Hi Zizai,

Thanks for the note and the interest. You are correct about that α_{12} and α_{21} need not both be < 1 to allow for possible coexistence, and, likewise that α_{12} and α_{21} need not both be > 1 to allow for precedence. Rather, it is their respective products that should meet these terms. Some summary thoughts here...

General rules

For coexistence to be possible, we stated that the system must satisfy the conditions that $\frac{K_1}{\alpha_{12}} > K_2$ and $\frac{K_2}{\alpha_{21}} > K_1$. Rearranging, it then follows that $\alpha_{12}\alpha_{21} < 1$.

For precedence (e.g. winner determined by starting conditions) to occur, we instead stated that the system must satisfy the conditions that $\frac{K_1}{\alpha_{12}} < K_2$ and $\frac{K_2}{\alpha_{21}} < K_1$. Rearranging, it then follows that $\alpha_{12}\alpha_{21} > 1$.

This product of α_{12} and α_{21} is what I meant by the strength of interspecific interactions.

Intra- vs. Inter-species interactions

It is possible to quantify intra- vs. interspecies interactions more explicitly by representing A_{11} and A_{22} as the strength of each species' effect on itself, where $A_{11} = \frac{r_1}{K_1}$ and $A_{22} = \frac{r_2}{K_2}$.

Correspondingly, we can then represent A_{12} and A_{21} as the strength of each species' effect on each other, where $A_{12} = \frac{r_1\alpha_{12}}{K_1}$ and $A_{21} = \frac{r_2\alpha_{21}}{K_2}$.

Under the conditions for stable coexistence listed above ($\frac{K_1}{\alpha_{12}} > K_2$ and $\frac{K_2}{\alpha_{21}} > K_1$), we see that the following must be satisfied: $A_{12}A_{21} < A_{11}A_{22}$. Thus, the strength of intraspecific interactions is greater than the strength of interspecific interactions.

Your examples

The examples you provided actually correspond not to coexistence or precedence but to case 3 from the lecture, in which one species goes extinct while the other settles to carrying capacity.

You first suggested $K_1 = 10; K_2 = 100; \alpha_{12} = \alpha_{21} = 0.5$. This satisfies the conditions $\frac{K_1}{\alpha_{12}} < K_2$ and $\frac{K_2}{\alpha_{21}} > K_1$, whereby species 2 always wins.

You next suggested $K_1 = 10; K_2 = 100; \alpha_{12} = \alpha_{21} = 2$. This still satisfies the conditions $\frac{K_1}{\alpha_{12}} < K_2$ and $\frac{K_2}{\alpha_{21}} > K_1$, so again, species 2 always wins. If you plot the nullclines for both of these examples, they do not overlap.

My examples

A more relevant example for case 1 would be the following conditions for coexistence: $K_1 = 10; K_2 = 15; \alpha_{12} = 0.5; \alpha_{21} = 1.2$. The conditions for coexistence are satisfied: $\frac{K_1}{\alpha_{12}} > K_2$ and $\frac{K_2}{\alpha_{21}} > K_1$. You are correct that $\alpha_{21} > 1$, but, as stated above, $\alpha_{12}\alpha_{21} < 1$.

Similarly, in case 4 (precedence), try: $K_1 = 10; K_2 = 22; \alpha_{12} = 0.5; \alpha_{21} = 11$. Here, the conditions for unstable coexistence are met: $\frac{K_1}{\alpha_{12}} < K_2$ and $\frac{K_2}{\alpha_{21}} < K_1$ even though $\alpha_{12} < 1$. However, as stated above, $\alpha_{12}\alpha_{21} > 1$.

Close

I hope that helps! I'll make a few points of clarification in class on Tuesday.

Cheers, Cara