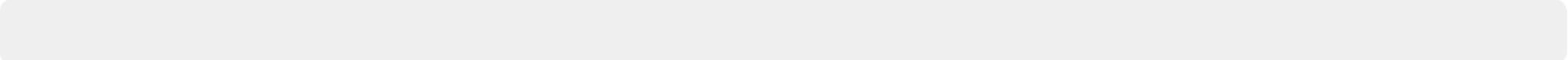


Are you ready for ecology?

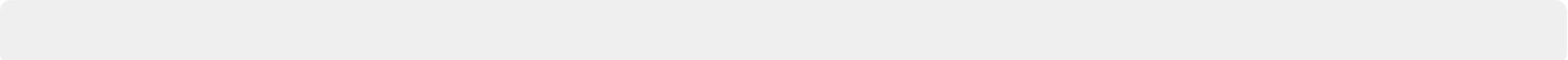
yes please!



0%

A horizontal grey progress bar with a darker grey filled section on the left side, representing 0% completion.

I need to evolve some more



0%

A horizontal grey progress bar with a darker grey filled section on the left side, representing 0% completion.

Fundamentals of Ecology

Week 5, Ecology Lecture 1

Cara Brook

February 1, 2024

Where are we headed?

Week	Mon/Tues Lab	Tues Lecture	Thurs Lecture
1	--	--	Jan 4: Evolution History
2	1: Intro R	Jan 9: Mutation and Variation	Jan 11: Hardy-Weinberg
3	2: Hardy-Weinberg	Jan 16: Migration and Drift	Jan 18: Natural Selection
4	3: Microevolution	Jan 23: Phylogenetics	Jan 25: Molecular Evolution & Sexual Selection
5	4: Phylogenetics	Jan 30: Kin Selection, Speciation	Feb 1: Ecology & Population Growth
6	5: Evolution paper	Feb 6: Single Species Population Growth & Regulation	Feb 8: Midterm
7	6: Population Growth	Feb 13: Species Interactions 1	Feb 15: Species Interactions 2
8	7: Population Regulation	Feb 20: Disease Dynamics as Population Biology 1	Feb 22: Disease Dynamics as Population Biology 2
9	8: Disease Dynamics	Feb 27: Community Assembly & Island Biogeography	Feb 29: Critical Transitions & Conservation Interventions

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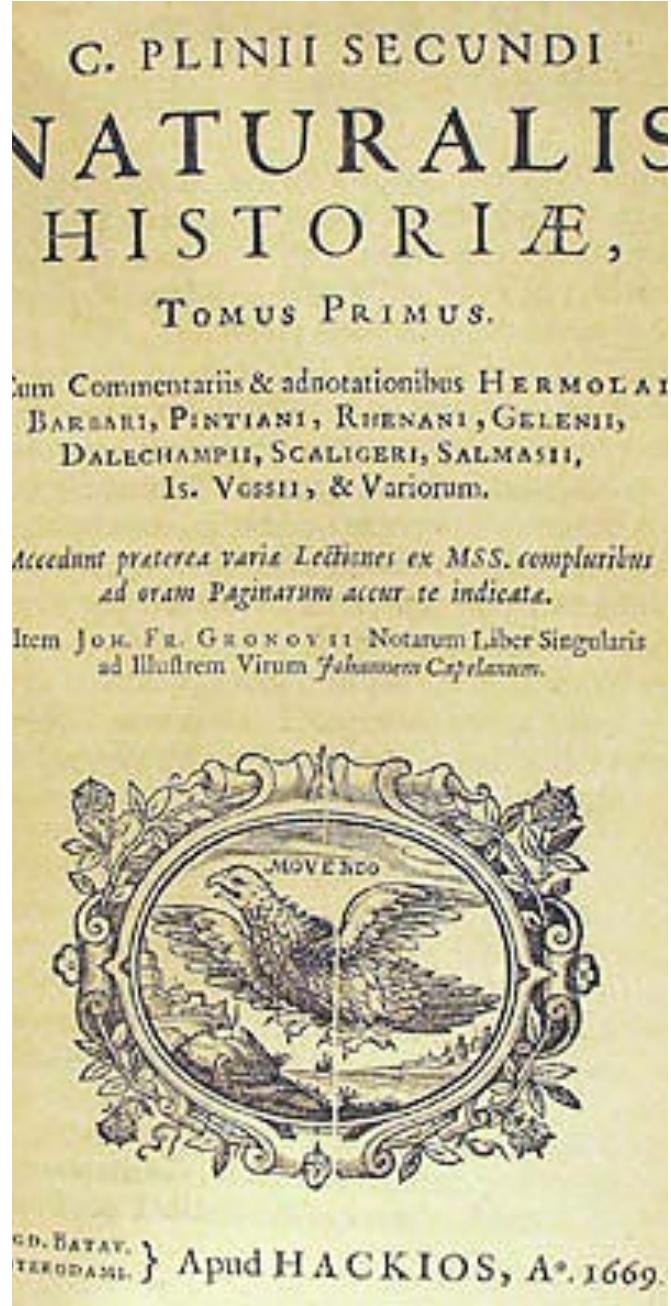
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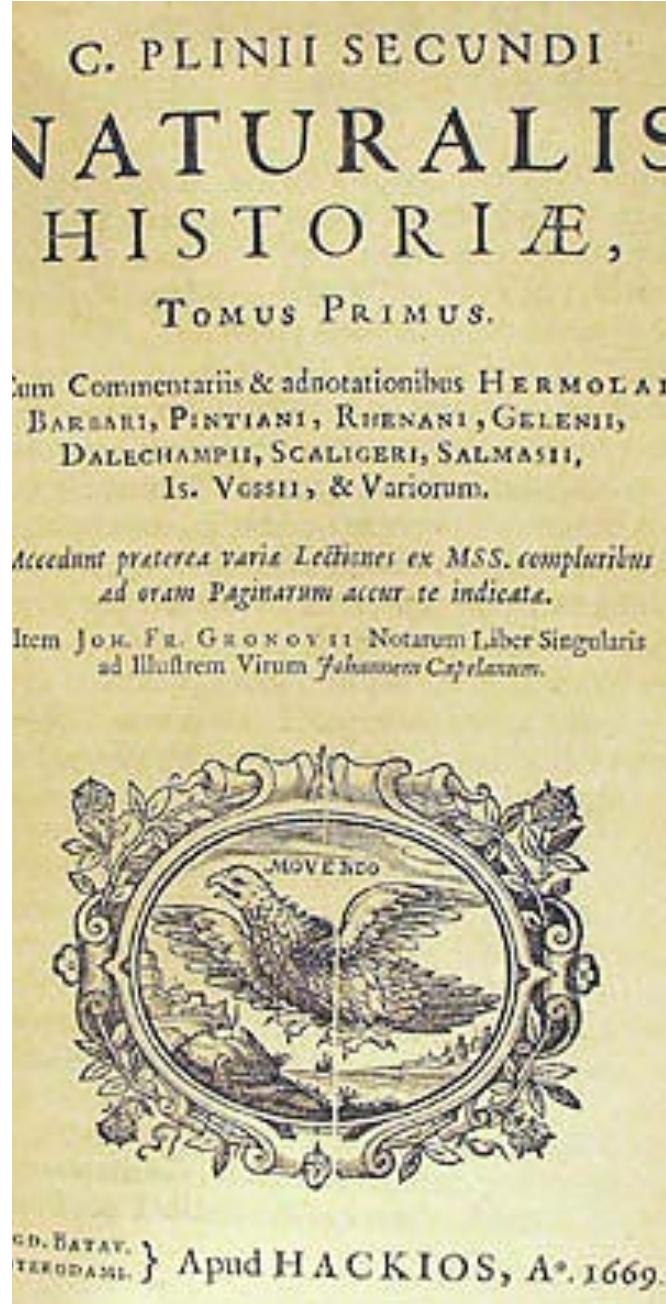


Natural history is the observational study of the living things on planet Earth.

Is natural history science?

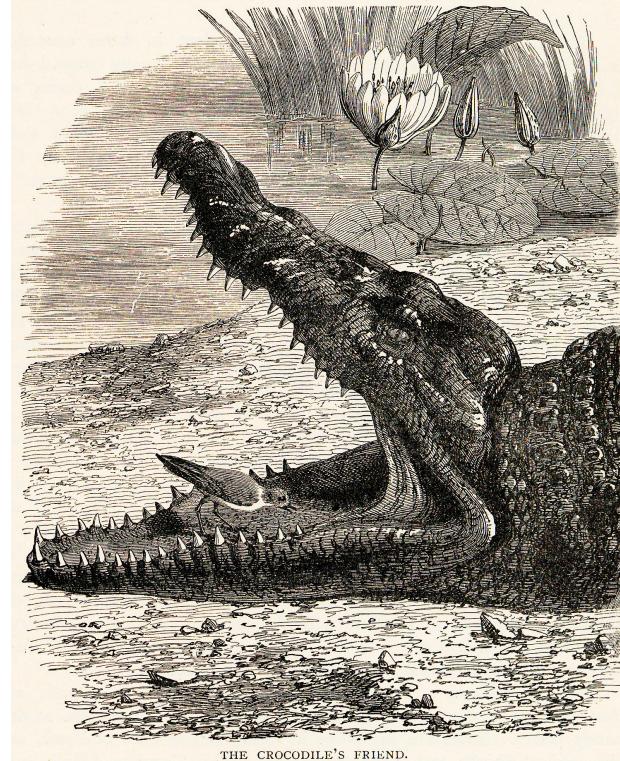
Science: the **systematic observation** of natural events and conditions in order to discover facts about them and to **formulate laws and principles** based on these facts.

– *Academic Press Dictionary of Science & Technology*

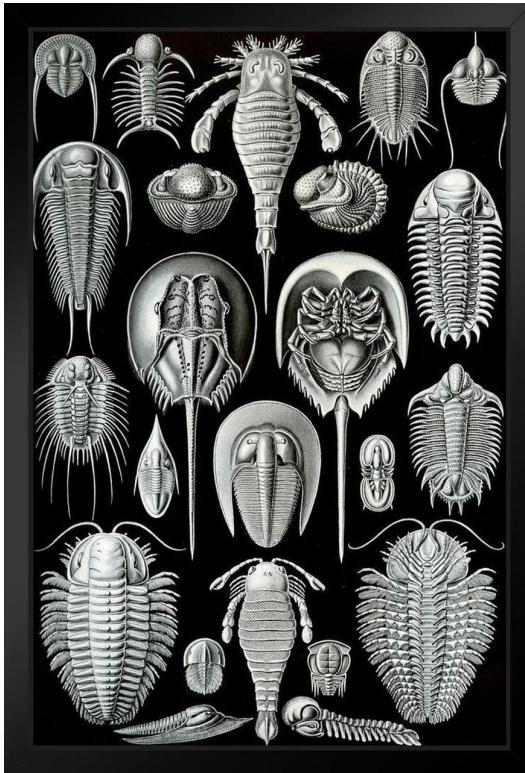


Natural history gave way to the **natural sciences** when those observations became **systematic** and were used to support **general laws and principles** describing the natural world.

Ecology is the study of the
interactions of **organisms** with
each other and their
environment.



Ecology is the study of the
interactions of **organisms** with
each other and their
environment.



oikos = 'house'
+ *logia* = 'study of'

ecology

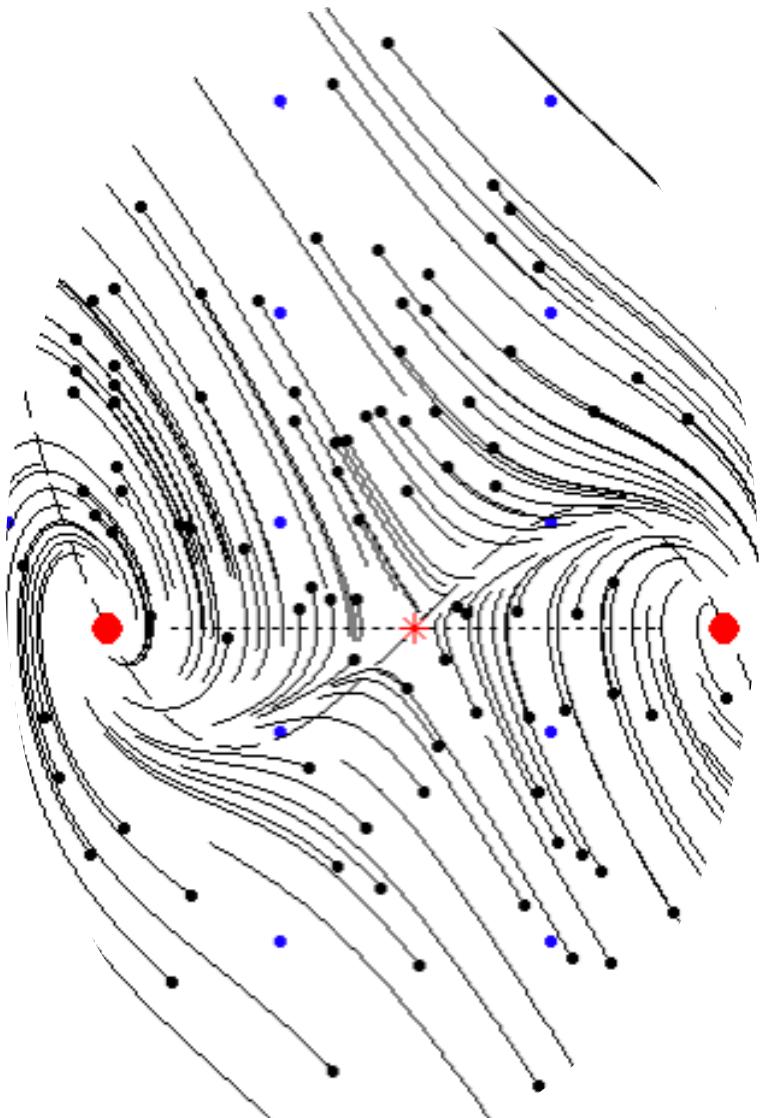
- 1866 Ernst Haeckel

“Ecology has a synonym which is **ALL.**”

-John Steinbeck

The Log from the Sea of Cortez (1941)





Ecology is the study of
the **interactions** of
organisms with each
other and their
environment.

As a science, ecology
uses **models** to formalize
general **laws and**
principles describing the
natural world.



What is a model? an abstract representation of a phenomenon

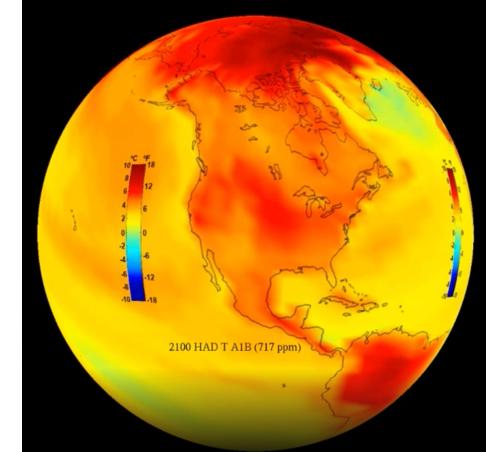
Human



Solar System



Climate



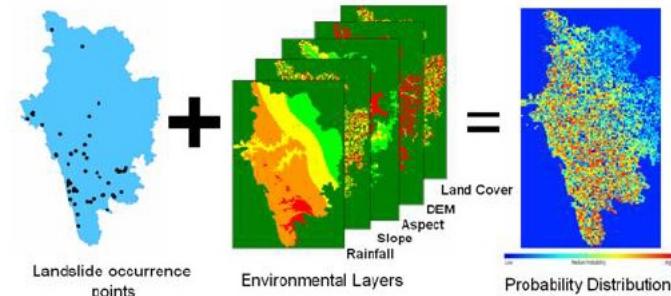
Human Genetics



Human Disease

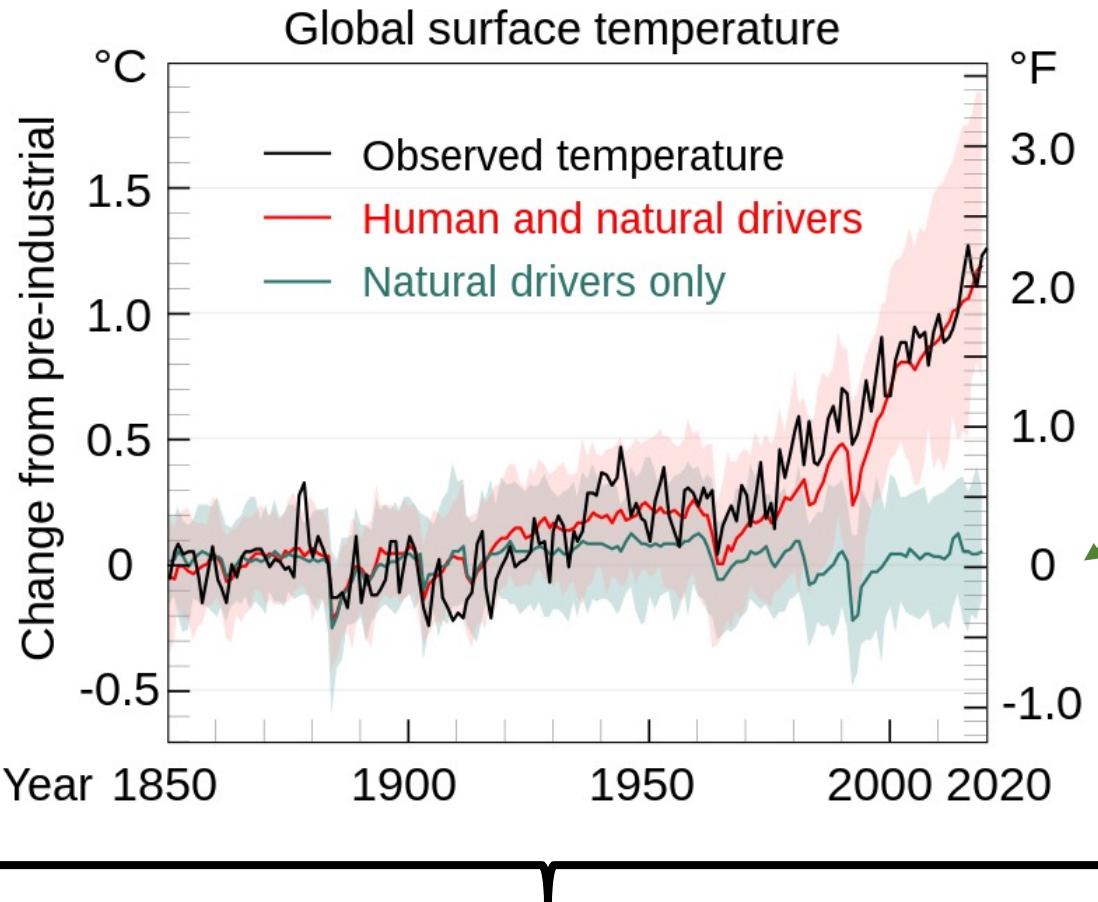


Species Distribution



Why build models?

to explain and predict



Models produce 'simulated data' that is compared against the observed data.

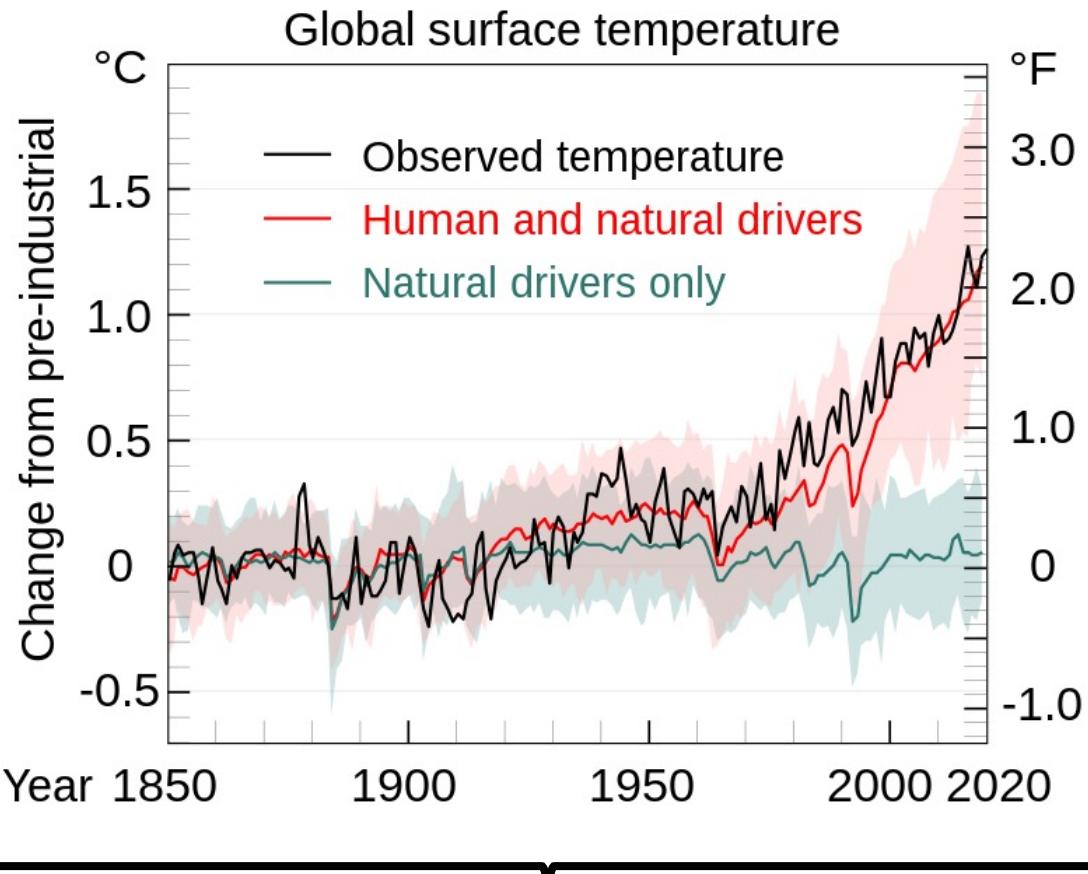
The process of adjusting the model to more closely match the data is called 'model fitting.'

explain

Why build models?

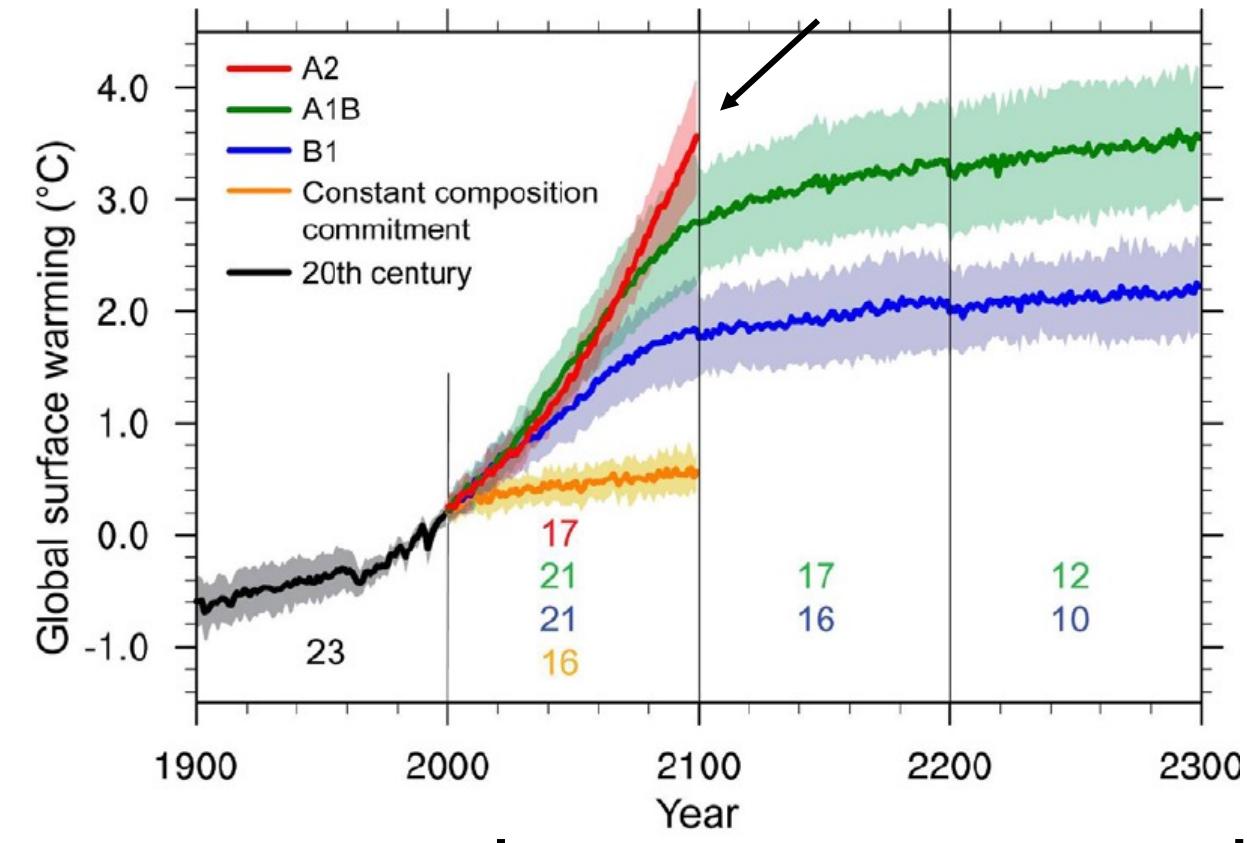
to explain and predict

Fitted models can then be used to “predict” the future, under different conditions.



(IPCC 2007)

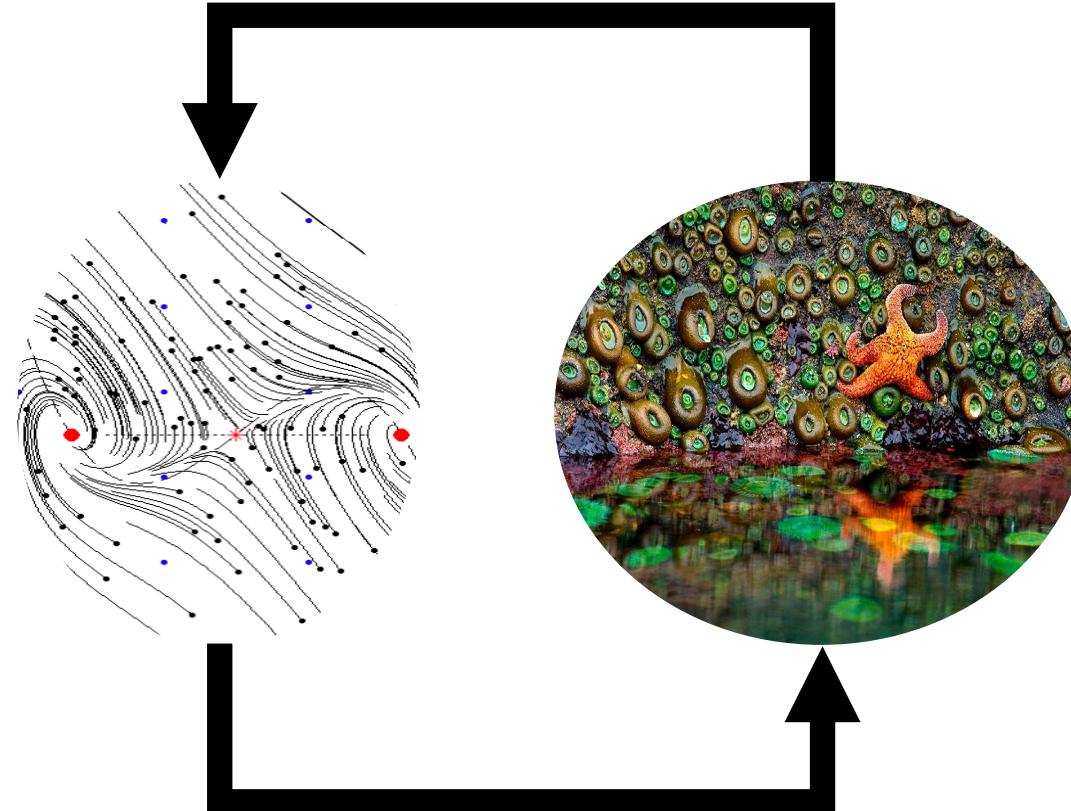
explain



predict

Two kinds of ecological models

Statistical Model
Pattern

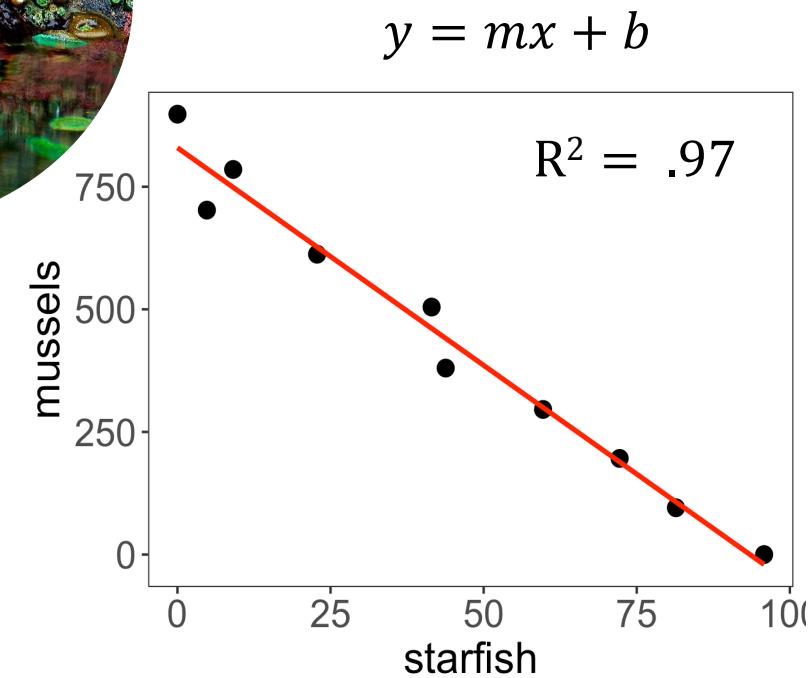
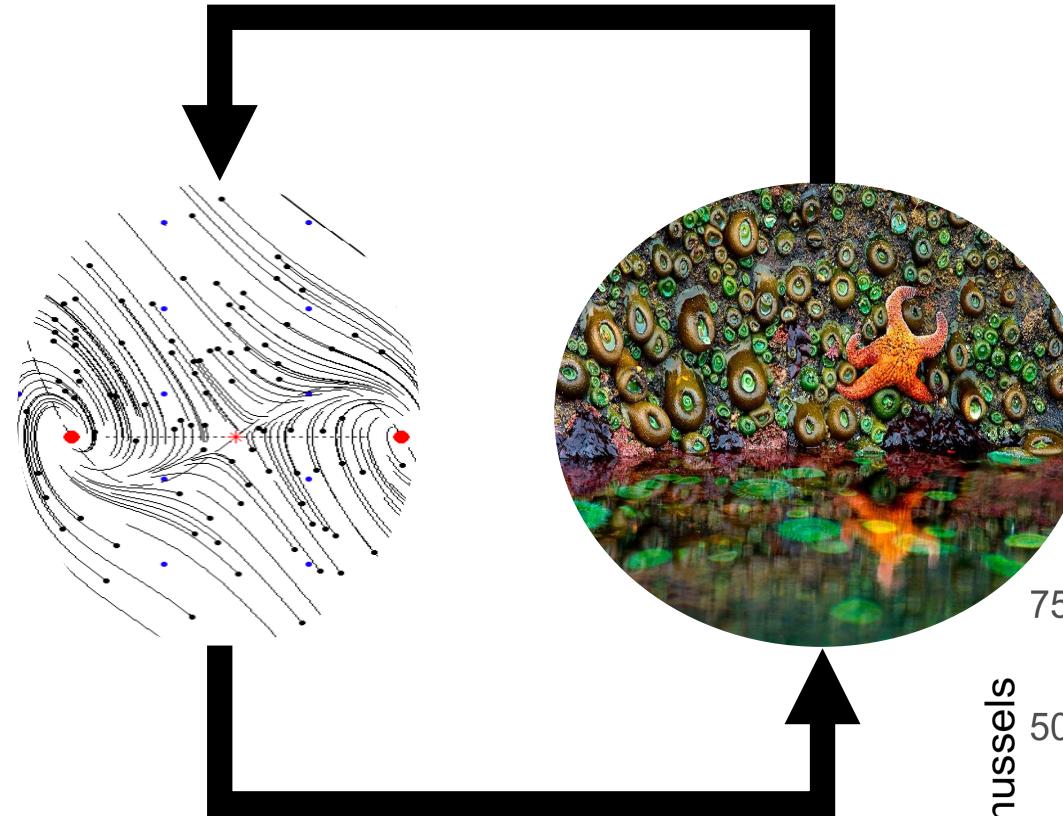


Population Model
Process

Two kinds of ecological models

Statistical Model *Pattern*

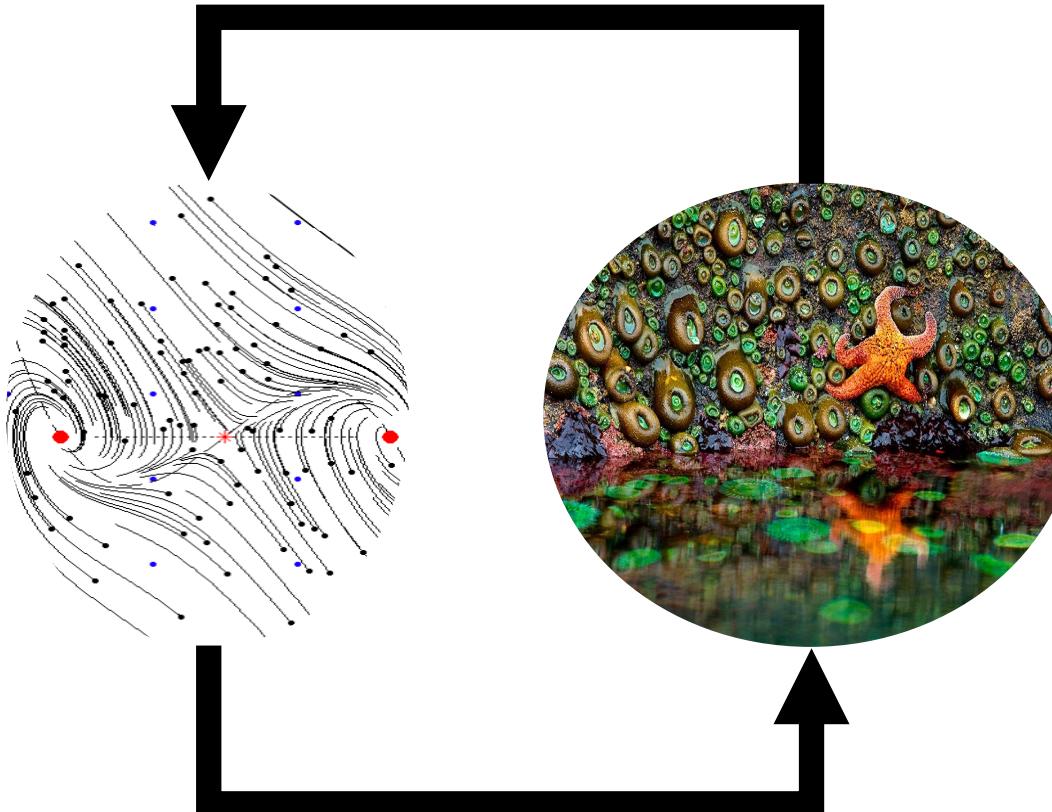
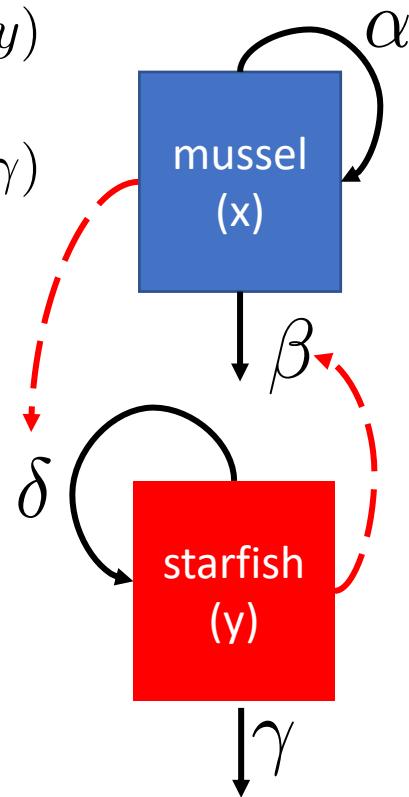
Population Model *Process*



Two kinds of ecological models

Statistical Model *Pattern*

$$\frac{dx}{dt} = x(\alpha - \beta y)$$
$$\frac{dy}{dt} = y(\delta x - \gamma)$$

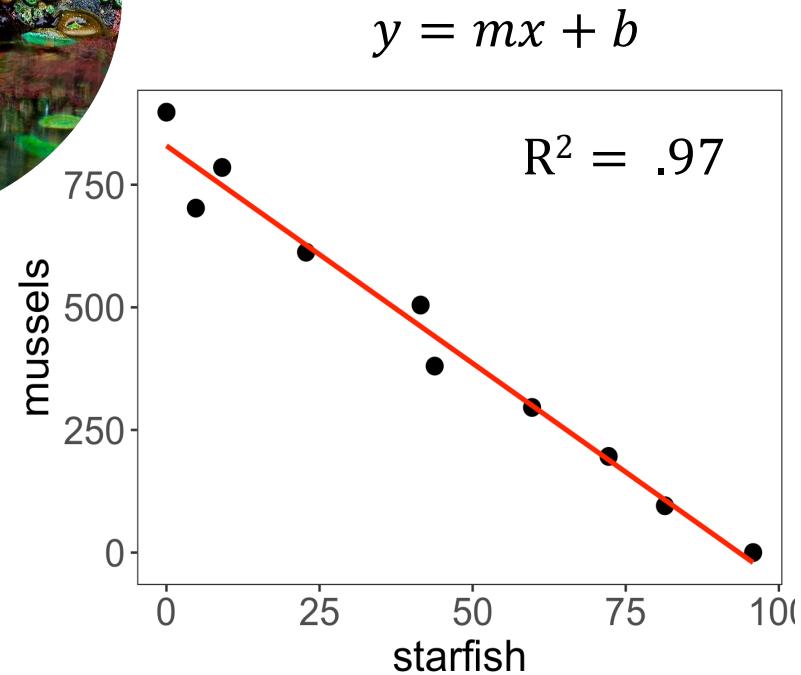
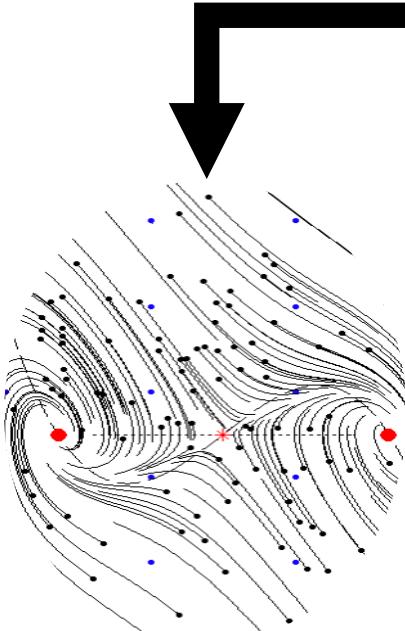
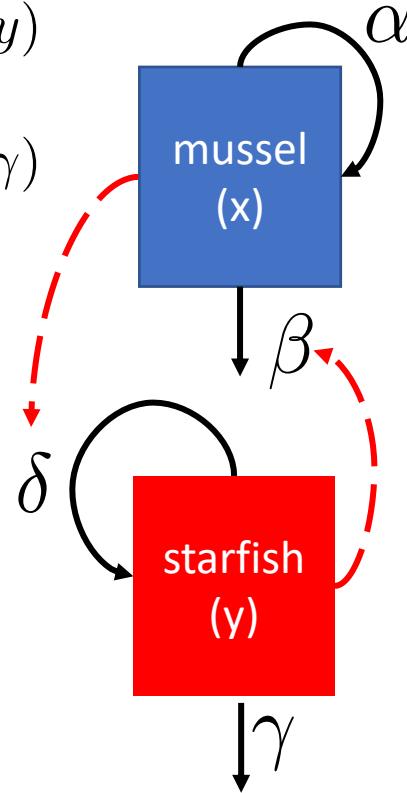


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Two kinds of ecological models

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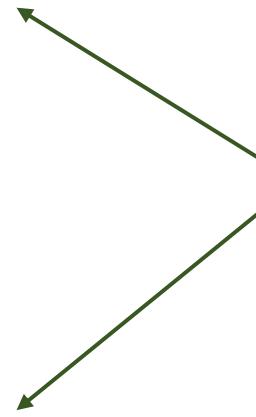


Population Model
Process

How to construct a population model

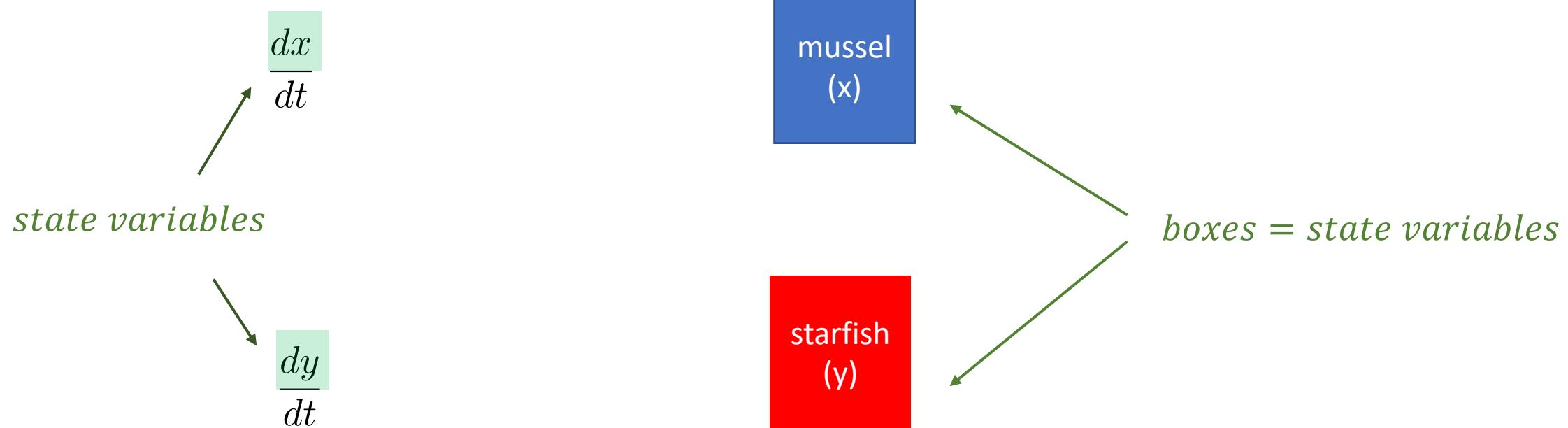
mussel
(x)

starfish
(y)

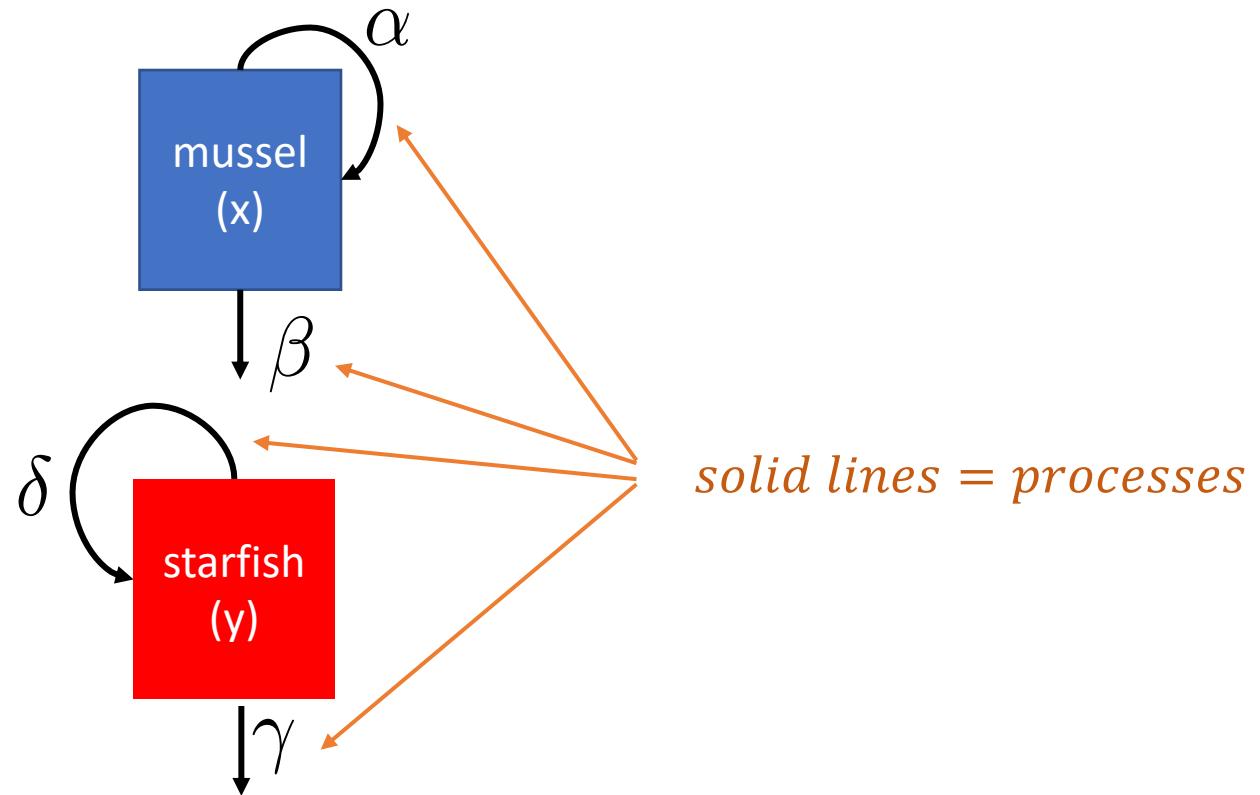


boxes = state variables

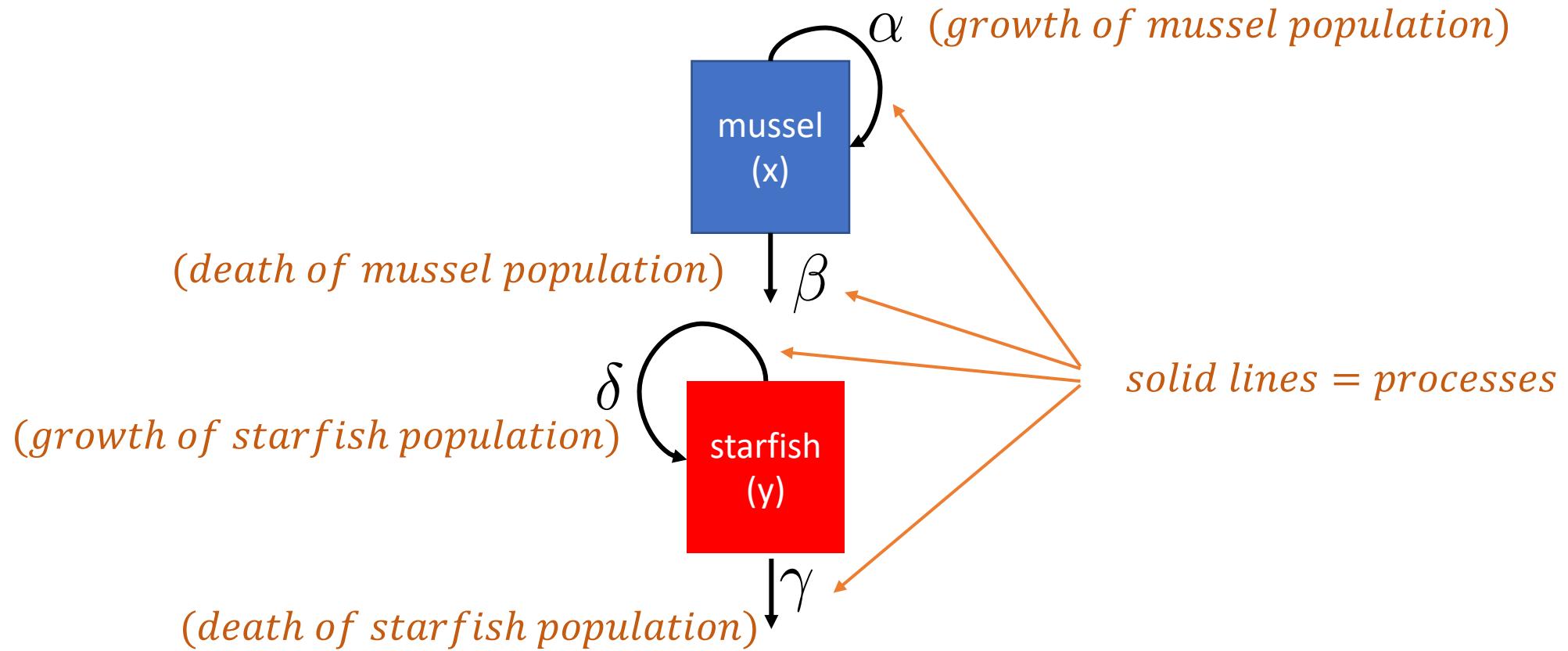
How to construct a population model



How to construct a population model



How to construct a population model

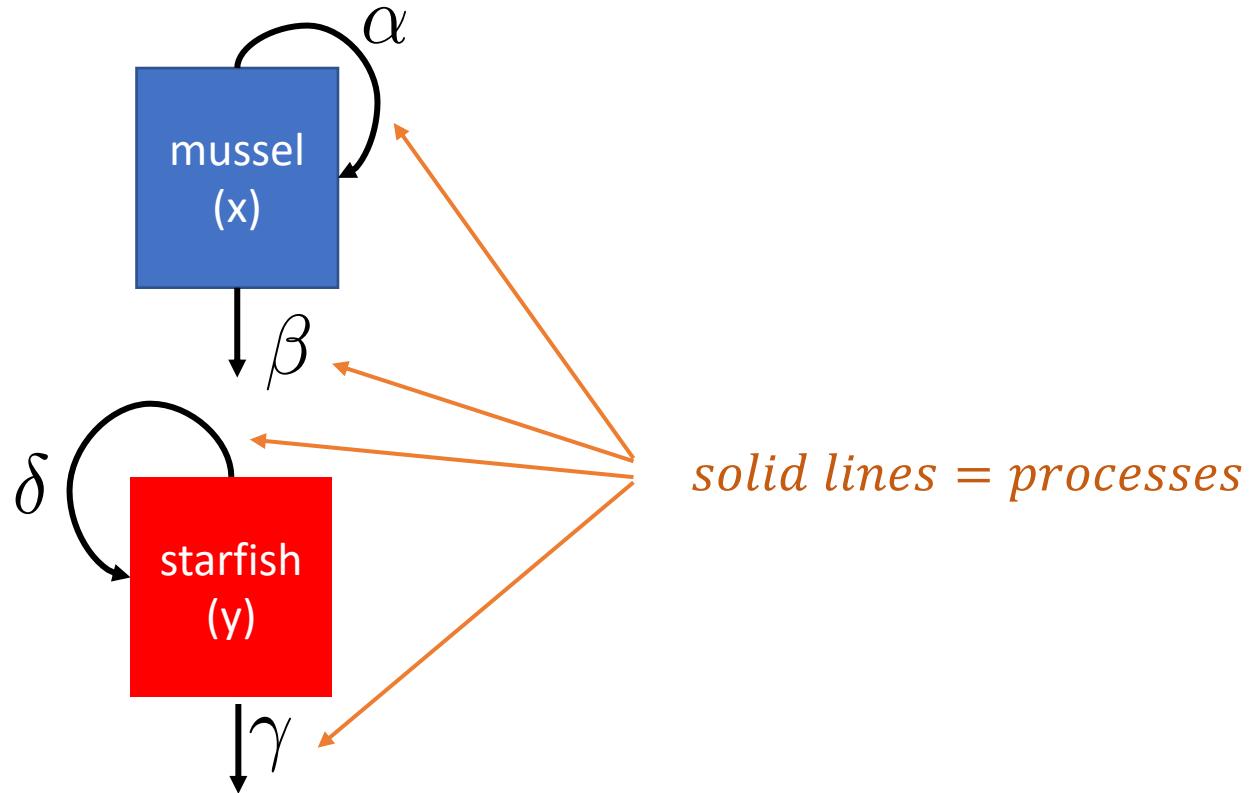


How to construct a population model

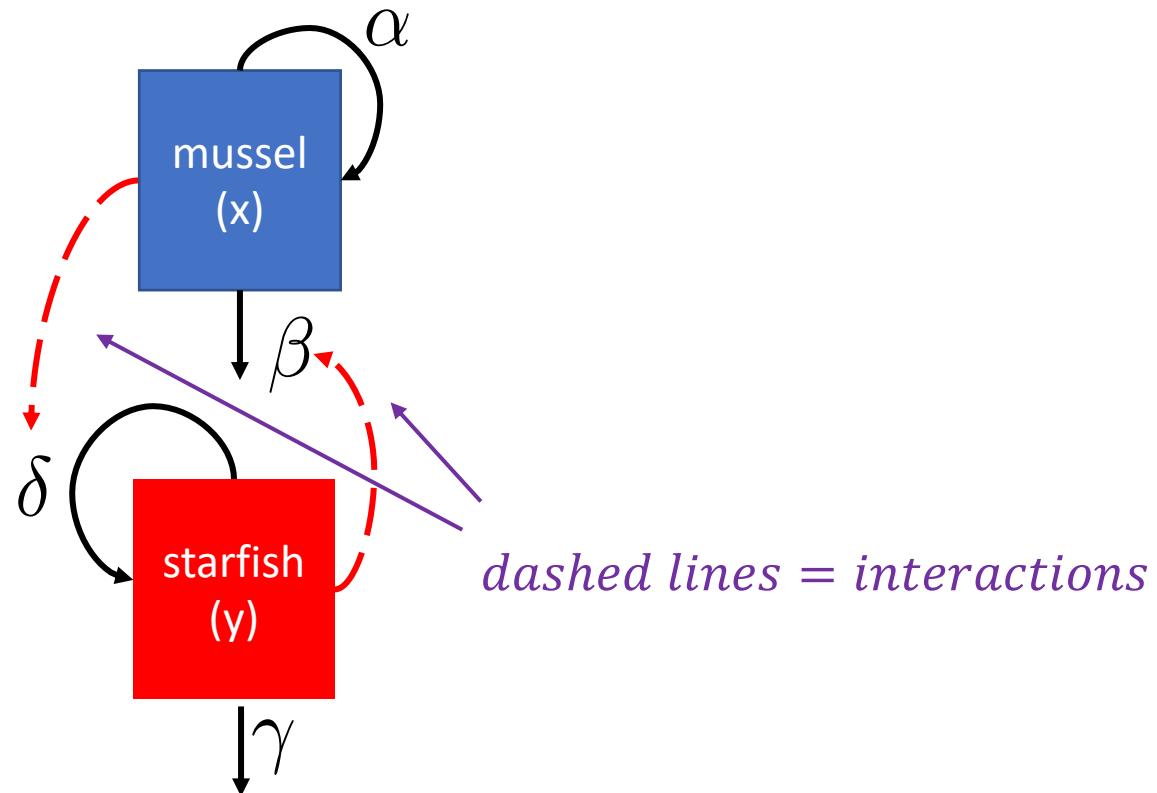
$$\frac{dx}{dt} = x(\alpha - \beta y)$$

processes

$$\frac{dy}{dt} = y(\delta x - \gamma)$$



How to construct a population model



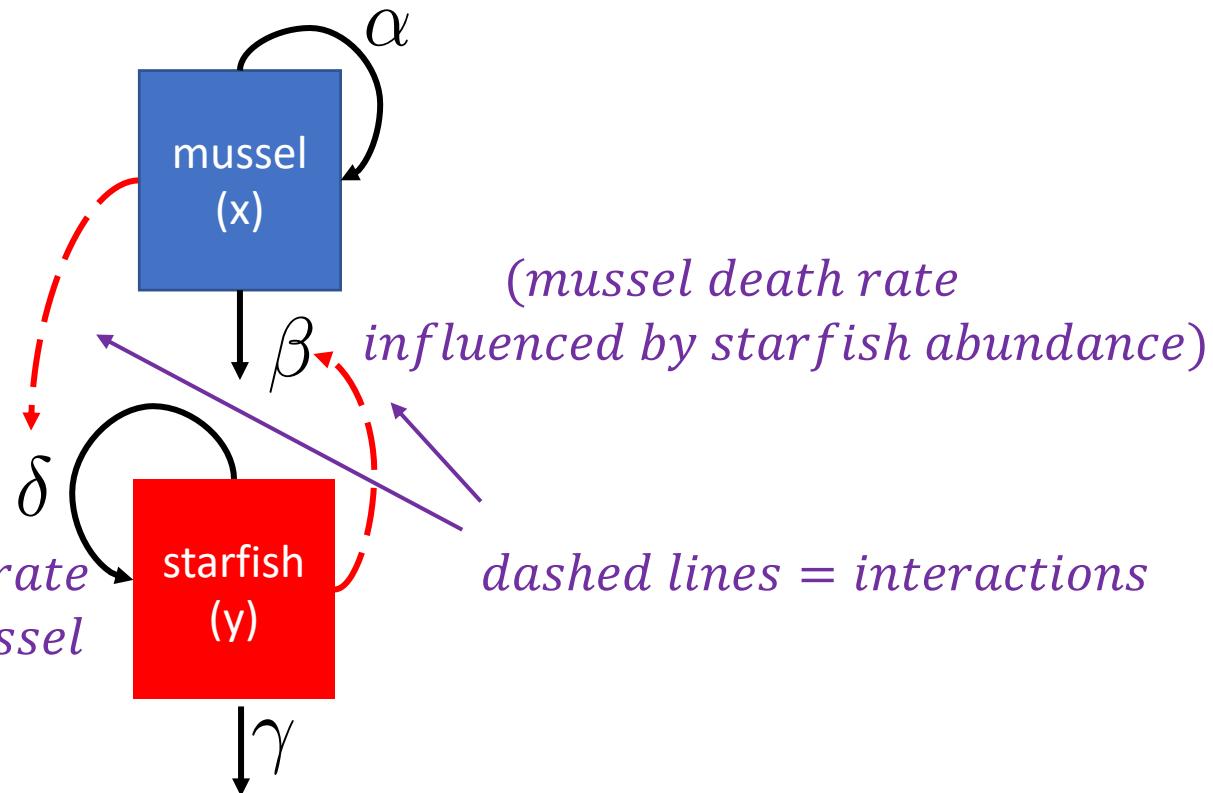
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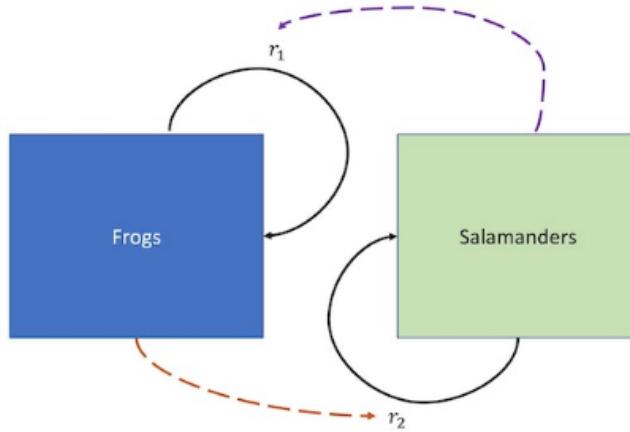
interactions

$$\frac{dy}{dt} = y(\delta x - \gamma)$$

*(starfish growth rate
influenced by mussel
abundance)*



What does the dashed purple line in this model diagram indicate?



The growth rate of the salamanders

0%

The growth rate of the frogs

0%

The influence of salamander abundance on the growth rate of the frogs

0%

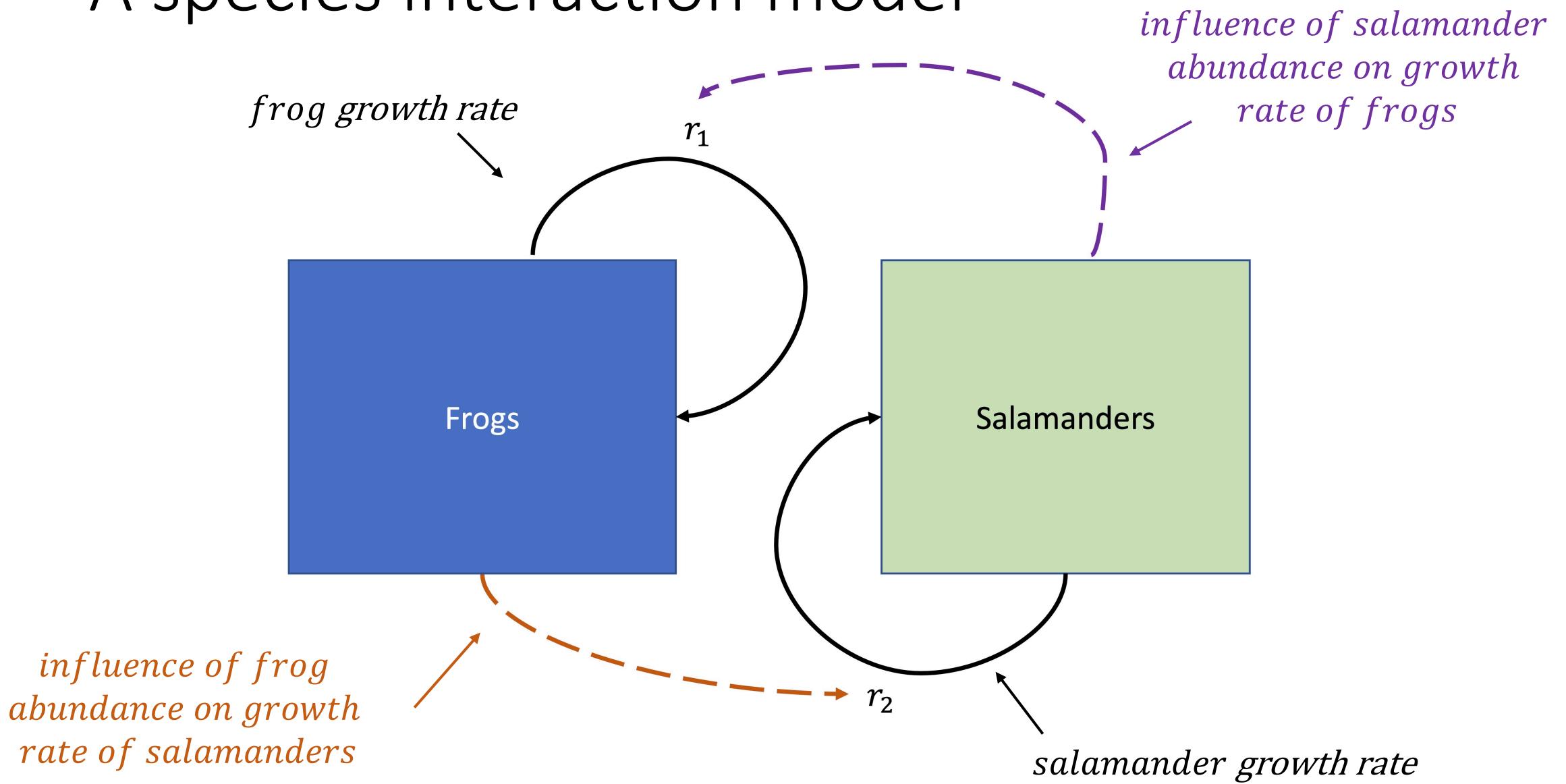
The influence of the frog abundance on the growth rate of the salamanders

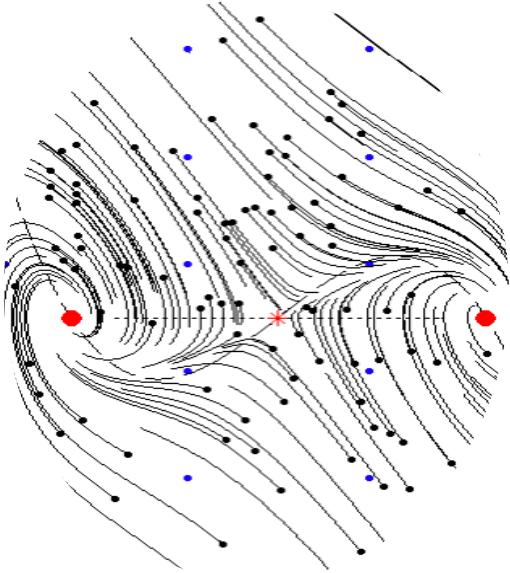
0%

The population size of the frogs

0%

A species interaction model





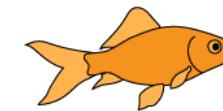
Ecology is the study of
the **interactions** of
organisms with each
other and their
environment.

dynamic
frequently changing

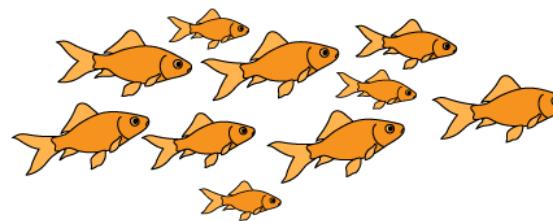
complex
many interacting parties



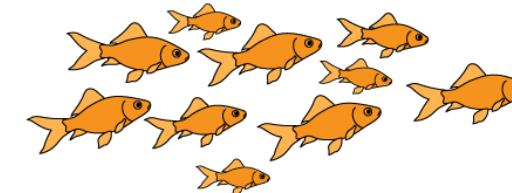
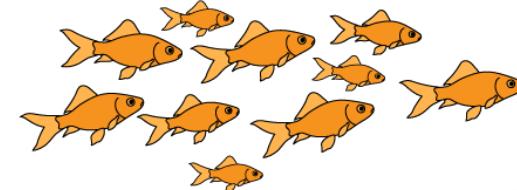
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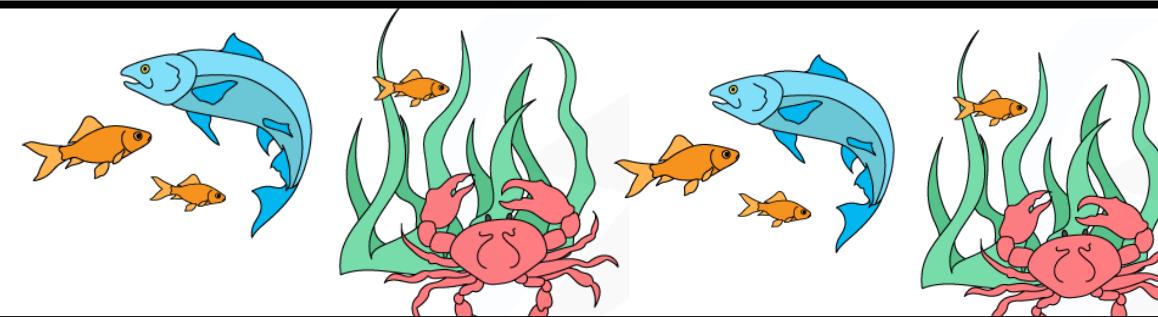
individual



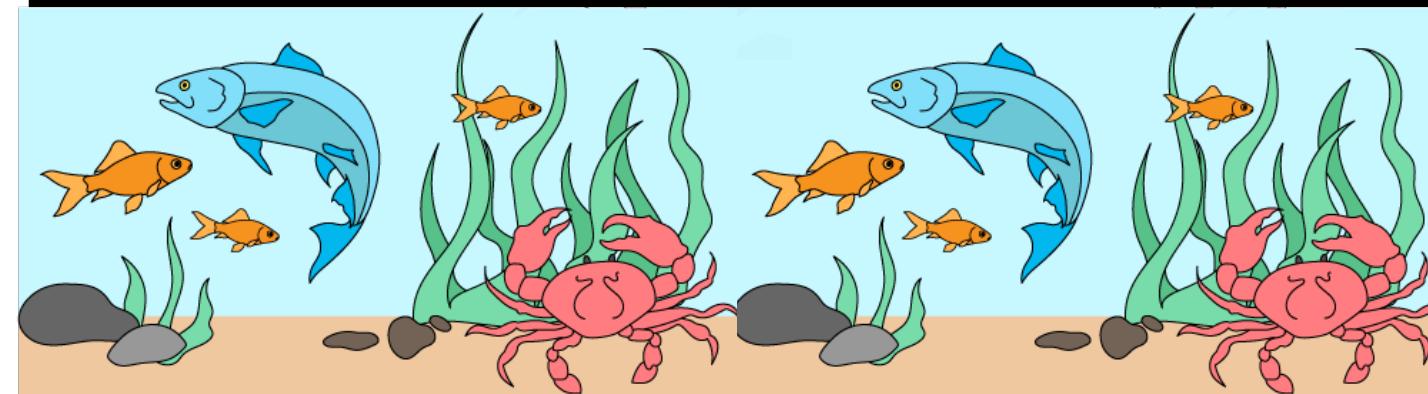
population



metapopulation

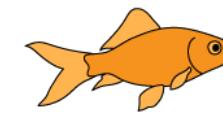


community



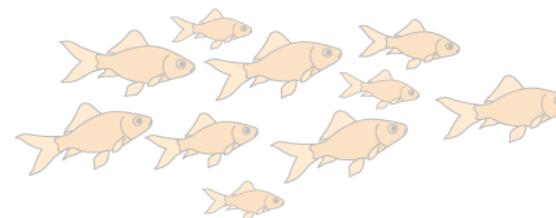
ecosystem

individual

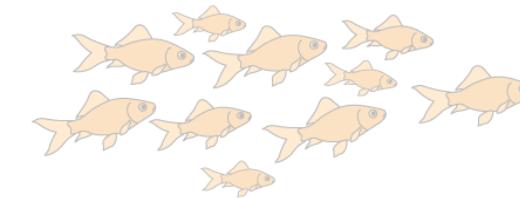
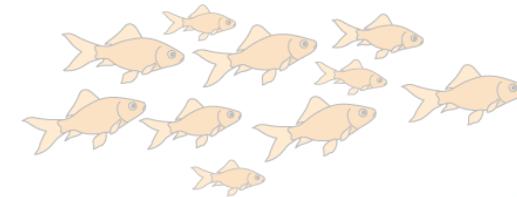


Individual:
metabolism, behavior,
life history.

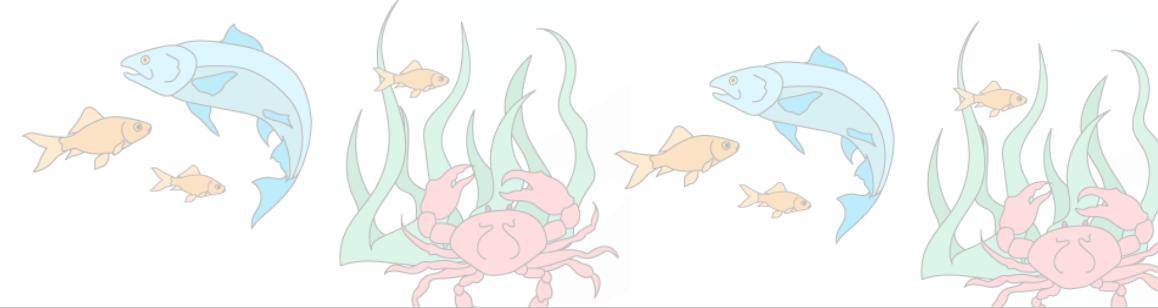
interactions of an
individual with the
environment



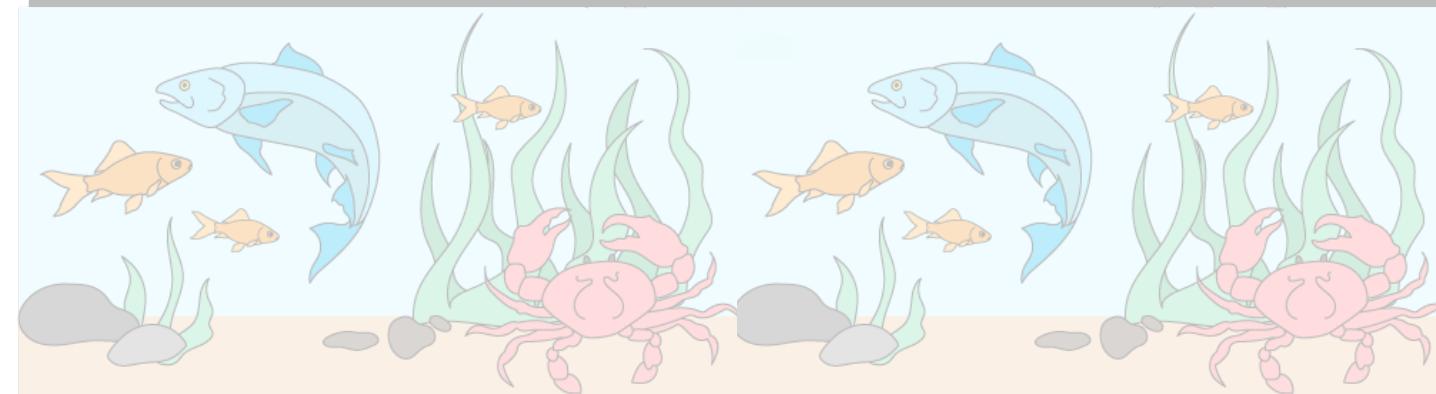
population



metapopulation

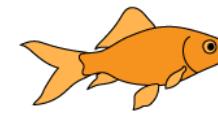


community



ecosystem

individual



Individual:
metabolism,
behavior, life history.

population

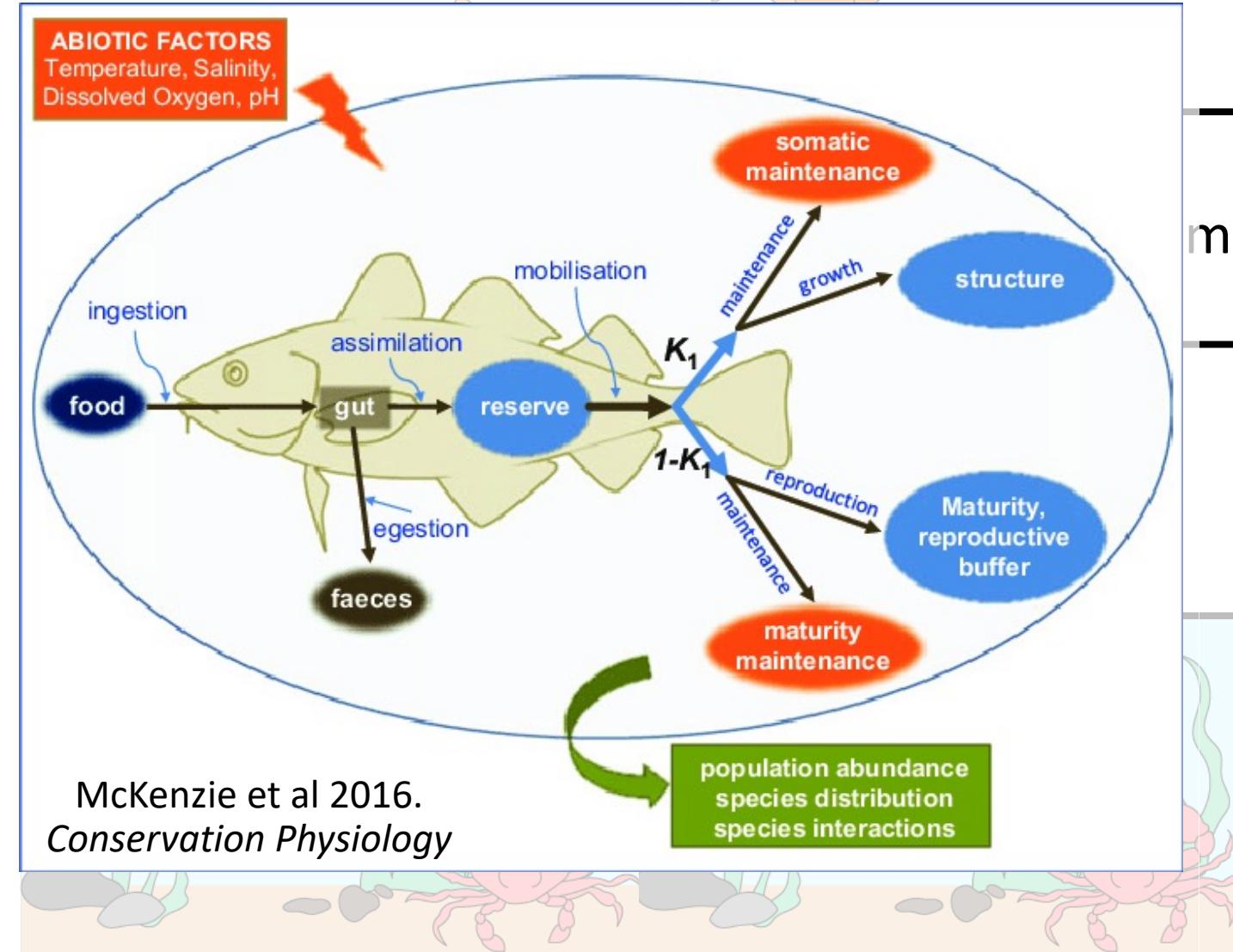
Dynamic Energy
Budget (DEB) Model

metapopulation

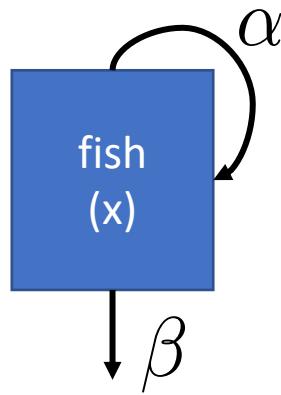
*How does a fish's
metabolism **change**
with temperature?*

community

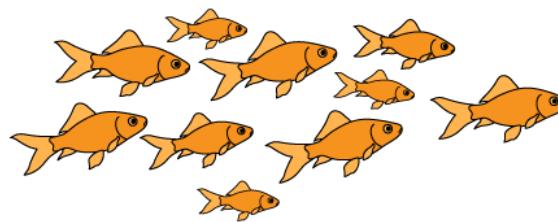
ecosystem



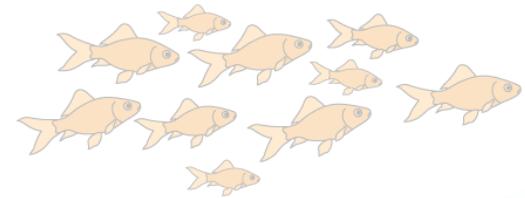
Population = multiple individuals of the same species (**conspecifics**) in the same habitat



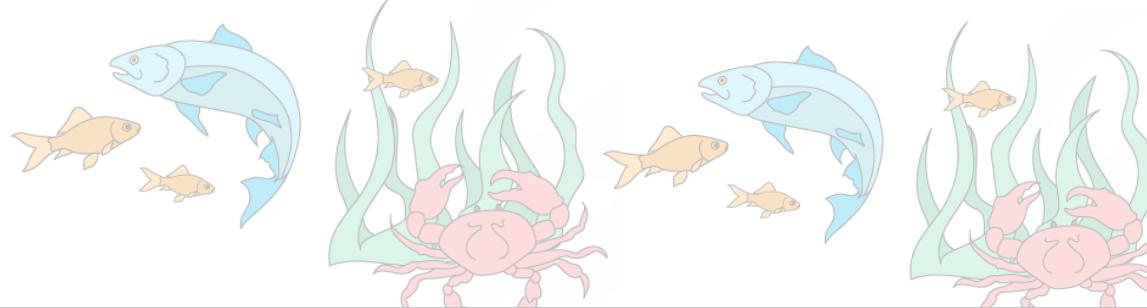
individual



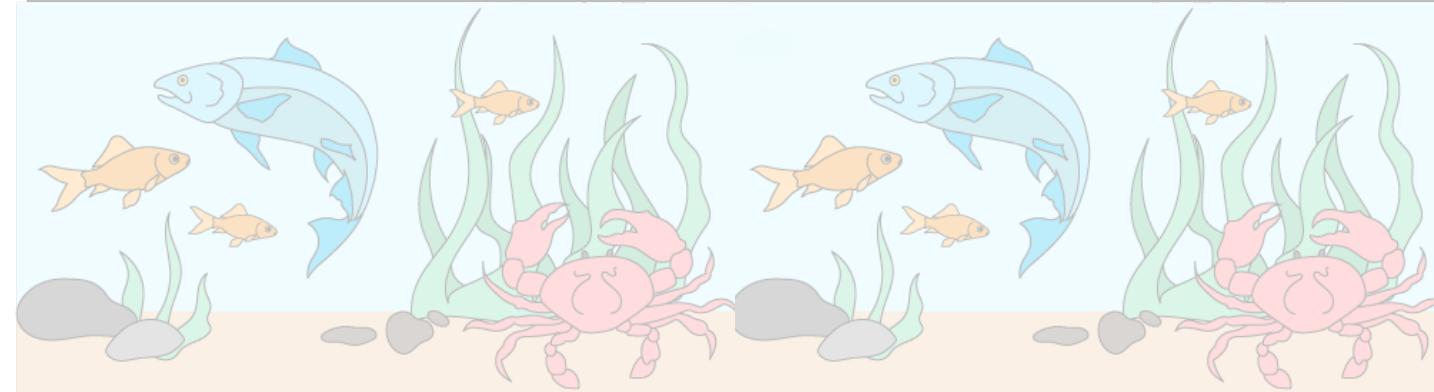
population



metapopulation



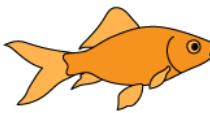
community



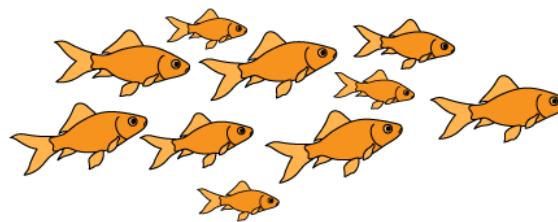
ecosystem

*How does the abundance of fish **change** through time?*

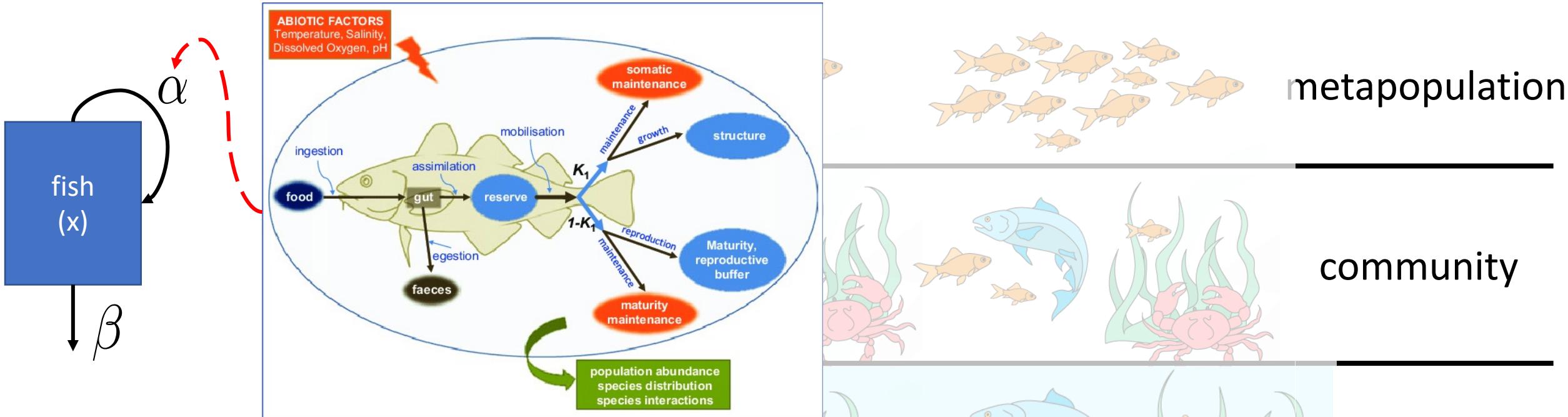
Nested Models, including a class of model known as **Integral Projection Models** (IPMs), link individual- and population-level processes



individual

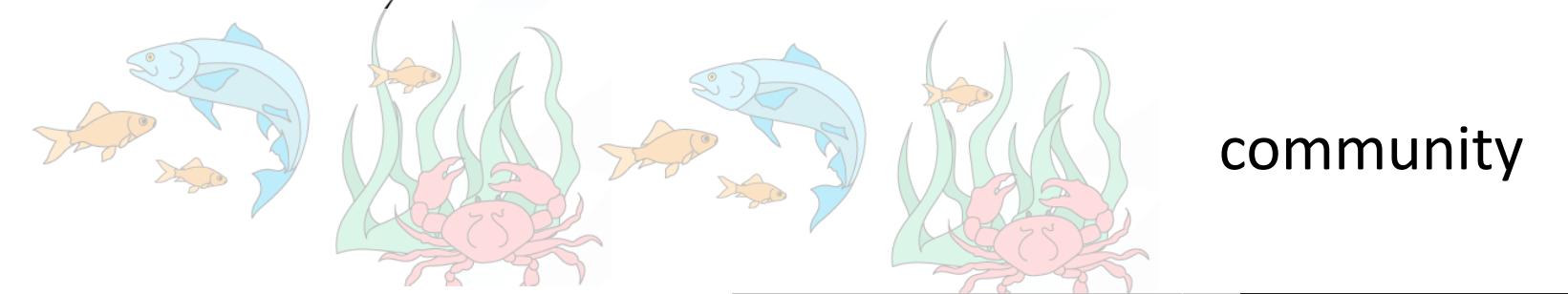
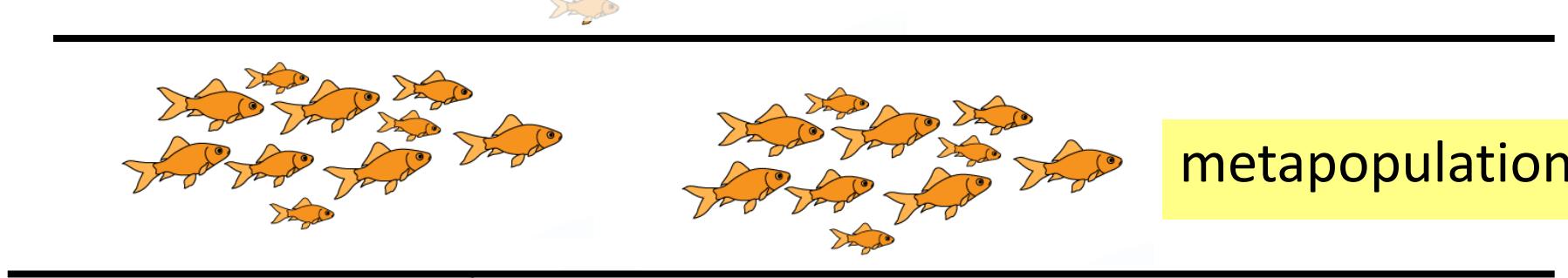
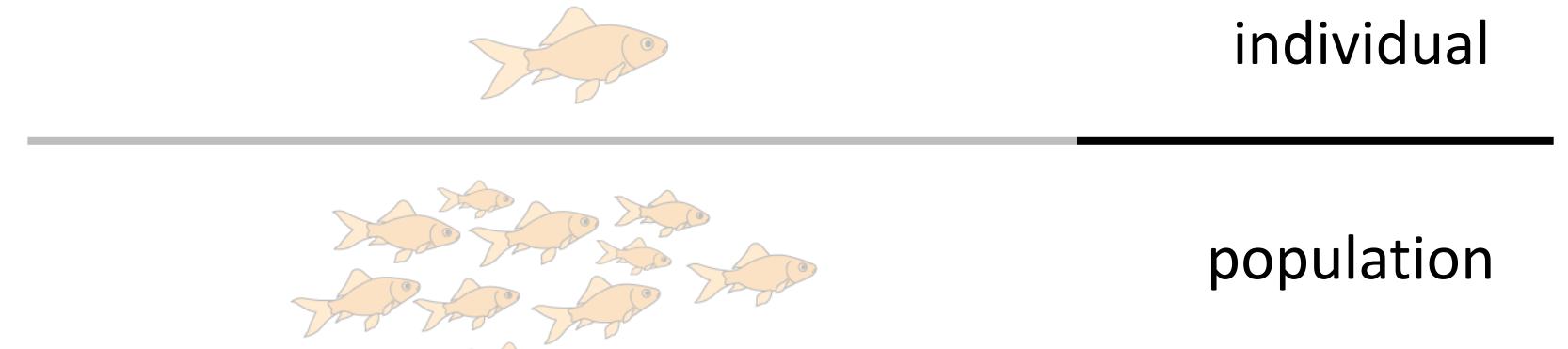
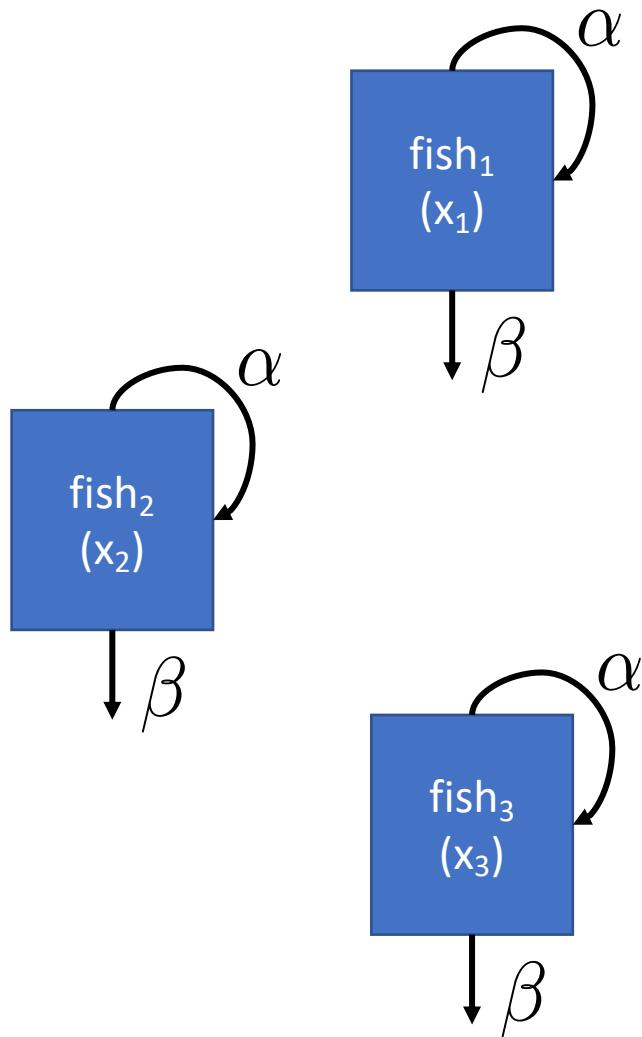


population

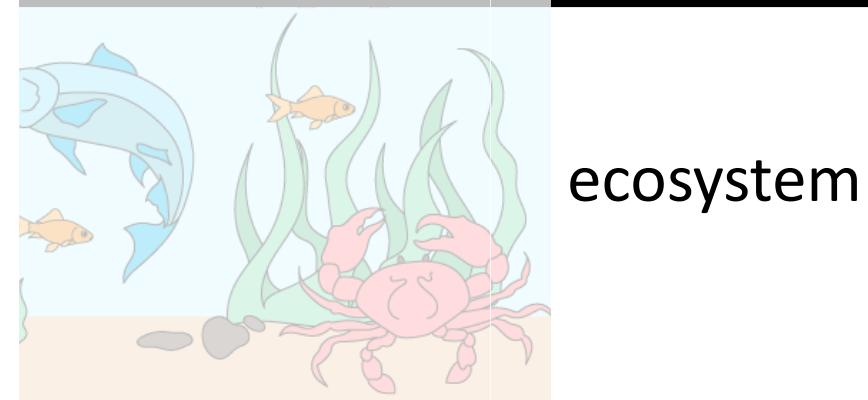


How does the abundance of fish **change** through time as temperature **changes** metabolism?

Metapopulation = sub-populations of conspecifics connected by migration or dispersal

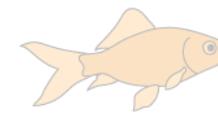


*How do the abundances of different subpopulations of the same fish **vary** in space [and time]?*

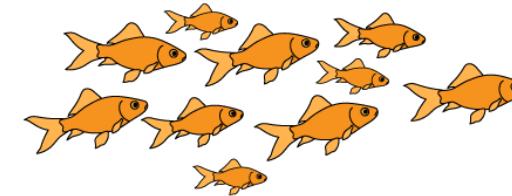
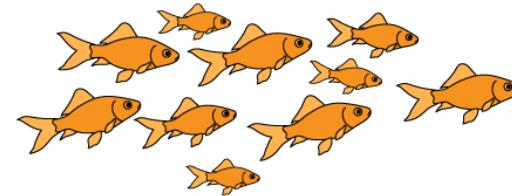
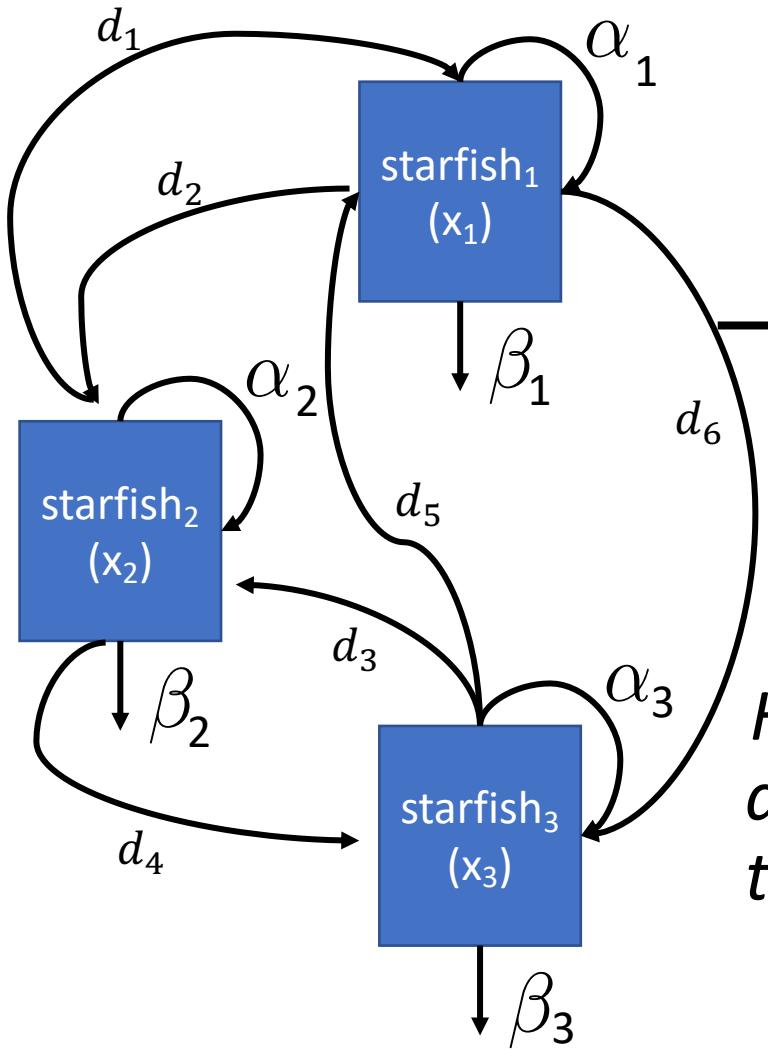
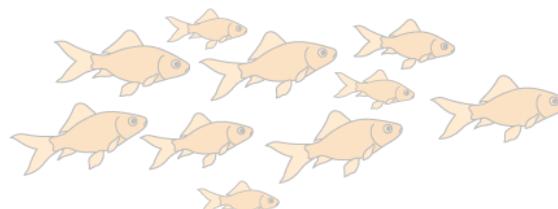


Metapopulation = sub-populations of conspecifics connected by migration or dispersal

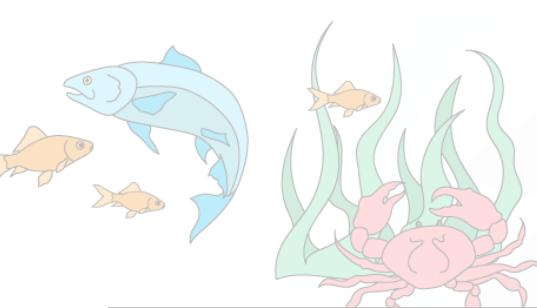
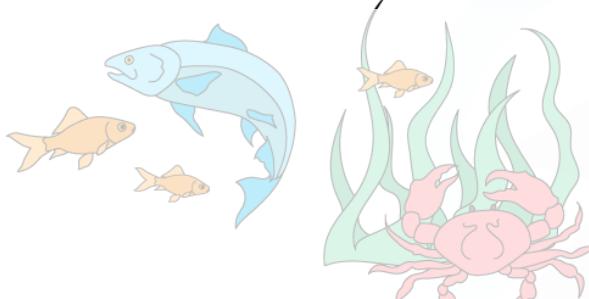
individual



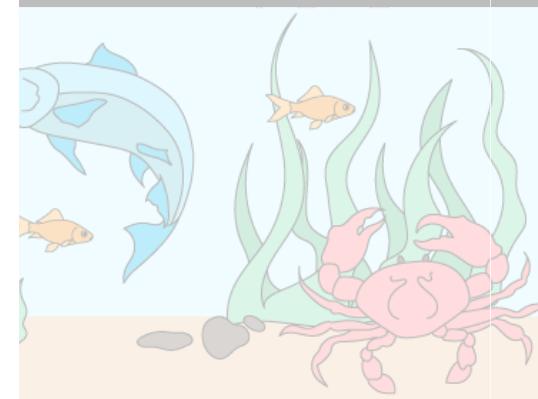
population



metapopulation



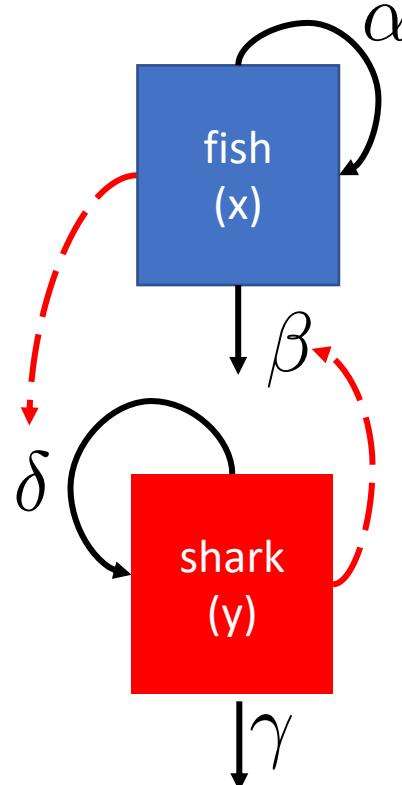
community



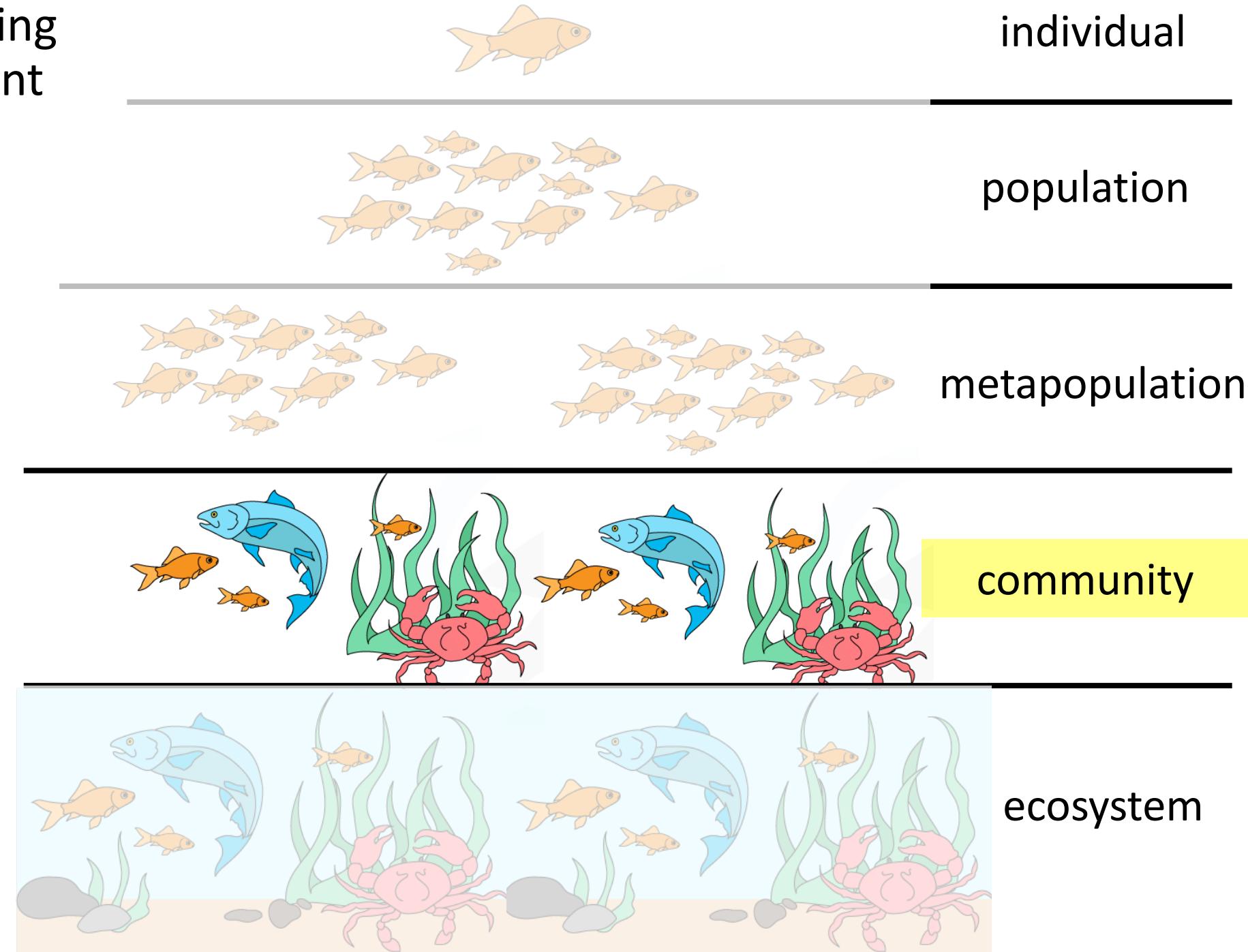
ecosystem

*How do the abundances of different subpopulations of the same fish **vary** in space [and time]?*

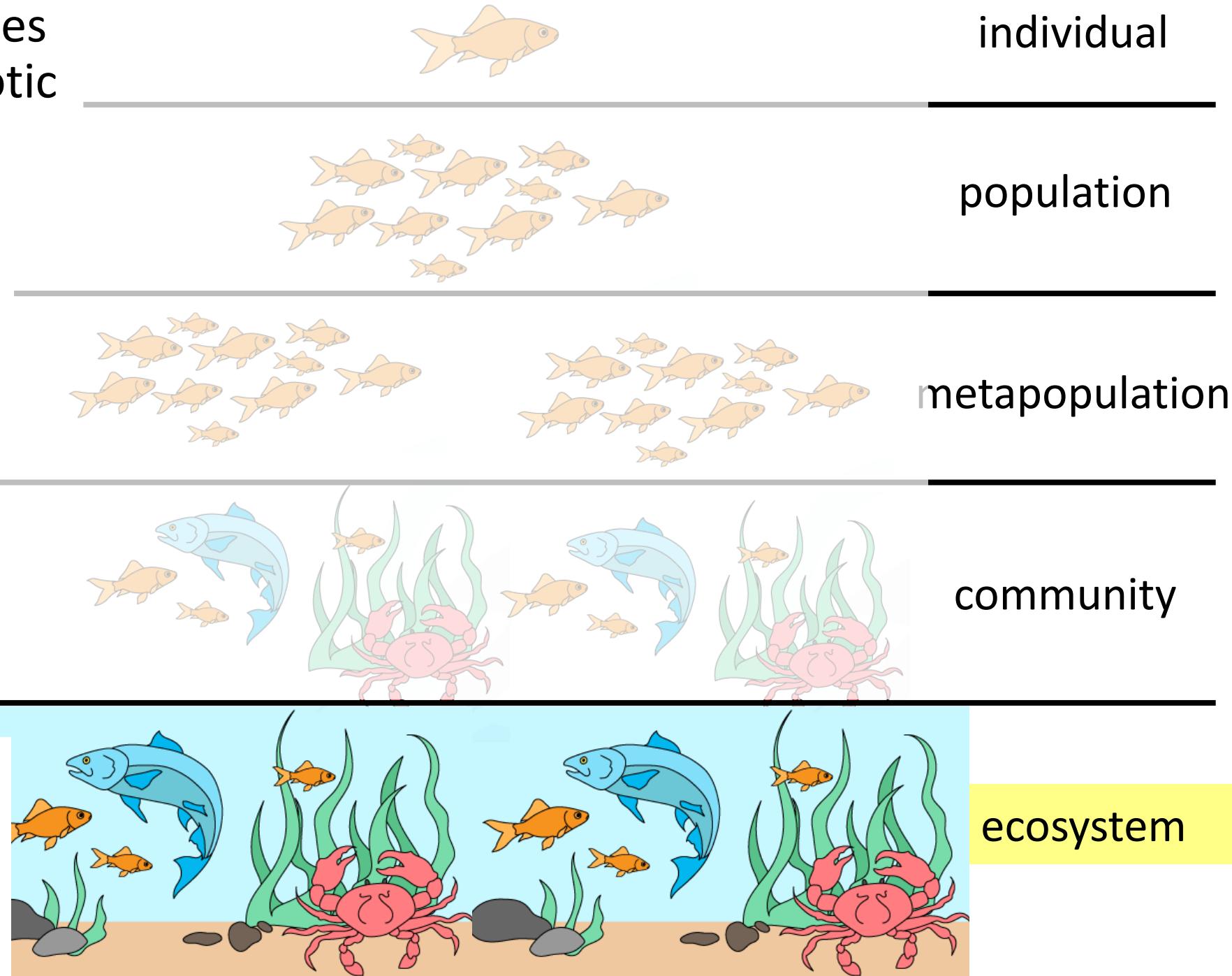
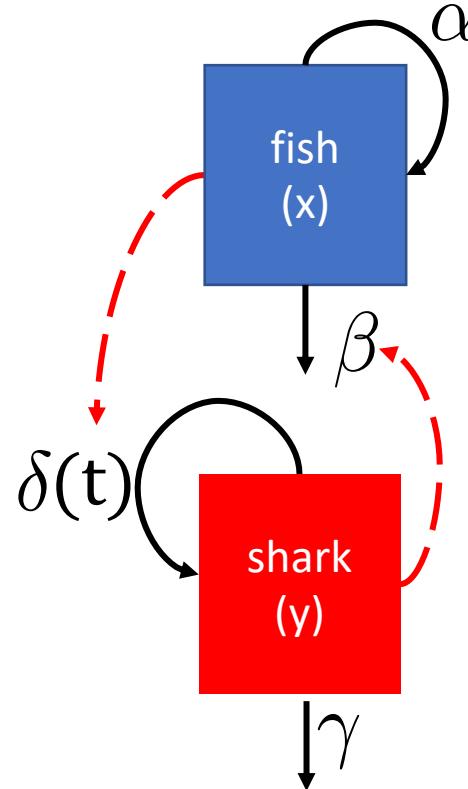
Community = interacting populations of different species



How does fish abundance **vary** with changes in shark abundance?



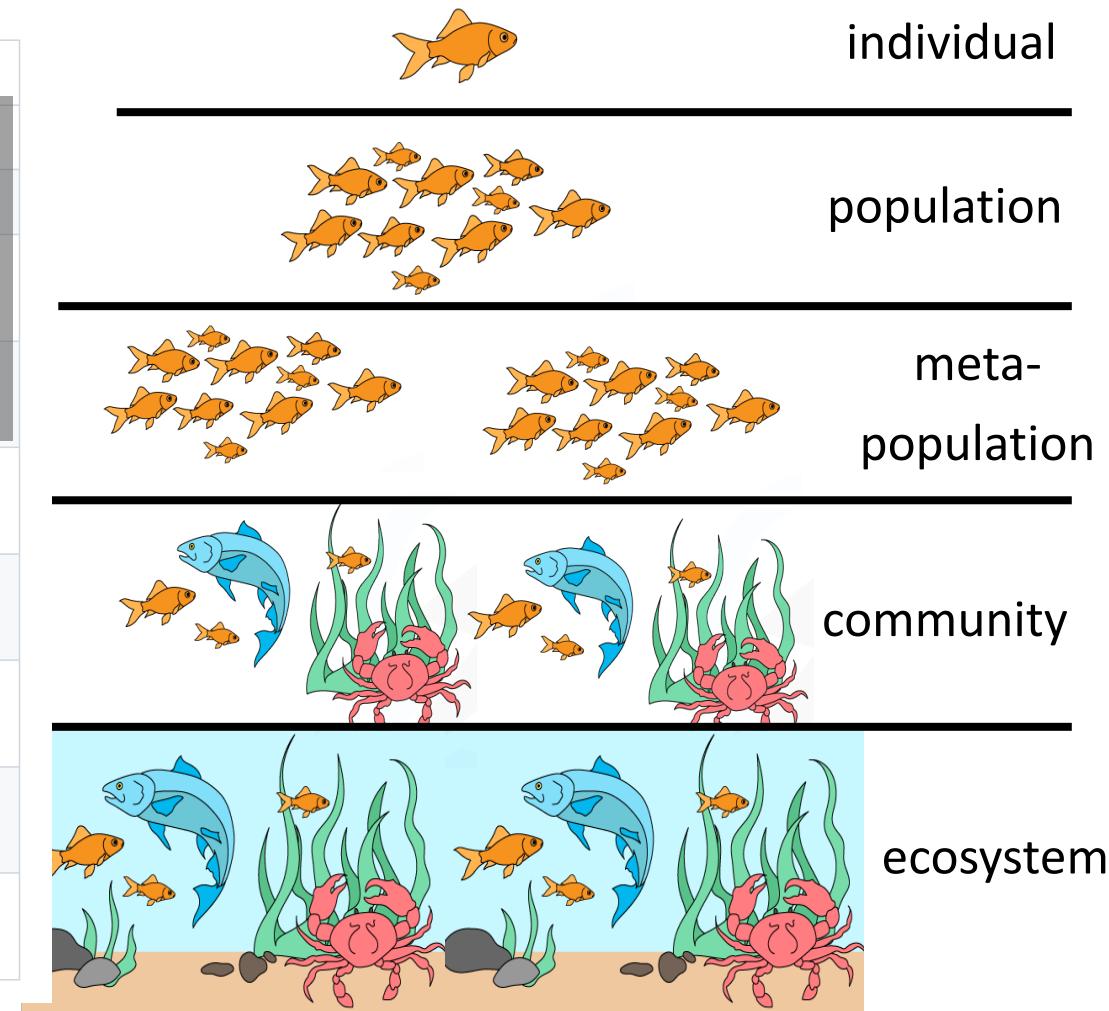
Ecosystem = communities interacting with the abiotic environment



*How does fish abundance **vary** with changes in shark birth rates with **temperature**?*

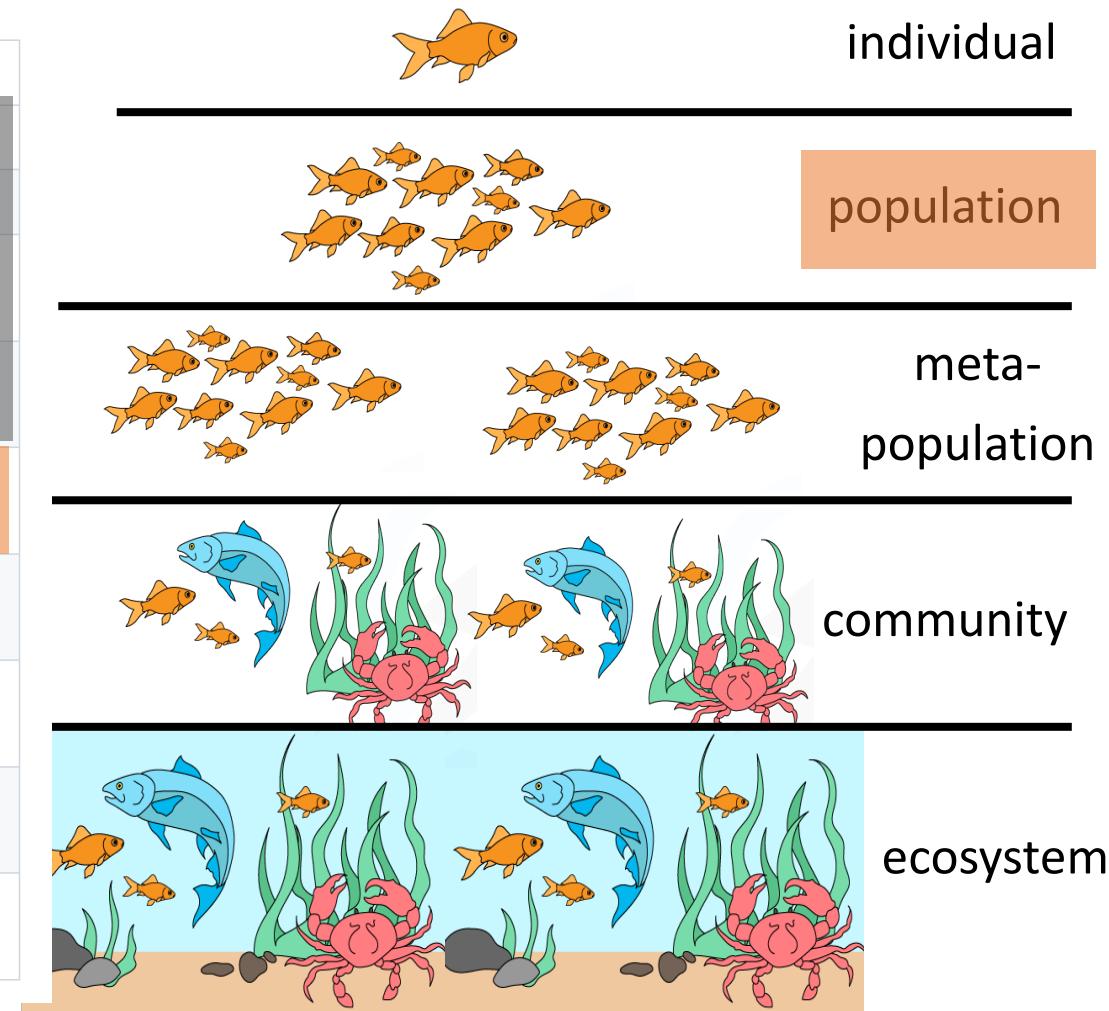
Where are we headed?

Week	Mon/Tues Lab	Tues Lecture	Thurs Lecture
1	--	--	Jan 4: Evolution History
2	1: Intro R	Jan 9: Mutation and Variation	Jan 11: Hardy-Weinberg
3	2: Hardy-Weinberg	Jan 16: Migration and Drift	Jan 18: Natural Selection
4	3: Microevolution	Jan 23: Phylogenetics	Jan 25: Molecular Evolution & Sexual Selection
5	4: Phylogenetics	Jan 30: Kin Selection, Speciation	Feb 1: Ecology & Population Growth
6	5: Evolution paper	Feb 6: Single Species Population Growth & Regulation	Feb 8: Midterm
7	6: Population Growth	Feb 13: Species Interactions 1	Feb 15: Species Interactions 2
8	7: Population Regulation	Feb 20: Disease Dynamics as Population Biology 1	Feb 22: Disease Dynamics as Population Biology 2
9	8: Disease Dynamics	Feb 27: Community Assembly & Island Biogeography	Feb 29: Critical Transitions & Conservation Interventions



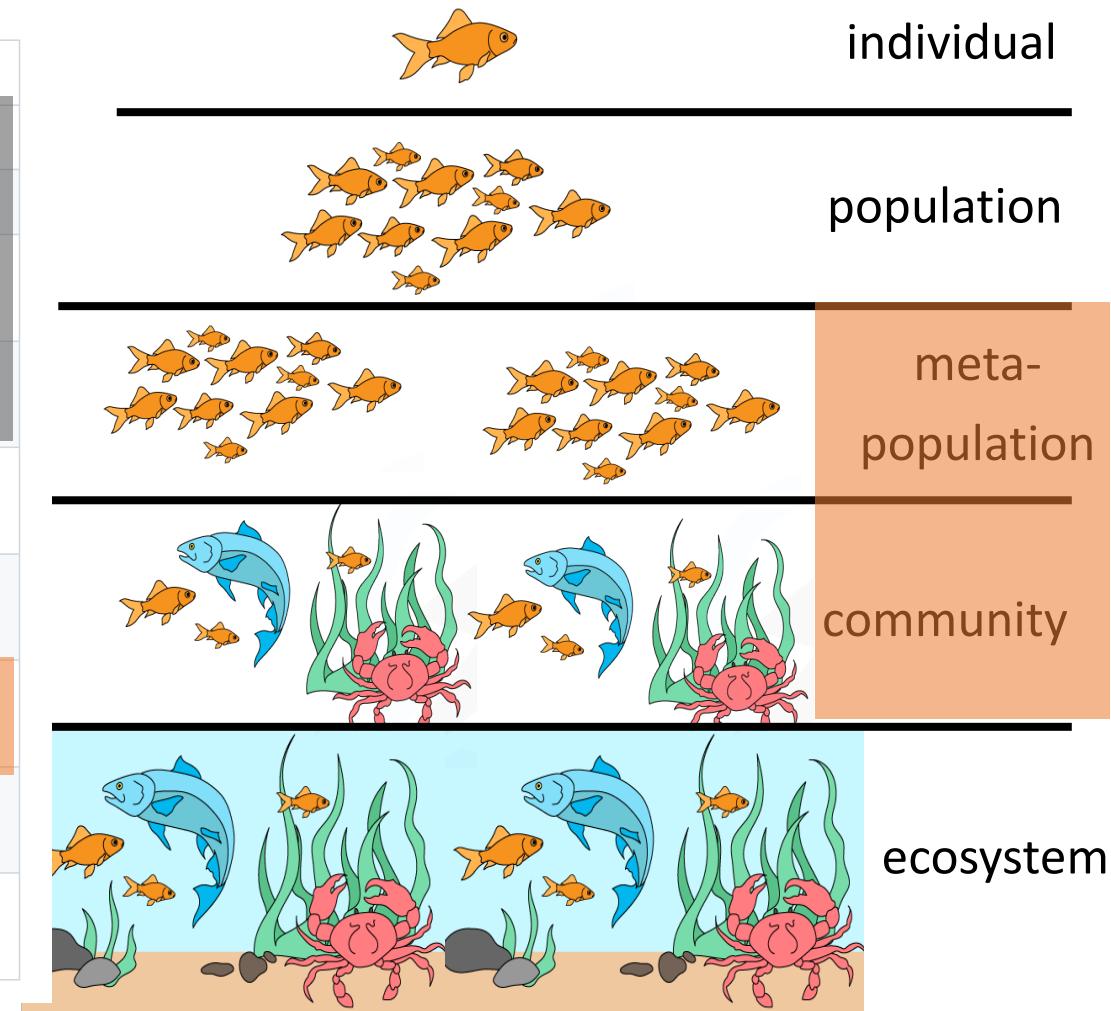
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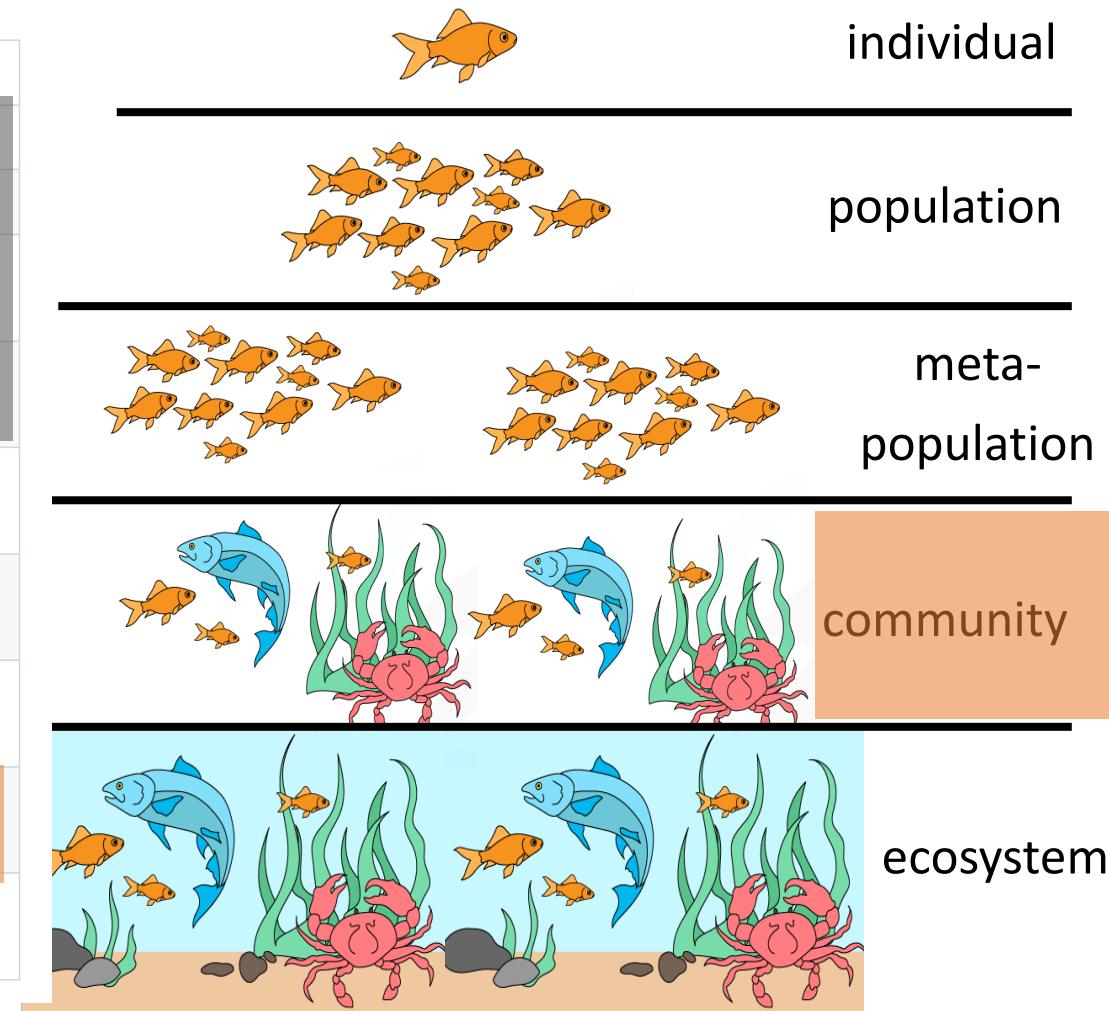
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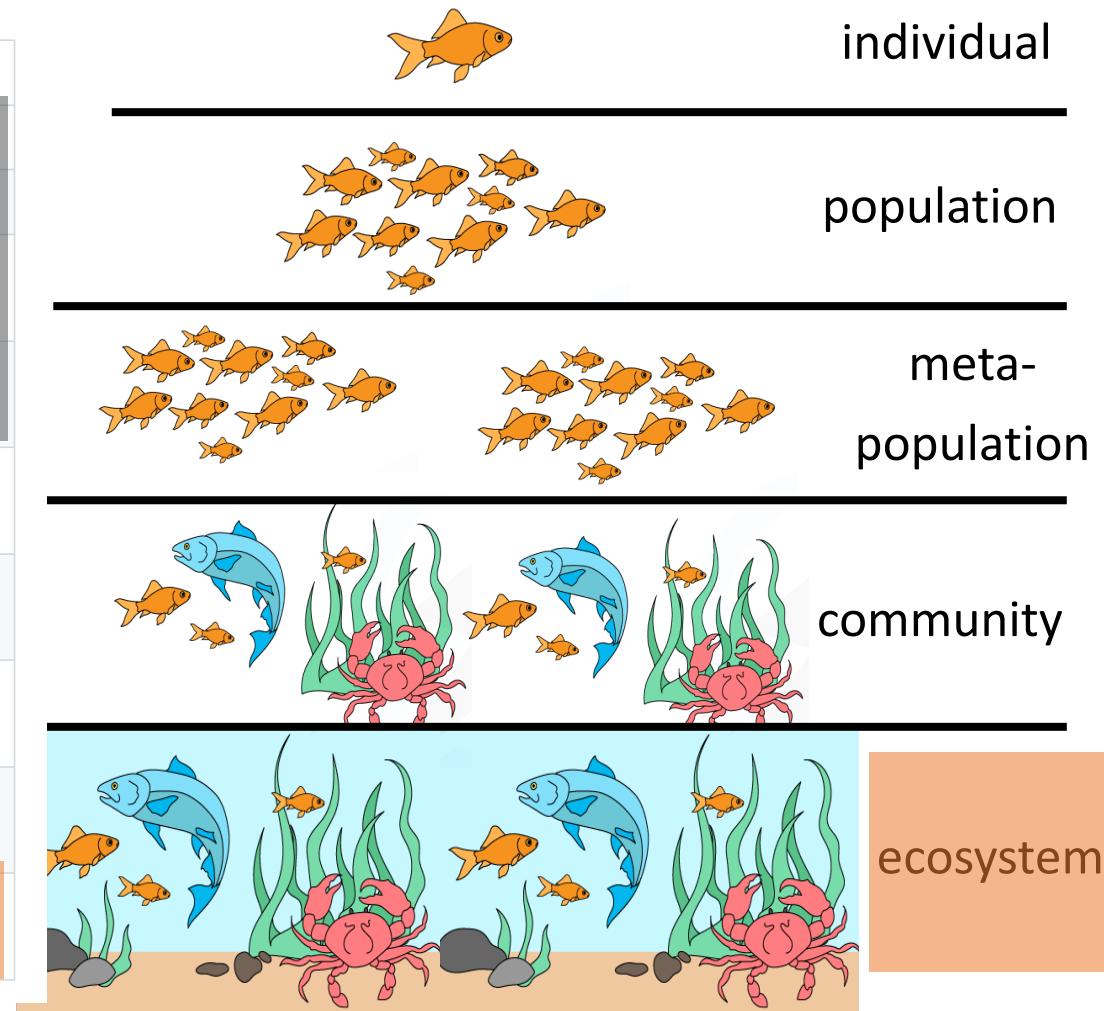
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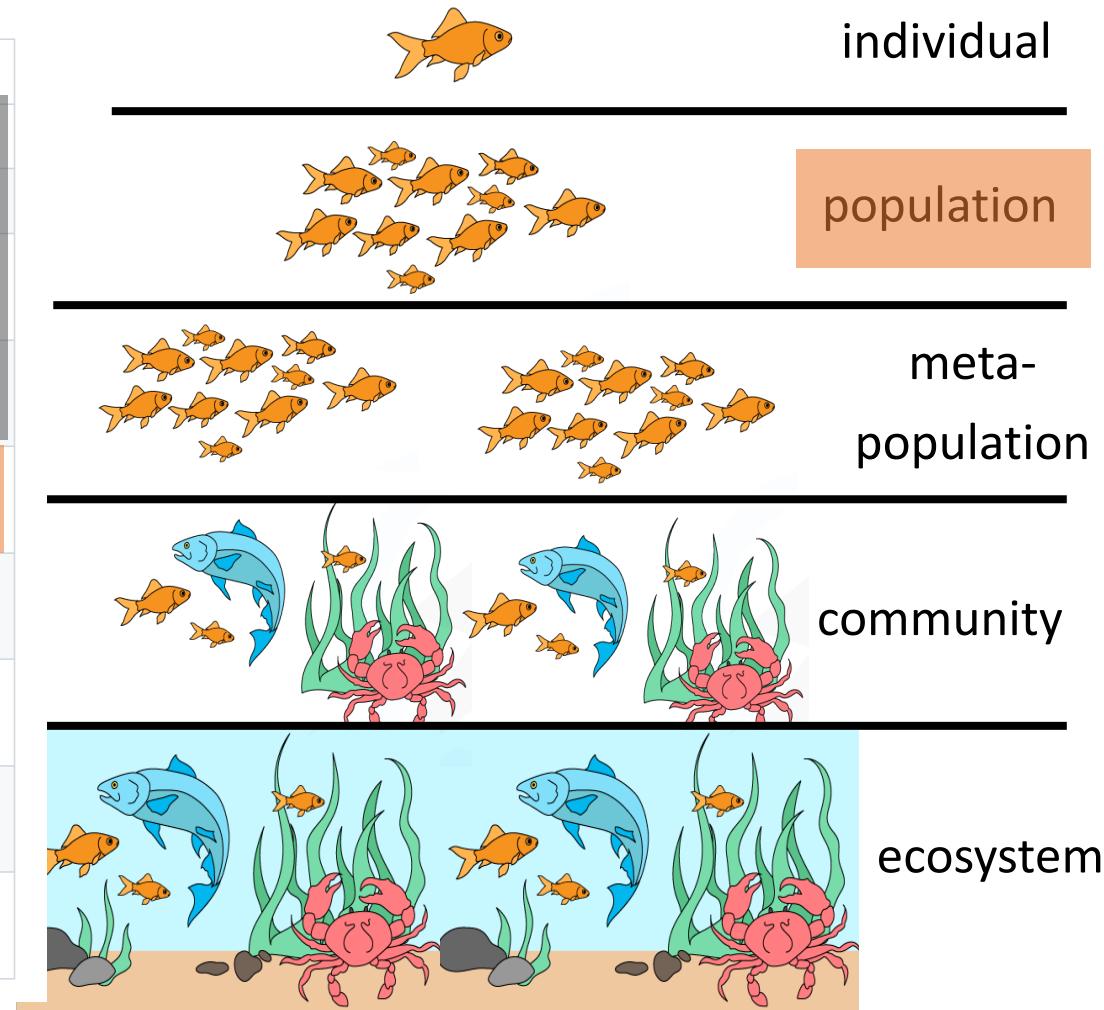
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Population Growth and Regulation

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Why do we care how populations grow?

(Shafer 1980)

DETERMINING MINIMUM VIABLE POPULATION SIZES FOR THE GRIZZLY BEAR¹

MARK L. SHAFFER,² School of Forestry and Environmental Studies, Duke University, Durham, NC 27706

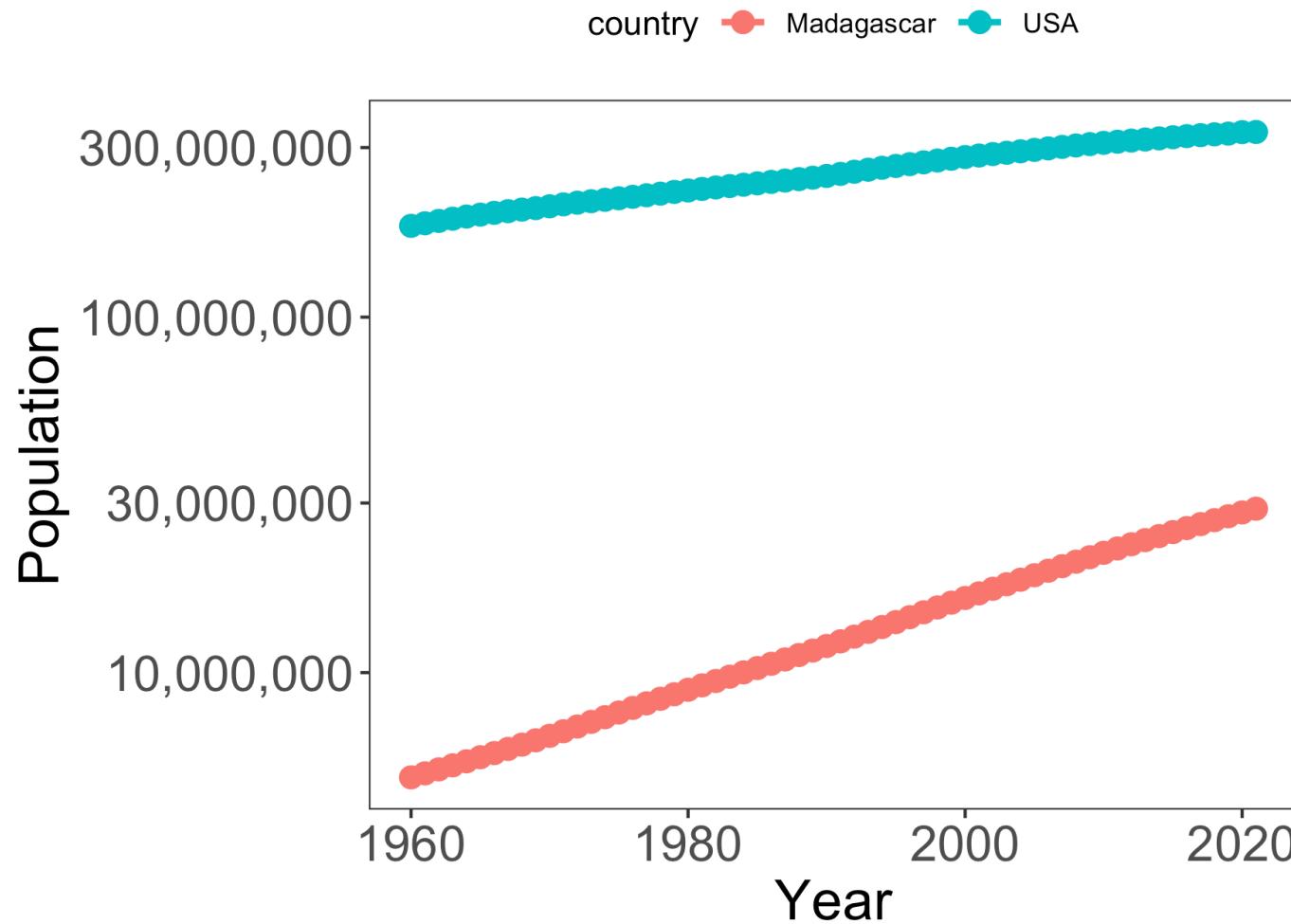
Abstract: A stochastic computer simulation is presented for use in determining the relationship of population size to extinction probabilities for populations of grizzly bears (*Ursus arctos*). Published data on numbers, age, sex, reproduction, and mortality for the grizzly bear population of Yellowstone National Park were used to develop and test several simulation models. The results indicate that, for the Yellowstone grizzlies, 35 to 70 bears constitute a minimum viable population (the smallest population with a 95% probability of surviving at least 100 years). Minimum area requirements for populations of this size range from 700 to 10,000 km.²

Int. Conf. Bear Res. and Manage. 5:133–139



to protect populations from extinction

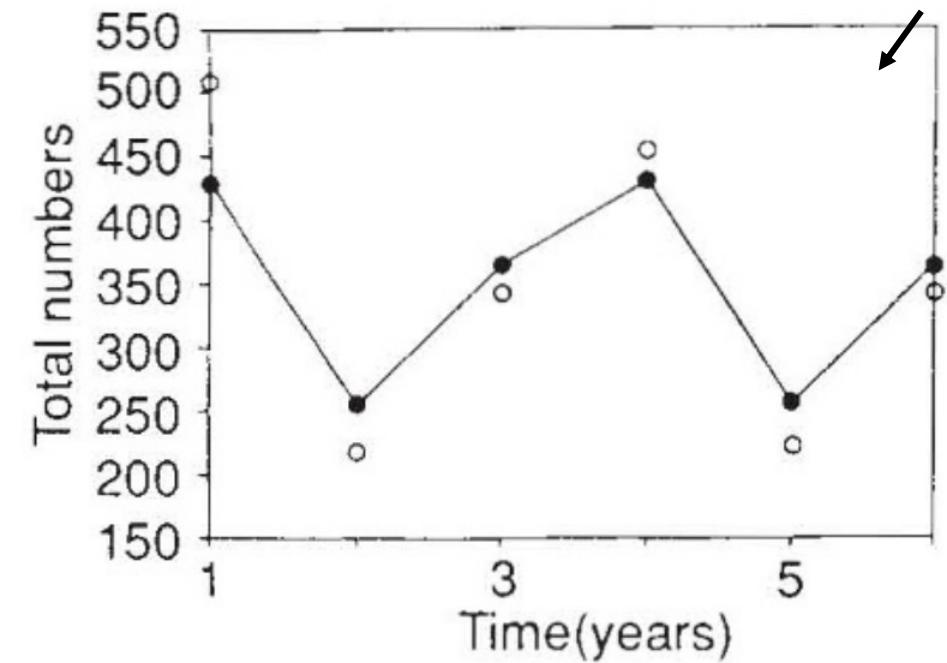
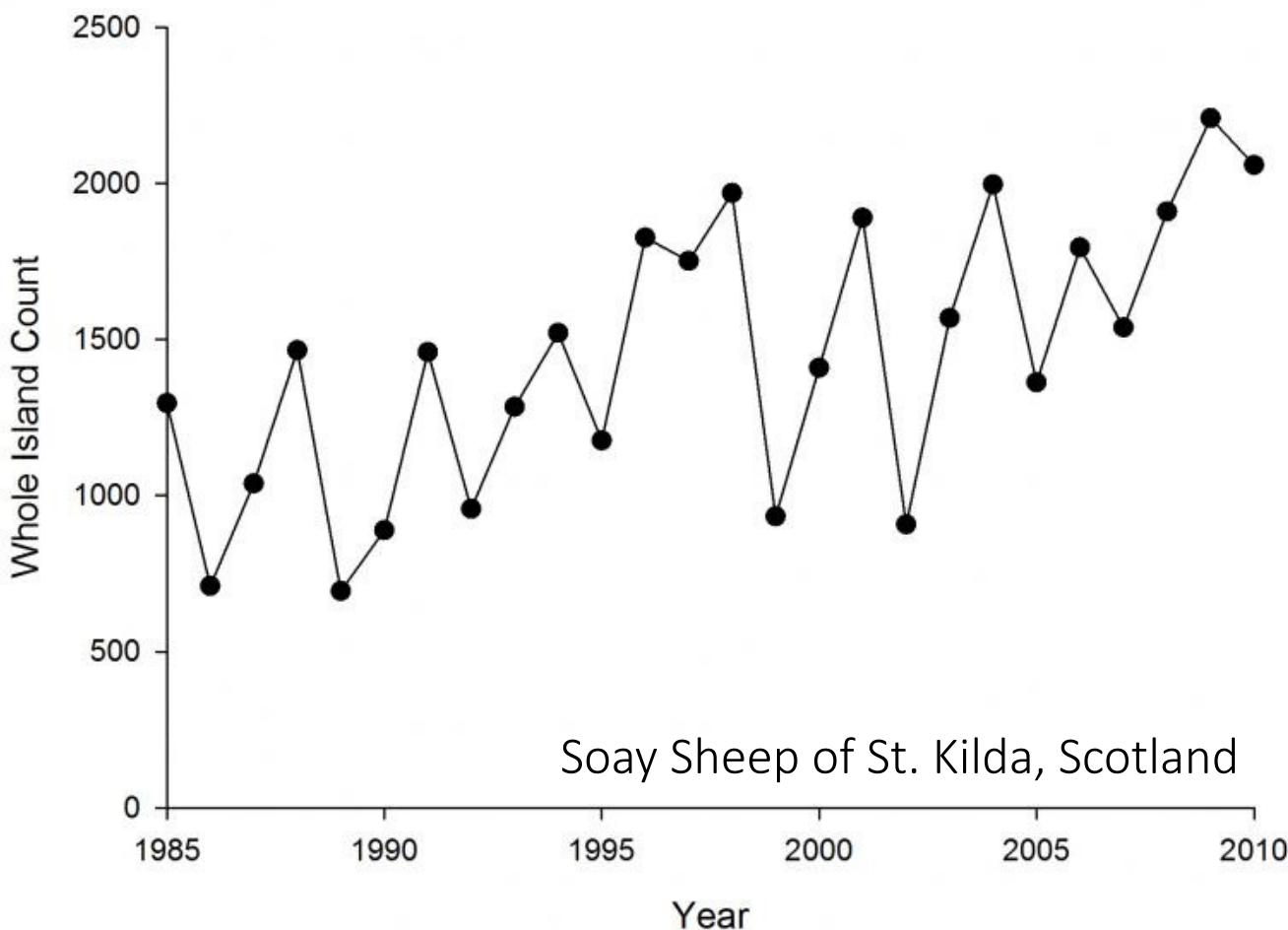
Why do we care how populations grow?



to forecast resource use

Why do we care how populations grow?

to understand past phenomena



Overcompensation and population cycles in an ungulate

[B. T. Grenfell, O. F. Price, S. D. Albon & T. H. Glutton-Brock](#)

[Nature](#) 355, 823–826 (1992) | [Cite this article](#)

589 Accesses | 107 Citations | [Metrics](#)

Model
compared
against
data

The simplest population model

1. Populations are divided into compartments
2. Individuals within a compartment are homogeneously mixed
3. Compartments and transition rates are determined by biological systems
4. Rates of transferring between compartments are expressed mathematically



The simplest population model

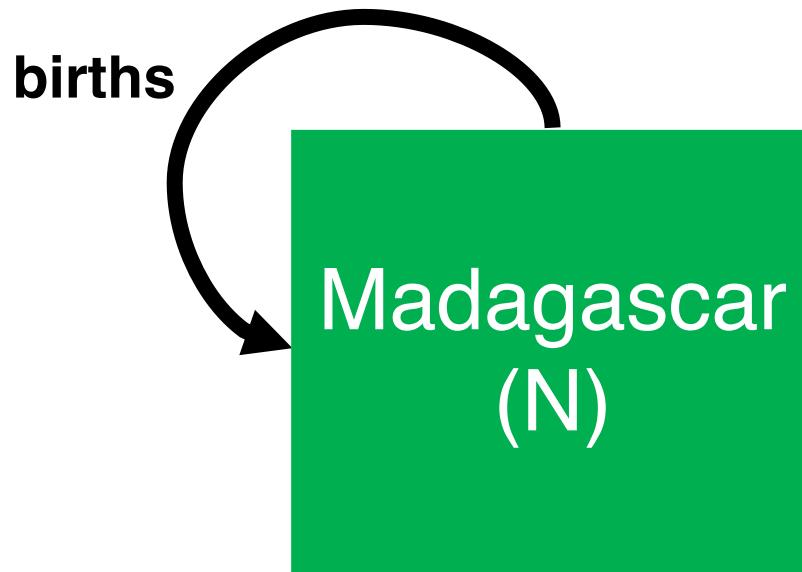
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Madagascar
(N)

How does the population grow?



The simplest population model

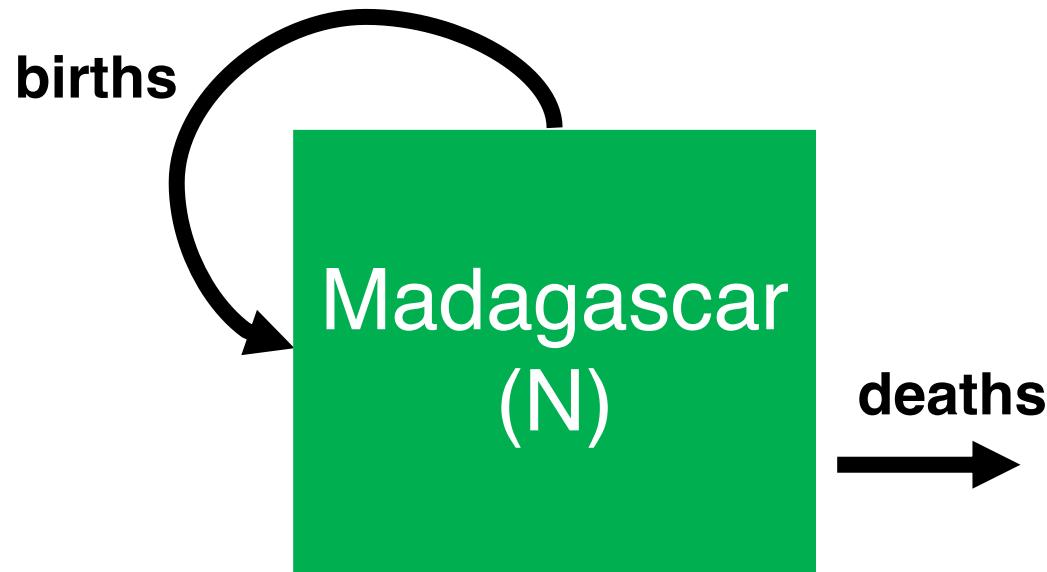


1. Populations are divided into compartments
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How does the population grow?

The simplest population model



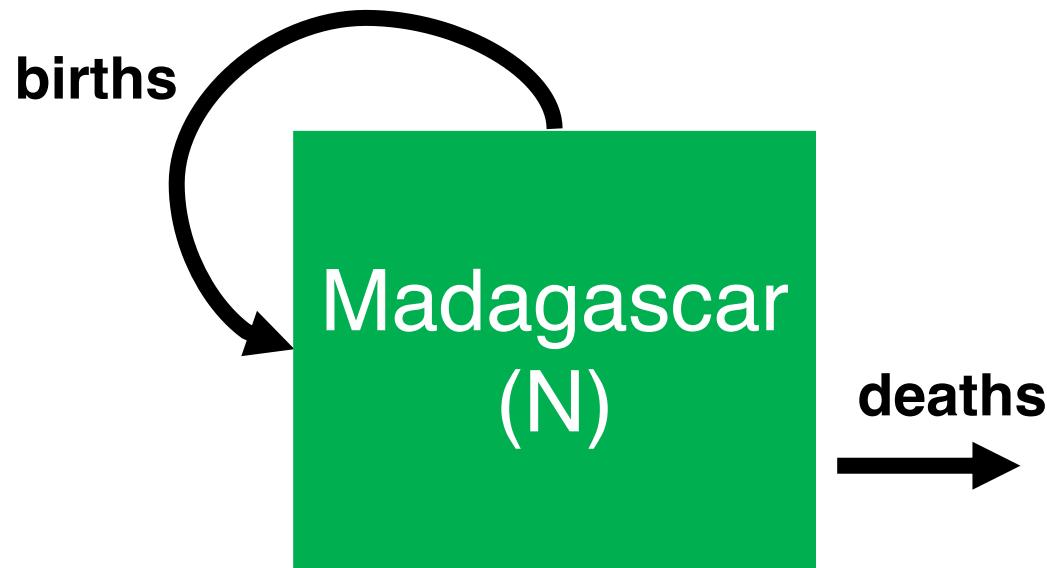
1. Populations are divided into compartments
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4. Rates of transferring between compartments are expressed mathematically



How does the population decrease?

The simplest population model

1. Populations are divided into compartments
2. Individuals within a compartment are homogenously mixed
3. Compartments and transition rates are determined by biological systems
4. Rates of transferring between compartments are expressed mathematically



$$N_{t+1} = N_t + \text{births} * N_t - \text{deaths} * N_t$$

$$N_{t+1} = N_t + R * N_t$$

$$N_{t+1} = (1 + R) * N_t$$

$$N_{t+1} = \lambda * N_t$$

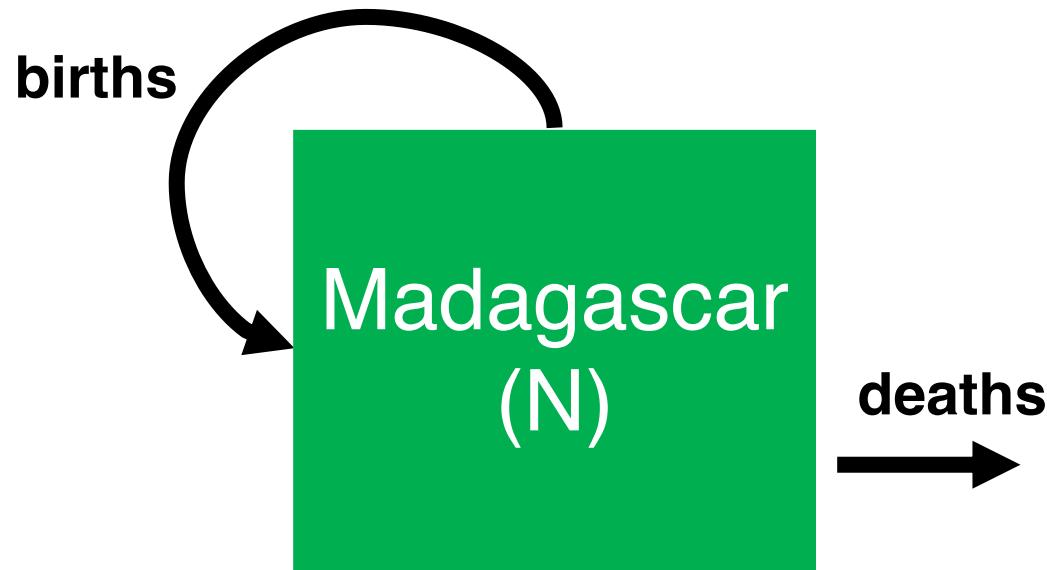


λ = population rate of increase
(finite growth rate)

$$\lambda = 1 + R$$

The simplest population model

1. Populations are divided into compartments
2. Individuals within a compartment are homogenously mixed
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4. Rates of transferring between compartments are expressed mathematically



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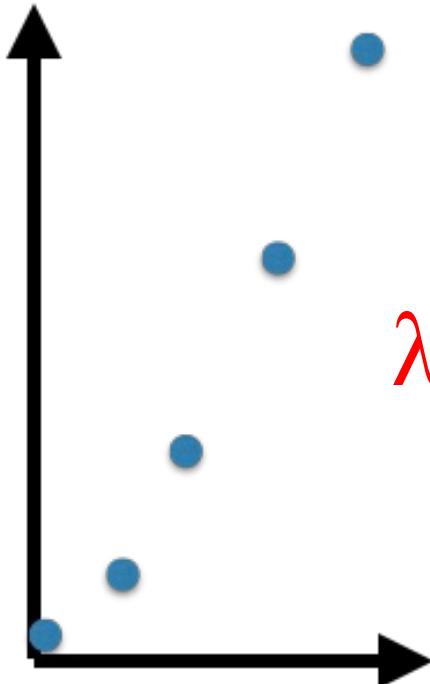
**λ = population rate of increase
(finite growth rate)**

(pop grows @ $\lambda > 1$ & declines @ $\lambda < 1$)

Geometric growth



Geometric growth is measured in discrete time



$$\lambda = N_{t+1}/N_t$$

$$N_1 = \lambda N_0$$

$$N_2 = \lambda[\lambda N_0] = \lambda^2 N_0$$

$$N_3 = \lambda^3 N_0$$

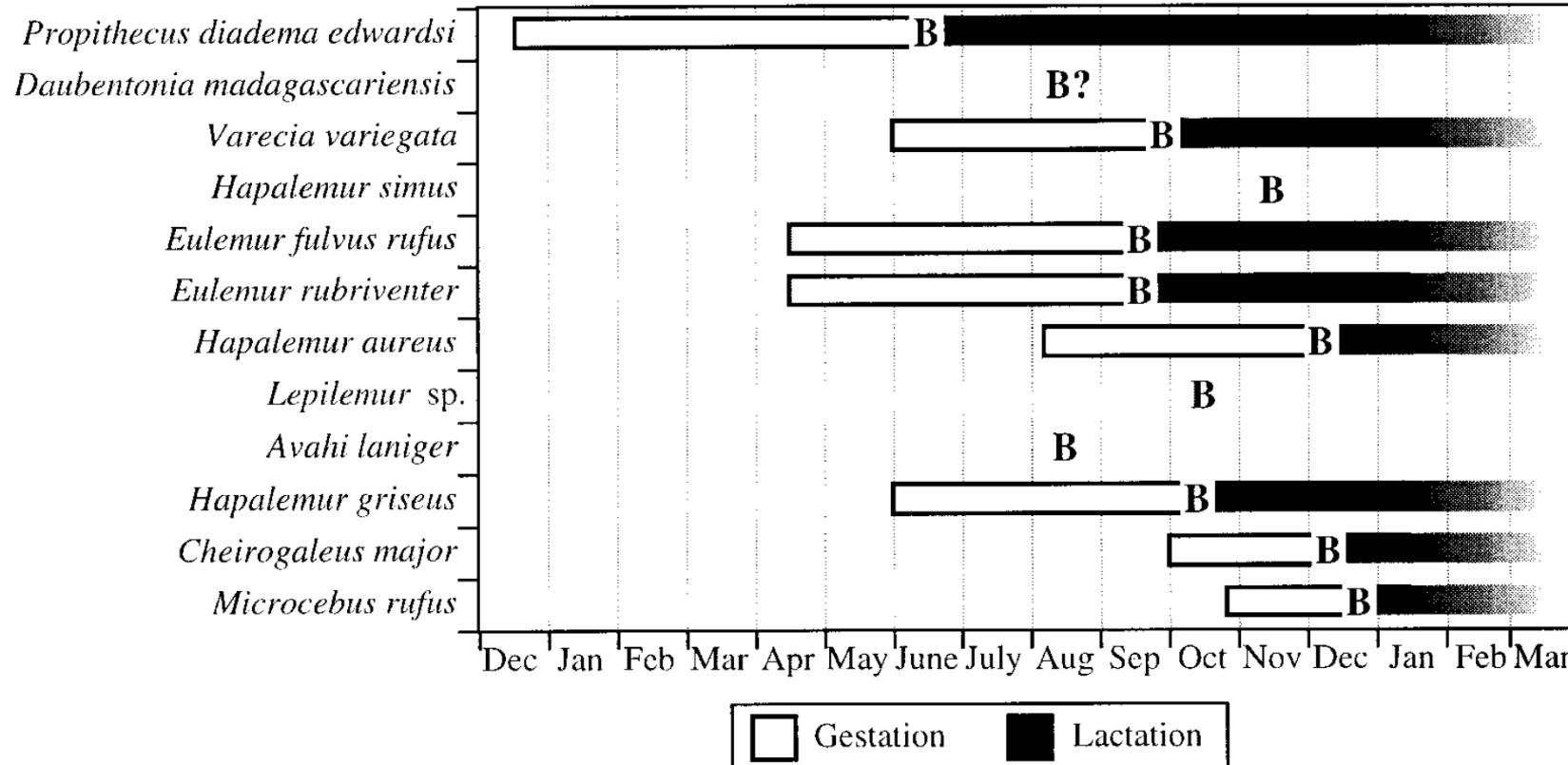
$$N_t = \lambda^t N_0$$

λ = population rate of increase

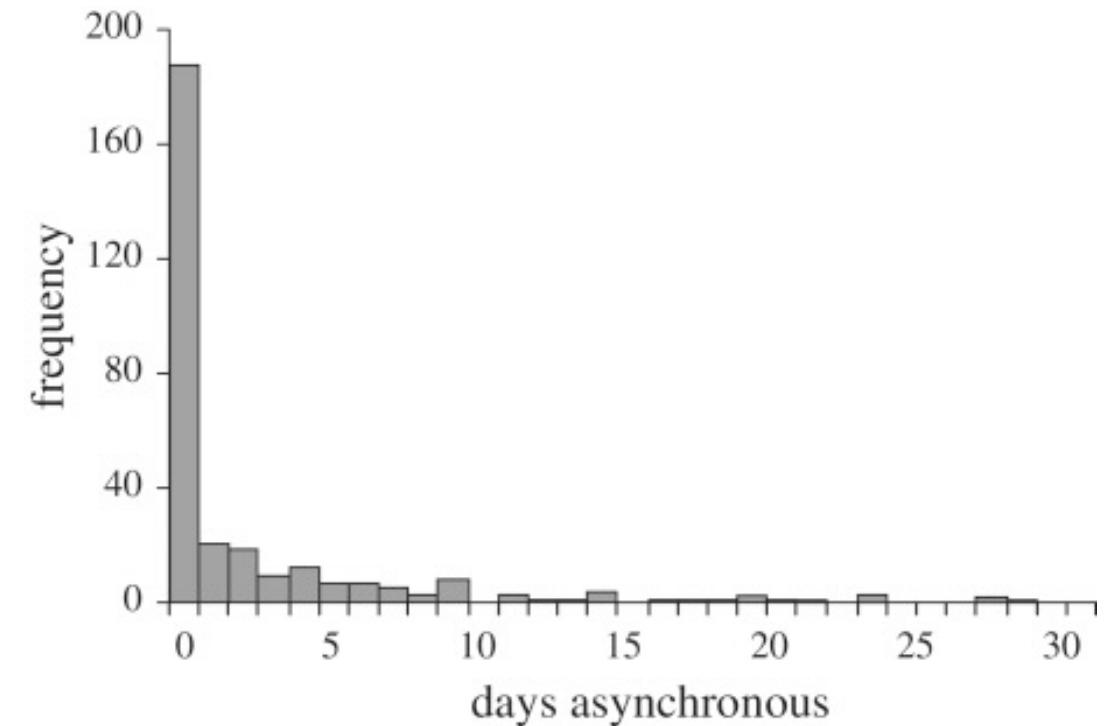
Is discrete time realistic for population growth?



Is discrete time realistic for population growth?

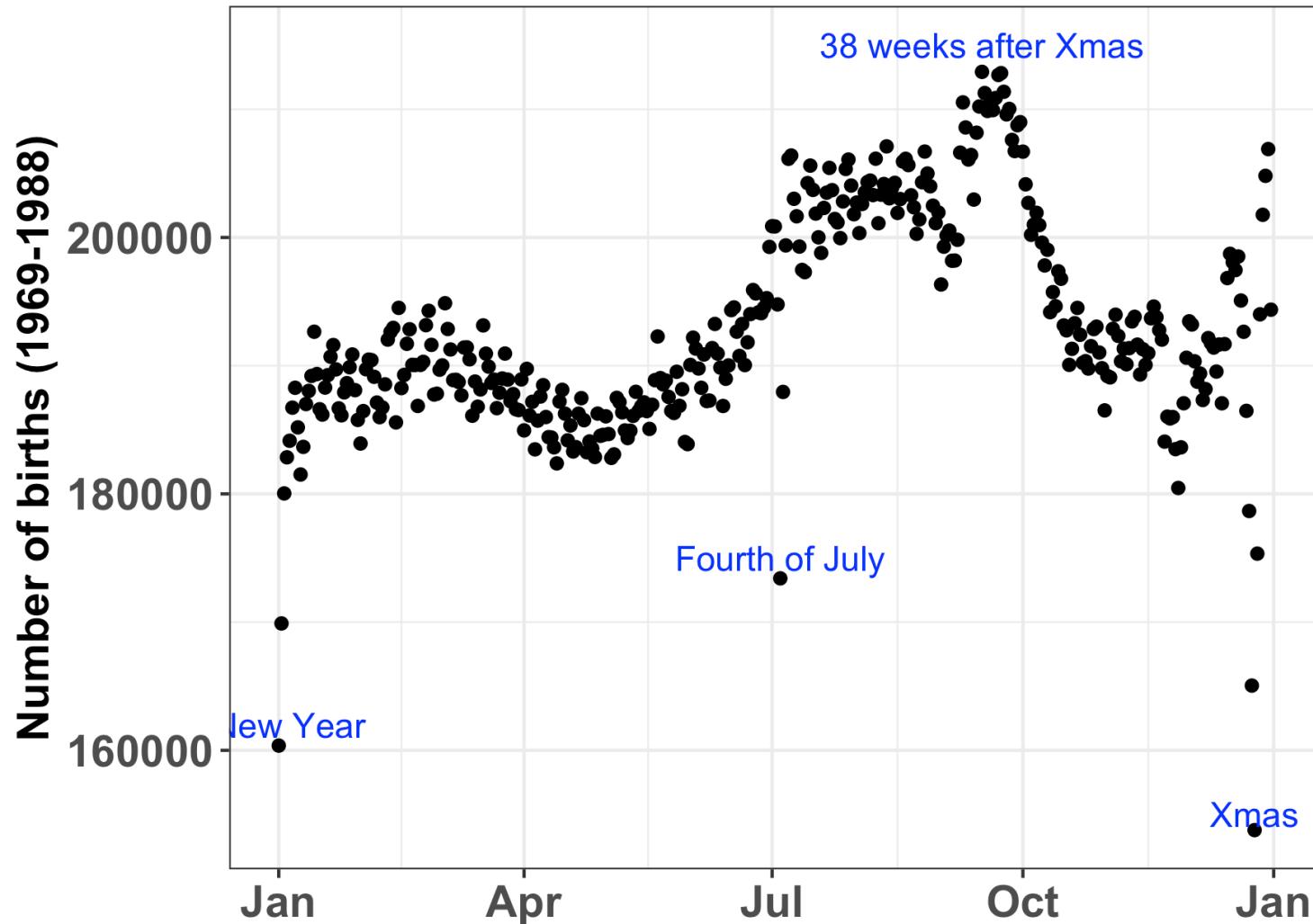


Is discrete time realistic for population growth?



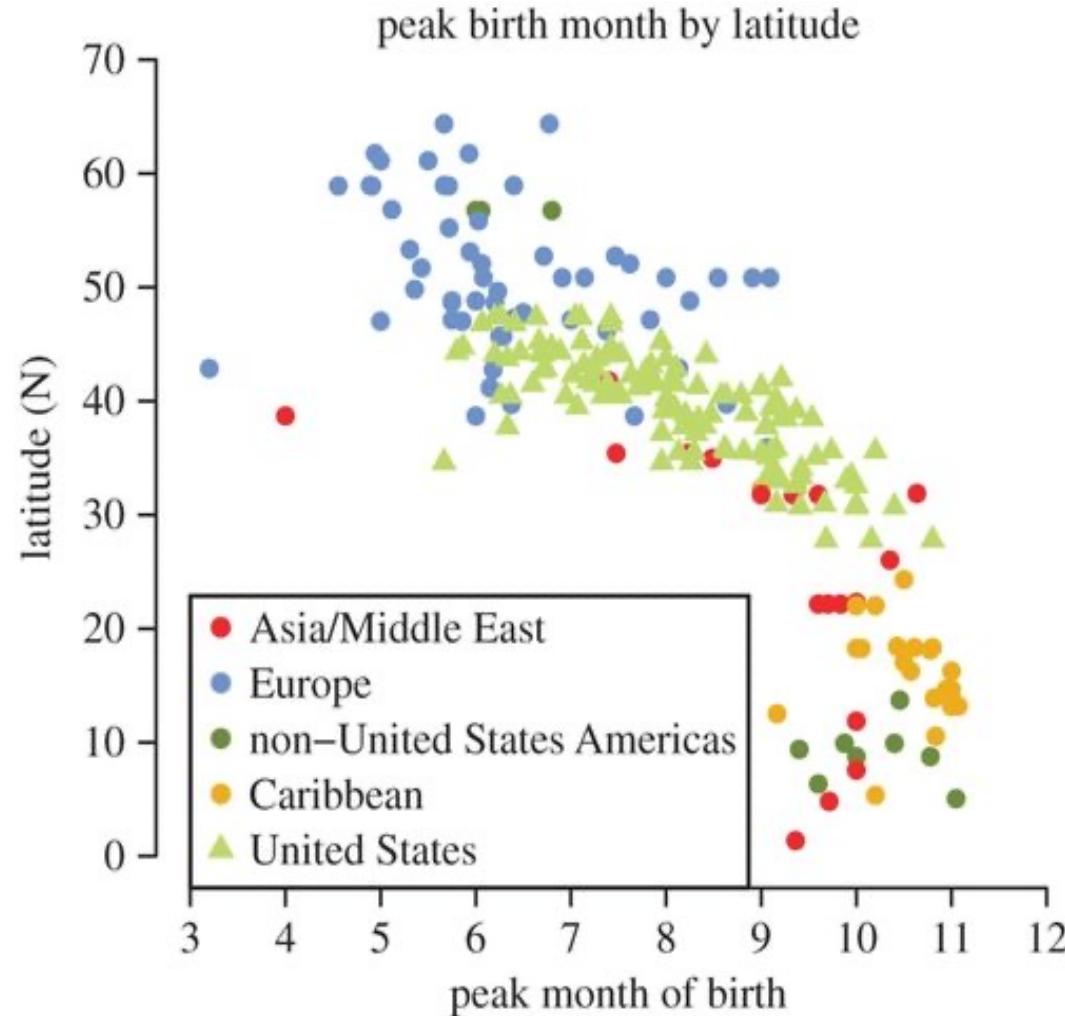
64% of banded mongoose pups are born on the same day.

Is discrete time realistic for population growth?



Human births by day in the US (1969-1988)

Is discrete time realistic for population growth?

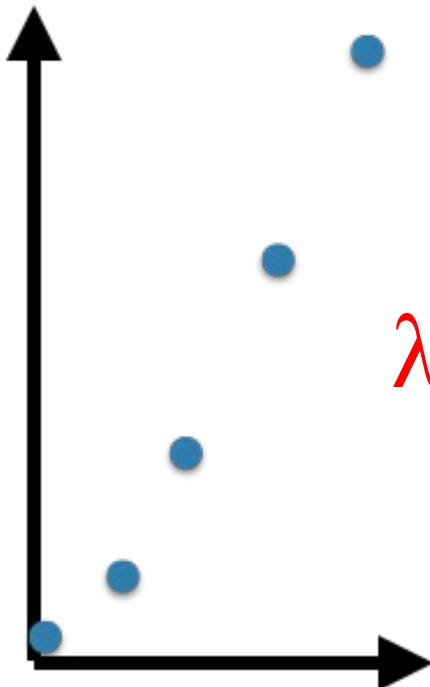


Many populations (including human) are better approximated by an assumption of a continuous birth rate

Geometric vs. exponential growth



Geometric growth



$$\lambda = N_{t+1}/N_t$$

$$N_1 = \lambda N_0$$

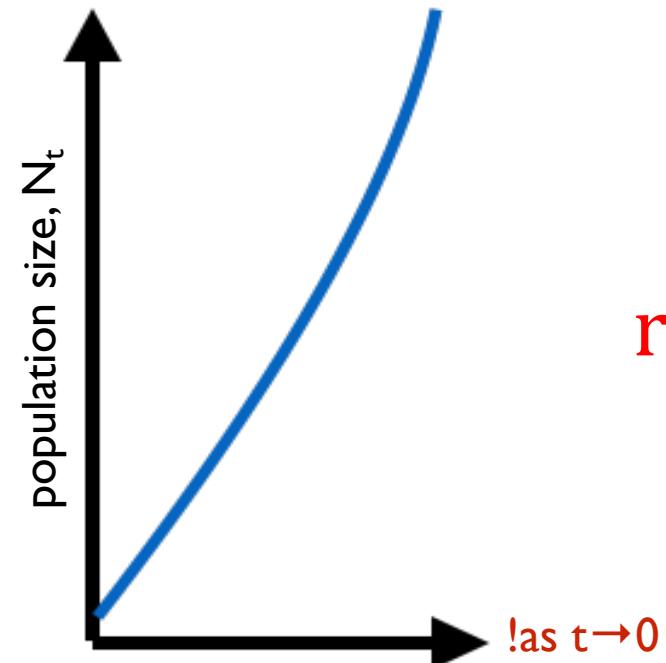
$$N_2 = \lambda[\lambda N_0] = \lambda^2 N_0$$

$$N_3 = \lambda^3 N_0$$

$$N_t = \lambda^t N_0$$

λ = population rate of increase

Exponential growth is measured in continuous time



$$r = \frac{\ln\left(\frac{N_t}{N_0}\right)}{t}$$

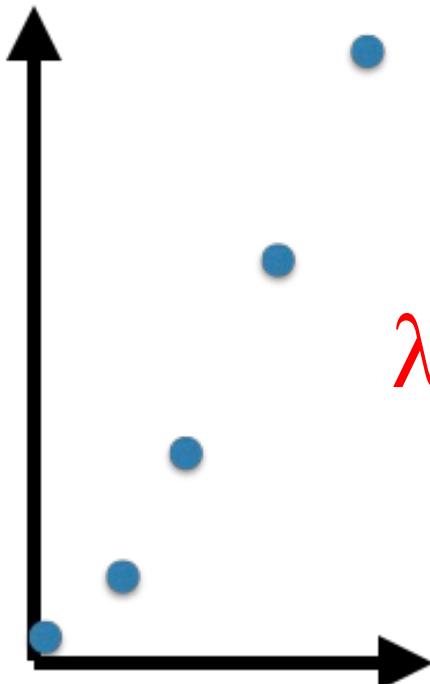
$$dN(t)/dt = rN(t)$$

r = intrinsic (instantaneous) rate of increase

Geometric vs. exponential growth



Discrete time



$$\lambda = N_{t+1}/N_t$$

Continuous time

$$dN(t)/dt = rN(t)$$

Separation of variables:
 $dN(t)/N(t) = r dt$

Integrate both sides:
 $\int dN(t)/N(t) = \int r dt$

By definition:
 $\log(N(t)) = rt + c$

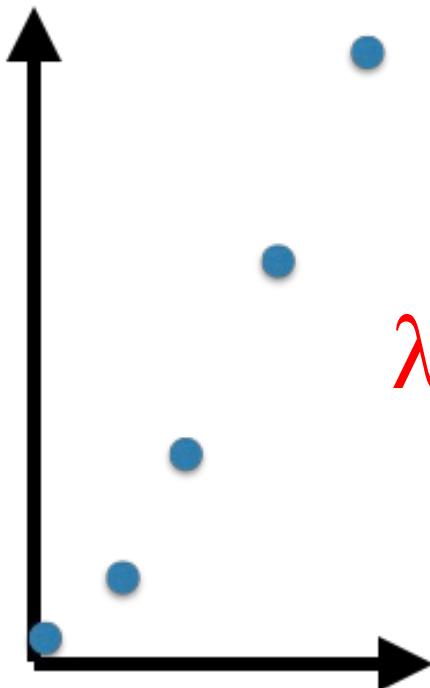
Take exponentials:
 $N(t) = e^{rt+c} = Ce^{rt}$
 $N(t) = N(0)e^{rt}$

$$\begin{aligned}N_1 &= \lambda N_0 \\N_2 &= \lambda[\lambda N_0] = \lambda^2 N_0 \\N_3 &= \lambda^3 N_0 \\N_t &= \lambda^t N_0\end{aligned}$$

Geometric vs. exponential growth

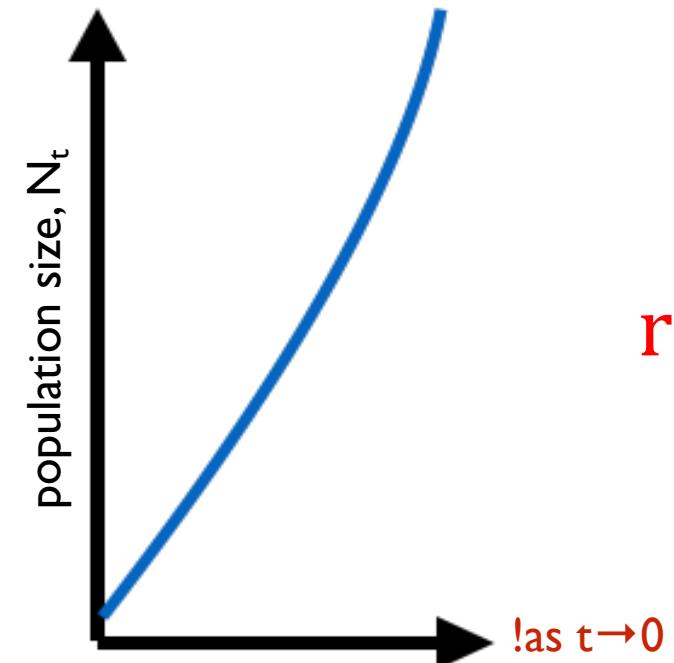


Discrete time



$$\lambda = N_{t+1}/N_t$$

Continuous time



$$r = \frac{\ln \left(\frac{N_t}{N_0} \right)}{t}$$

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λ =population rate of increase

(pop grows @ $\lambda > 1$ & declines @ $\lambda < 1$)

$$N_t = N_0 e^{rt}$$

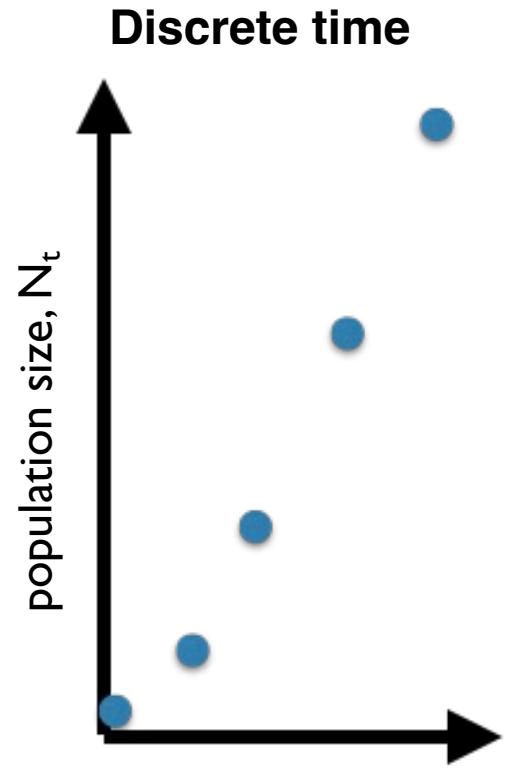
r =intrinsic (instantaneous) rate of increase

(pop grows @ $r > 0$ & declines @ $r < 0$)

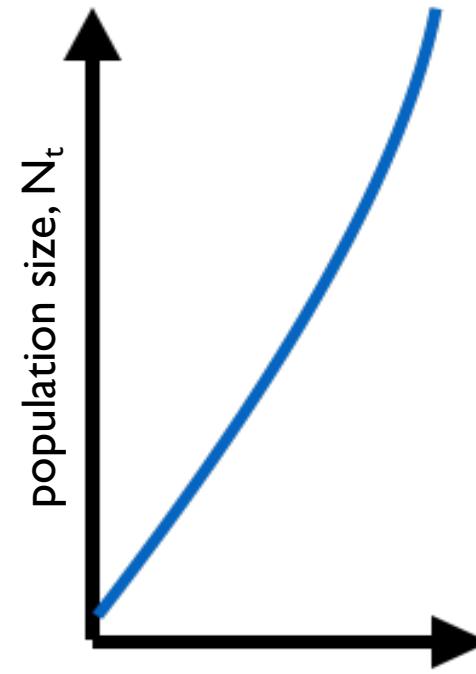
Geometric vs. exponential growth

geometric
 $N_t = \lambda^t N_0$

exponential
 $N_t = N_0 e^{rt}$



Continuous time



Geometric vs. exponential growth

geometric

$$N_t = \lambda^t N_0$$

exponential

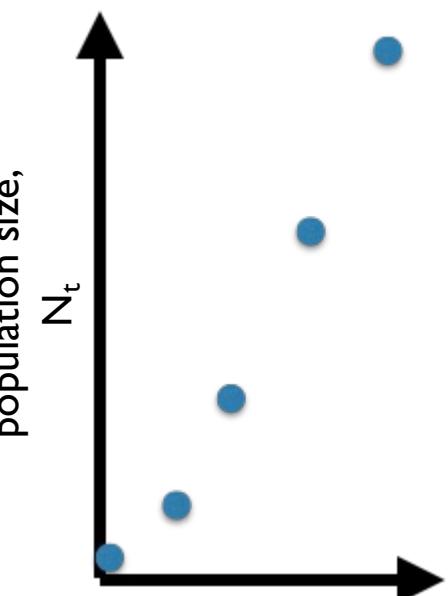
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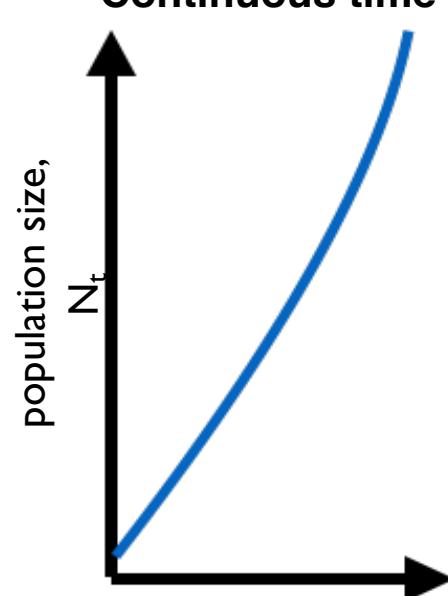
$$\lambda^t = e^{rt}$$

$$\lambda = e^r$$

Discrete time



Continuous time



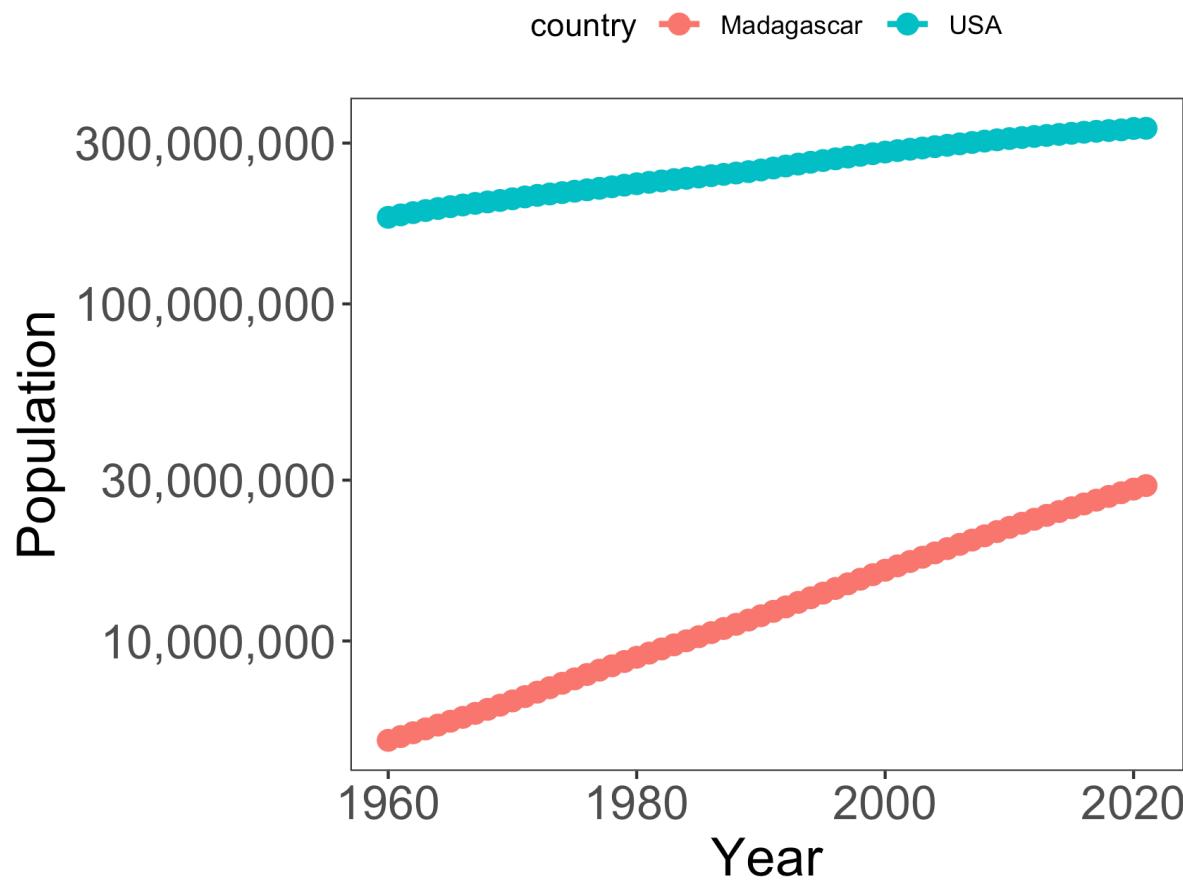
Continuous models can be discretized.

Discrete models can be approximated in continuous time.

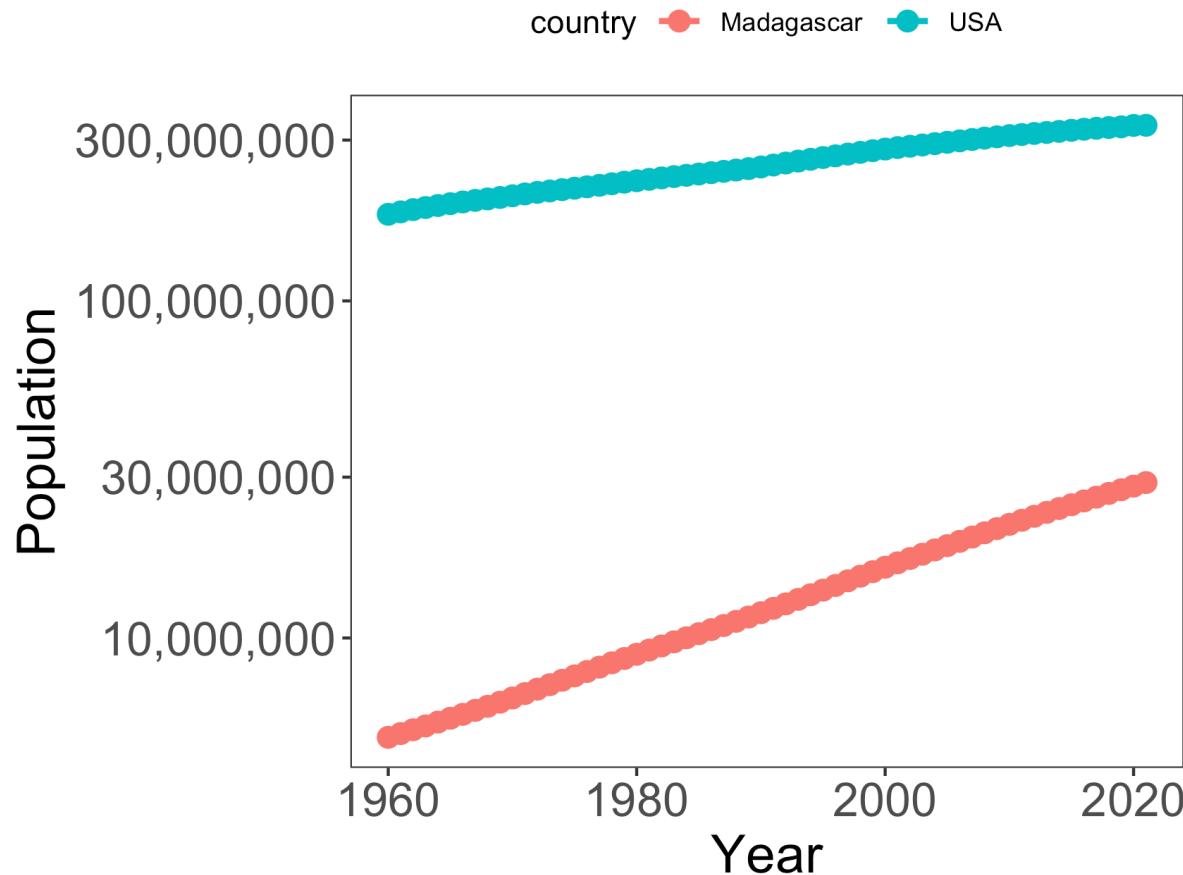
How to choose what to do?

The answer depends on the data and the question at hand!

Geometric vs. exponential growth



Geometric vs. exponential growth

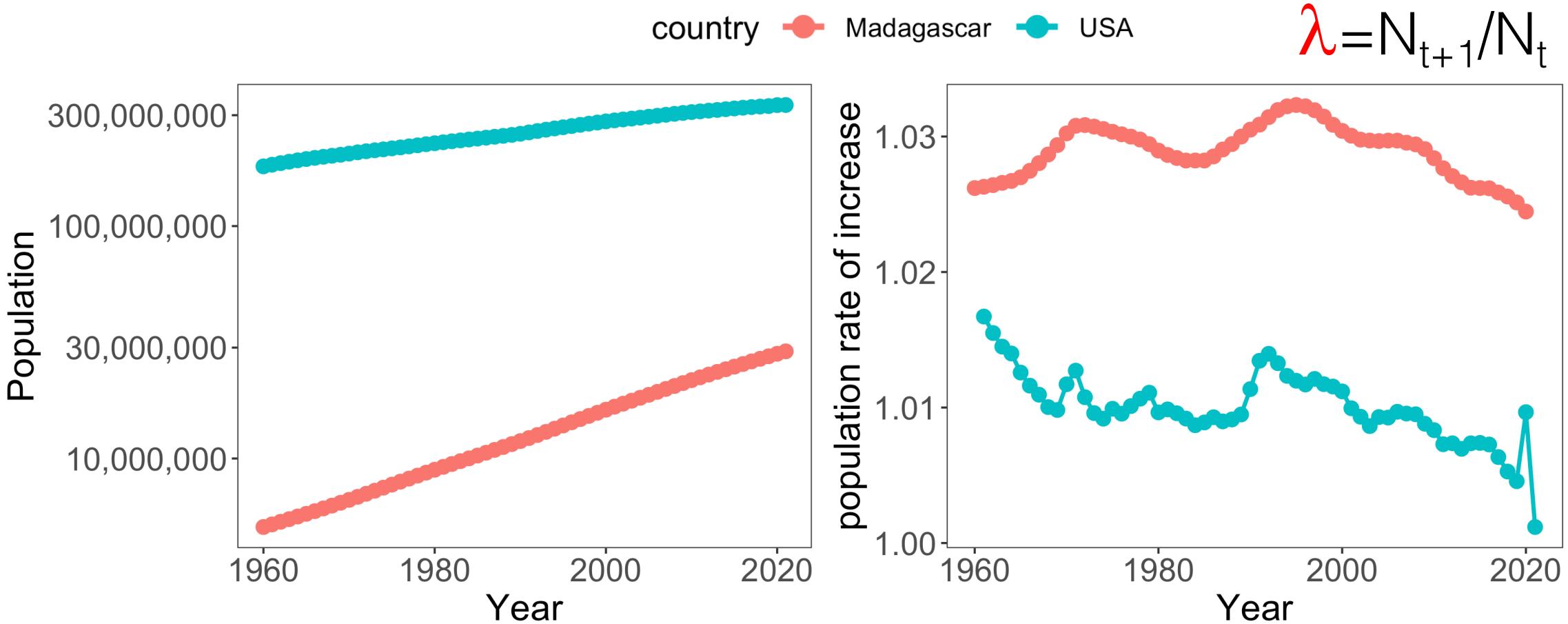


$$\lambda = N_{t+1}/N_t$$

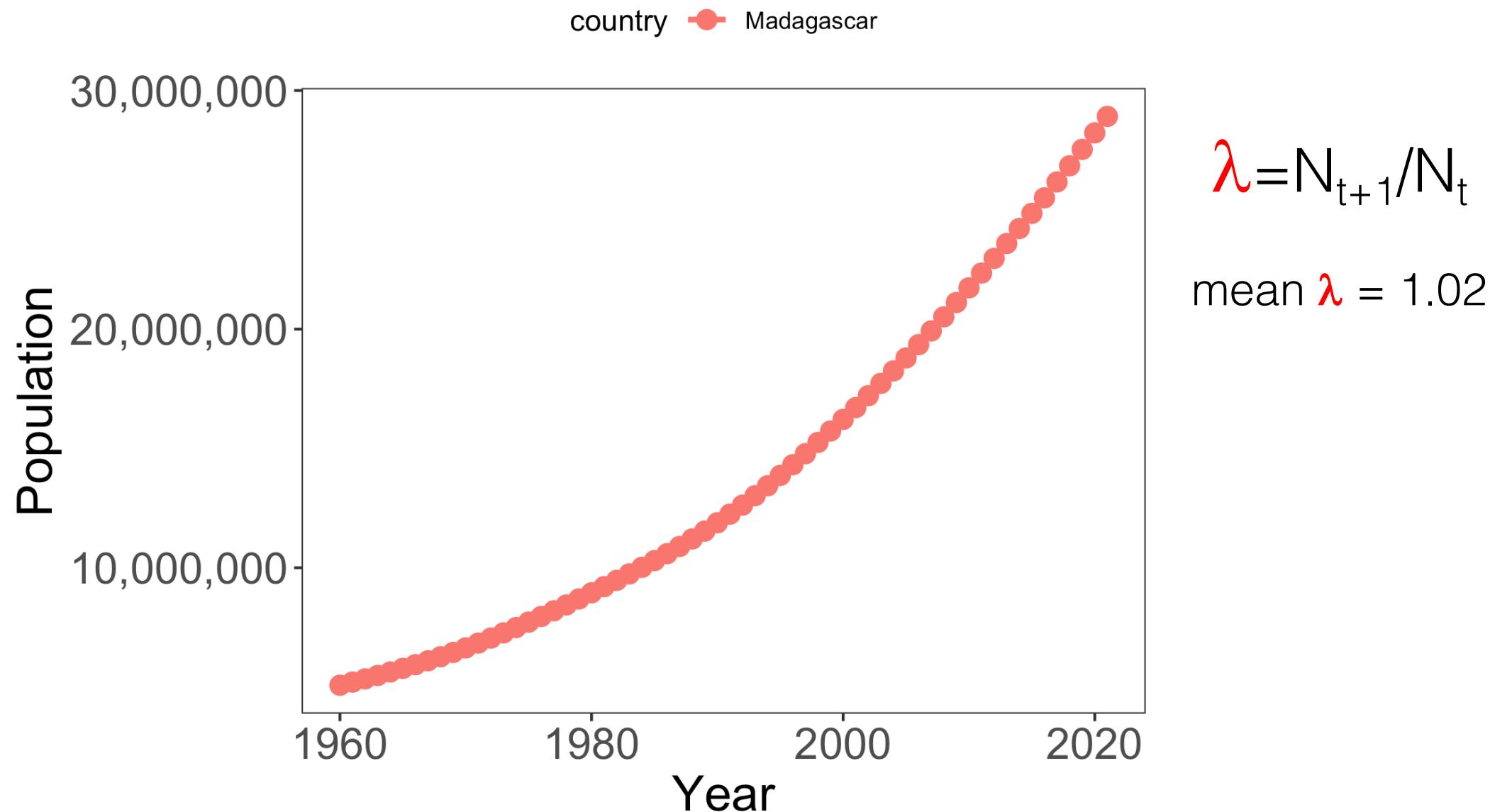
where $t = 1$ year

Which country has the higher growth rate?

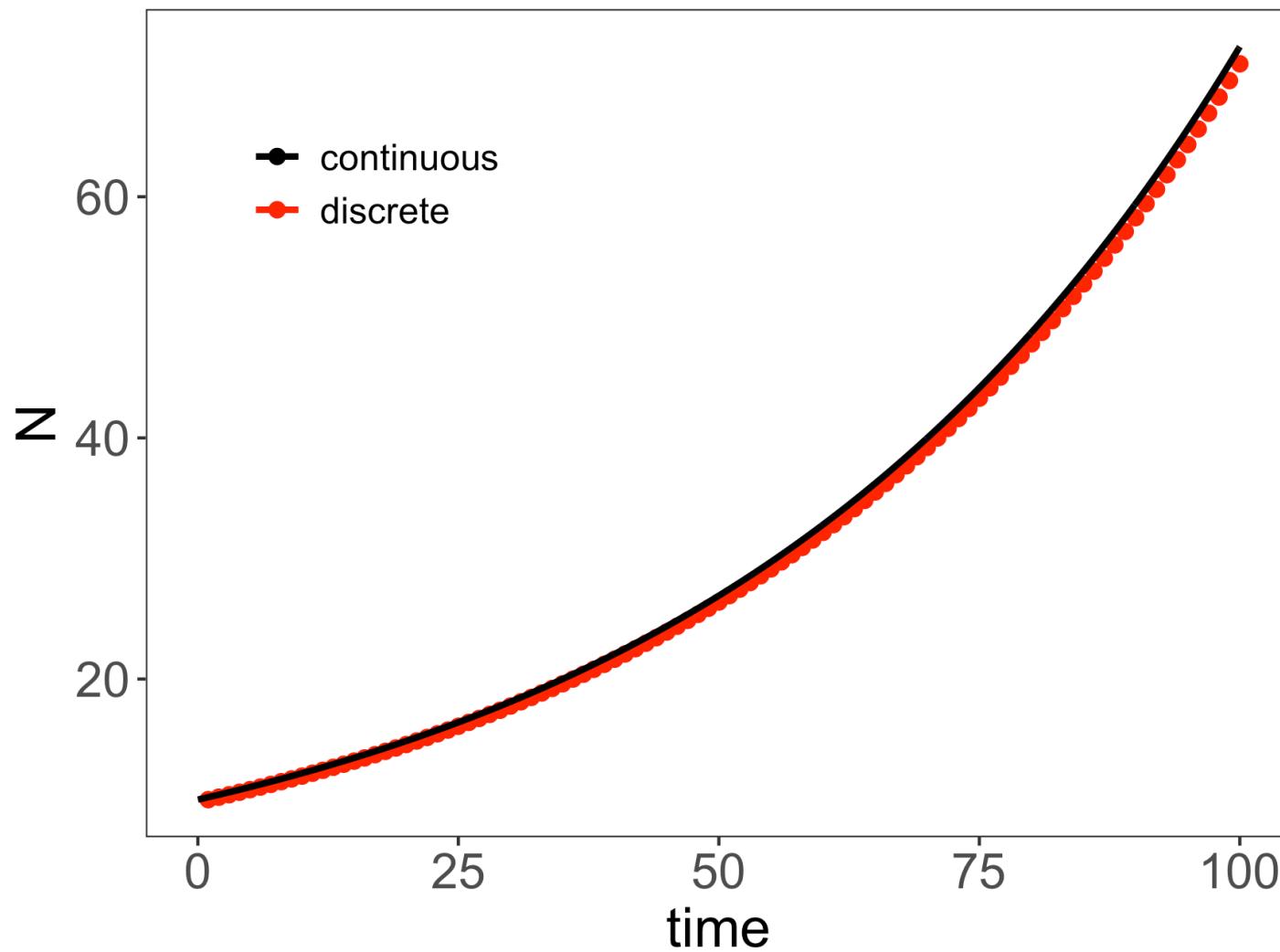
Geometric vs. exponential growth



Geometric growth approximation of Madagascar's population



Projecting population under geometric vs. exponential growth



Geometric: $\lambda = 1.02$

Exponential: $r = \log(1.02) = 0.02$

Models are similar over
short time horizons
and/or when discrete
timesteps are small!

We can estimate growth rates by ‘fitting’ a model to data

