

# Fundamentals of Ecology

Week 5, Ecology Lecture 1

Cara Brook

February 4, 2025

# Where are we headed?

Week	Mon/Tues Lab	Tues Lecture	Thurs Lecture
1	--	Jan 7: Evolution History	Jan 9: Mutation and Variation
2	1: Intro R	Jan 14: Hardy-Weinberg	Jan 16: Migration and Drift
3	2: Hardy-Weinberg	Jan 21: Natural Selection	Jan 23: Phylogenetics
4	3: Microevolution	Jan 28: Molecular Evolution & Sexual Selection	Jan 30: Kin Selection, Speciation
5	4: Phylogenetics	Feb 4: Ecology & Population Growth	Feb 6: Single Species Population Growth & Regulation
6	5: Evolution paper	Feb 11: Species Interactions 1	Feb 13: Midterm
7	6: Population Growth	Feb 18: Species Interactions 2	Feb 20: Disease Dynamics as Population Biology 1
8	7: Population Regulation	Feb 25: Disease Dynamics as Population Biology 2	Feb 27: Community Assembly & Island Biogeography
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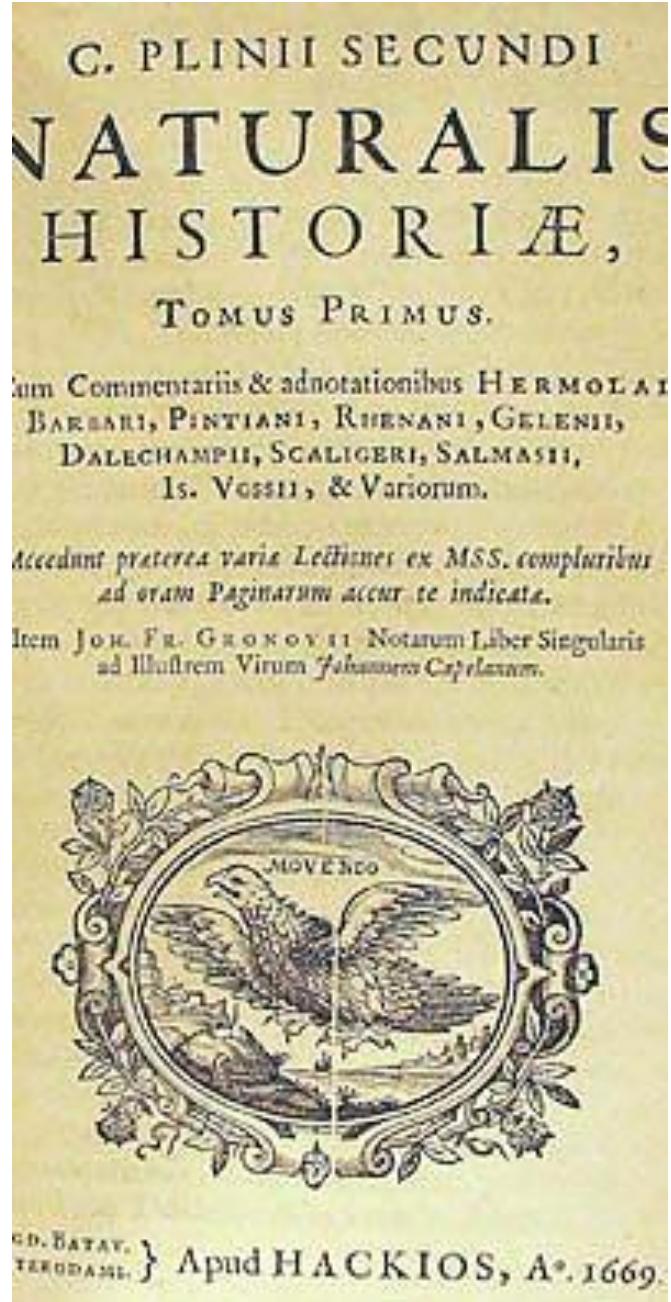
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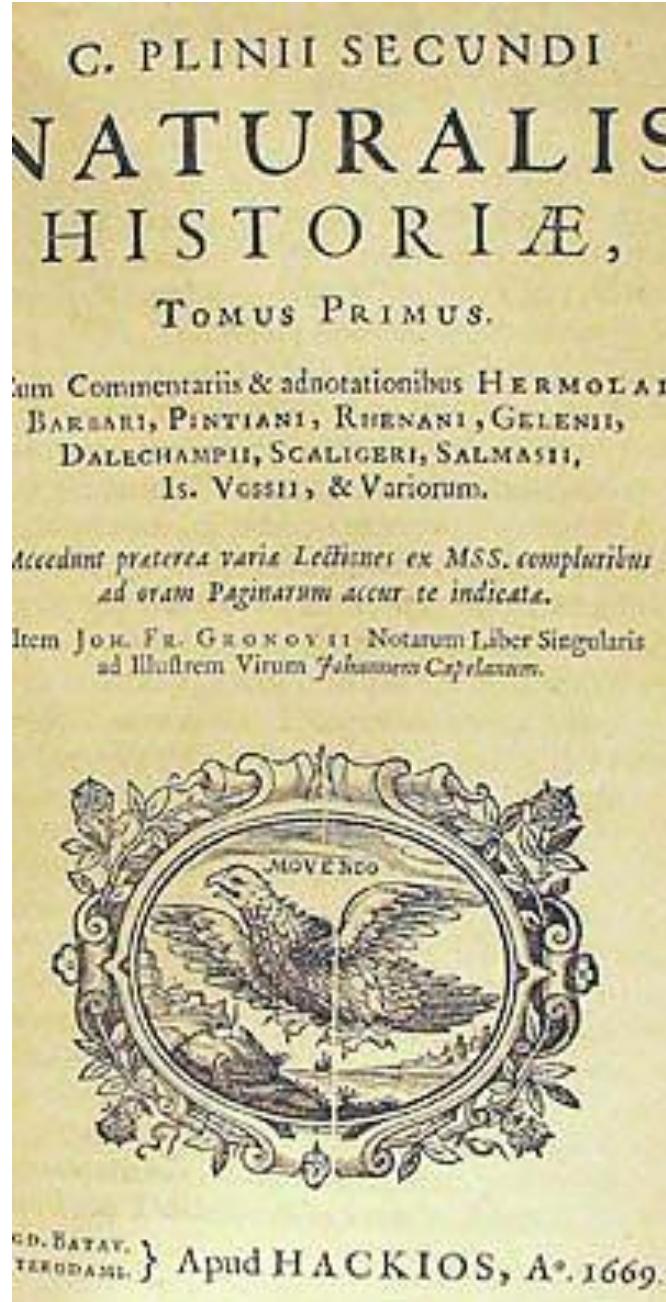
**Natural history** is the observational study of the living things on planet Earth.

# Is natural history science?

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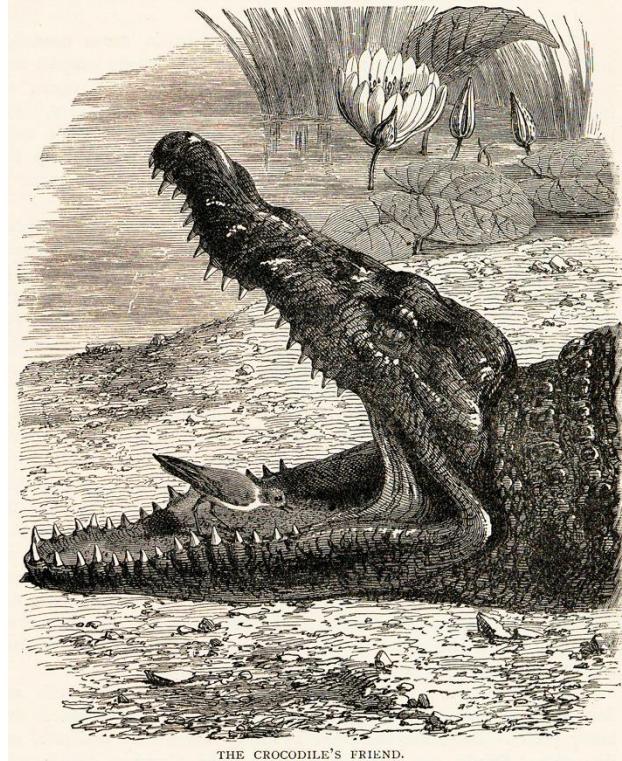
**Science:** the **systematic observation** of natural events and conditions in order to discover facts about them and to **formulate laws and principles** based on these facts.

– *Academic Press Dictionary of Science & Technology*

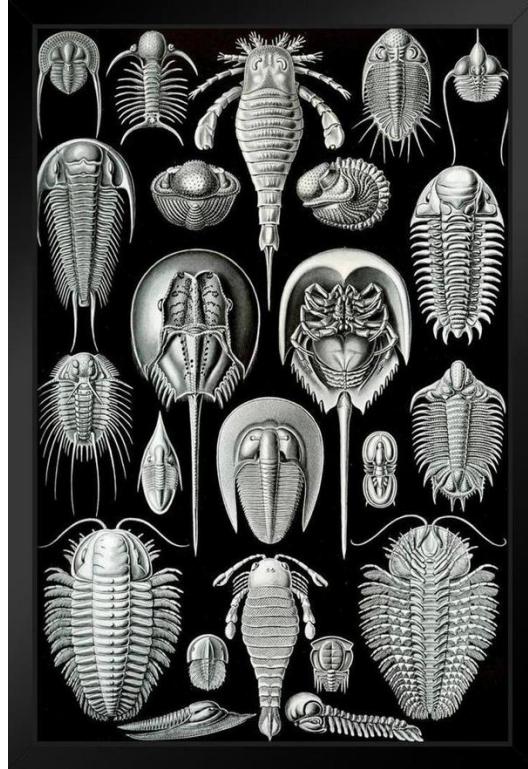


Natural history gave way to the **natural sciences** when those observations became **systematic** and were used to support **general laws and principles** describing the natural world.

Ecology is the study of the  
**interactions** of **organisms** with  
each other and their  
**environment.**



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*oikos* = 'house'  
+ *logia* = 'study of'

**ecology**

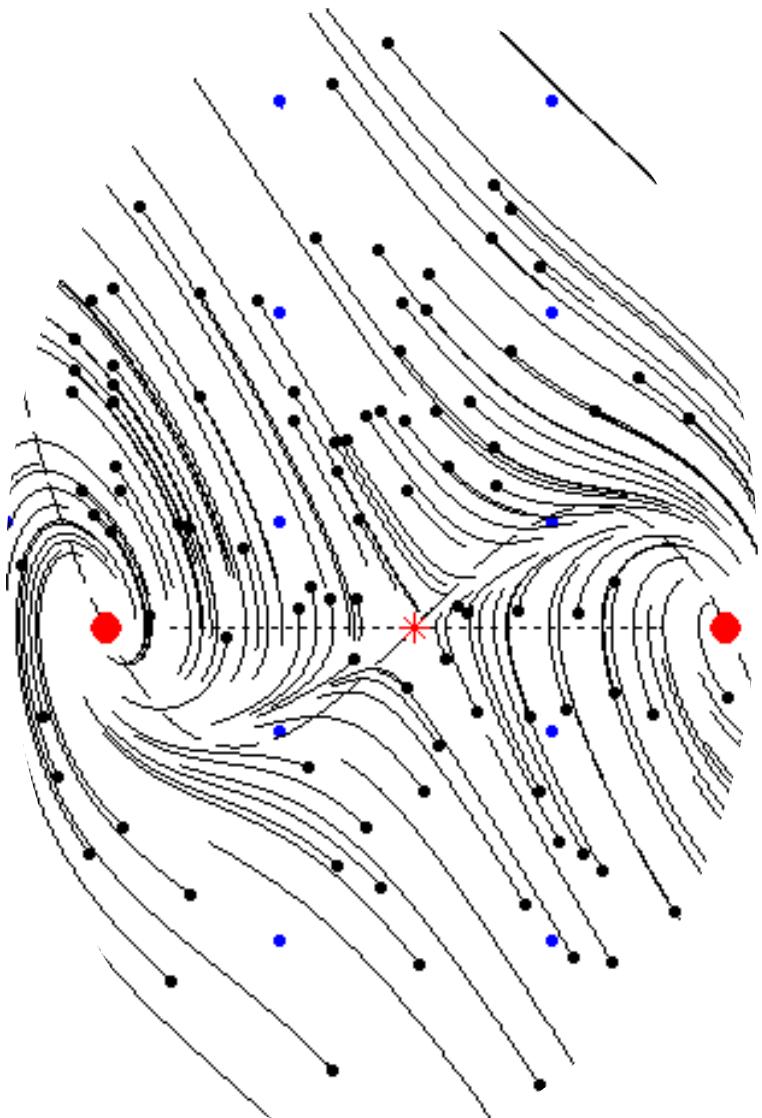
- 1866 Ernst Haeckel

**“Ecology** has a synonym which is **ALL.**”

-John Steinbeck

*The Log from the Sea of Cortez* (1941)





Ecology is the study of  
the **interactions** of  
**organisms** with each  
other and their  
**environment**.

As a science, ecology  
uses **models** to formalize  
general **laws and**  
**principles** describing the  
natural world.



# What is a model? an abstract representation of a phenomenon

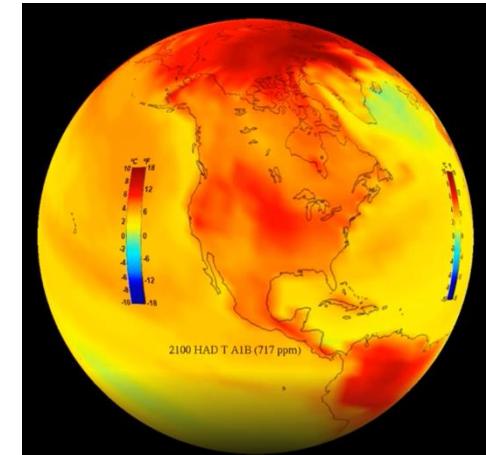
Human



Solar System



Climate



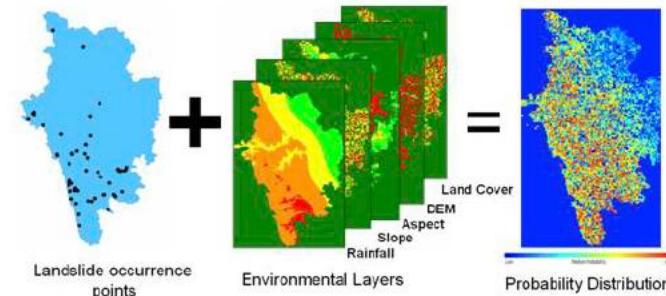
Human Genetics



Human Disease

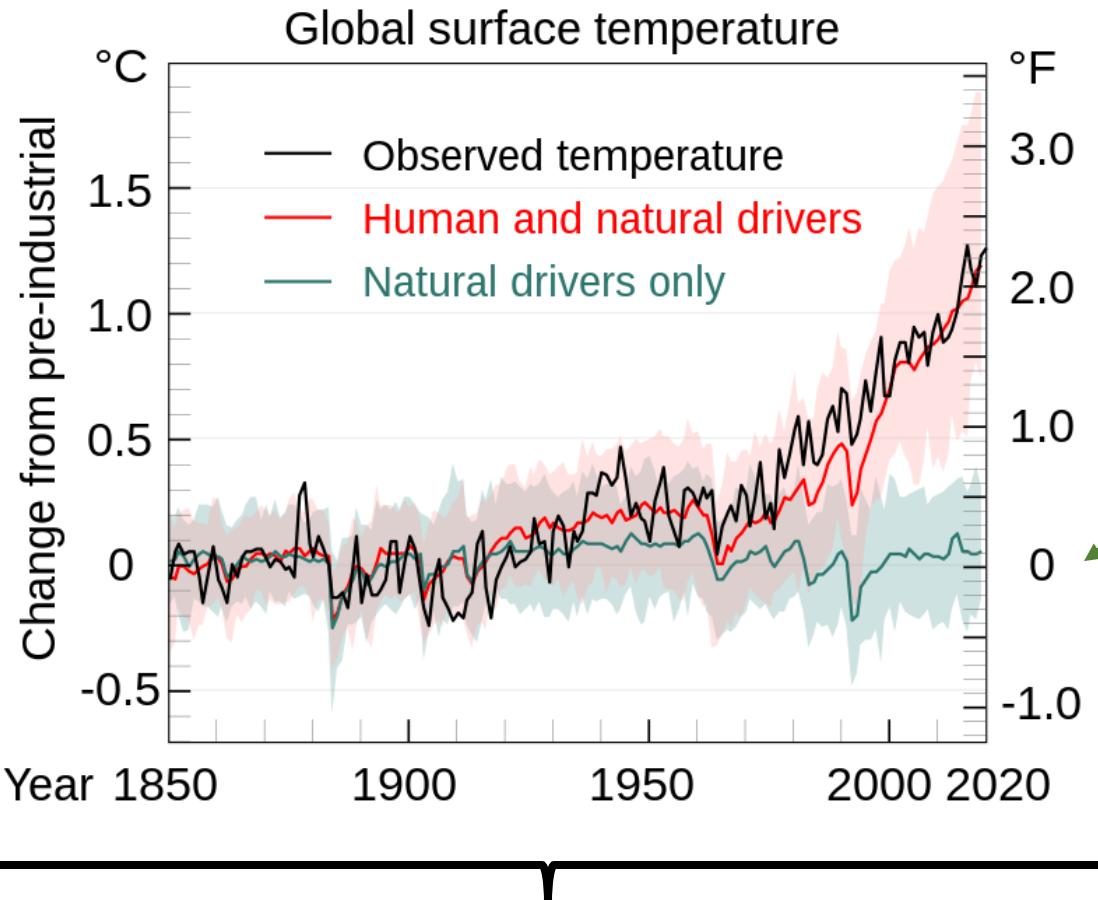


Species Distribution



# Why build models?

to explain and predict



explain

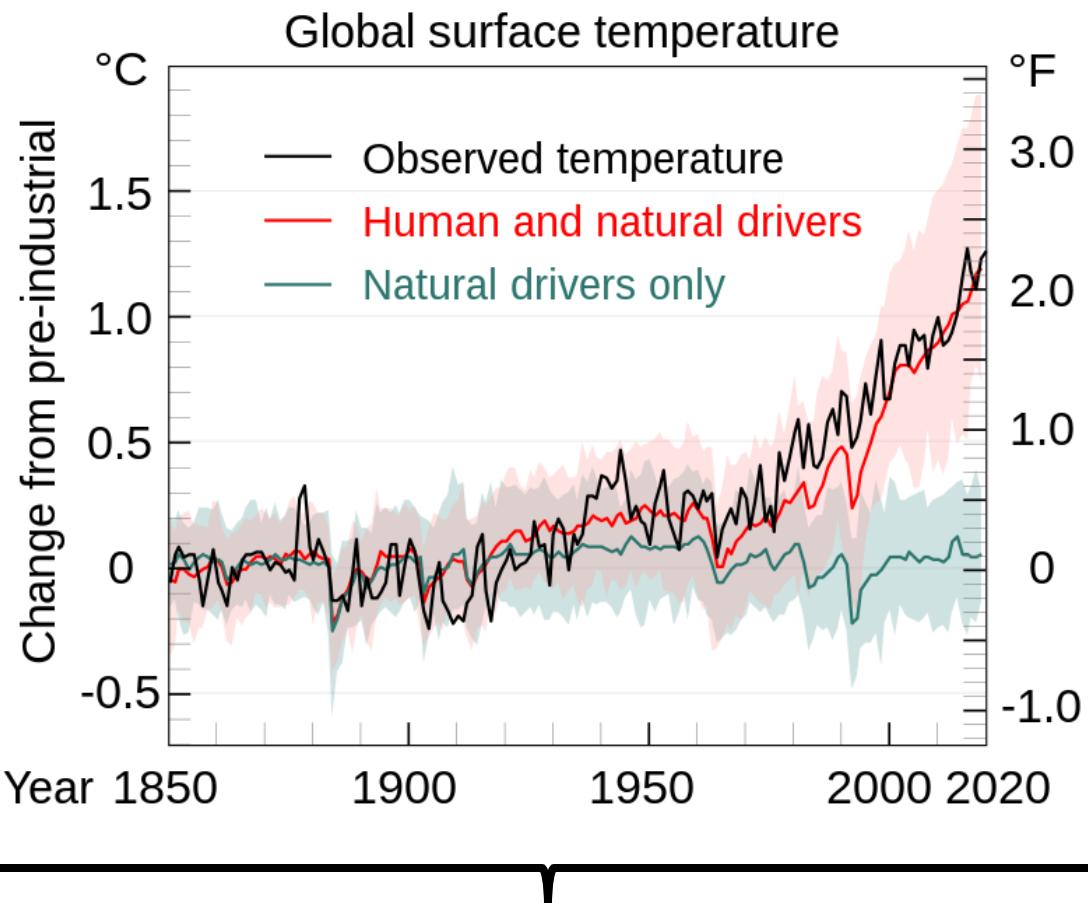
Models produce 'simulated data' that is compared against the observed data.

The process of adjusting the model to more closely match the data is called 'model fitting.'

# Why build models?

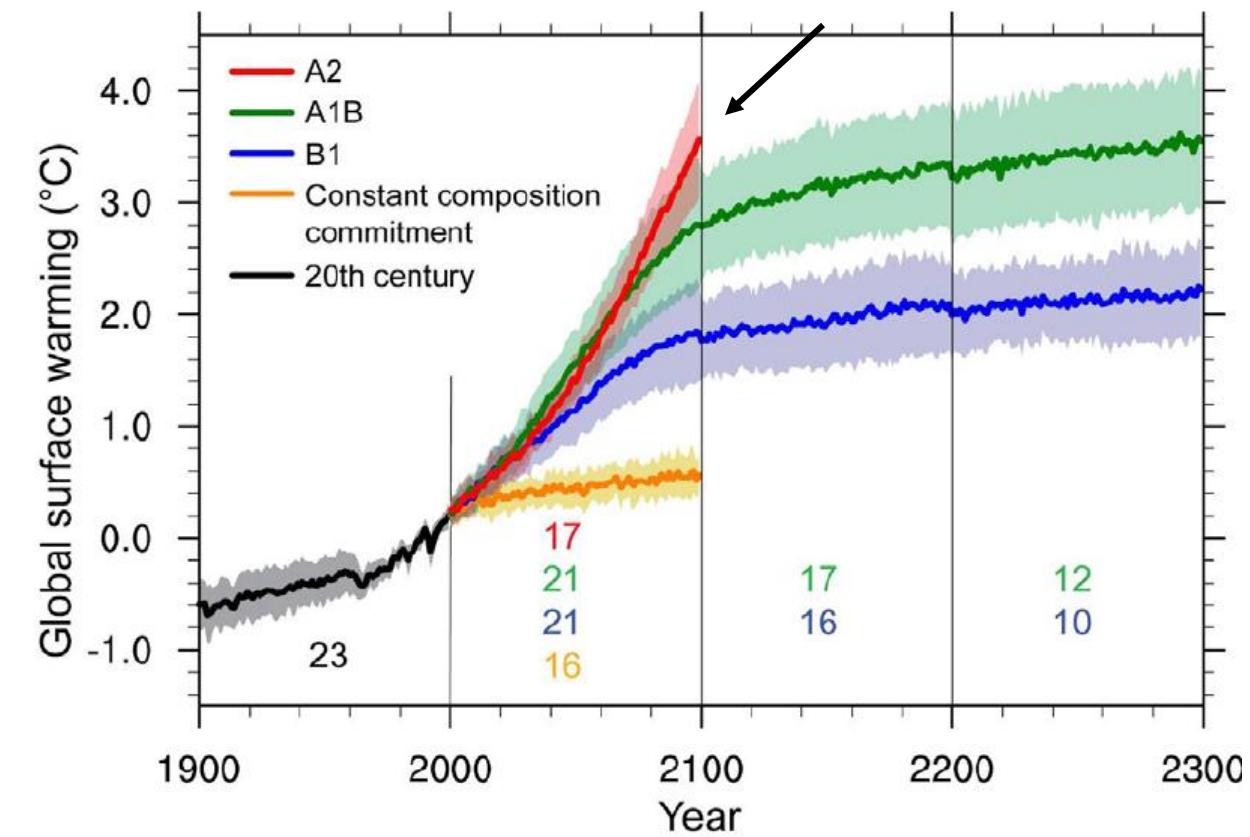
to explain and predict

Fitted models can then be used to “predict” the future, under different conditions.



(IPCC 2007)

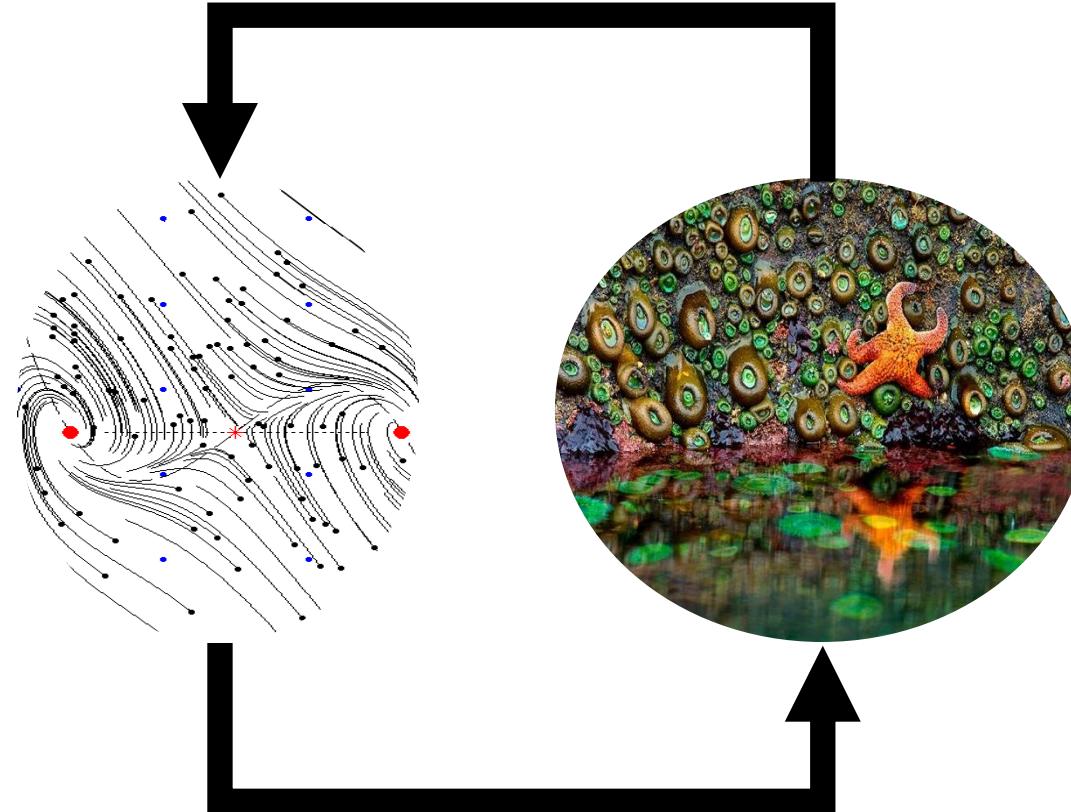
explain



predict

# Two kinds of ecological models

**Statistical Model**  
*Pattern*

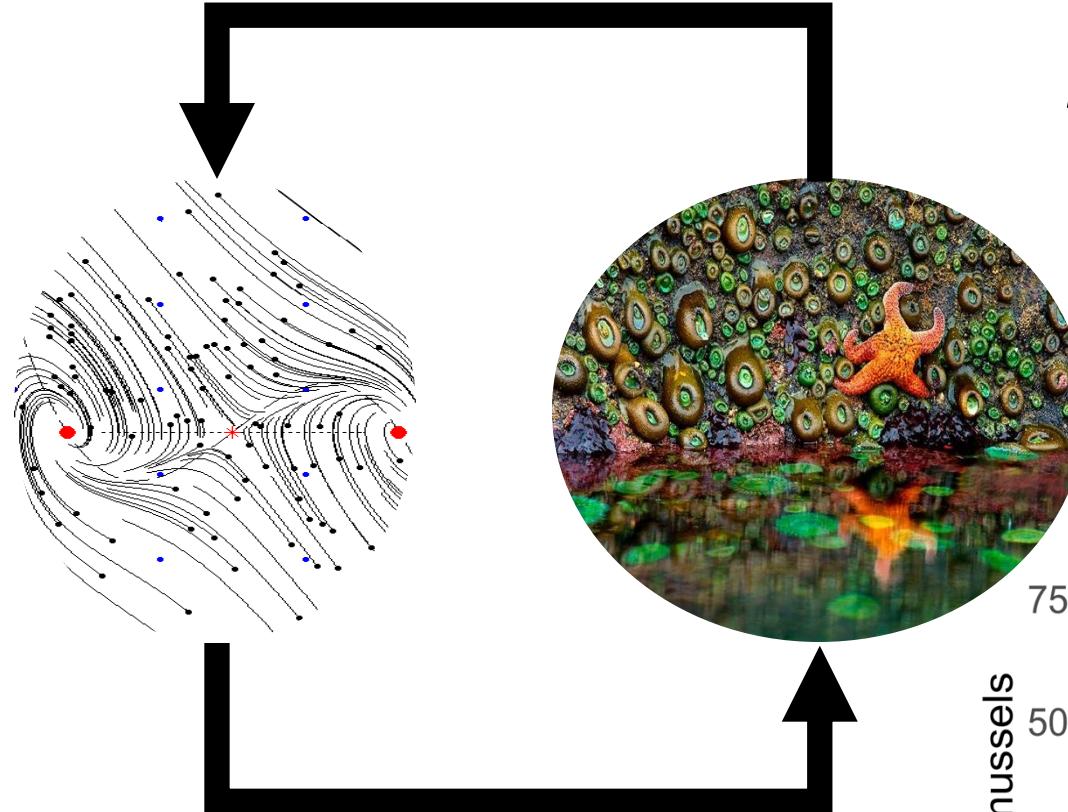


**Population Model**  
*Process*

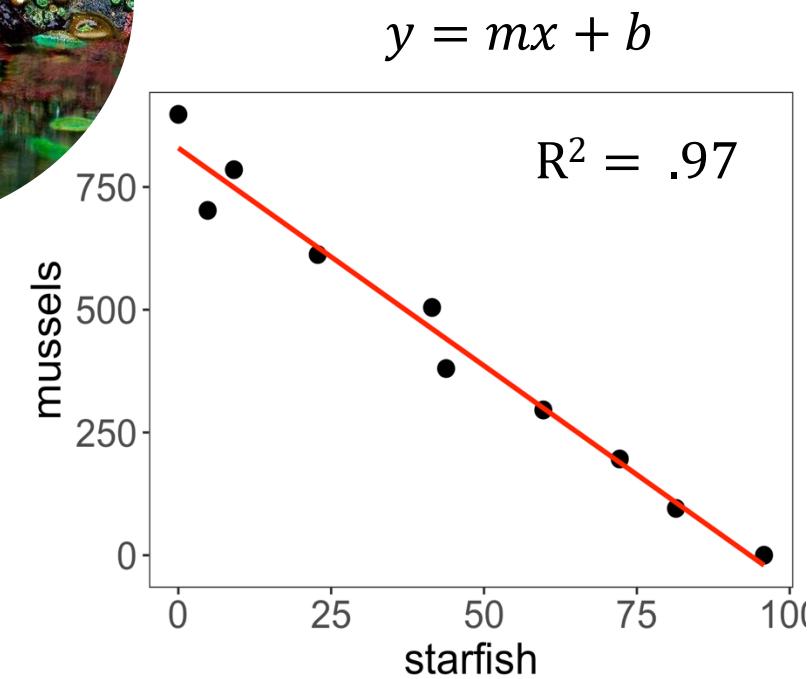
# Two kinds of ecological models

## Statistical Model Pattern

Population Model  
*Process*



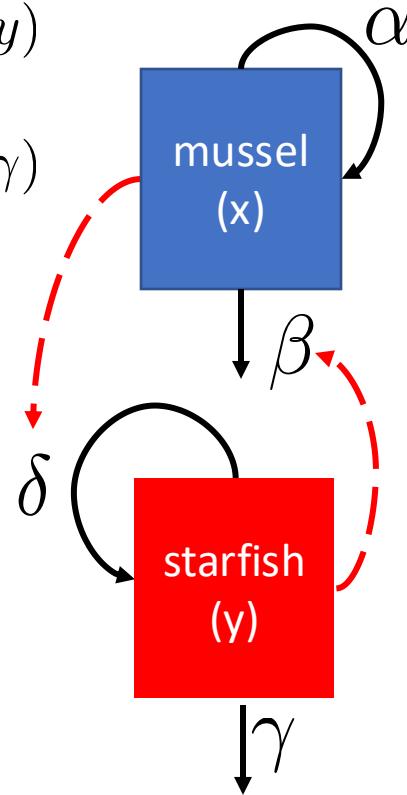
**What** is the relationship between the abundance of starfish and the abundance of mussels?



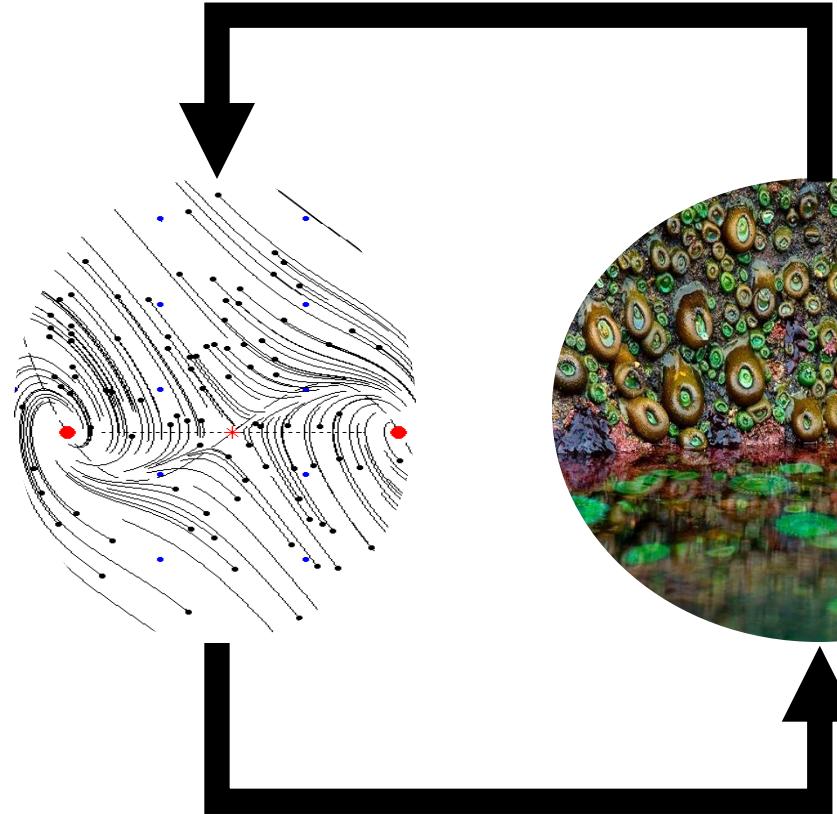
# Two kinds of ecological models

## Statistical Model Pattern

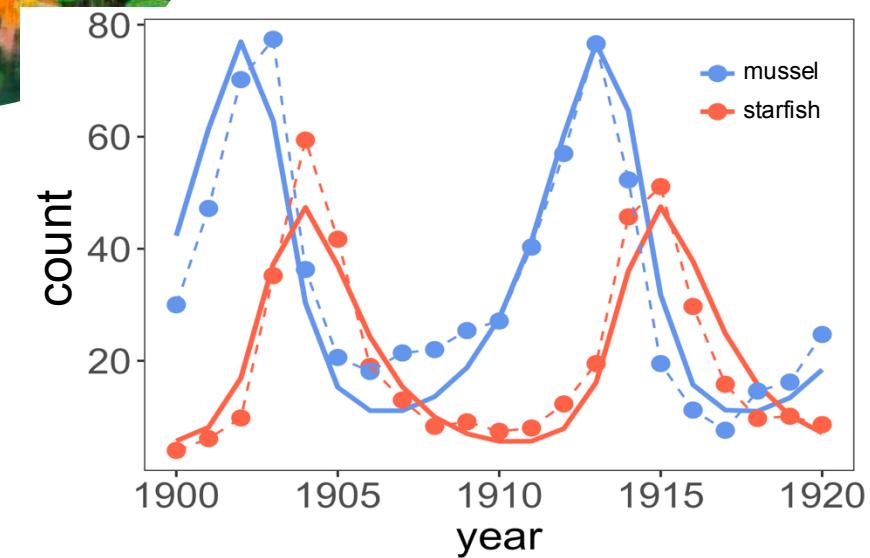
$$\frac{dx}{dt} = x(\alpha - \beta y)$$
$$\frac{dy}{dt} = y(\delta x - \gamma)$$



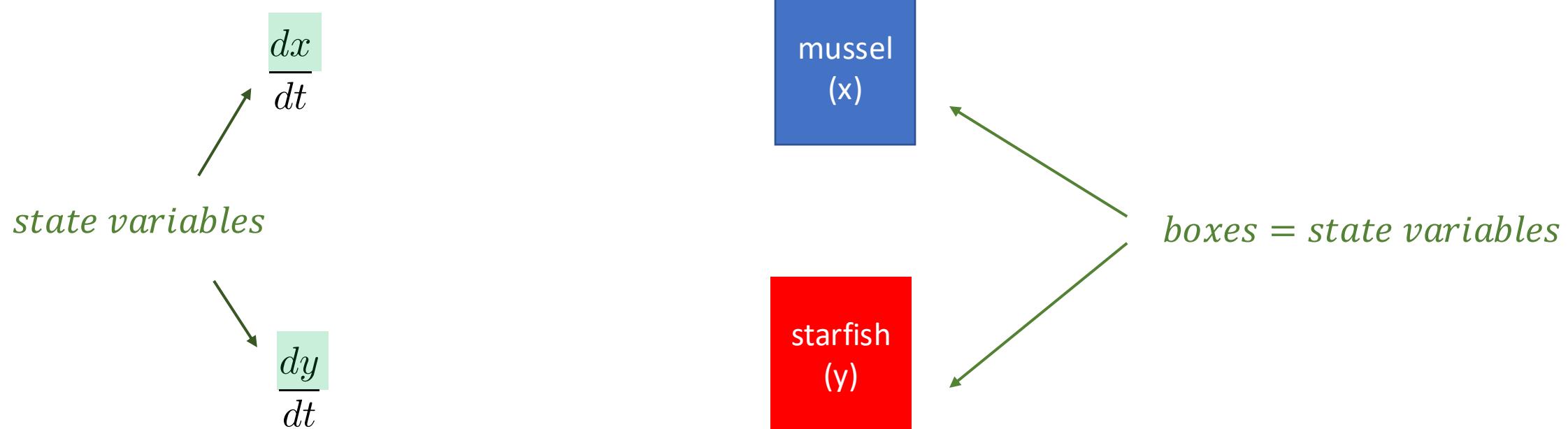
**Population Model  
Process**



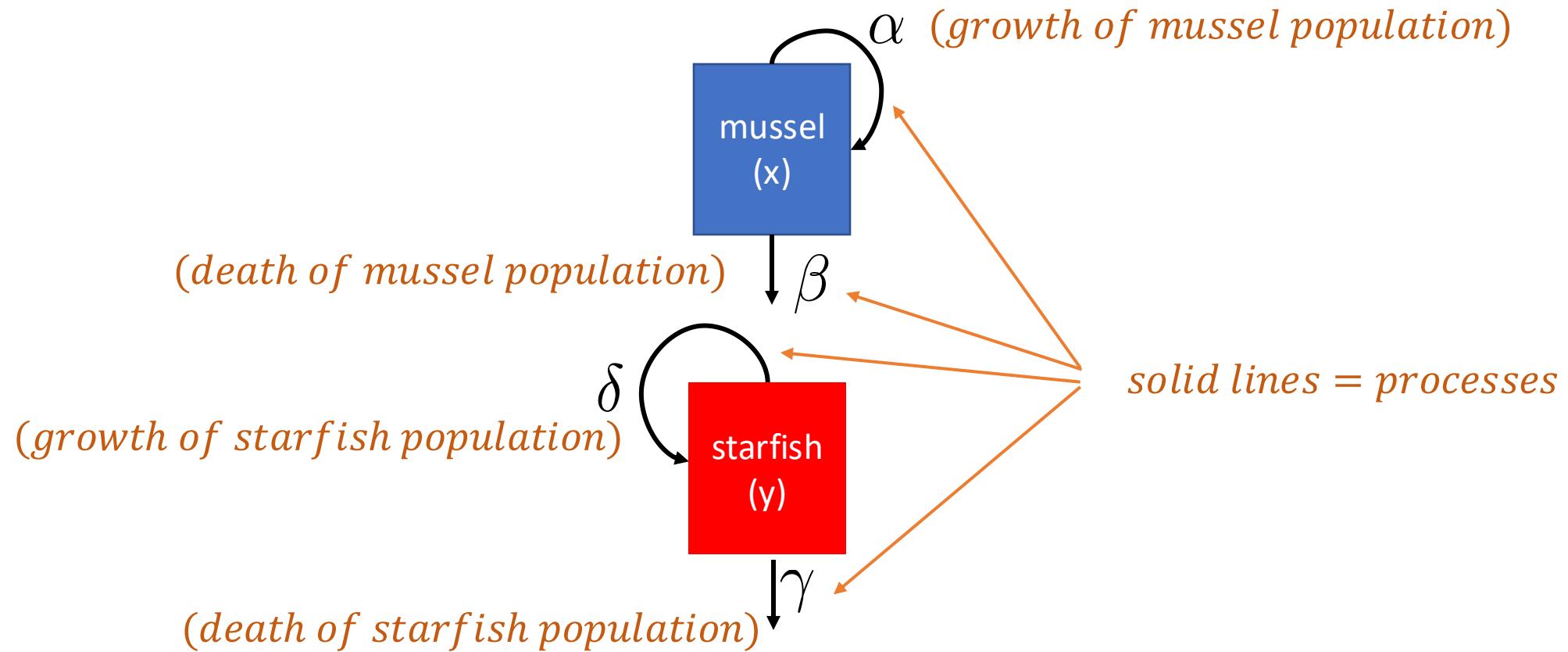
**How does the abundance of starfish change *as a result of* the abundance of mussels?**



# How to construct a population model



# How to construct a population model

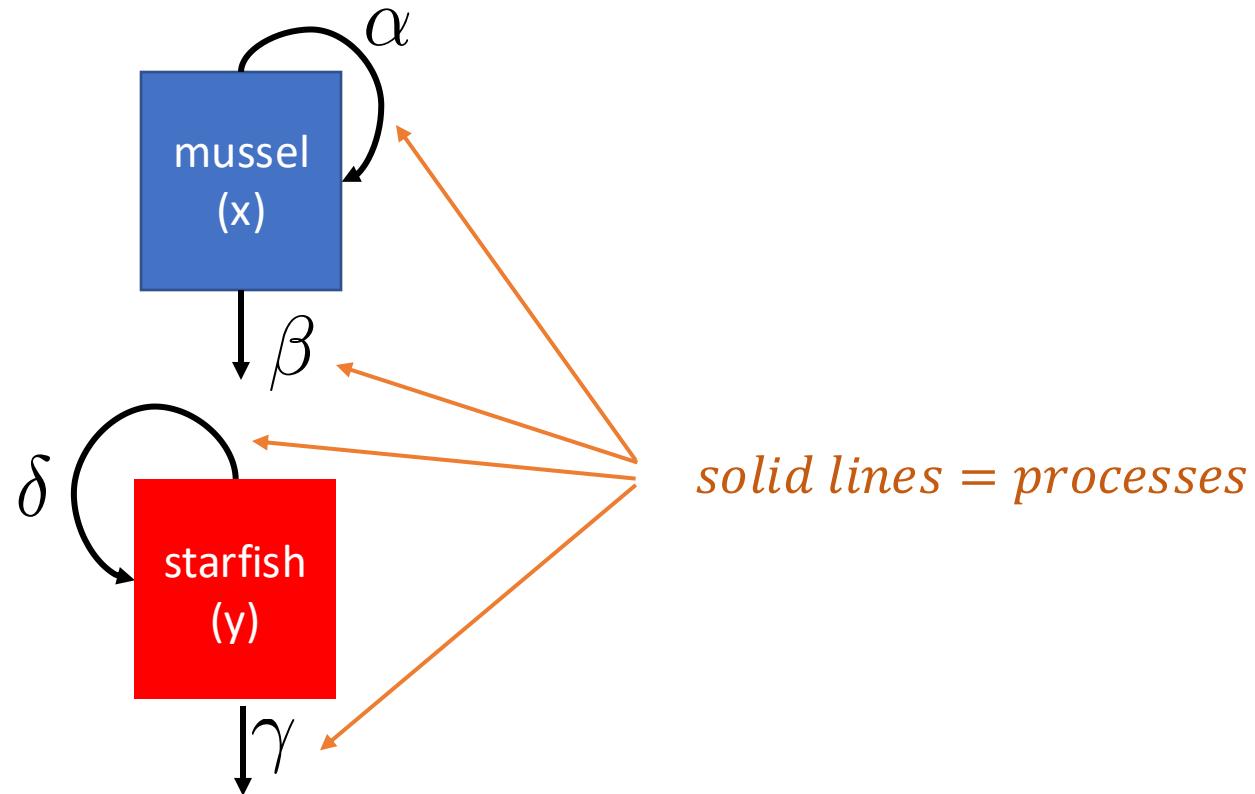


# How to construct a population model

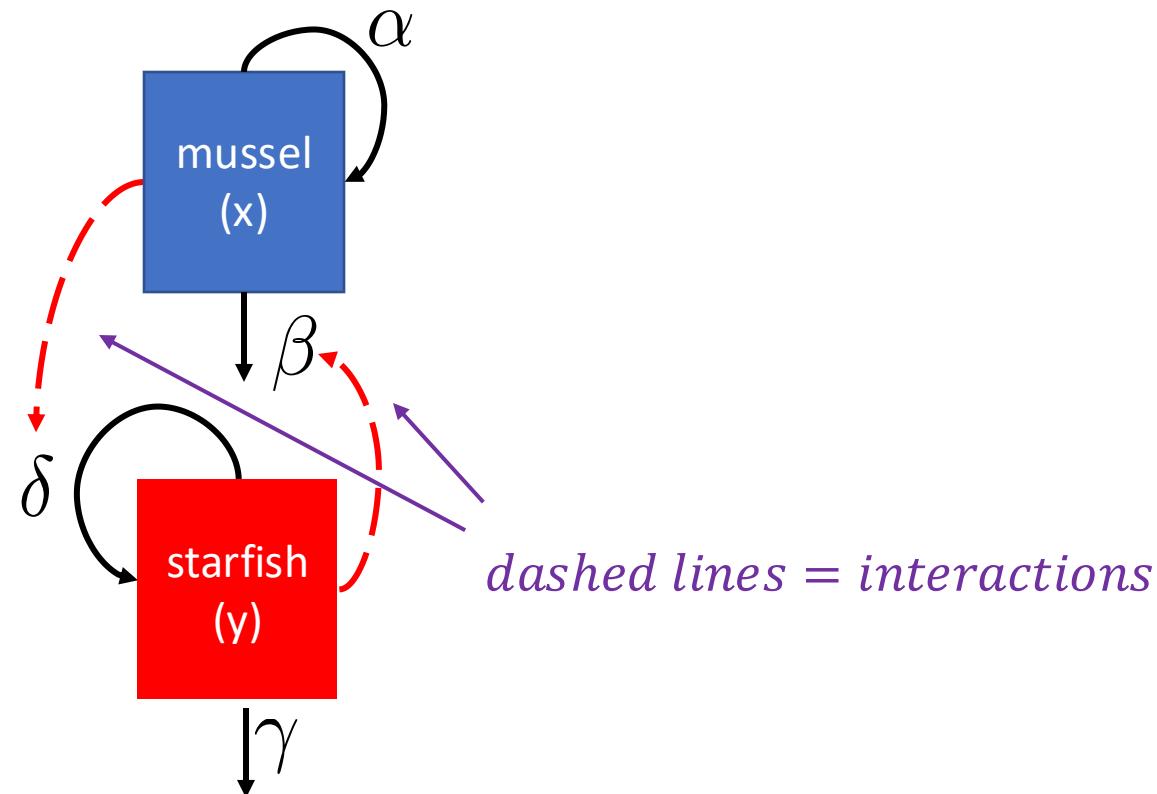
$$\frac{dx}{dt} = x(\alpha - \beta y)$$

*processes*

$$\frac{dy}{dt} = y(\delta x - \gamma)$$



# How to construct a population model



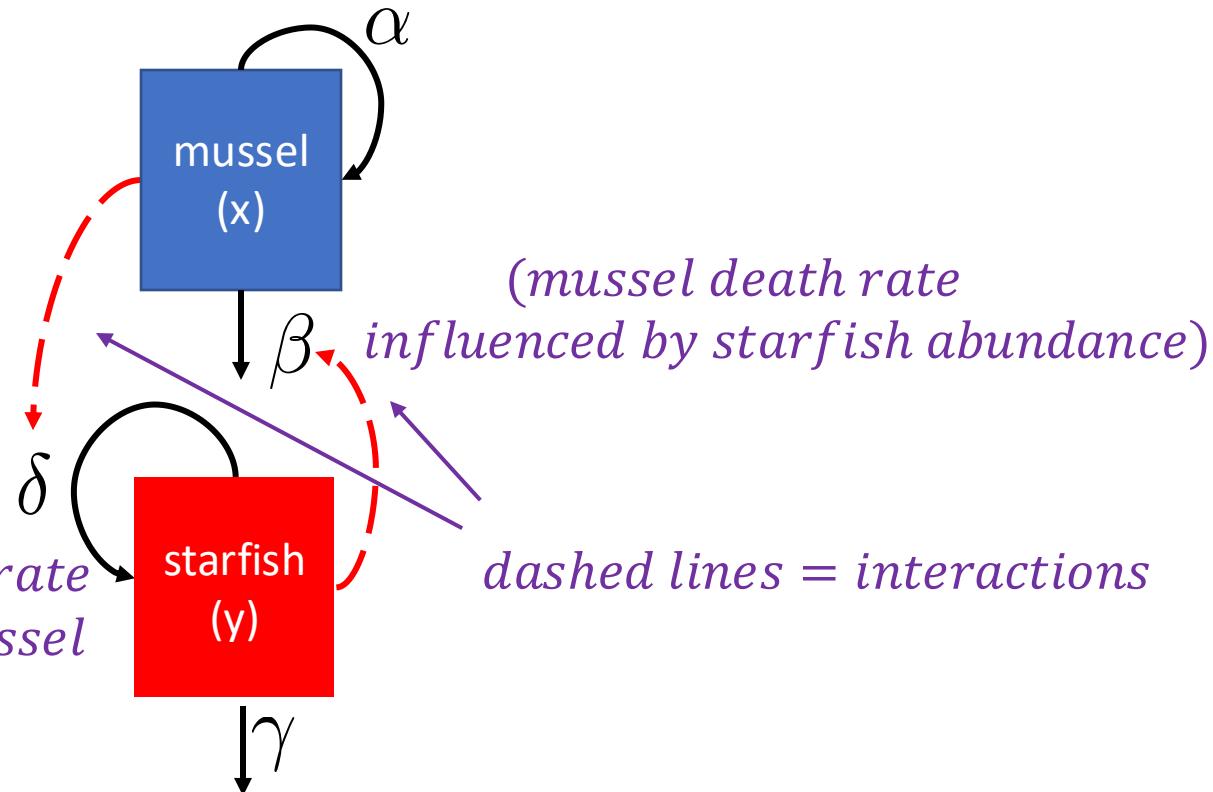
# How to construct a population model

$$\frac{dx}{dt} = x(\alpha - \beta y)$$

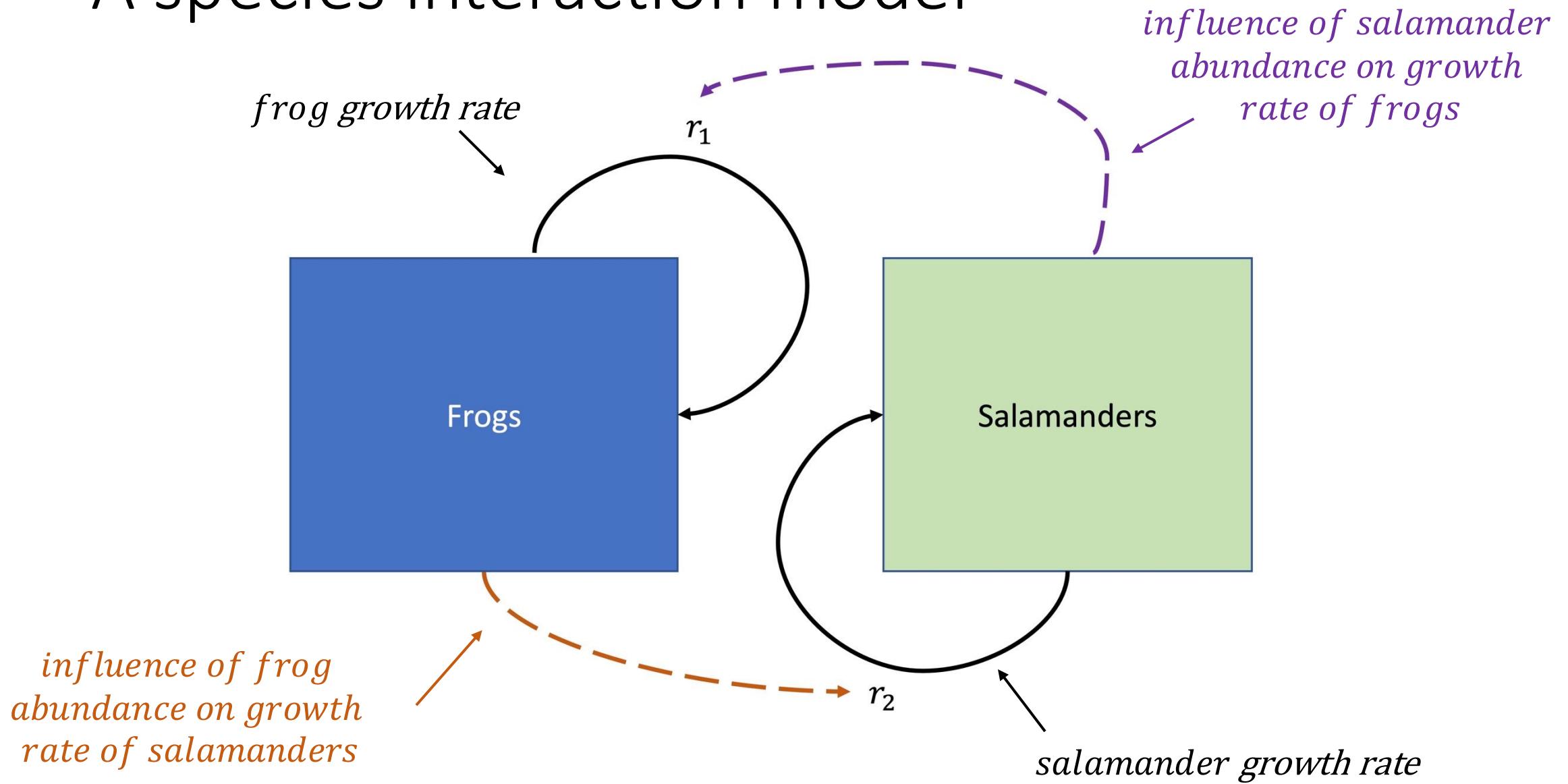
*interactions*

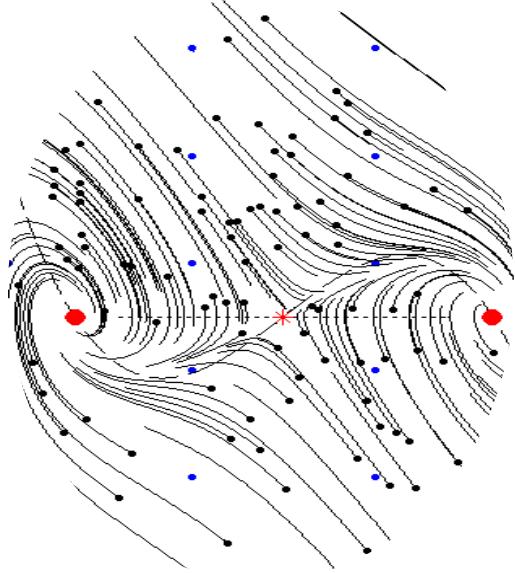
$$\frac{dy}{dt} = y(\delta x - \gamma)$$

*(starfish growth rate  
influenced by mussel  
abundance)*



# A species interaction model





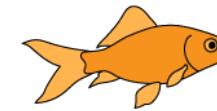
Ecology is the study of  
the **interactions** of  
**organisms** with each  
other and their  
**environment.**

**dynamic**  
*frequently changing*

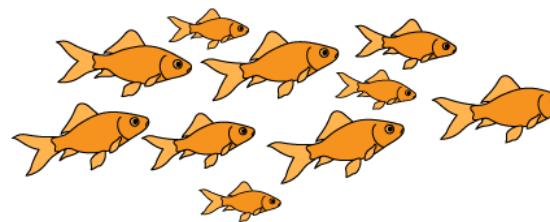
**complex**  
*many interacting parties*



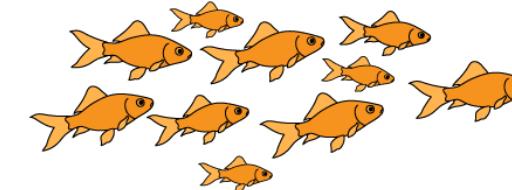
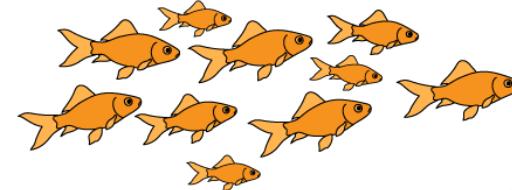
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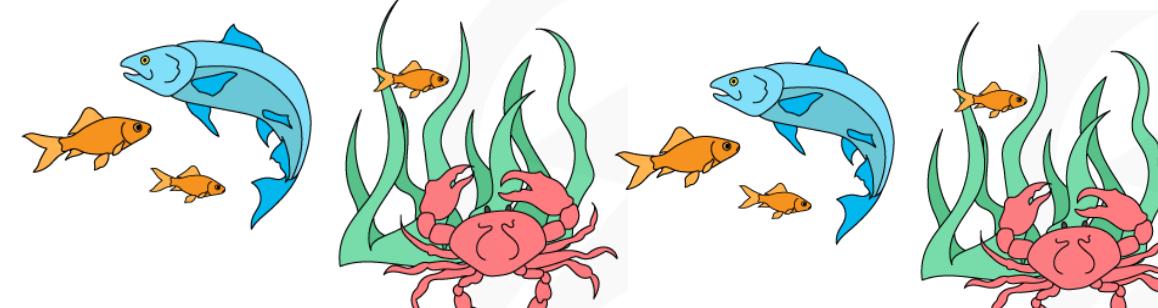
individual



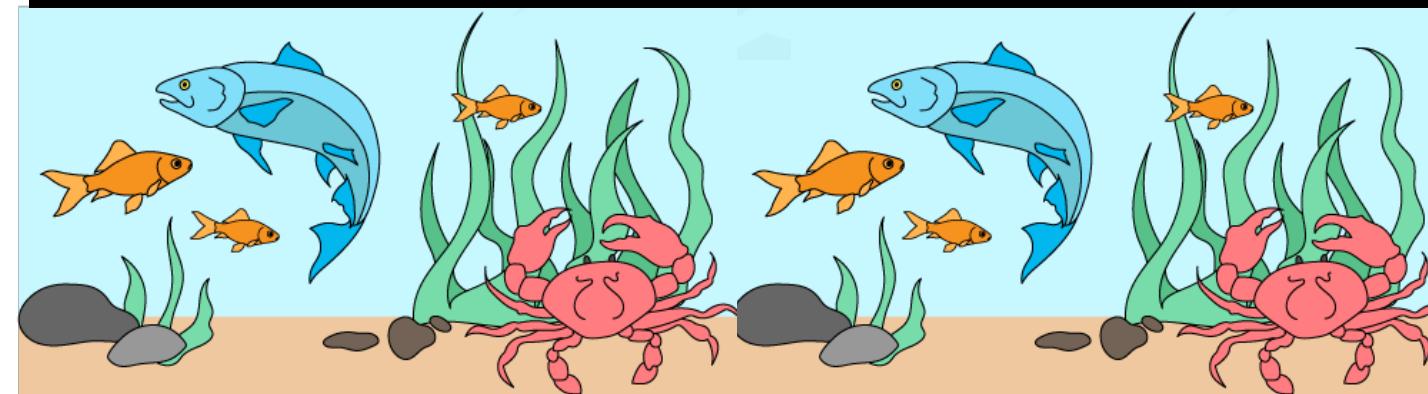
population



metapopulation

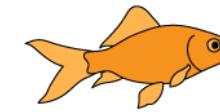


community



ecosystem

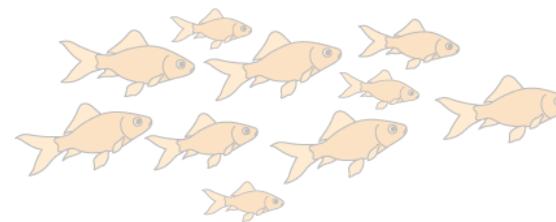
individual



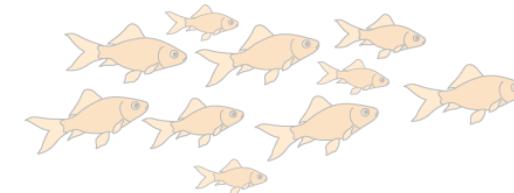
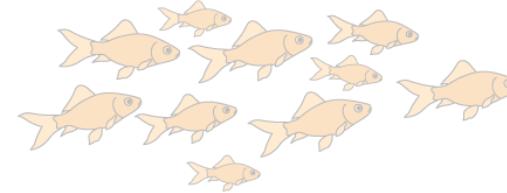
## Individual:

metabolism, behavior,  
life history.

interactions of an  
individual with the  
environment



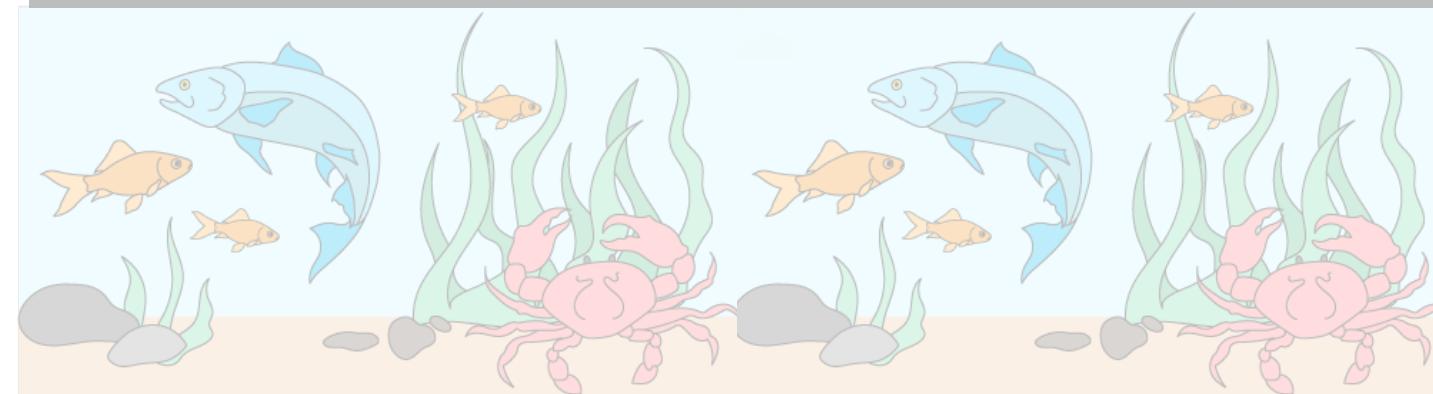
population



metapopulation

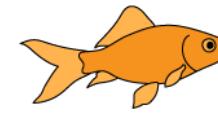


community



ecosystem

individual



Individual:  
metabolism,  
behavior, life history.

population

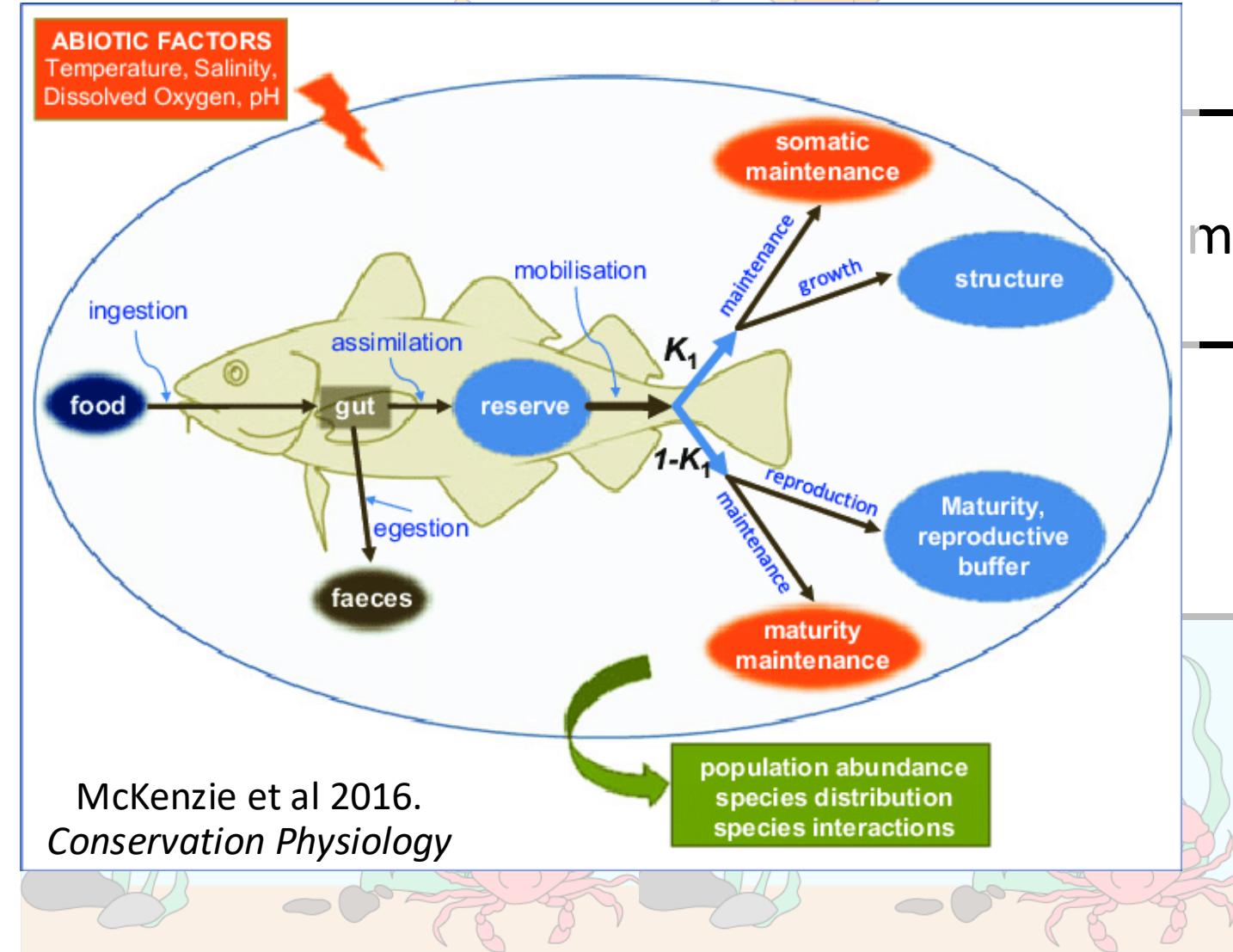
metapopulation

community

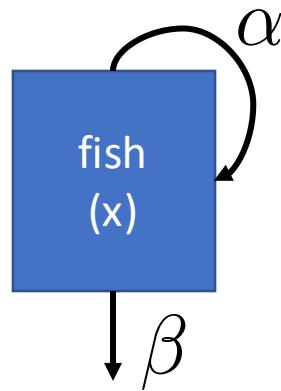
ecosystem

## Dynamic Energy Budget (DEB) Model

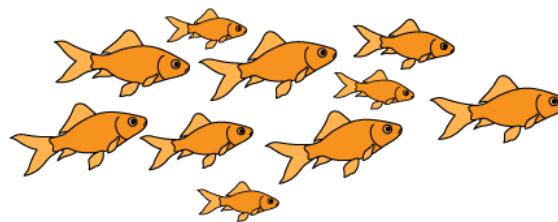
*How does a fish's  
metabolism **change**  
with temperature?*



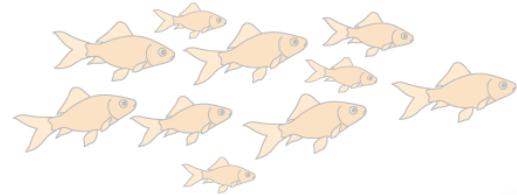
**Population** = multiple individuals of the same species (**conspecifics**) in the same habitat



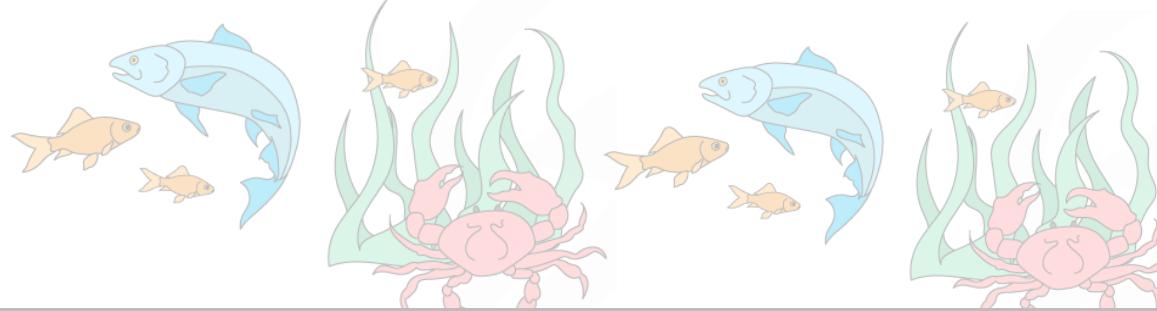
individual



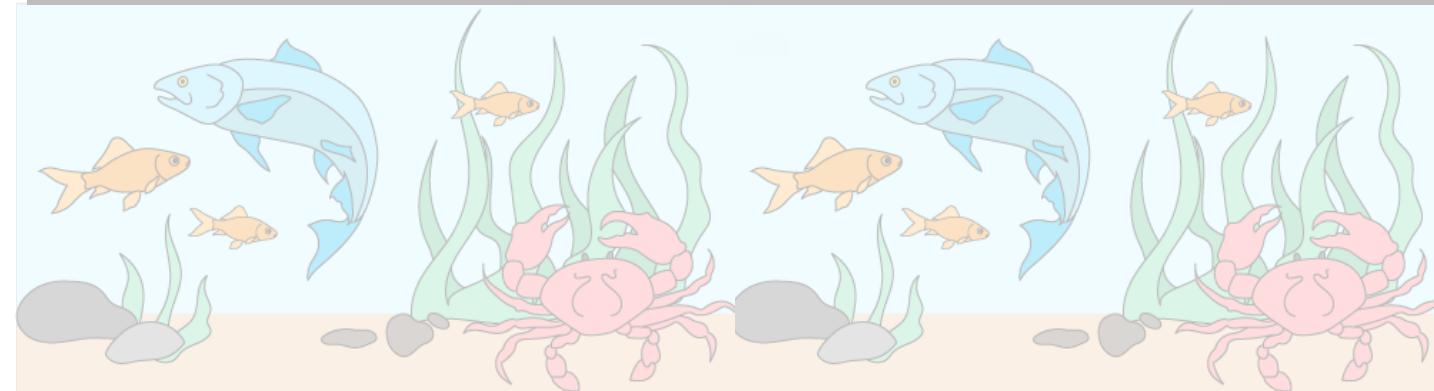
population



metapopulation



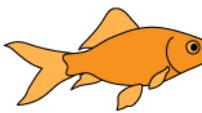
community



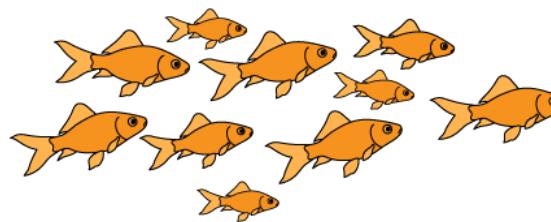
ecosystem

*How does the abundance of fish **change** through time?*

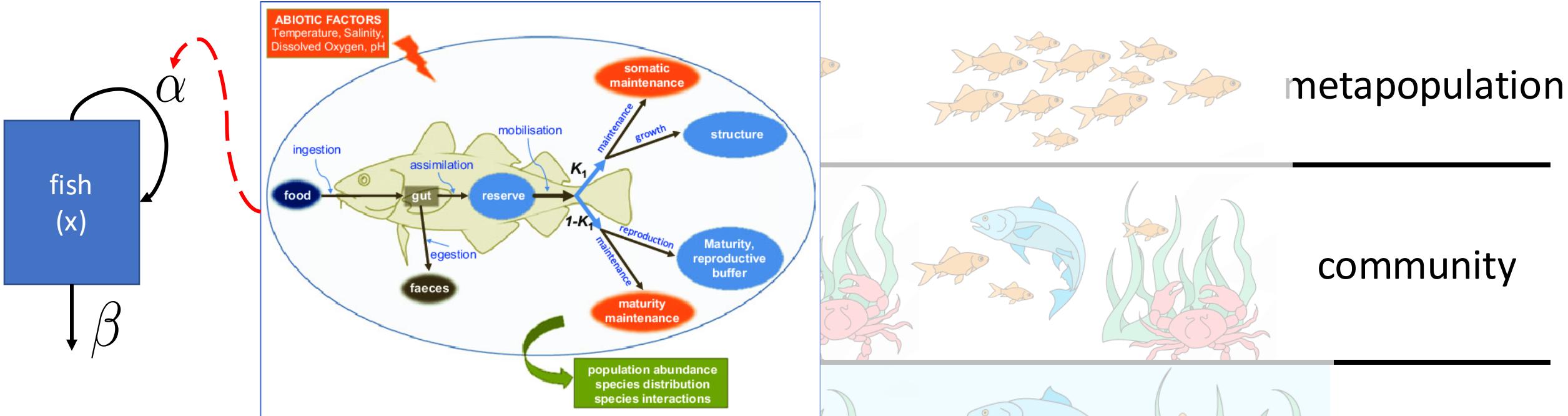
**Nested Models**, including a class of model known as **Integral Projection Models** (IPMs), link individual- and population-level processes



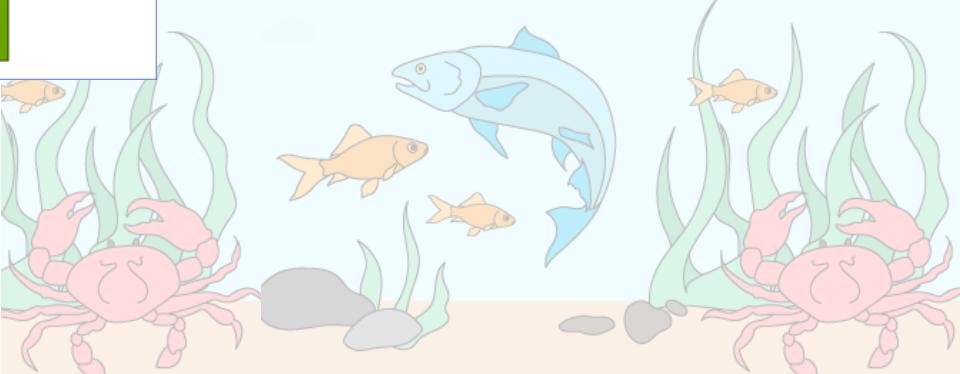
individual



population



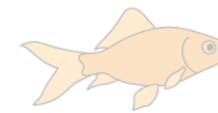
*How does the abundance of fish change through time as temperature changes metabolism?*



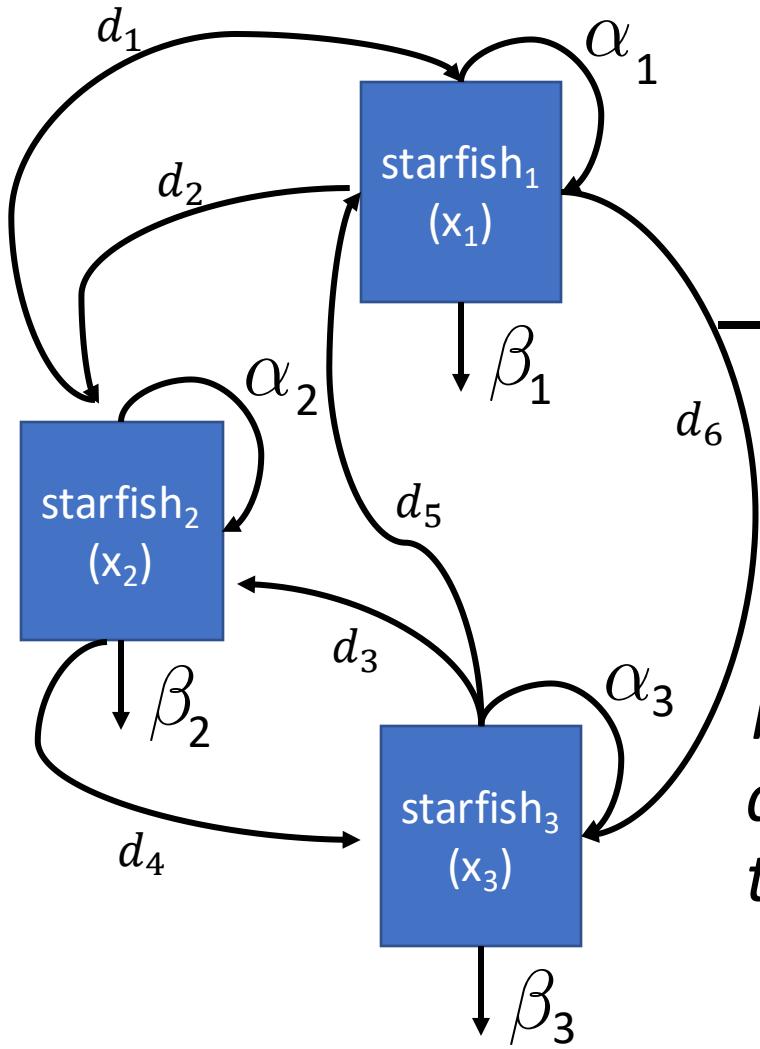
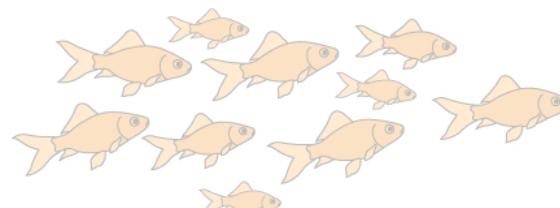
ecosystem

**Metapopulation** = sub-populations of conspecifics connected by migration or dispersal

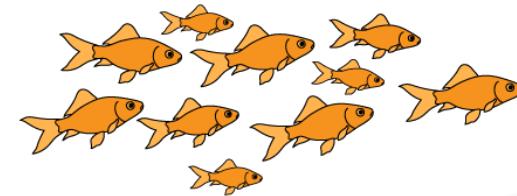
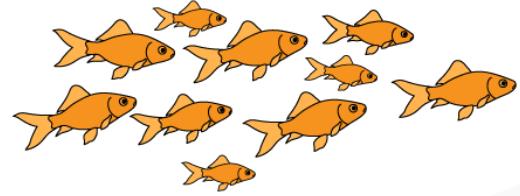
individual



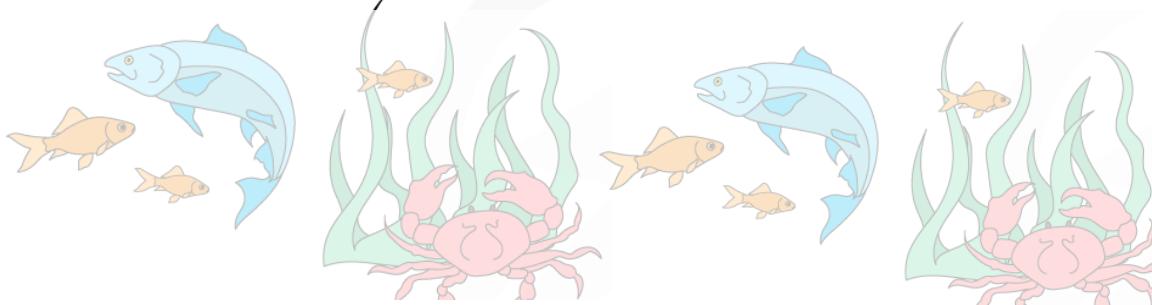
population



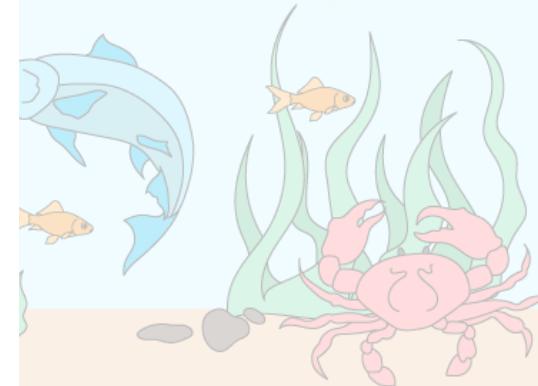
metapopulation



community

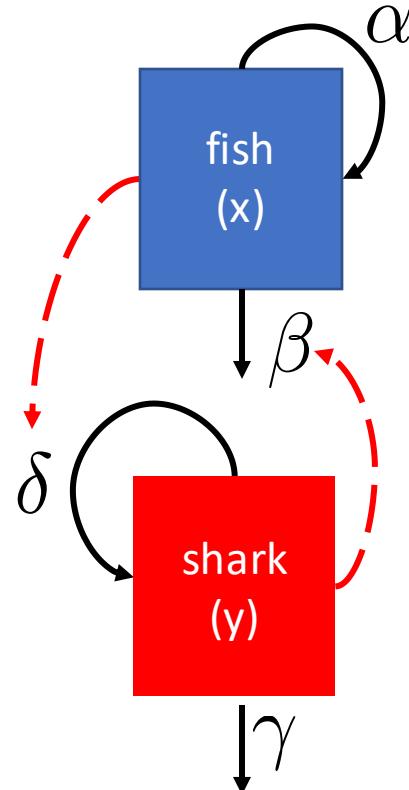


ecosystem

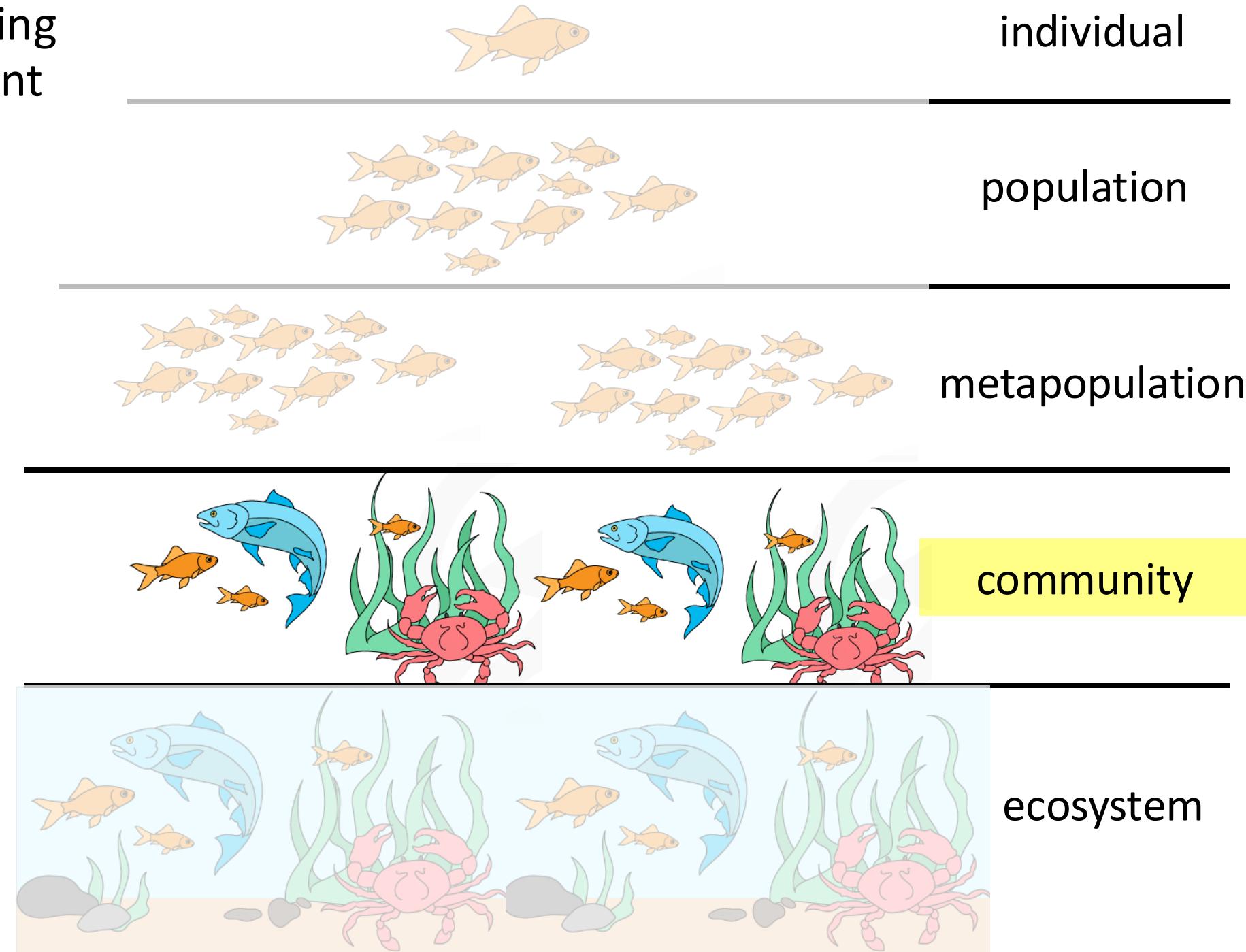


*How do the abundances of different subpopulations of the same fish **vary** in space [and time]?*

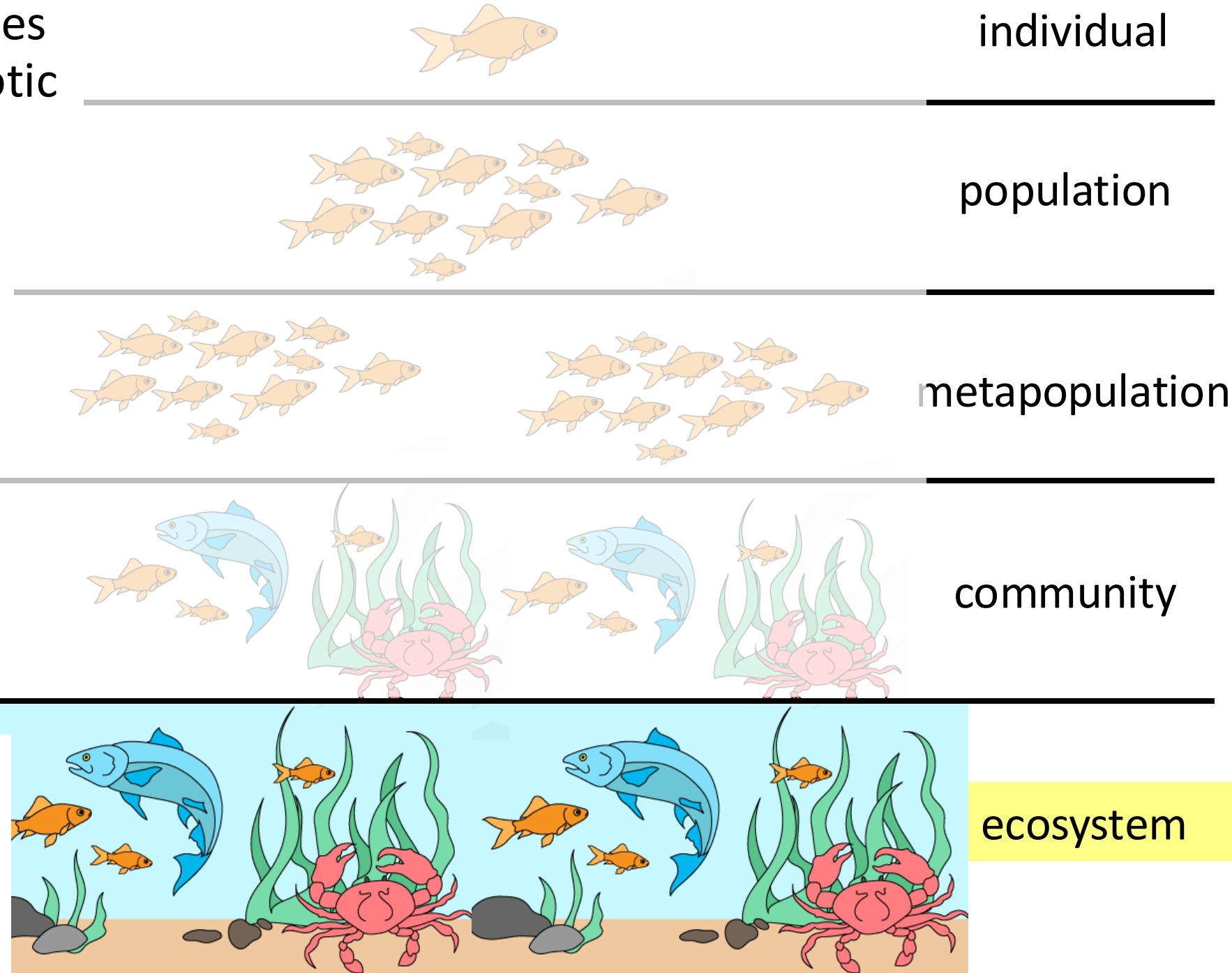
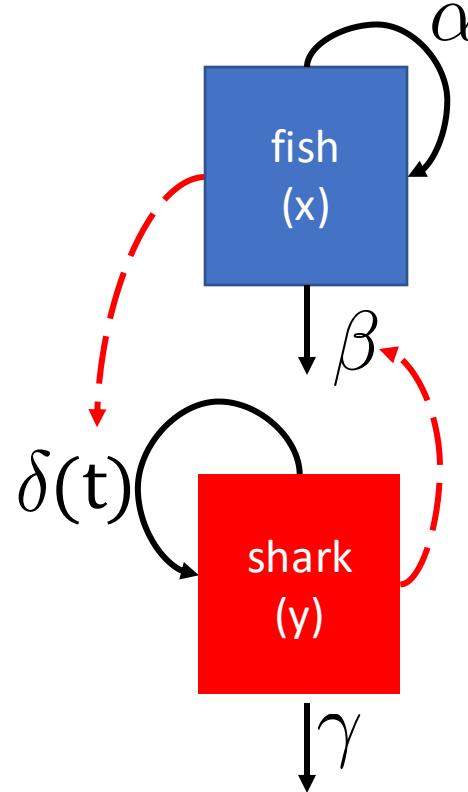
**Community** = interacting populations of different species



How does fish abundance **vary** with changes in shark abundance?



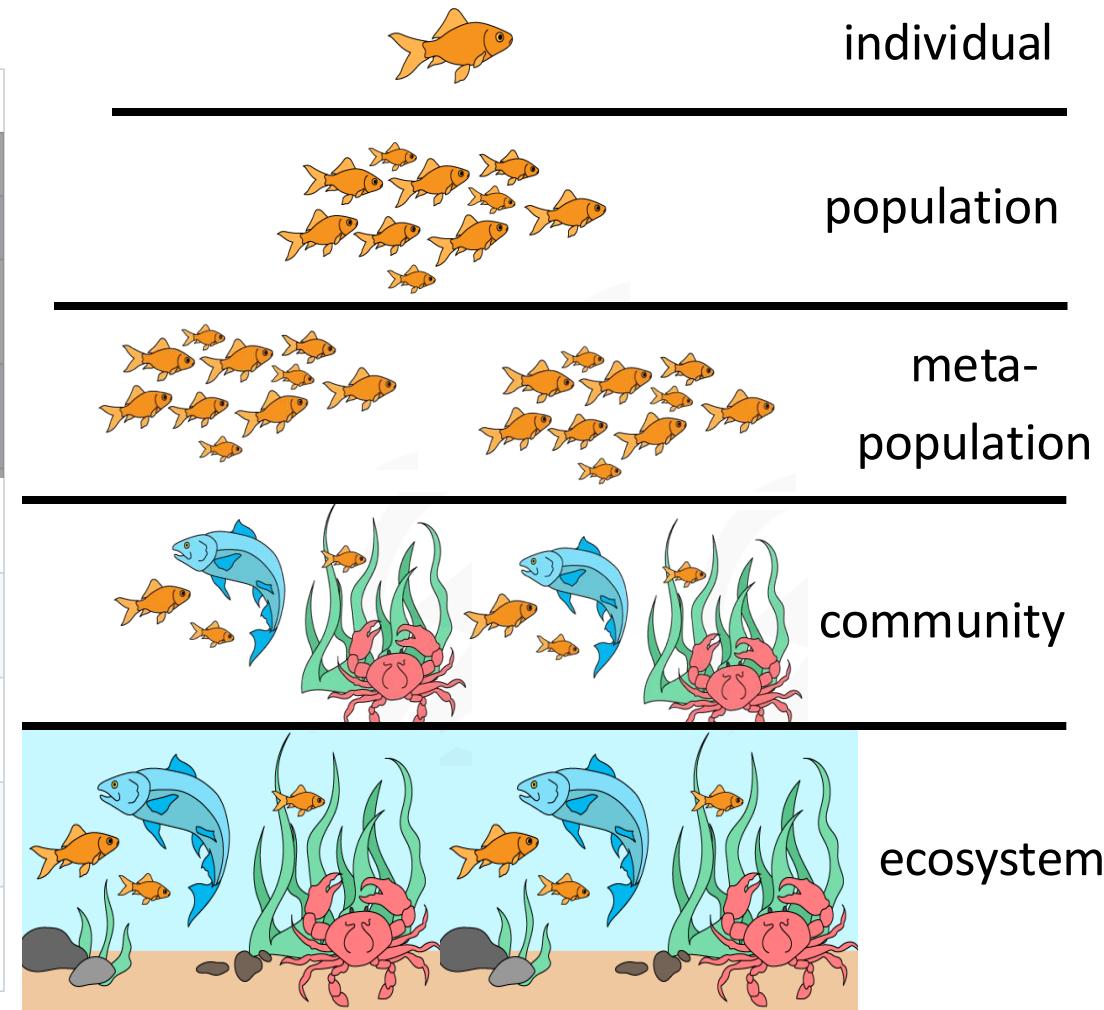
**Ecosystem** = communities interacting with the abiotic environment



How does fish abundance **vary** with changes in shark birth rates with **temperature**?

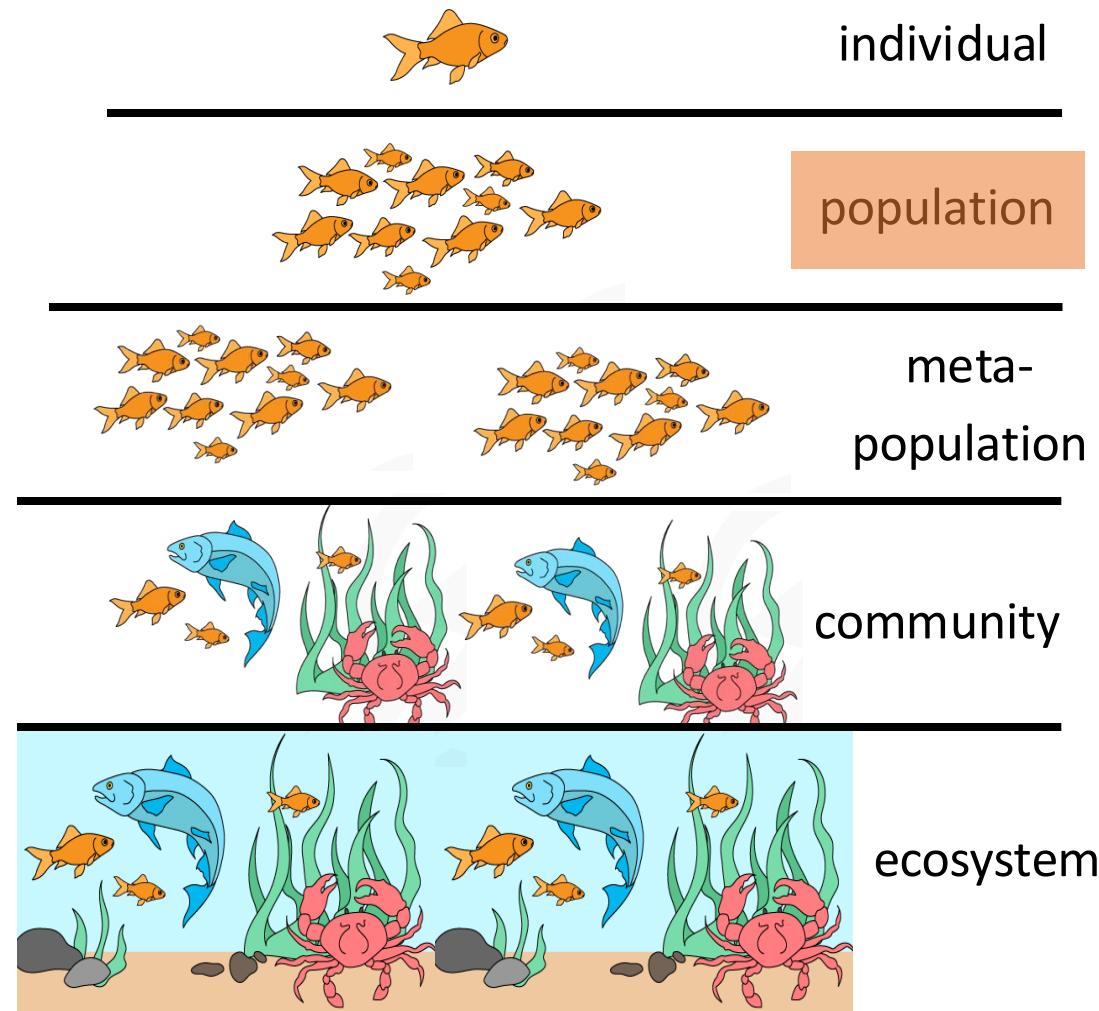
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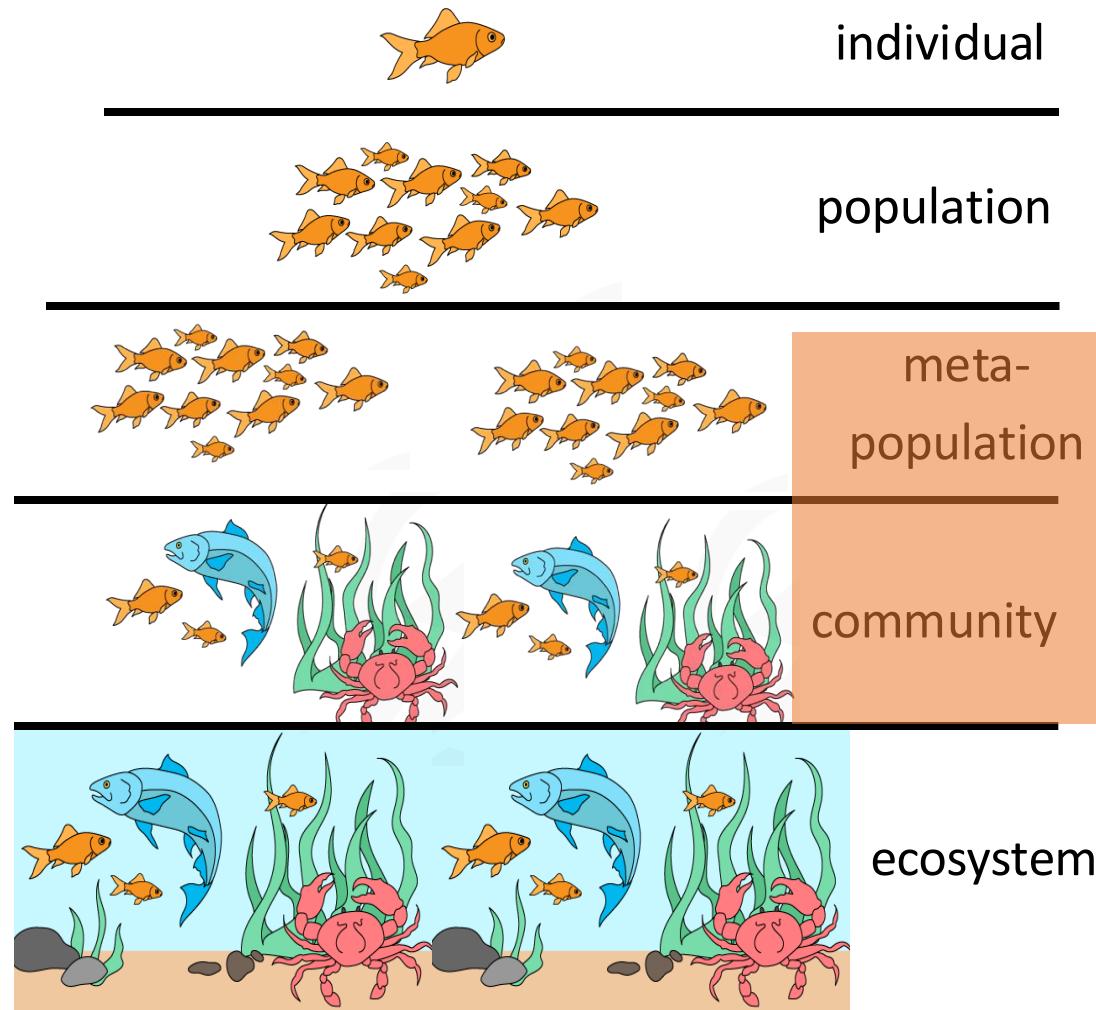
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7	6: Population Growth	Feb 18: Species Interactions 2	Feb 20: Disease Dynamics as Population Biology 1
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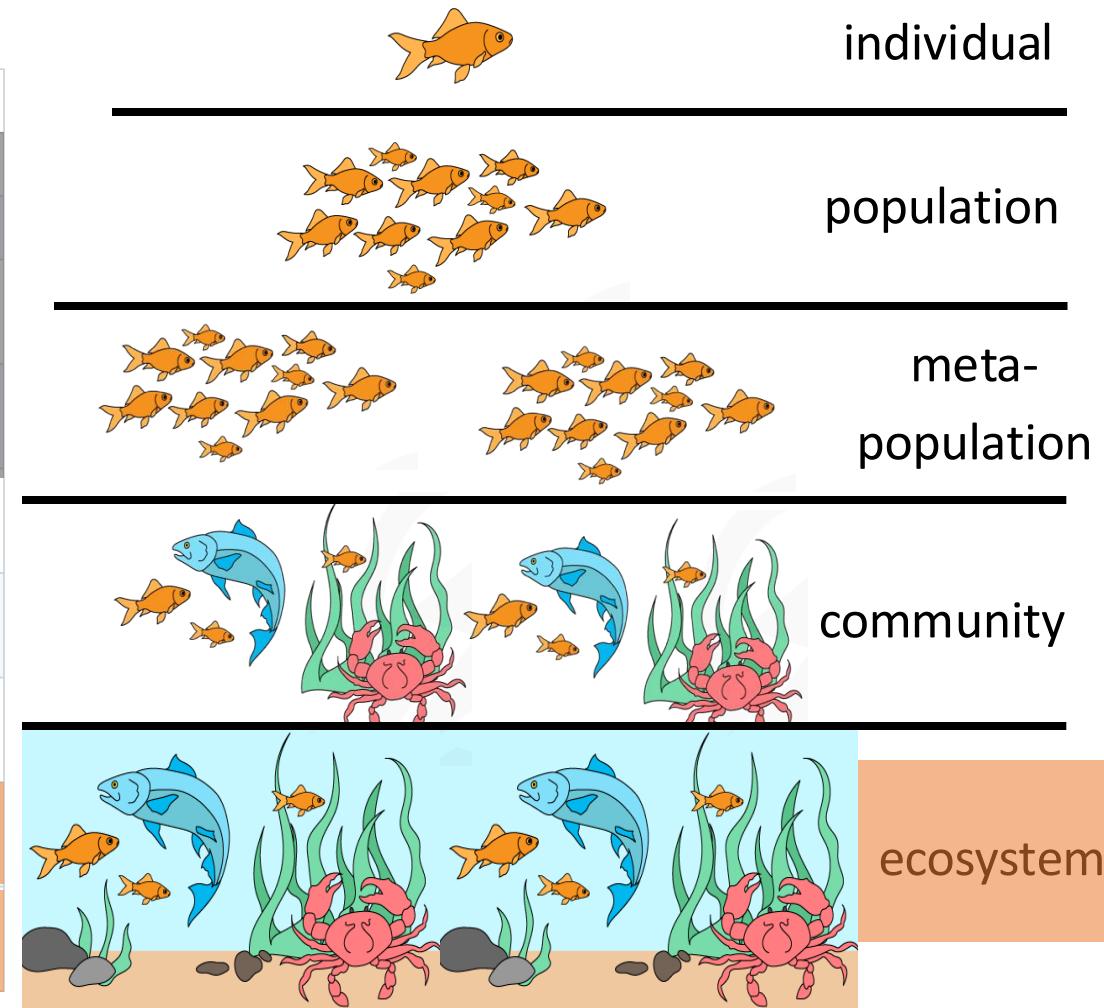
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Week	Mon/Tues Lab	Tues Lecture	Thurs Lecture
1	--	Jan 7: Evolution History	Jan 9: Mutation and Variation
2	1: Intro R	Jan 14: Hardy-Weinberg	Jan 16: Migration and Drift
3	2: Hardy-Weinberg	Jan 21: Natural Selection	Jan 23: Phylogenetics
4	3: Microevolution	Jan 28: Molecular Evolution & Sexual Selection	Jan 30: Kin Selection, Speciation
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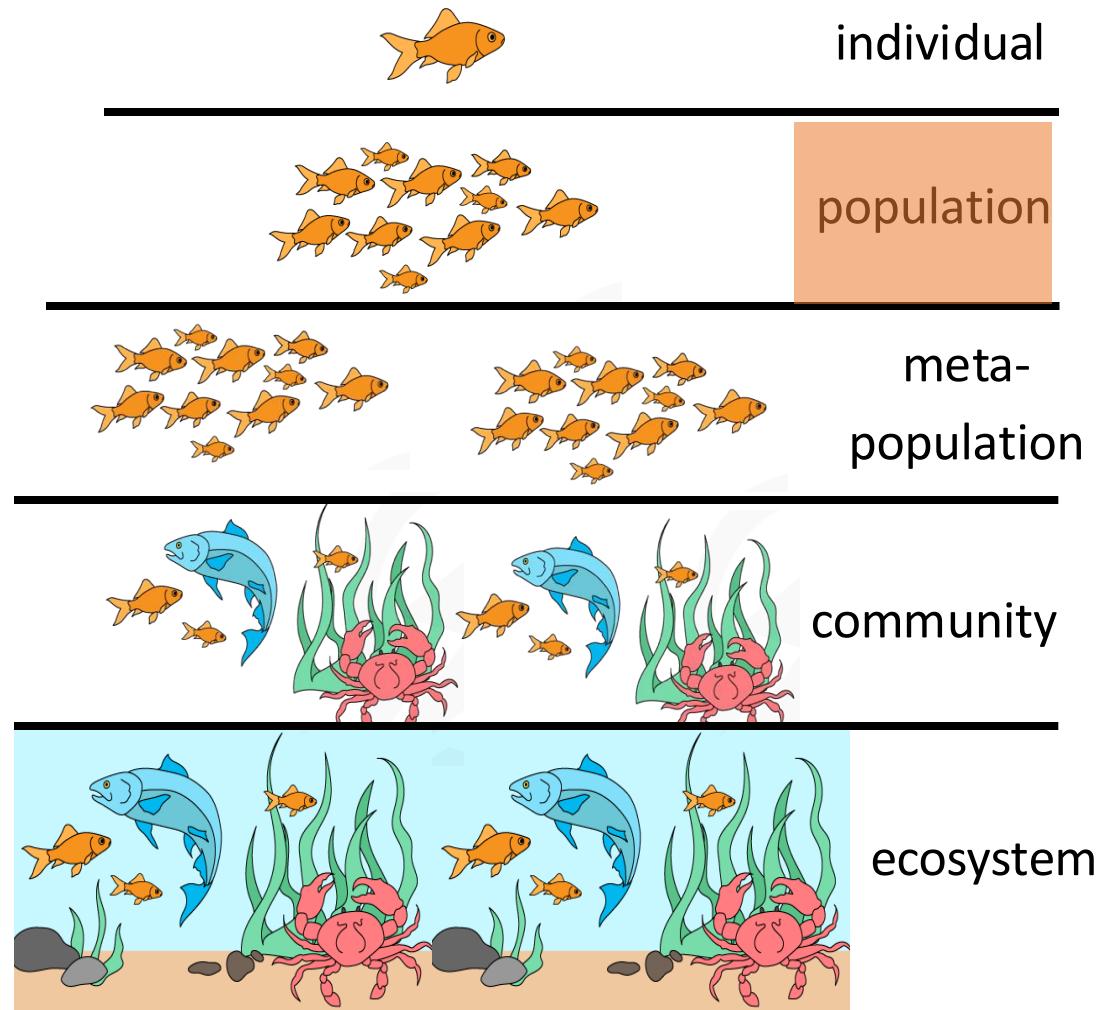
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# Why do we care how populations grow?

(Shafer 1980)

## **DETERMINING MINIMUM VIABLE POPULATION SIZES FOR THE GRIZZLY BEAR<sup>1</sup>**

MARK L. SHAFFER,<sup>2</sup> School of Forestry and Environmental Studies, Duke University, Durham, NC 27706

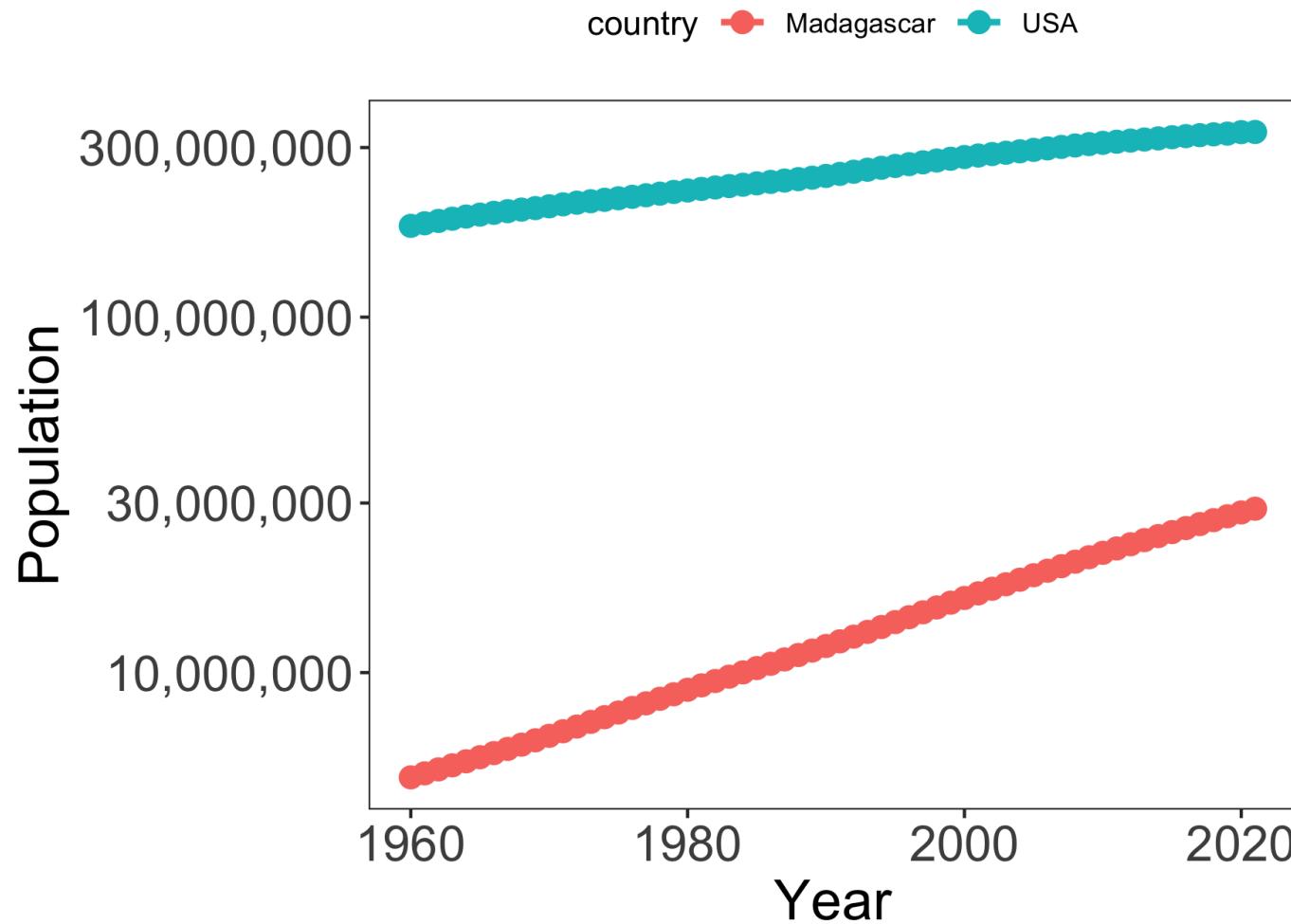
*Abstract:* A stochastic computer simulation is presented for use in determining the relationship of population size to extinction probabilities for populations of grizzly bears (*Ursus arctos*). Published data on numbers, age, sex, reproduction, and mortality for the grizzly bear population of Yellowstone National Park were used to develop and test several simulation models. The results indicate that, for the Yellowstone grizzlies, 35 to 70 bears constitute a minimum viable population (the smallest population with a 95% probability of surviving at least 100 years). Minimum area requirements for populations of this size range from 700 to 10,000 km.<sup>2</sup>

*Int. Conf. Bear Res. and Manage.* 5:133–139



to protect populations from extinction

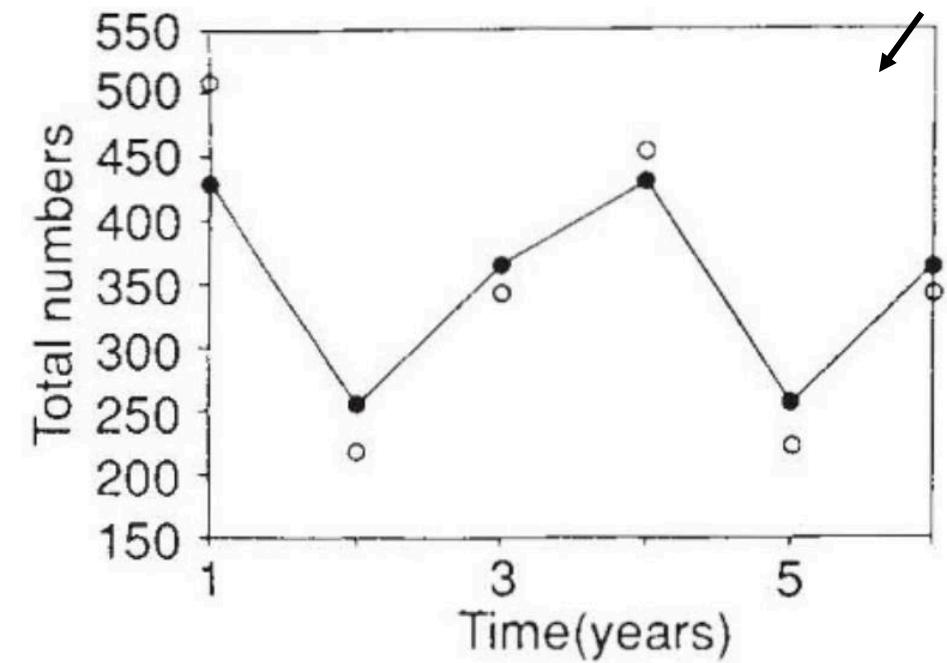
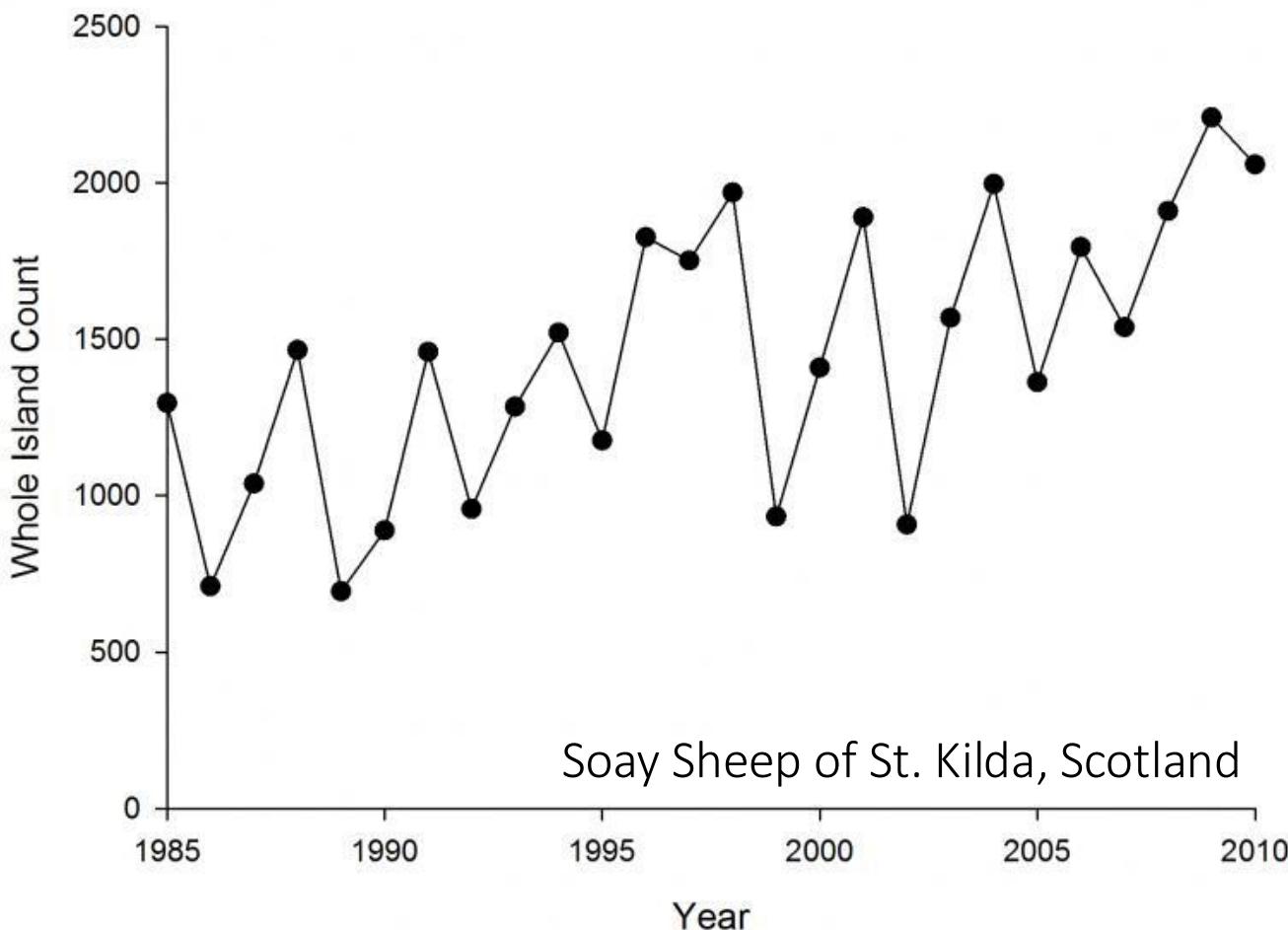
# Why do we care how populations grow?



to forecast resource use

# Why do we care how populations grow?

to understand past phenomena



Model compared against data

Overcompensation and population cycles in an ungulate

B. T. Grenfell, O. F. Price, S. D. Albon & T. H. Glutton-Brock

Nature 355, 823–826 (1992) | Cite this article

589 Accesses | 107 Citations | Metrics

# The simplest population model

1. Populations are divided into compartments
2. Individuals within a compartment are homogenously mixed
3. Compartments and transition rates are determined by biological systems
4. Rates of transferring between compartments are expressed mathematically



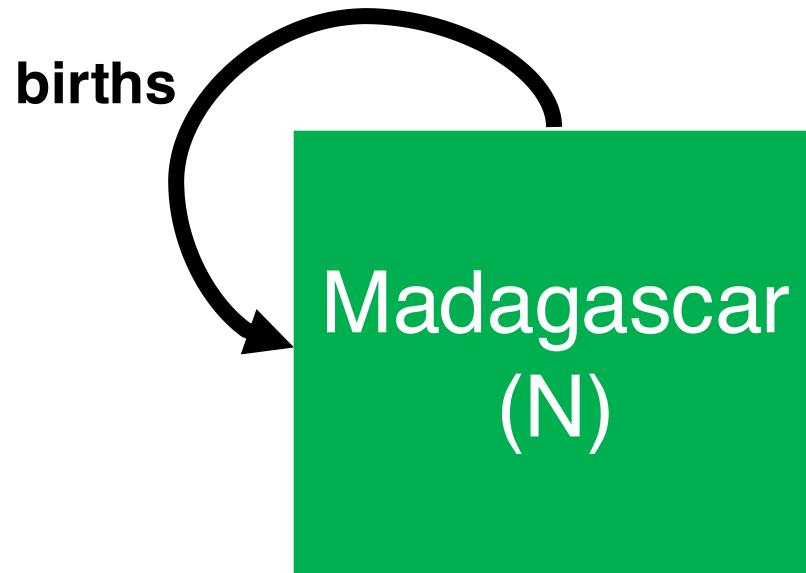
# The simplest population model

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Madagascar  
(N)



# The simplest population model

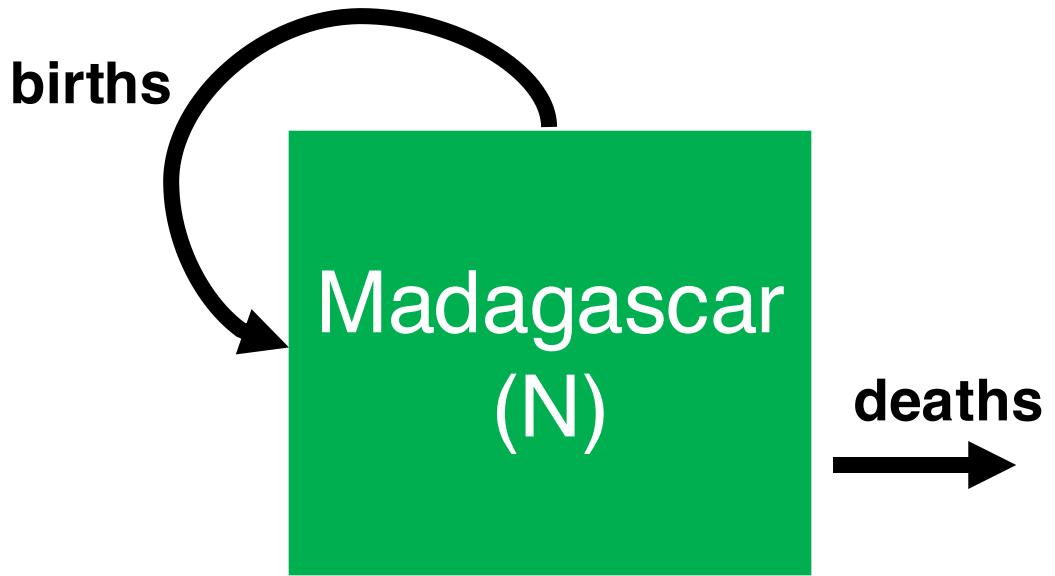


**How does the population grow?**

1. Populations are divided into compartments
2. Individuals within a compartment are homogenously mixed
3. Compartments and transition rates are determined by biological systems
4. Rates of transferring between compartments are expressed mathematically



# The simplest population model

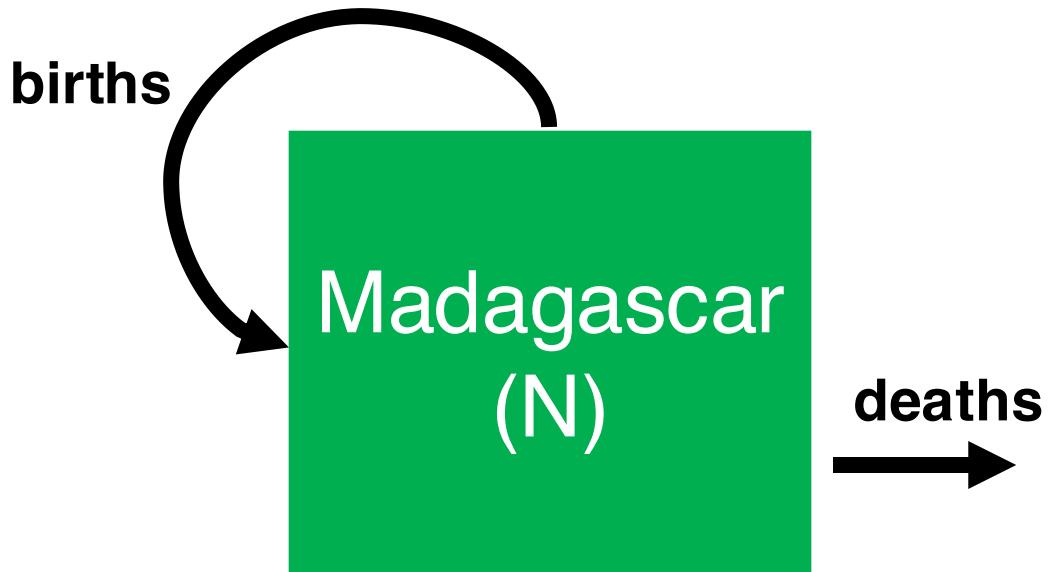


1. Populations are divided into compartments
2. Individuals within a compartment are homogenously mixed
3. Compartments and transition rates are determined by biological systems
4. Rates of transferring between compartments are expressed mathematically



**How does the population decrease?**

# The simplest population model



$$N_{t+1} = N_t + \text{births} * N_t - \text{deaths} * N_t$$

$$N_{t+1} = N_t + (\text{births} - \text{deaths}) * N_t$$

$$N_{t+1} = N_t + R * N_t$$

1. Populations are divided into compartments
2. Individuals within a compartment are homogenously mixed
3. Compartments and transition rates are determined by biological systems
4. Rates of transferring between compartments are expressed mathematically

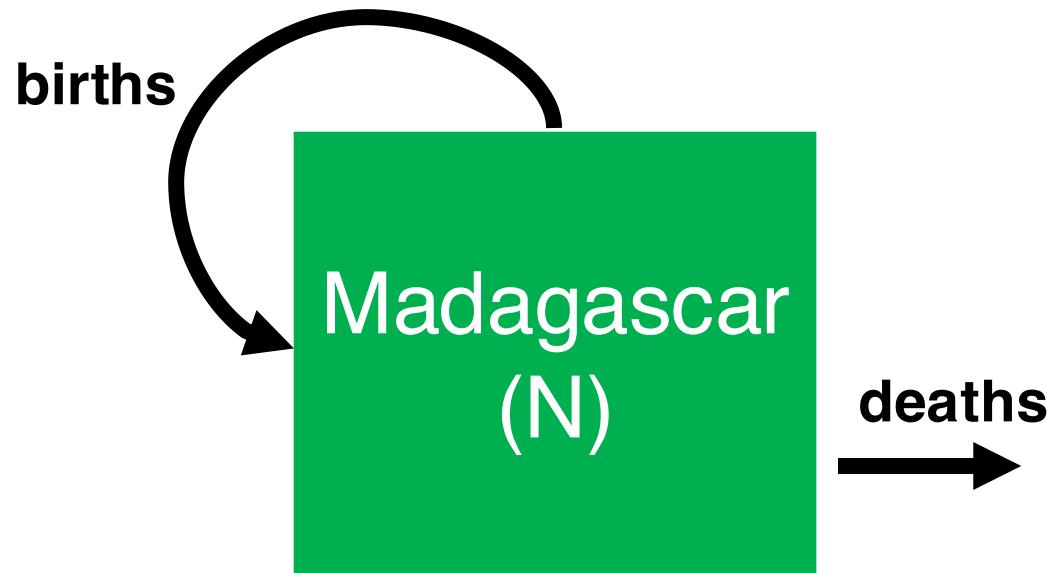


**R=geometric rate of increase**  
**R = (births – deaths)**

*(pop grows @ R > 0 & declines @ R < 0 )*

# The simplest population model

1. Populations are divided into compartments
2. Individuals within a compartment are homogenously mixed
3. Compartments and transition rates are determined by biological systems
4. Rates of transferring between compartments are expressed mathematically



$$N_{t+1} = N_t + \text{births} * N_t - \text{deaths} * N_t$$

$$N_{t+1} = N_t + R * N_t$$

$$N_{t+1} = (1 + R) * N_t$$

$$N_{t+1} = \lambda * N_t$$

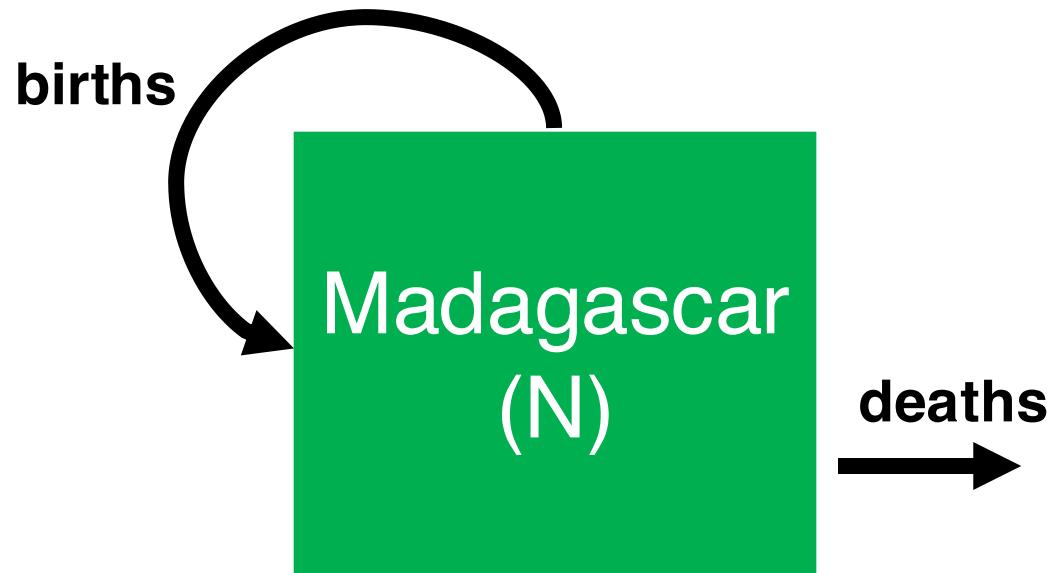


$\lambda$  = population rate of increase  
(finite growth rate)

$$\lambda = 1 + R$$

# The simplest population model

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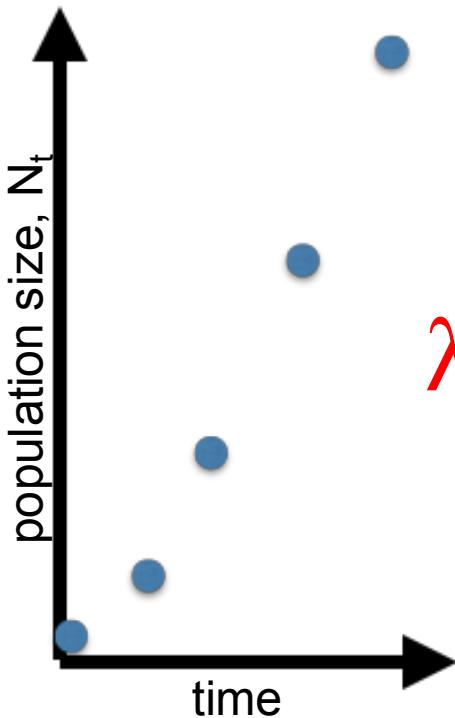


**$\lambda$  = population rate of increase  
(finite growth rate)**  
*(pop grows @  $\lambda > 1$  & declines @  $\lambda < 1$ )*

# Geometric growth



Geometric growth is measured in discrete time



$$\lambda = N_{t+1}/N_t$$

$$N_1 = \lambda N_0$$

$$N_2 = \lambda[\lambda N_0] = \lambda^2 N_0$$

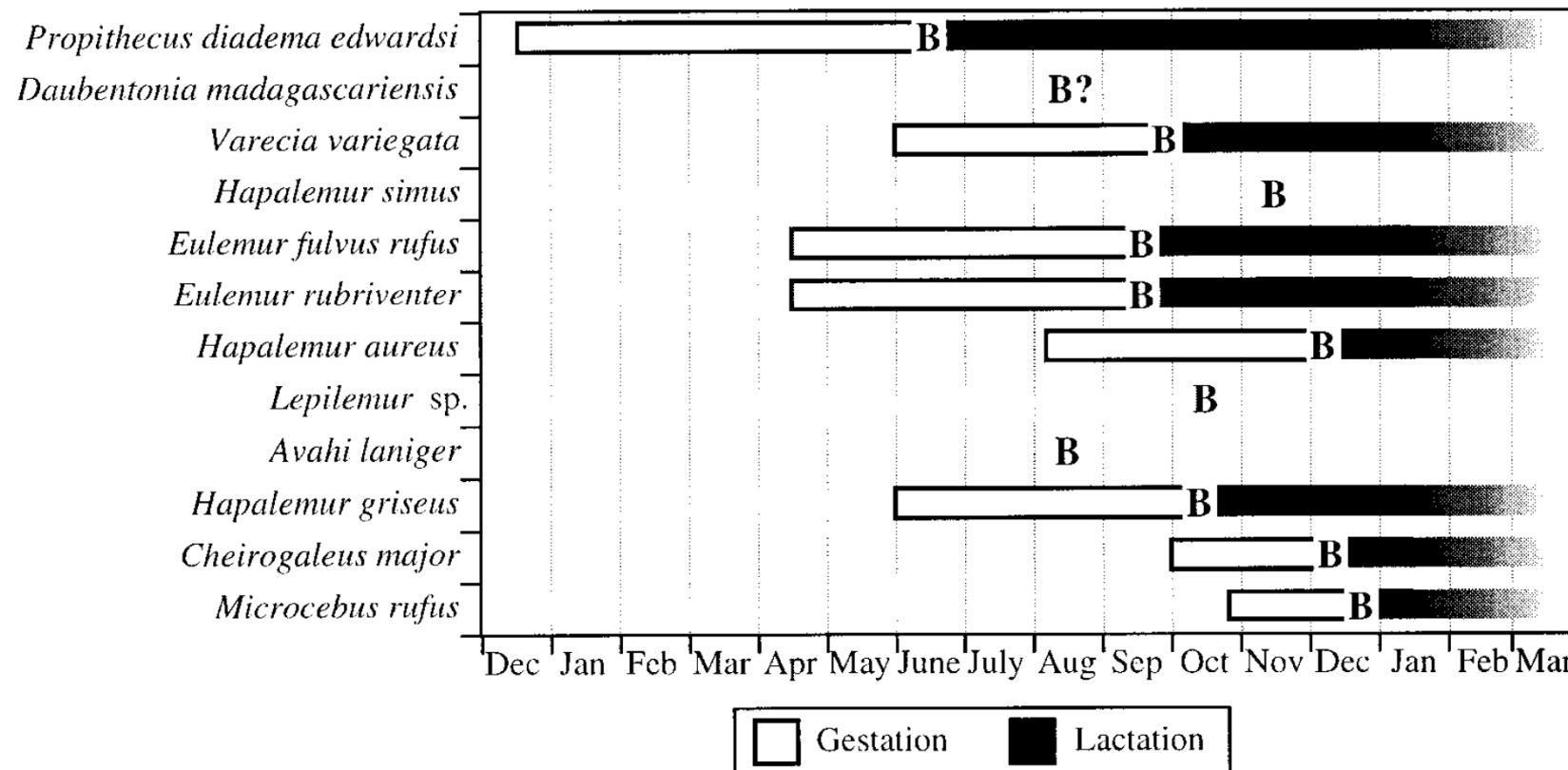
$$N_3 = \lambda^3 N_0$$

$$N_t = \lambda^t N_0$$

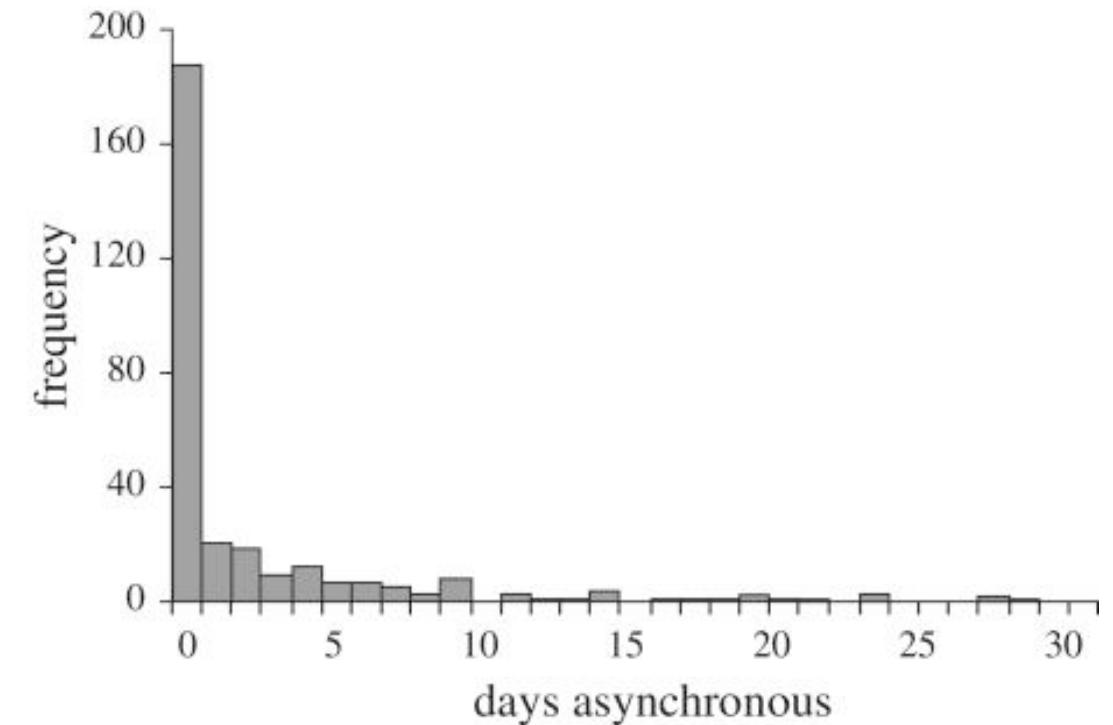
$\lambda$  = population rate of increase



# Is discrete time realistic for population growth?

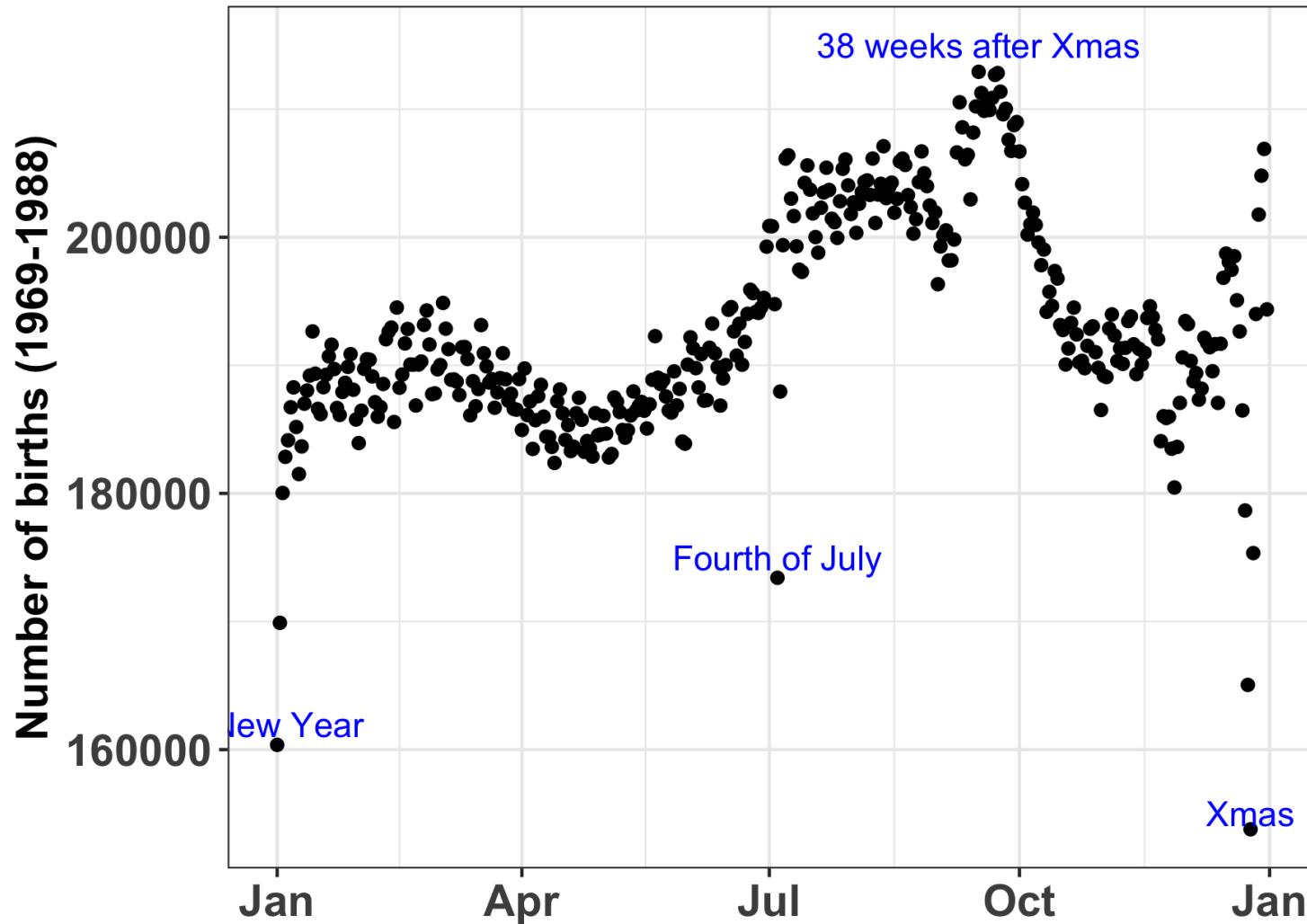


# Is discrete time realistic for population growth?



64% of banded mongoose pups are born on the same day.

# Is discrete time realistic for population growth?



Human births by day in the US (1969-1988)

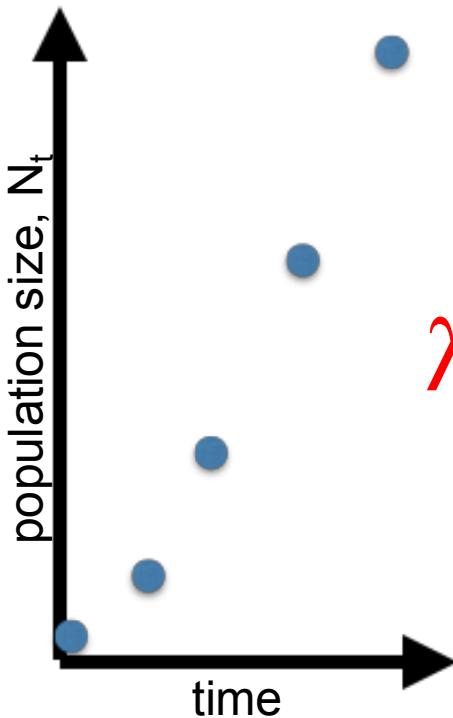
# Is discrete time realistic for population growth?

- Many **continuously breeding populations** (including human) are better approximated by an assumption of a **continuous birth rate**
- **Univoltine/bivoltine species** may be better approximated by **discrete time growth rates**
  - Voltinism = number of broods (generations) produced per year
- Ultimately **the best choice model depends** on the **available data!**
  - For example, discrete time model may work well for human populations if censuses are conducted only annually

# Geometric vs. exponential growth



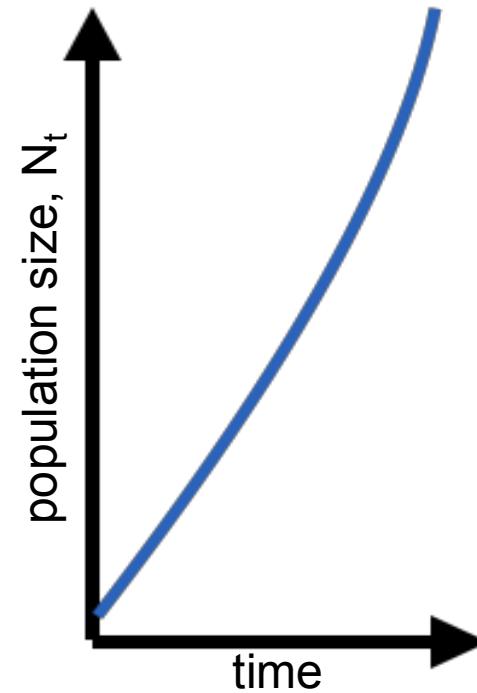
Geometric growth



$$\begin{aligned}N_1 &= \lambda N_0 \\N_2 &= \lambda[\lambda N_0] = \lambda^2 N_0 \\N_3 &= \lambda^3 N_0 \\N_t &= \lambda^t N_0\end{aligned}$$

$\lambda$  = population rate of increase

Exponential growth is measured in continuous time



$$r = \frac{\ln \left( \frac{N_t}{N_0} \right)}{t}$$

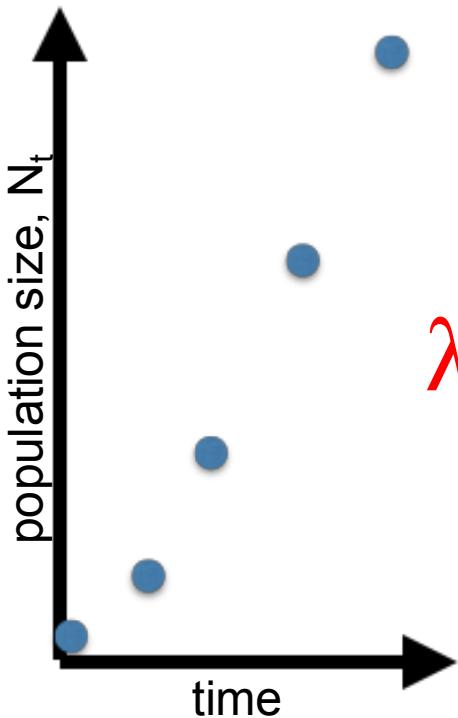
$$dN(t)/dt = rN(t)$$

$r$  = intrinsic (instantaneous) rate of increase



# Geometric vs. exponential growth

**Discrete time**



$$\begin{aligned}N_1 &= \lambda N_0 \\N_2 &= \lambda[\lambda N_0] = \lambda^2 N_0 \\N_3 &= \lambda^3 N_0 \\N_t &= \lambda^t N_0\end{aligned}$$

**Continuous time**

$$dN(t)/dt = rN(t)$$

*Separation of variables:*  
 $dN(t)/N(t) = r dt$

*Integrate both sides:*  
 $\int dN(t)/N(t) = \int r dt$

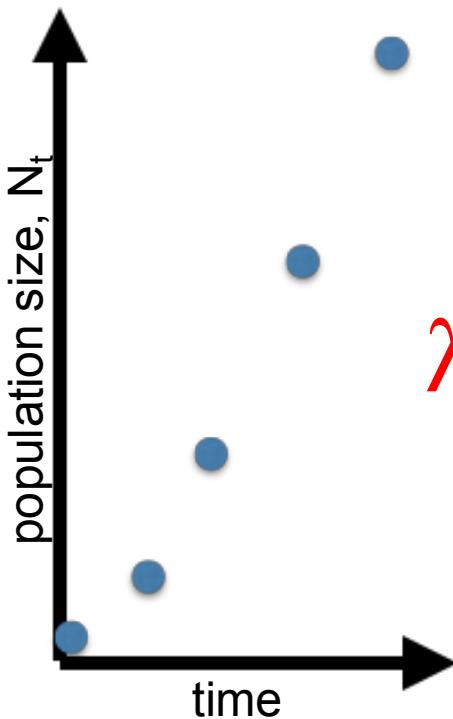
*By definition:*  
 $\ln(N(t)) = rt + c$

*Take exponentials:*  
 $N(t) = e^{rt+c} = Ce^{rt}$   
 $N(t) = N(0)e^{rt}$

# Geometric vs. exponential growth



Discrete time

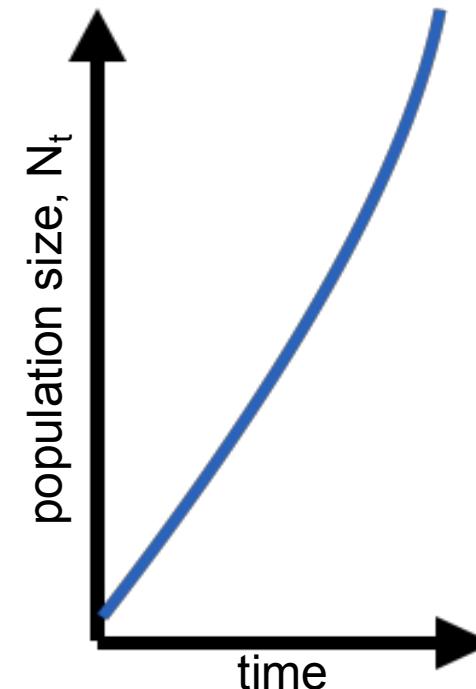


$$\begin{aligned} N_1 &= \lambda N_0 \\ N_2 &= \lambda[\lambda N_0] = \lambda^2 N_0 \\ N_3 &= \lambda^3 N_0 \\ N_t &= \lambda^t N_0 \end{aligned}$$

$\lambda$ =population rate of increase

(pop grows @  $\lambda > 1$  & declines @  $\lambda < 1$ )

Continuous time



$$r = \frac{\ln\left(\frac{N_t}{N_0}\right)}{t}$$

$$N_t = N_0 e^{rt}$$

$r$ =intrinsic (instantaneous) rate of increase  
(pop grows @  $r > 0$  & declines @  $r < 0$ )

# Geometric vs. exponential growth

geometric

$$N_t = \lambda^t N_0$$

exponential

$$N_t = N_0 e^{rt}$$

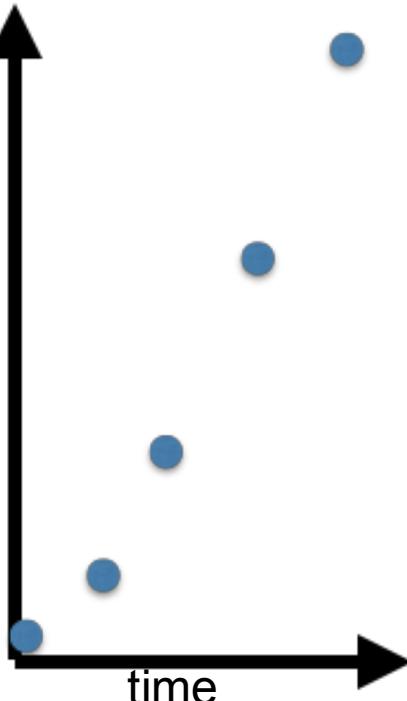
$$\lambda^t N_0 = N_0 e^{rt}$$

$$\lambda^t = e^{rt}$$

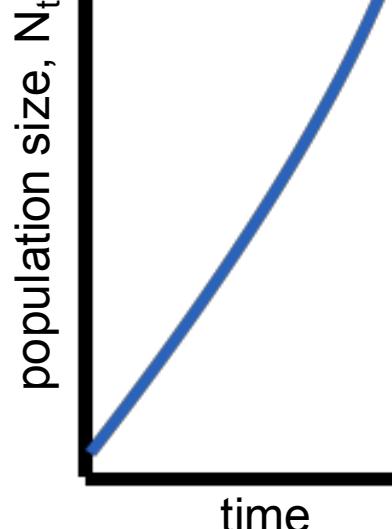
$$\lambda = e^r$$

Discrete time

population size,  $N_t$



Continuous time



# Geometric vs. exponential growth

geometric

$$N_t = \lambda^t N_0$$

exponential

$$N_t = N_0 e^{rt}$$

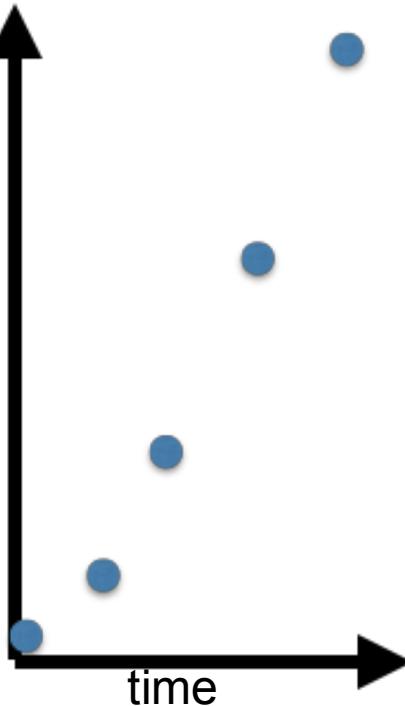
$$\lambda^t N_0 = N_0 e^{rt}$$

$$\lambda^t = e^{rt}$$

$$\lambda = e^r$$

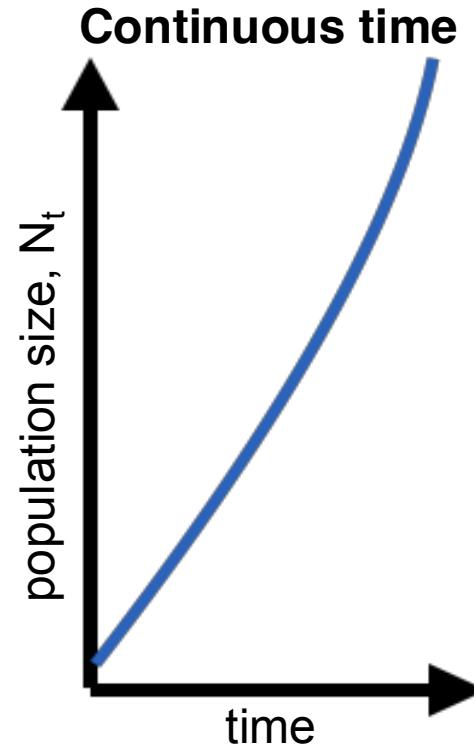
Discrete time

population size,  $N_t$



Continuous time

population size,  $N_t$



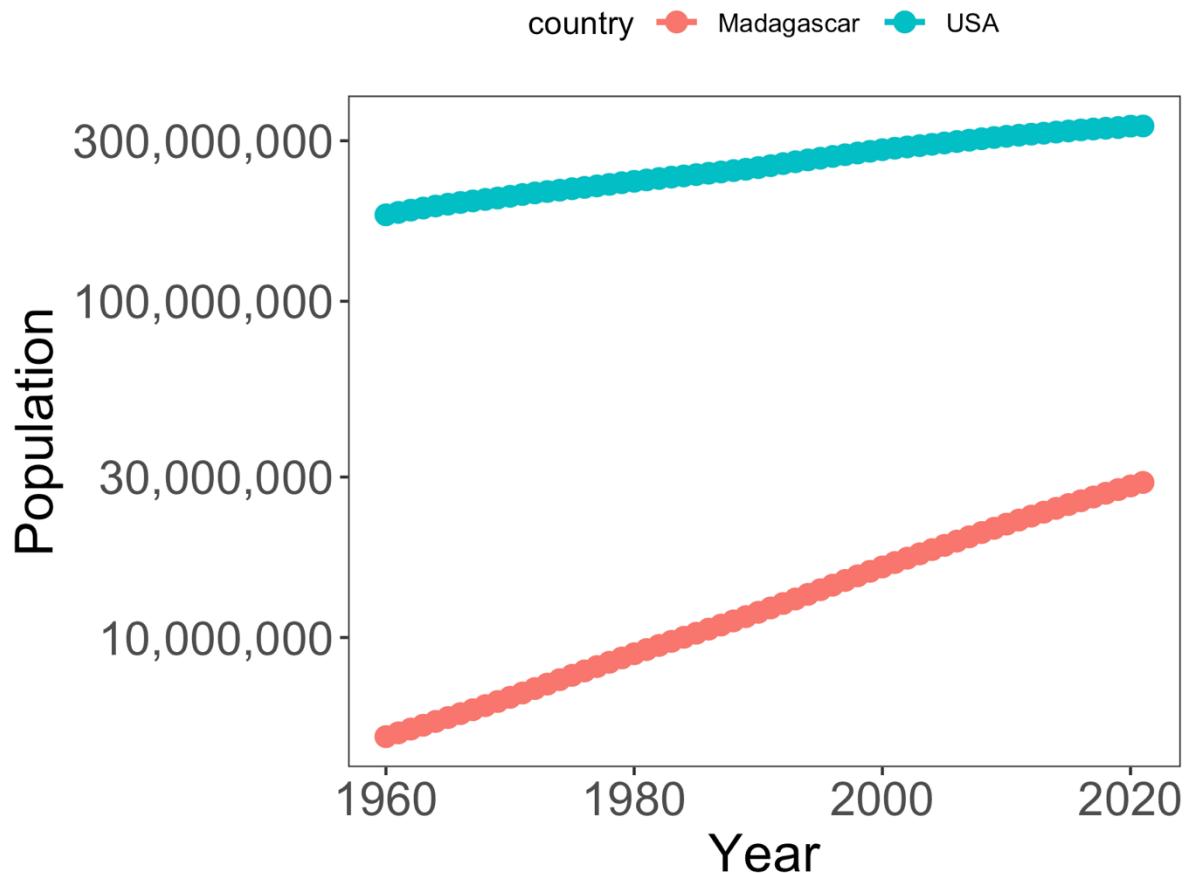
Continuous models can be discretized.

Discrete models can be approximated in continuous time.

How to choose what to do?

The answer depends on the data and the question at hand!

# Geometric vs. exponential growth

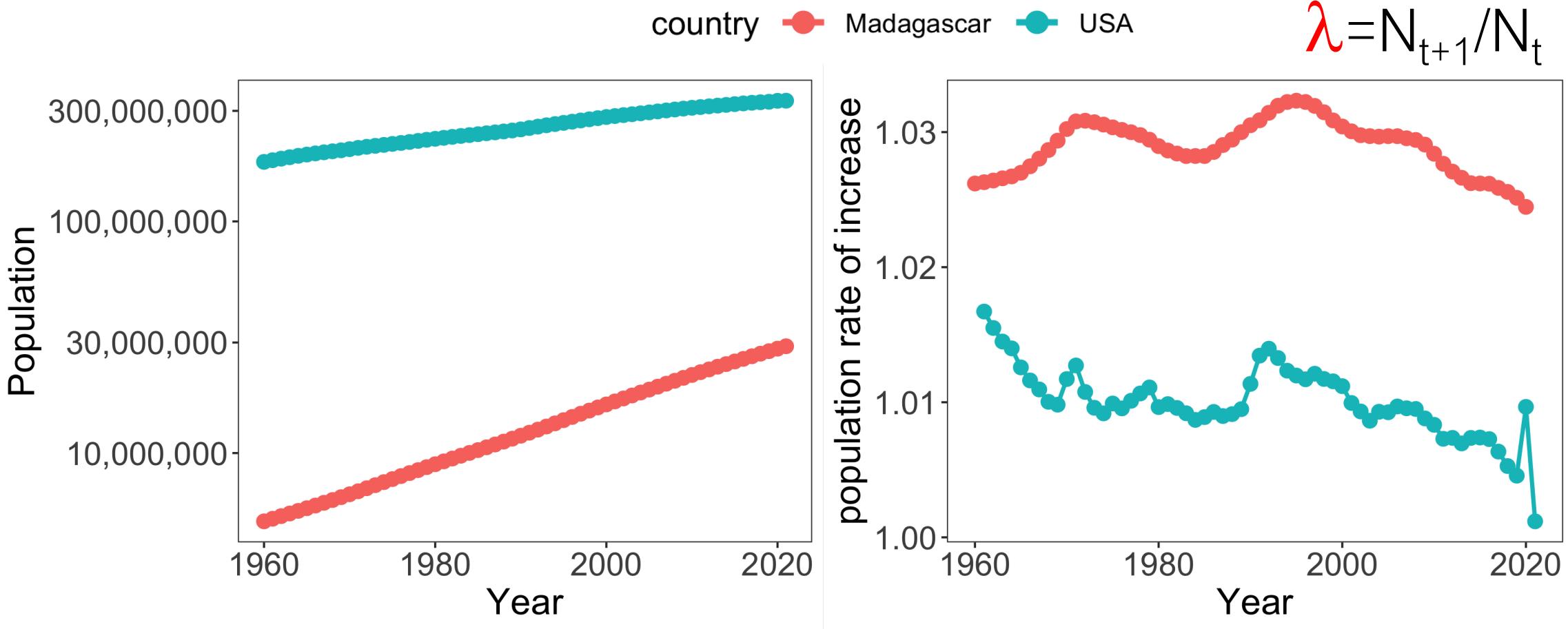


$$\lambda = N_{t+1} / N_t$$

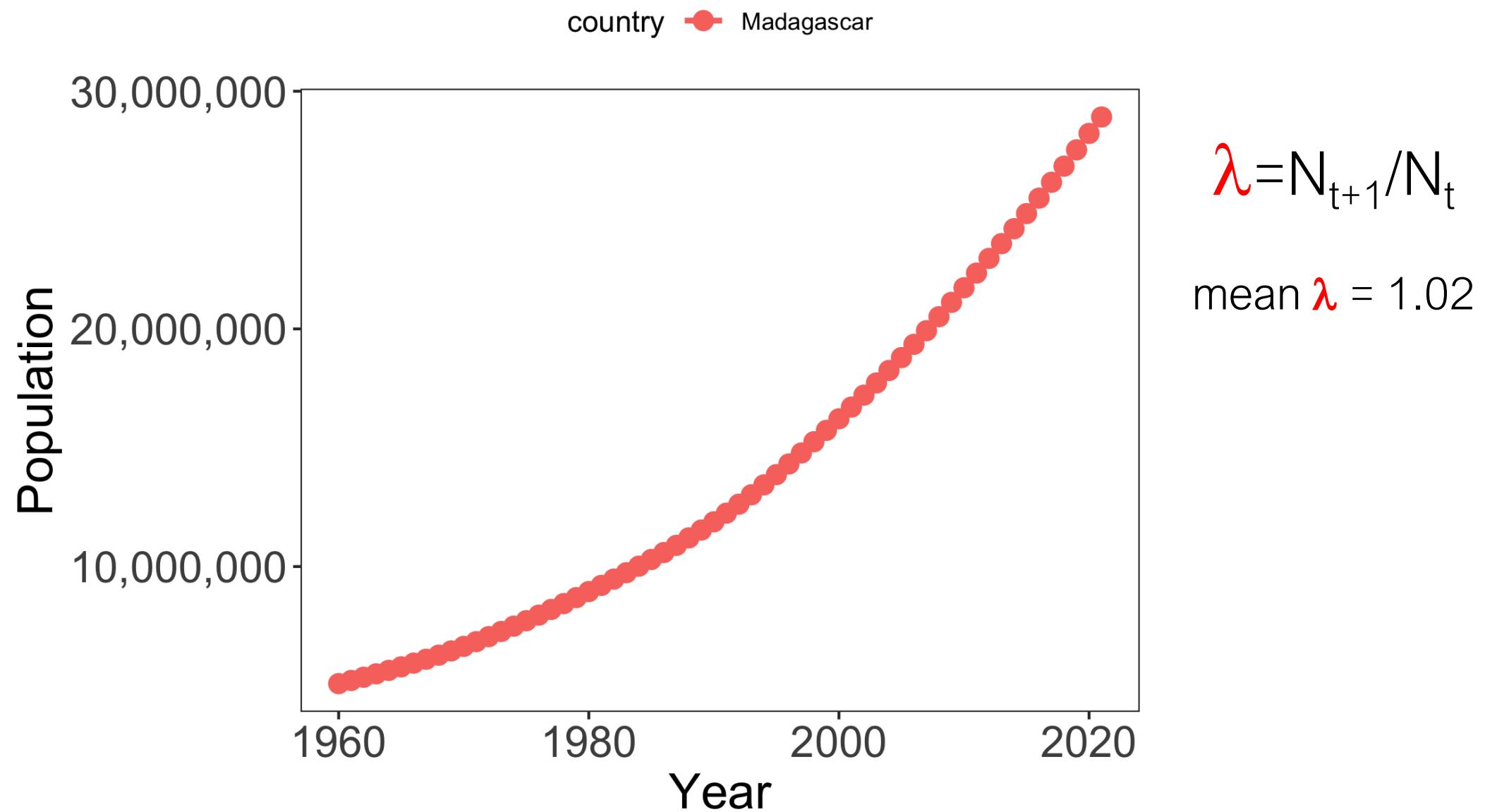
where  $t = 1$  year

Which country has the higher growth rate?

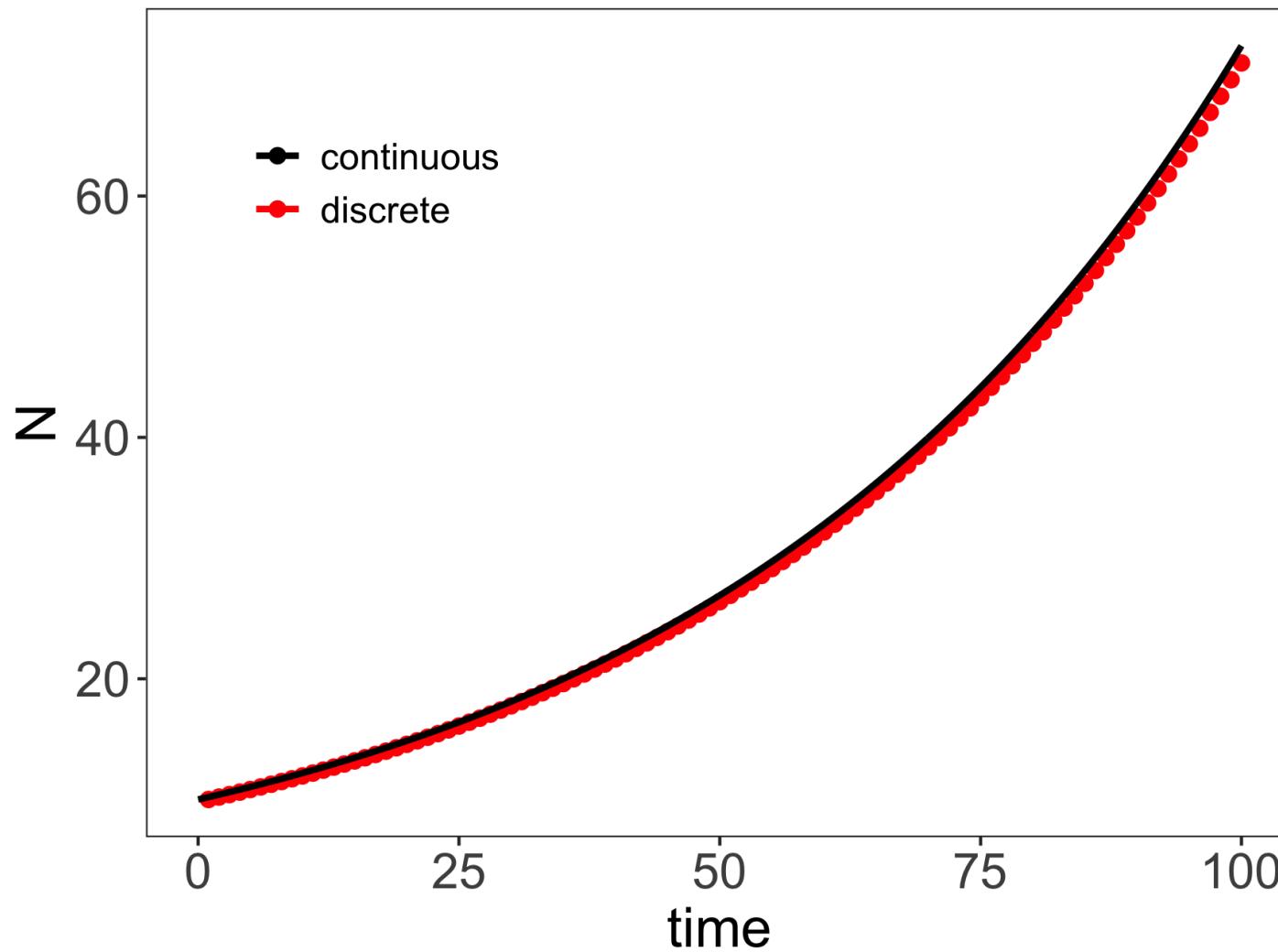
# Geometric vs. exponential growth



# Geometric growth approximation of Madagascar's population



# Projecting population under geometric vs. exponential growth

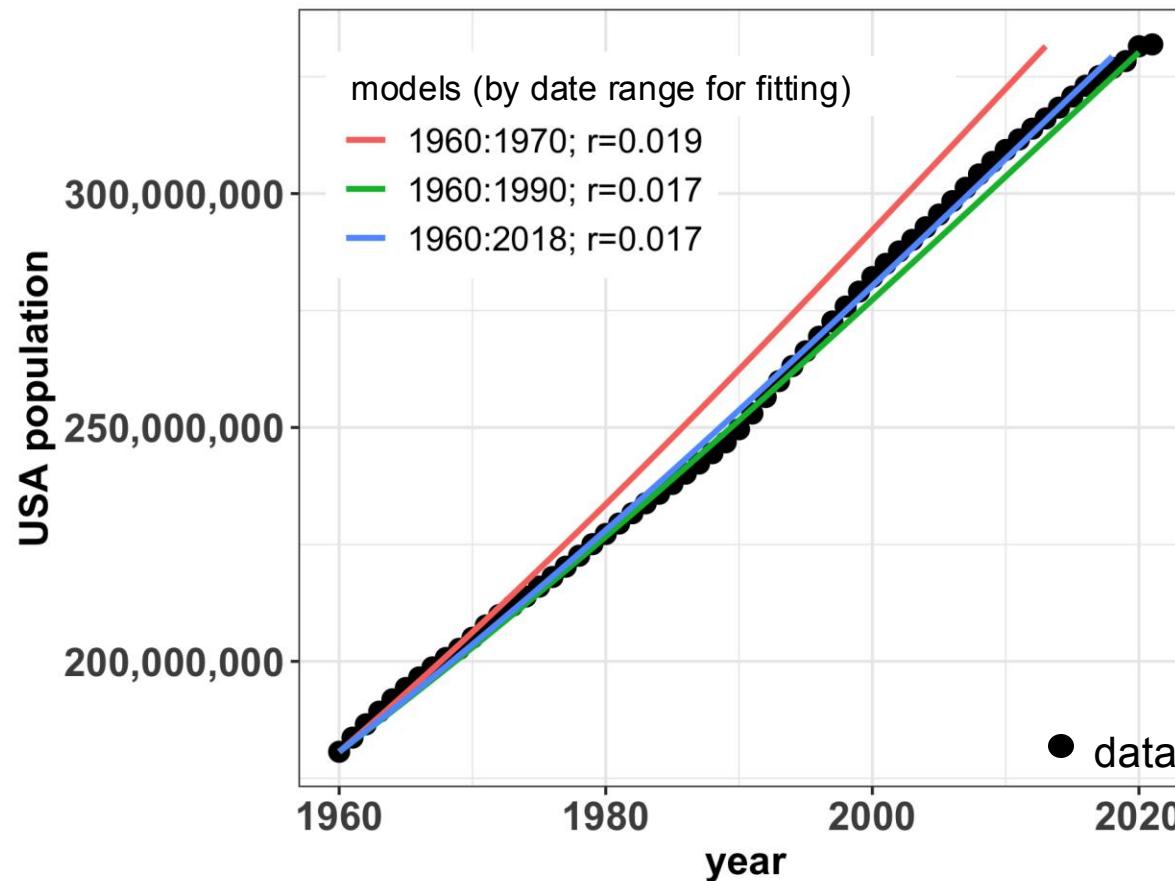


Geometric:  $\lambda = 1.02$

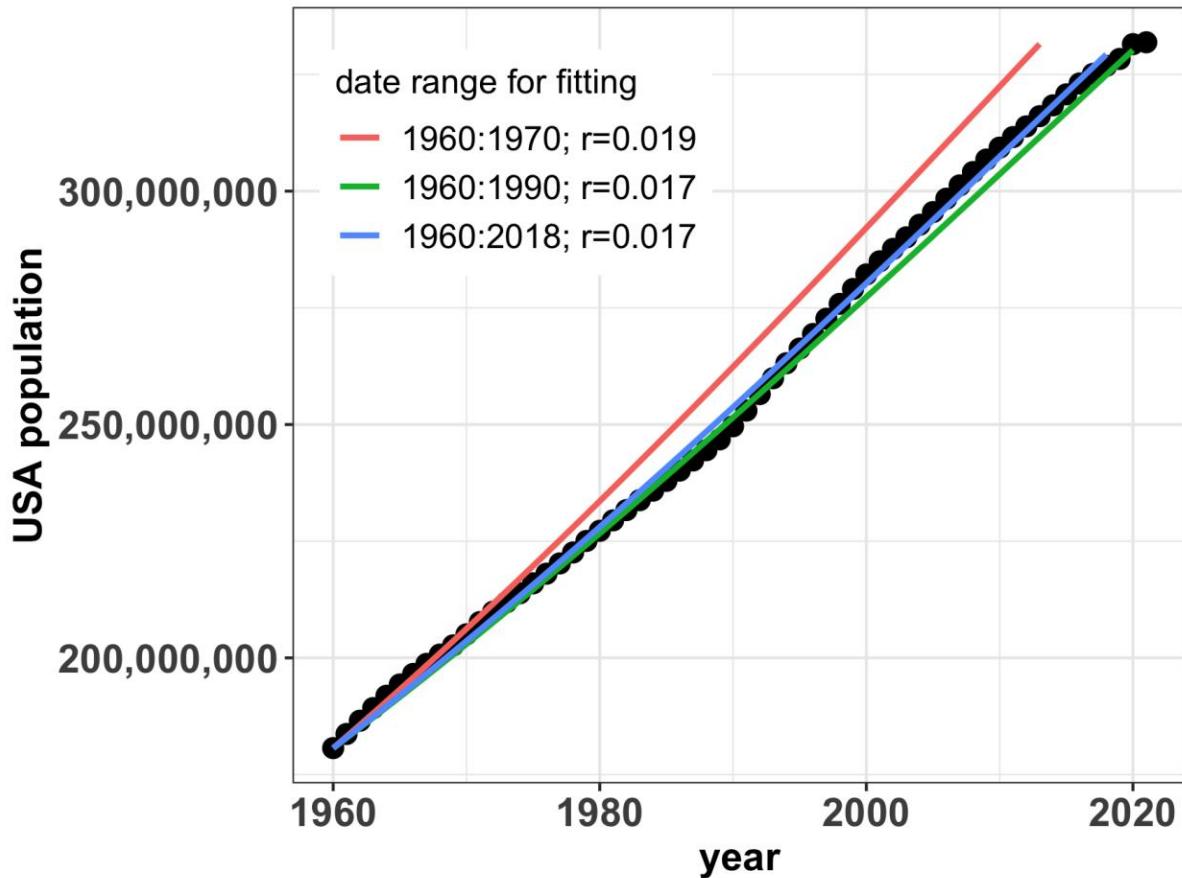
Exponential:  $r = \ln(1.02) = 0.02$

Models are similar over short time horizons and/or when discrete timesteps are small!

We can estimate growth rates by ‘fitting’ a model to data



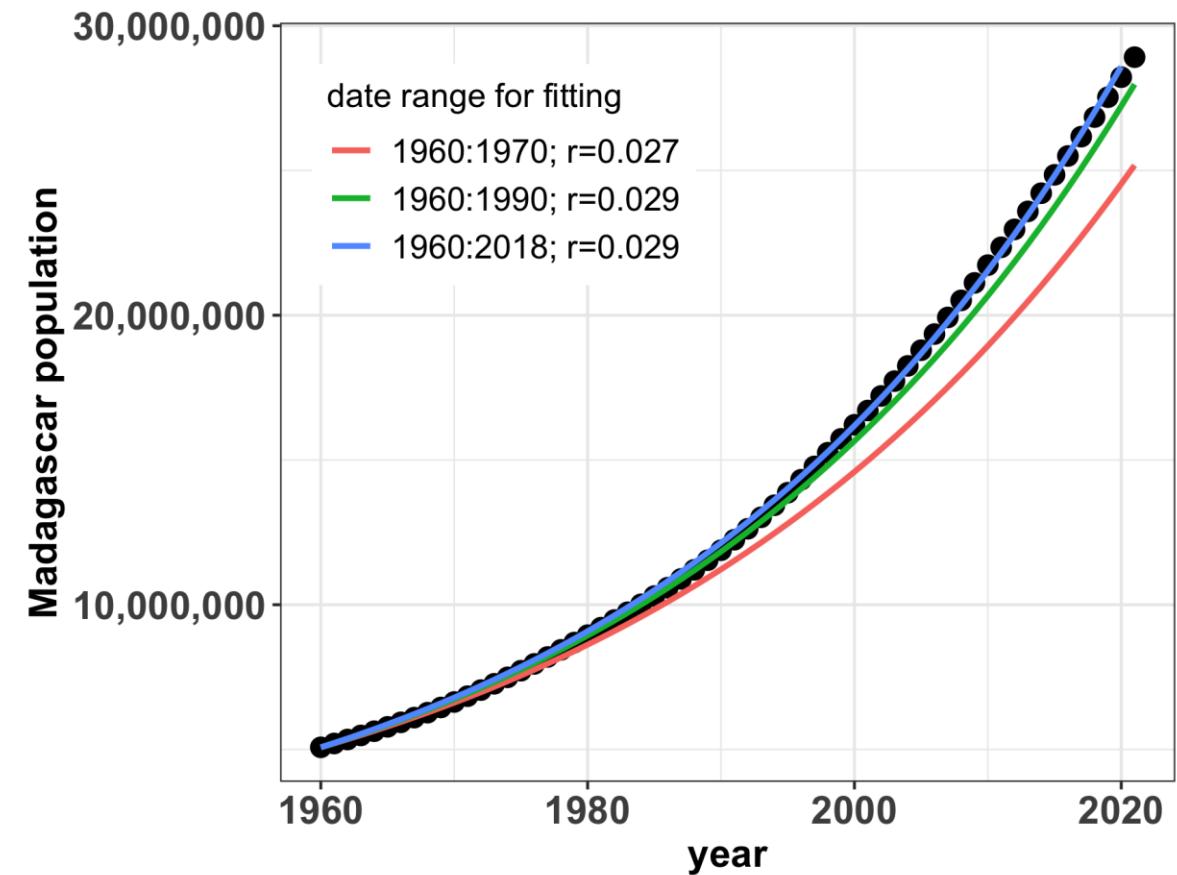
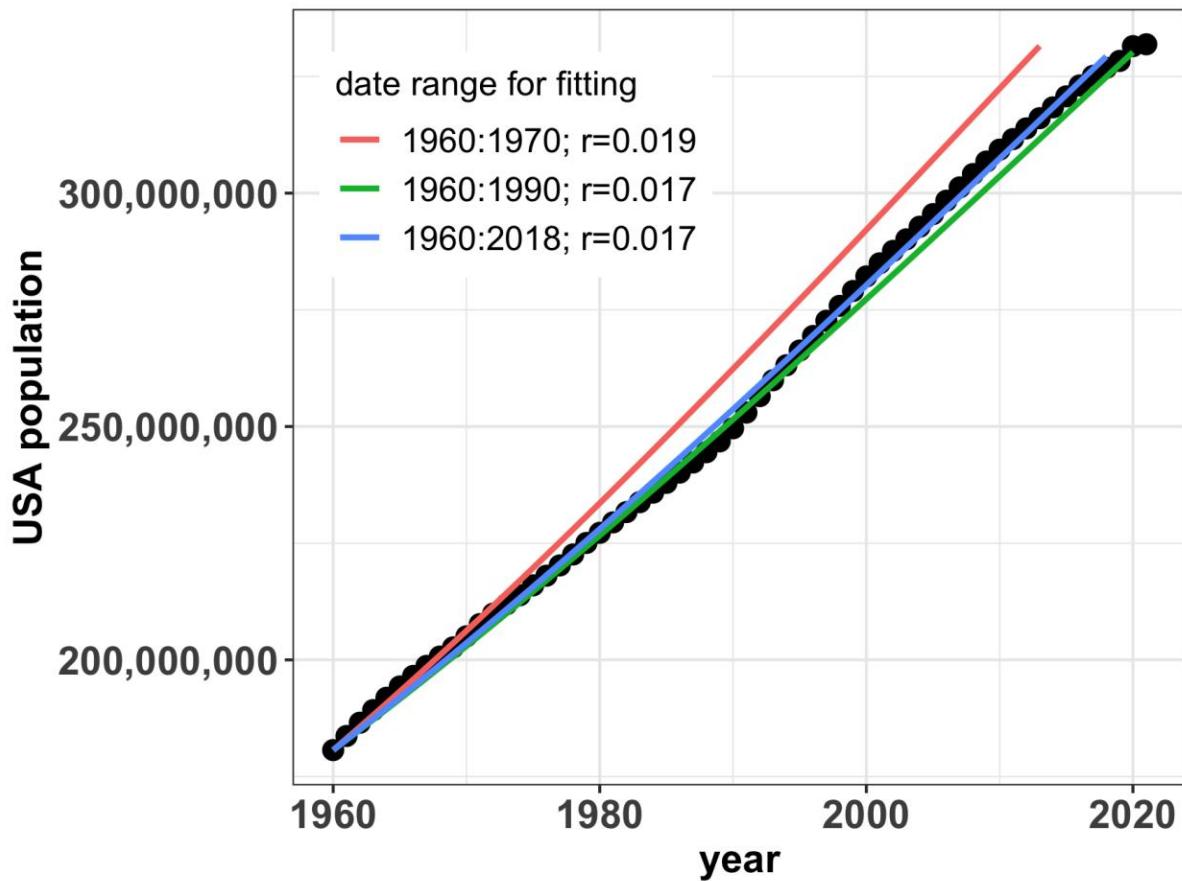
We can estimate growth rates by ‘fitting’ a model to data



US growth rates slowed over the time period!

Model fits to data from earlier years **overestimate** current population growth rates

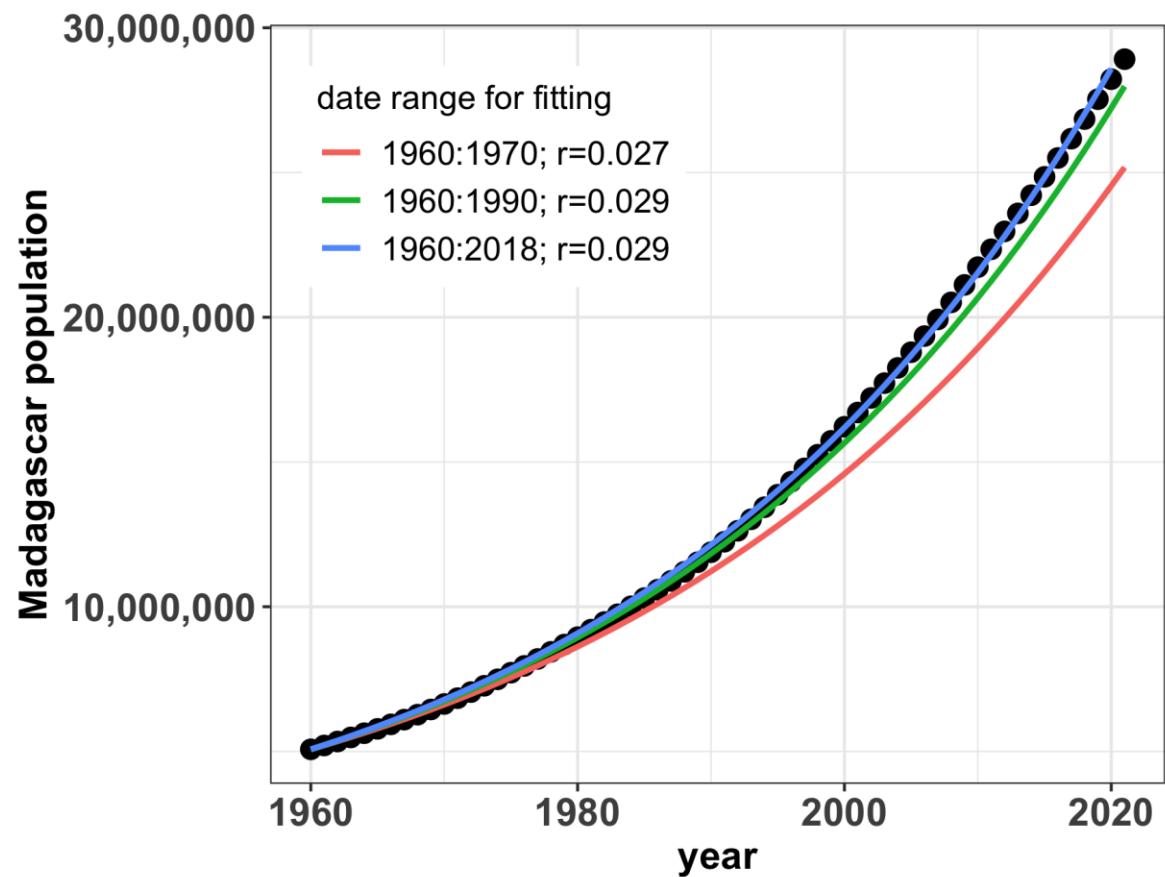
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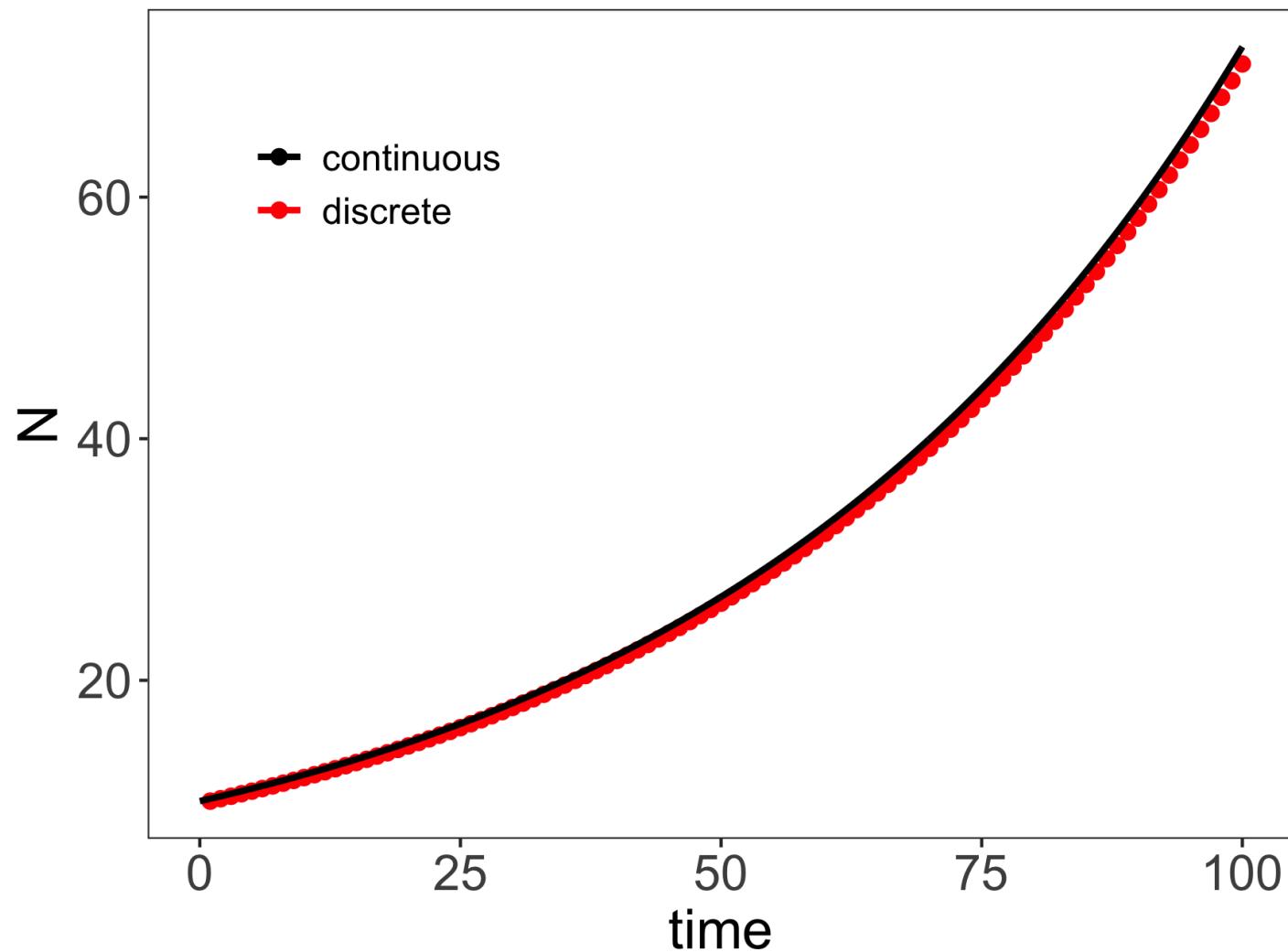
We can estimate growth rates by ‘fitting’ a model to data

Madagascar growth rates  
accelerated over the time  
period!

Model fits to data from earlier  
years **underestimate** current  
population growth rates



Both geometric and exponential growth are **unchecked**.



# Malthus proposed some limits to population growth

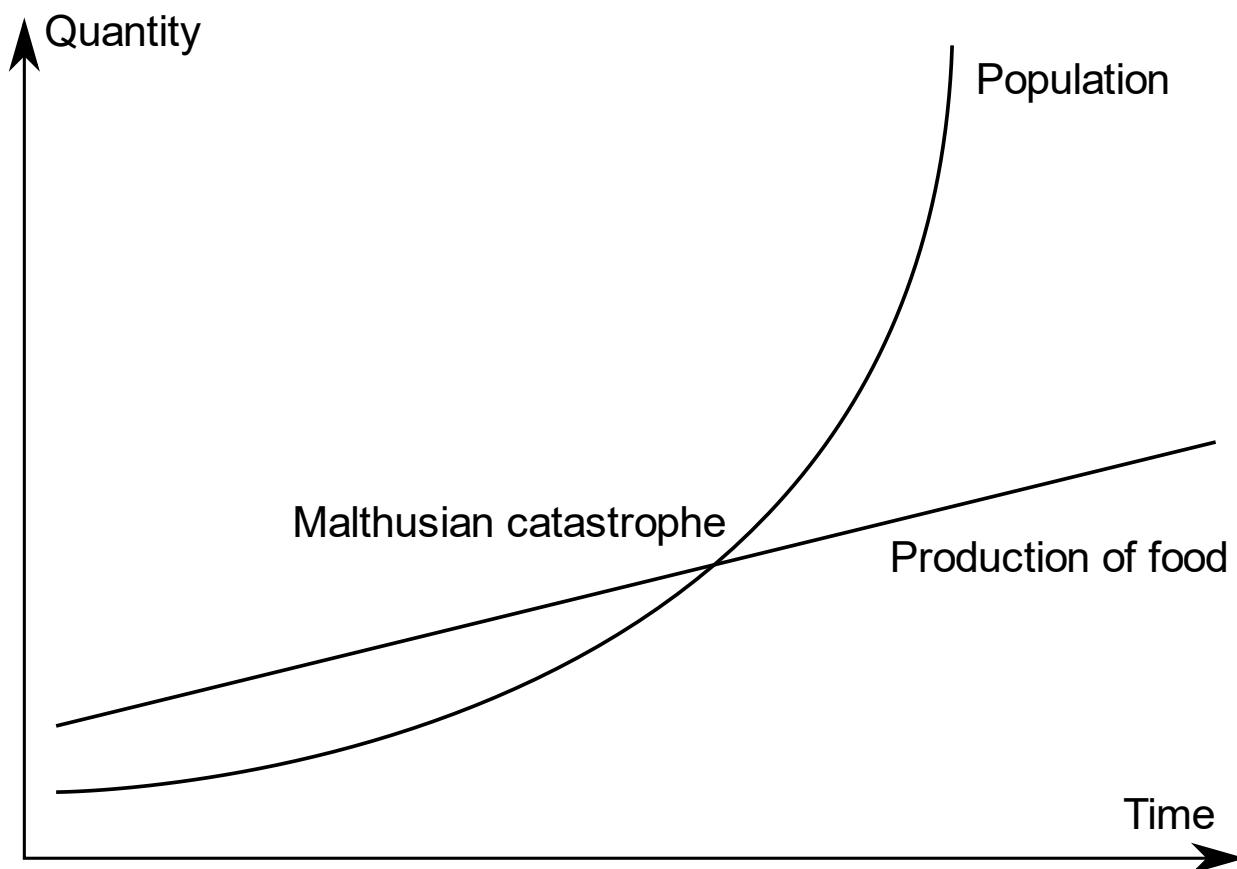
*[. . .] the power of population is indefinitely greater than the power in the earth to produce subsistence for man. Population, when unchecked, increases in a geometrical ratio. Subsistence increases only in an arithmetical ratio. A slight acquaintance with numbers will shew the immensity of the first power in comparison of the second. By that law of our nature which makes food necessary to the life of man, the effects of these two unequal powers must be kept equal. This implies a strong and constantly operating check on population from the difficulty of subsistence. This difficulty must fall somewhere; and must necessarily be severely felt by a large portion of mankind."*

- Thomas Malthus (1798)

*An Essay on the Principle of Population as it Effects the Future Improvement of Society, With Remarks on the Speculations of Mr Godwin, Mr. Condorcet and Other Writers*



# Malthus proposed some limits to population growth



**Problem!**  
No basis for  
assumption of  
arithmetical ratio  
for food.



- Thomas Malthus (1798)

*An Essay on the Principle of Population as it Effects the Future Improvement of Society,  
With Remarks on the Speculations of Mr. Godwin, Mr. Condorcet and Other Writers*

# Logistic growth equation

change in population  
abundance per unit time

$$\frac{dN}{dt} = rN \left(1 - \frac{N}{K}\right)$$

↑                      ↑

population size

intrinsic growth    **carrying capacity**  
rate

*"We shall not insist on the hypothesis of geometric progression, given that it can hold only in very special circumstances; for example, when a fertile territory of almost unlimited size happens to be inhabited by people..."*

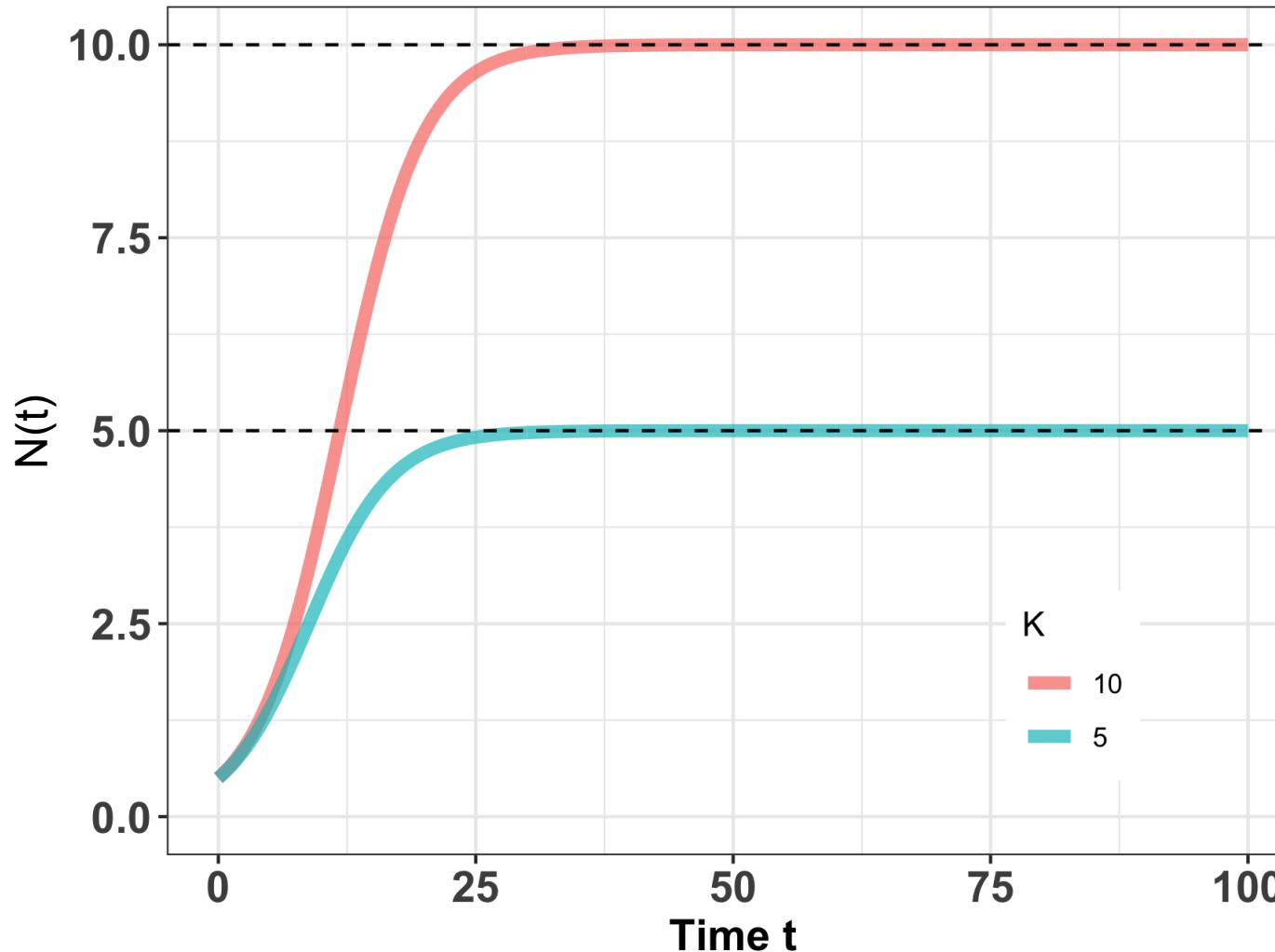
- Pierre-Francois Verhulst (1838)

Population growth slows as abundance (**N**)  
approaches **carrying capacity (K)**.

Population growth is **density-dependent**.

# Logistic growth equation

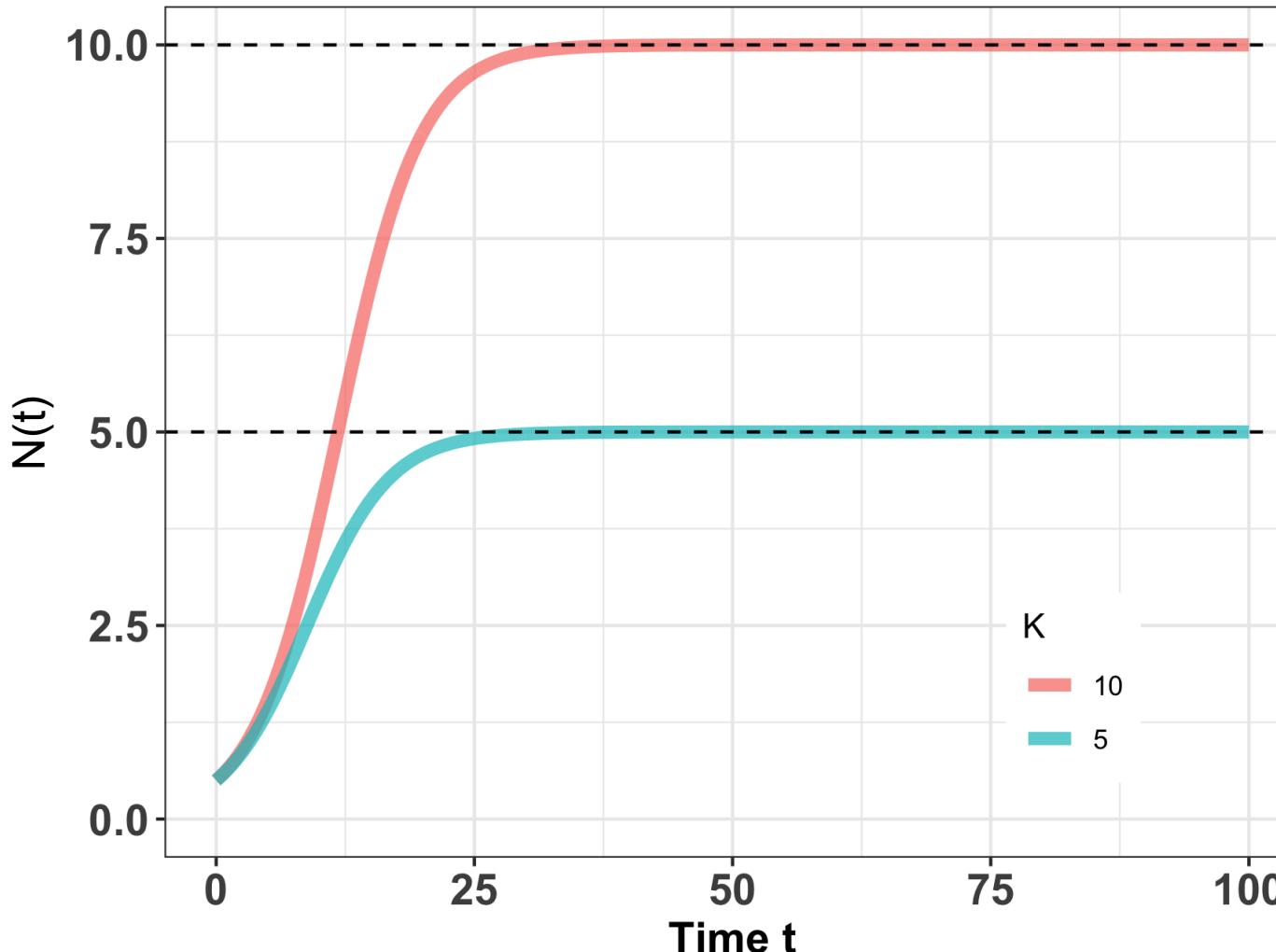
$$\frac{dN}{dt} = rN \left(1 - \frac{N}{K}\right)$$



Population growth slows to 0 as  $N$  approaches **K**, or – in other words – as the total population size approaches **carrying capacity**.

# Logistic growth equation

$$\frac{dN}{dt} = rN \left(1 - \frac{N}{K}\right)$$



**Carrying capacity=**  
maximum population size  
an environment can  
sustain indefinitely.

Ecology = organisms  
interacting with each other  
and the **environment**

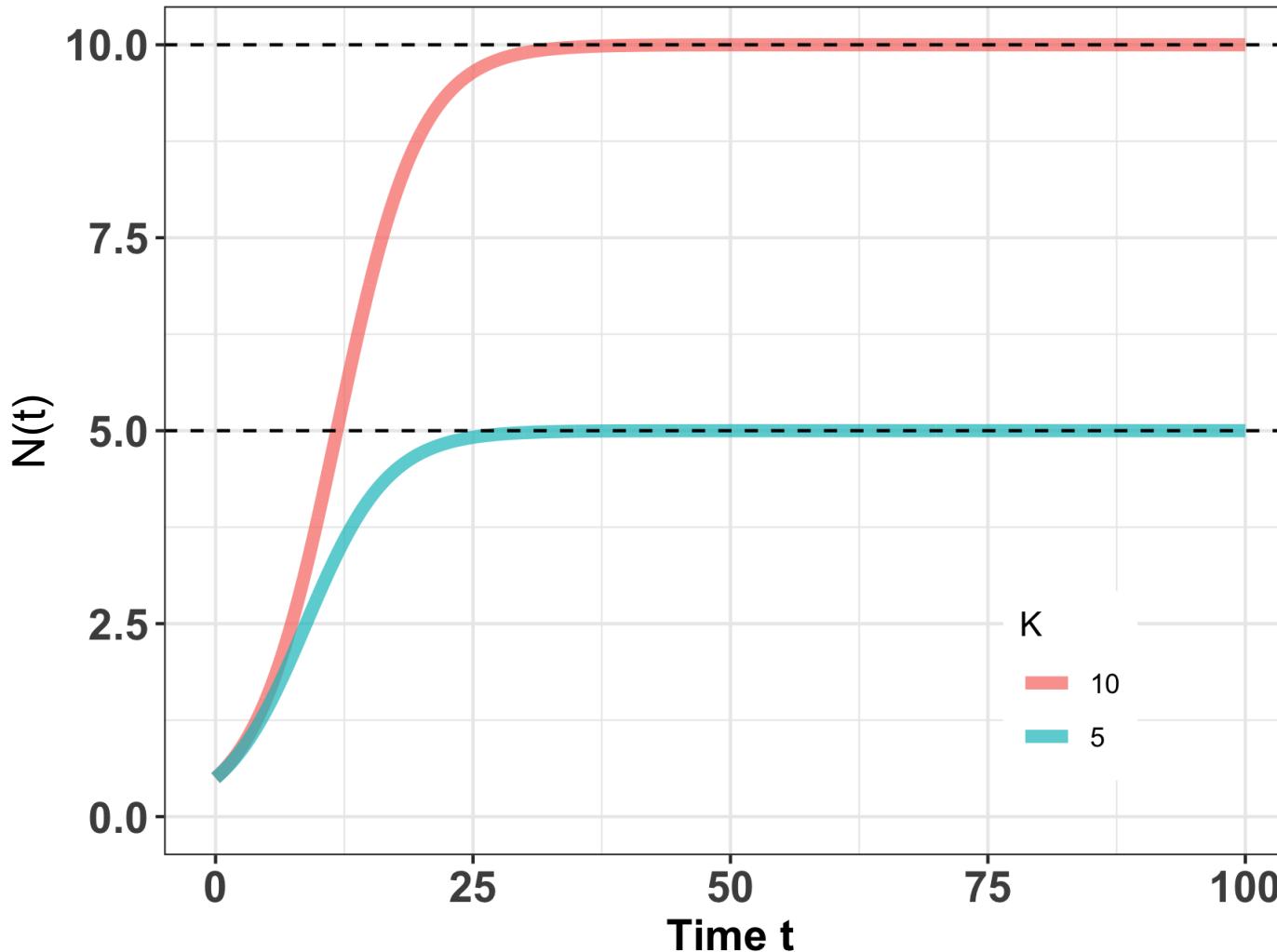
**K** can change!

'r' vs. '**K**'-selected species

# Logistic growth and equilibrium

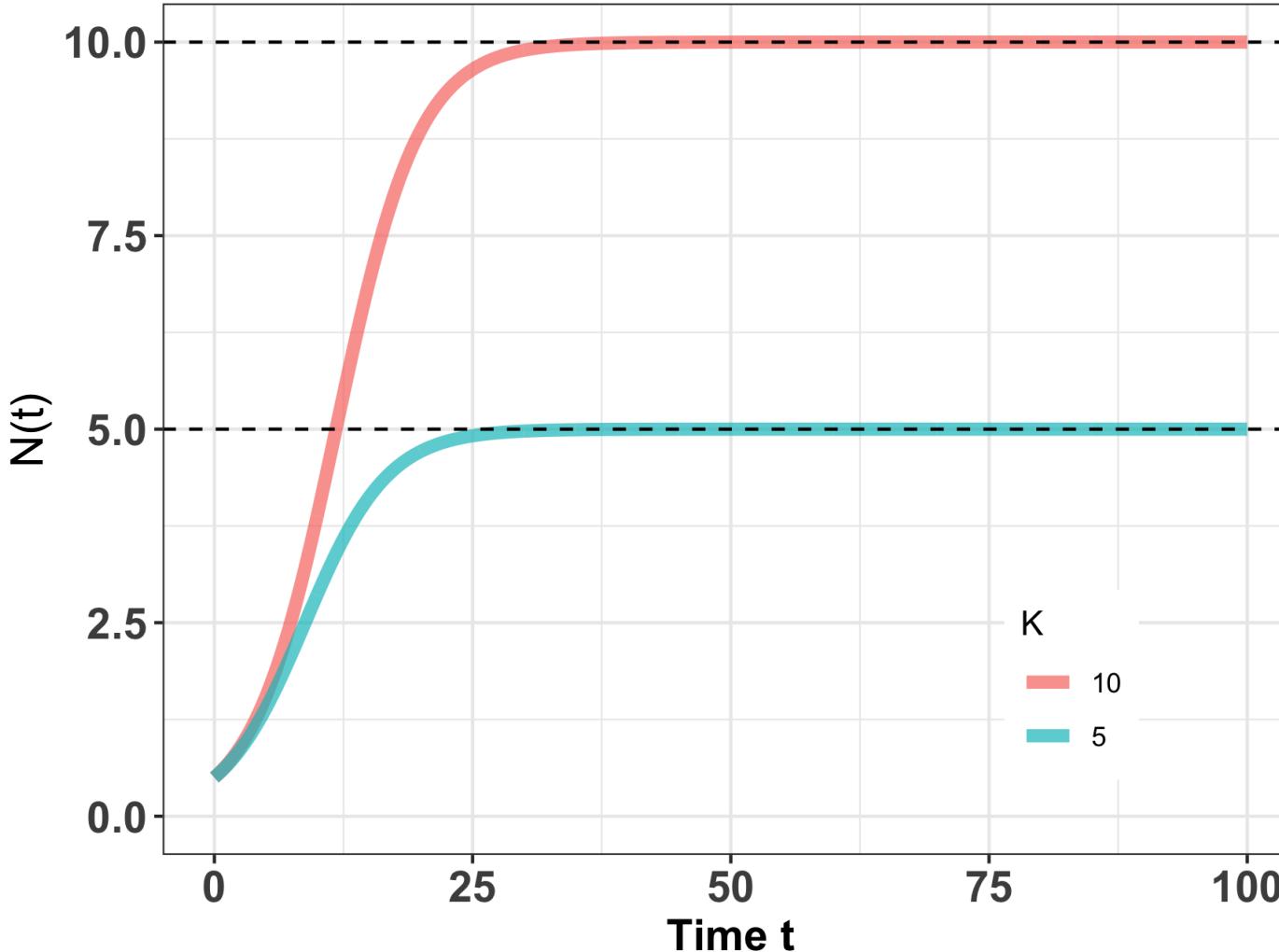
$$\frac{dN}{dt} = rN \left(1 - \frac{N}{K}\right)$$

$$\frac{dN}{dt} = 0$$



When population size is not changing, the population is said to be at **equilibrium**.

# Logistic growth and equilibrium



$$\frac{dN}{dt} = rN \left(1 - \frac{N}{K}\right)$$

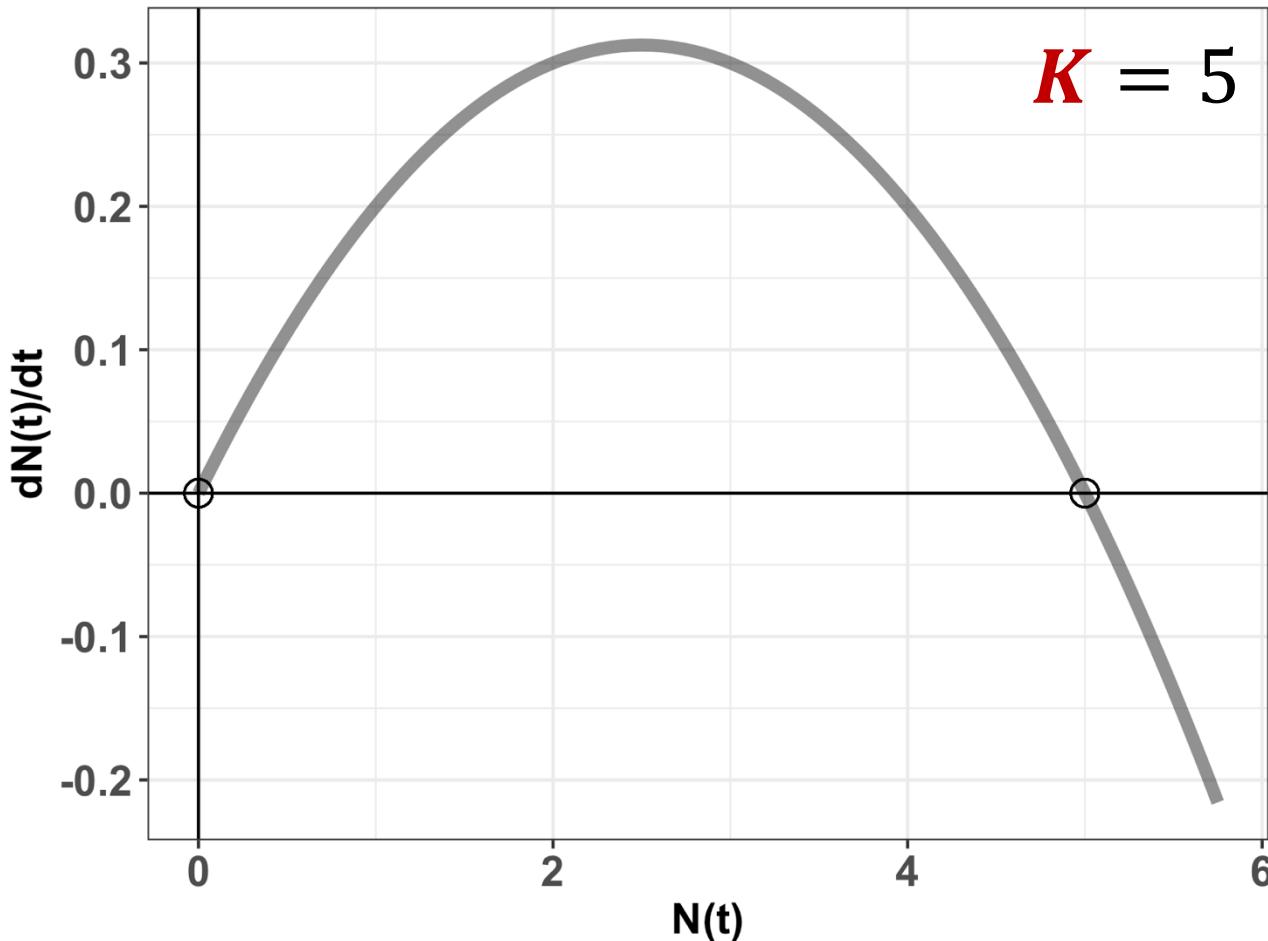
$$\frac{dN}{dt} = 0$$

$$0 = rN \left(1 - \frac{N}{K}\right)$$

$$N = K$$

A little algebra shows that the population at carrying capacity is at **equilibrium**.

# Logistic growth and equilibrium



What is  $K$ ?

$$\frac{dN}{dt} = rN \left(1 - \frac{N}{K}\right)$$

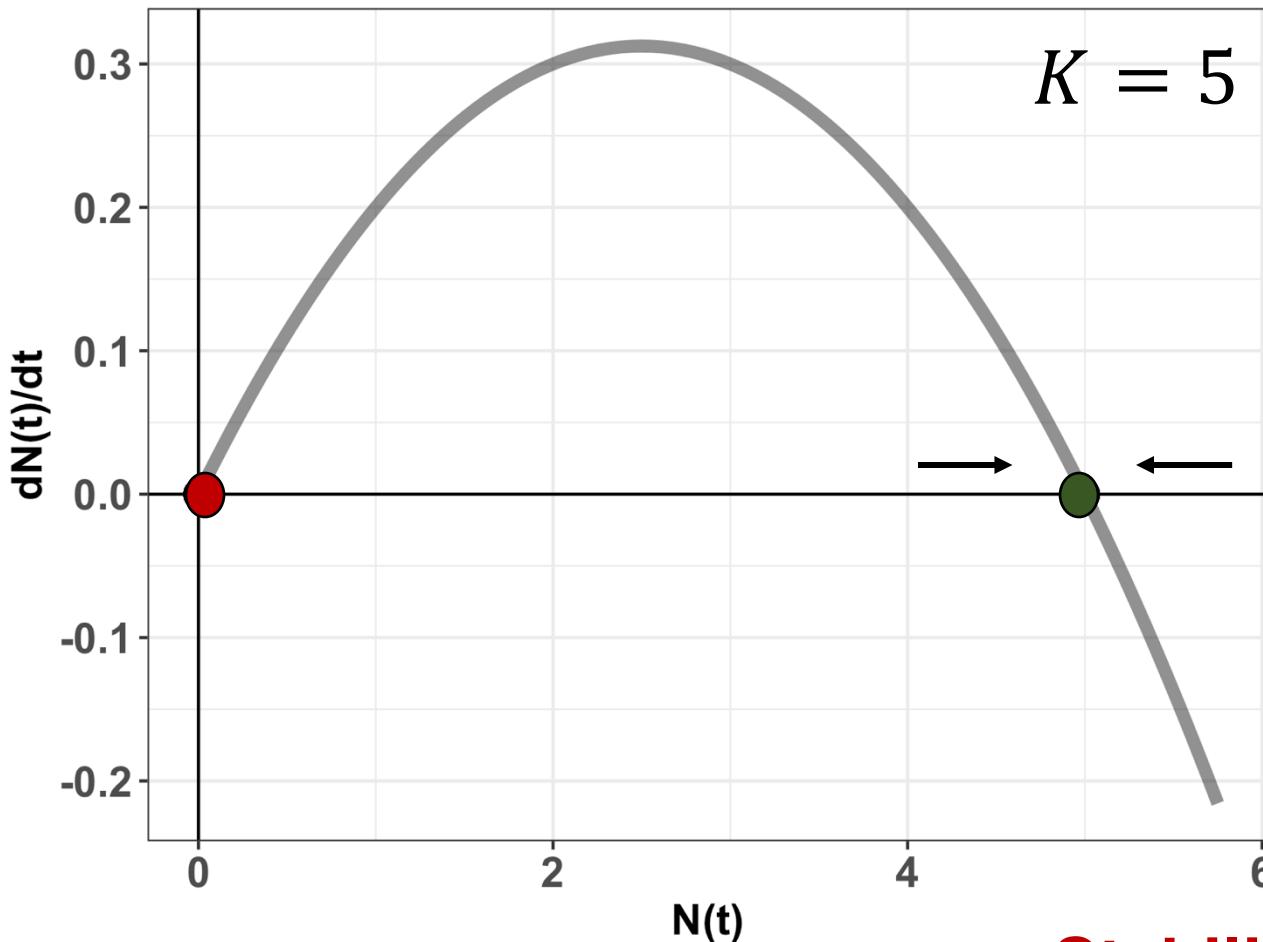
$$\frac{dN}{dt} = 0$$

$$0 = rN \left(1 - \frac{N}{K}\right)$$

$N = K$  or  
 $N = 0$

logistic  
growth  
**equilibria**

# Logistic growth and equilibrium



$$\frac{dN}{dt} = rN \left(1 - \frac{N}{K}\right)$$

$$\frac{dN}{dt} = 0$$

$$0 = rN \left(1 - \frac{N}{K}\right)$$

$N = K$  or  
 $N = 0$

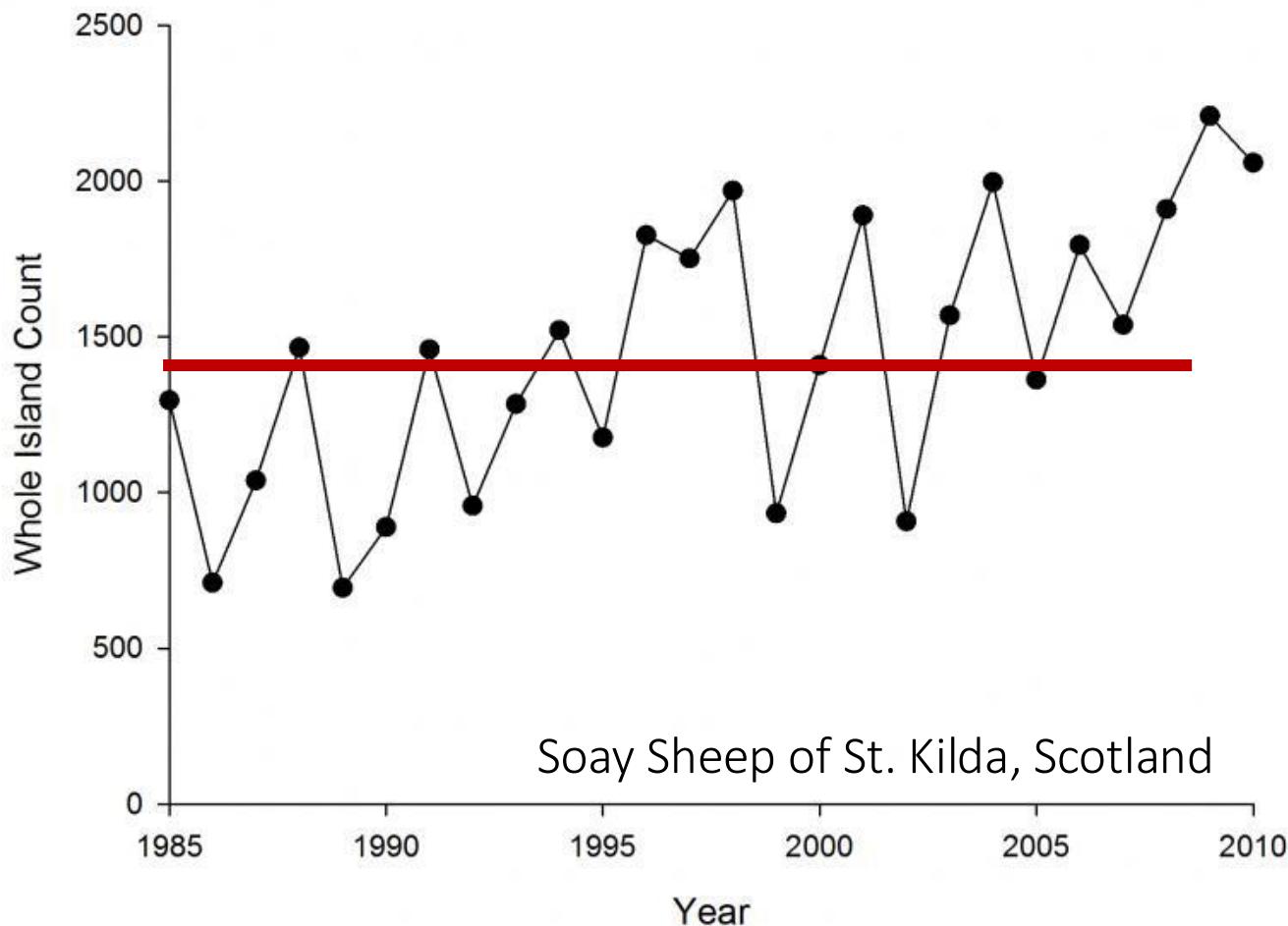
logistic  
growth  
**equilibria**

**Stability:** If the population is perturbed, will it return to equilibrium?

Many populations will fluctuate above or below carrying capacity.



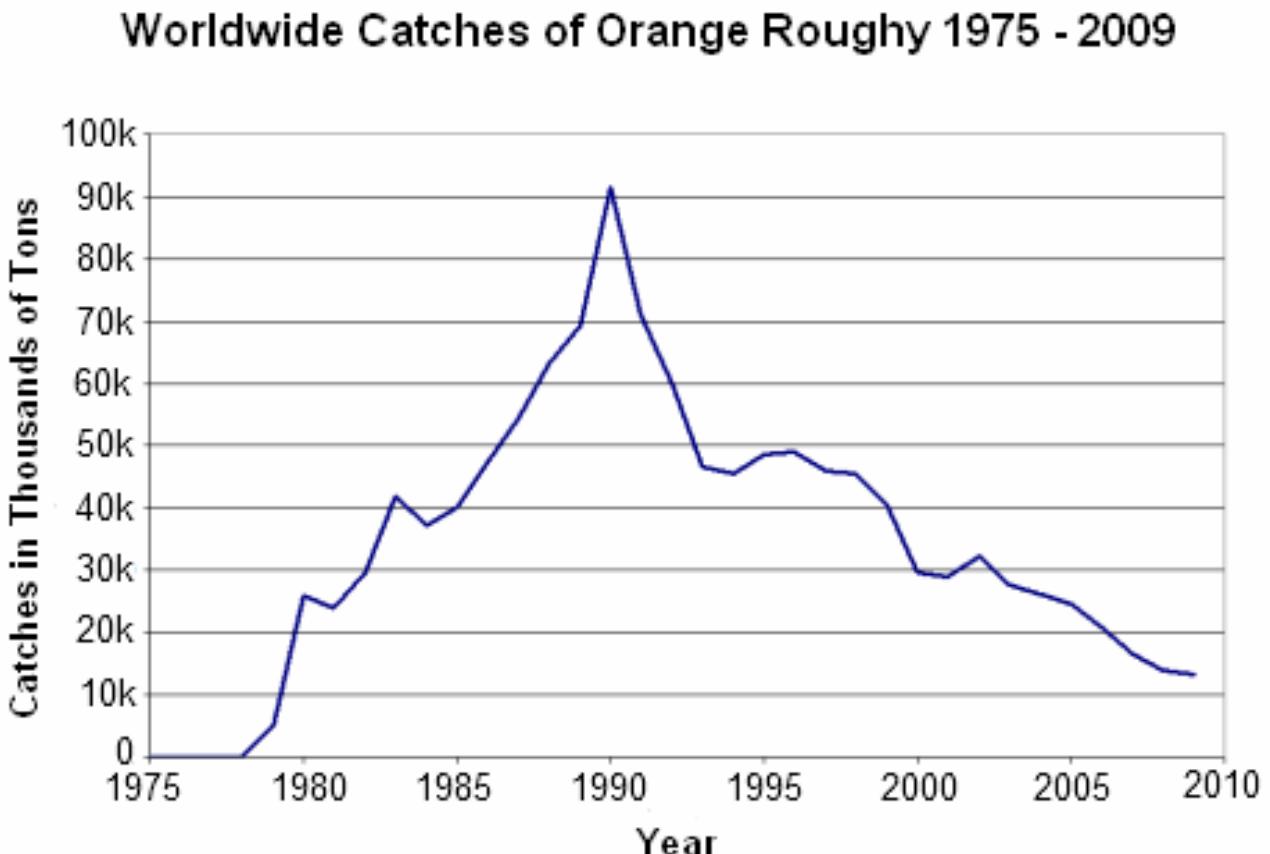
Many populations will fluctuate above or below carrying capacity.



But they can still be stable populations if they return to **equilibrium**.

In some cases, it is not possible to recover.

Many populations will fluctuate above or below carrying capacity.



Source: FAO (Fisheries and Agriculture Organisation of the United Nations) Fisheries and Aquaculture Information and Statistics Service. © L. Baumont



But they can still be stable populations if they return to **equilibrium**.

In some cases, it is not possible to recover.

Logistic growth still does not describe human populations well.

