Fundamentals of Ecology

Week 5, Ecology Lecture 1

Cara Brook

February 4, 2025

Where are we headed?

Week	Mon/Tues Lab	Tues Lecture	Thurs Lecture
1	--	Jan 7: Evolution Intro	Jan 9: Genetic Variation
2	1: Intro R	Jan 14: Evolution without selection: Genetic drift & migration	Jan 16: Natural selection 1
3	2: Hardy-Weinberg	Jan 21: Natural Selection 2	Jan 23: Sexual selection & genetic conflicts
4	3: Microevolution	Jan 28: Speciation	Jan 30: Phylogenetics & biodiversity
5	4: Phylogenetics	Feb 4: Ecology & Population Growth	Feb 6: Single Species Population Growth & Regulation
6	5: Population Growth	Feb 11: Species Interactions 1	Feb 13: Midterm
7	6: Population Regulation	Feb 18: Species Interactions 2	Feb 20: Disease Dynamics as Population Biology 1
8	7: Predation & Competition	Feb 25: Disease Dynamics as Population Biology 2	Feb 27: Community Assembly & Island Biogeography
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Cum Commentariis & adnotationibus HERMOLAI
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Item JOH. FR. GRONOVII Notarum Liber Singularis
ad Illustrem Virum *Johannem Capelanum*.

GD. BATAV. TERODAMI. } Apud HACKIOS, A°. 1669.

Natural history is the observational study of the living things on planet Earth.

Is natural history science?

Is natural history science?

Science: the **systematic observation** of natural events and conditions in order to discover facts about them and to **formulate laws and principles** based on these facts.

– *Academic Press Dictionary of Science & Technology*


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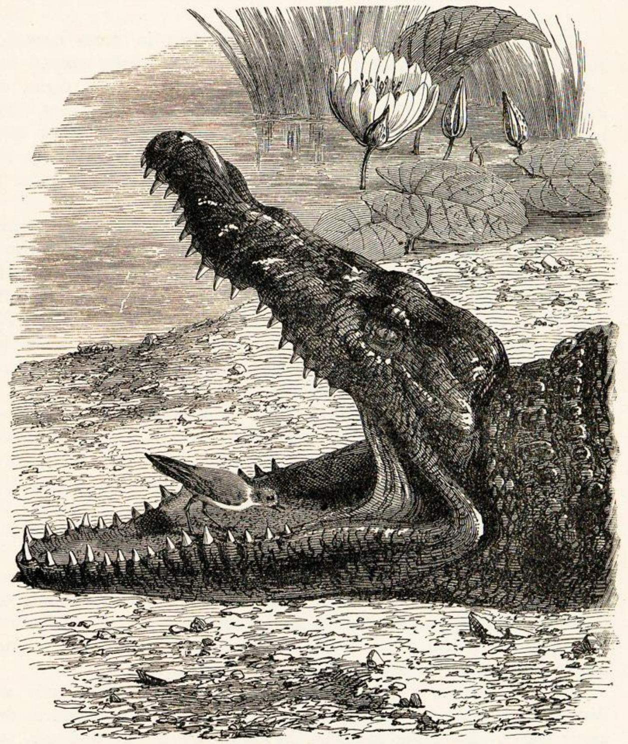
MOVENDO

GD. BATAV. TERODAMI. } Apud HACKIOS, A°. 1669.

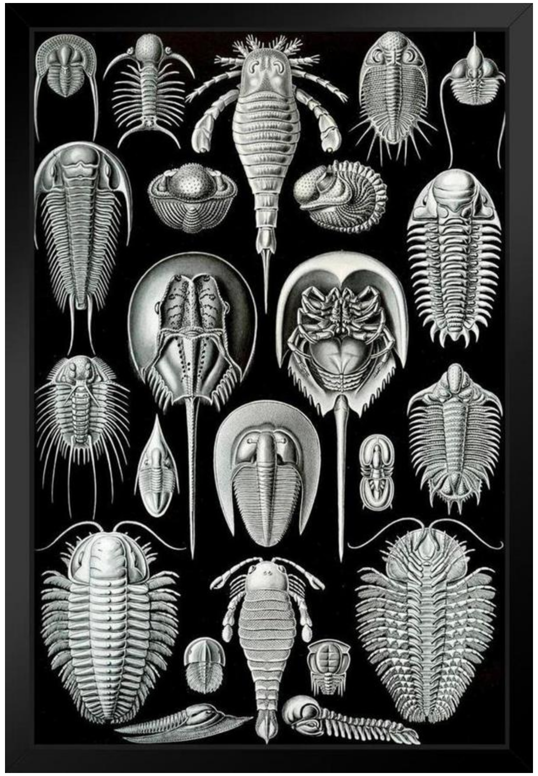
Natural history gave way to the **natural sciences** when those observations became **systematic** and were used to support **general laws and principles** describing the natural world.

Ecology is the study of the **interactions** of **organisms** with each other and their **environment**.



THE CROCODILE'S FRIEND.

Ecology is the study of the **interactions** of **organisms** with each other and their **environment.**



oikos = 'house'

\+ *logia* = 'study of'

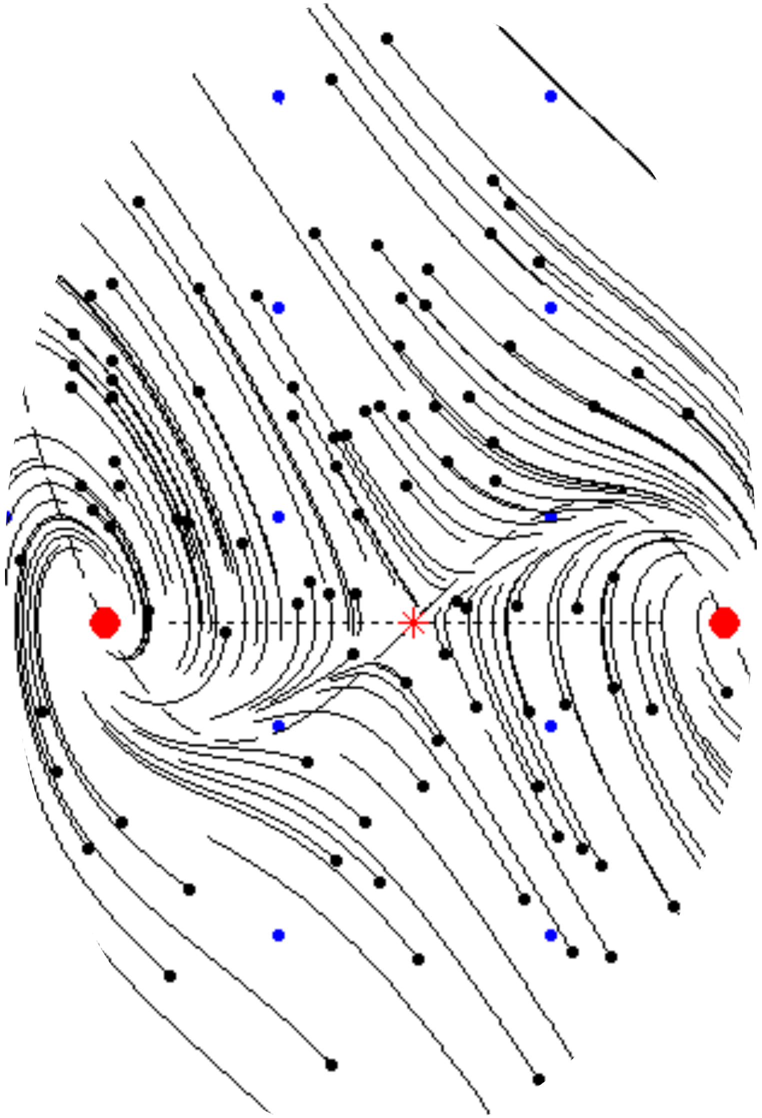
ecology

\- 1866 Ernst Haeckel

“**Ecology** has a synonym which is **ALL**.”

-John Steinbeck

The Log from the Sea of Cortez (1941)



Ecology is the study of the **interactions** of **organisms** with each other and their **environment**.

As a science, ecology uses **models** to formalize general **laws and principles** describing the natural world.

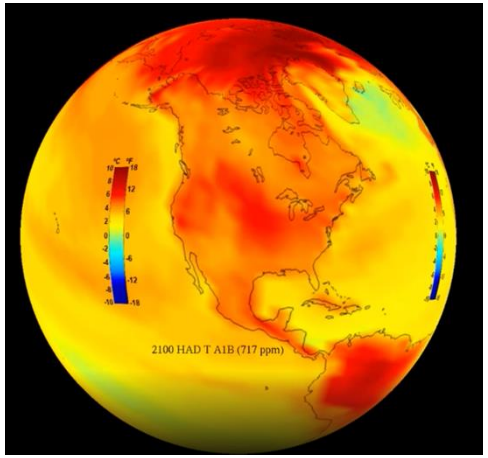
What is a model?

an abstract representation of a phenomenon

Human

Solar System

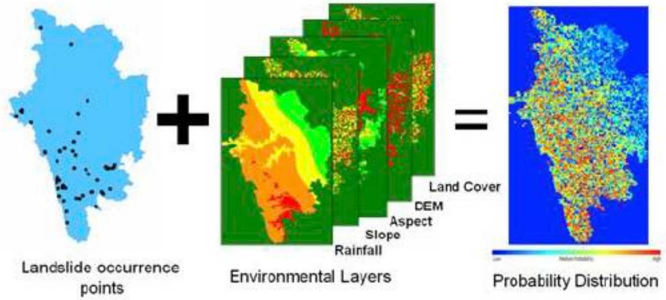
Climate



Human Genetics

Human Disease

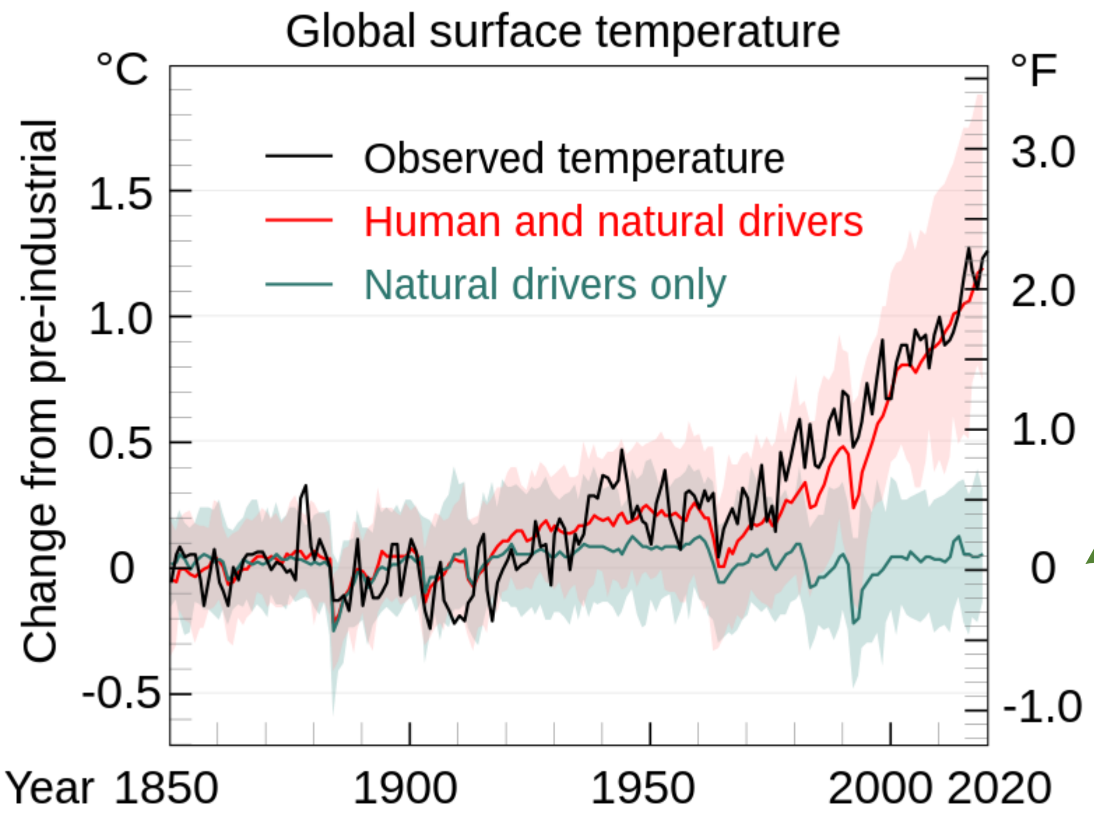
Species Distribution



(IPCC 2007)

Why build models?

to explain and predict



explain

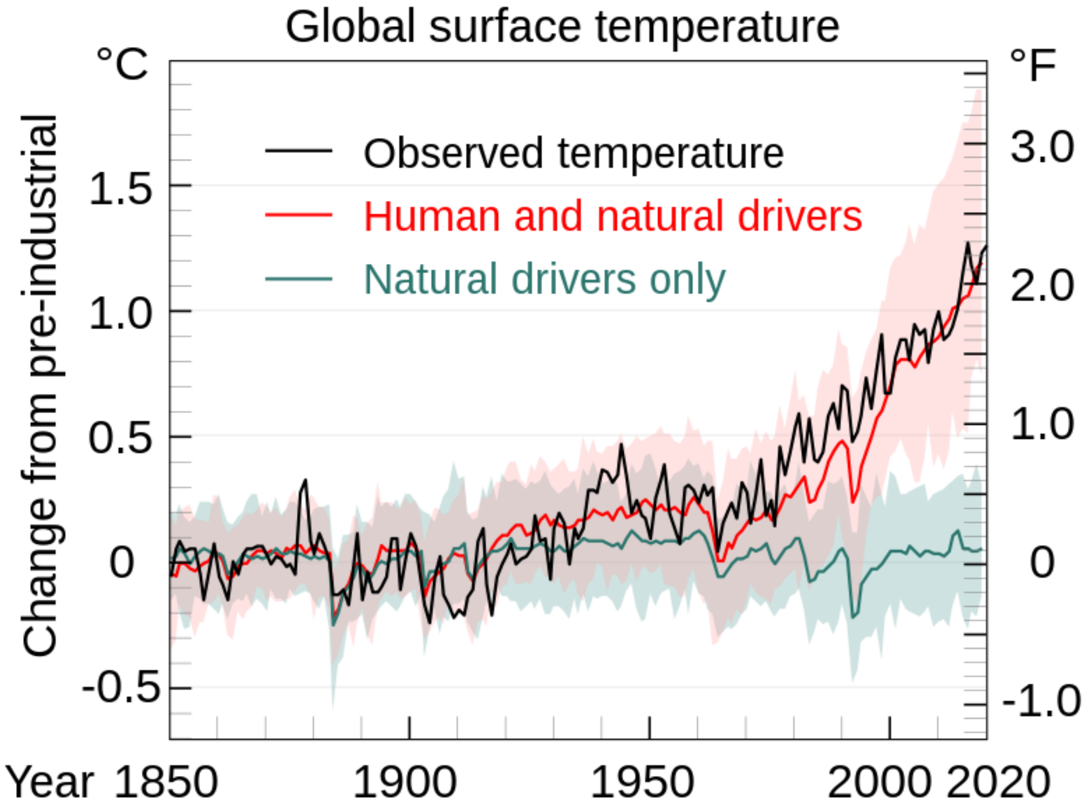
Models produce 'simulated data' that is compared against the observed data.

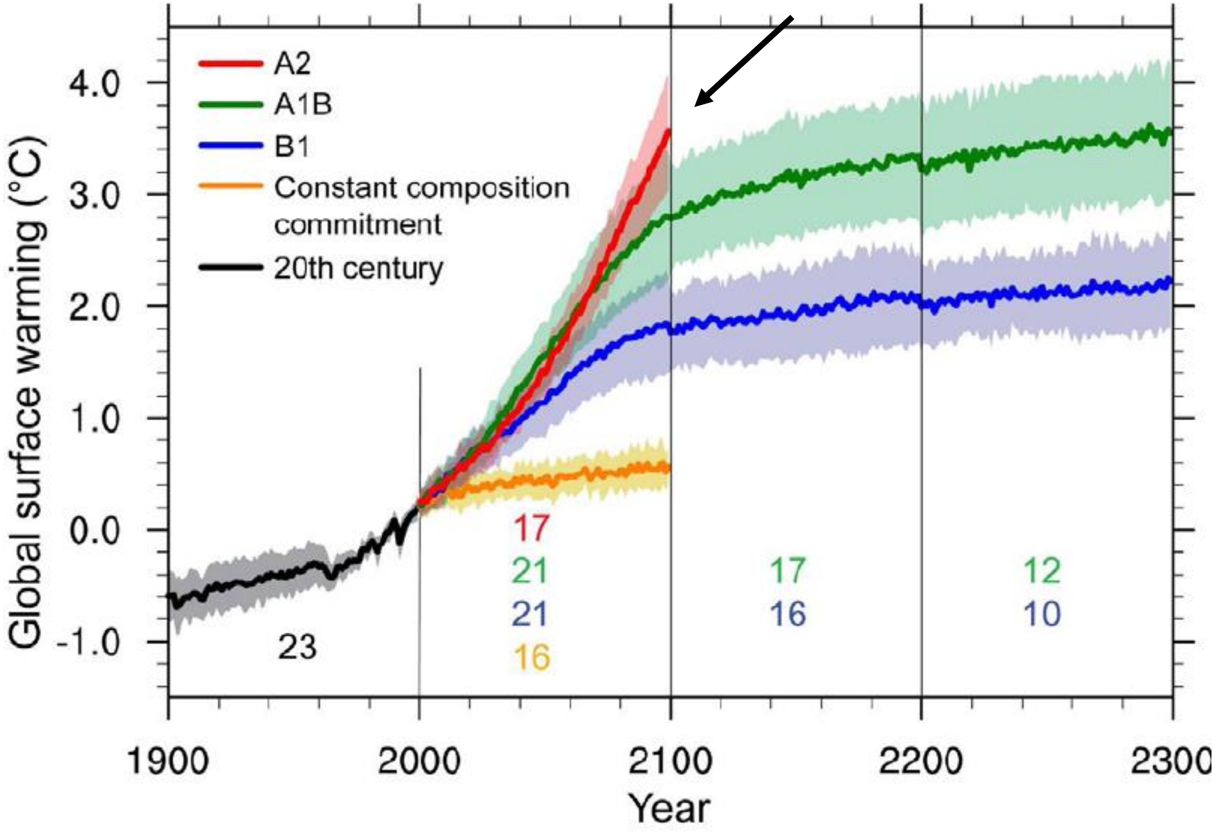
The process of adjusting the model to more closely match the data is called 'model fitting.'

Why build models?

to <u>explain</u> and <u>predict</u>

Fitted models can then be used to "predict" the future, under different conditions.





<u>explain</u>

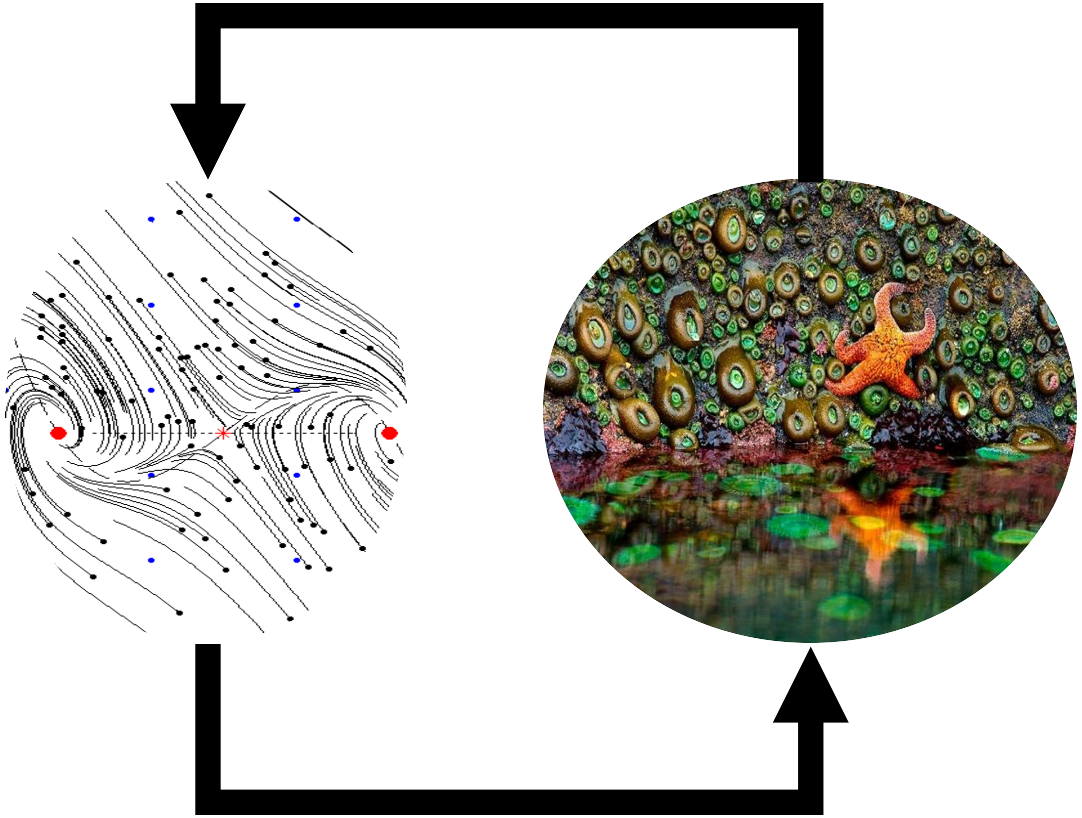
<u>predict</u>

(IPCC 2007)

Two kinds of ecological models

Statistical Model

Pattern



Population Model

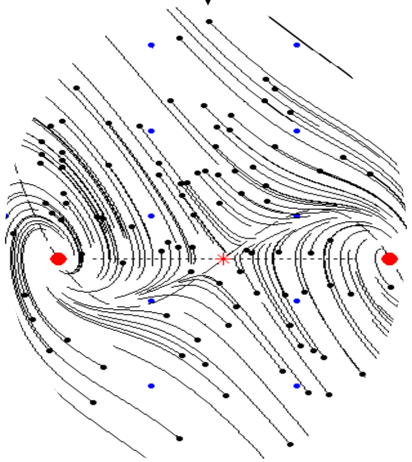
Process

Two kinds of ecological models

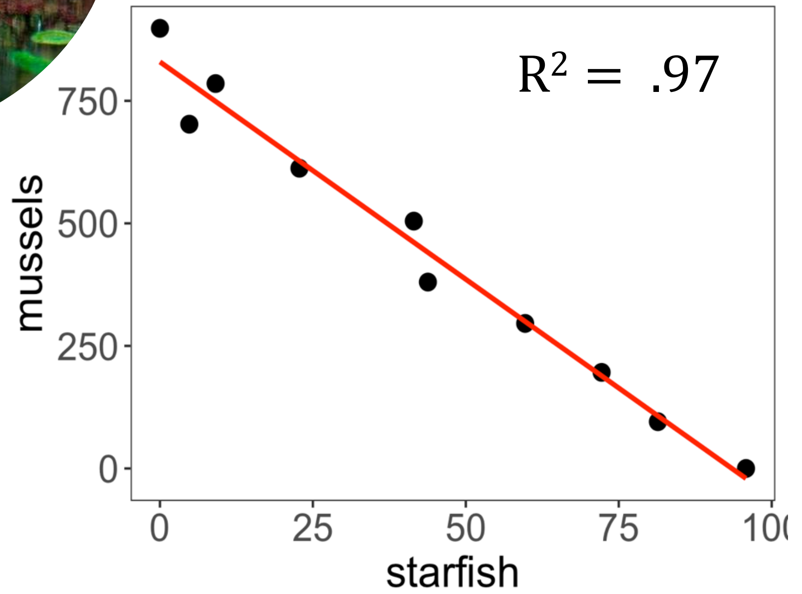
Statistical Model

Pattern

What *is the relationship between the abundance of starfish and the abundance of mussels?*



$y = mx + b$



Population Model

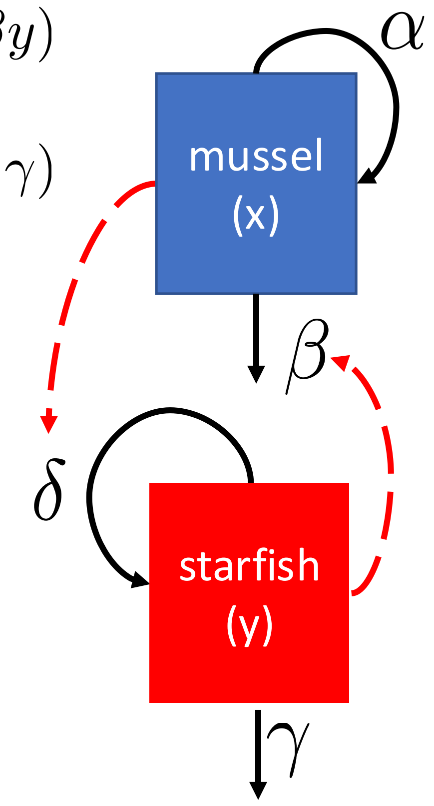
Process

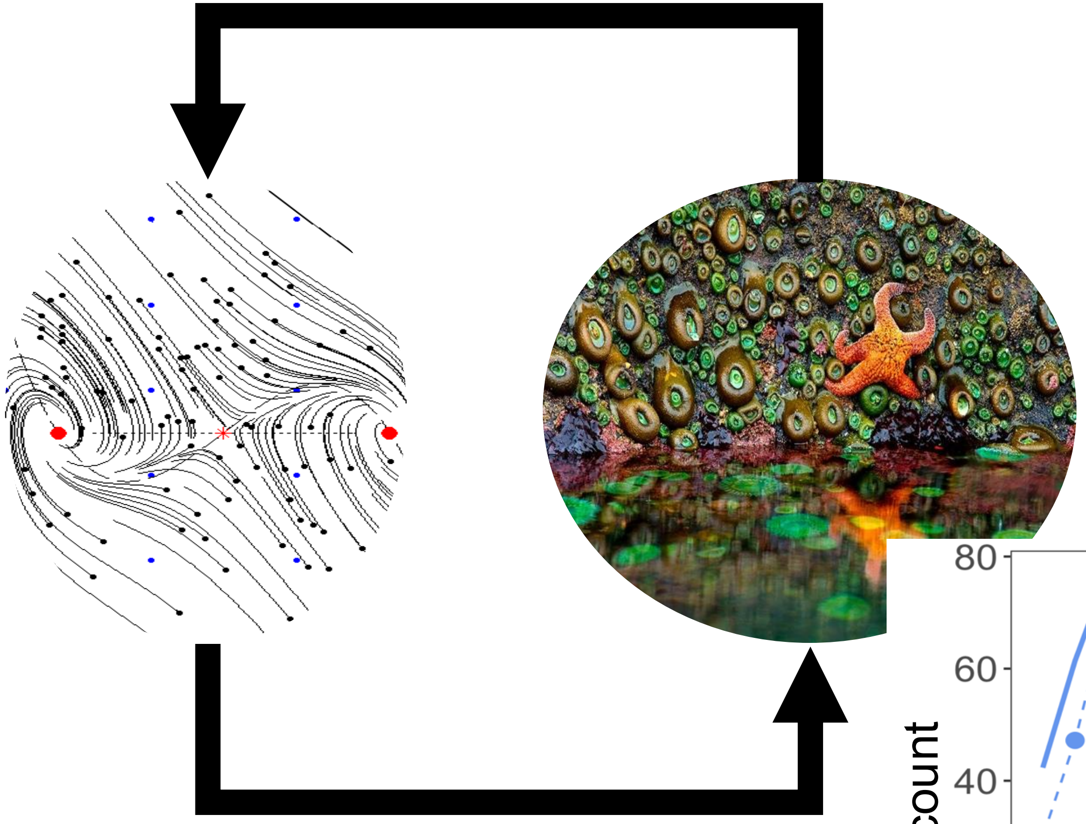
Two kinds of ecological models

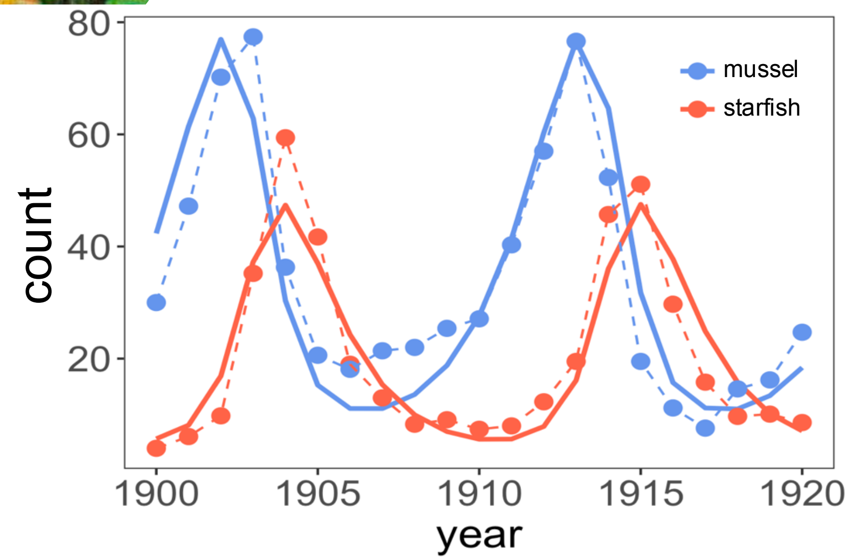
Statistical Model

Pattern

$$\frac{dx}{dt} = x(\alpha - \beta y)$$
$$\frac{dy}{dt} = y(\delta x - \gamma)$$







Population Model

Process

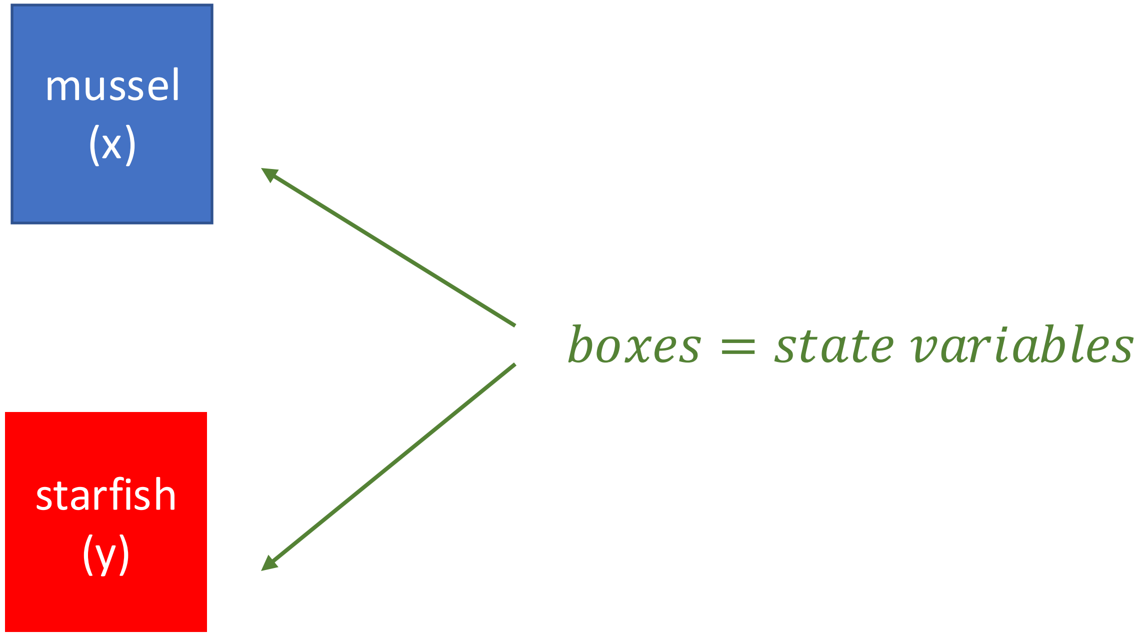
***How** does the abundance of starfish change **as a result of** the abundance of mussels?*

How to construct a population model

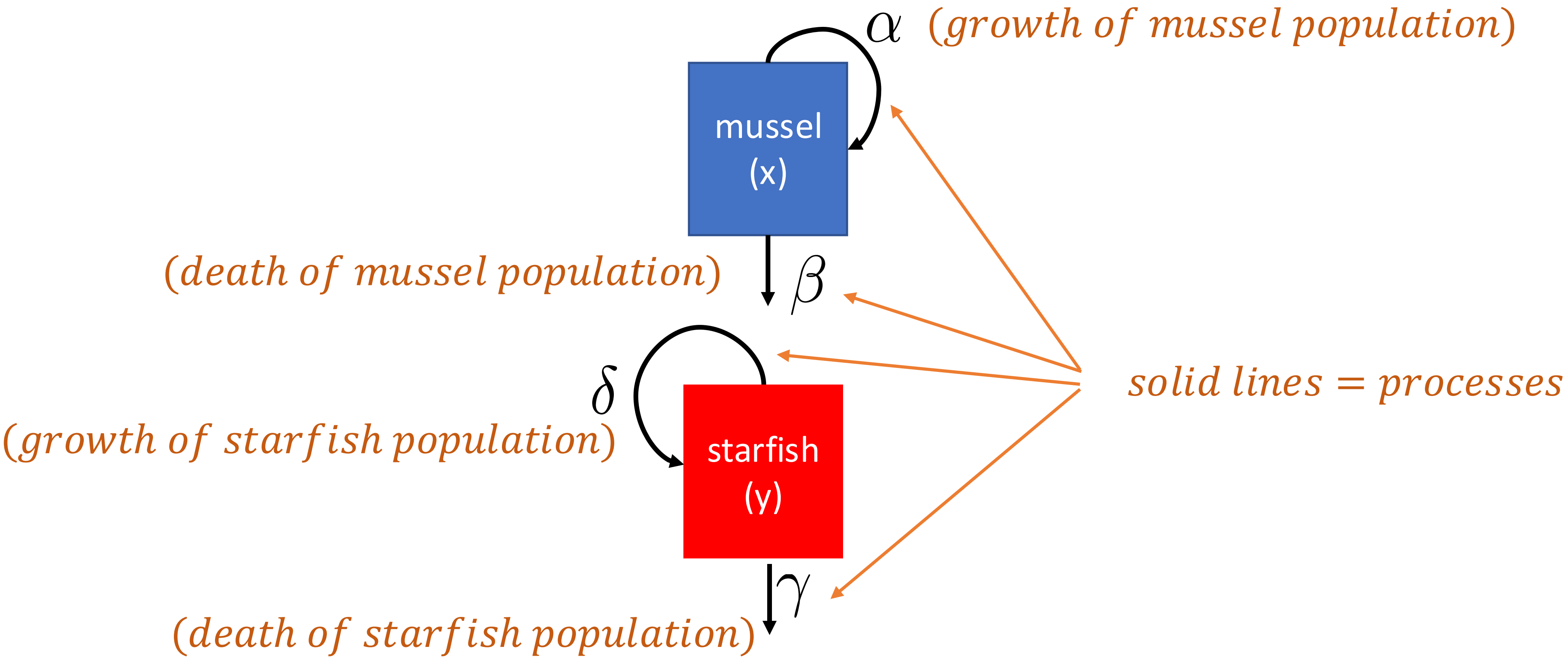
$$\frac{dx}{dt}$$

state variables

$$\frac{dy}{dt}$$



How to construct a population model

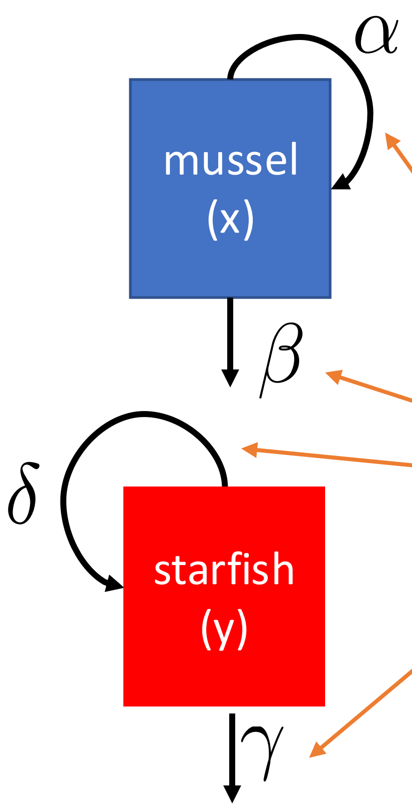


How to construct a population model

$$\frac{dx}{dt} = x(\alpha - \beta y)$$

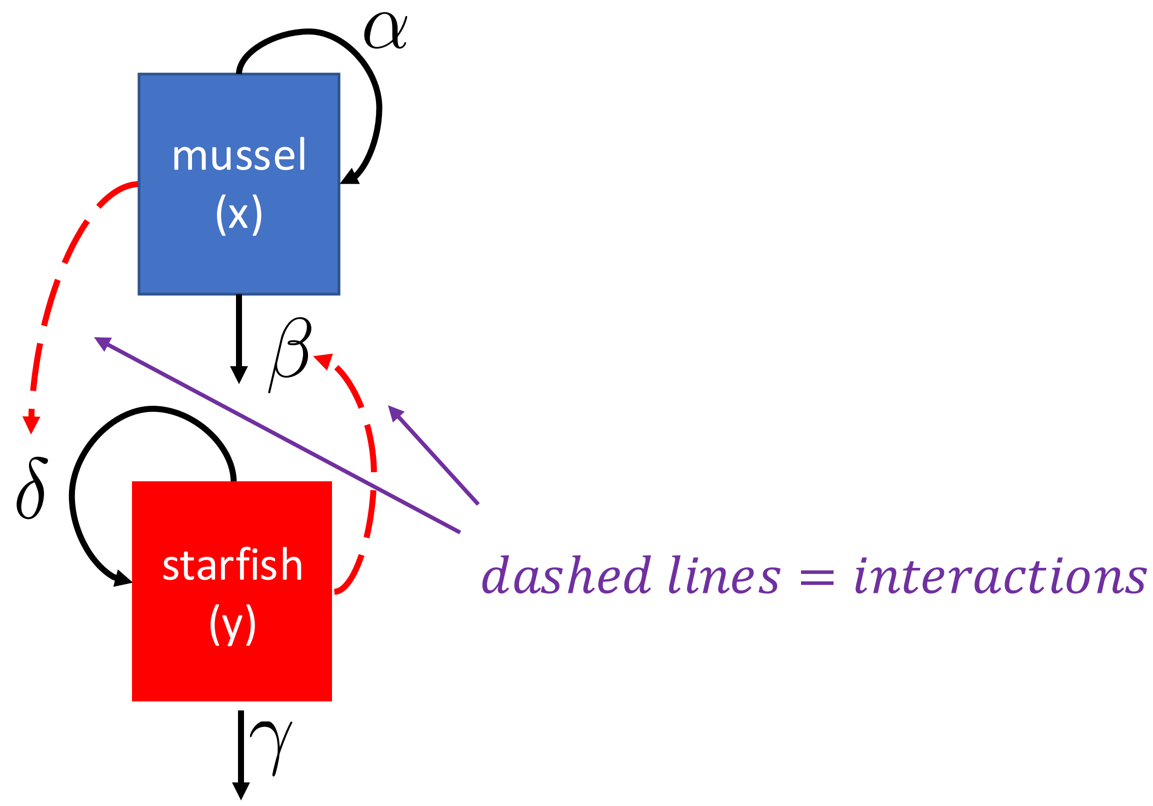
processes

$$\frac{dy}{dt} = y(\delta x - \gamma)$$



solid lines = processes

How to construct a population model



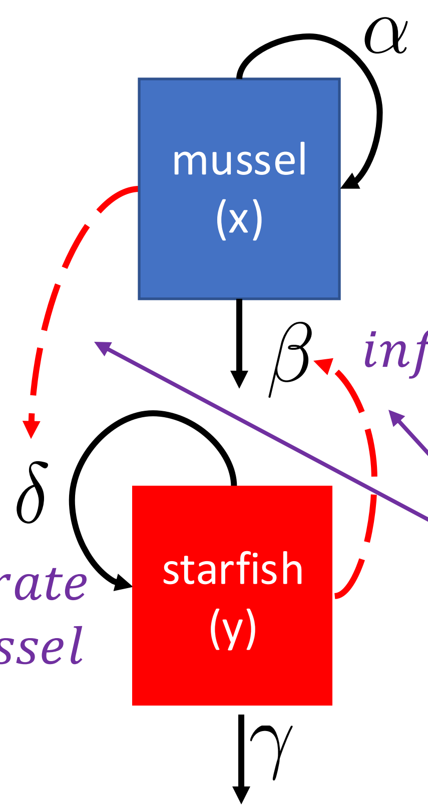
How to construct a population model

$$\frac{dx}{dt} = x(\alpha - \beta y)$$

interactions

$$\frac{dy}{dt} = y(\delta x - \gamma)$$

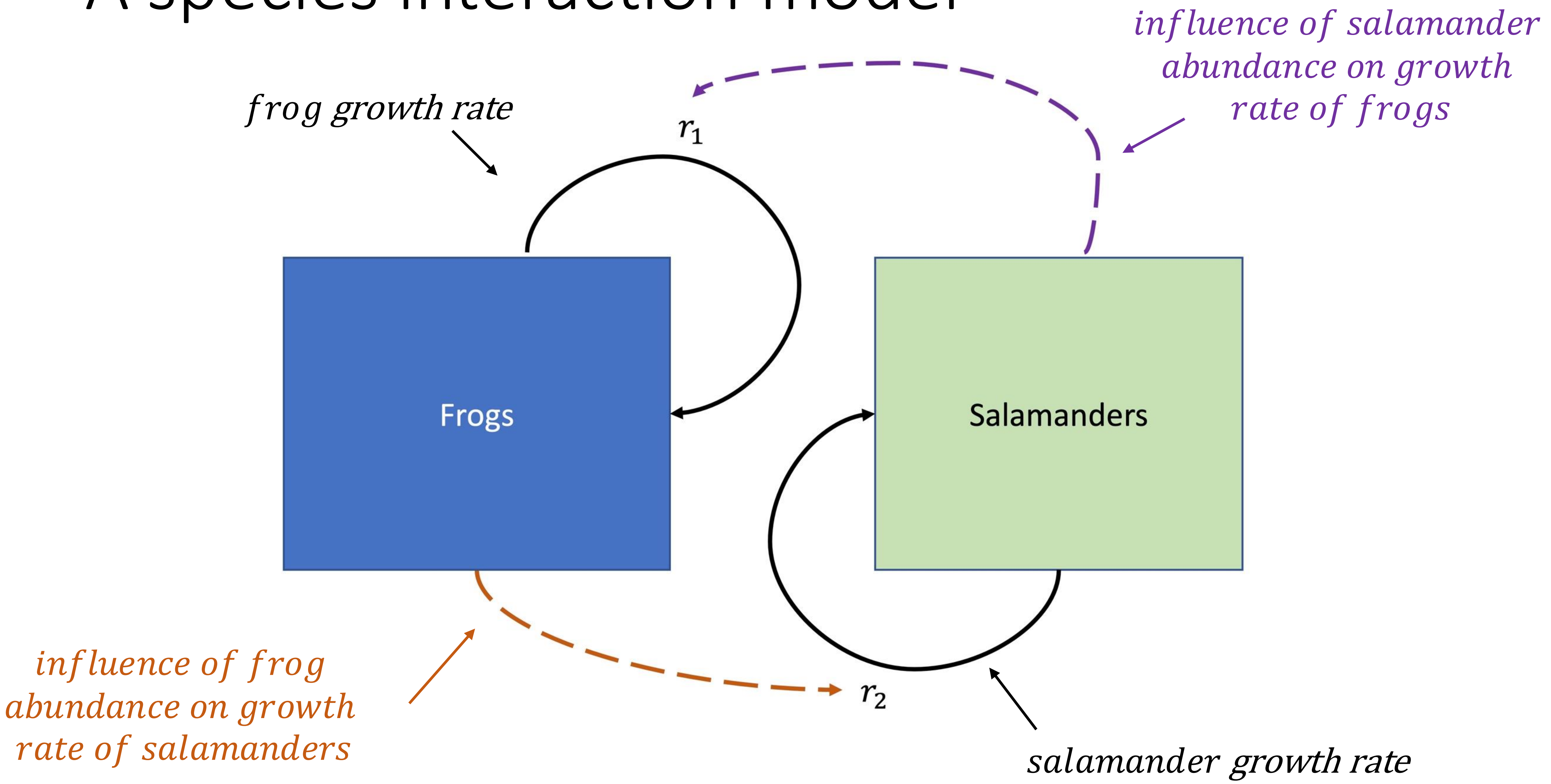
(starfish growth rate influenced by mussel abundance)

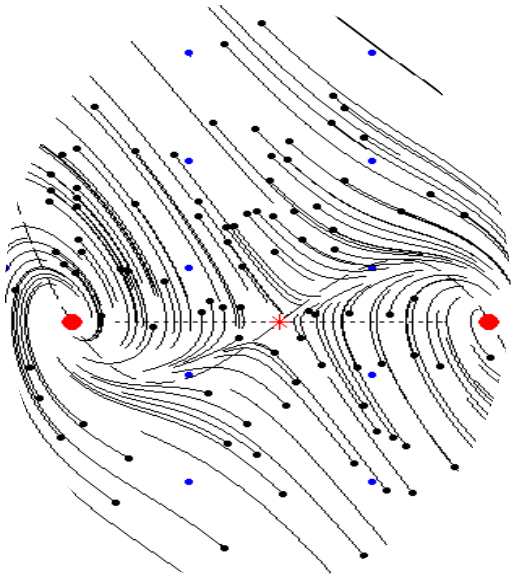


(mussel death rate influenced by starfish abundance)

dashed lines = interactions

A species interaction model





Ecology is the study of the **interactions** of **organisms** with each other and their **environment**.

dynamic

frequently changing

complex

many interacting parties

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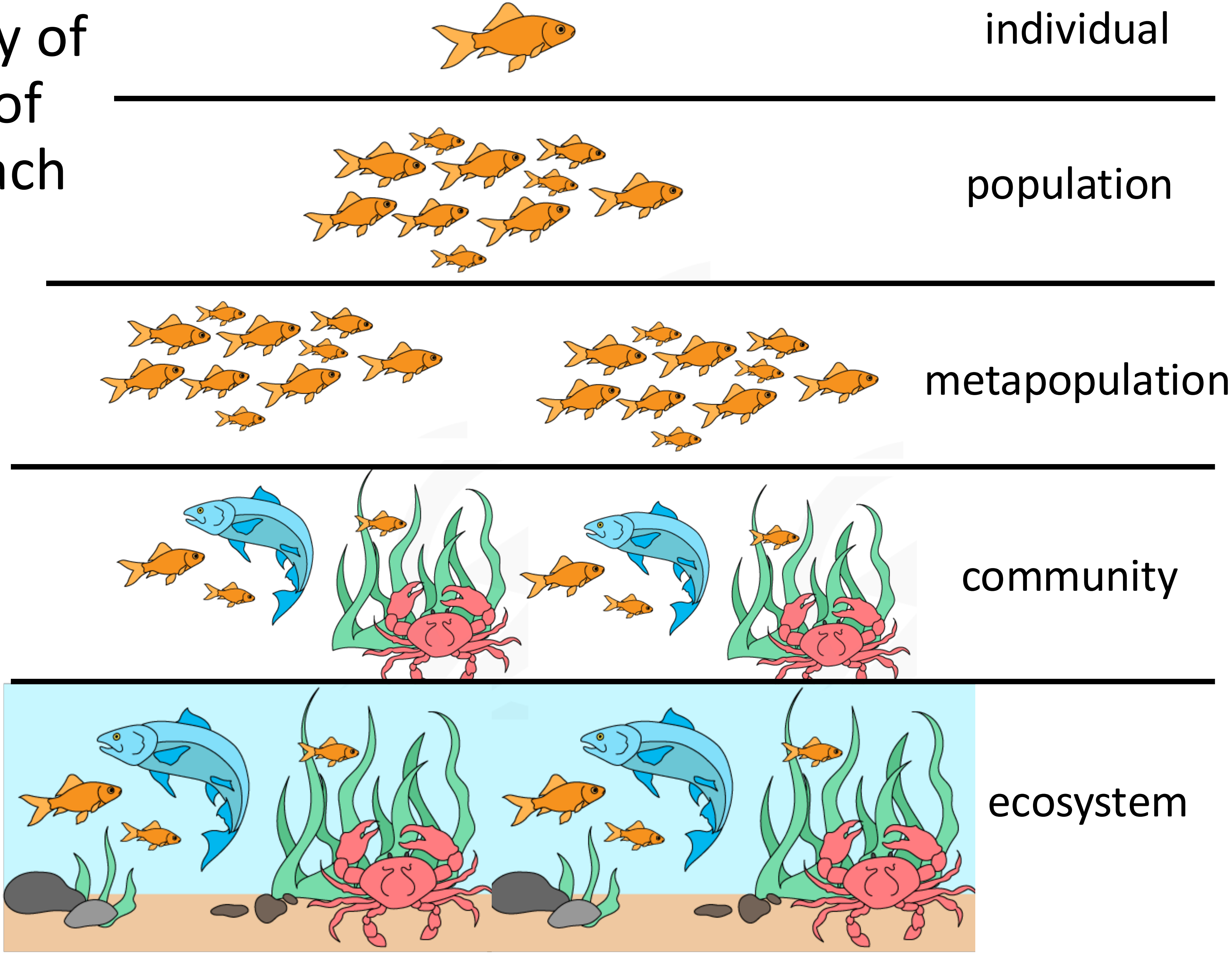
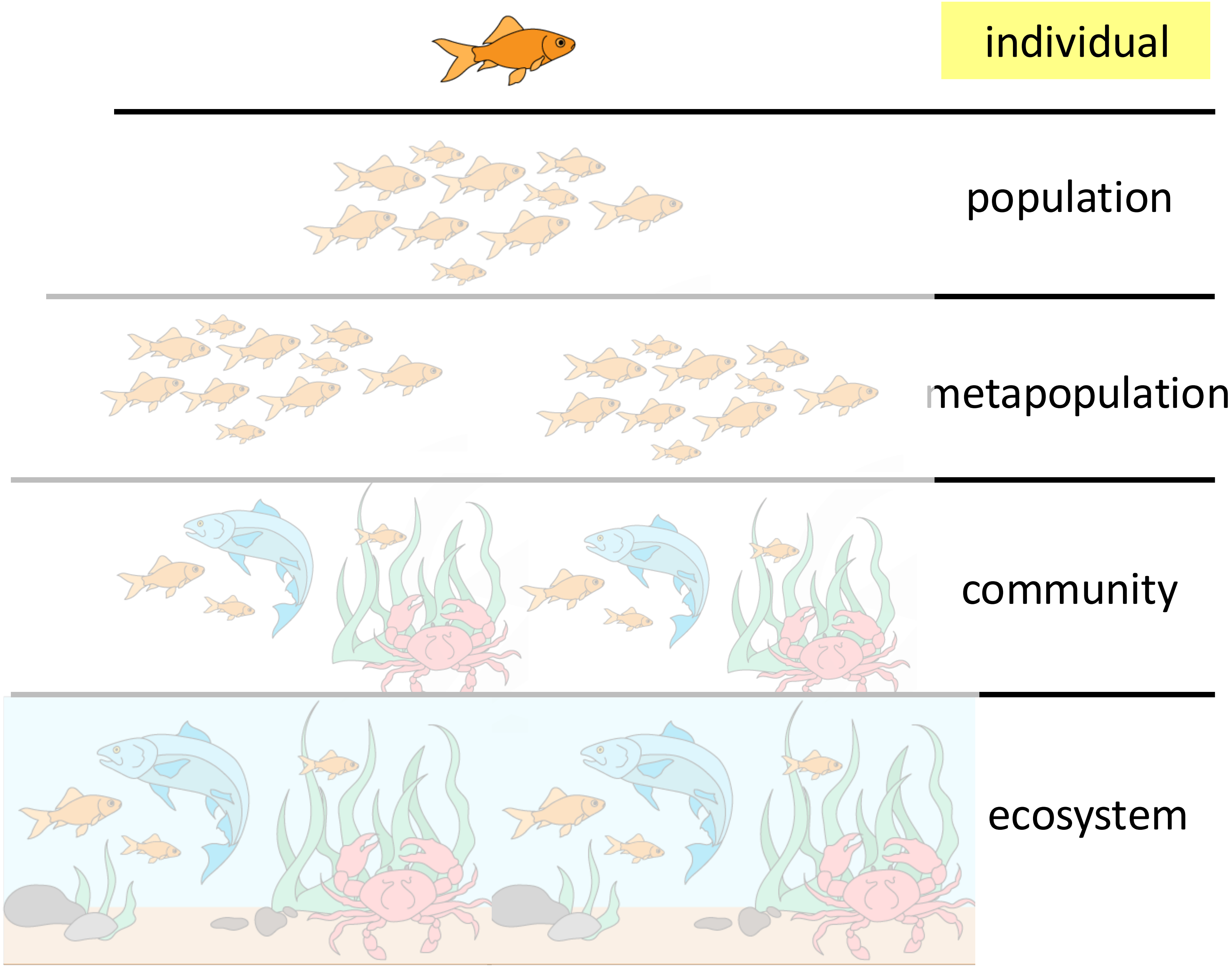


Individual:

metabolism, behavior, life history.

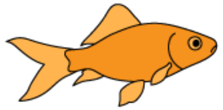
interactions of an individual with the environment



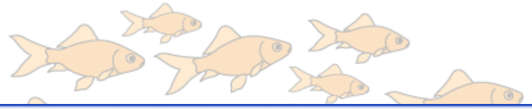
Individual: metabolism, behavior, life history.

Dynamic Energy Budget (DEB) Model

*How does a fish's metabolism **change** with temperature?*



individual

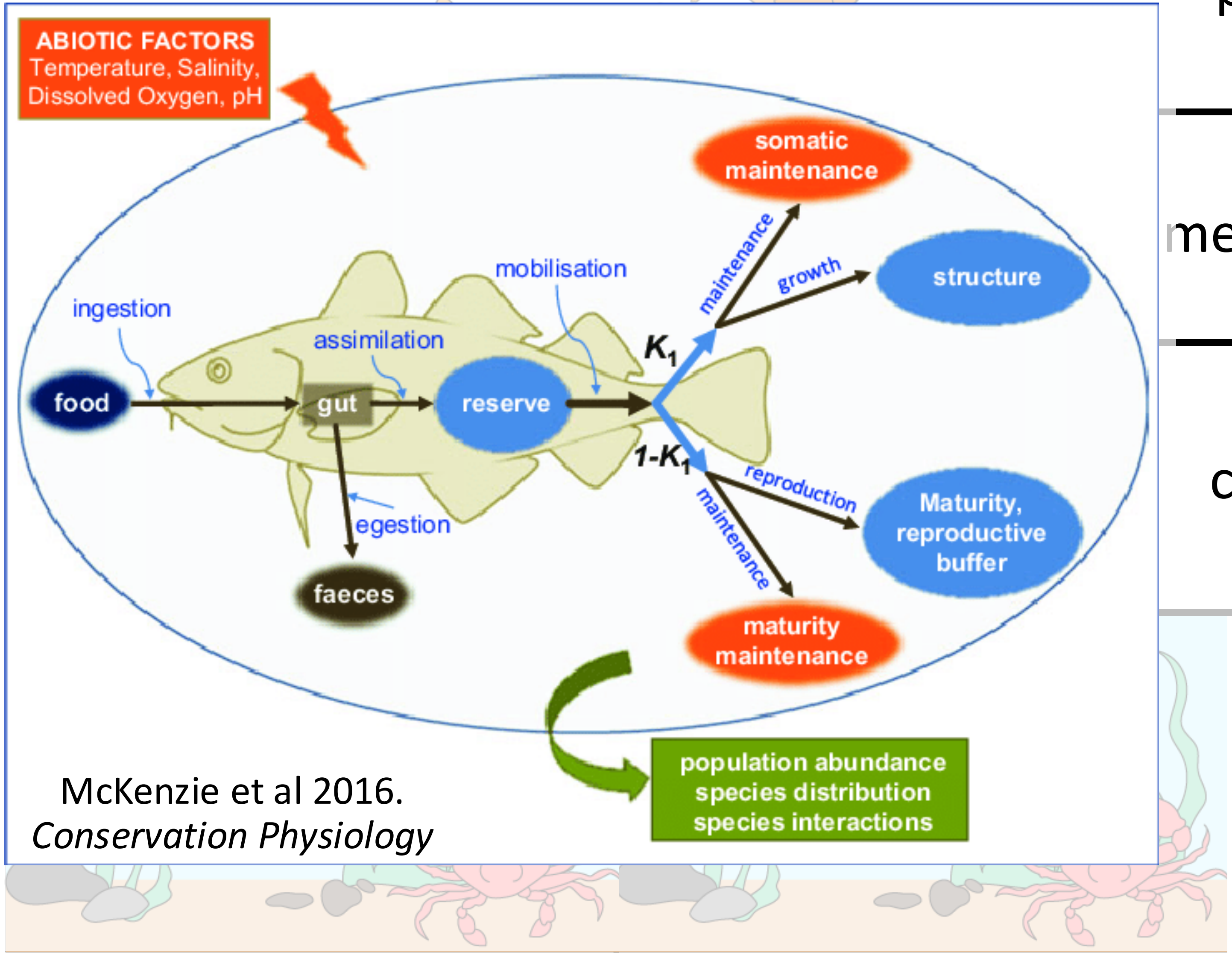


population

metapopulation

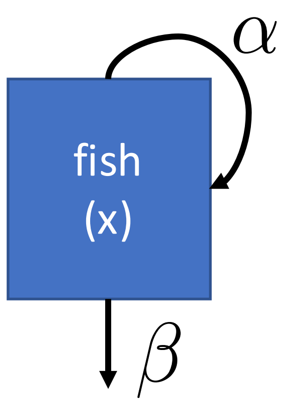
community

ecosystem

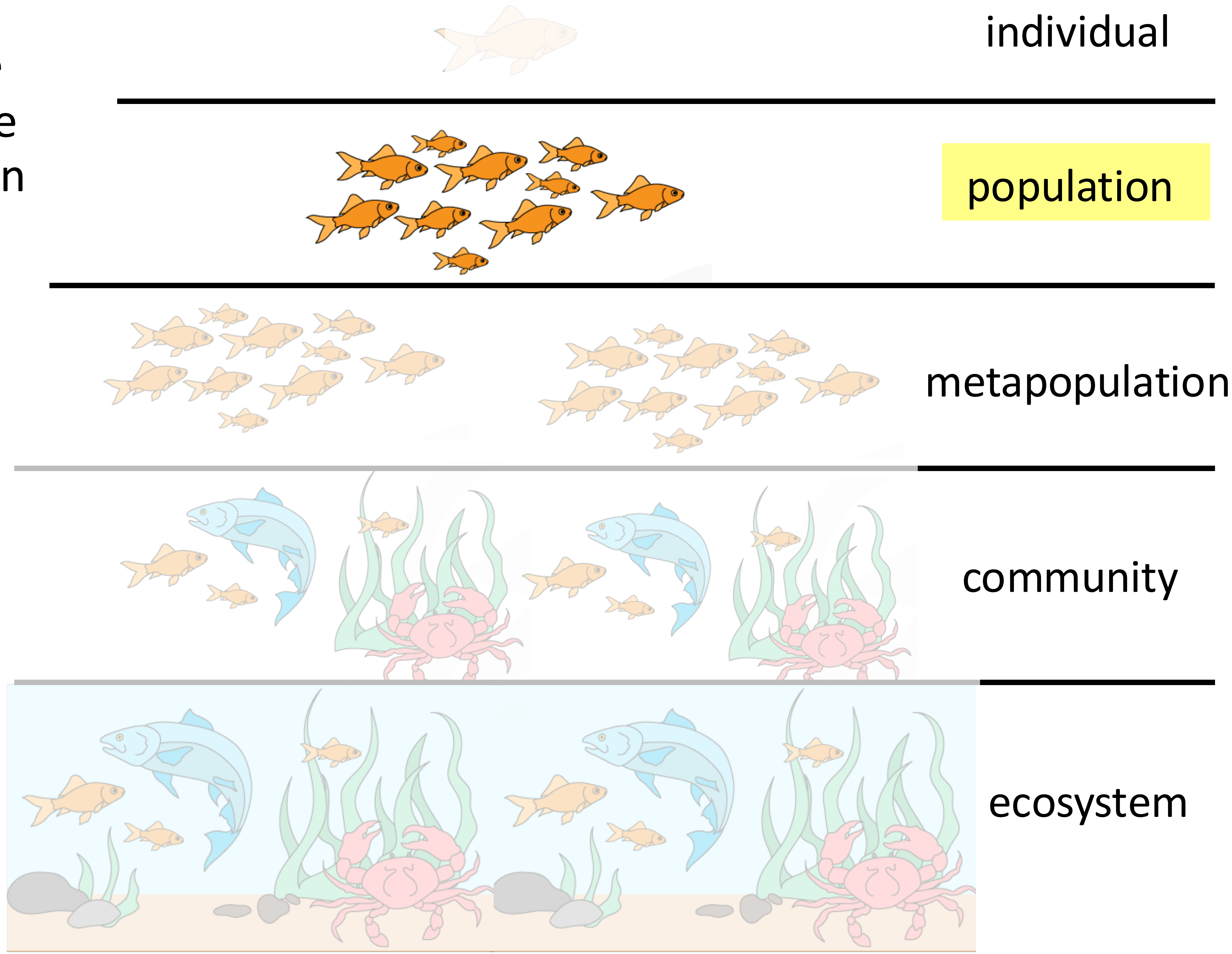


McKenzie et al 2016. *Conservation Physiology*

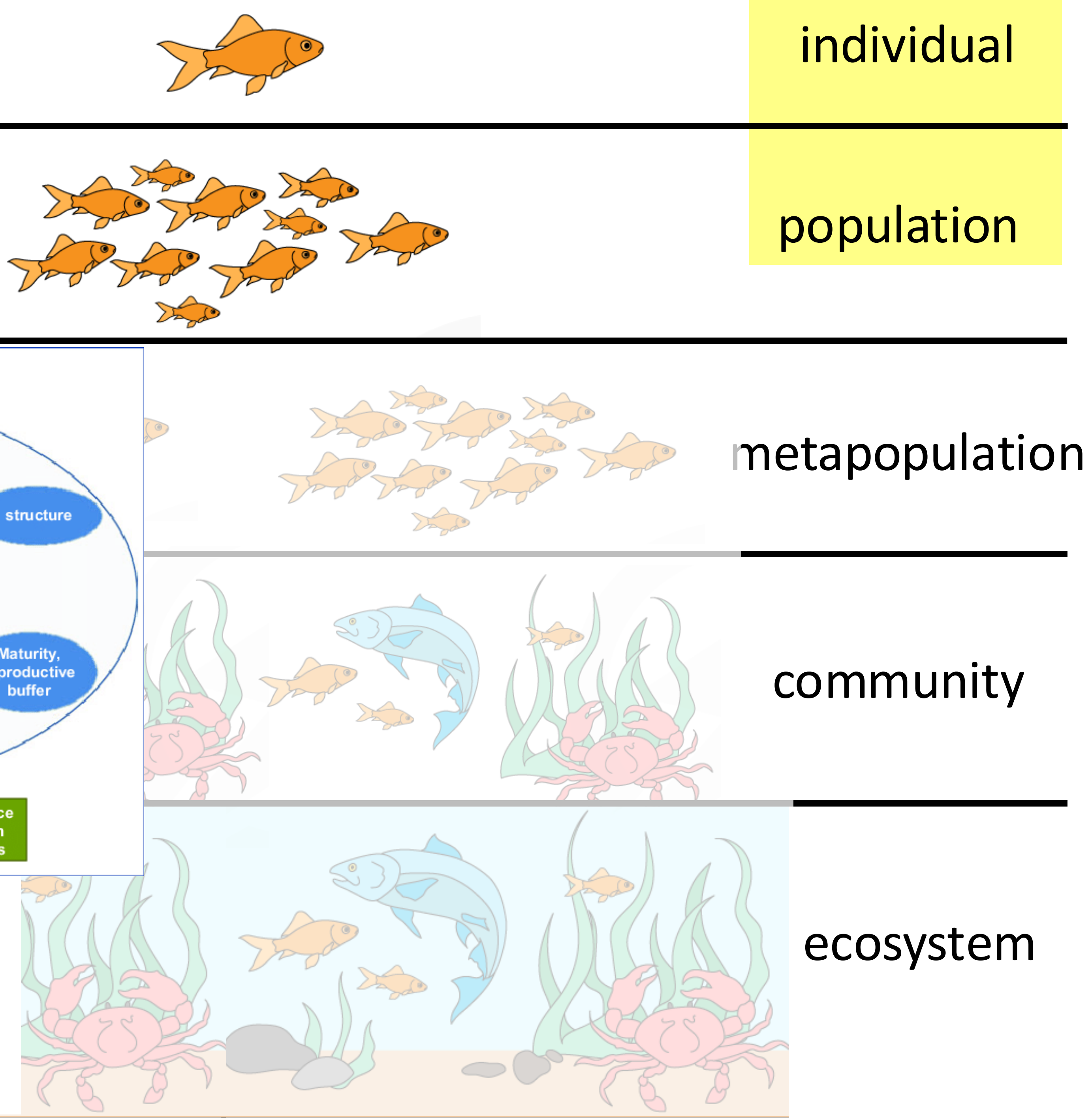
Population = multiple individuals of the same species (**conspecifics**) in the same habitat

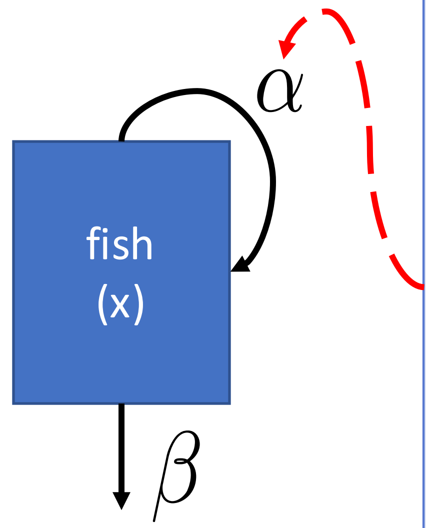


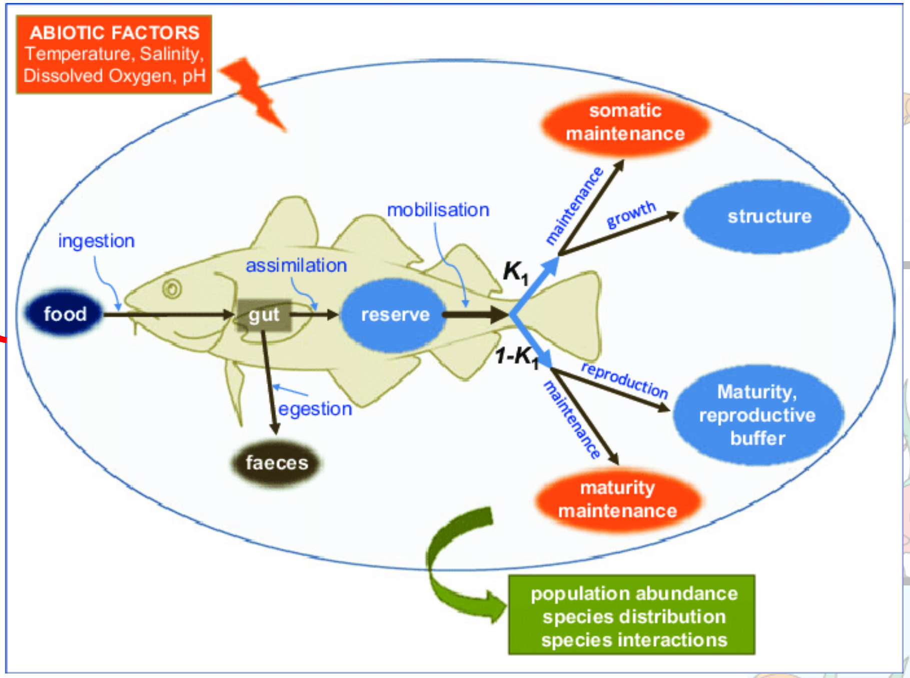
*How does the abundance of fish **change** through time?*



Nested Models, including a class of model known as **Integral Projection Models** (IPMs), link individual- and population-level processes

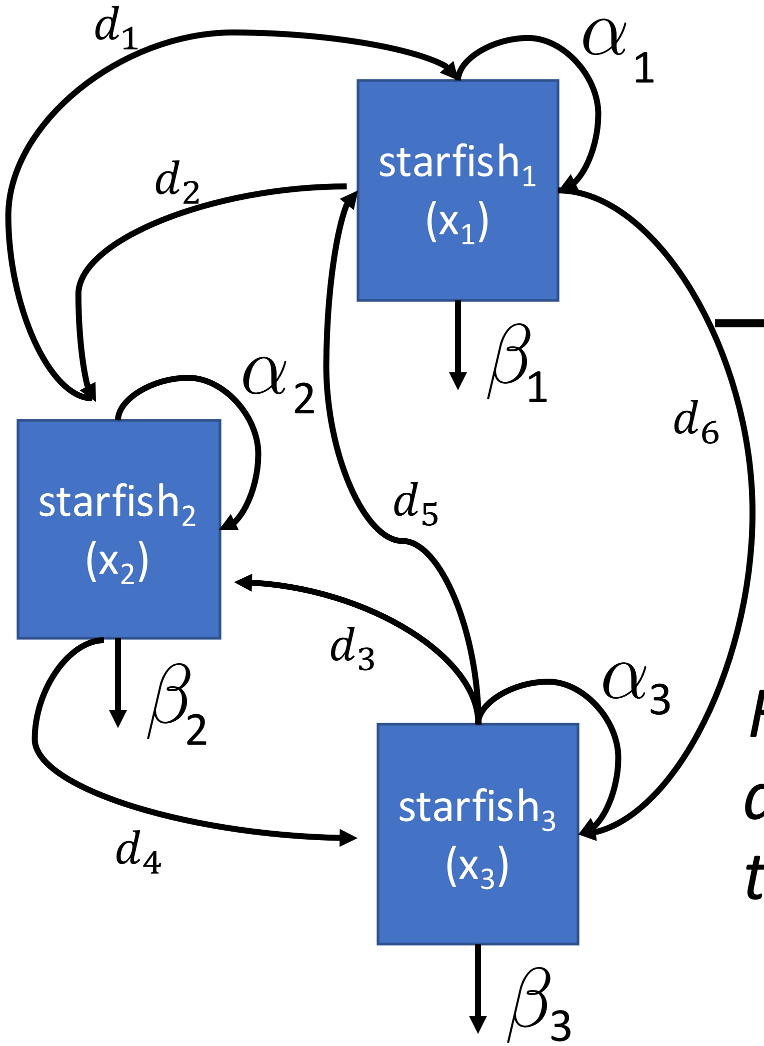


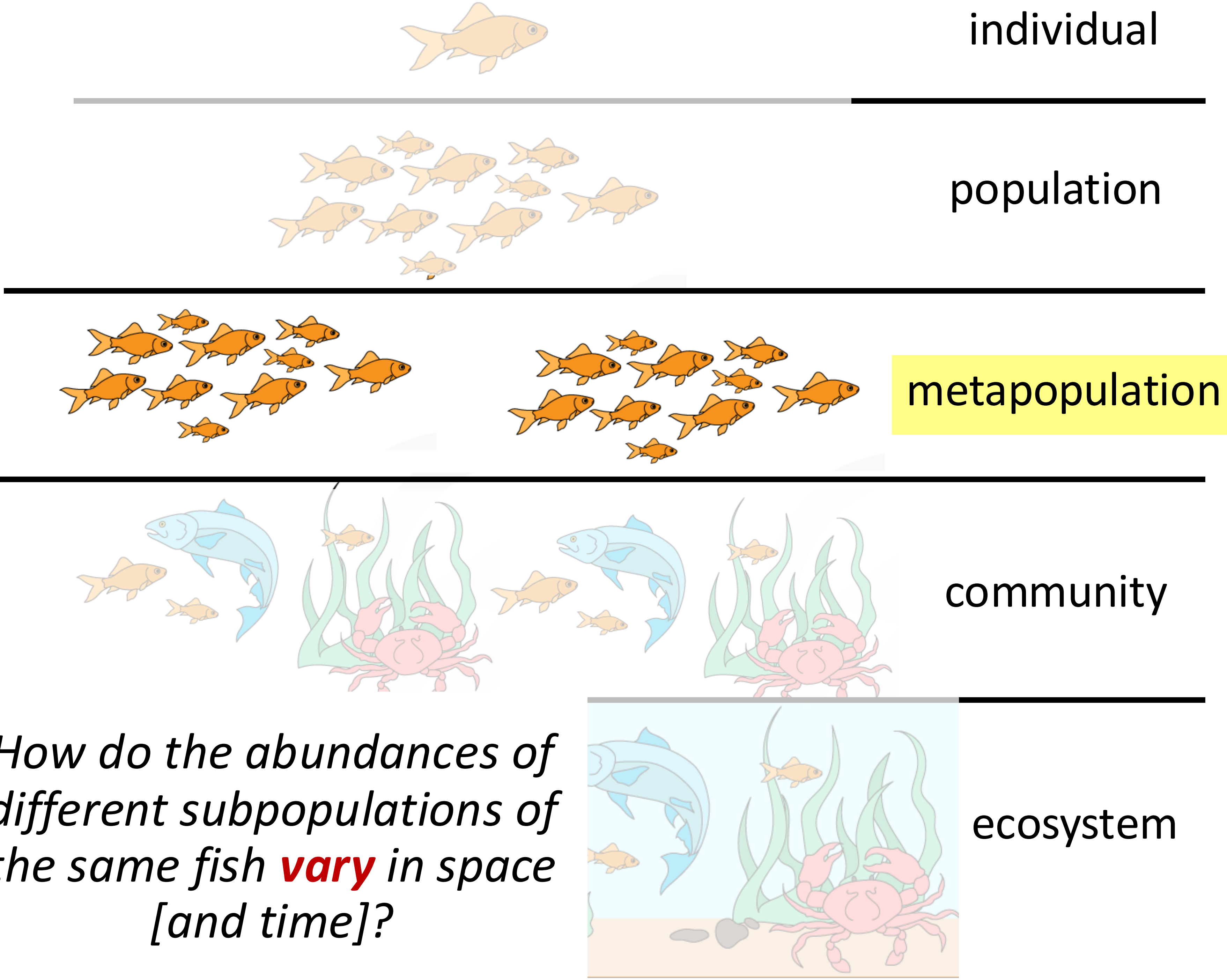




*How does the abundance of fish **change** through time as temperature **changes** metabolism?*

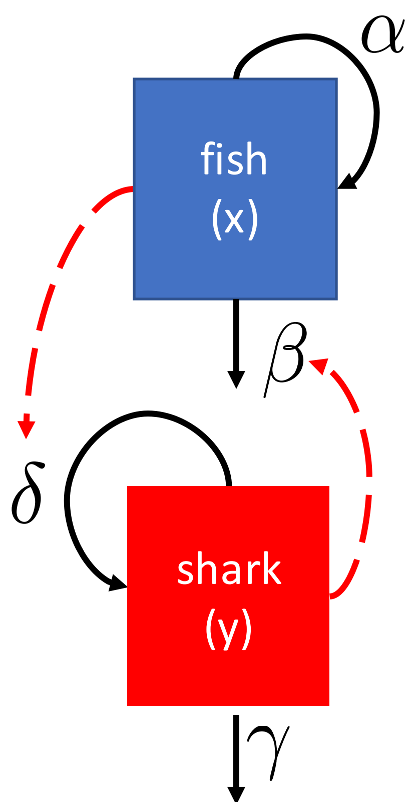
Metapopulation = sub-populations of conspecifics connected by migration or dispersal



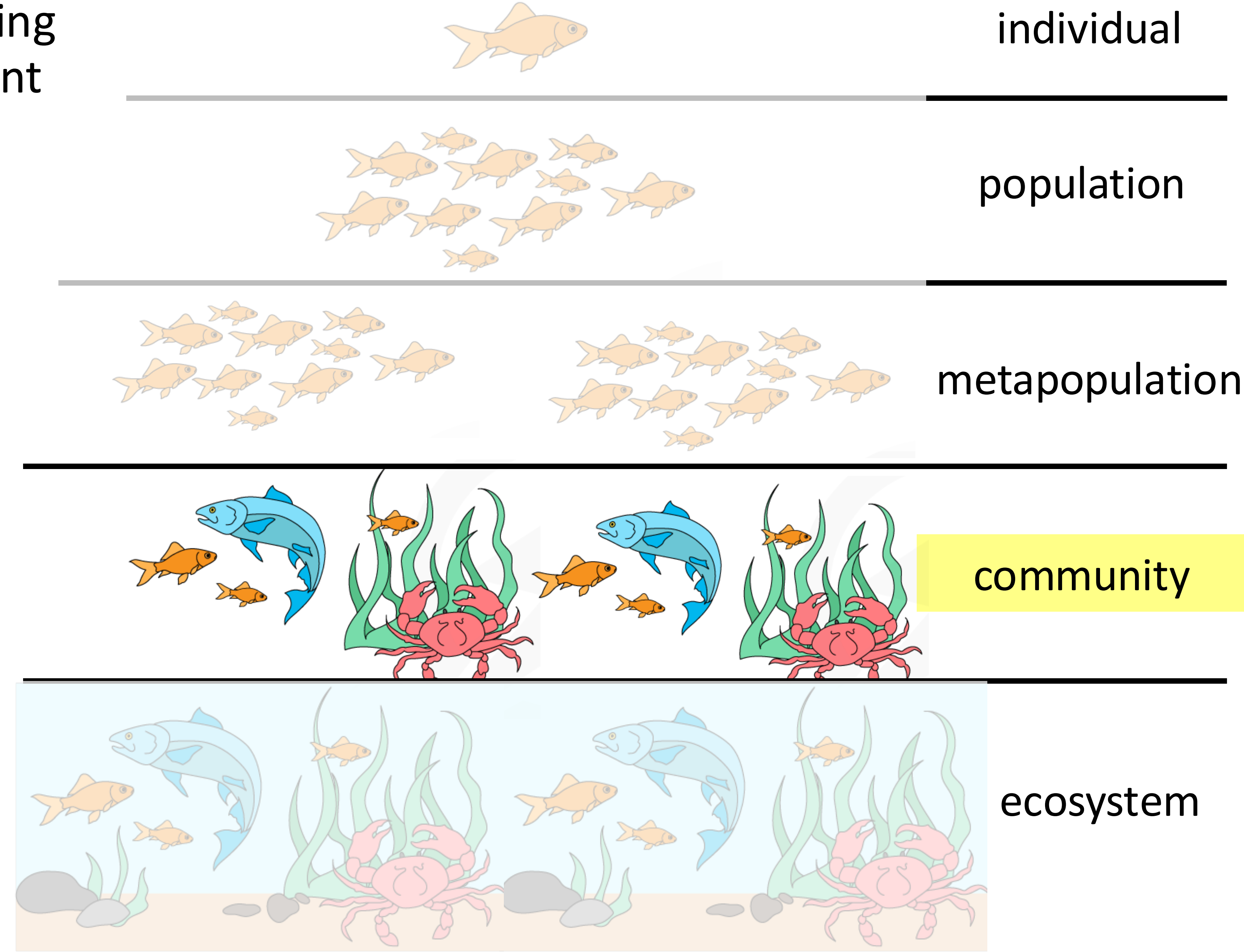


*How do the abundances of different subpopulations of the same fish **vary** in space [and time]?*

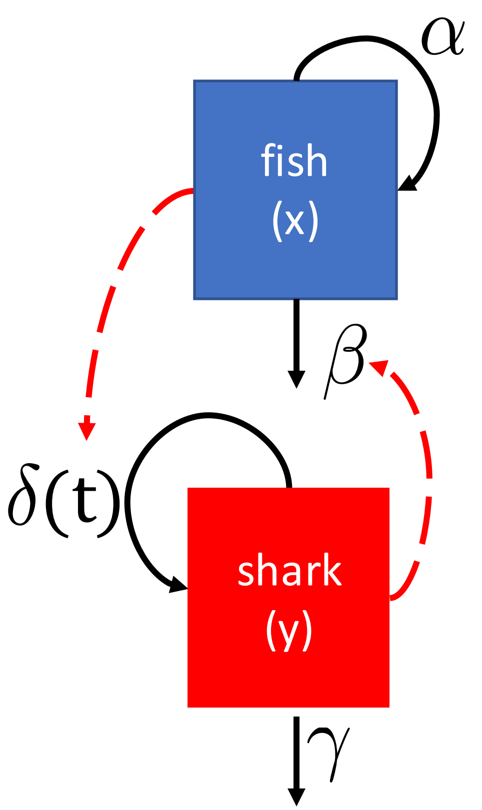
Community = interacting populations of different species



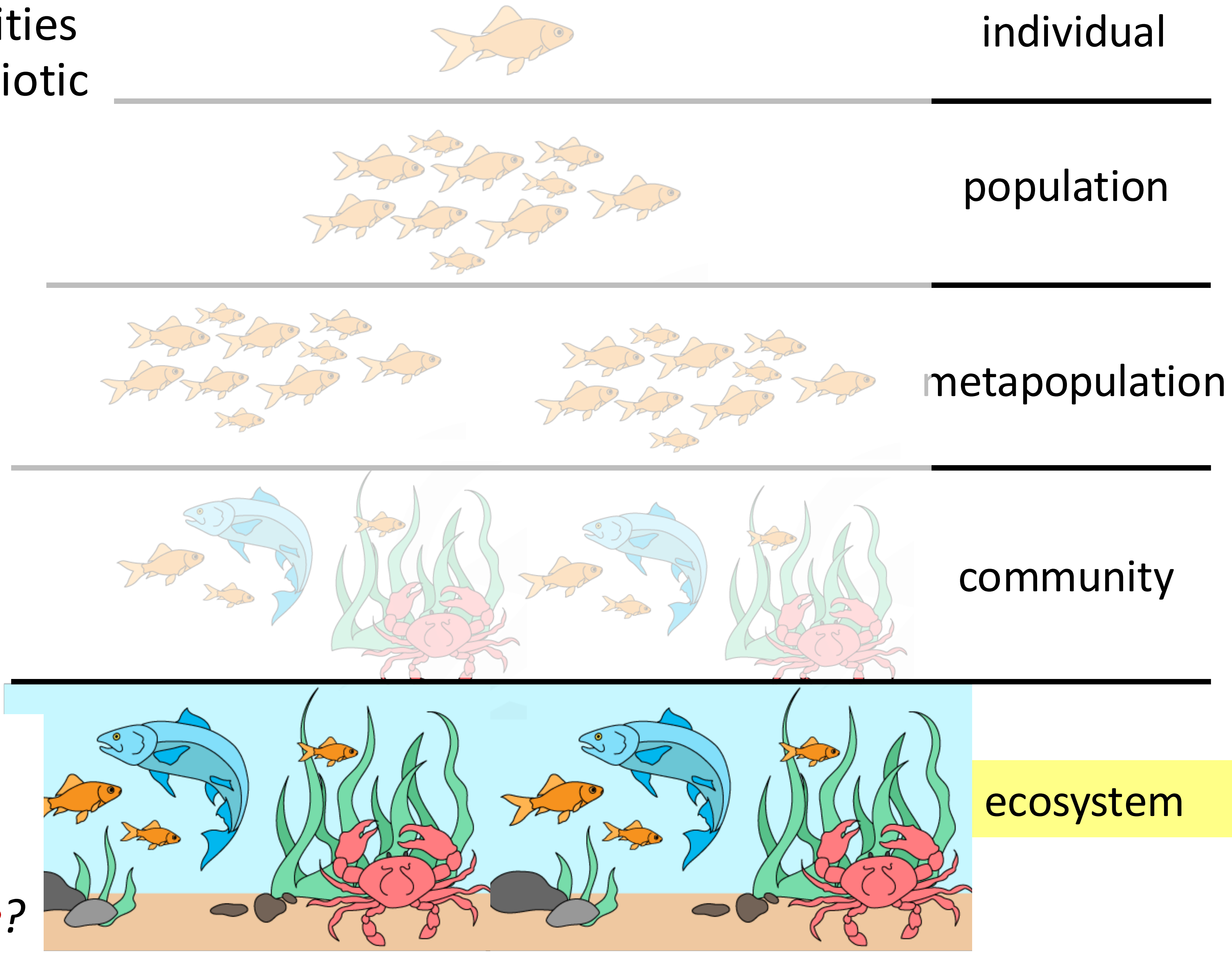
*How does fish abundance **vary** with **changes** in shark abundance?*



Ecosystem = communities interacting with the abiotic environment



*How does fish abundance **vary** with **changes** in shark birth rates with **temperature**?*



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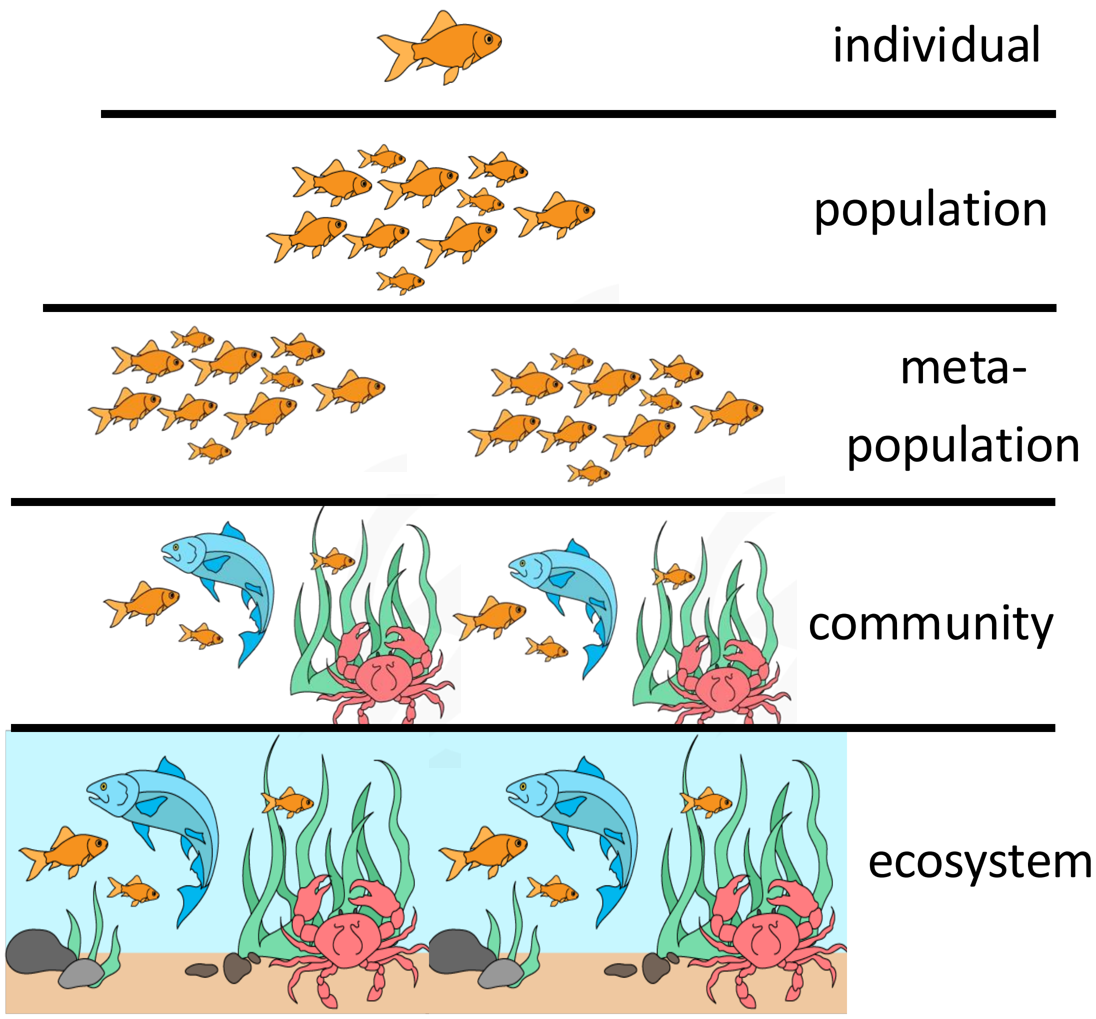

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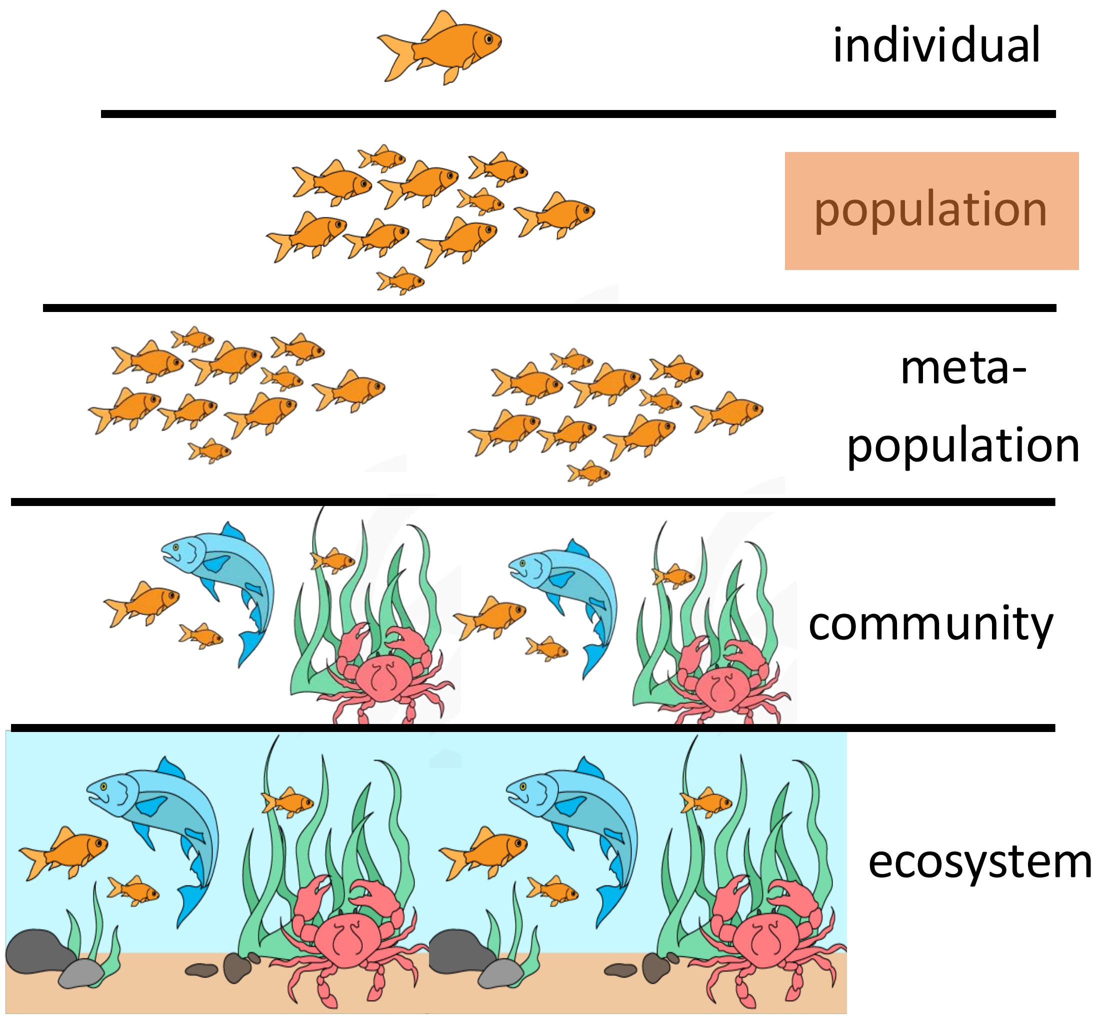

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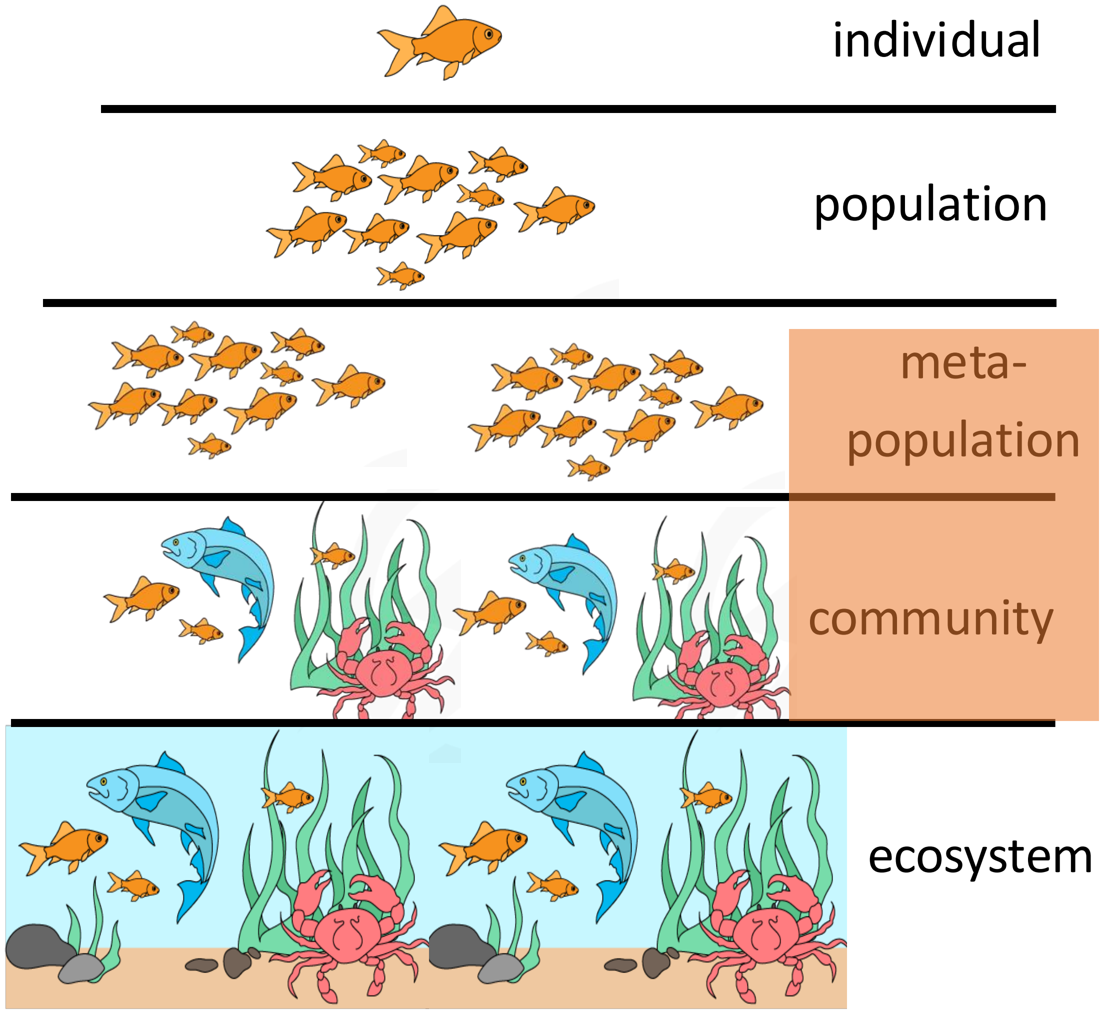

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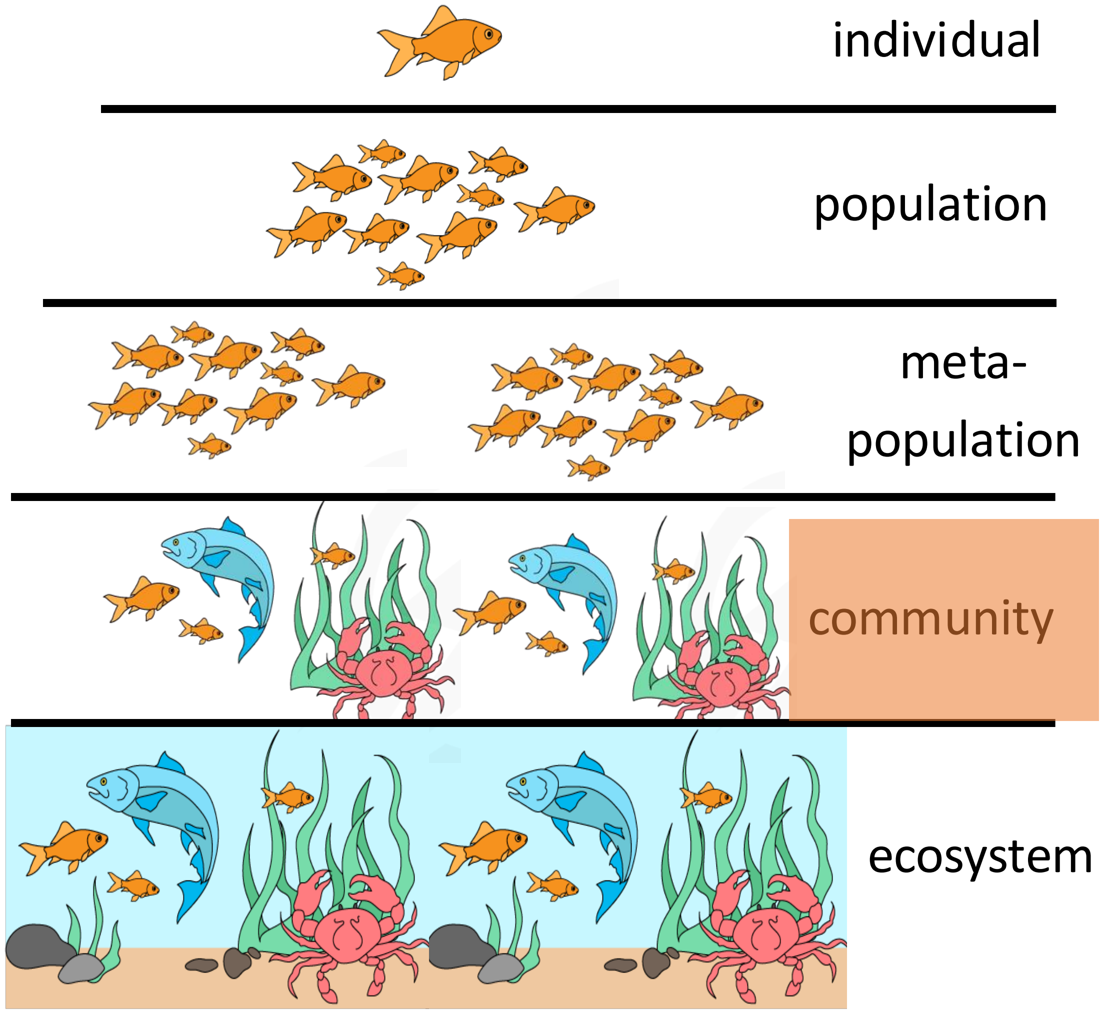
7	6: Population Regulation	Feb 18: Species Interactions 2	Feb 20: Disease Dynamics as Population Biology 1
8	7: Predation & Competition	Feb 25: Disease Dynamics as Population Biology 2	Feb 27: Community Assembly & Island Biogeography
9	8: Disease Dynamics	Mar 4: Conservation Biology 1	Mar 6: Conservation Biology 2



Where are we headed?

Week	Mon/Tues Lab	Tues Lecture	Thurs Lecture
1	--	Jan 7: Evolution Intro	Jan 9: Genetic Variation
2	1: Intro R	Jan 14: Evolution without selection: Genetic drift & migration	Jan 16: Natural selection 1
3	2: Hardy-Weinberg	Jan 21: Natural Selection 2	Jan 23: Sexual selection & genetic conflicts
4	3: Microevolution	Jan 28: Speciation	Jan 30: Phylogenetics & biodiversity
5	4: Phylogenetics	Feb 4: Ecology & Population Growth	Feb 6: Single Species Population Growth & Regulation
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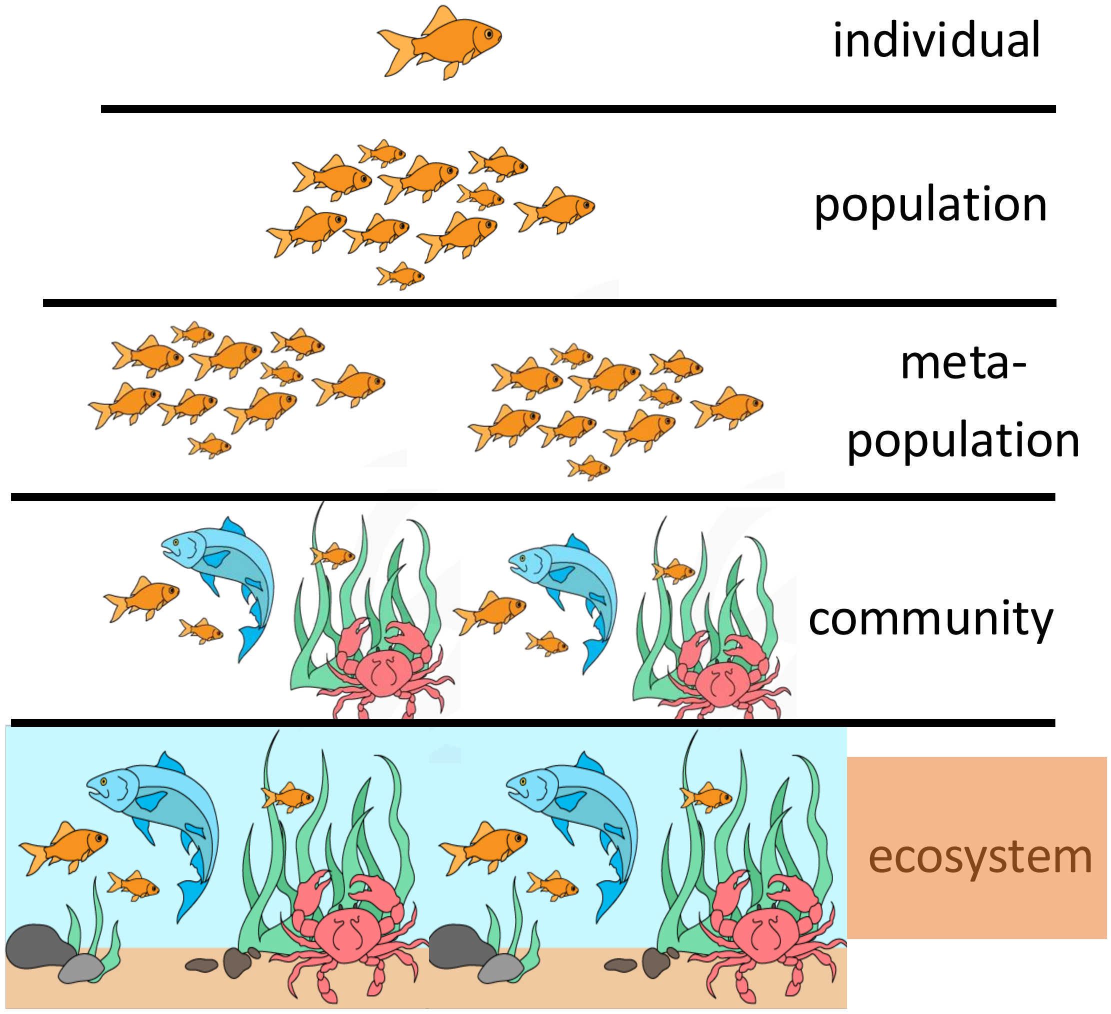

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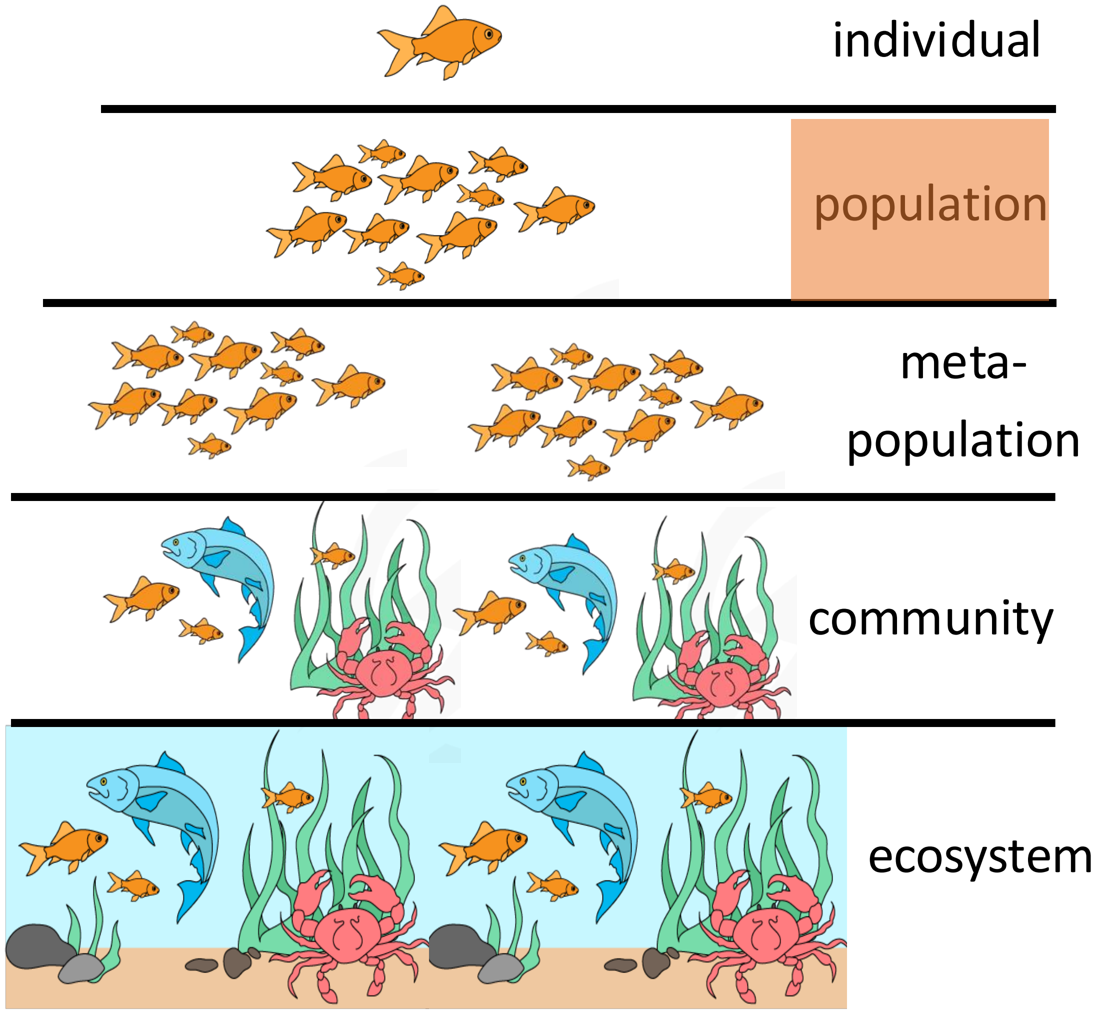


Why do we care how populations grow?

(Shafer 1980)

DETERMINING MINIMUM VIABLE POPULATION SIZES FOR THE GRIZZLY BEAR[1]

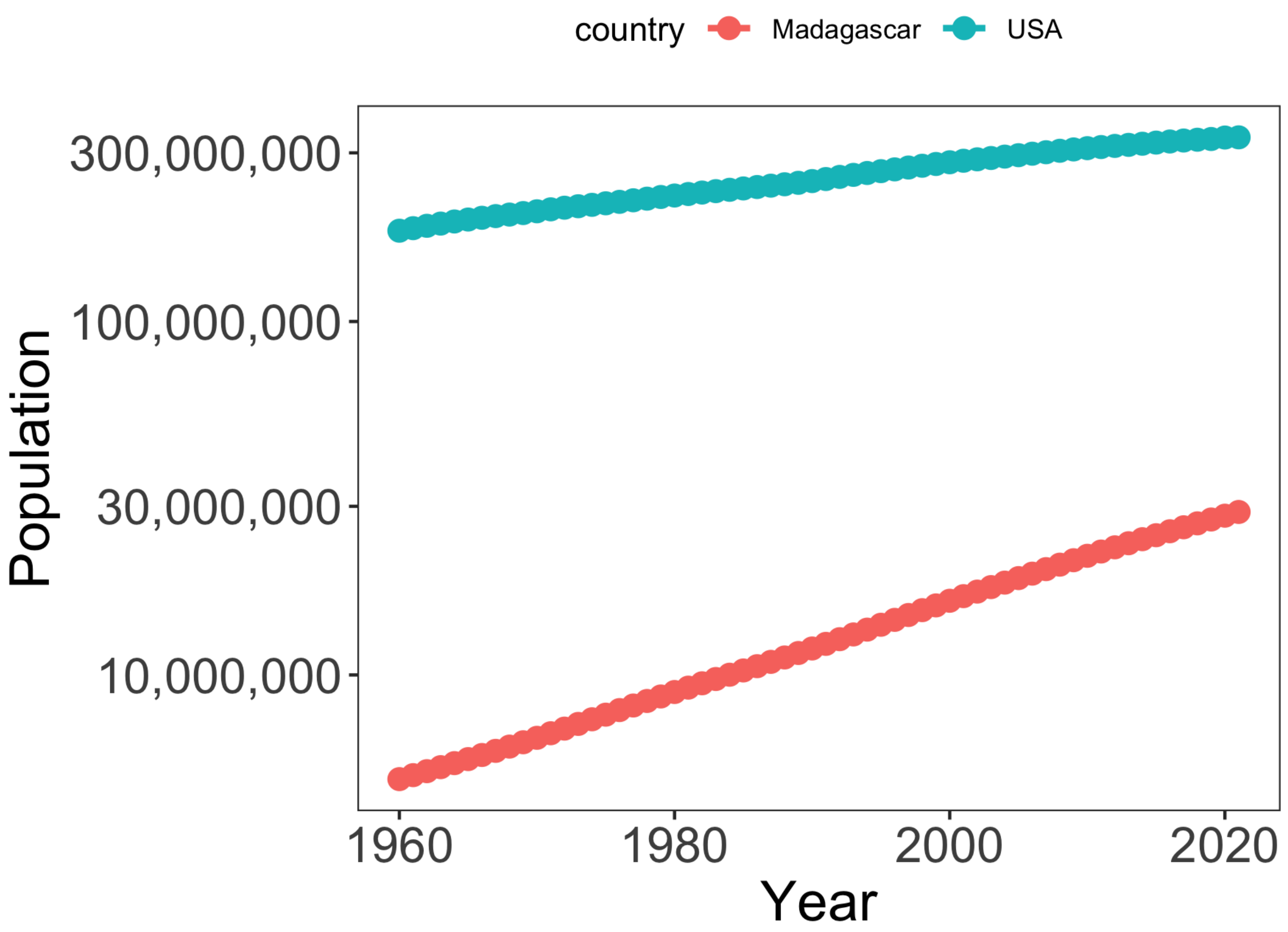
MARK L. SHAFFER,[2] School of Forestry and Environmental Studies, Duke University, Durham, NC 27706

Abstract: A stochastic computer simulation is presented for use in determining the relationship of population size to extinction probabilities for populations of grizzly bears (*Ursus arctos*). Published data on numbers, age, sex, reproduction, and mortality for the grizzly bear population of Yellowstone National Park were used to develop and test several simulation models. The results indicate that, for the Yellowstone grizzlies, 35 to 70 bears constitute a minimum viable population (the smallest population with a 95% probability of surviving at least 100 years). Minimum area requirements for populations of this size range from 700 to 10,000 km.[2]

Int. Conf. Bear Res. and Manage. 5:133 – 139

to protect populations from extinction

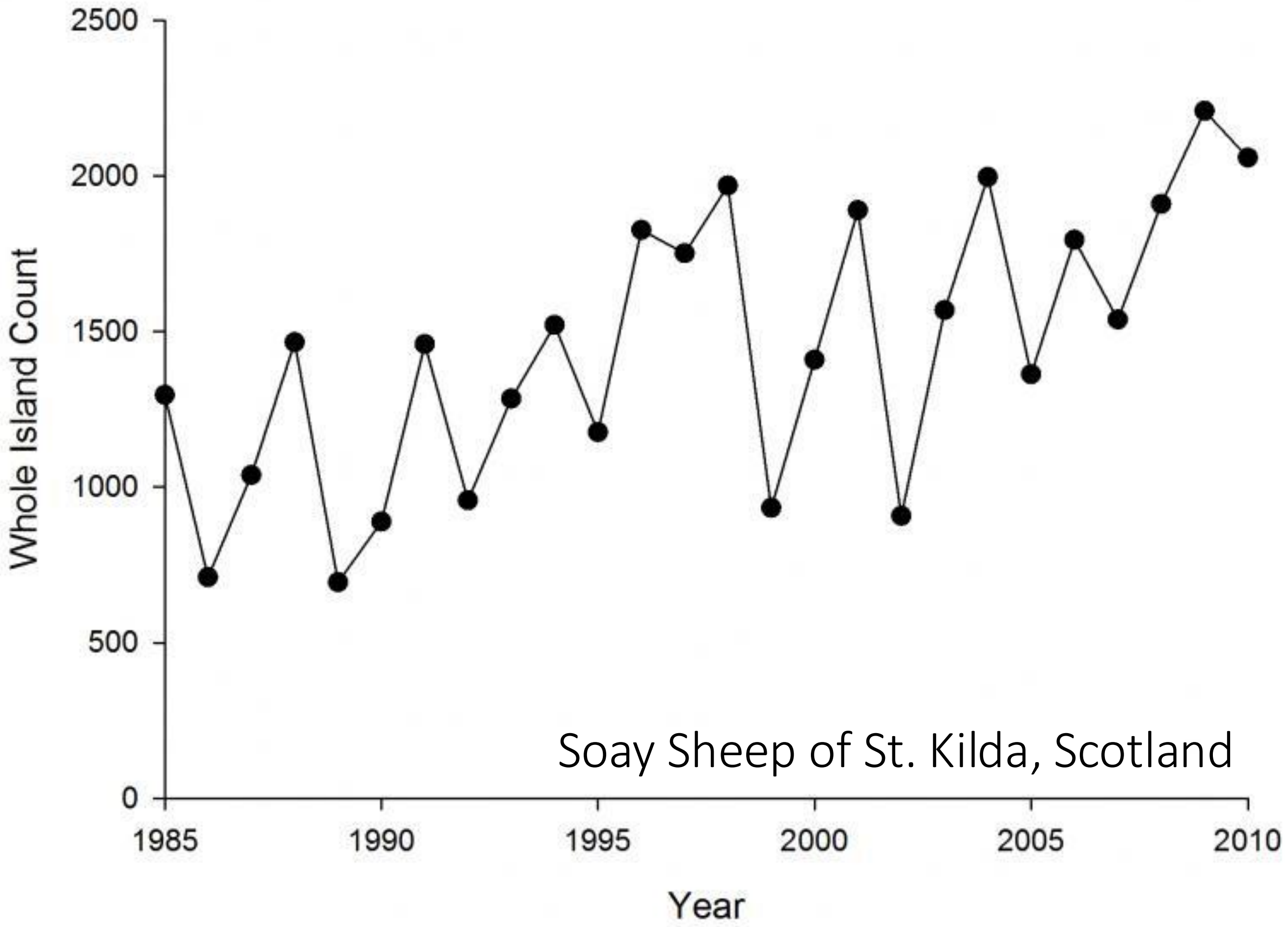
Why do we care how populations grow?



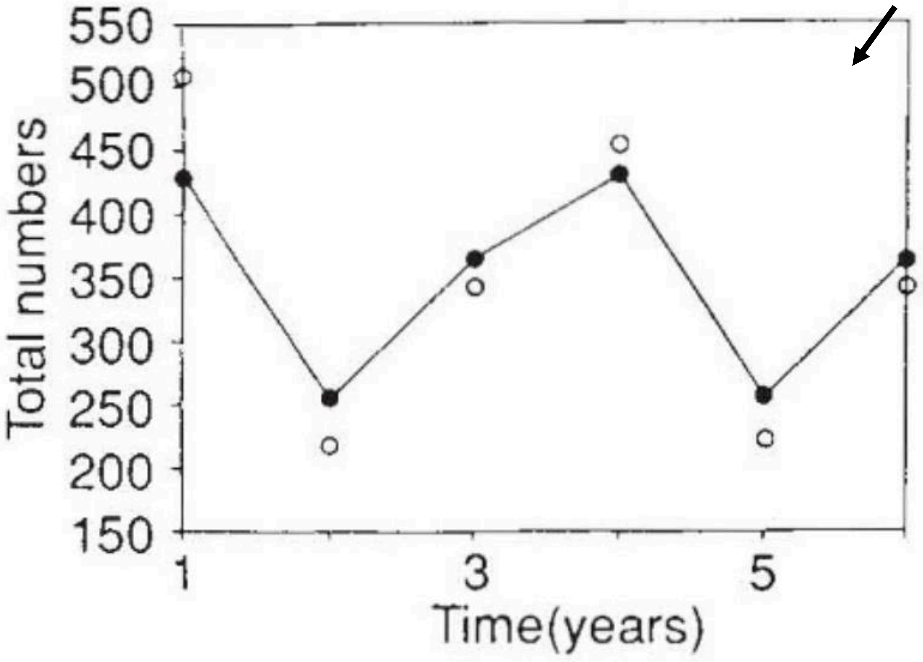
to forecast resource use

Why do we care how populations grow?

to understand past phenomena



Model compared against data



Overcompensation and population cycles in an ungulate

B. T. Grenfell, O. F. Price, S. D. Albon & T. H. Glutton-Brock

Nature **355**, 823–826 (1992) | Cite this article

The simplest population model

1. Populations are divided into compartments
2. Individuals within a compartment are homogenously mixed
3. Compartments and transition rates are determined by biological systems
4. Rates of transferring between compartments are expressed mathematically

Madagascar
(N)

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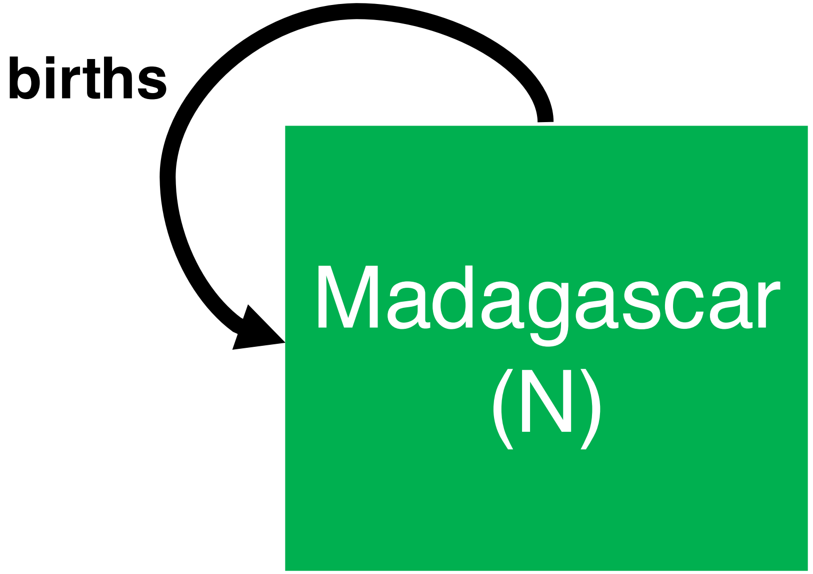

How does the population grow?

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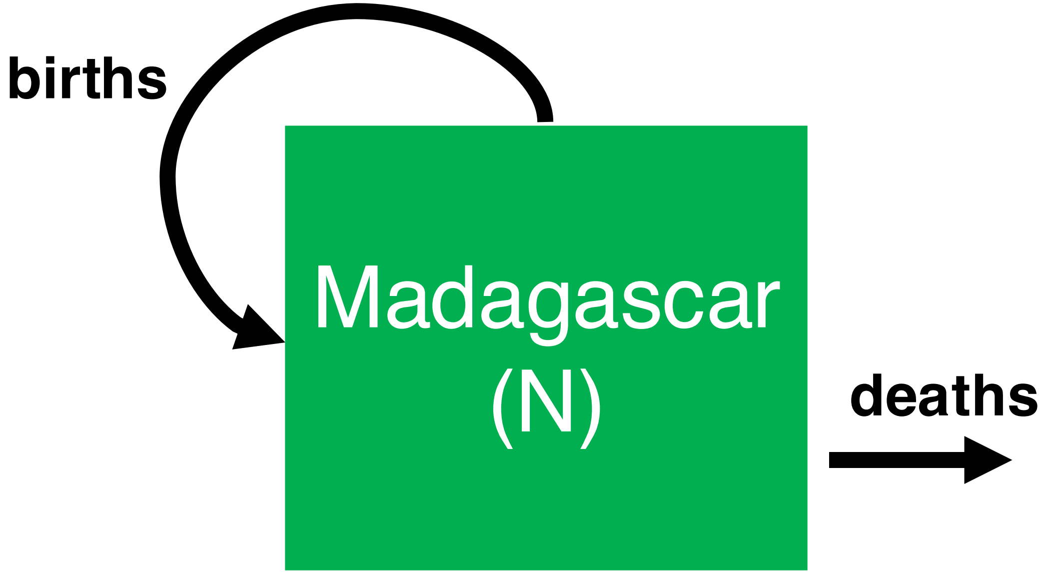

How does the population decrease?

The simplest population model

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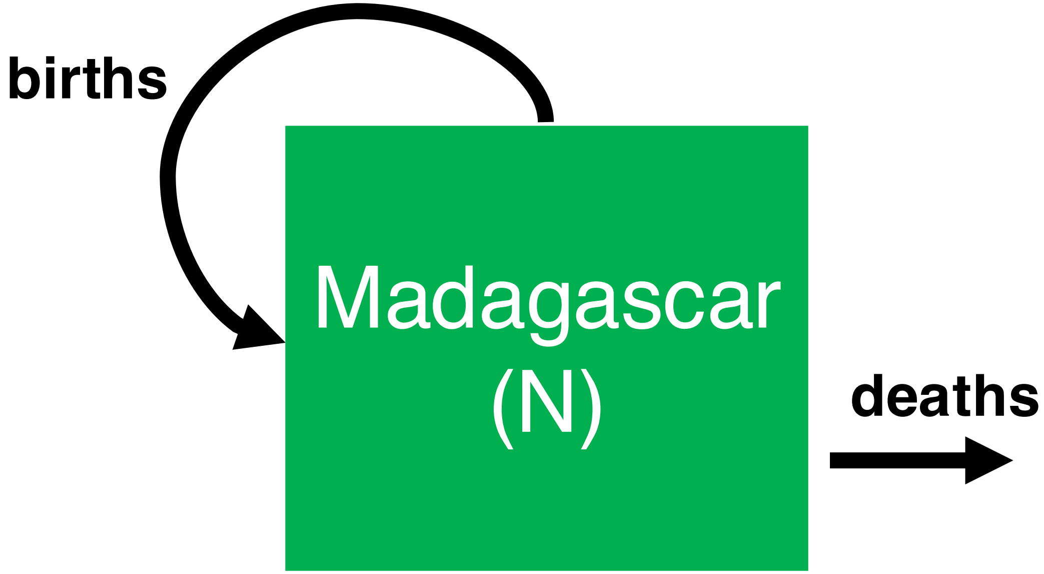

$$N_{t+1} = N_t + \text{births}*N_t - \text{deaths}*N_t$$
$$N_{t+1} = N_t + (\text{births}-\text{deaths})*N_t$$
$$N_{t+1} = N_t + R*N_t$$

R=geometric rate of increase

R = (births – deaths)

(pop grows @ R > 0 & declines @ R < 0)

The simplest population model

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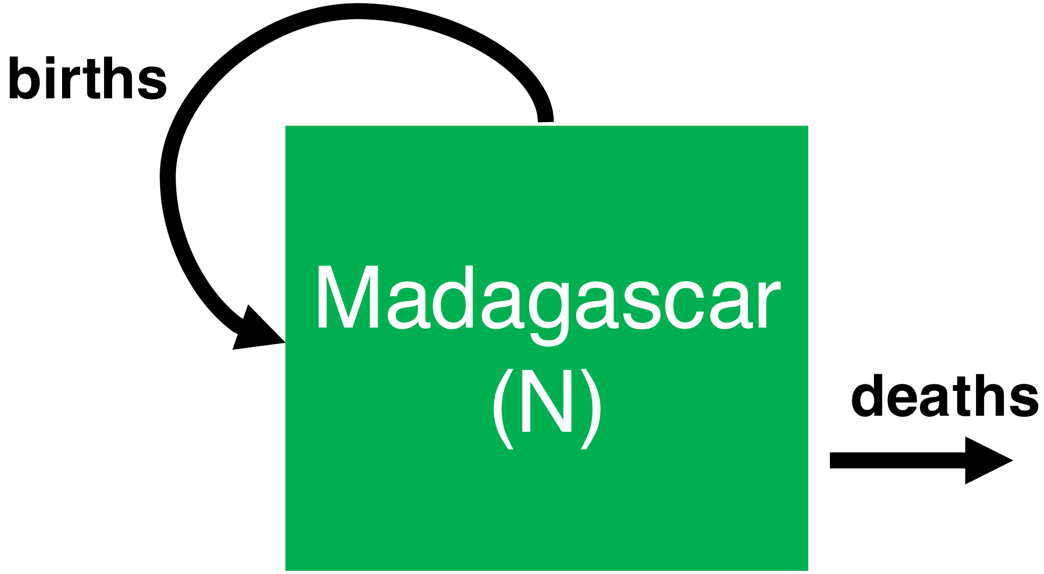

$$N_{t+1} = N_t + births*N_t - deaths*N_t$$

$$N_{t+1} = N_t + R*N_t$$

$$N_{t+1} = (1 + R)*N_t$$

$$N_{t+1} = \lambda*N_t$$

λ= population rate of increase (finite growth rate)

λ=1+R

The simplest population model

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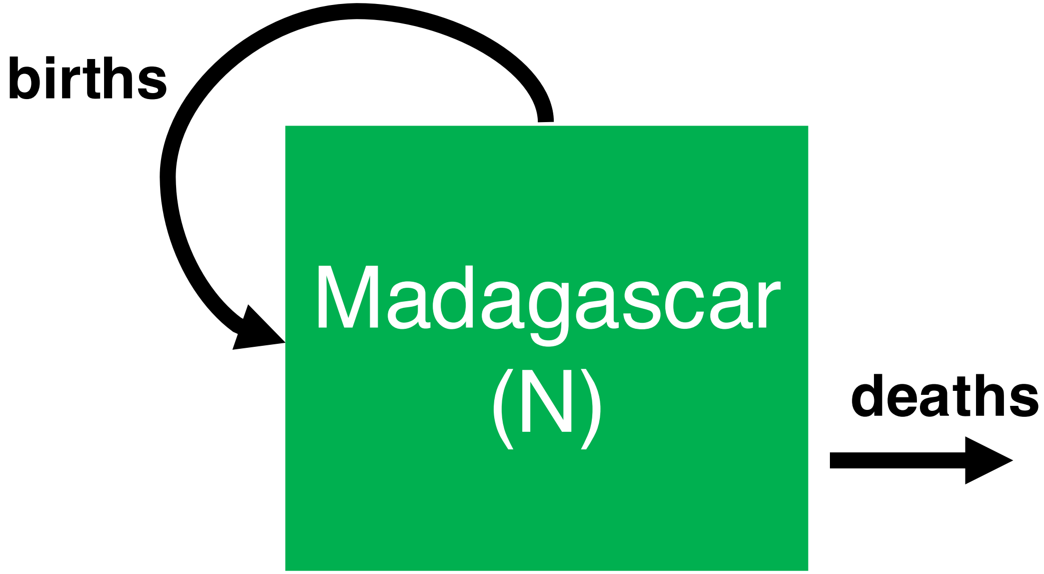

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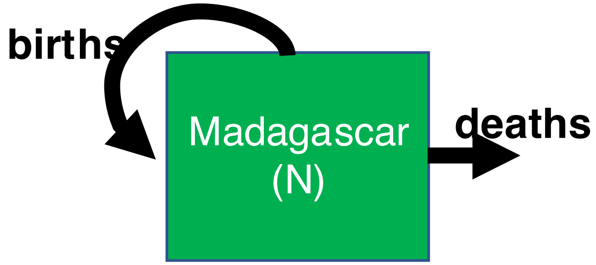
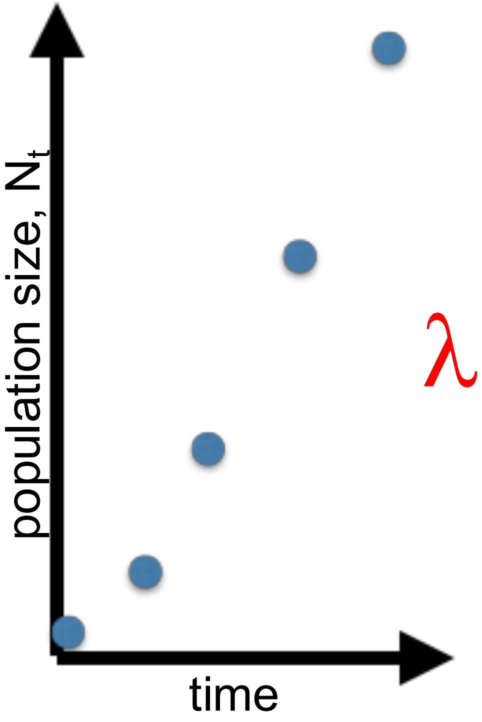
(pop grows @ $\lambda > 1$ & declines @ $\lambda < 1$)

Geometric growth



Geometric growth is measured in __discrete__ time

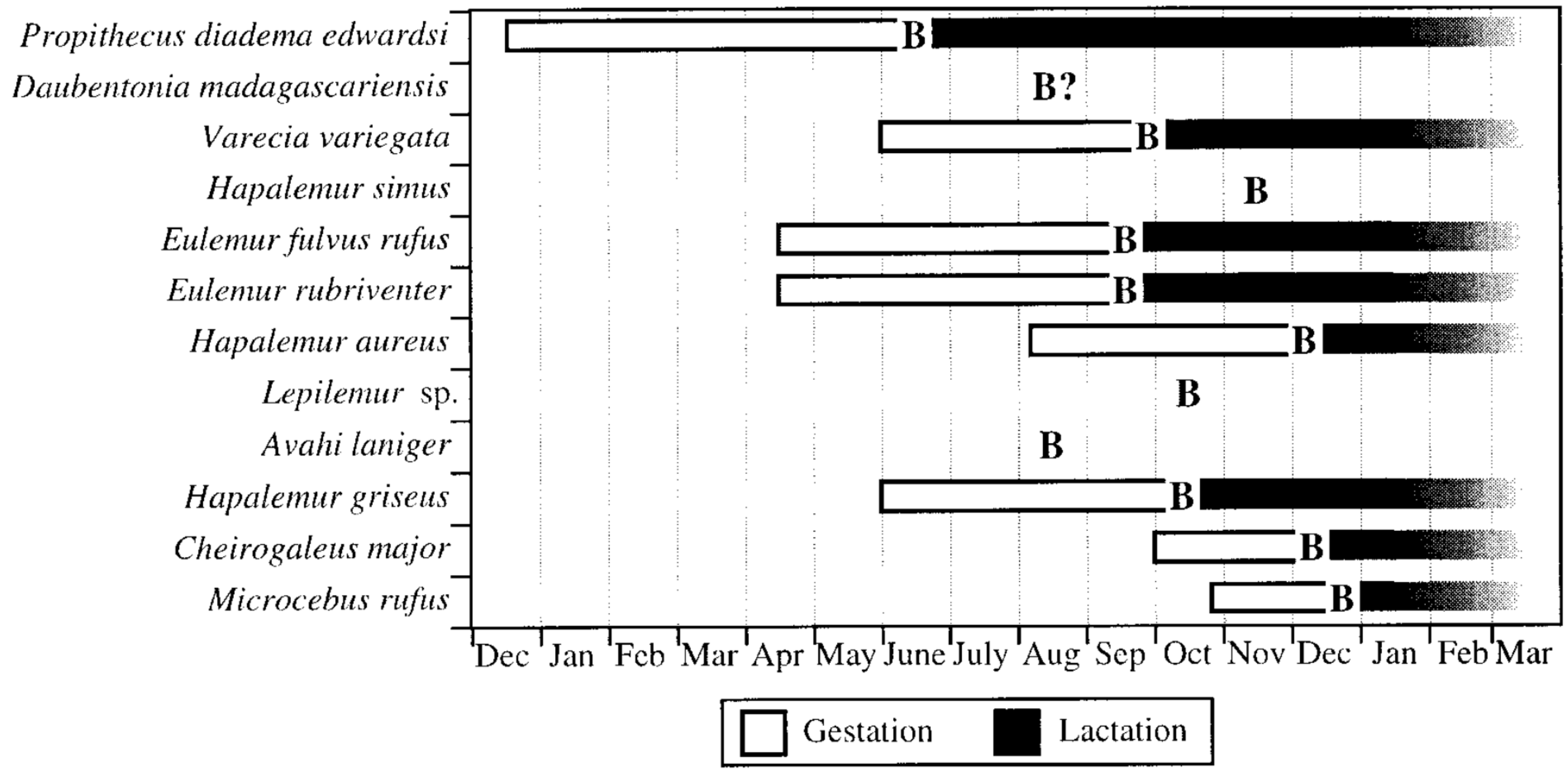


$$\lambda = N_{t+1}/N_t$$

$N_1 = \lambda N_0$
$N_2 = \lambda[\lambda N_0] = \lambda^2 N_0$
$N_3 = \lambda^3 N_0$
$N_t = \lambda^t N_0$

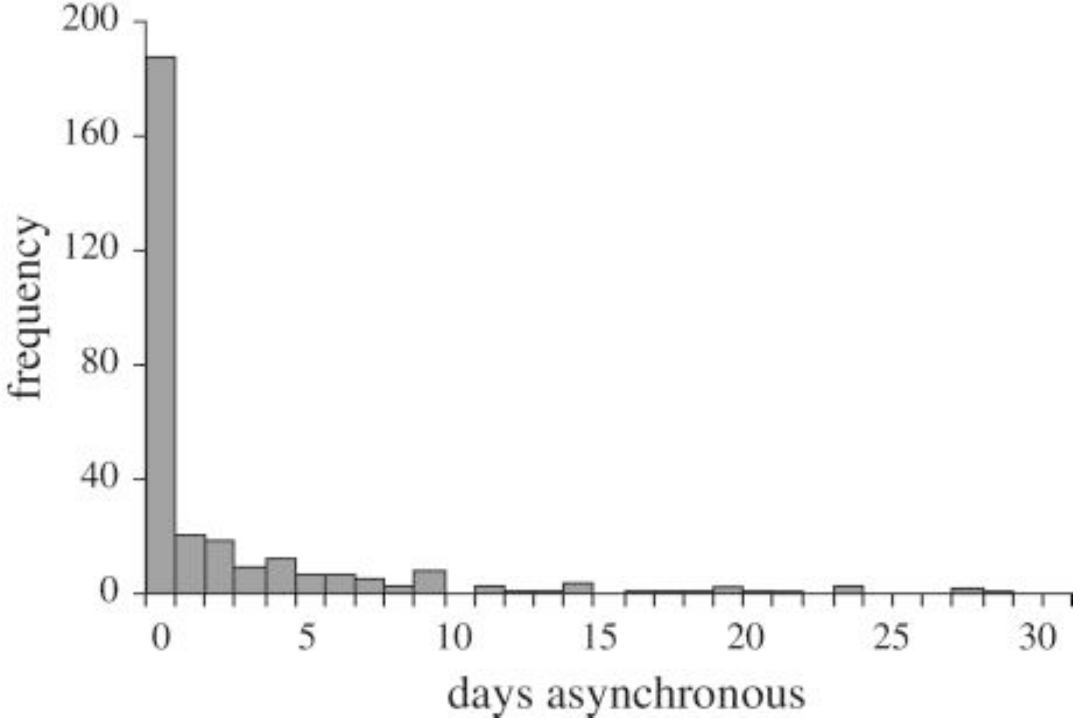
λ=population rate of increase

Is discrete time realistic for population growth?



Wright 1999. *Am J of Phys Anthr.*

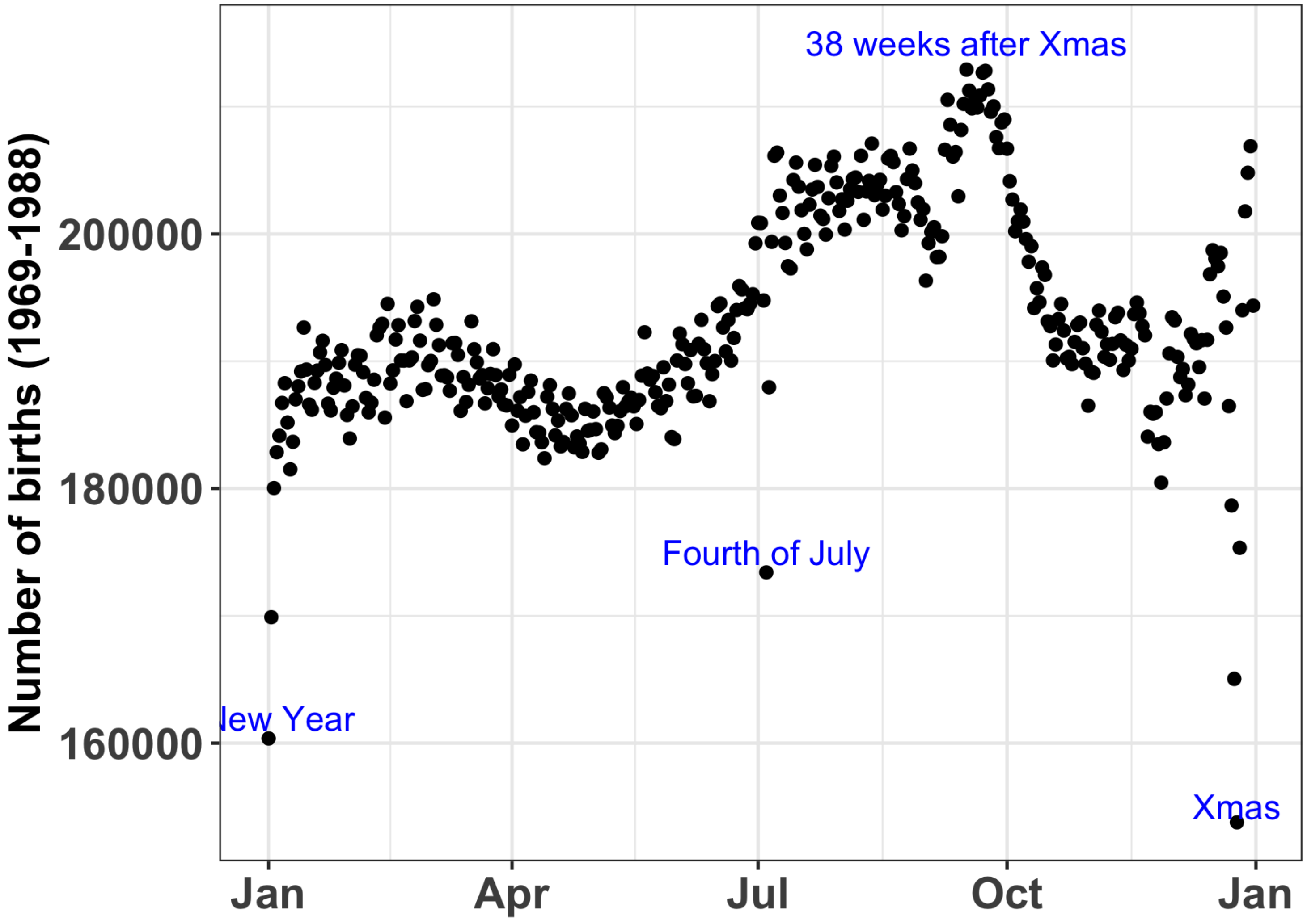
Is discrete time realistic for population growth?



64% of banded mongoose pups are born on the same day.

Hodge et al. 2011. *Biology Letters*.

Is discrete time realistic for population growth?

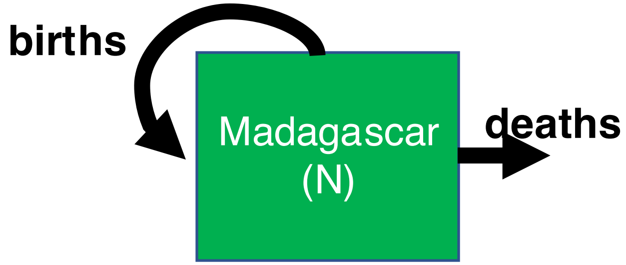


Human births by day in the US (1969-1988)

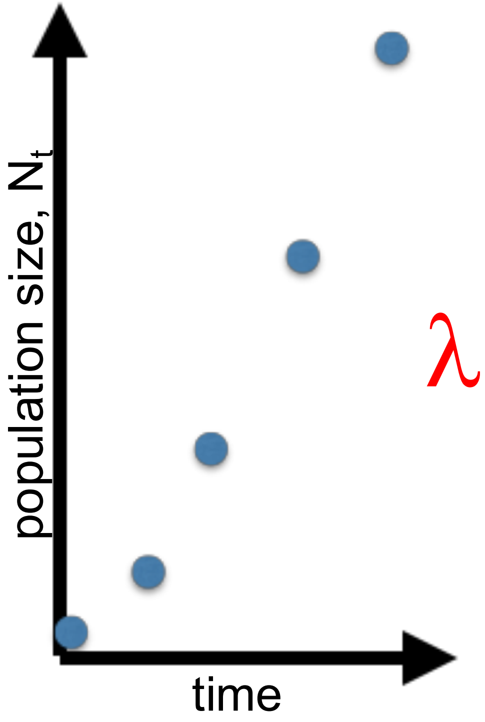
Is discrete time realistic for population growth?

- Many **continuously breeding populations** (including human) are better approximated by an assumption of a **continuous birth rate**
- **Univoltine/bivoltine species** may be better approximated by **discrete time growth rates**
  - Voltinism = number of broods (generations) produced per year
- Ultimately **the best choice model depends** on the **available data**!
    - For example, discrete time model may work well for human populations if censuses are conducted only annually

Geometric vs. exponential growth



Geometric growth



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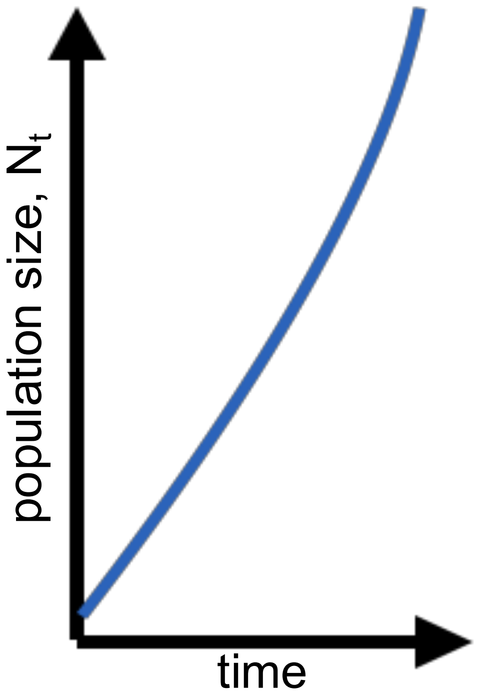
λ=population rate of increase

Exponential growth is measured in <u>continuous</u> time

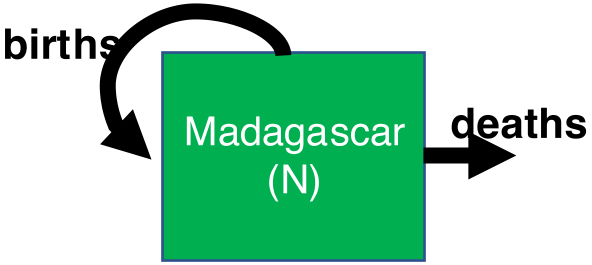


$dN(t)/dt = rN(t)$

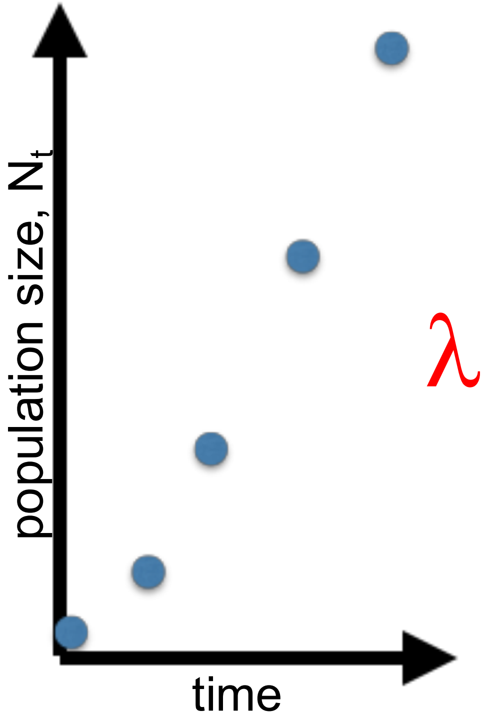
$$r = \frac{\ln\left(\frac{N_t}{N_0}\right)}{t}$$

r=intrinsic (instantaneous) rate of increase

Geometric vs. exponential growth



Discrete time



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$N_t = \lambda^t N_0$

Continuous time

$dN(t)/dt = rN(t)$

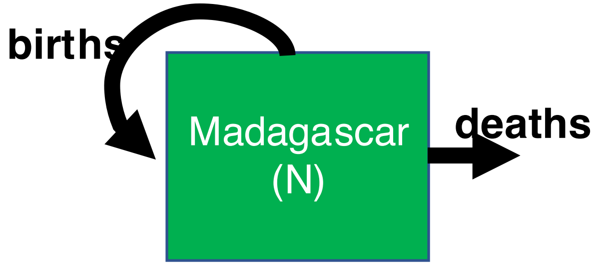
Separation of variables:
$dN(t)/N(t) = r\, dt$

Integrate both sides:
$\int dN(t)/N(t) = \int r\, dt$

By definition:
$\ln(N(t)) = rt + c$

Take exponentials:
$N(t) = e^{rt + c} = Ce^{rt}$
$N(t) = N(0)e^{rt}$

Geometric vs. exponential growth



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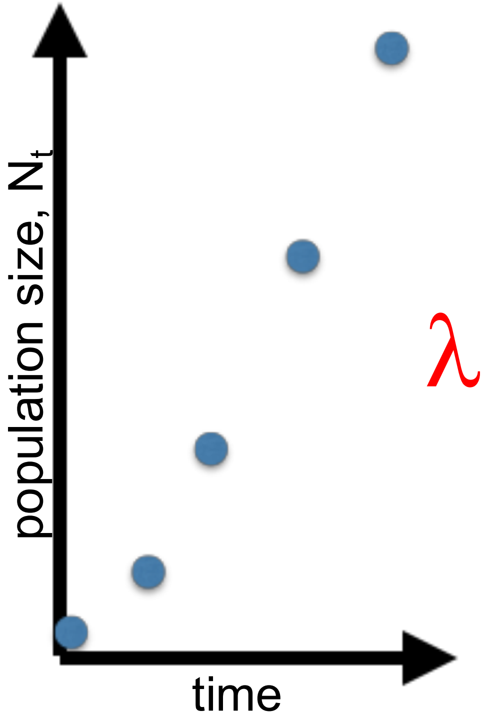

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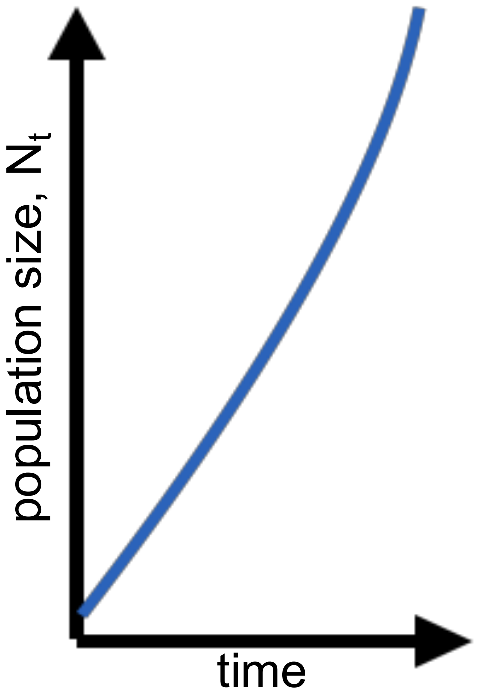
Continuous time



$N_t = N_0 e^{rt}$

$$r = \frac{\ln\left(\frac{N_t}{N_0}\right)}{t}$$

λ=population rate of increase

(pop grows @ $\lambda > 1$ & declines @ $\lambda < 1$)

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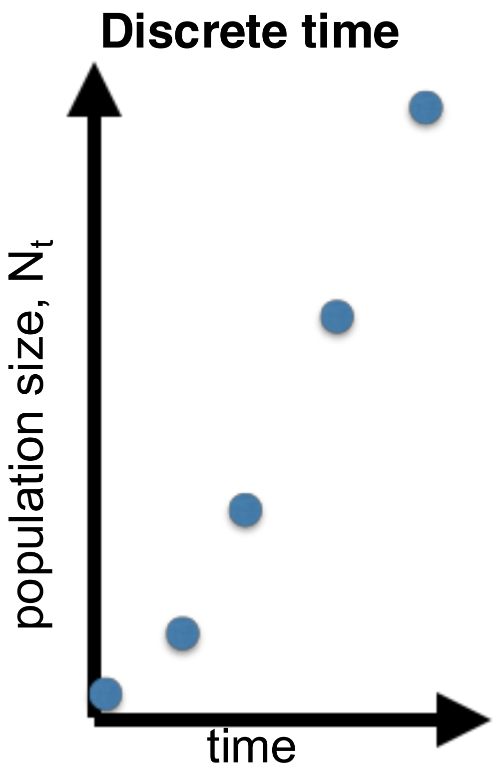
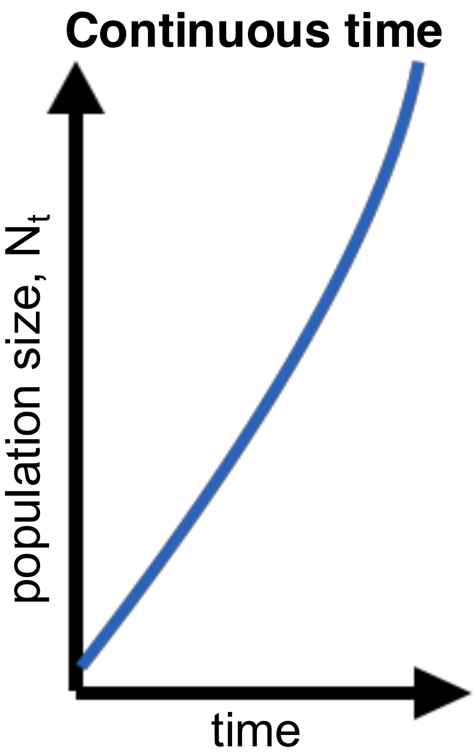
Geometric vs. exponential growth

geometric
$N_t = \lambda^t N_0$

exponential
$N_t = N_0 e^{rt}$

$\lambda^t N_0 = N_0 e^{rt}$
$\lambda^t = e^{rt}$
$\lambda = e^r$





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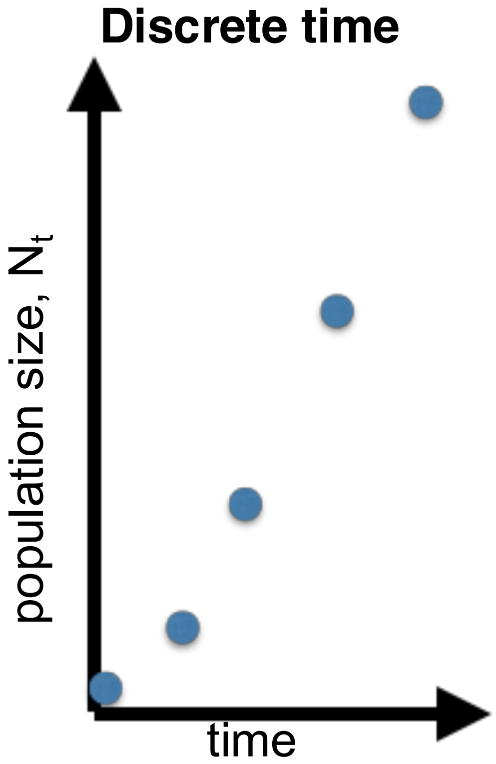
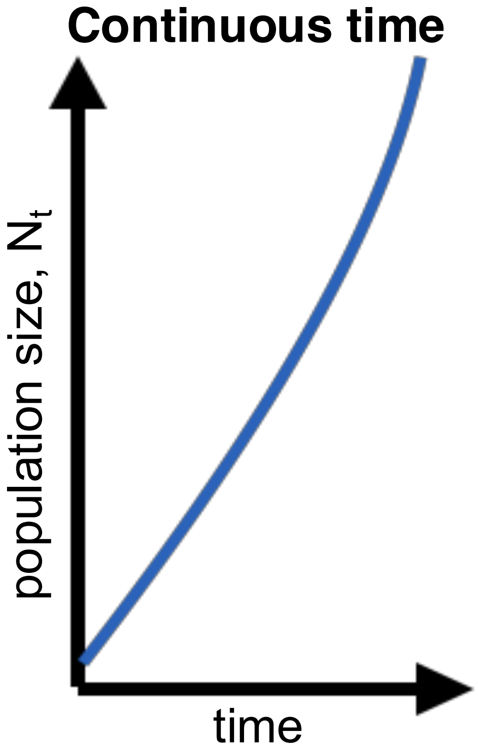




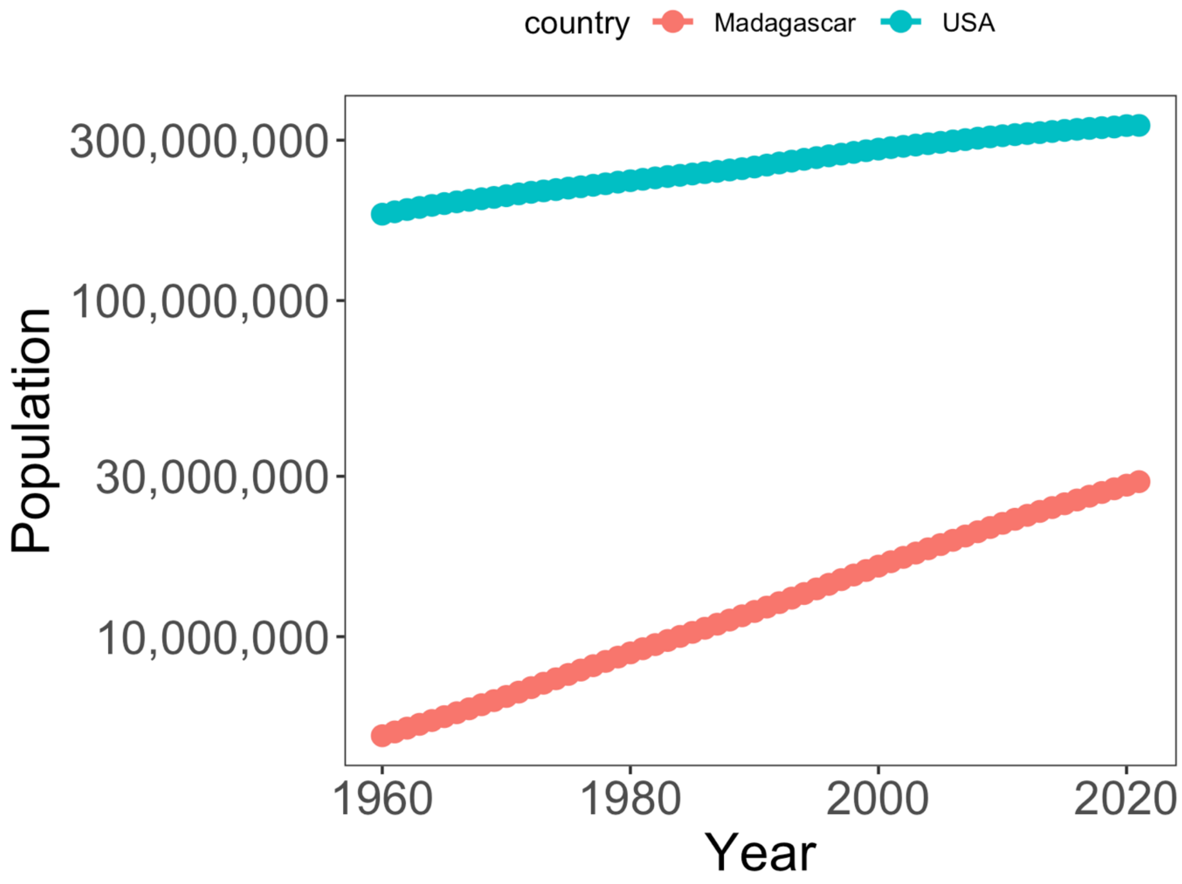
Continuous models can be discretized.

Discrete models can be approximated in continuous time.

How to choose what to do?

The answer depends on the data and the question at hand!

Geometric vs. exponential growth



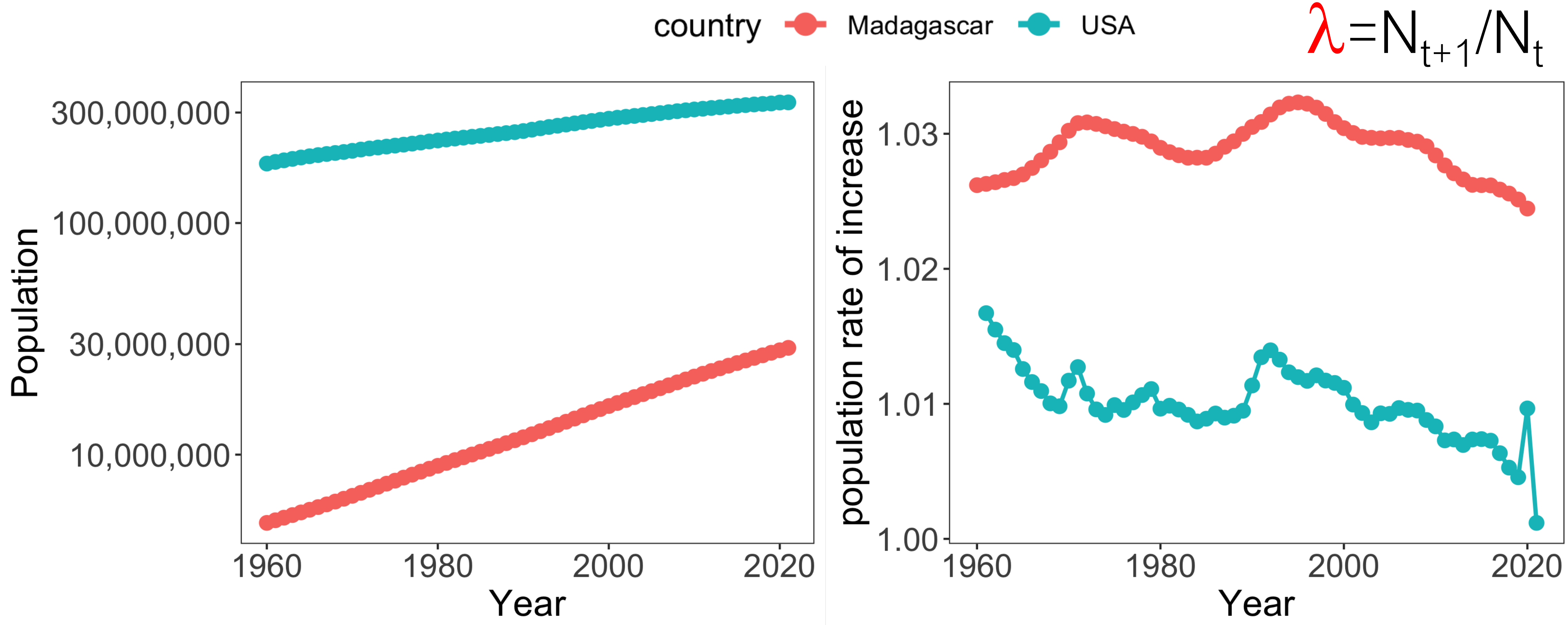
$\lambda = N_{t+1}/N_t$

where t = 1 year

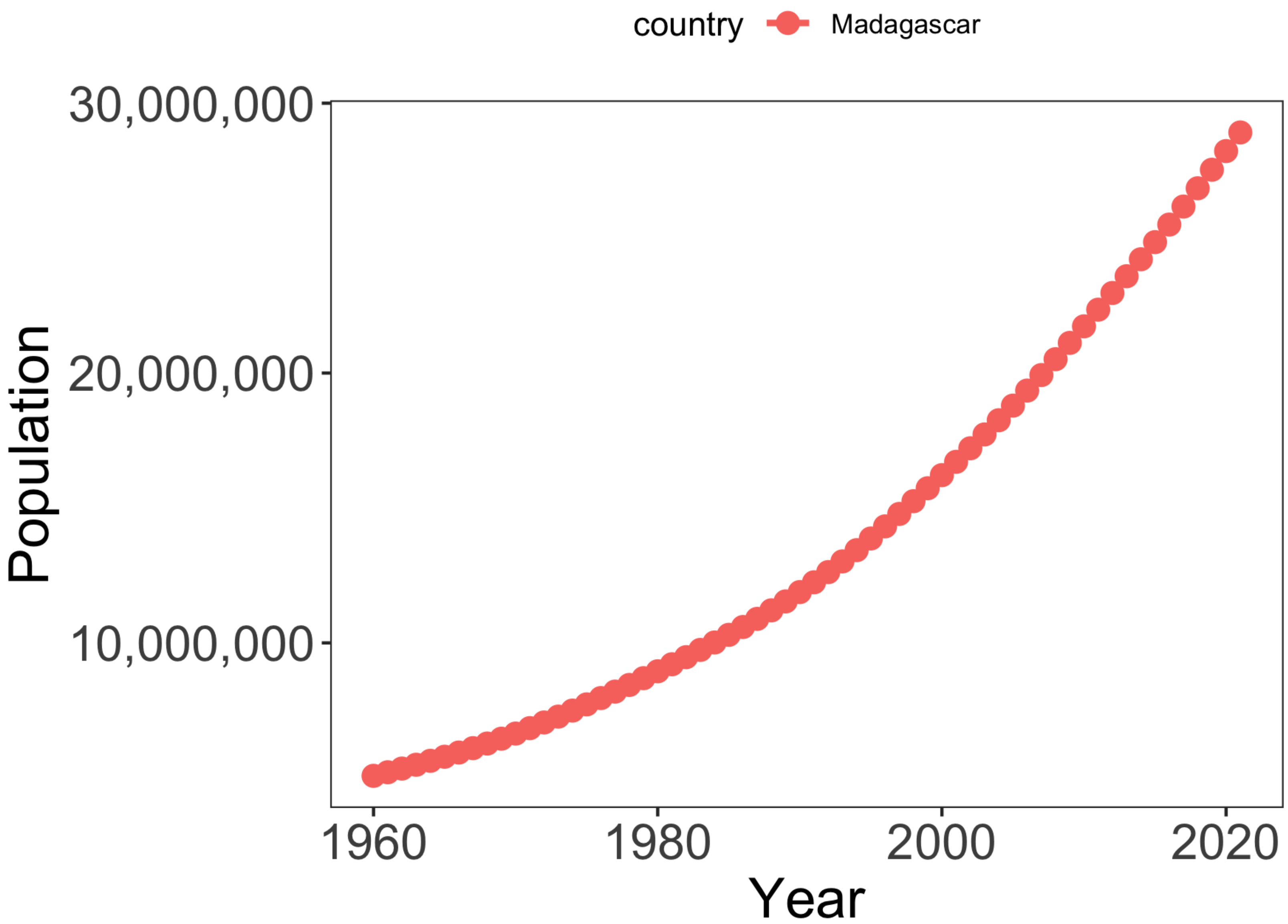
Which country has the higher growth rate?

Geometric vs. exponential growth

$\lambda = N_{t+1}/N_t$



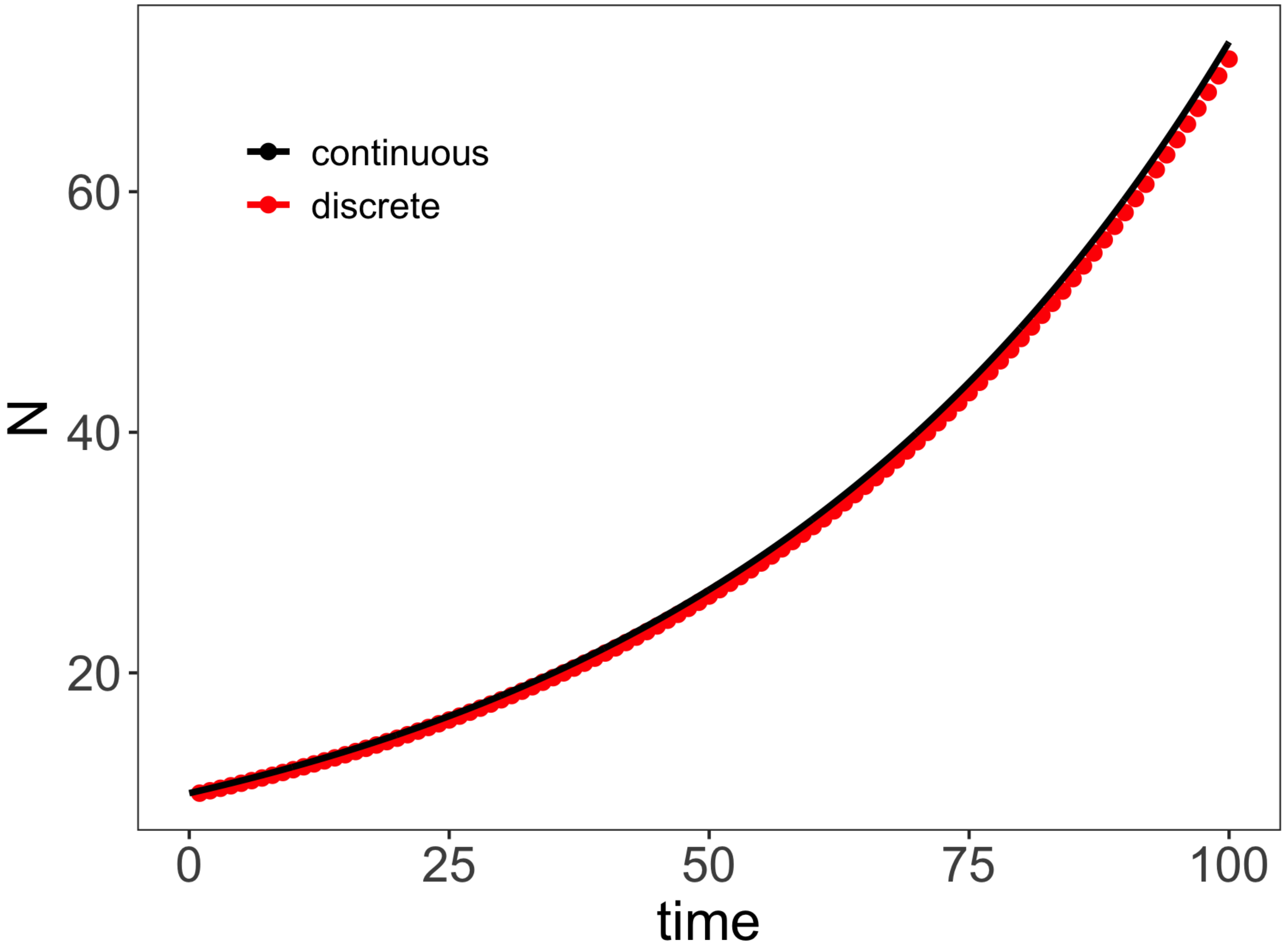
Geometric growth approximation of Madagascar's population



$\lambda = N_{t+1}/N_t$

mean λ = 1.02

Projecting population under geometric vs. exponential growth

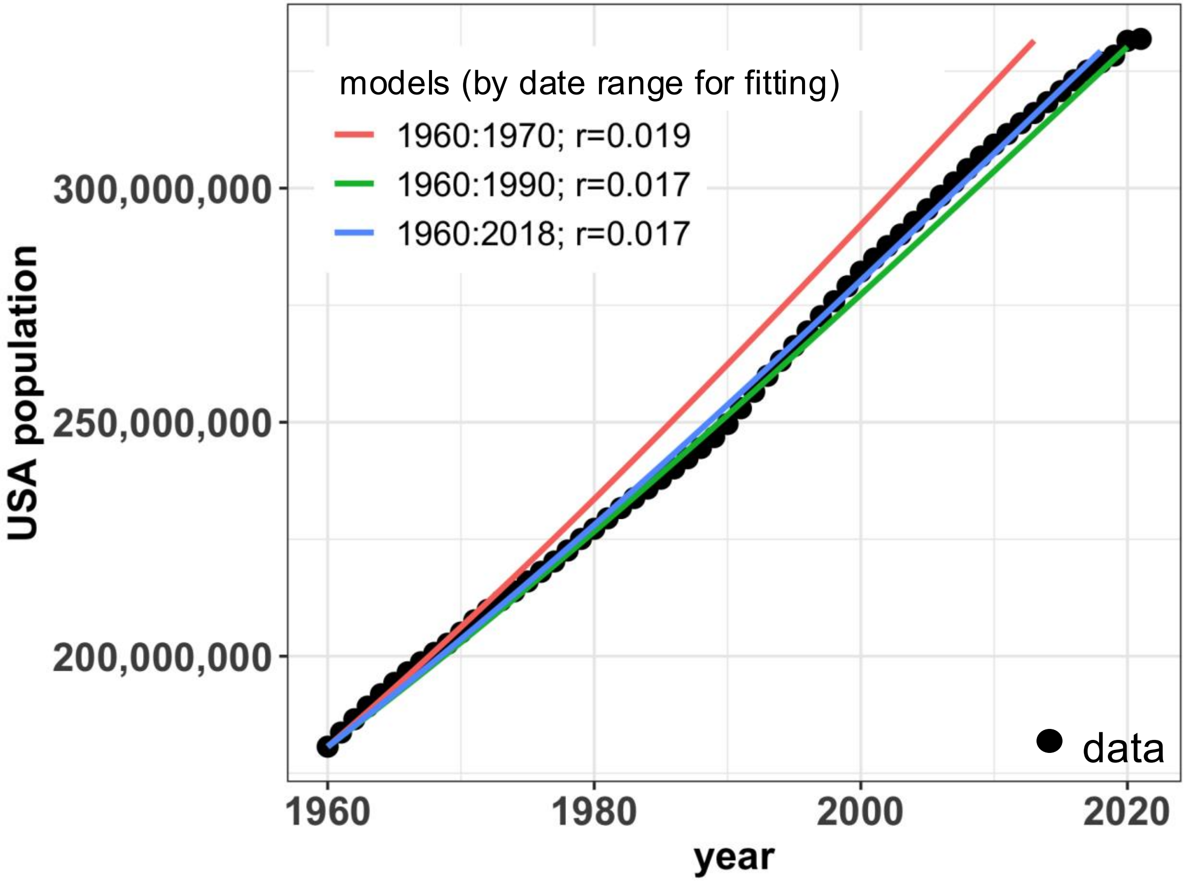


Geometric: λ = 1.02

Exponential: r = ln(1.02) = 0.02

Models are similar over short time horizons and/or when discrete timesteps are small!

We can estimate growth rates by 'fitting' a model to data



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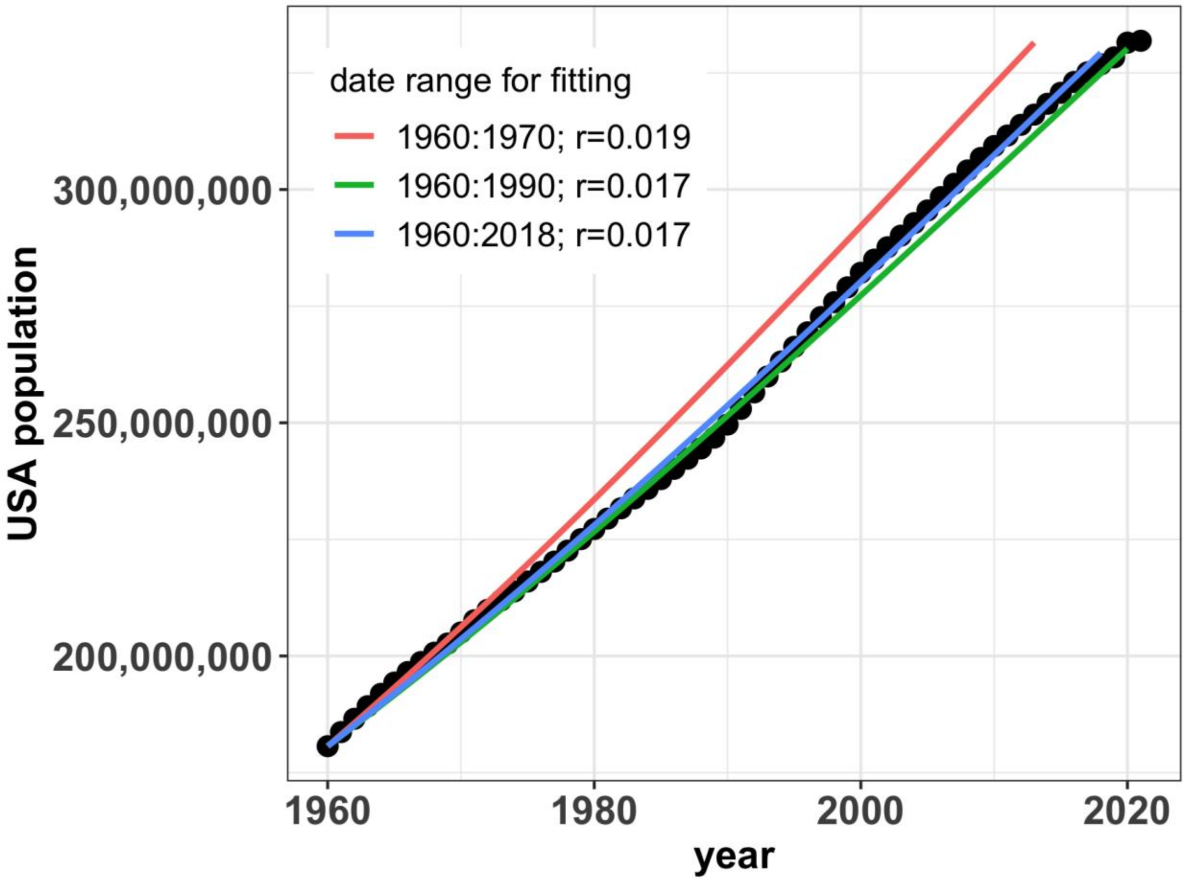
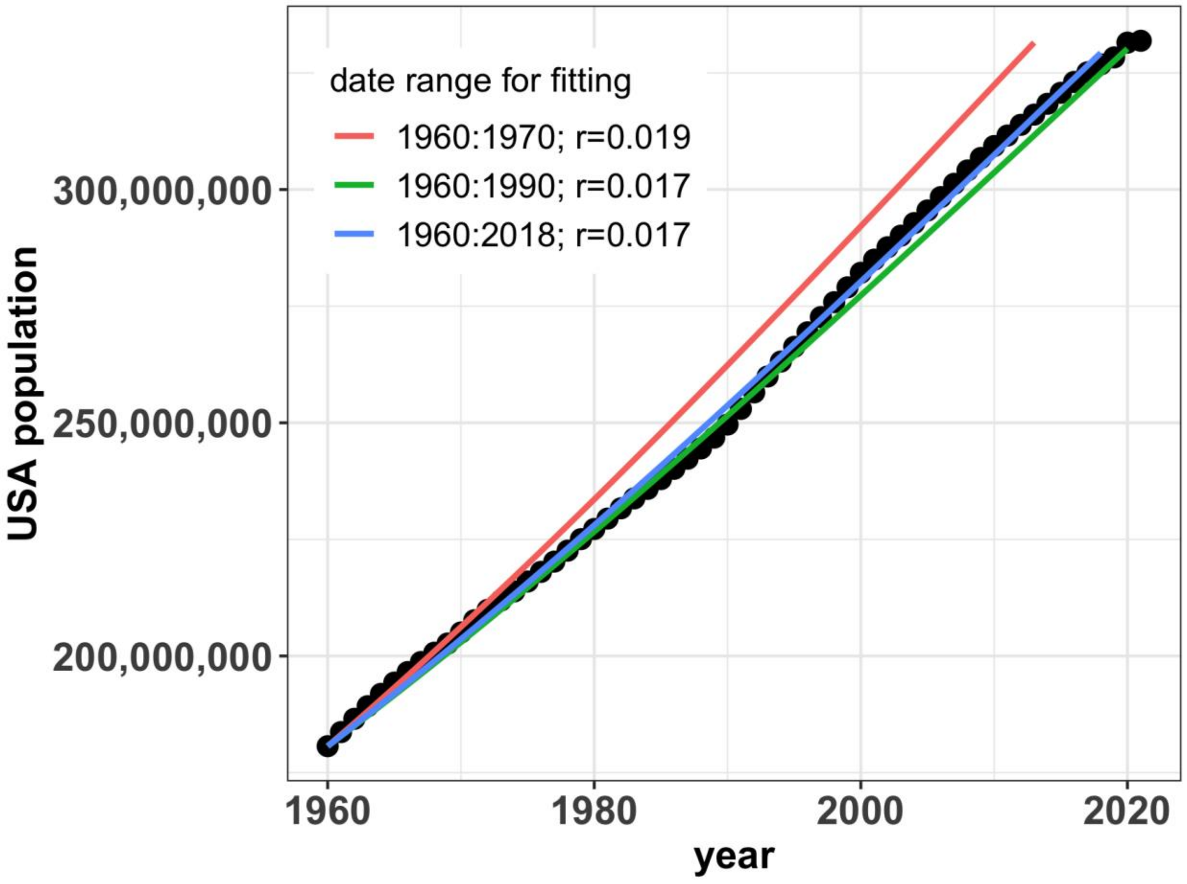
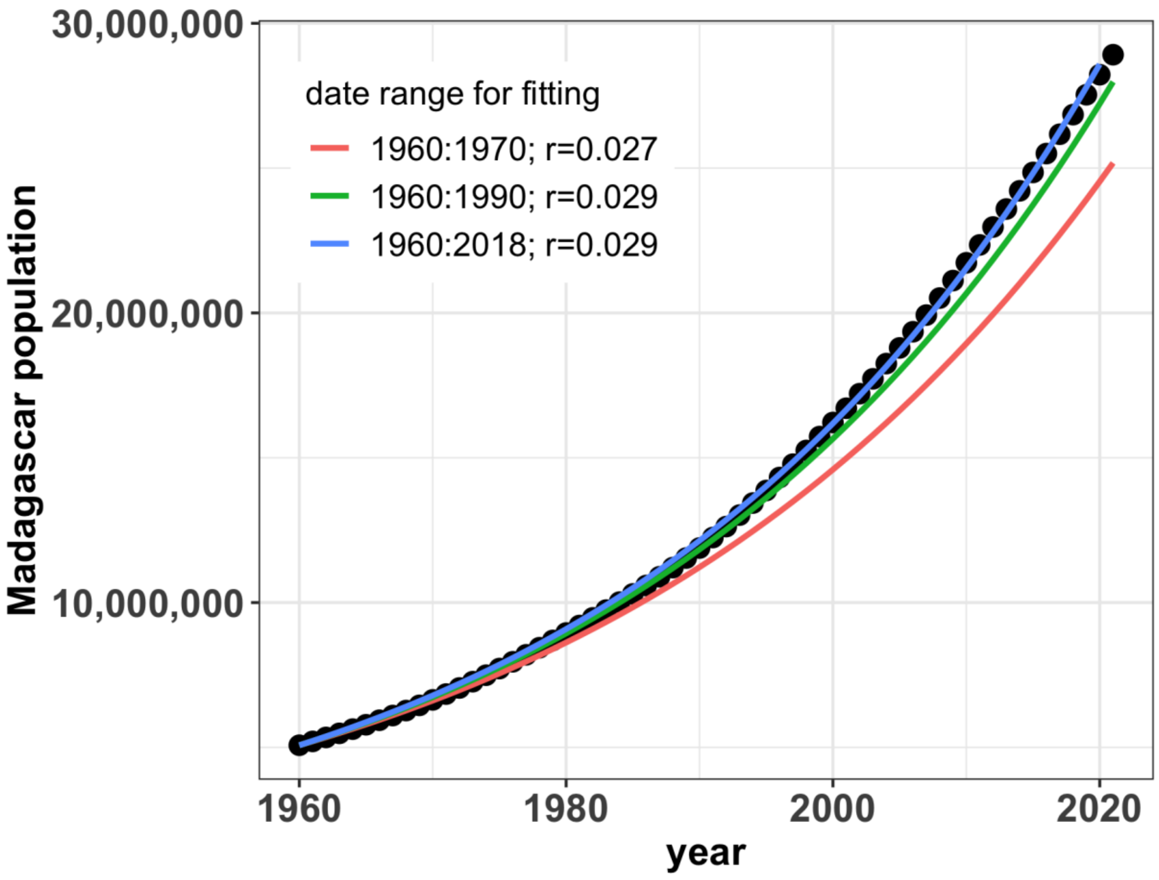


US growth rates slowed over the time period!

Model fits to data from earlier years **overestimate** current population growth rates

We can estimate growth rates by ‘fitting’ a model to data

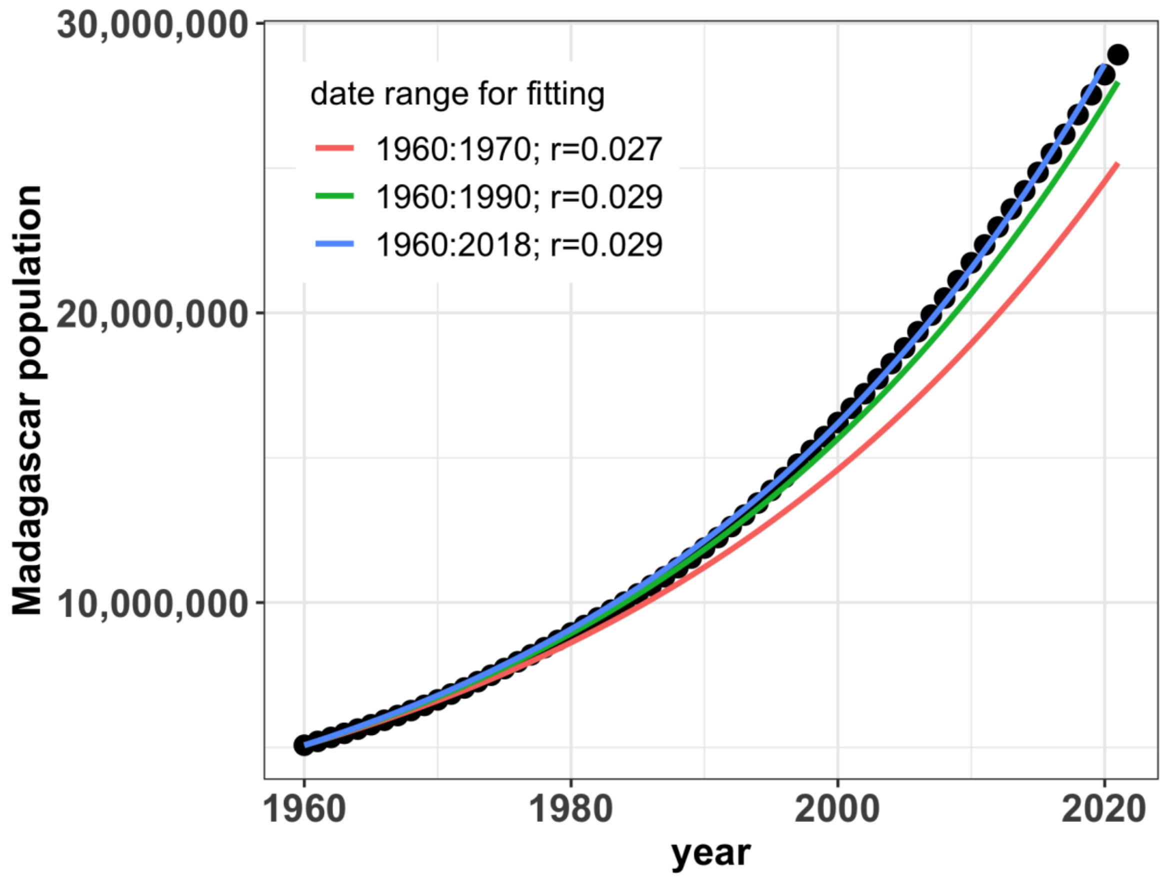




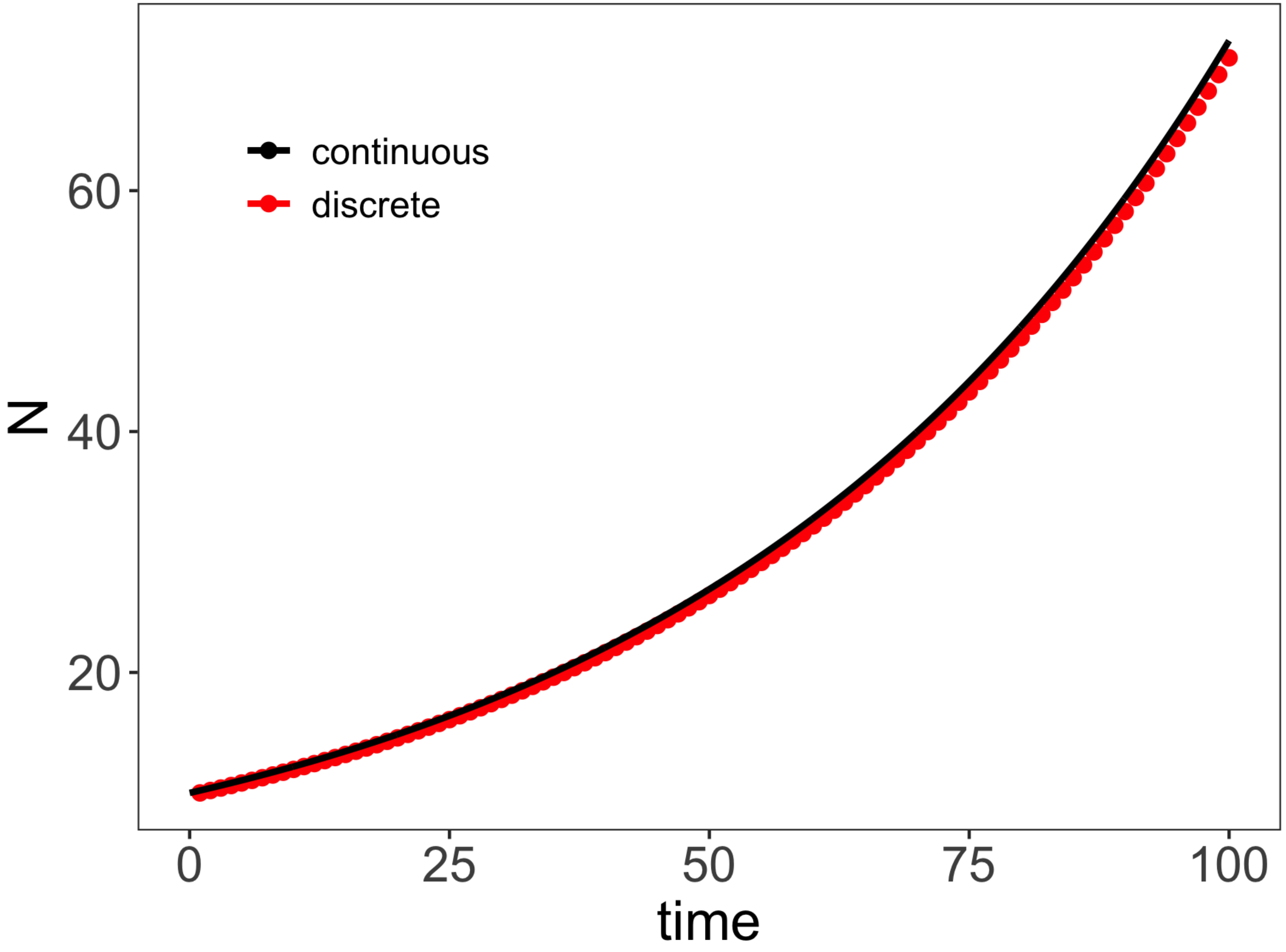
We can estimate growth rates by ‘fitting’ a model to data

Madagascar growth rates accelerated over the time period!

Model fits to data from earlier years **underestimate** current population growth rates



Both geometric and exponential growth are unchecked.



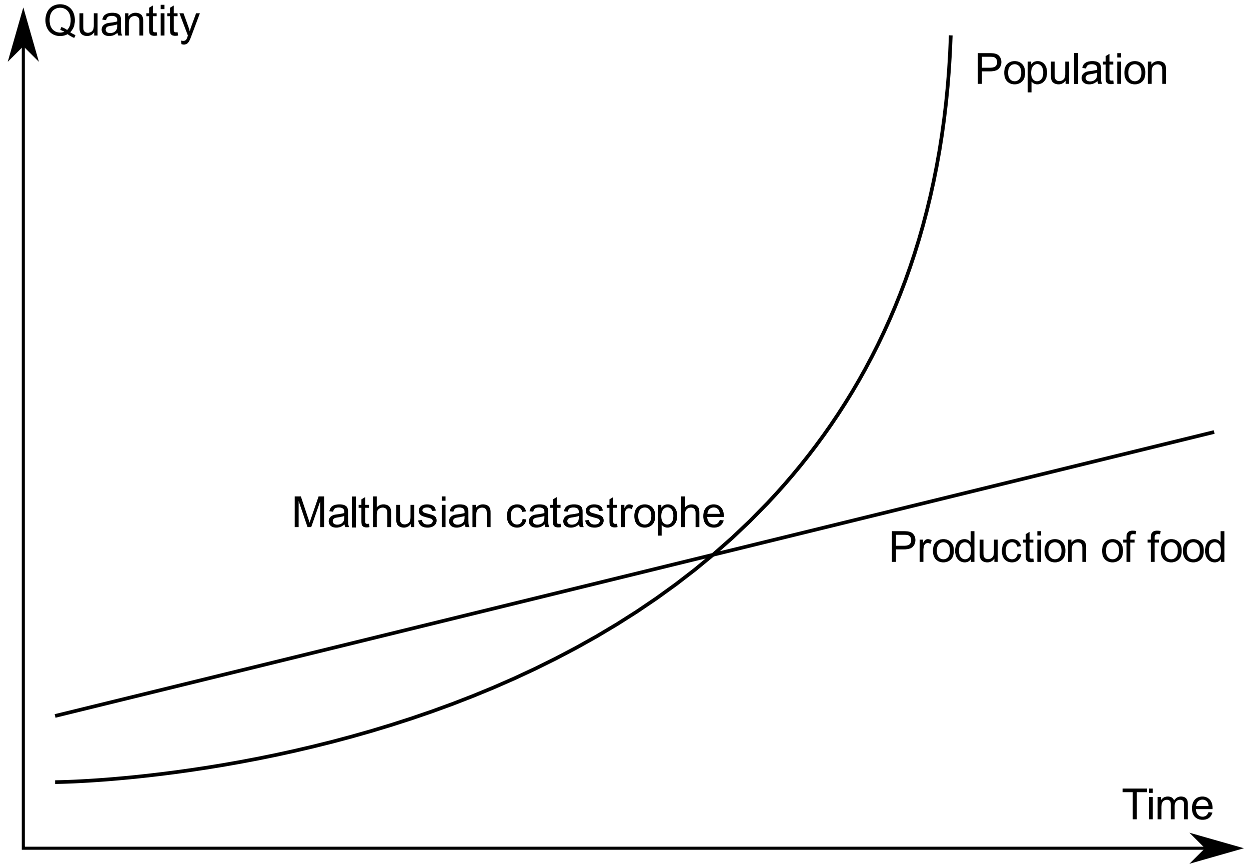
Malthus proposed some limits to population growth

"[. . .] the power of population is indefinitely greater than the power in the earth to produce subsistence for man. Population, when unchecked, increases in a geometrical ratio. Subsistence increases only in an arithmetical ratio. A slight acquaintance with numbers will shew the immensity of the first power in comparison of the second. By that law of our nature which makes food necessary to the life of man, the effects of these two unequal powers must be kept equal. This implies a strong and constantly operating check on population from the difficulty of subsistence. This difficulty must fall somewhere; and must necessarily be severely felt by a large portion of mankind."

- Thomas Malthus (1798)

An Essay on the Principle of Population as it Effects the Future Improvement of Society, With Remarks on the Speculations of Mr Godwin, Mr. Condorcet and Other Writers

Malthus proposed some limits to population growth



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An Essay on the Principle of Population as it Effects the Future Improvement of Society, With Remarks on the Speculations of Mr. Godwin, Mr. Condorcet and Other Writers

Problem!
No basis for assumption of arithmetical ratio for food.

Logistic growth equation

change in population abundance per unit time

$$\frac{dN}{dt} = rN\left(1 - \frac{N}{K}\right)$$

population size

intrinsic growth rate

carrying capacity

Population growth slows as abundance (**N**) approaches **carrying capacity** (**K**).

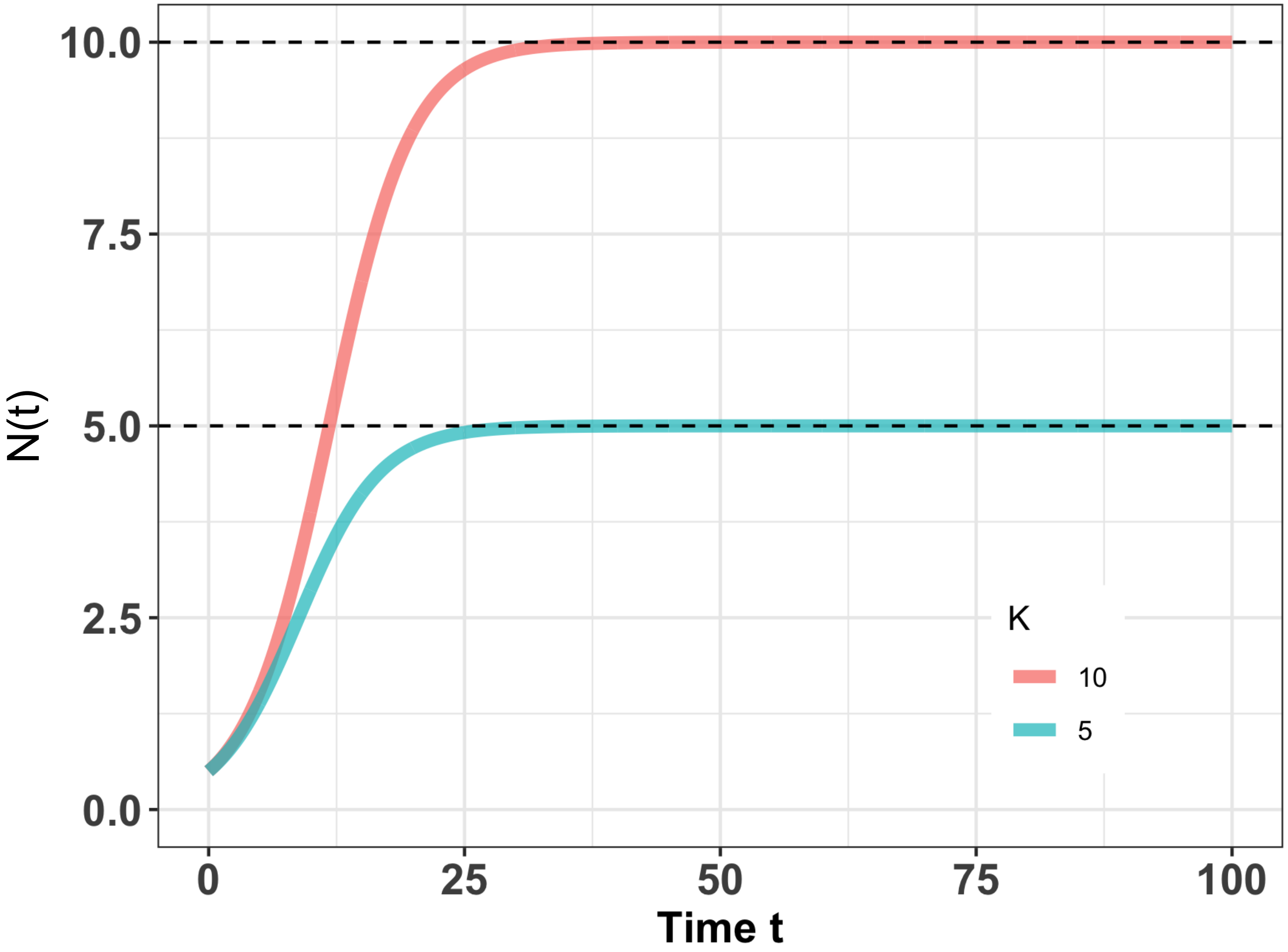
Population growth is **density-dependent**.

"We shall not insist on the hypothesis of geometric progression, given that it can hold only in very special circumstances; for example, when a fertile territory of almost unlimited size happens to be inhabited by people…"

- Pierre-Francois Verhulst (1838)

Logistic growth equation

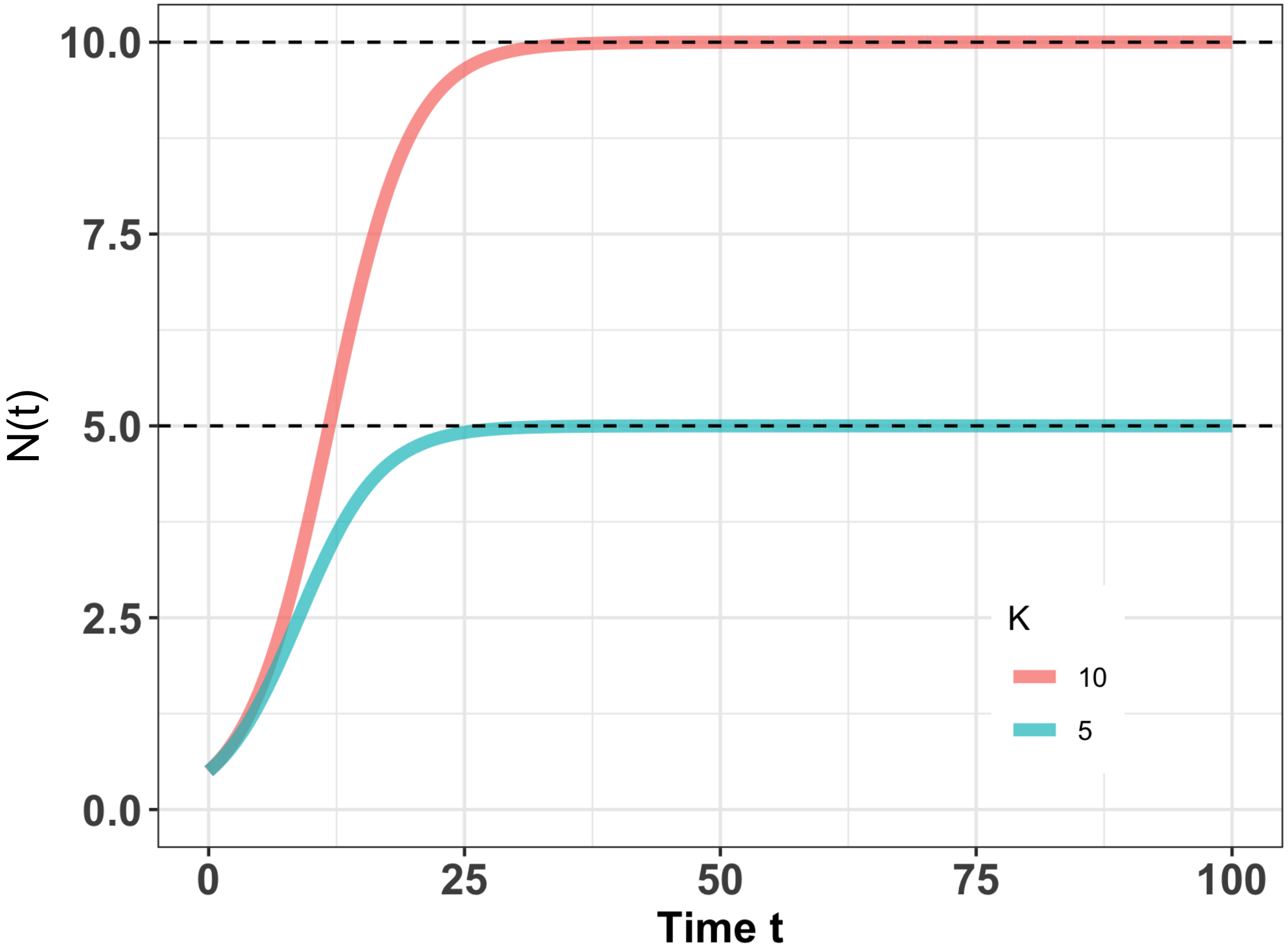
$$\frac{dN}{dt} = rN\left(1 - \frac{N}{K}\right)$$



Population growth slows to 0 as N approaches **K**, or – in other words – as the total population size approaches **carrying capacity**.

Logistic growth equation

$$\frac{dN}{dt} = rN\left(1 - \frac{N}{K}\right)$$



Carrying capacity= maximum population size an environment can sustain indefinitely.

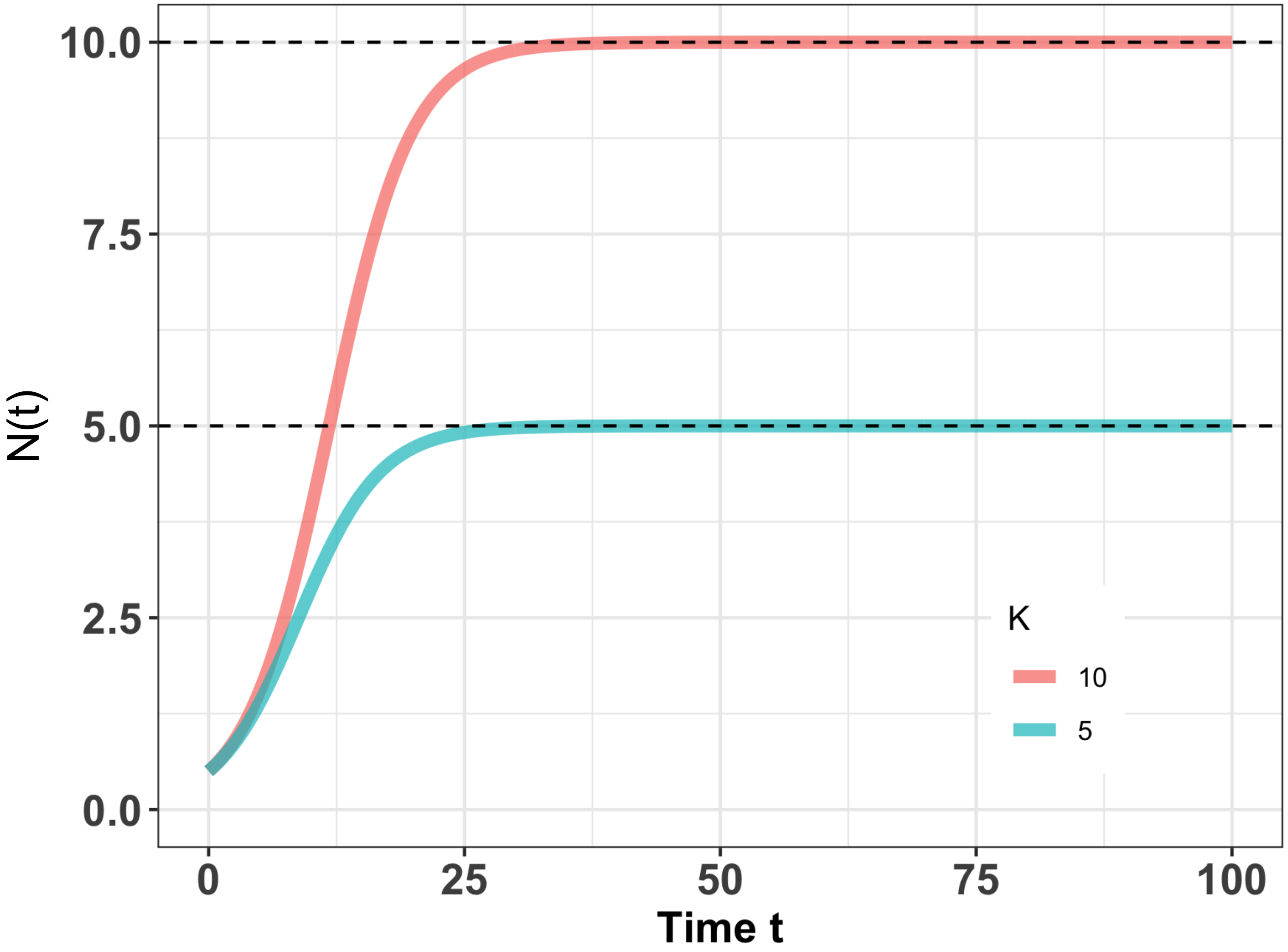
Ecology = organisms interacting with each other and the **environment**

K can change!

‘**r**’ vs. ‘**K**’-selected species

Logistic growth and equilibrium

$$\frac{dN}{dt} = rN\left(1 - \frac{N}{K}\right)$$



$$\frac{dN}{dt} = 0$$

When population size is not changing, the population is said to be at **equilibrium**.

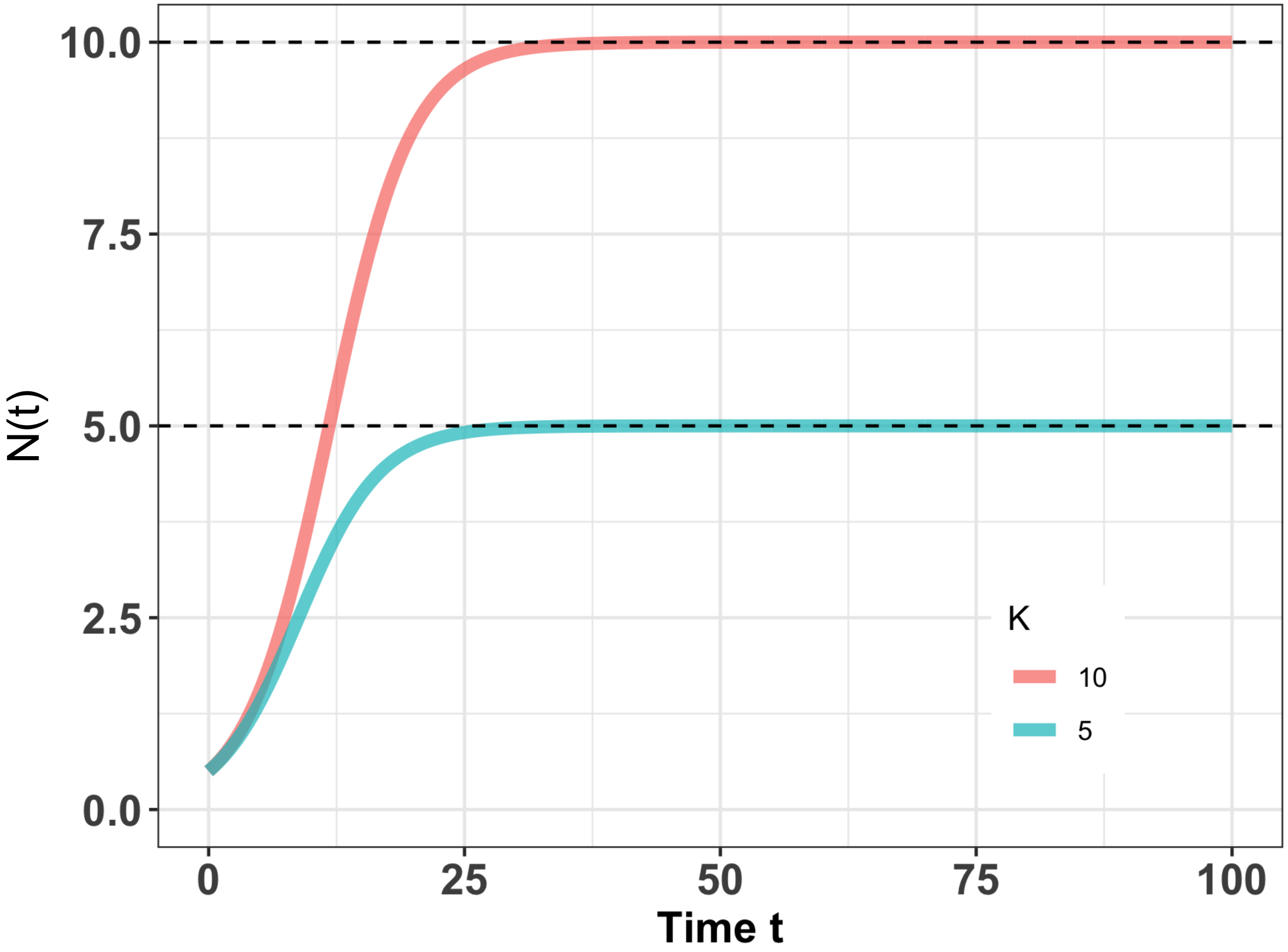
Logistic growth and equilibrium

$$\frac{dN}{dt} = rN\left(1 - \frac{N}{K}\right)$$

$$\frac{dN}{dt} = 0$$

$$0 = rN\left(1 - \frac{N}{K}\right)$$

$$N = K$$



A little algebra shows that the population at carrying capacity is at **equilibrium**.

Logistic growth and equilibrium

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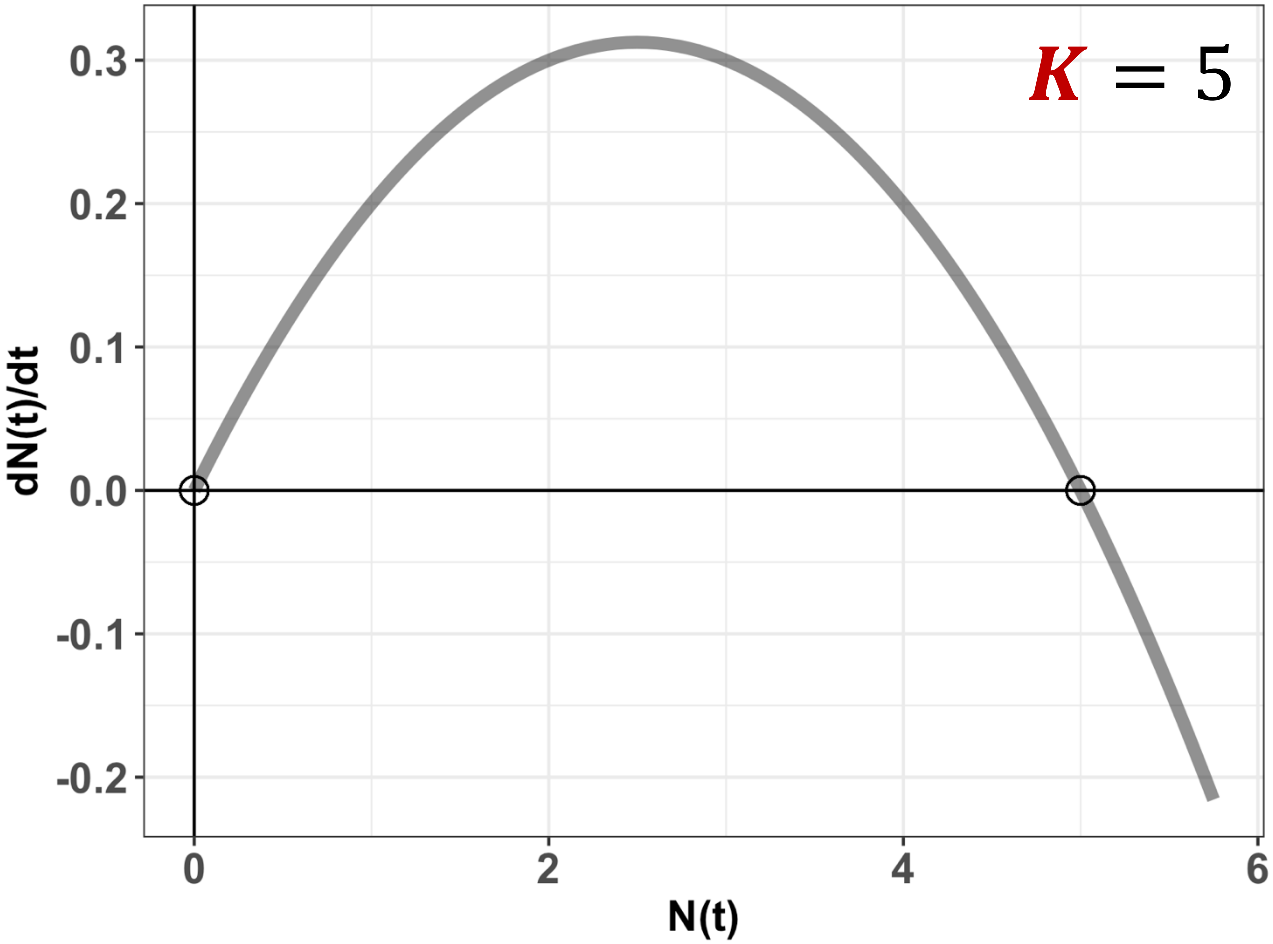

$$\left.\begin{array}{l} N = K \text{ or} \\ N = 0 \end{array}\right\}$$ logistic growth **equilibria**

What is **K**?

Logistic growth and equilibrium

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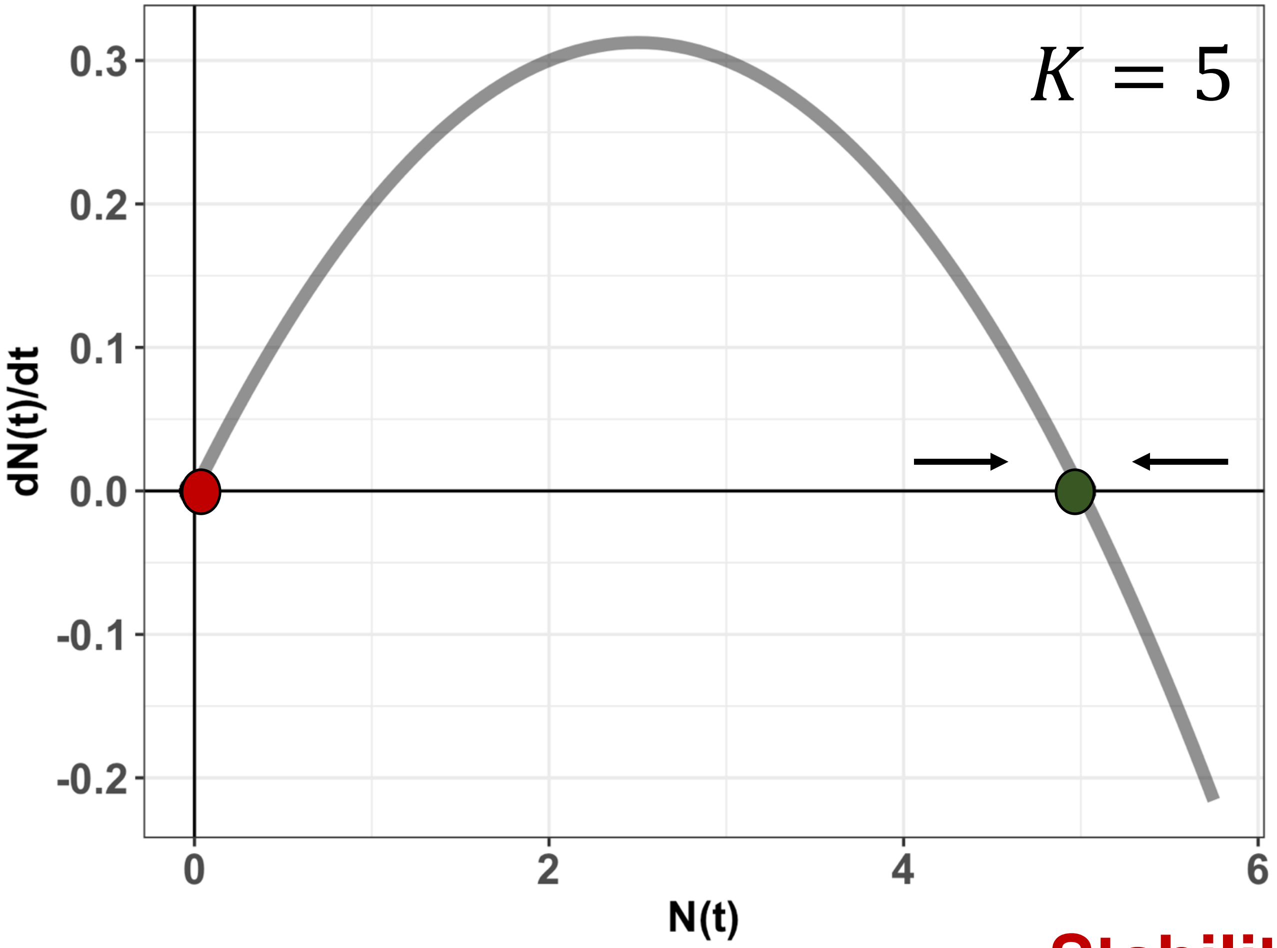

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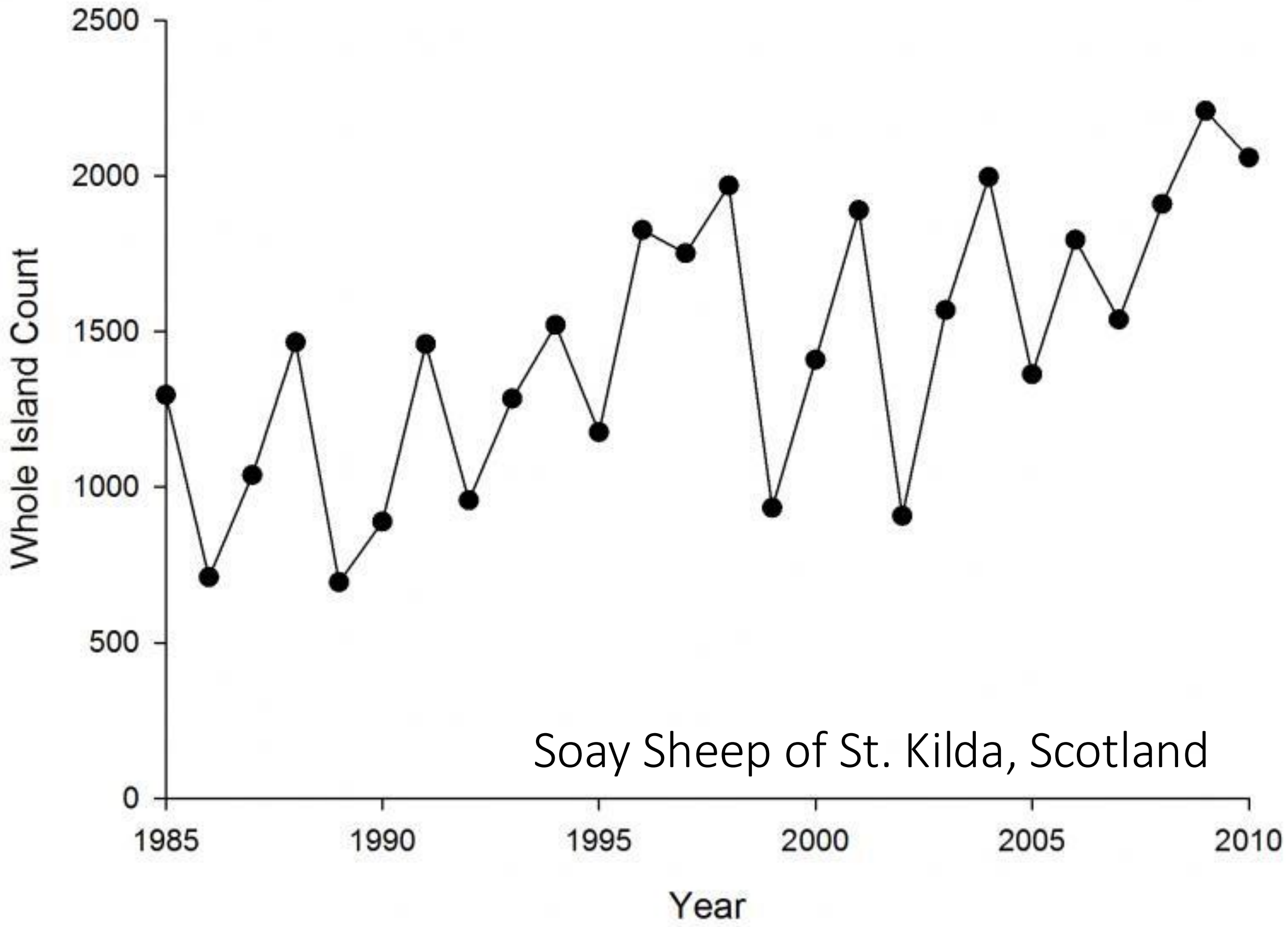
$$\left.\begin{array}{l} N = K \text{ or} \\ N = 0 \end{array}\right\}$$ logistic growth **equilibria**

Stability: If the population is perturbed, will it return to equilibrium?

Many populations will fluctuate above or below carrying capacity.



Soay Sheep of St. Kilda, Scotland

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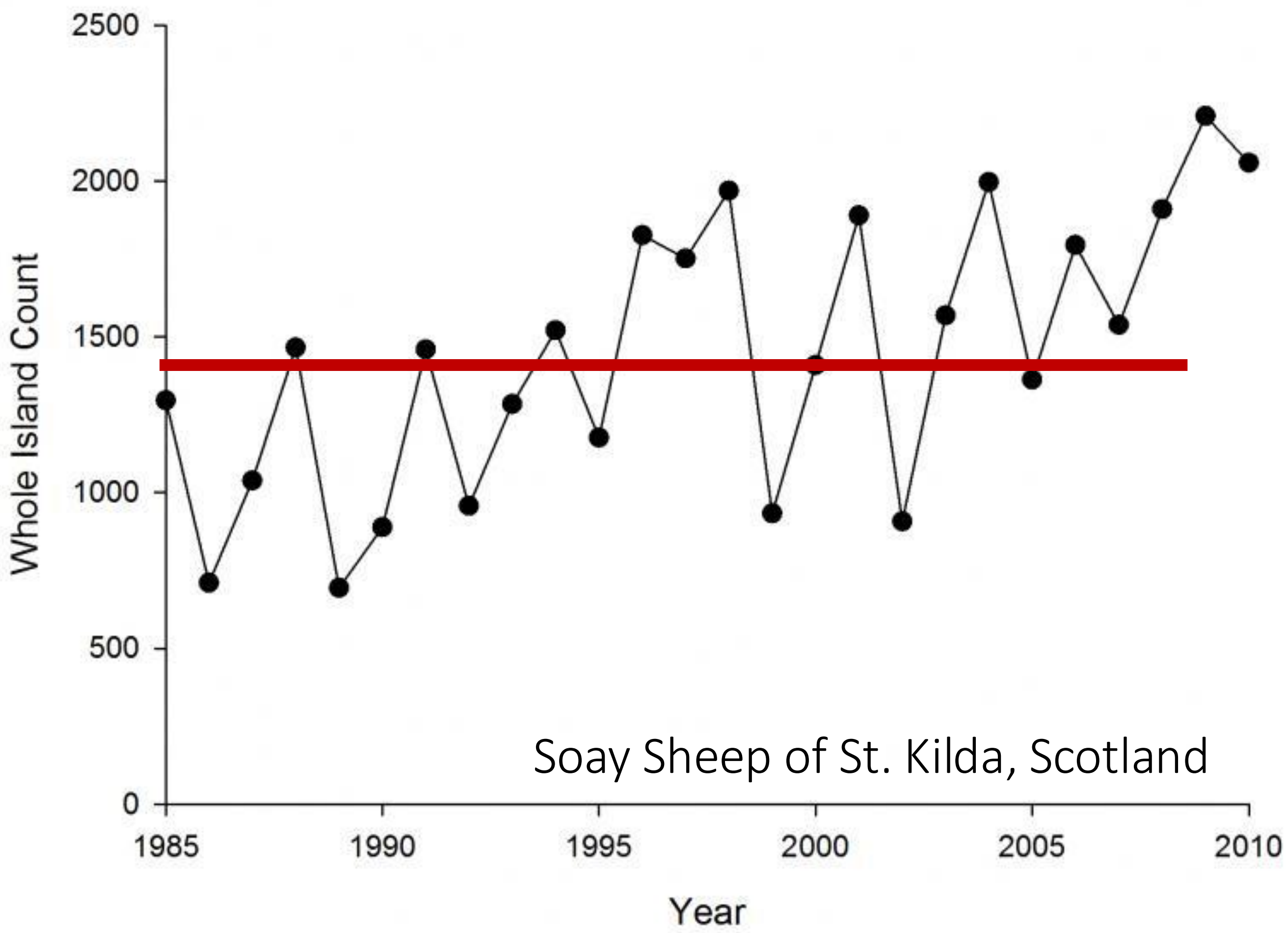

But they can still be stable populations if they return to **equilibrium**.

In some cases, it is not possible to recover.

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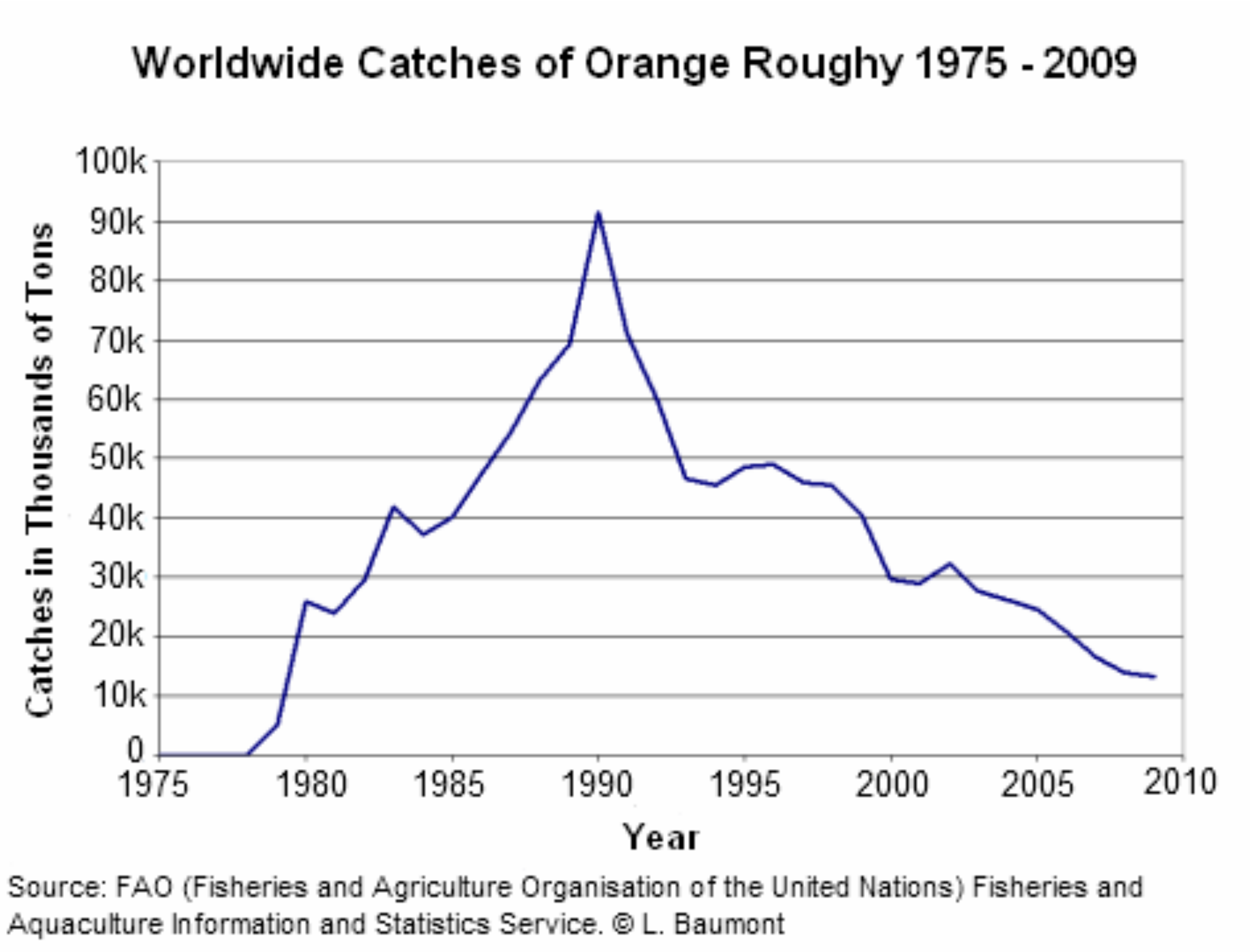

But they can still be stable populations if they return to **equilibrium**.

In some cases, it is not possible to recover.

Logistic growth still does not describe human populations well.