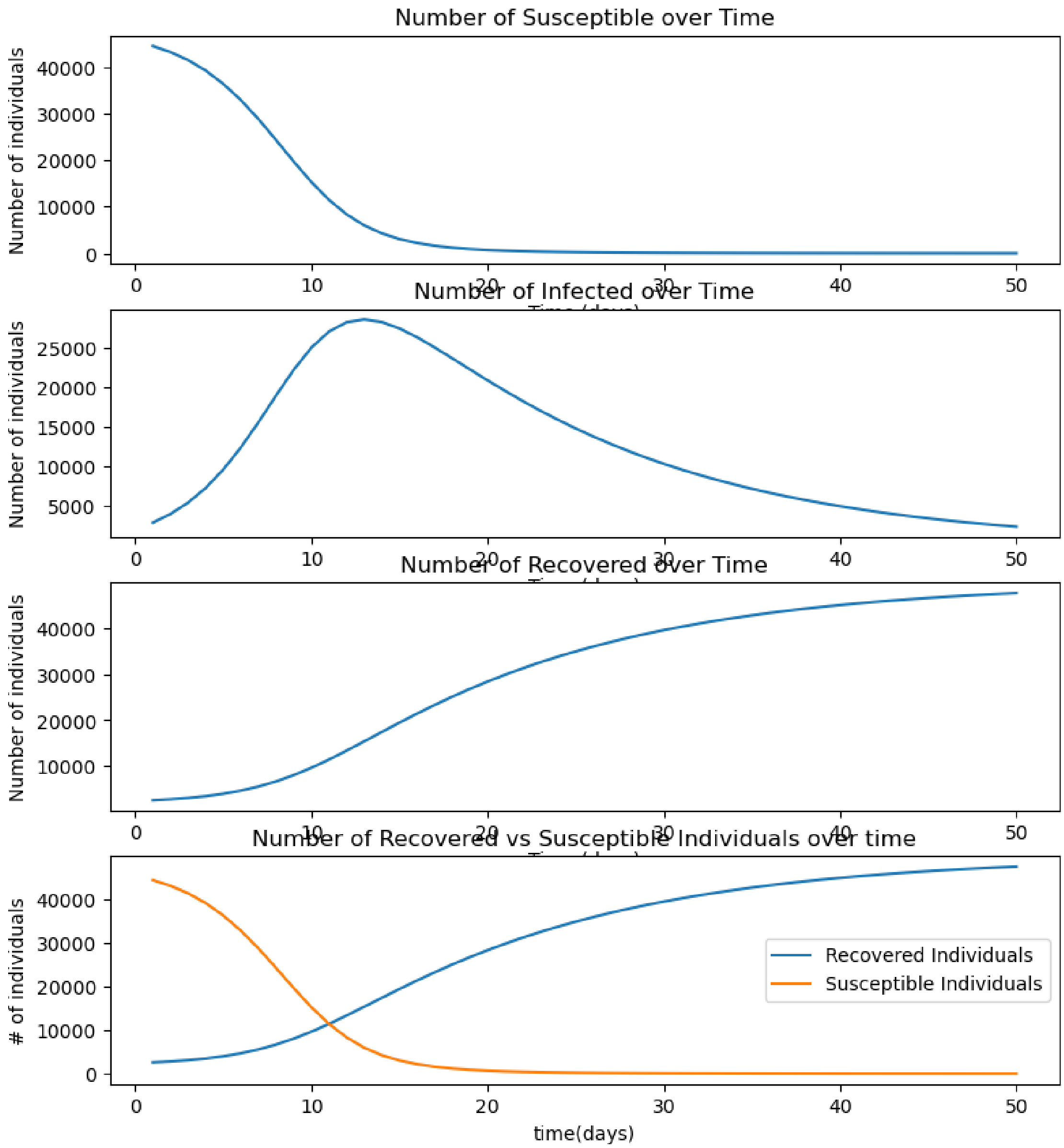
```
        Ilist.append(I)
        R = R + dR
        if (R >24000 and R < 26000):
            print(R, t)
        Rlist.append(R)
    print("Sum of infected individuals day 1-20:", np.sum(Ilist[:20]))
    print("MaxDI = ", maxDI, "on day", maxt)
    fig, ax = plt.subplots(4, 1, figsize = (9, 10))
    ax[0].plot(tlist, Slist)
    ax[0].set_title("Number of Susceptible over Time")
    ax[0].set_xlabel("Time (days)")
    ax[0].set_ylabel("Number of individuals")
    ax[1].plot(tlist, Ilist)
    ax[1].set_title("Number of Infected over Time")
    ax[1].set_xlabel("Time (days)")
    ax[1].set_ylabel("Number of individuals")
    ax[2].plot(tlist, Rlist)
    ax[2].set_title("Number of Recovered over Time")
    ax[2].set_xlabel("Time (days)")
    ax[2].set_ylabel("Number of individuals")
    ax[3].plot(tlist, Rlist, label = "Recovered Individuals")
    ax[3].plot(tlist, Slist, label = "Susceptible Individuals")
    ax[3].set_title("Number of Recovered vs Susceptible Individuals over time")
    ax[3].set_xlabel("time(days)")
    ax[3].set_ylabel("# of individuals")
    ax[3].legend()

    plt.show()
```

[47]:

```
SIR(0, 15, 45400, 2100, 2500)
plotSIR(0, 50, 45400, 2100, 2500)
```

```
After 15 days, the number of Susceptible is 3074.404523808178, the number of
Infected is 27444.500237176504, and the number of Recovered is
19481.095239015318
25110.17273744357 18
Sum of infected individuals day 1-20: 381238.6360292902
MaxDI =  3375.684665212726 on day 7
```



0.0.2 1.1 (1):

The infection will peak at around day 12, and there are about 13000 people infected. ### 1.1 (2): Initially, the Susceptible population count is 45400. It takes around 16 days for ### 1.1 (3): The Recovered population reaches 25000 people around day 30. (Based on the plot in the book, in the plots above this number is reached at day 17. The number of Recovered people will approach the initial number of Susceptible people, 45400. ### 1.1 (4): The size of the infected population is growing the fastest around day 7. We can tell because this is where the highest positive rate of change (slope) in the infected population is. ### 1.1 (5): The number of people who caught the disease (had the infected status) on or before day 20 is the sum of all people who had the infected condition on the days prior or on day 20. So, we need to first find the number of people with the infected condition on each day 1-20, then add all of those numbers together. I did this in the code above, and found that around 381200 were infected. ### 1.1 (6): (See fig. 4 above, I superimposed the graph of R on S with a timescale of 50) If the plot of S over time was placed on top of a graph of R over time with a timescale of 50 vs 100, then S would appear more compressed horizontally towards the left side of the plot. In the version I did above, my plot of R on a timescale of 50 as contrasted with 100 in the book, appears stretched horizontally.

0.0.3 1.1 (7):

(a): Using the code below, I found that 10 days after Wednesday, using the equation $S' = -470$, there were 14830 Susceptible individuals left ##### (b): Using the code below, I found that the susceptible population reaches 0 on day 42 ##### (c): Assuming that Wednesday is t = 0, then the following Sunday would be t = -3. We can use the equation S = 20000 -470(t) to get the value of S on day t = -3, which is 21410 individuals. ##### (d): Using the same strategy and equation as above, we can use the equation 30,000 = 20000 - 470(t) to find a value of t that satisfies this equation. In this case, $t = \frac{10000}{-470} \approx -21$

[48]:
```python
def Susceptible(si, ti, tf):
    dt = 1
    S = si
    empty = 0
    for i in range(ti, tf):
        Sprime = -470
        dS = Sprime * dt
        S = S + dS
        if (S <= 0 and empty == 0):
            empty = 1
            print("Population reaches 0 on day ", i)

    return(S, tf)

Susceptible(20000, 0, 50)

# ez equation version:
S2 = 20000 - 470*(-3)
print(S2)
print(10000/-470)
inflect = (1/14)/0.000005
print(inflect)
```

```
Population reaches 0 on day  42
21410
-21.27659574468085
14285.714285714284
```

0.0.4 1.1 (19)

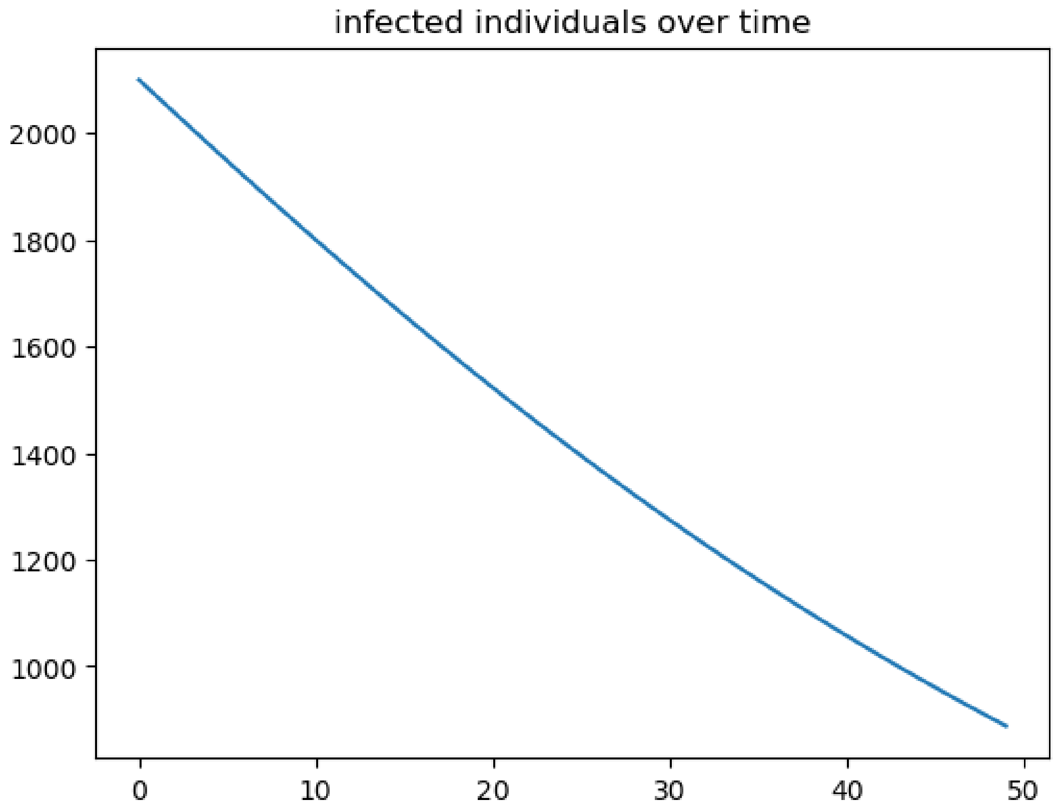
(a): The original transmission coefficient was $a = 0.00001$. By cutting this in half the new transmission coefficient is $a1 = 0.000005$. ##### (b): Given the new transmission coefficient, the slope of the infection curve will be 0 if $I = 0$ or if $(0.000005)S - \frac{1}{14} = 0$. Solving the second equation gives us a new inflection point of 14286 individuals. ##### (c): In the cell below, I modify the code used in the first 6 problems to determine if the number of infected individuals increases over time. Unfortunately, the rate does still increase. ##### (d): Modifying the code again, I divide the transmission coefficient by 2, 3, and 4, and find that a coefficient of 0.00000125 obtained by dividing 0.000005 by 4 ends the epidemic by preventing any more individuals from being infected. ##### (e): I accidentally did this in the previous problem. You would need to divide the original transmission coefficient of 0.00001 by 8 in order to have an effective quarantine.

[45]:
```
def SIR2(ti, tf, Si, Ii, Ri):
    dt = 1
    Ilist = []
    tlist = []
    for i in range (0, tf):
        Ilist.append(Ii)
        tlist.append(i)
        Sprime = -0.00000125 * Si * Ii
        Iprime = (0.00000125* Si*Ii) - (Ii/14)
        Rprime = (Ii/14)
        dS = Sprime * dt
        dI = Iprime * dt
        dR = Rprime * dt
        ti = ti + dt
        Si = Si + dS
        Ii = Ii + dI
        Ri = Ri + dR
    print(f"After {ti} days, the number of Susceptible is {Si}, the number of␣
 ↪Infected is {Ii}, and the number of Recovered is {Ri}")
    plt.plot(tlist, Ilist)
    plt.title("infected individuals over time")
    return (Si, Ii, Ri)

SIR2(0, 50, 45400, 2100, 2500)
```

```
After 50 days, the number of Susceptible is 41494.90769915957, the number of
Infected is 870.4929414116277, and the number of Recovered is 7634.599359428769
```

[45]:
```
(41494.90769915957, 870.4929414116277, 7634.599359428769)
```



0.0.5 1.1 (20)

(a): Based on the equations given in the book, because the slope of the recovery is 0.08, then we know that the infected period is about $\frac{8}{100} = 12.5$ days because one group out of 12.5 will recover each day. ### (b) In order for the slope of the "infected" curve to be positive, the Susceptible population must be modeled by:

$$(0.00002SI - 0.08I > 0$$

$$(0.00002S - 0.08)(I) > 0$$

If I is not 0 then

$$S > \frac{0.08}{0.00002}$$
$$S > 4000$$

(c) According to the model, if 100 people are sick one day then 8 will recover the following day ##### (d) If there are 30 new cases in the same 24 hours, then we know that S' is -30 on that date, and has a higher magnitude than R', and can be modeled as $-30 = -0.00002S*(100-8+30) = -0.00002S*122$ ##### (e) Yes, using the model from part d,

$$-30 = -0.00002S * 122$$

$$-0.00002S = -0.246$$

$$S = 12295$$

(S is rounded down)

0.0.6 1.1 (21):

(a): For this model, the following information is given:

$$R' = 0.25I$$

$$TC = 0.003I * 0.16666$$

. So, our set of SIR equations is as follows:

$$S' = 0.0005SI$$

$$I' = 0.0005SI - 0.25I$$
$$R' = 0.25I$$

(b): In order for $I' \leq 0$,

$$0 \geq 0.0005SI - 0.25I$$

$$0 \geq (0.005S - 0.25)I$$

Assuming I is not 0,

$$0 \geq 0.0005S - 0.25$$
$$\frac{0.25}{0.0005} \geq S$$
$$500 \geq S$$

The susceptible populateion must be less than or equal to 500.

0.0.7 1.1(22):

(a): Given the previous problems we've done in this assignment, We can use the equation for the rate of change of infected individuals to find the threshold level for S. Because in order for the infection to be stopped, we know that $I' \leq 0$. Writing this out with the expression gives us

$$0 \geq aSI - bI$$

. We can simplify this to

$$0 \geq (aS - b)I$$

. If we solve for

$$0 = (aS - b)I$$

and assume that $I \neq 0$, then we get

$$0 = (aS - b)$$

. Finally, we can simplify this expression to

$$\frac{b}{a} \geq S$$

(b): The length of time that a person stays sick is what determines the coefficient b in this system of equations. If a person stays sick for longer, then a smaller proportion of the total infected population will recover with each increment in t, making the coefficient b smaller. If we refer to the equation

$$\frac{b}{a} \geq S$$

, then we find that with a smaller b with the same coefficient a, The disease with a longer illness time will have a lower threshold S

[ ]:

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