Wk7-Lect5-Feb20

Visual settings

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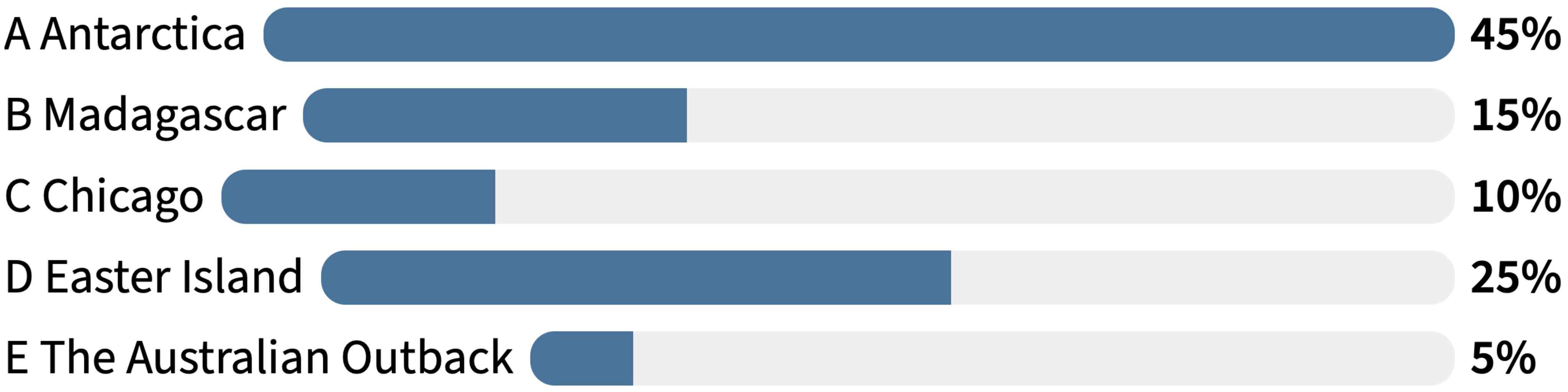
Which location is the best place to avoid COVID-19?



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Fundamentals of Ecology

Week 7, Ecology Lecture 5

Cara Brook

February 20, 2025

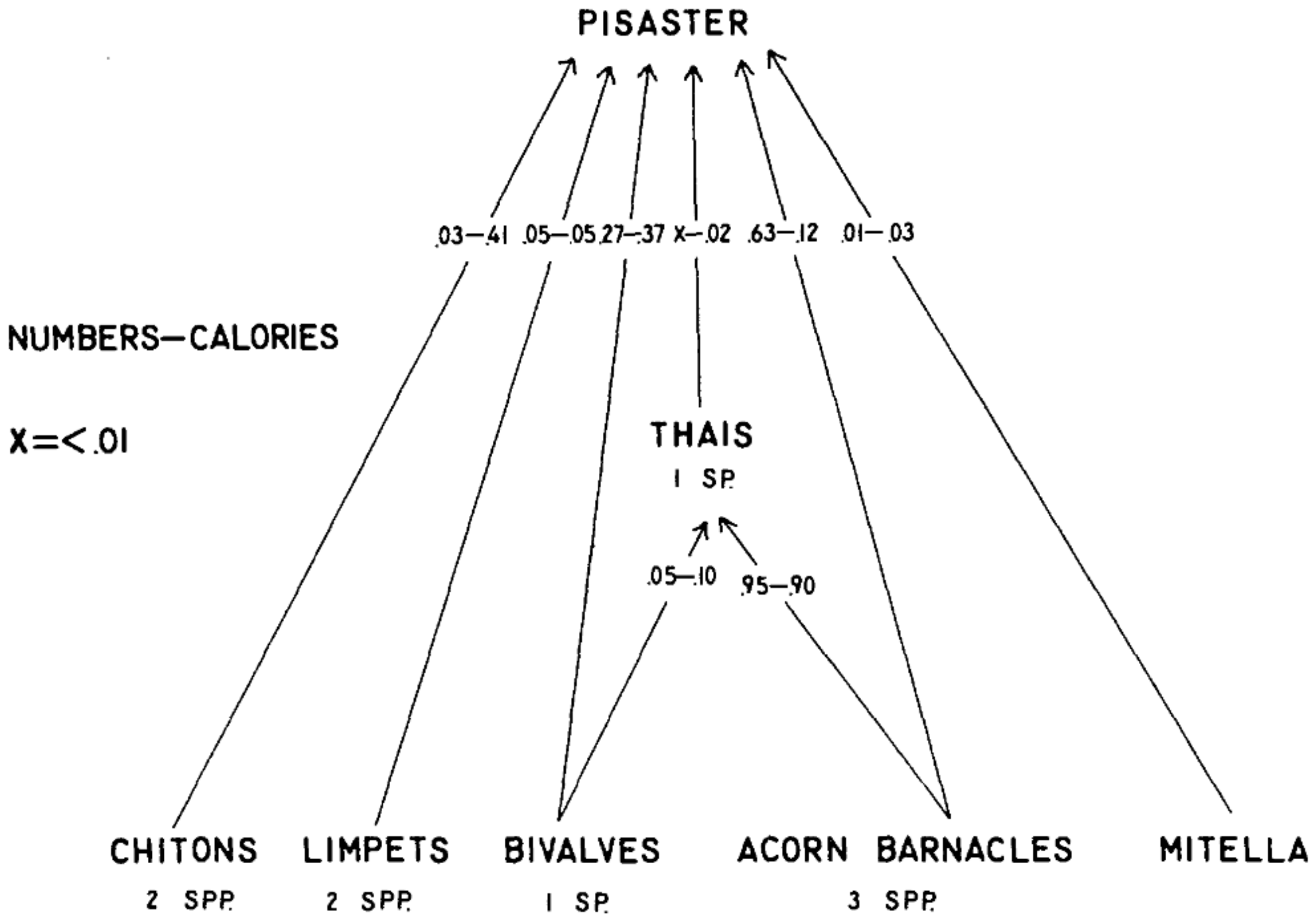
Office hours: On ZOOM
Friday, Feb 21, 2025
4-5pm
I will email out a link!

Learning objectives from Lecture 4

You should be able to:

- Understand and explain source-sink dynamics and an ecological trap
- Describe an Allee effect
- Understand trophic levels and know the direction of energy transfer, including what might happen to one level if there was a perturbation to a different level
- Know the ecosystem-level rules of HSS
- Understand the phase portrait in a Lotka-Volterra predator-prey model, including which direction to move forward in time
- Recognize a trophic cascade.

This work inspired empirical studies on **trophic cascades**: *Pisaster* removal on Tatoosh Island



Paine 1966. *The American Naturalist.*

Visual settings

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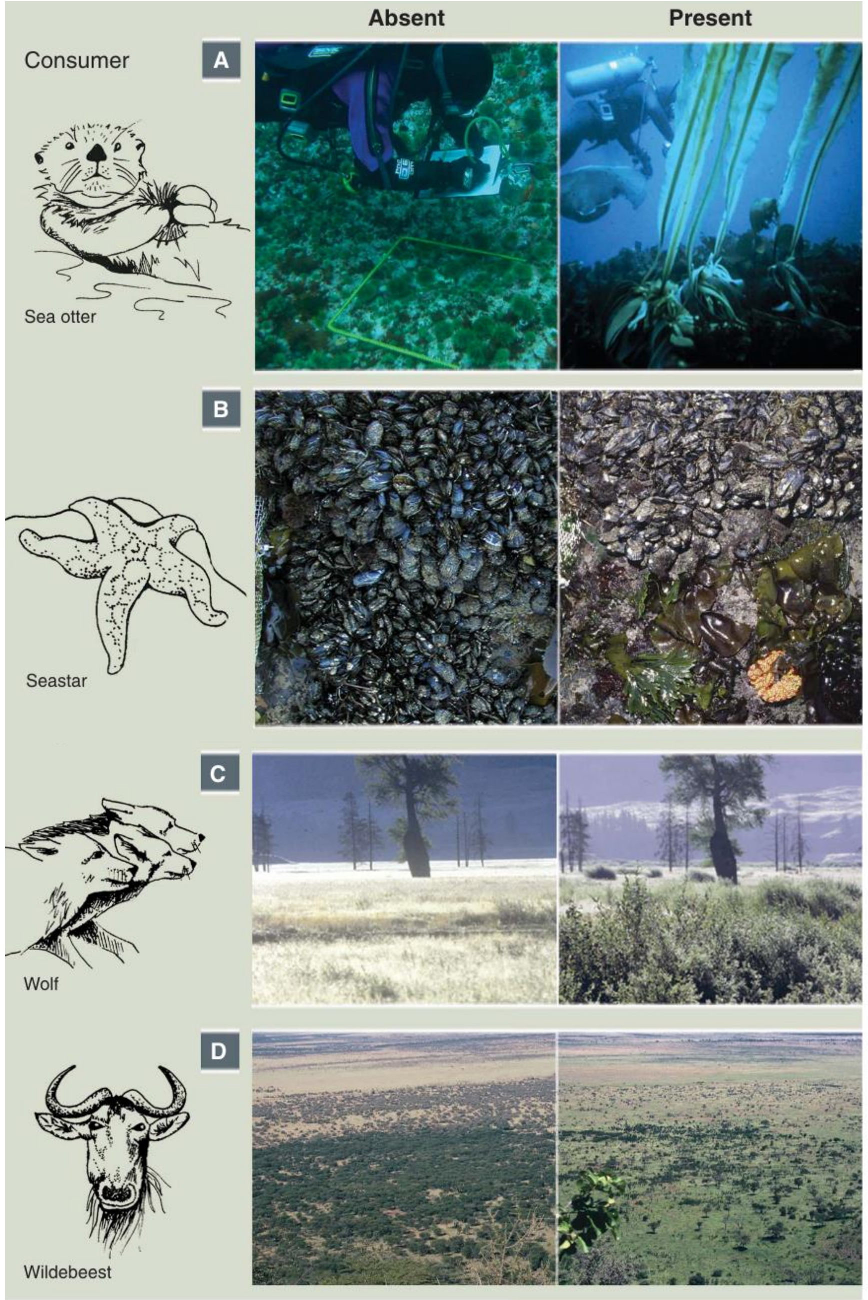
Which of the following is an example of a trophic cascade?

A. Lion and zebra populations cyclically track one another in population booms and busts in the…

10%

Other famous **trophic cascades:**

Estes et al. 2011. *Science.*

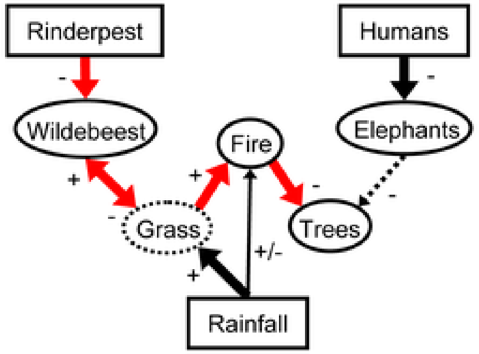


CA sea otters maintain kelp forest diversity by consuming herbivorous sea urchins. (Estes & Duggins 1995. *Ecological Monographs*)

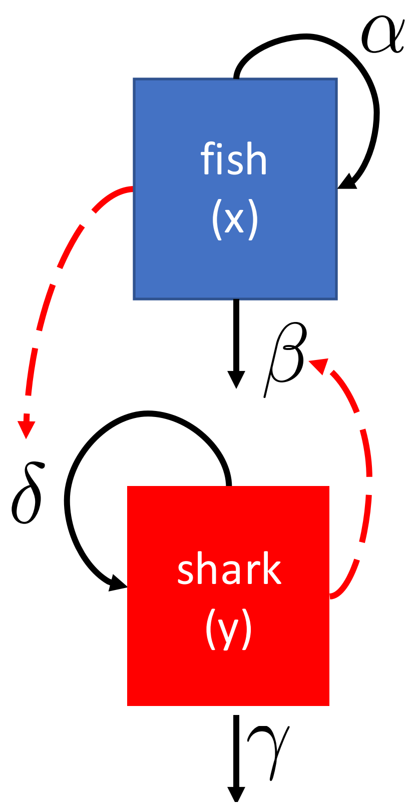
Starfish maintain diversity in Pacific intertidal by consuming space-dominating mussels. (Paine 1966 *The American* Naturalist)

Yellowstone wolves promote willow recovery by consuming overbrowsing elk (Ripple & Beschta 2005. *Forest Ecology & Management*)

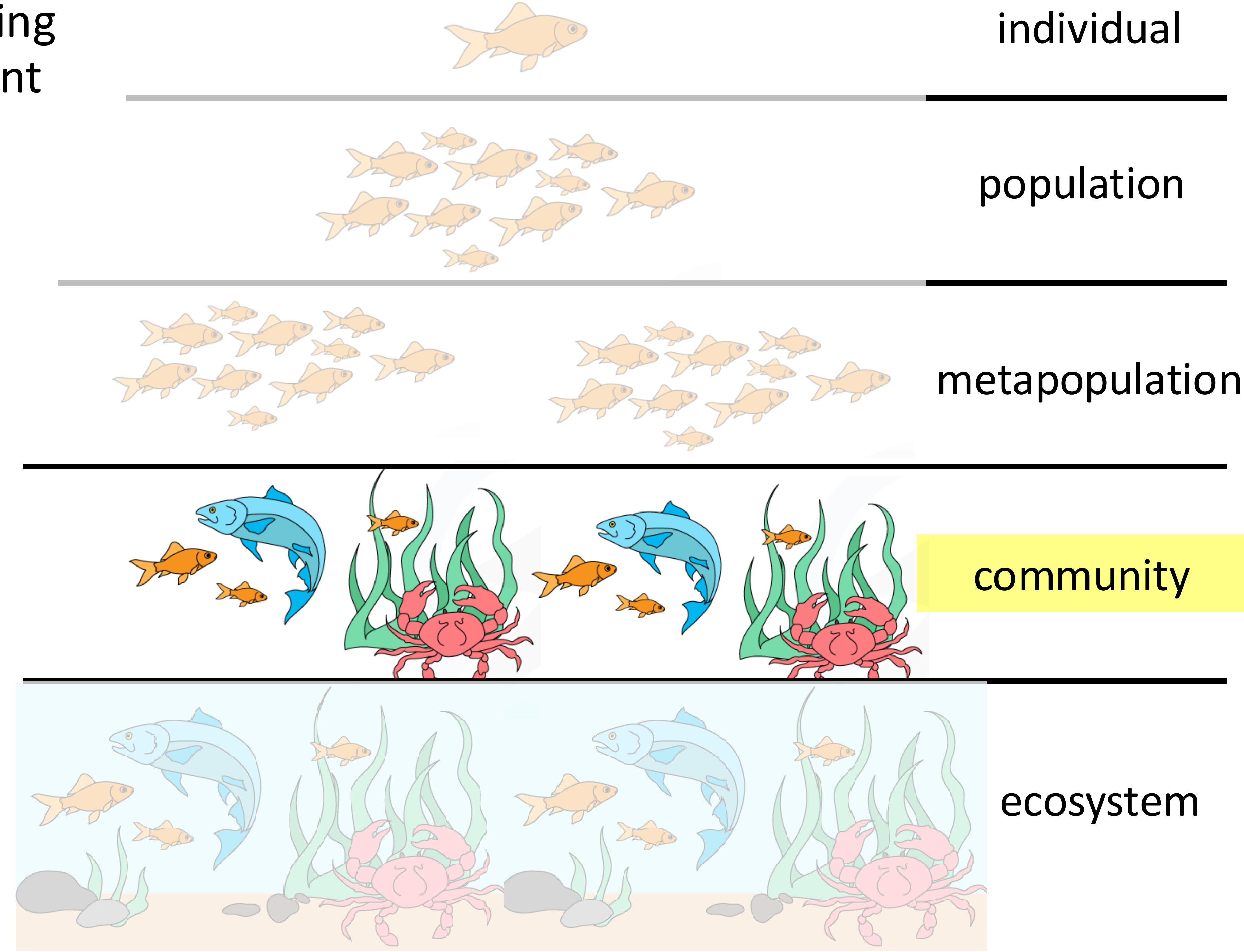
Rinderpest eradication releases wildebeest populations that control savanna, limit fire, and promote tree regrowth (Holdo et al. 2009. *PLoS Biology*)



Community = interacting populations of different species



*How does fish abundance **vary** with **changes** in shark abundance?*



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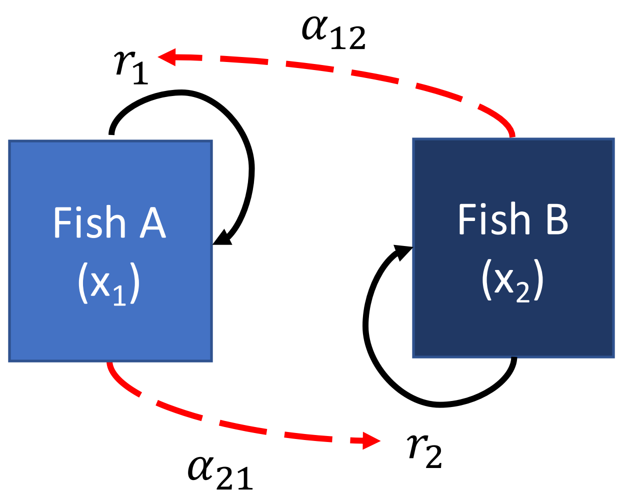

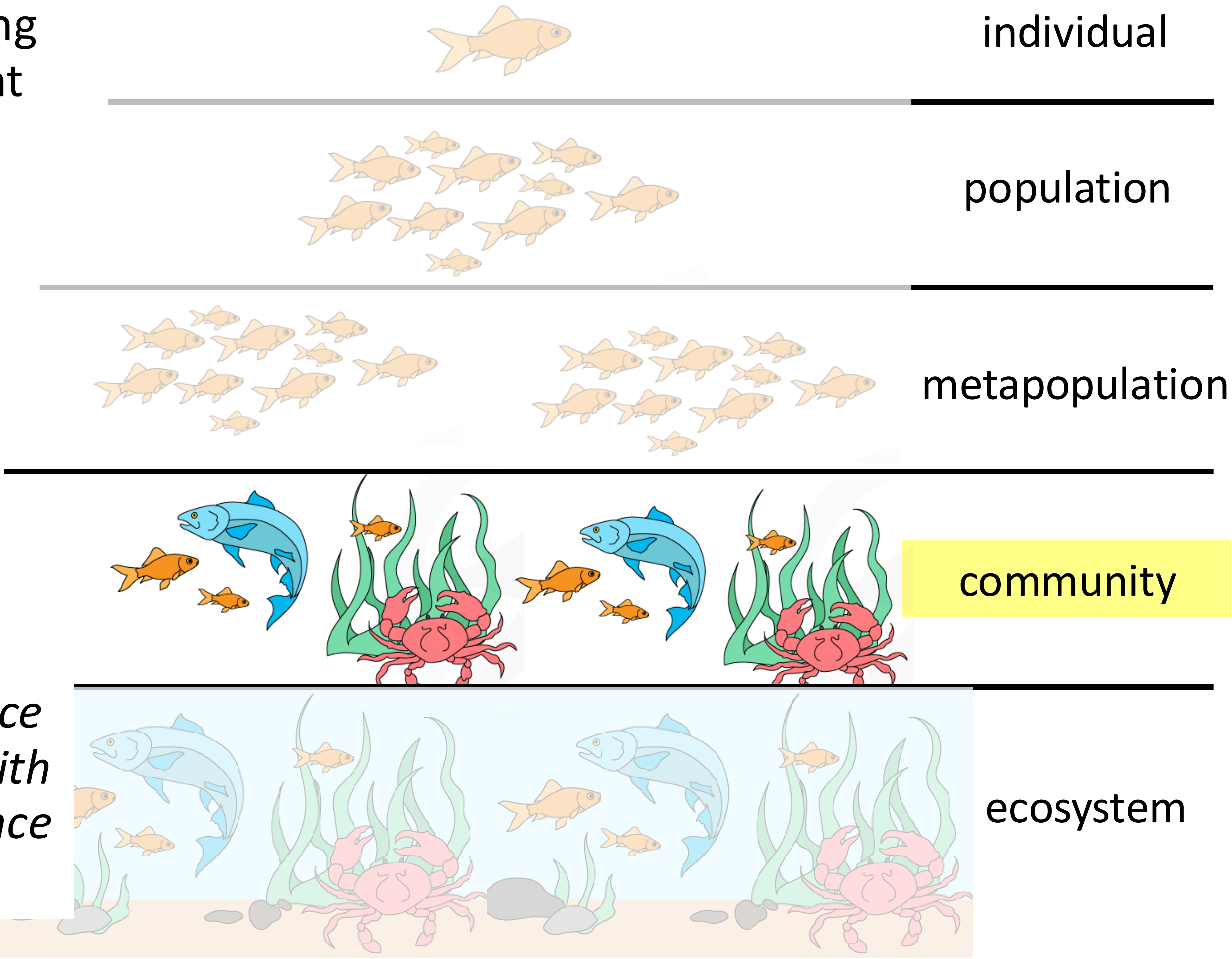

*How does the abundance of **fish species A** vary with changes in the abundance of **fish species B**?*

Lotka-Volterra equation can be modified for **interspecies competition.**

$$\frac{dx_1}{dt} = r_1 x_1 \left(1 - \frac{x_1 + \alpha_{12} x_2}{K_1}\right)$$

$$\frac{dx_2}{dt} = r_2 x_2 \left(1 - \frac{x_2 + \alpha_{21} x_1}{K_2}\right)$$

Lotka-Volterra equation can be modified for **interspecies competition.**

$$\frac{dx_1}{dt} = r_1x_1 - \frac{r_1(x_1)^2}{K_1} - \frac{r_1x_1x_2\alpha_{12}}{K_1}$$

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$$\frac{dx_2}{dt} = \underbrace{r_2 x_2 - \frac{r_2 (x_2)^2}{K_2}}_{\text{intraspecies competition of species 2}} - \frac{r_2 x_2 x_1 \alpha_{21}}{K_2}$$

Each species is **self-regulated** by logistic growth and its own carrying capacity (**K**) and growth rate (**r**).

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Each species is self-regulated by logistic growth and its own carrying capacity (K) and growth rate (r).

Each species is also **regulated by the density of its competitor** (e.g. for a specific resource).

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Each species is self-regulated by logistic growth and its own carrying capacity (K) and growth rate (r).

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$\boldsymbol{\alpha_{12}}$ = effect of species 2 on the population of species 1

$\boldsymbol{\alpha_{21}}$ = effect of species 1 on the population of species 2

The 2-species **Lotka-Volterra competition model** has **four equilibria**.

Remember: equilibrium occurs when neither population is changing!

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Four equilibria at:

$x_1^* = 0\,; x_2^* = 0$ Trivial.

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Species 1 extinct. Species 2 at carrying capacity.

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$$x_1^* = K_1;\; x_2^* = 0$$

$$x_1^* = \frac{K_1 - K_2 \alpha_{12}}{1 - \alpha_{21}\alpha_{12}}\,;\, x_2^* = \frac{K_2 - K_1 \alpha_{21}}{1 - \alpha_{12}\alpha_{21}}$$ Coexistence.

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Visual settings

Edit

In the competition equations below, what is α_{12}? The effect of:

01:00

$$\frac{dx_1}{dt} = r_1x_1 - \frac{r_1(x_1)^2}{K_1} - \frac{r_1x_1x_2\alpha_{12}}{K_1}$$

$$\frac{dx_2}{dt} = r_2x_2 - \frac{r_2(x_2)^2}{K_2} - \frac{r_2x_2x_1\alpha_{21}}{K_2}$$

A) Species 1 on its o…	6%
B) Species 2 on its o…	0%
C) Species 1 on the growth…	4%
D) Species 2 on the growth…	91%

Instructions

Responses

Correctness

More

Clear responses

Nullclines (or **isoclines**) of the **Lotka-Volterra competition model**

These are the lines that correspond to the conditions when the rate of change for **one species** is not changing!

Nullclines (or isoclines) of the Lotka-Volterra competition model

These are the lines that correspond to conditions when the rate of change for one species is 0.

- **Nullclines for species 1** occur at all conditions for which $\frac{dx_1}{dt} = 0$

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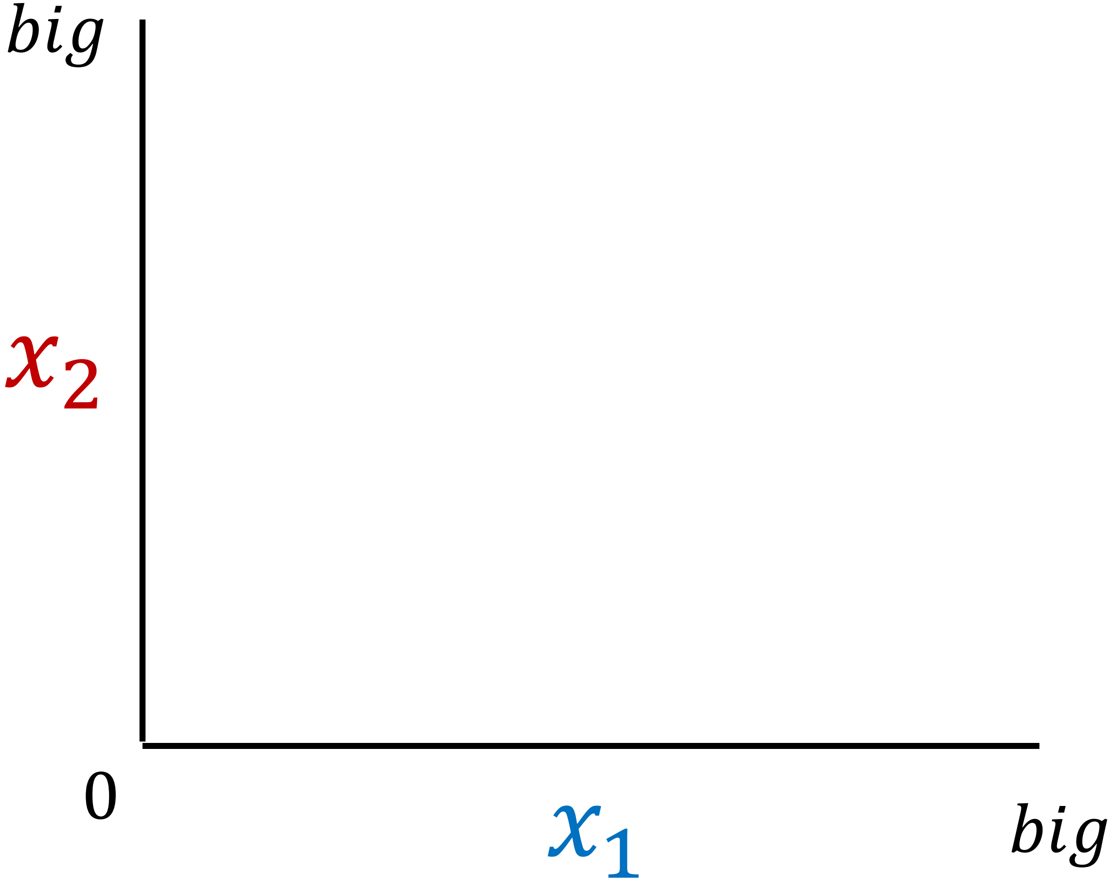

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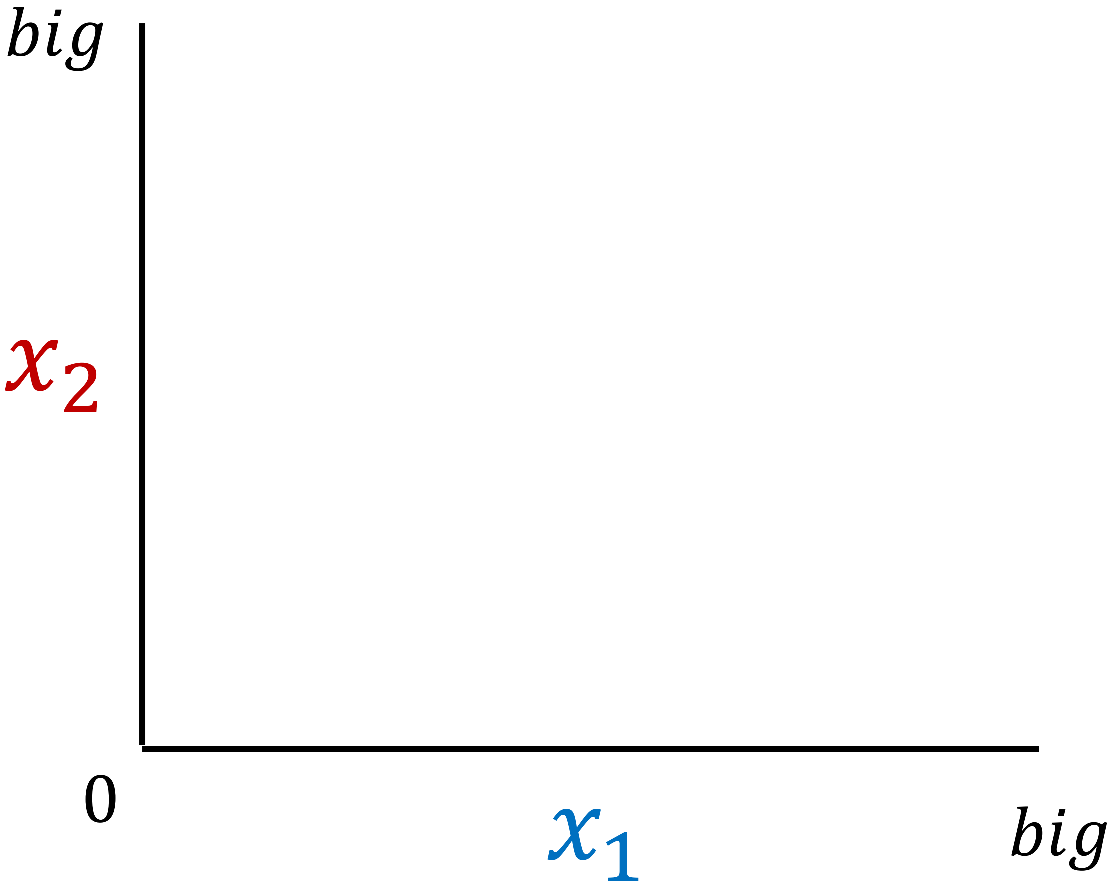

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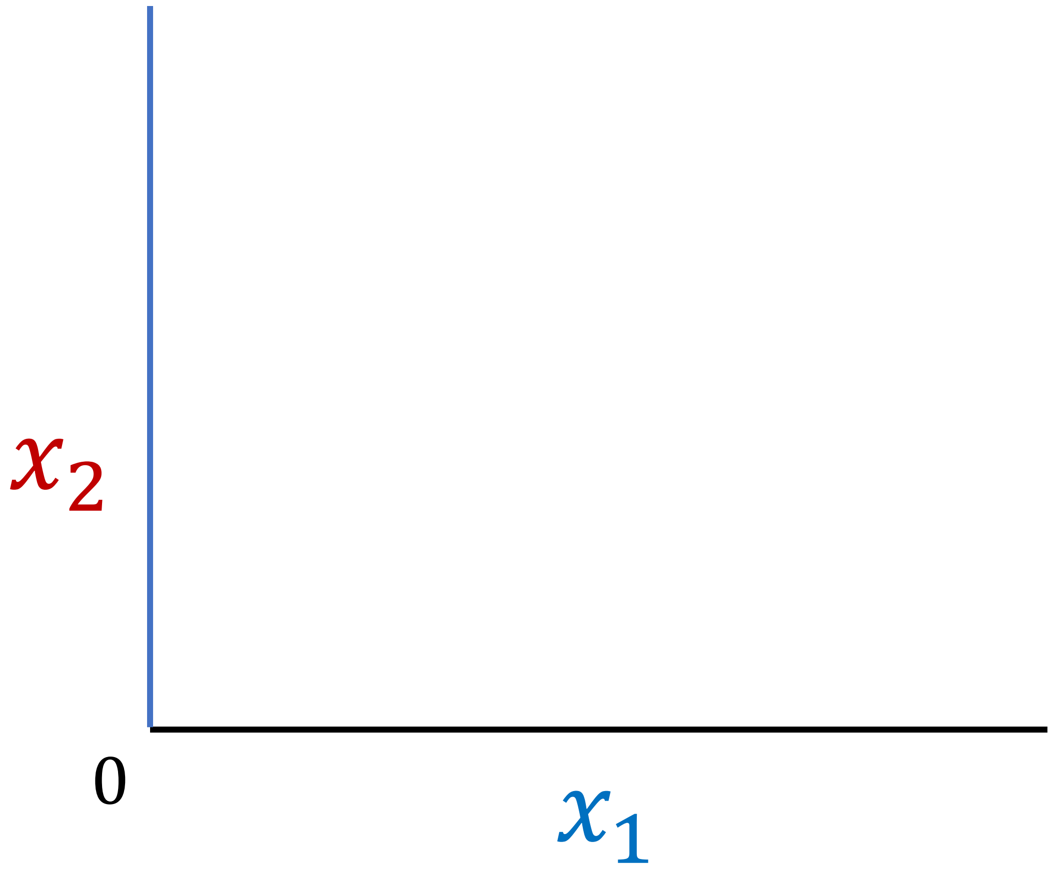
first nullcline at $x_1 = 0$



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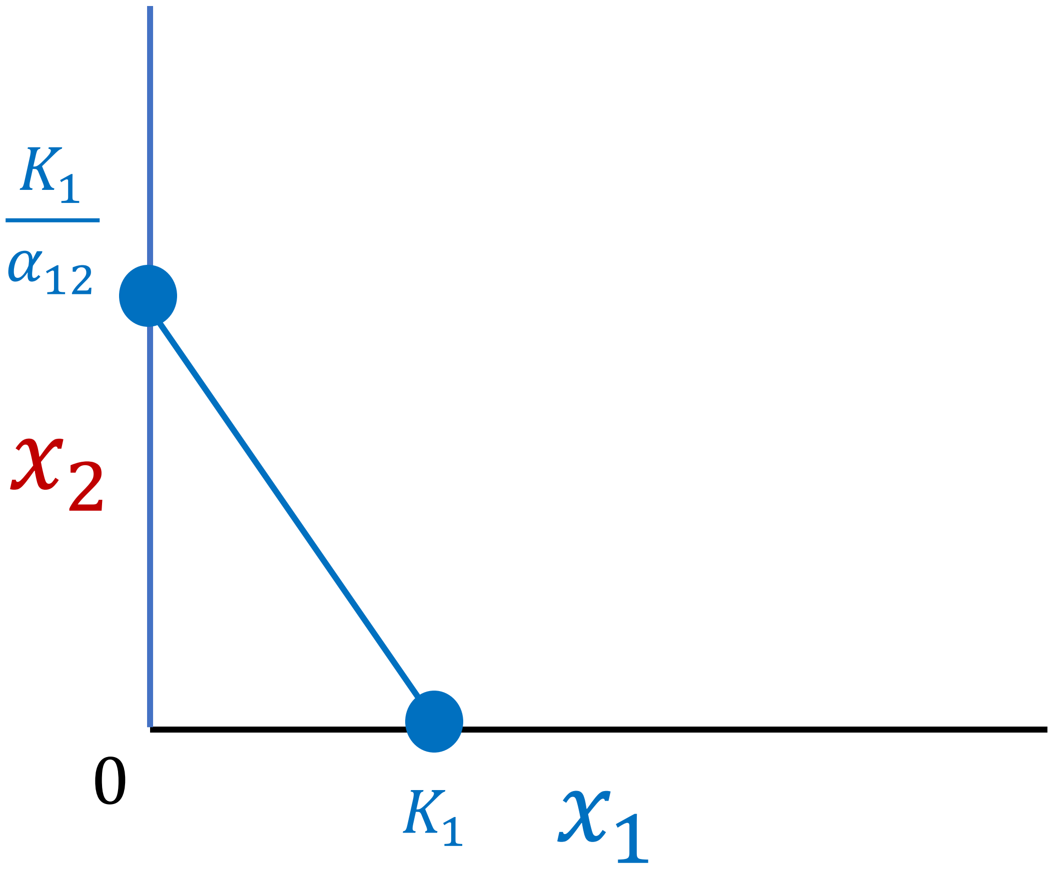
first nullcline at $x_1 = 0$

second nullcline at $x_1 = -\alpha_{12} x_2 + K_1$



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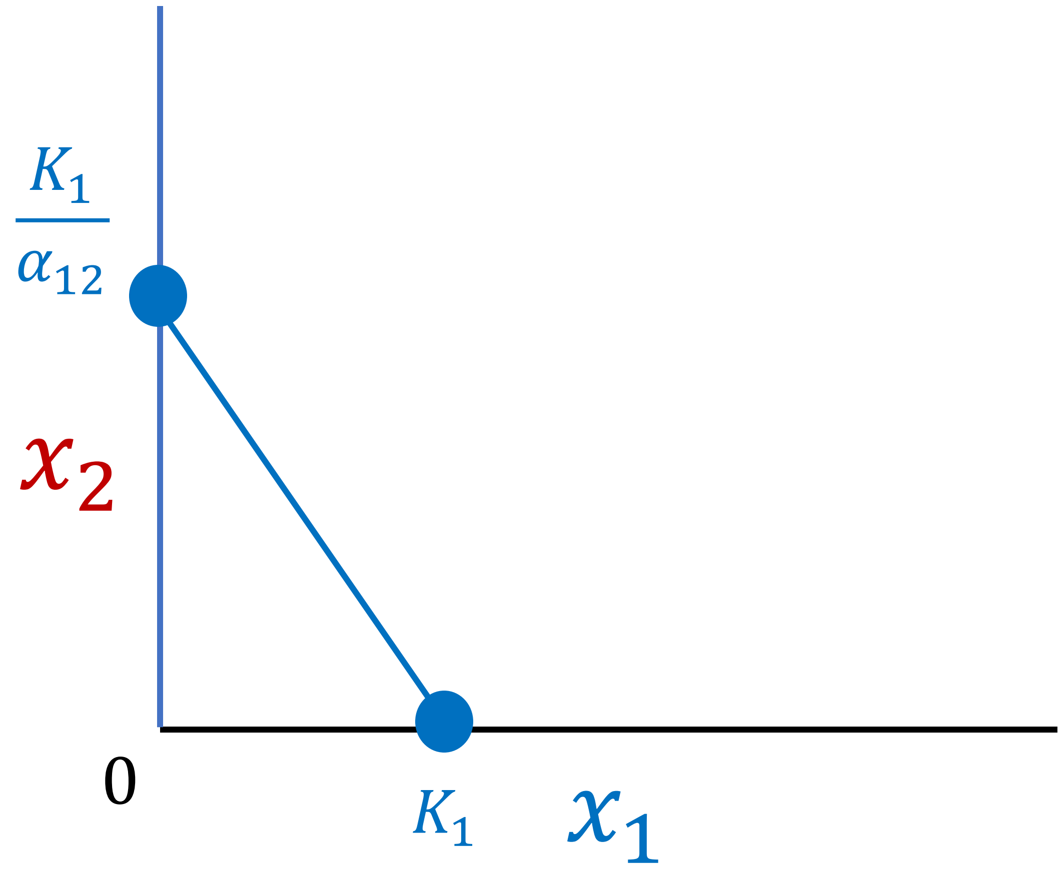

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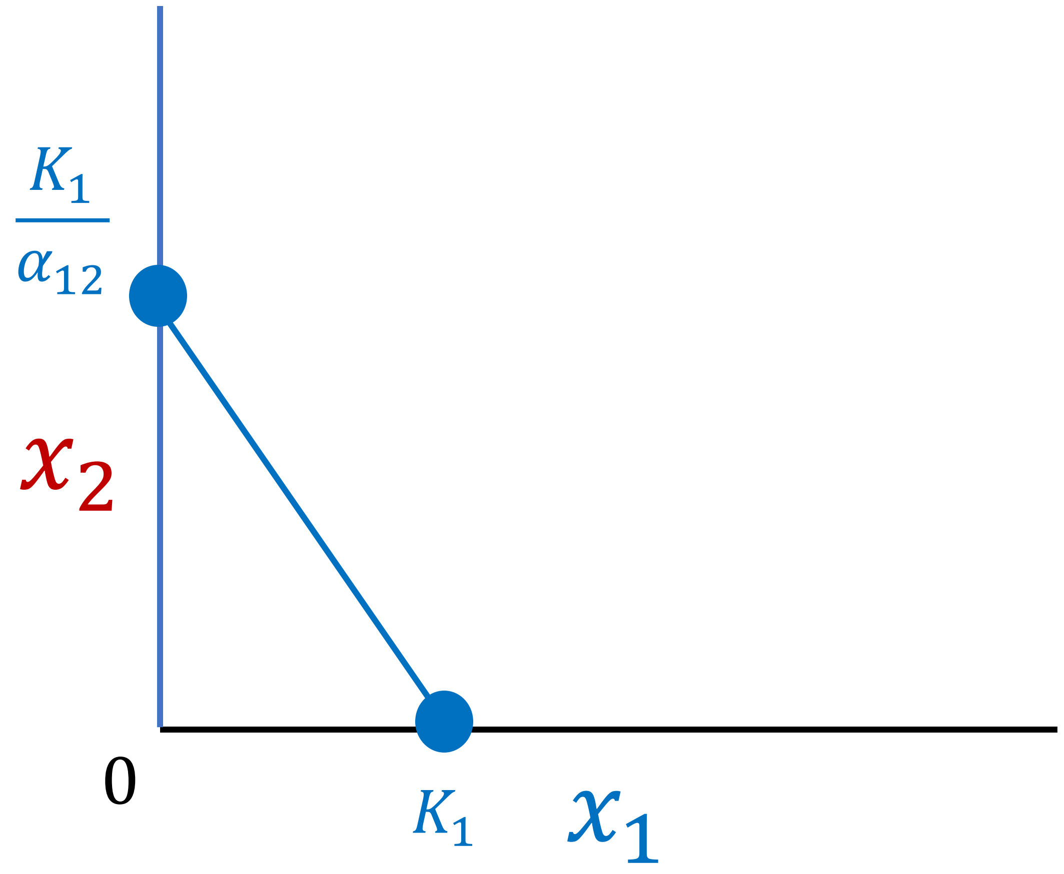

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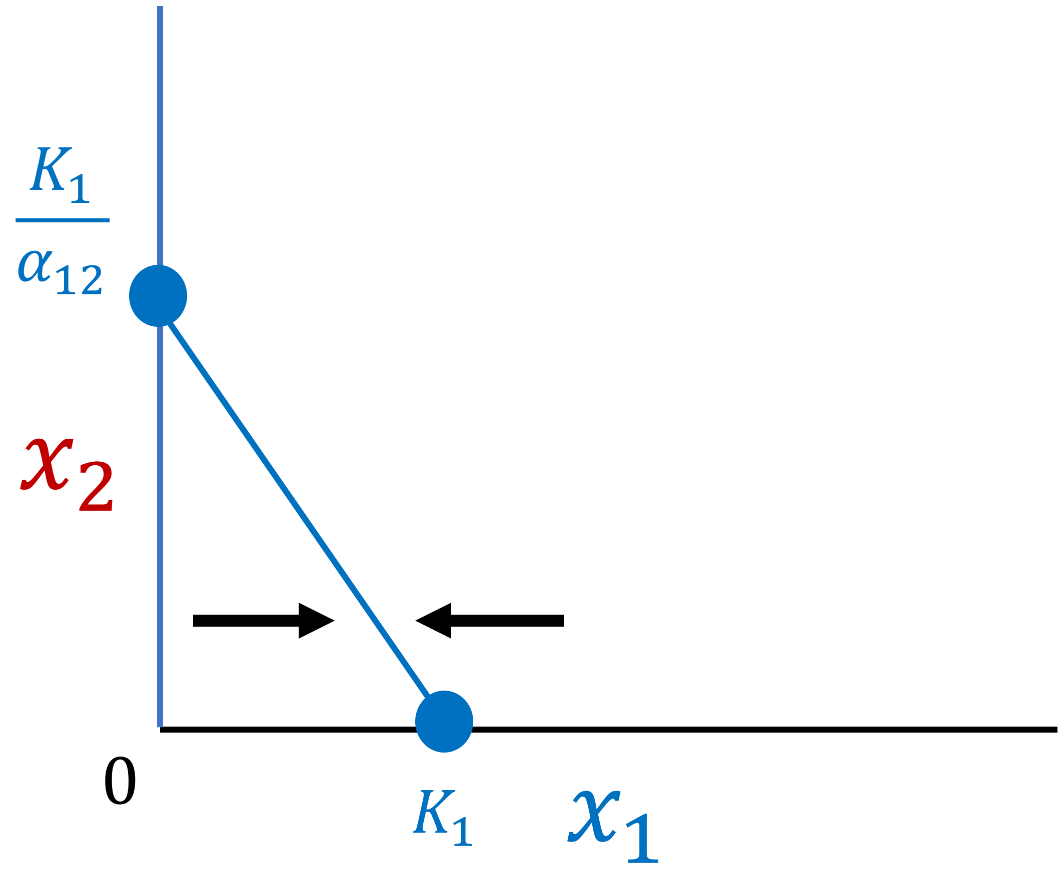

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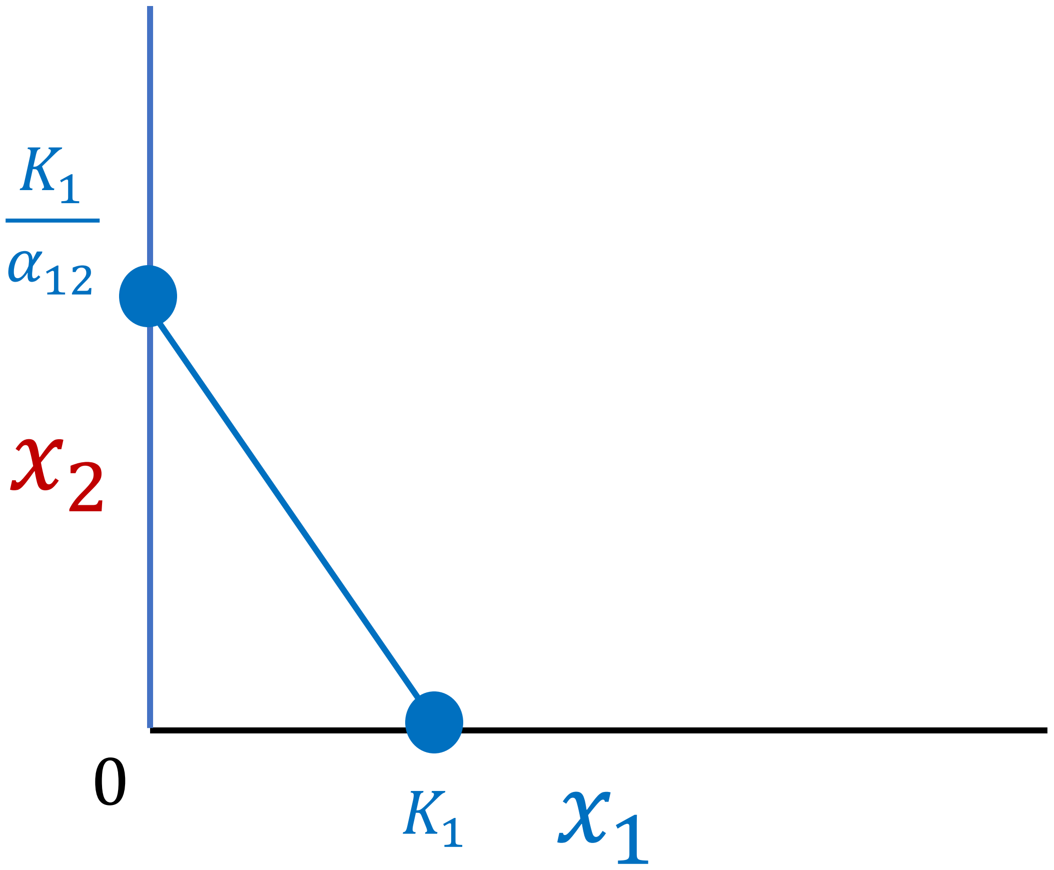

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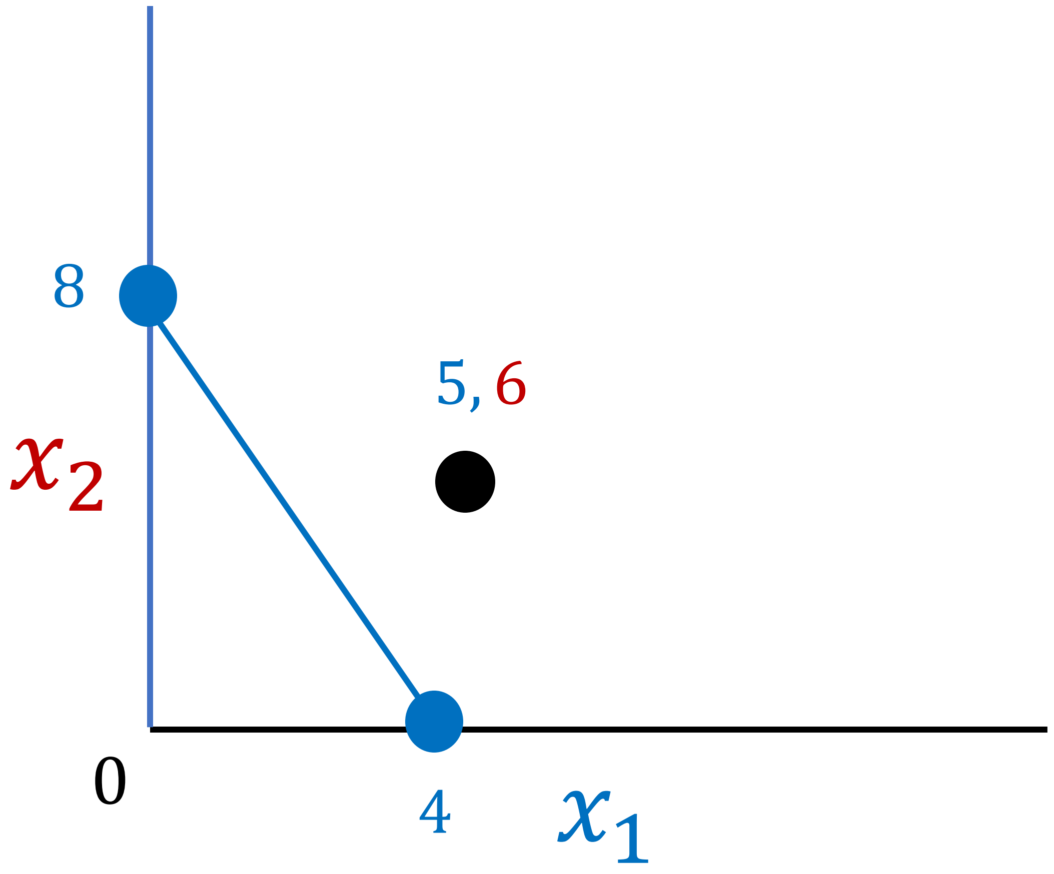

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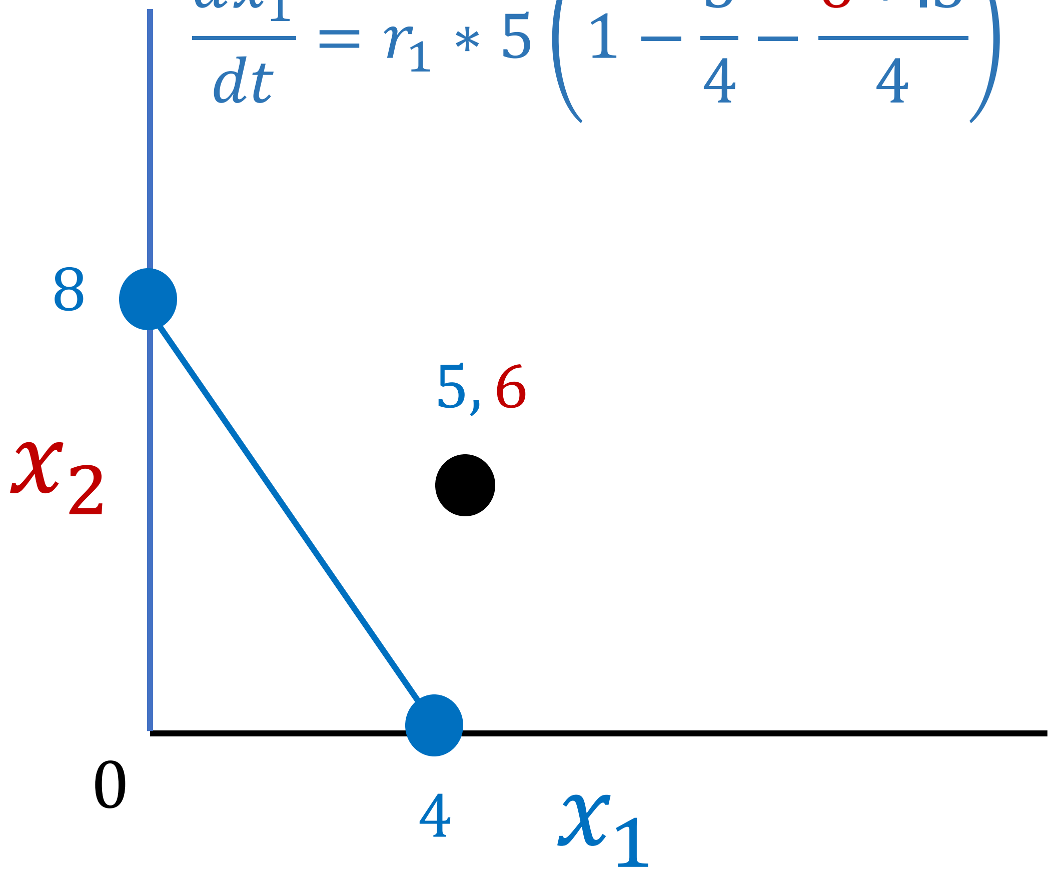

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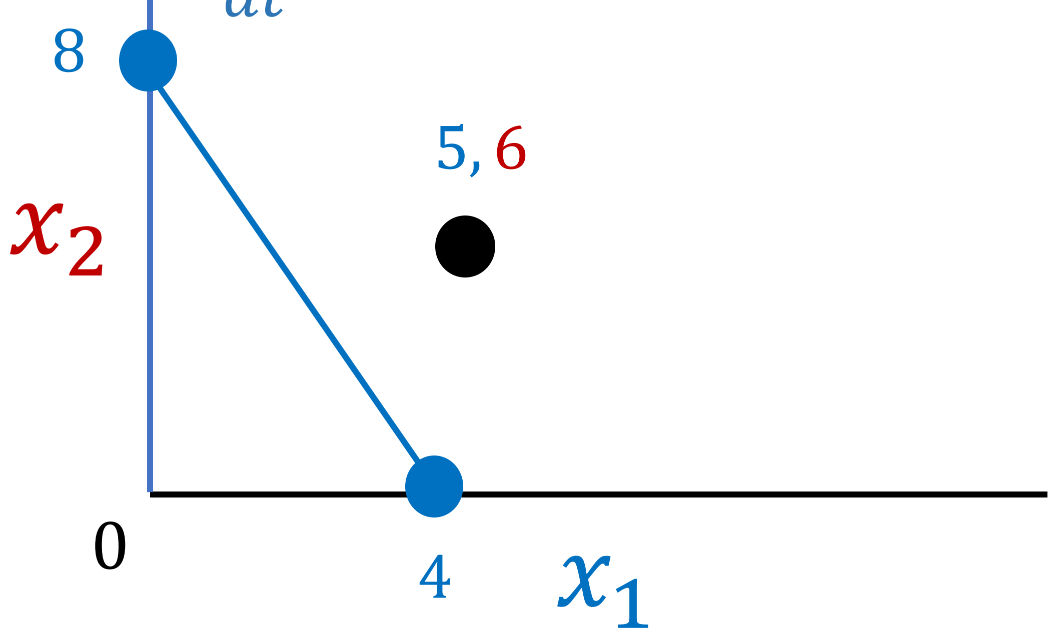
$$\frac{dx_1}{dt} = r_1 * 5(1 - 1.25 - .75)$$



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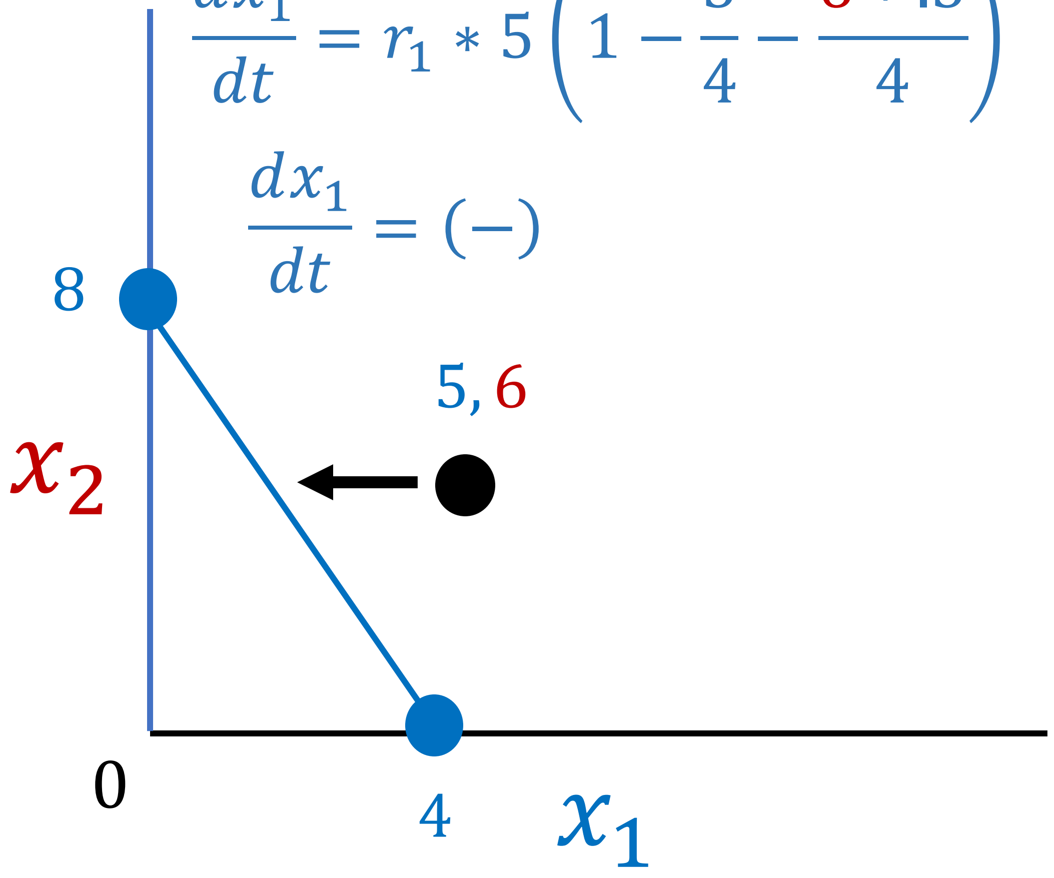
$$\frac{dx_1}{dt} = (-)$$



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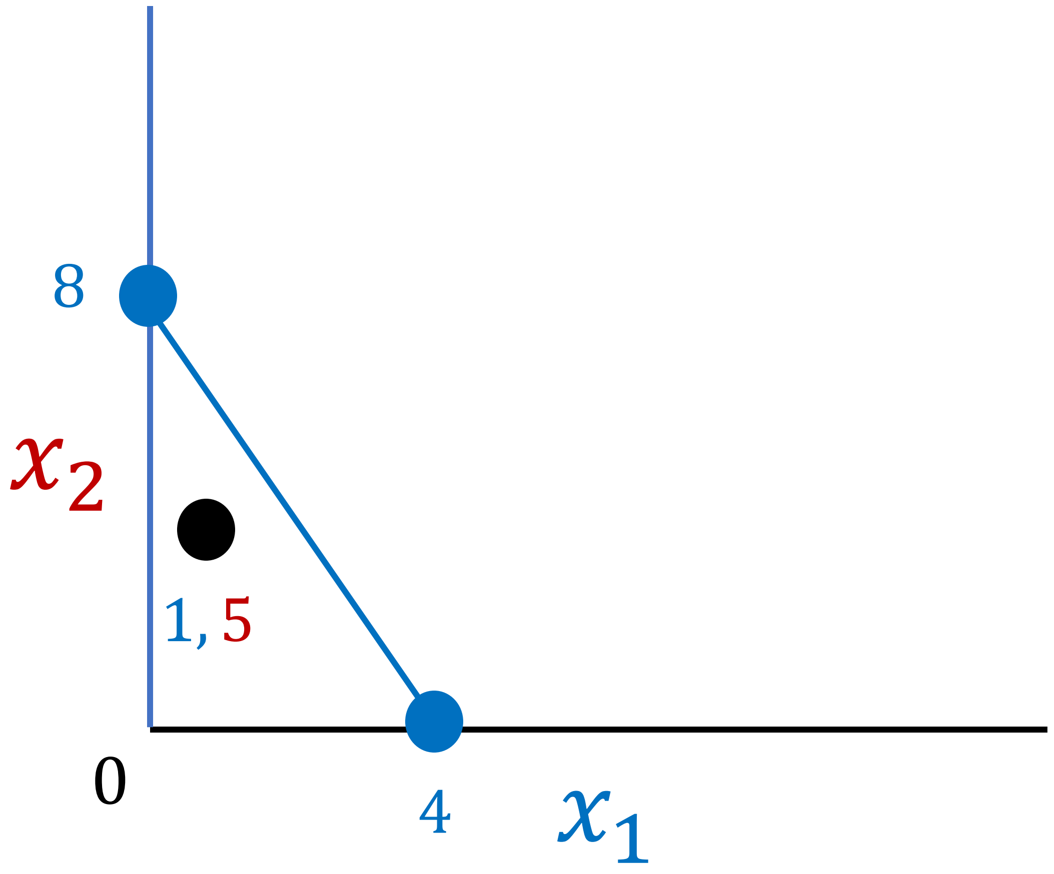

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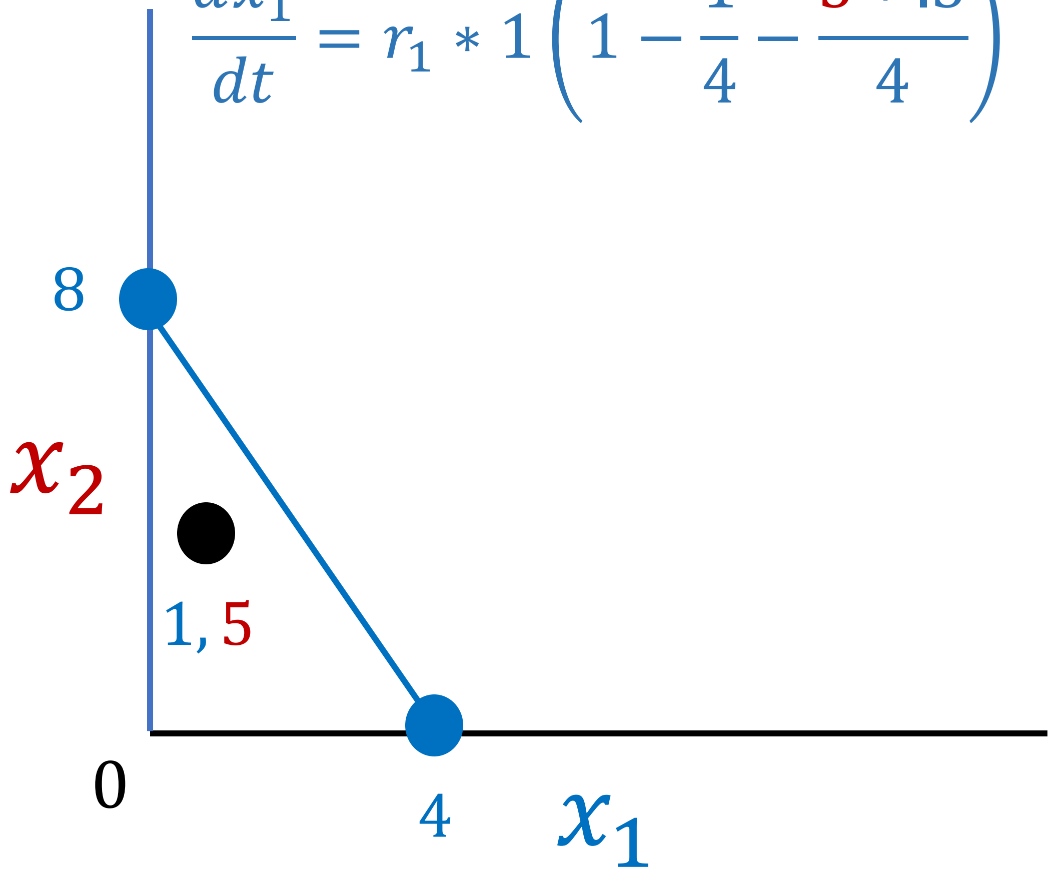

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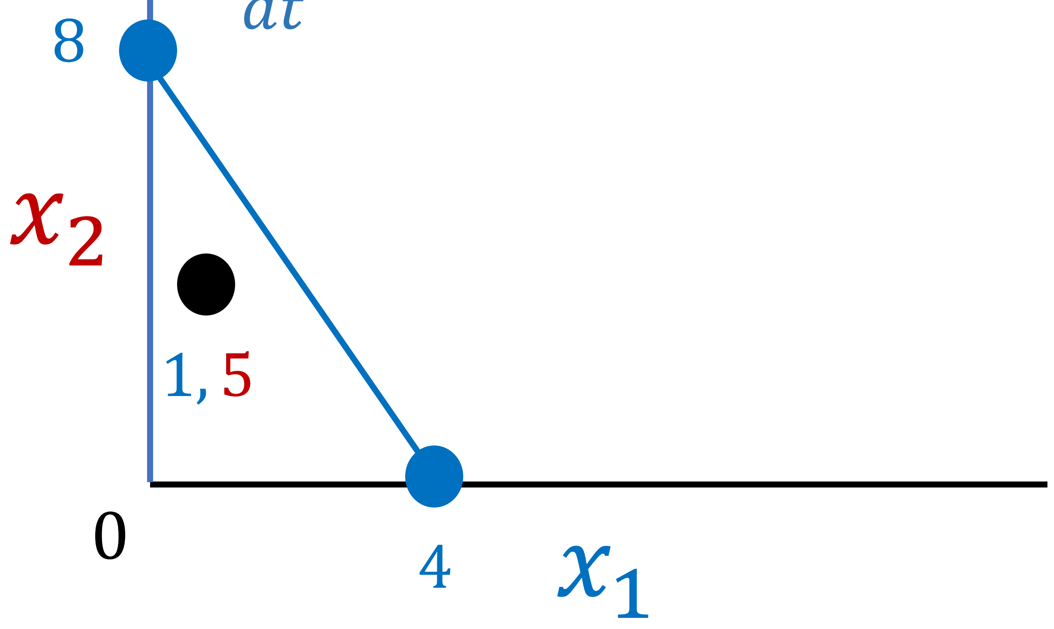
$$\frac{dx_1}{dt} = r_1 * 1(1 - .25 - .56)$$



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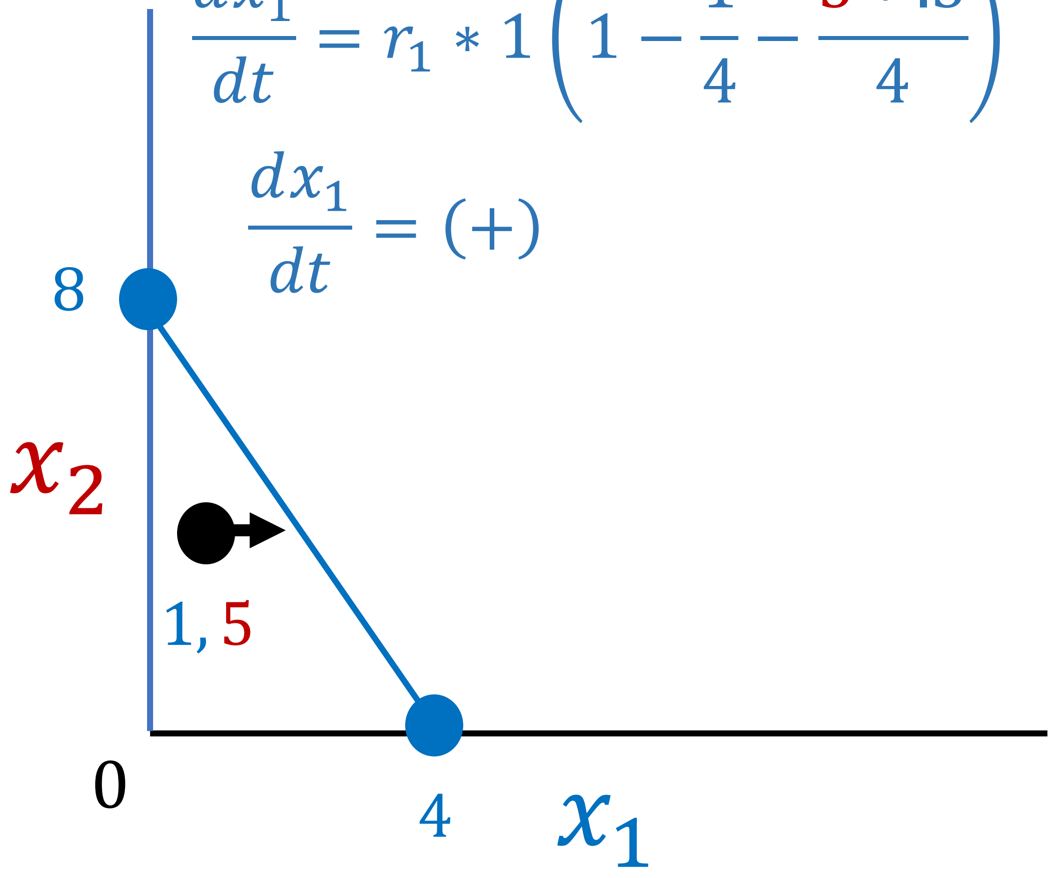
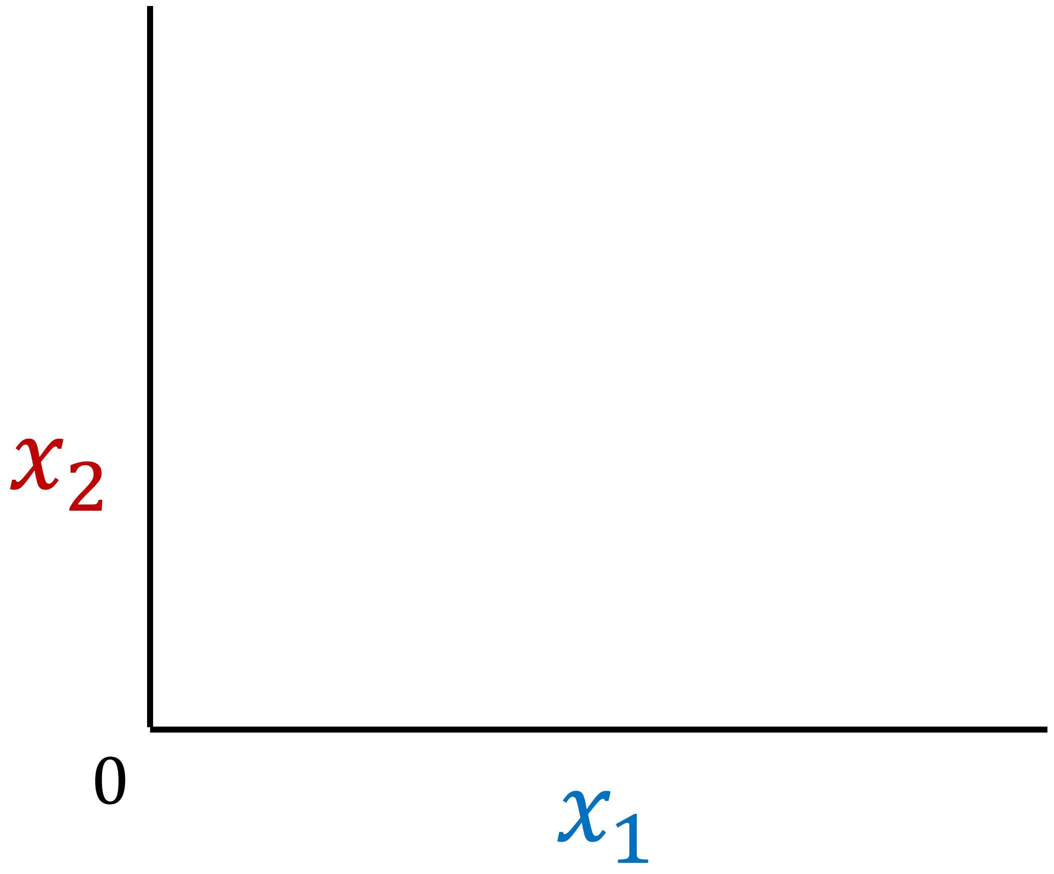


Nullclines (or **isoclines**) of the **Lotka-Volterra competition model**

These are the lines that correspond to conditions when the rate of change for one species is 0.

- **Nullclines for species 2** occur at all conditions for which $\frac{dx_2}{dt} = 0$



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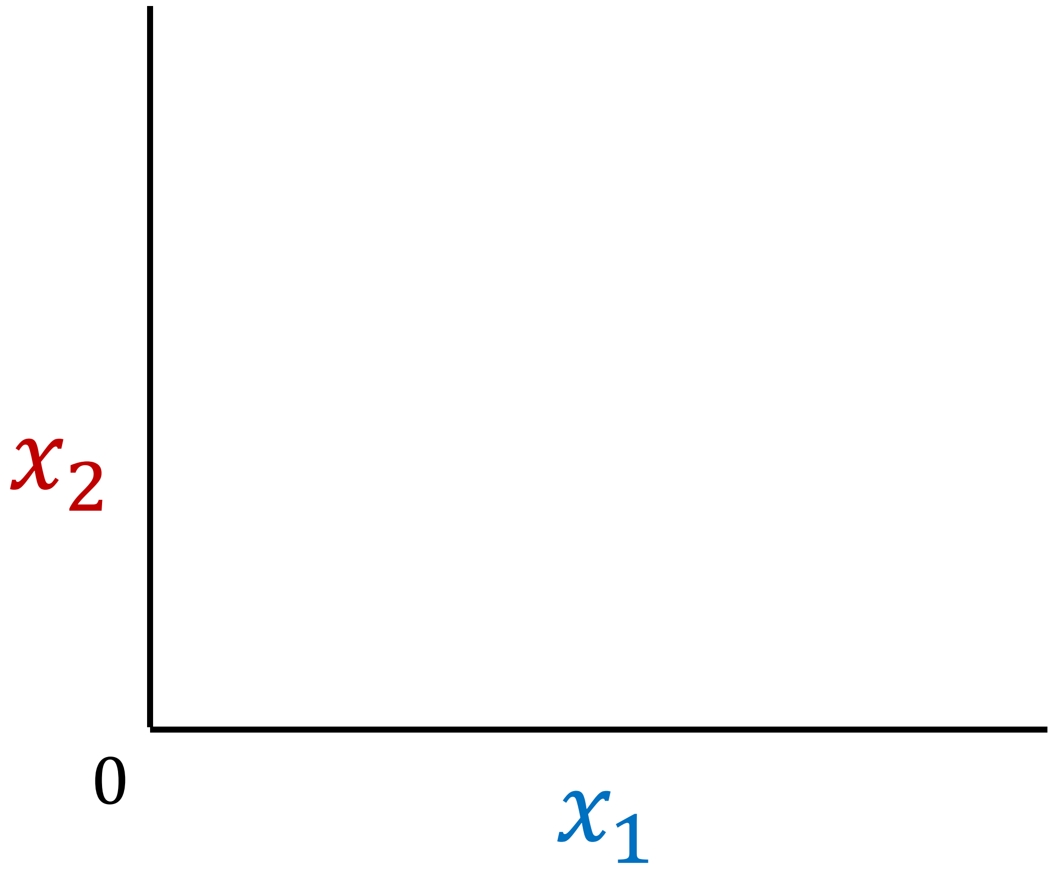

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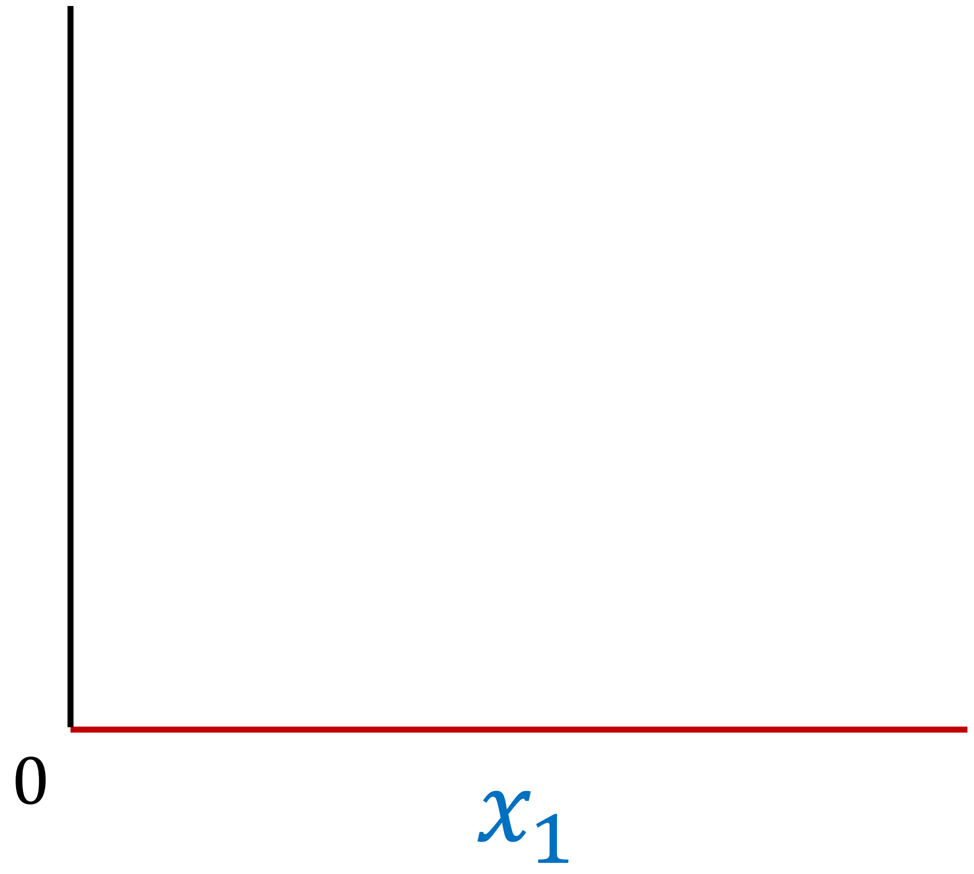

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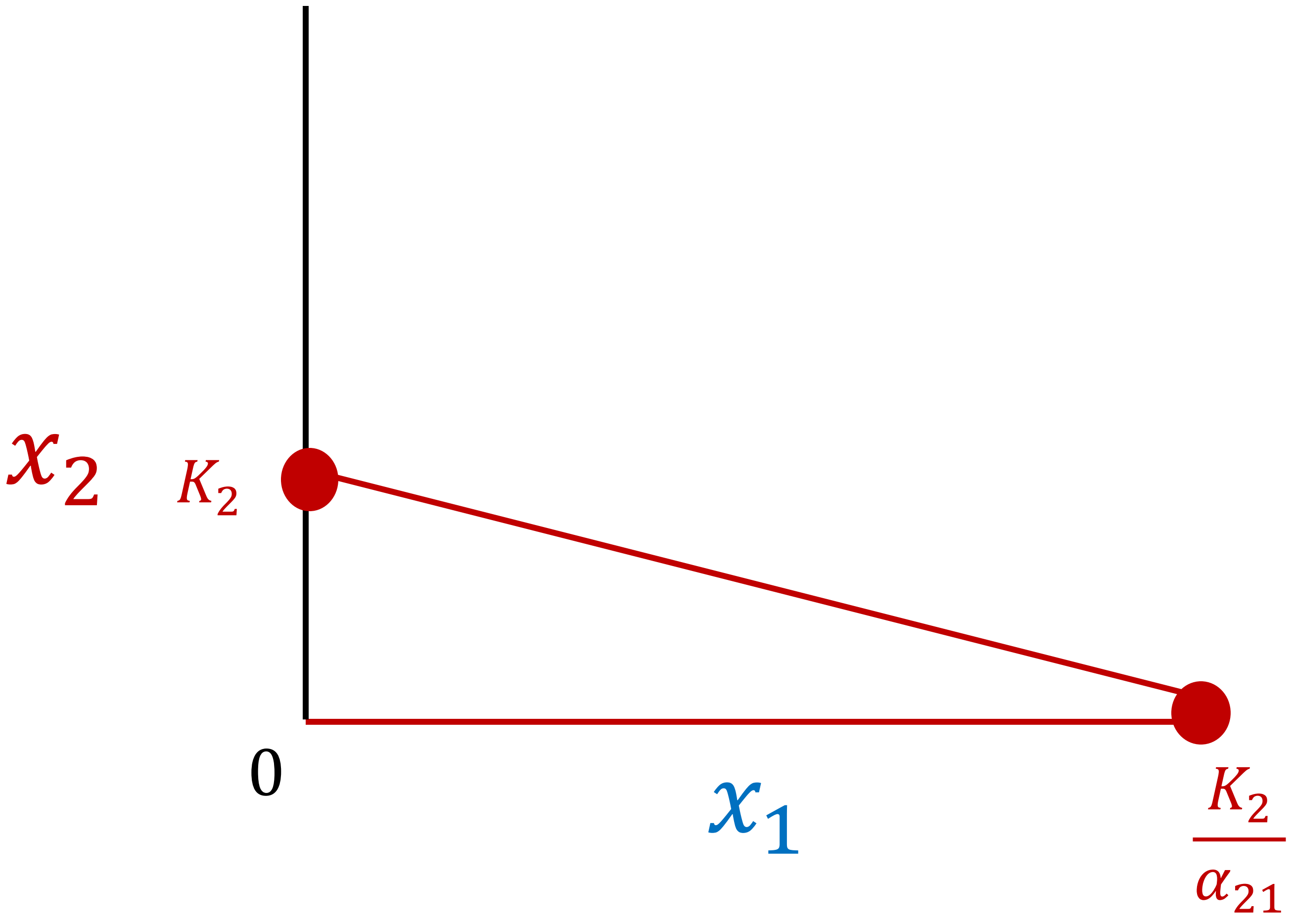
first nullcline at $x_2 = 0$

second nullcline at $x_2 = -\alpha_{21} x_1 + K_2$



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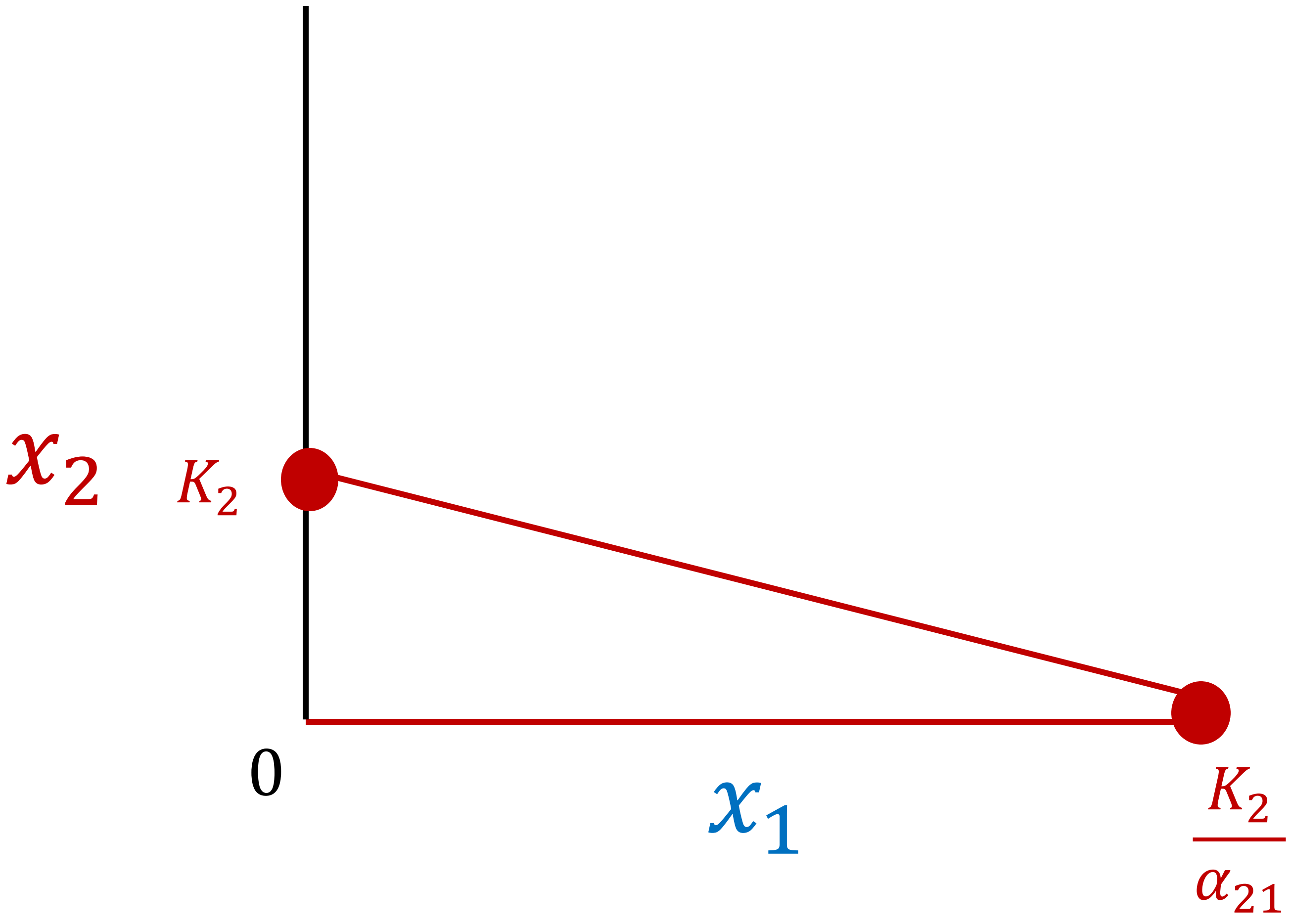

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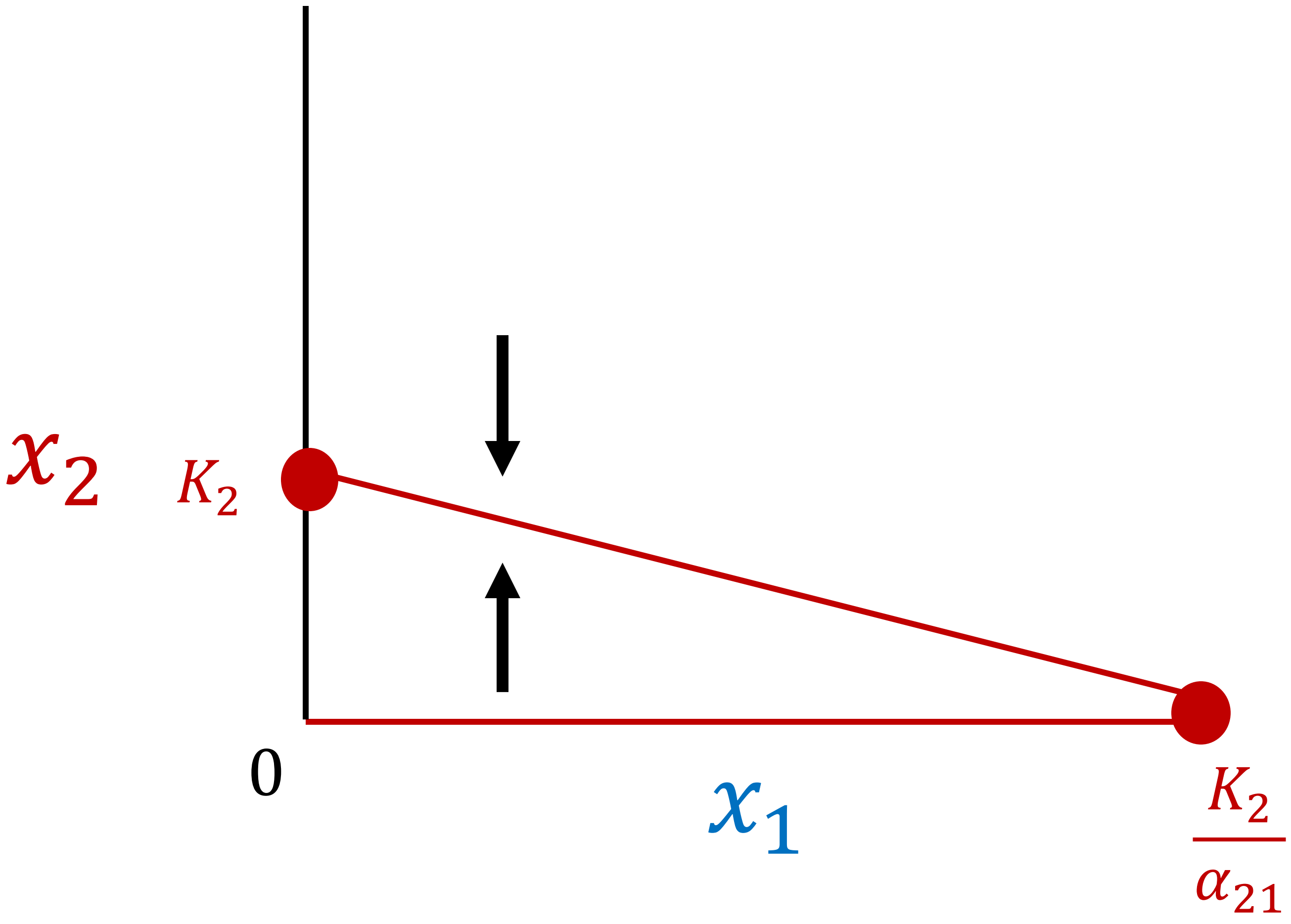

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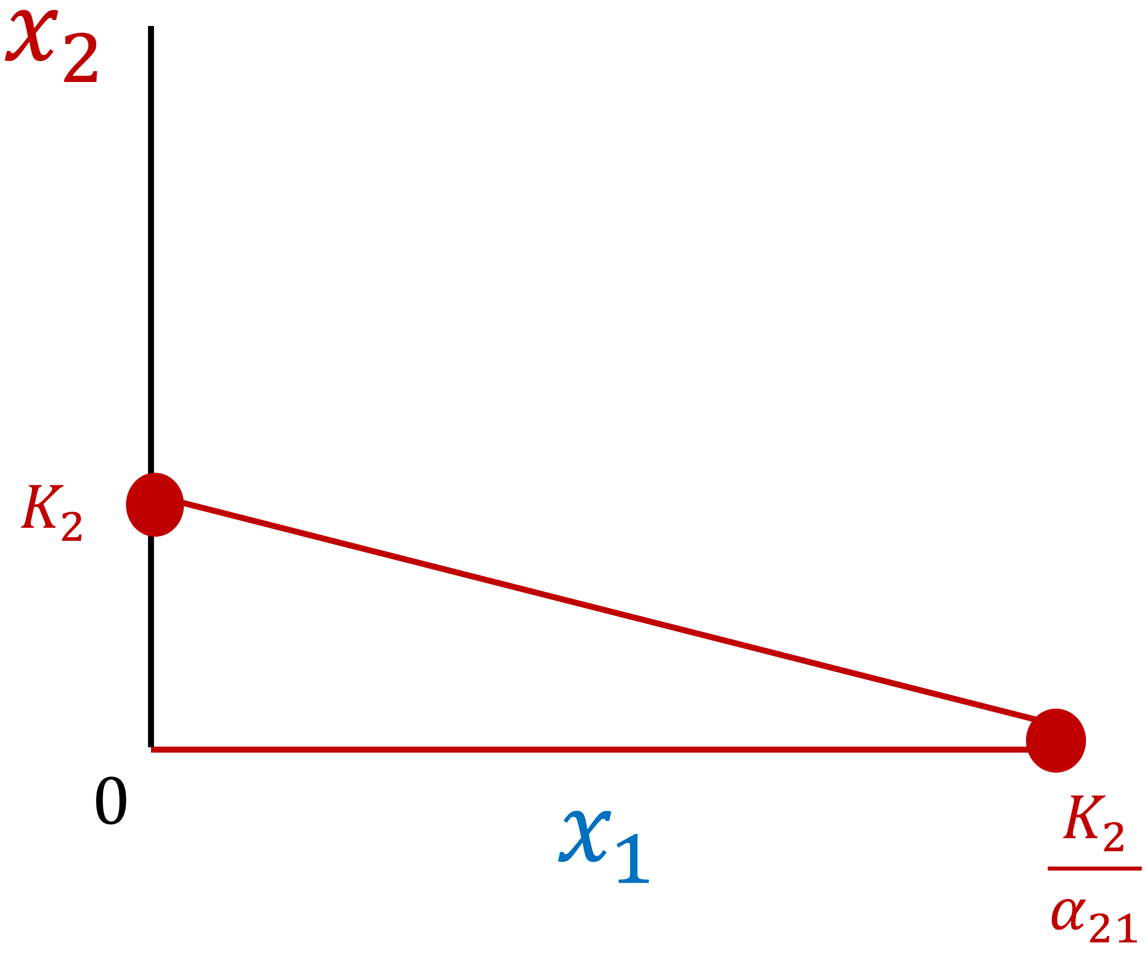

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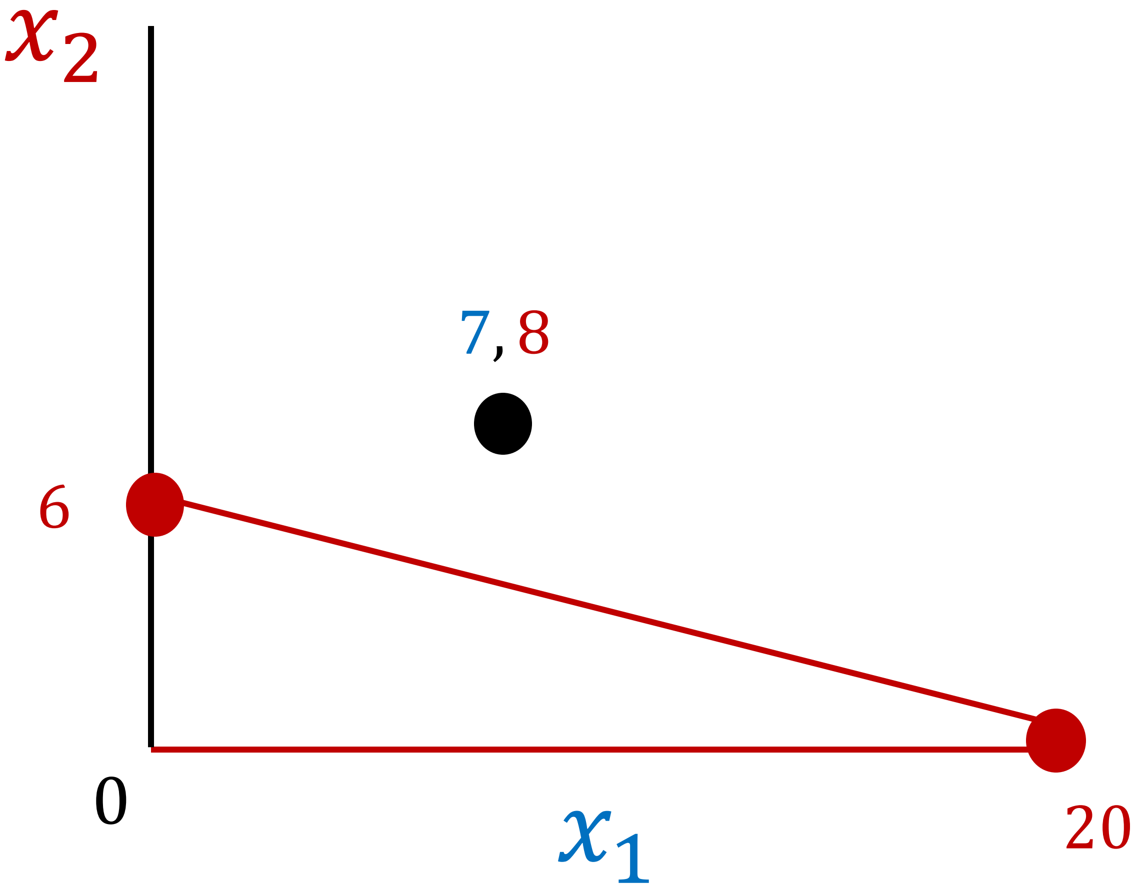

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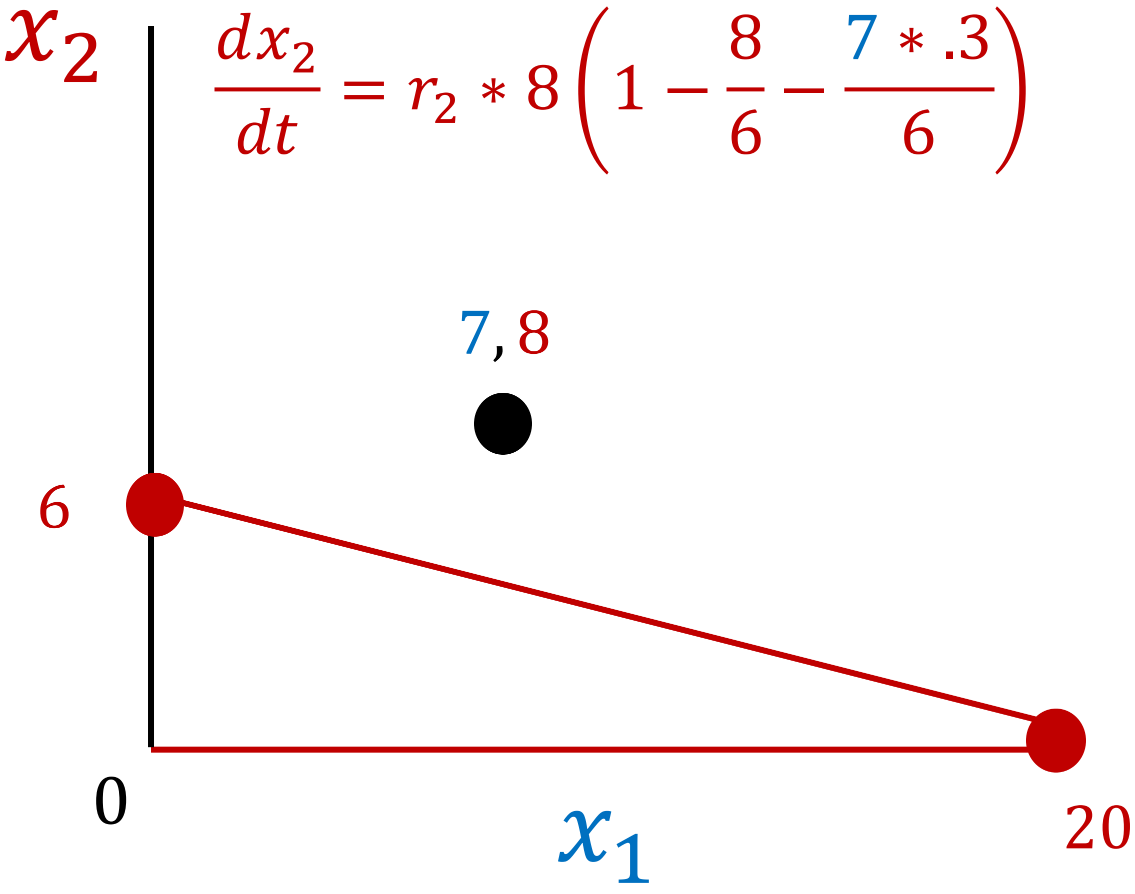

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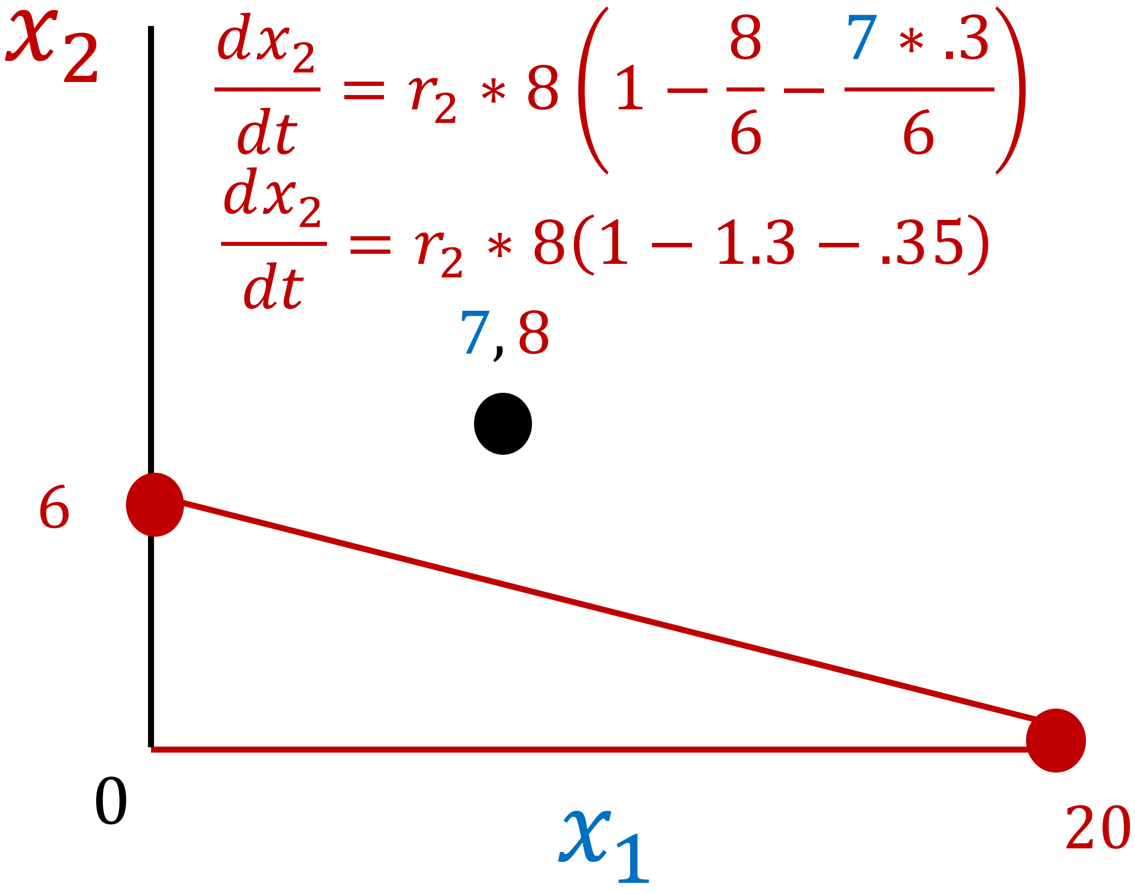

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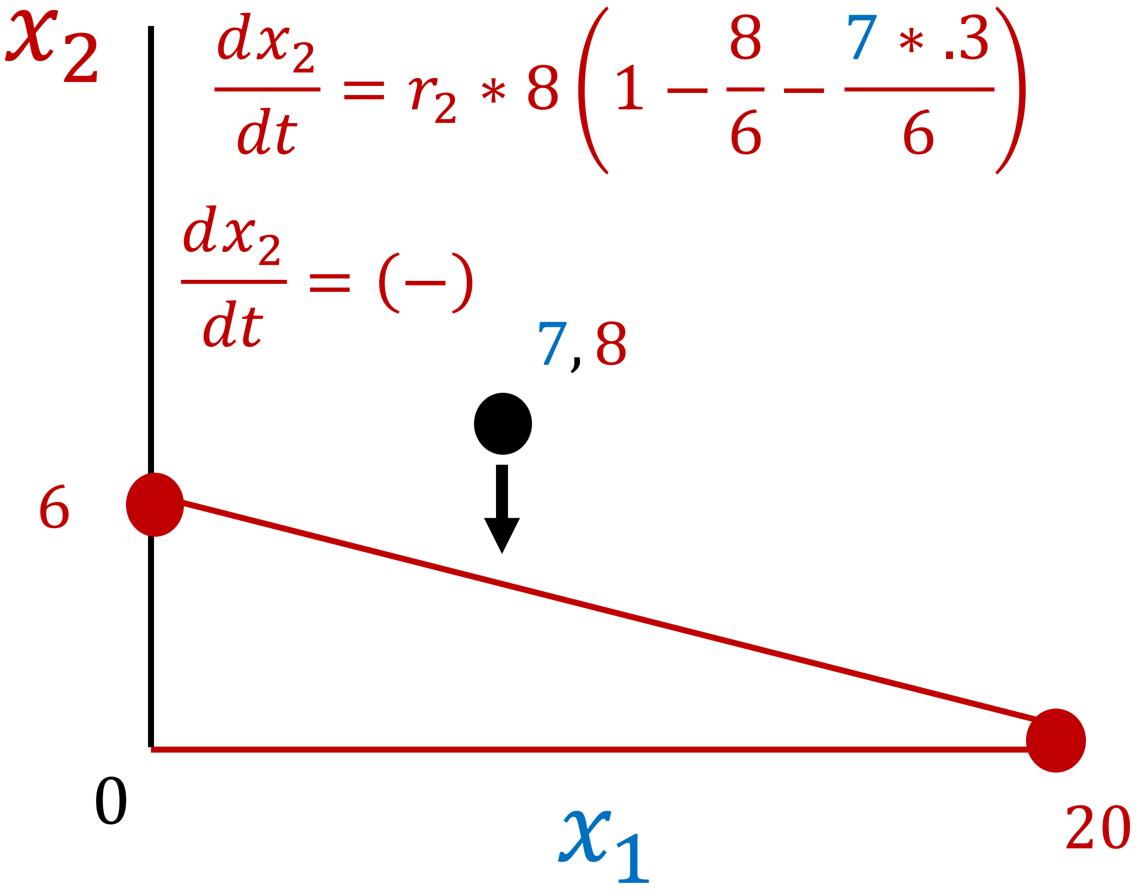

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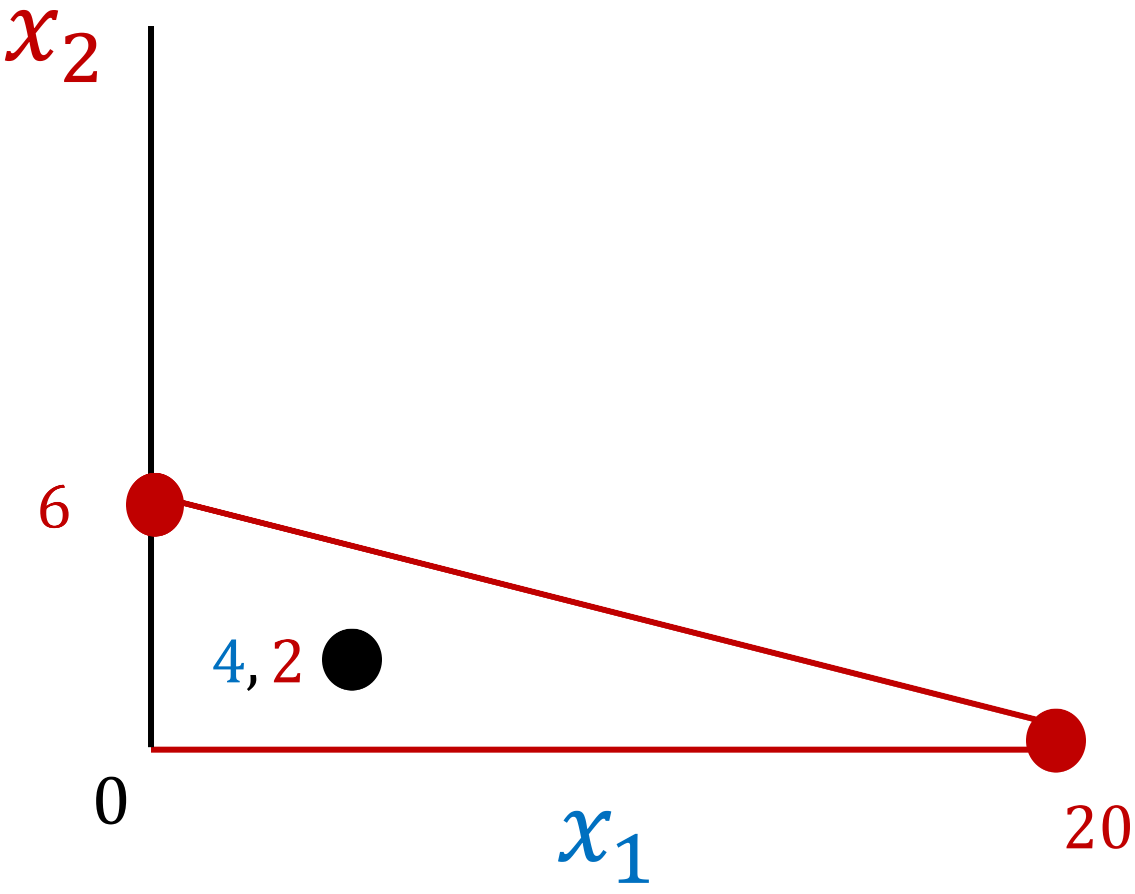

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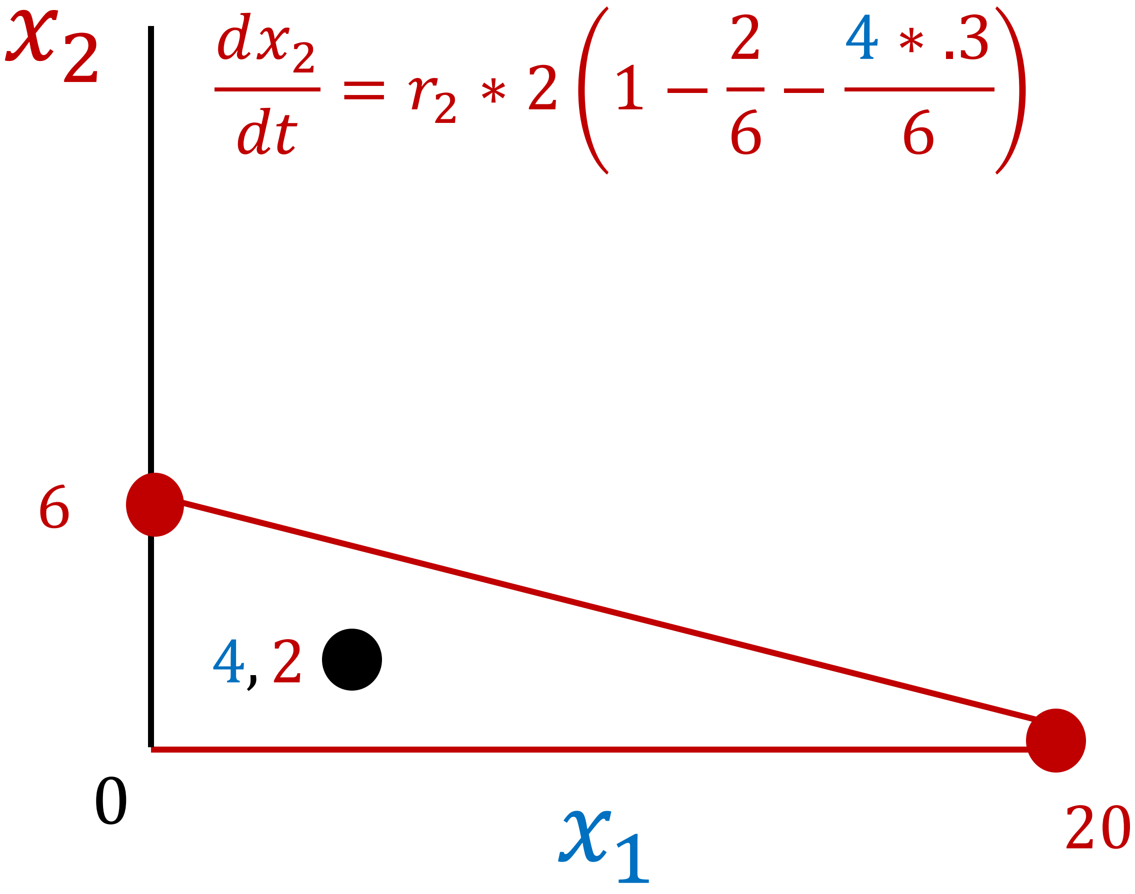

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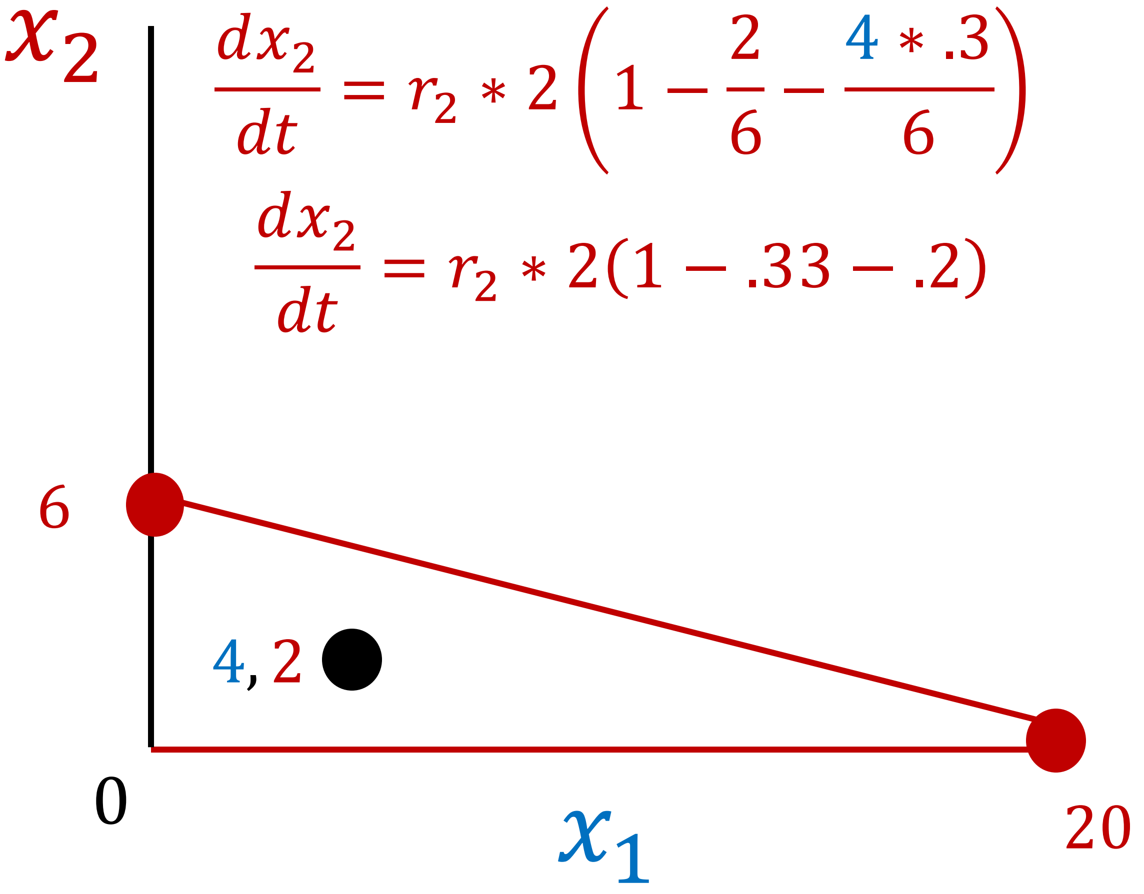

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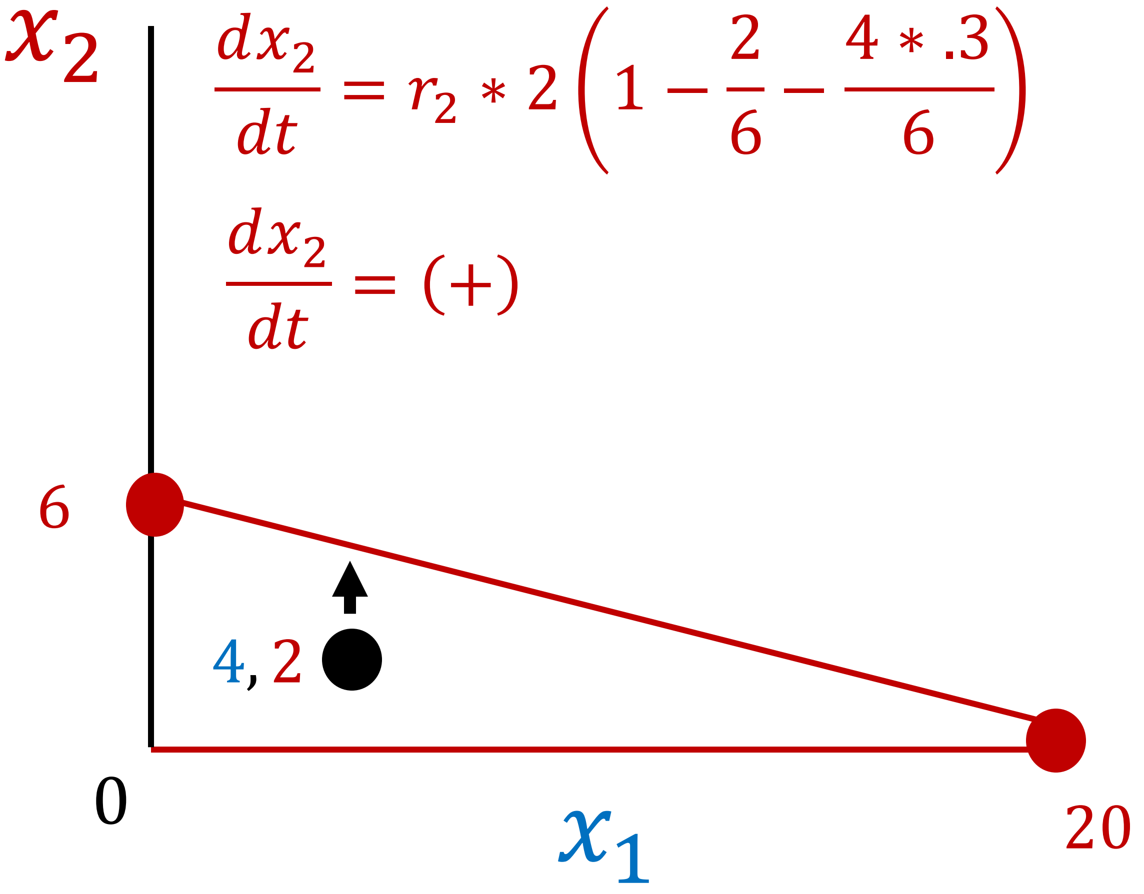
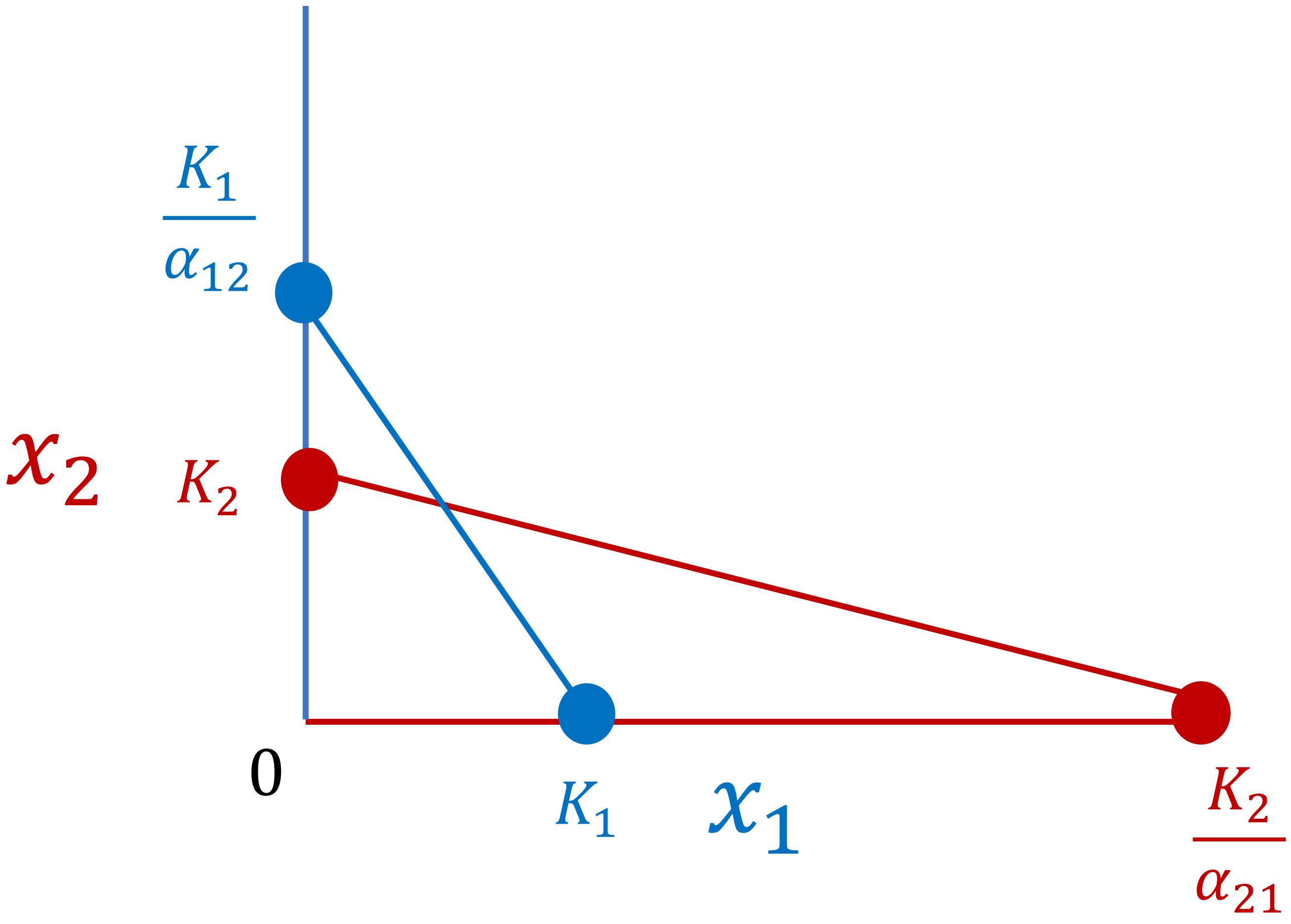
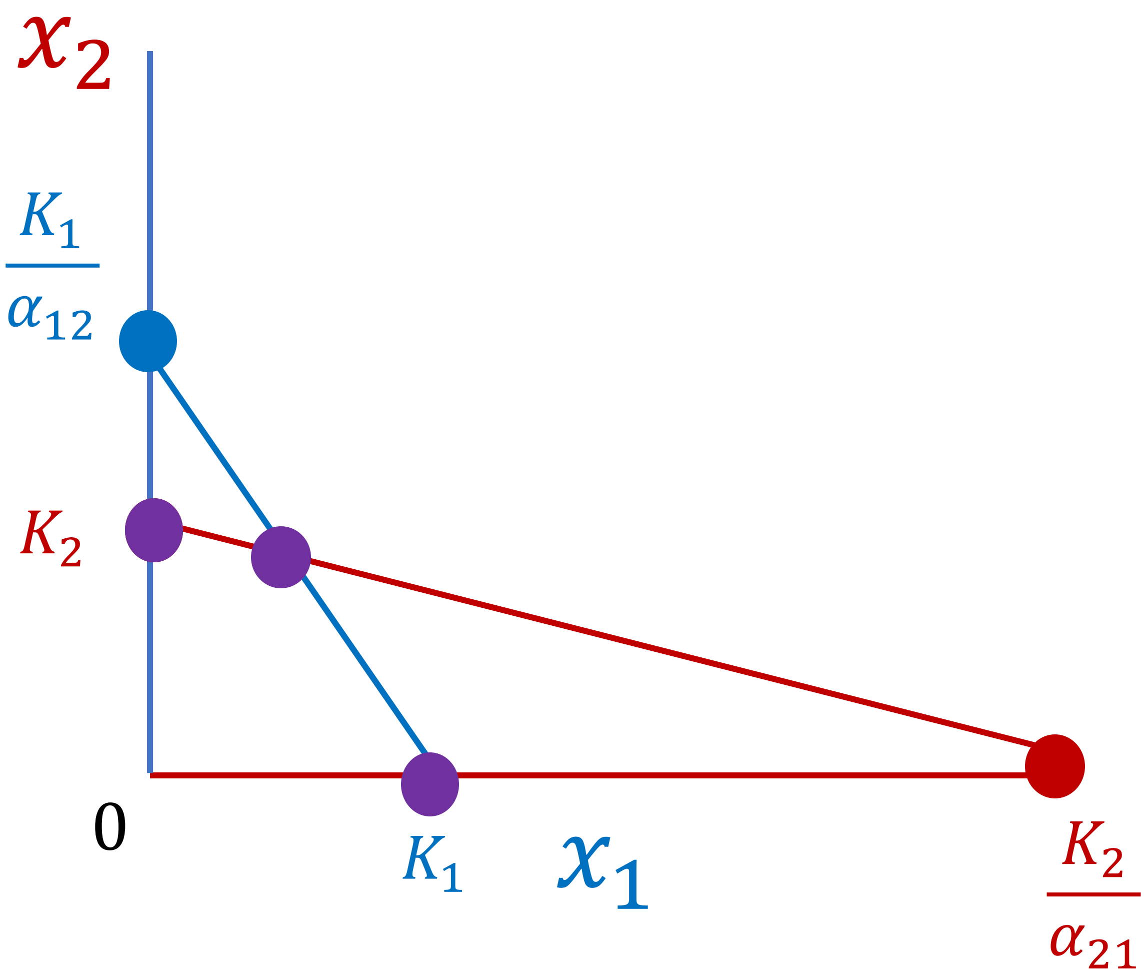


Given different combinations of x_1 and x_2 what is the outcome of competition?



Given different combinations of x_1 and x_2 what is the outcome of competition?

System will converge to one of its equilibria, depending on the values of the competition and carrying capacity parameters.



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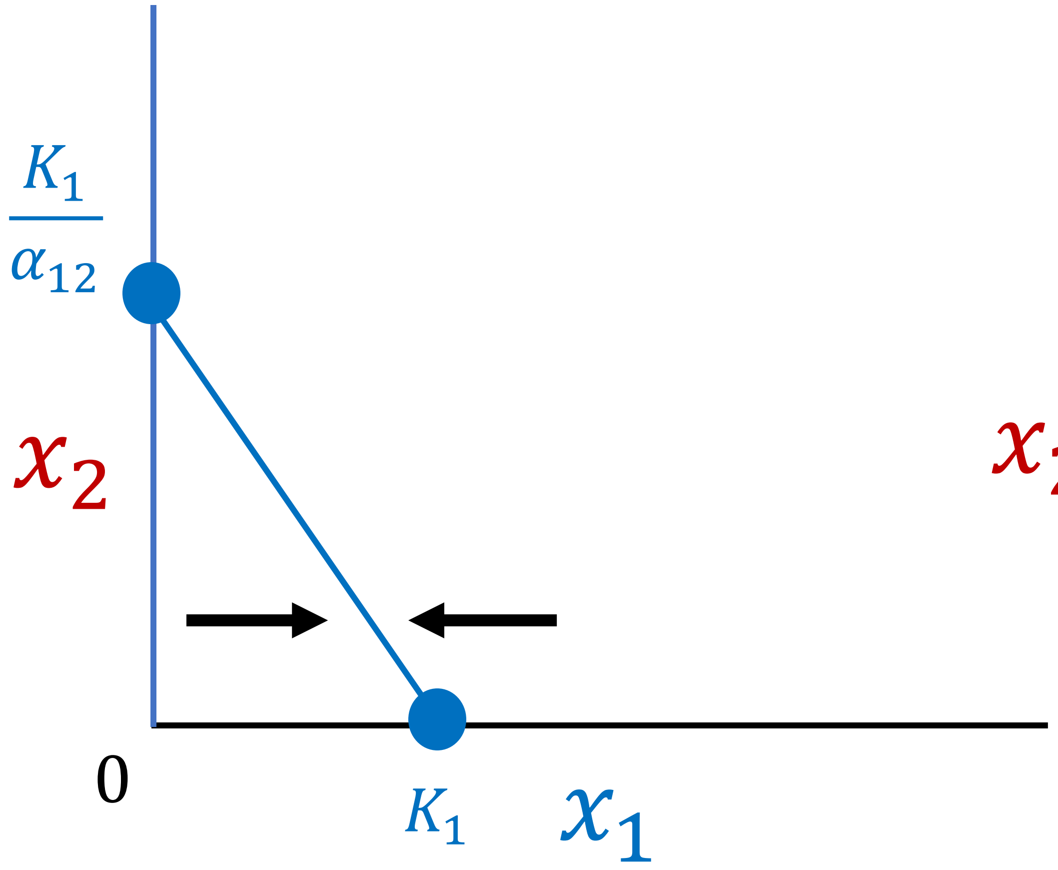

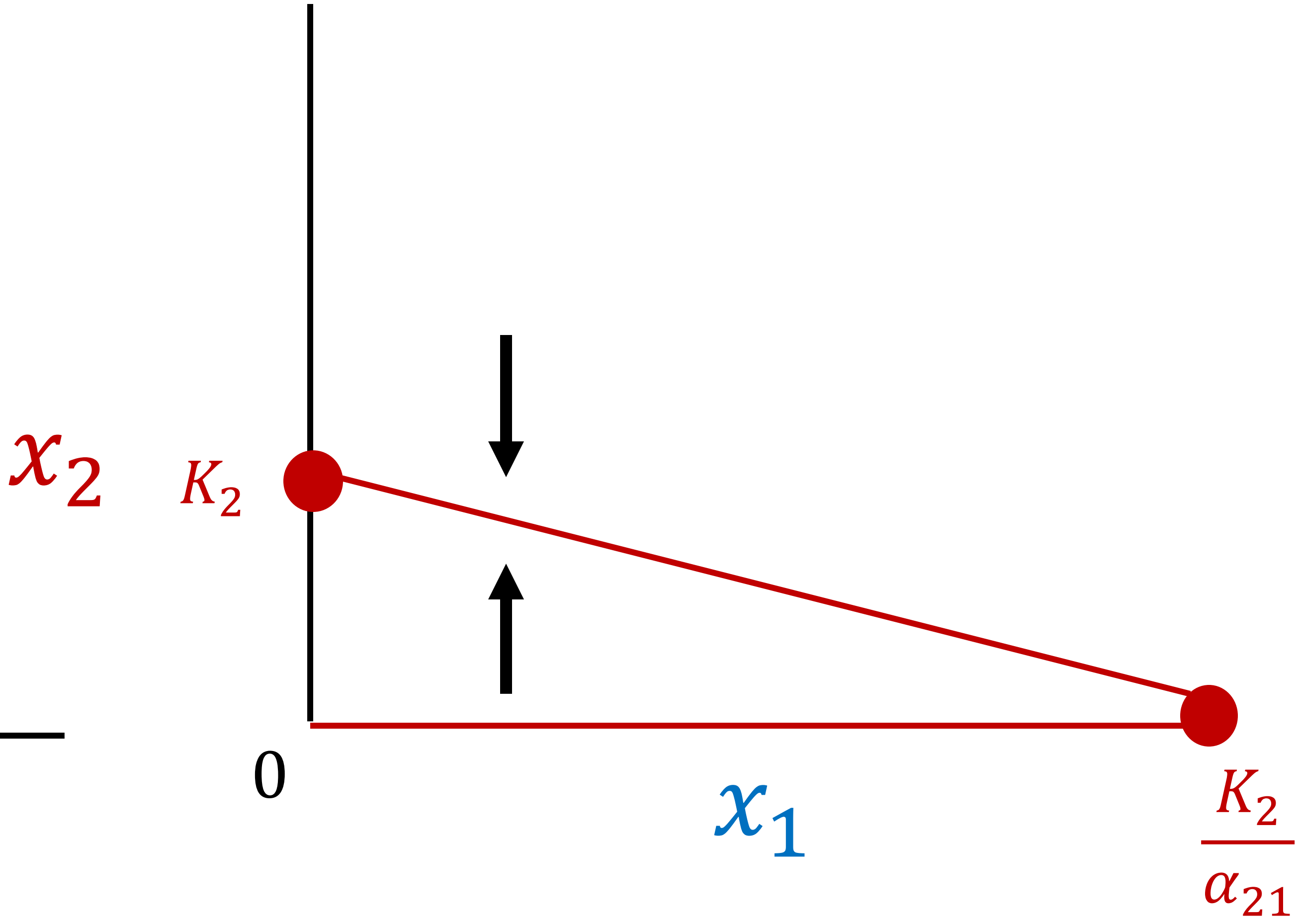

We can use vector addition from the individual nullclines to determine the outcome of competition.

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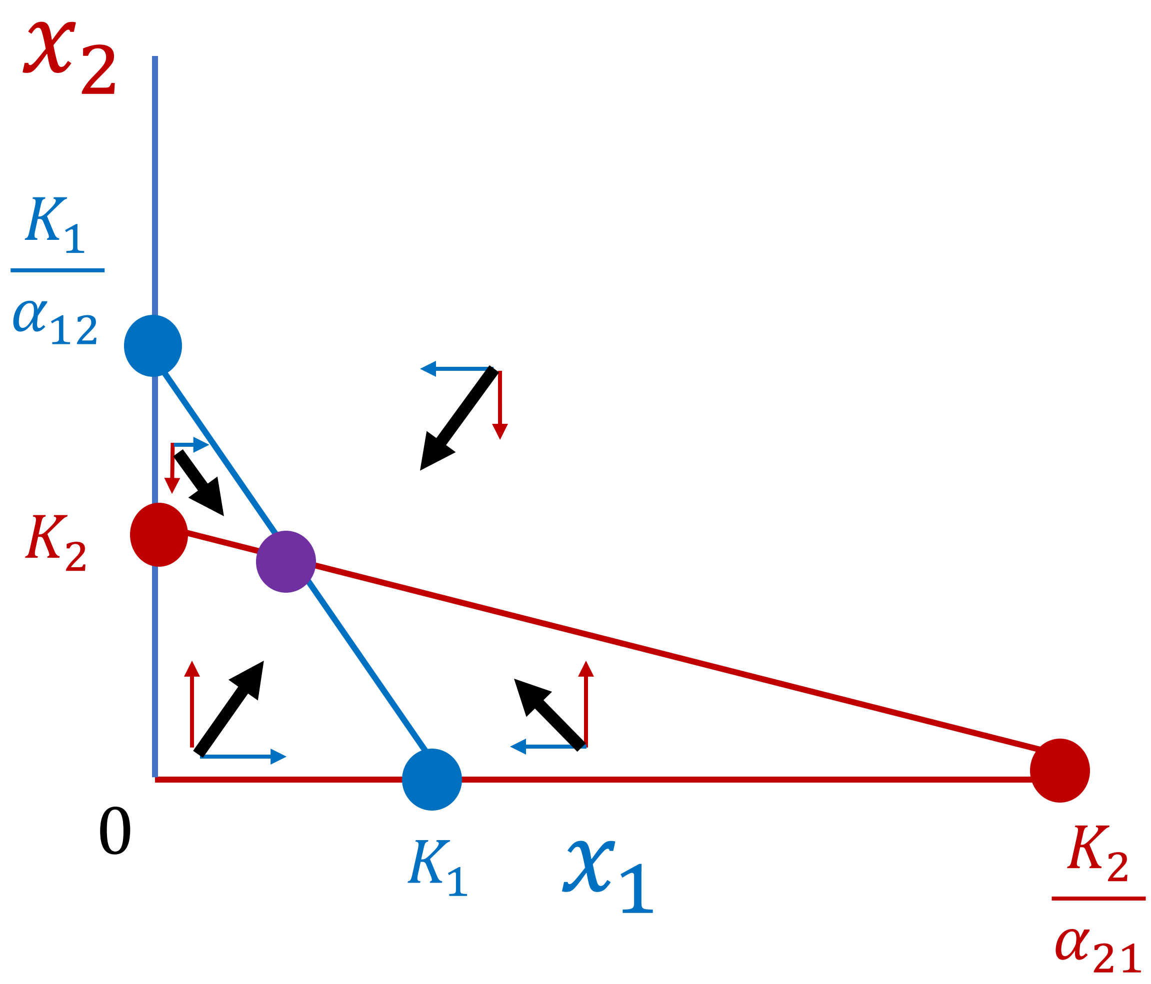
We can use vector addition **in each quadrat** from the individual nullclines to determine the outcome of competition.



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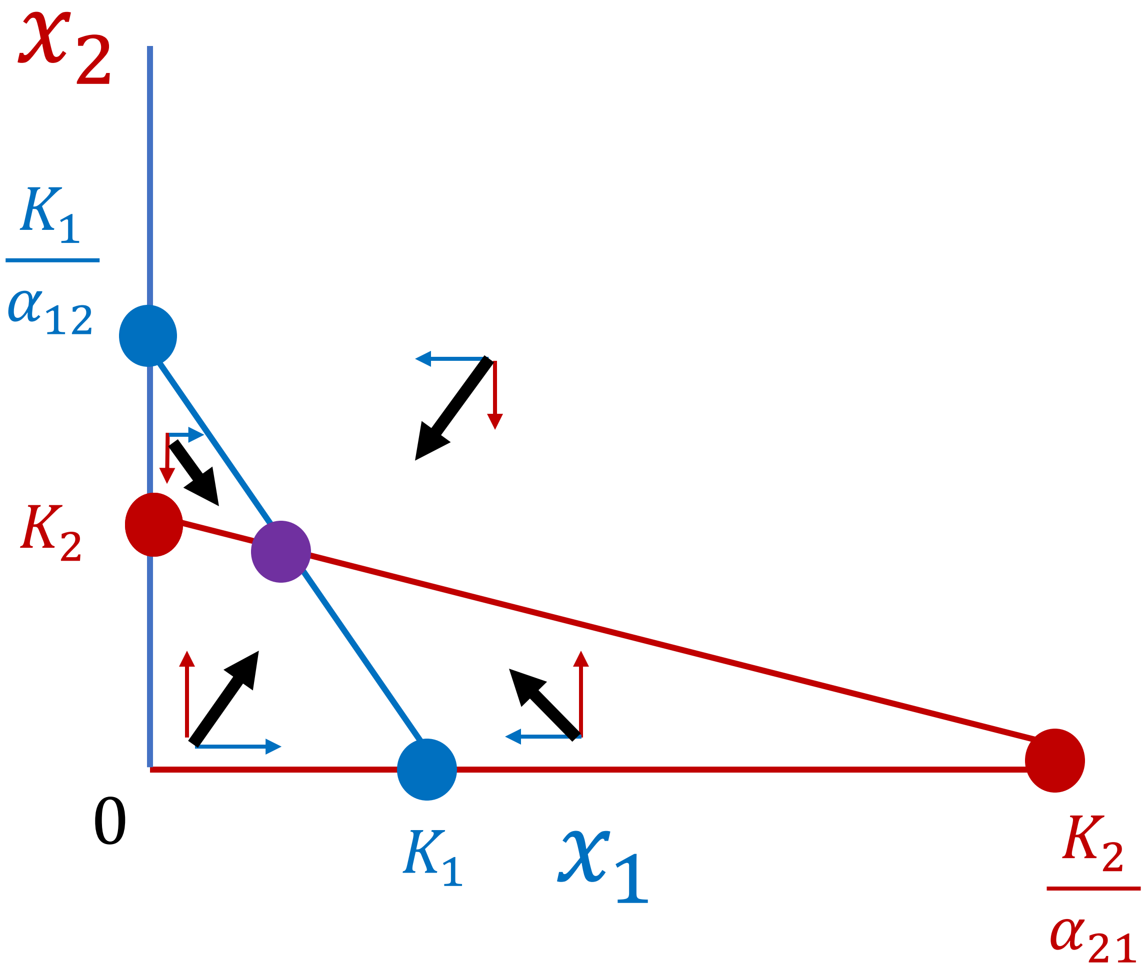
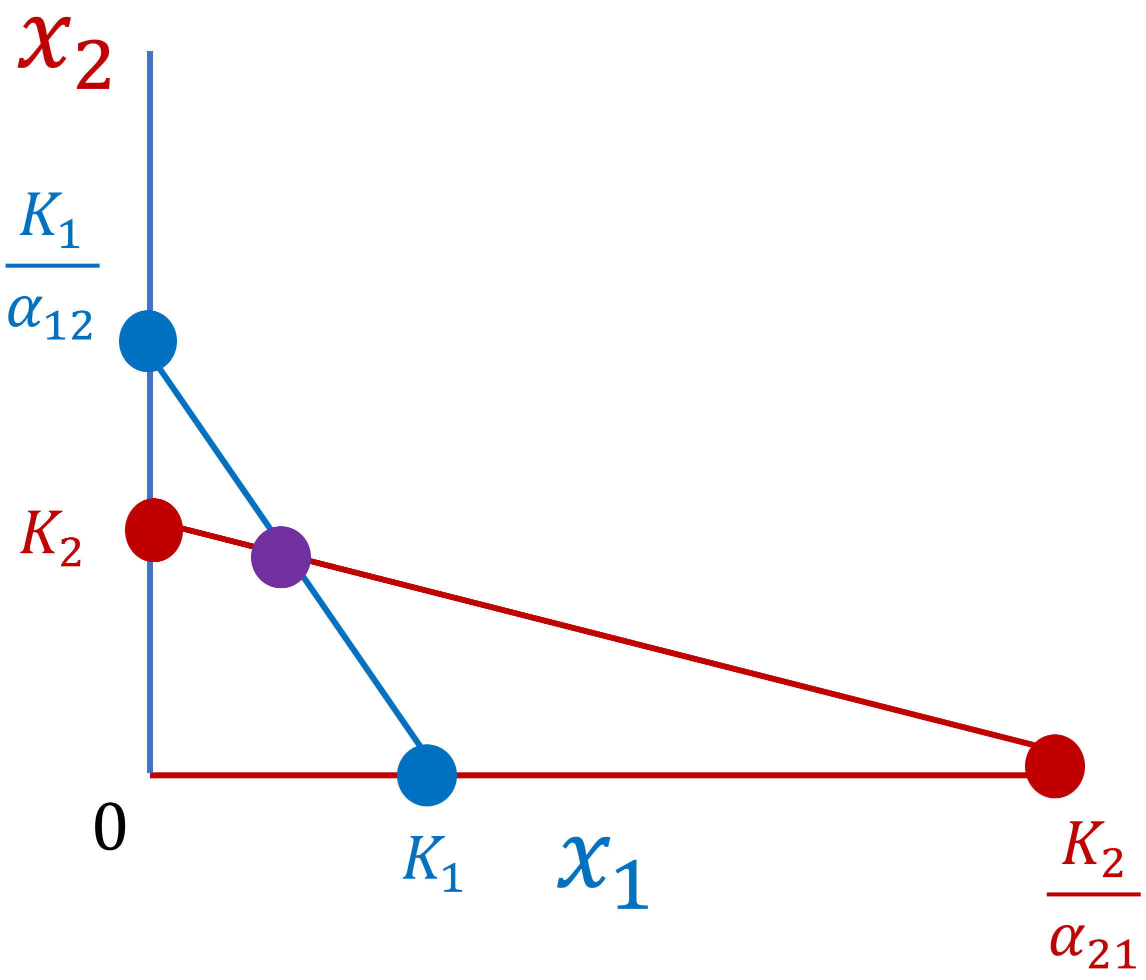
This configuration is a **stable equilibrium**, indicating **stable coexistence** at:

$$x_1^* = \frac{K_1 - K_2\alpha_{12}}{1 - \alpha_{21}\alpha_{12}} \quad x_2^* = \frac{K_2 - K_1\alpha_{21}}{1 - \alpha_{12}\alpha_{21}}$$



Given different combinations of x_1 and x_2 what is the outcome of competition?

Like before, we can also solve $\frac{dx_1}{dt}$ at different combinations of x_1 and x_2 to test the direction the x_1 population will move during competition (always along the x-axis).



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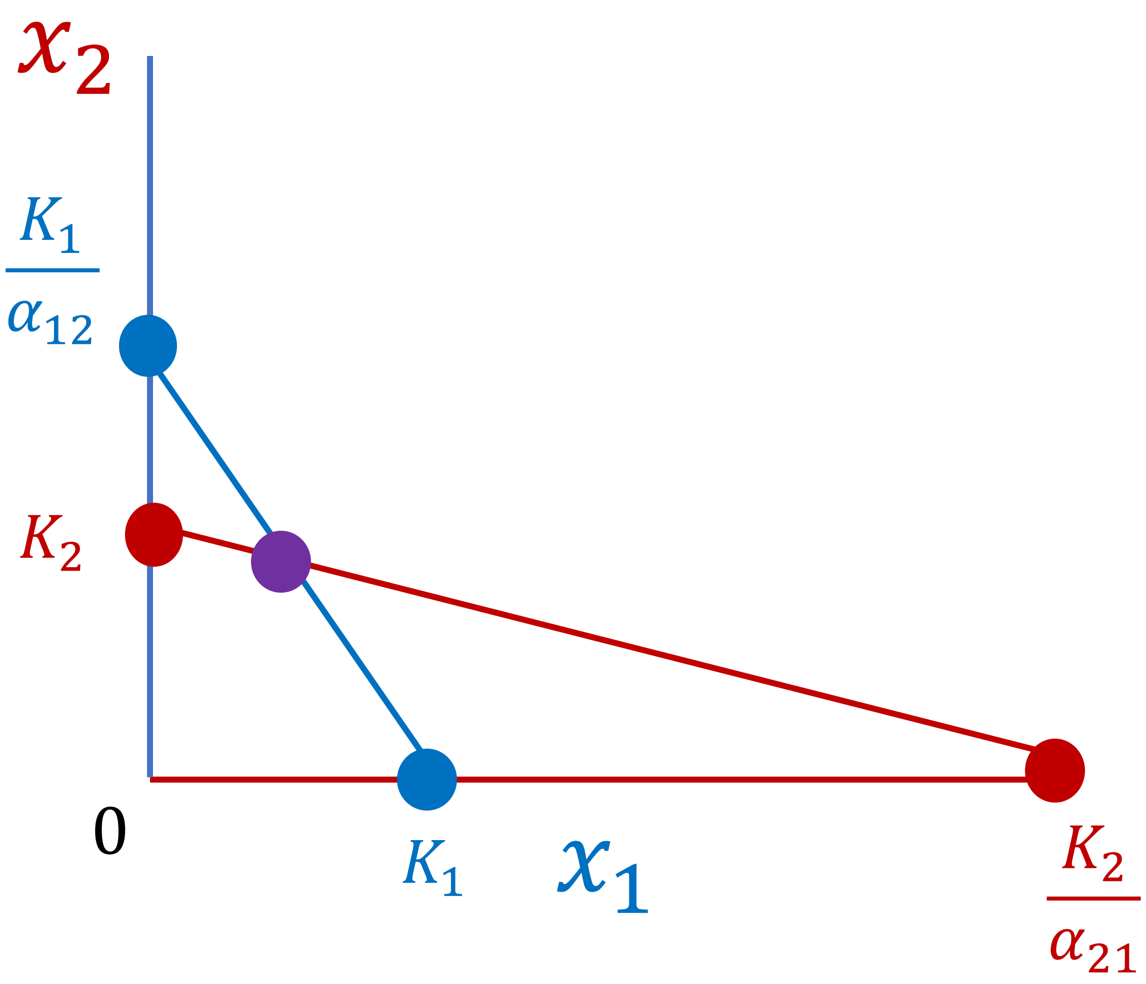
We can solve $\frac{dx_2}{dt}$ at different combinations of x_1 and x_2 to test the direction the x_2 population will move during competition (always along the y-axis).

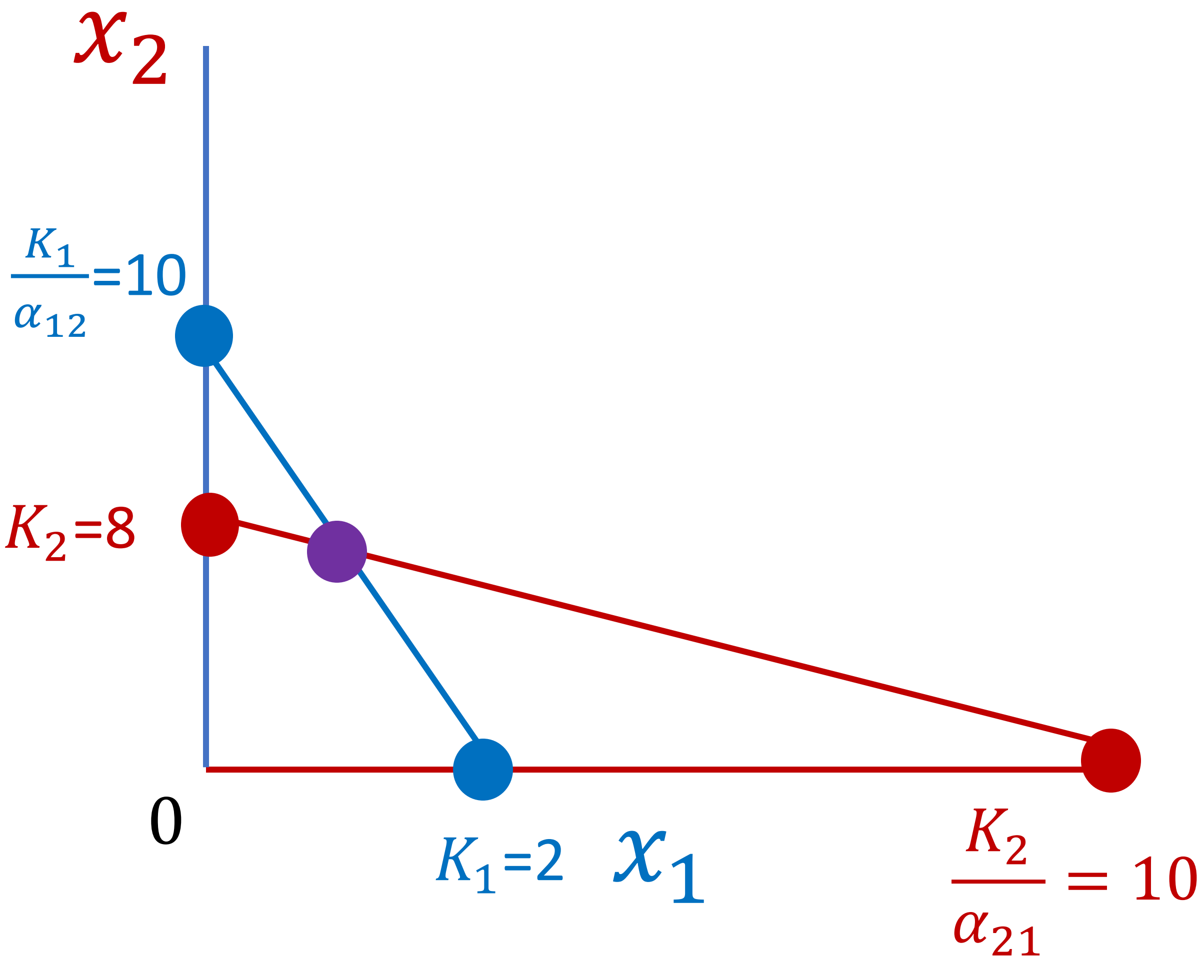


Given different combinations of x_1 and x_2 what is the outcome of competition?

Let's try it!

$K_1 = 2\ ; \alpha_{12} = .3$

$K_2 = 8\ ; \alpha_{21} = .8$



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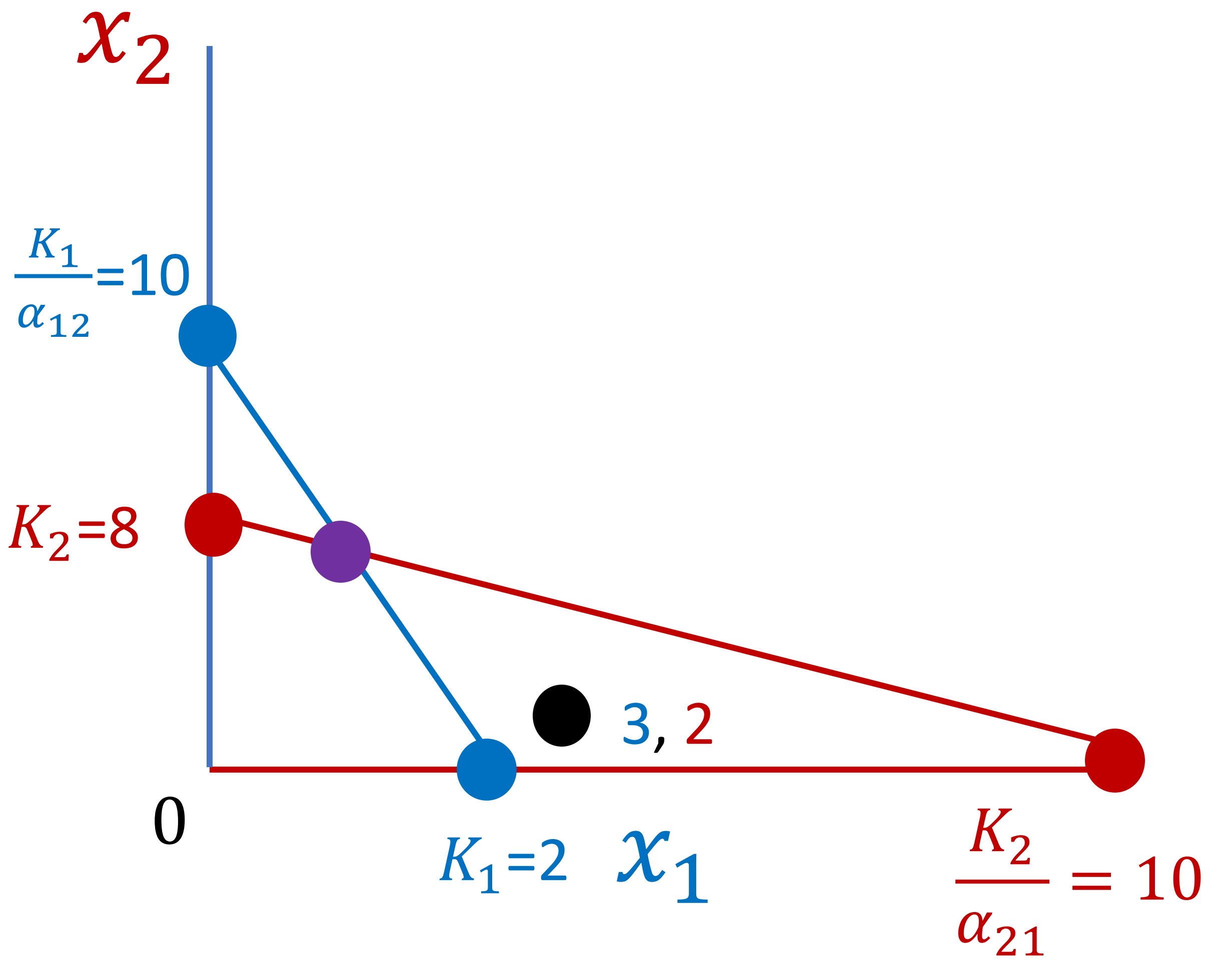

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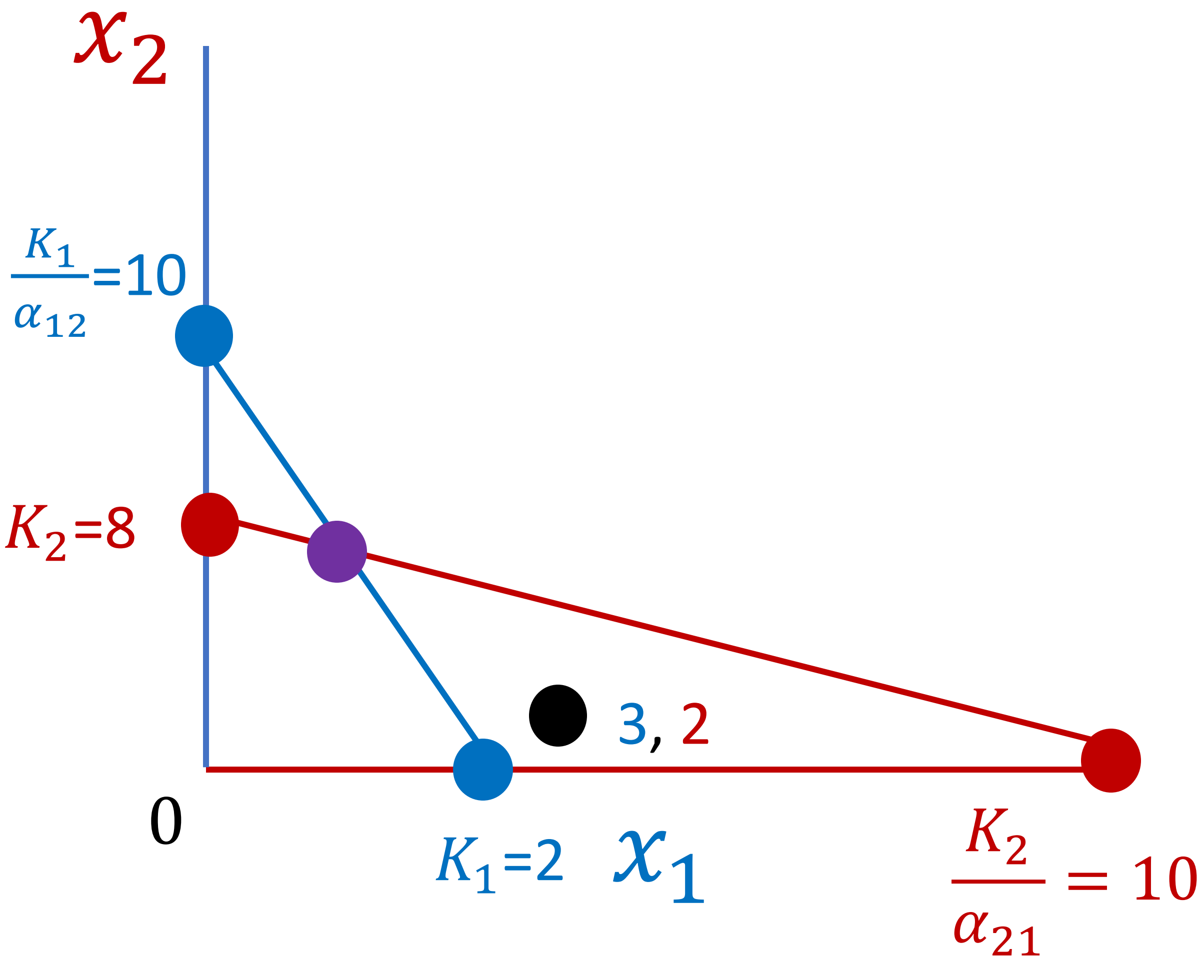

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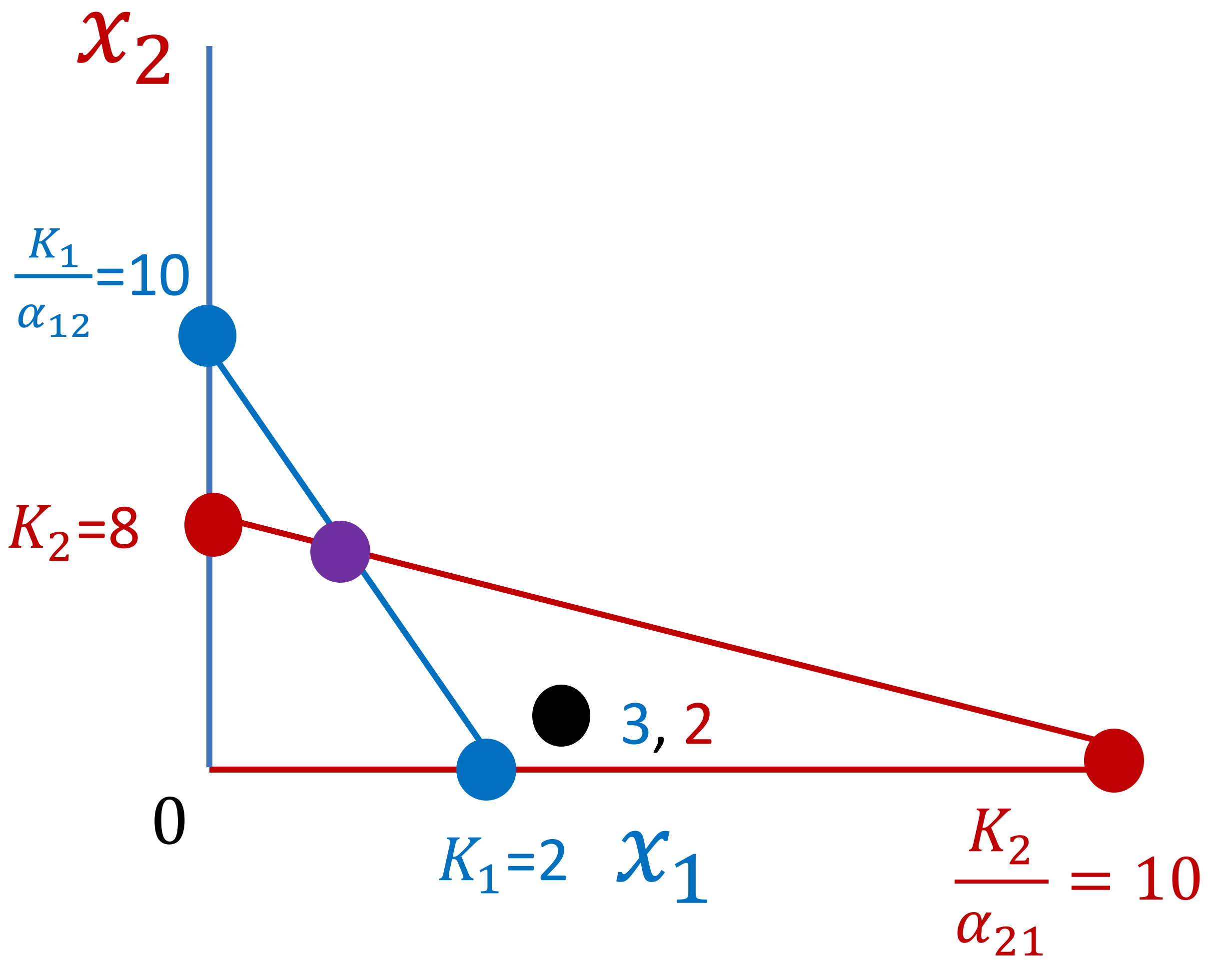

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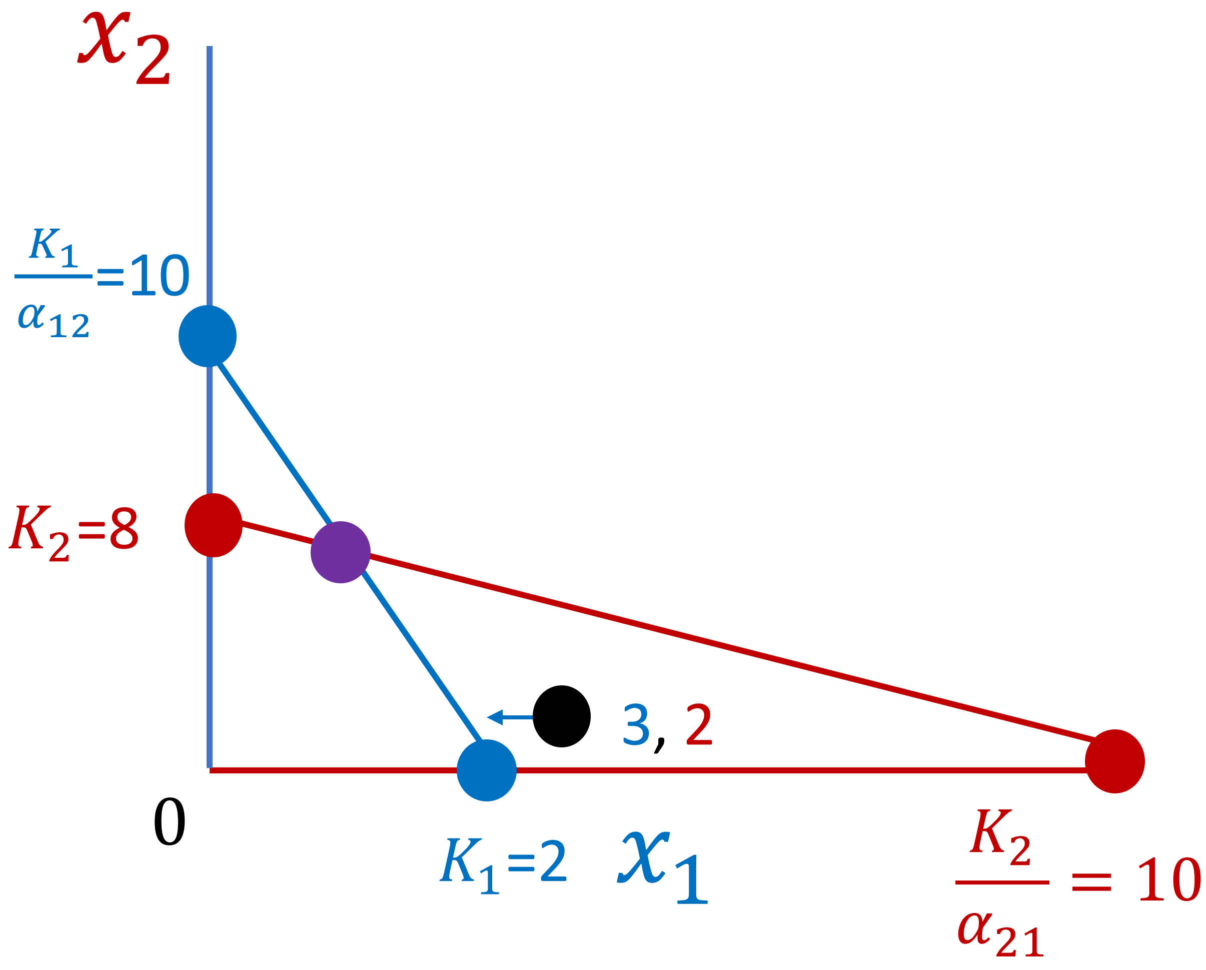

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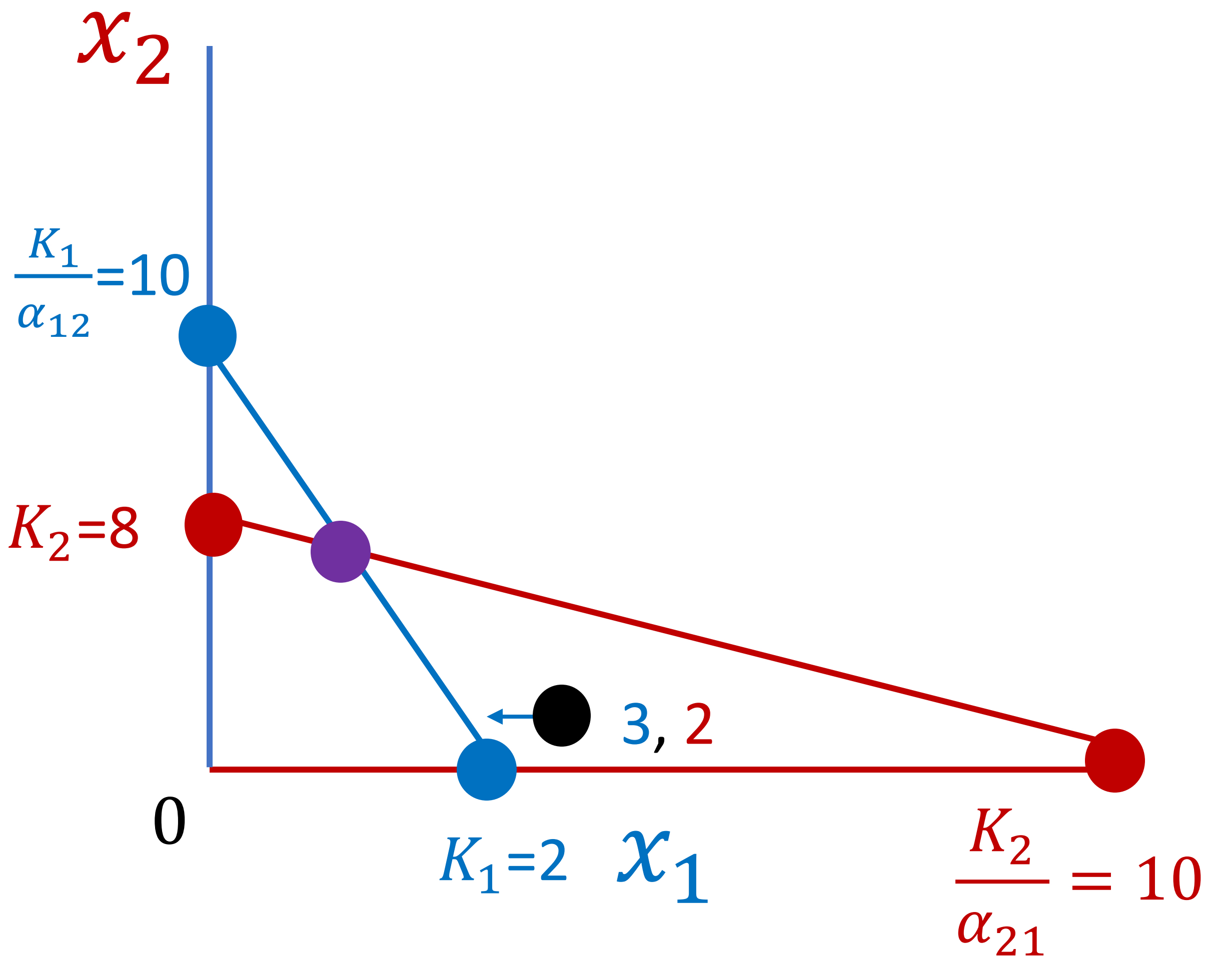

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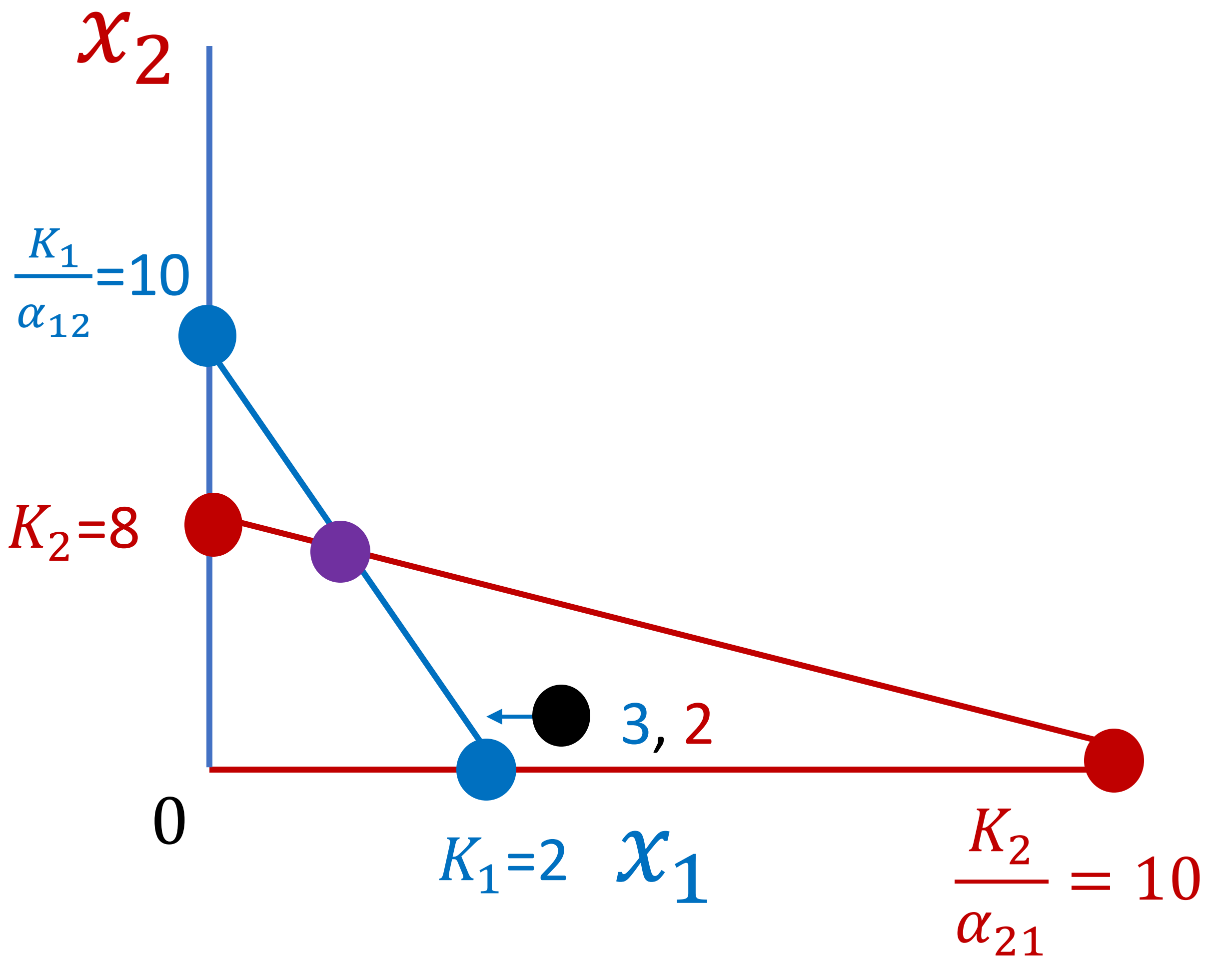

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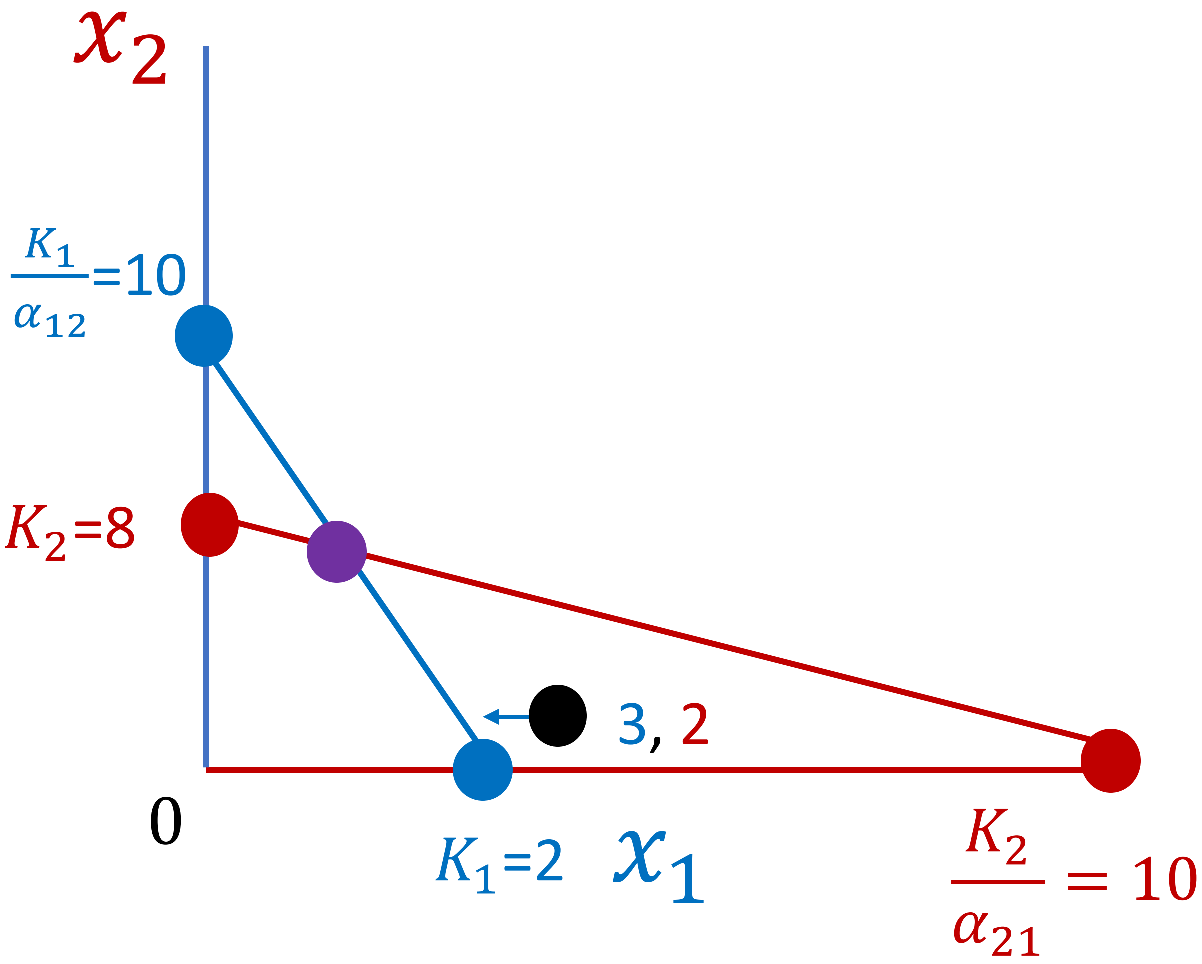

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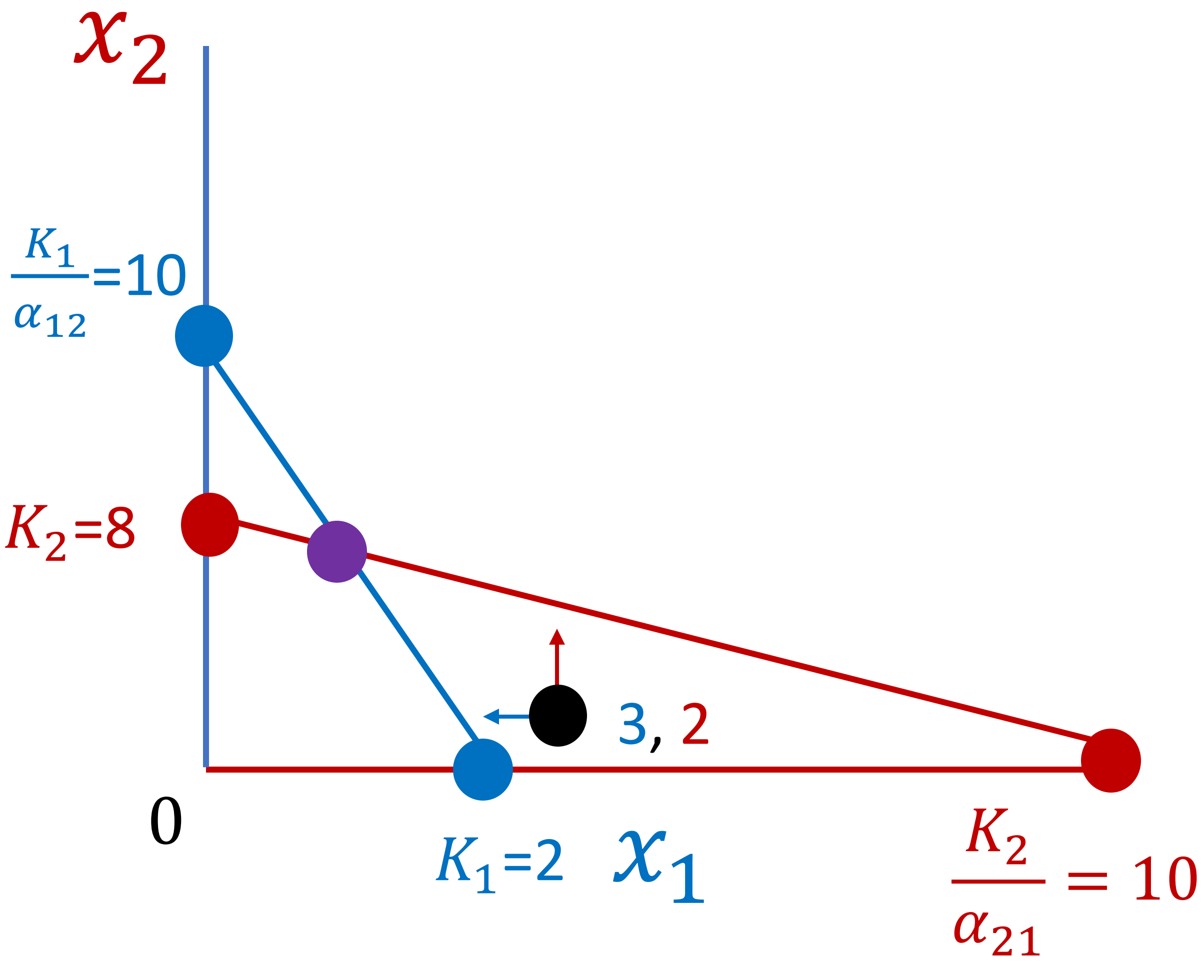

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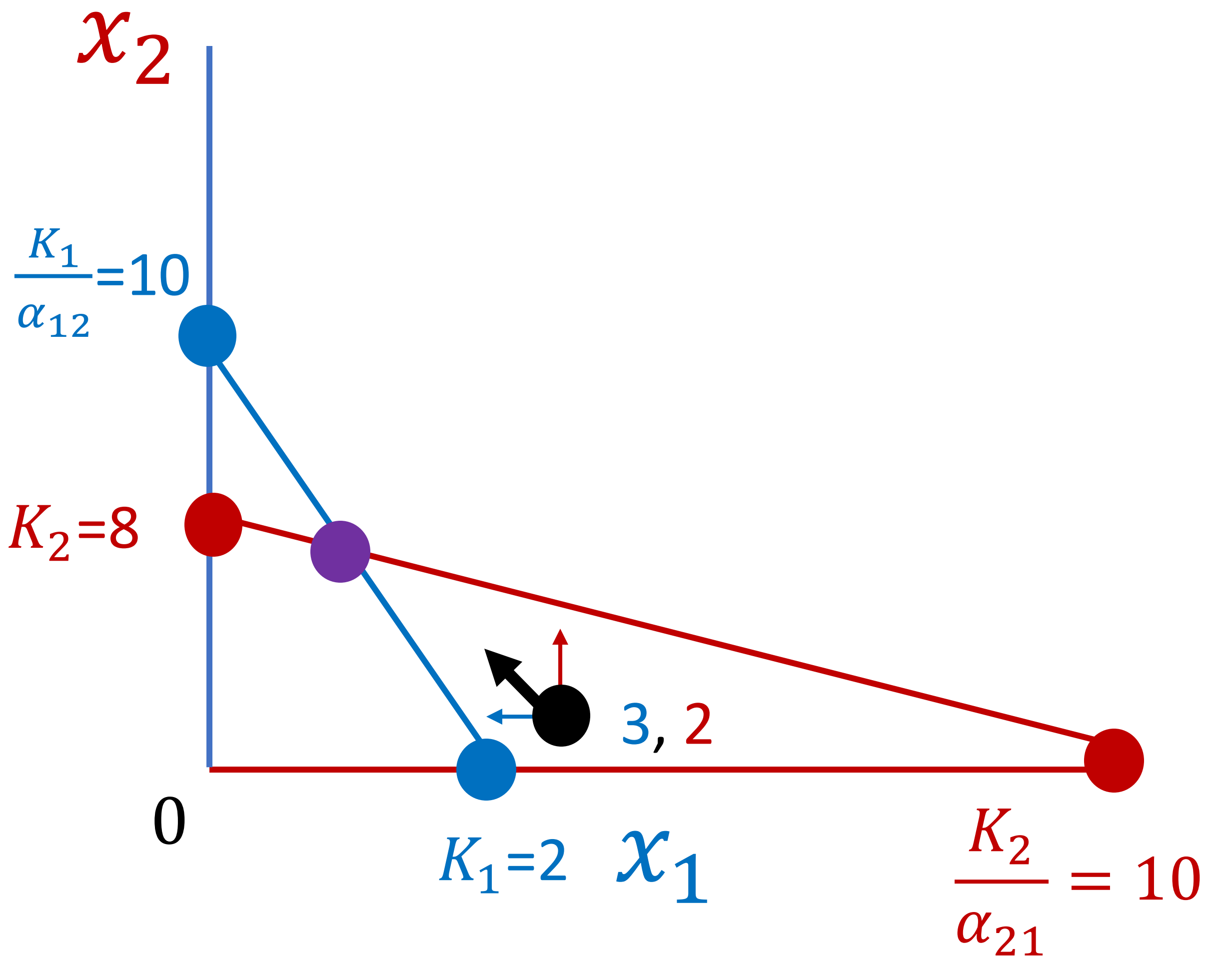
$K_2 = 8\ ;\ \alpha_{21} = .8$

Results replicate graphical approach!

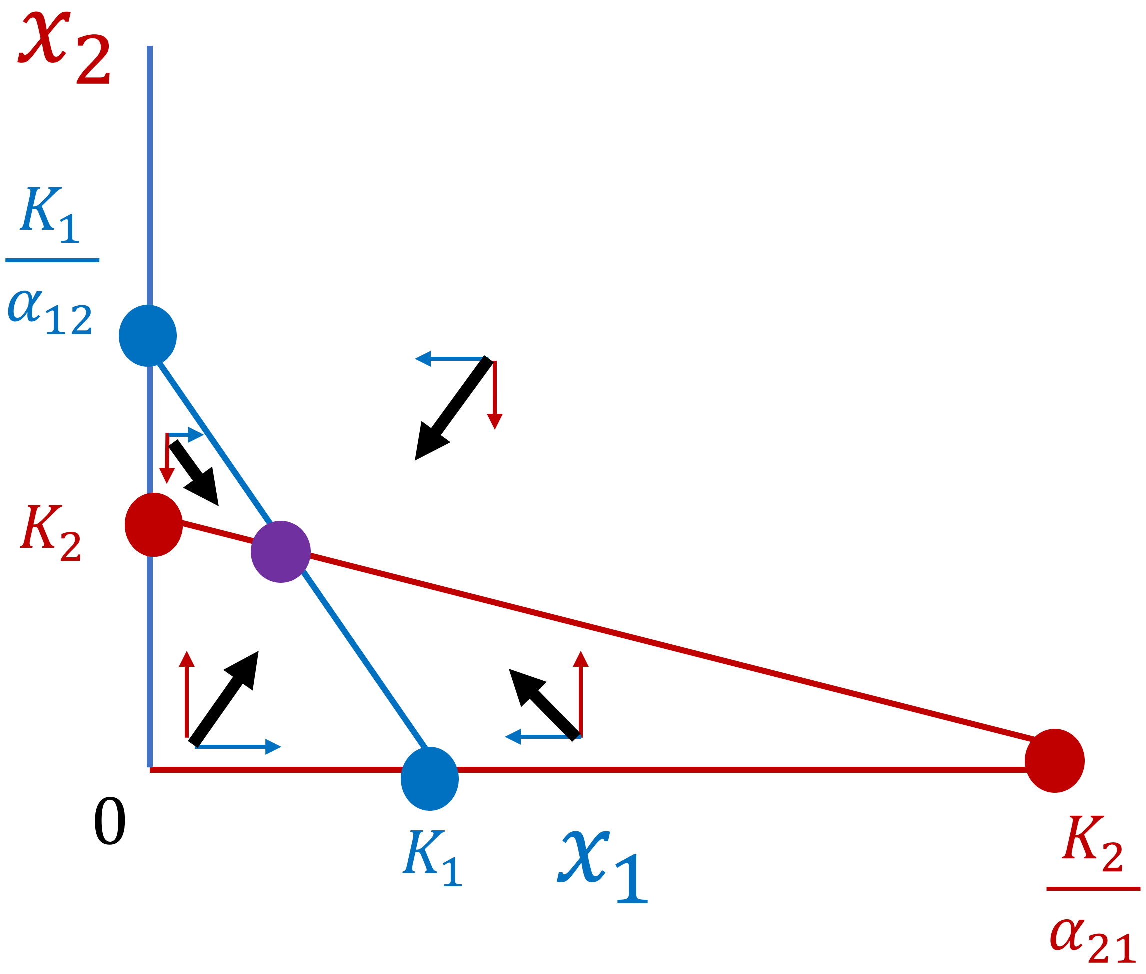


Four possible outcomes for competition

Case 1:

Stable equilibrium, with **coexistence** at:

$$x_1^* = \frac{K_1 - K_2\alpha_{12}}{1 - \alpha_{21}\alpha_{12}} \quad x_2^* = \frac{K_2 - K_1\alpha_{21}}{1 - \alpha_{12}\alpha_{21}}$$



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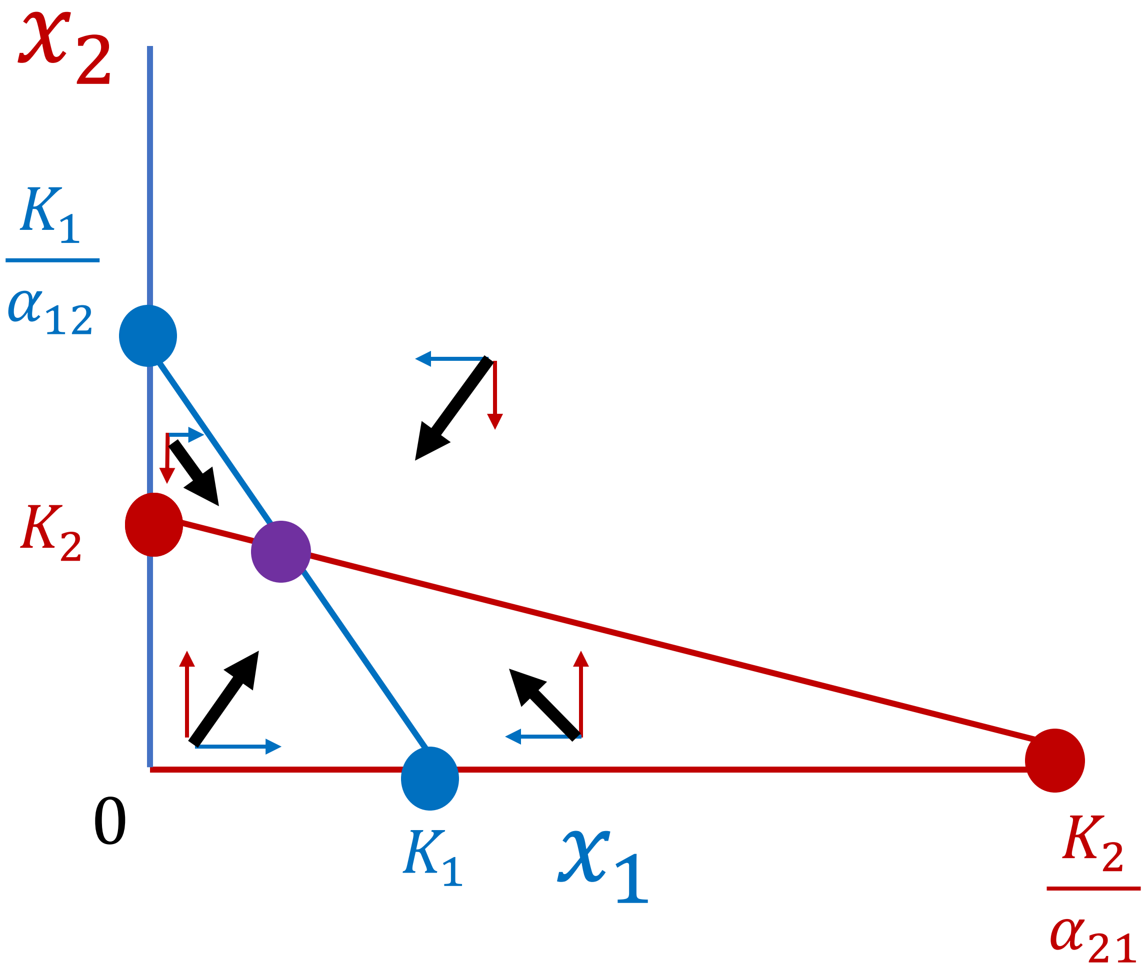
$$\frac{\boldsymbol{K_1}}{\boldsymbol{\alpha_{12}}} > \boldsymbol{K_2} \;\&\; \frac{\boldsymbol{K_2}}{\boldsymbol{\alpha_{21}}} > \boldsymbol{K_1}$$

$$(\alpha_{12} * \alpha_{21} < 1)$$



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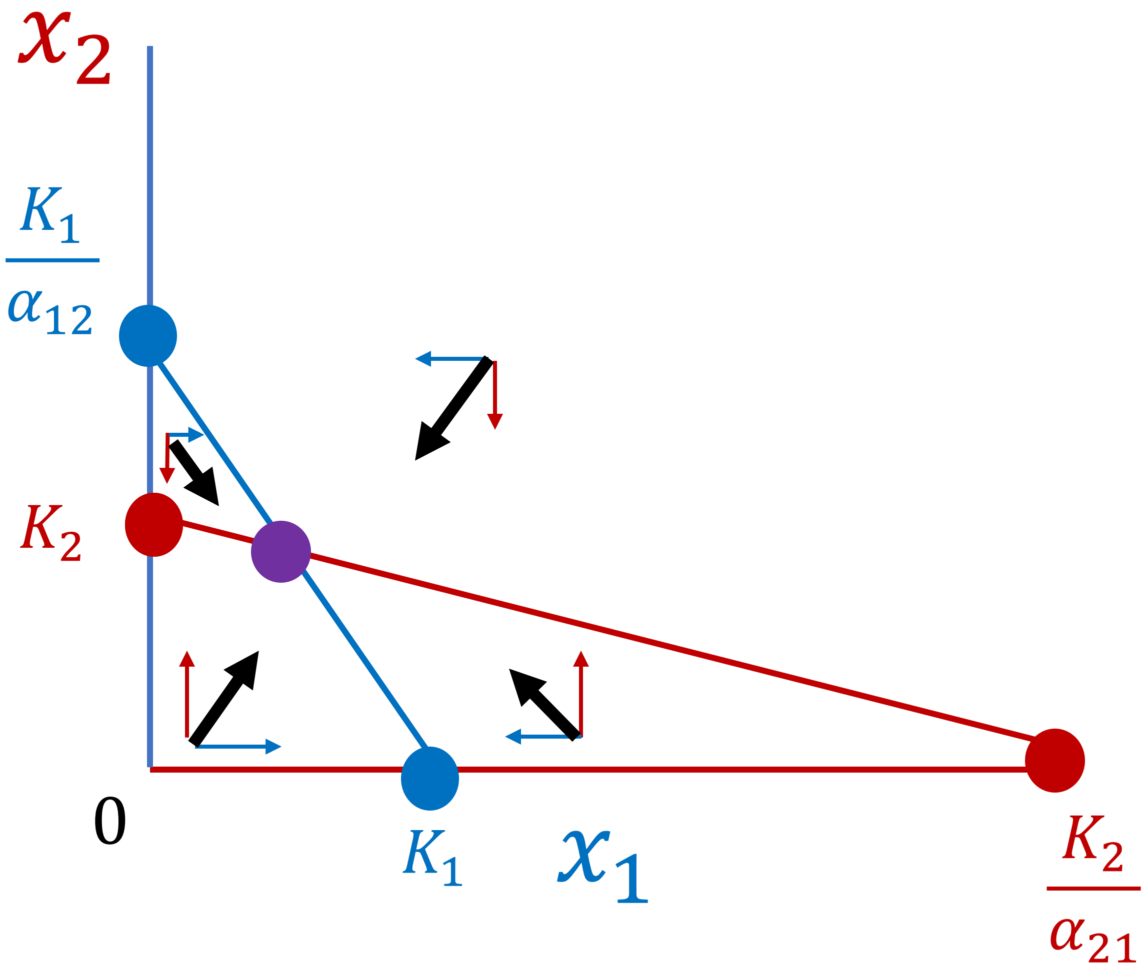
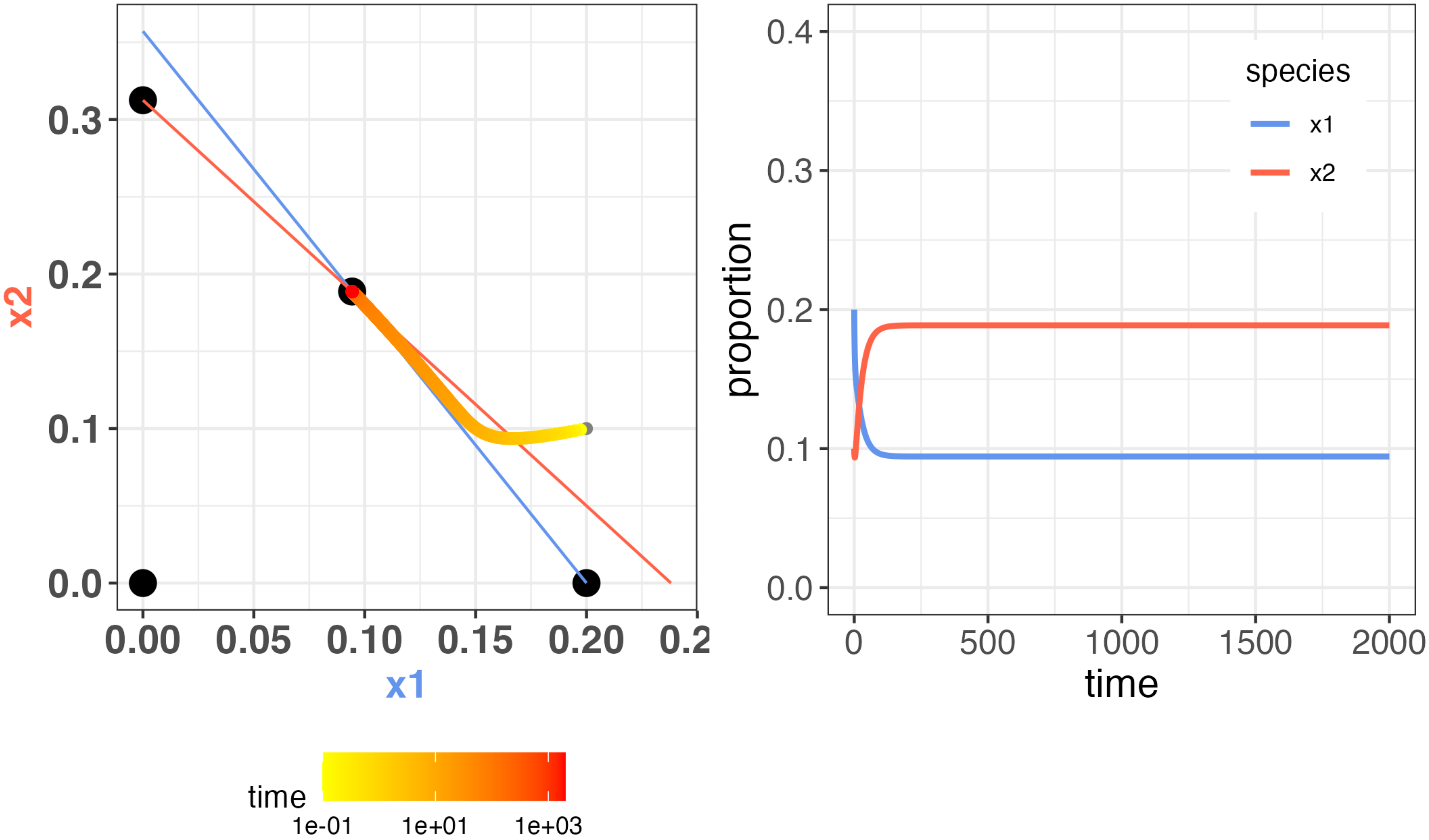


- Species coexist stably but **below carrying capacity** for each individual population.
- **Intraspecific** competition is stronger than **interspecific** competition.

Case 1:

Stable equilibrium, with **coexistence**:

$$\frac{K_1}{\alpha_{12}} > K_2 \;\&\; \frac{K_2}{\alpha_{21}} > K_1 \qquad (\alpha_{12} * \alpha_{21} < 1)$$

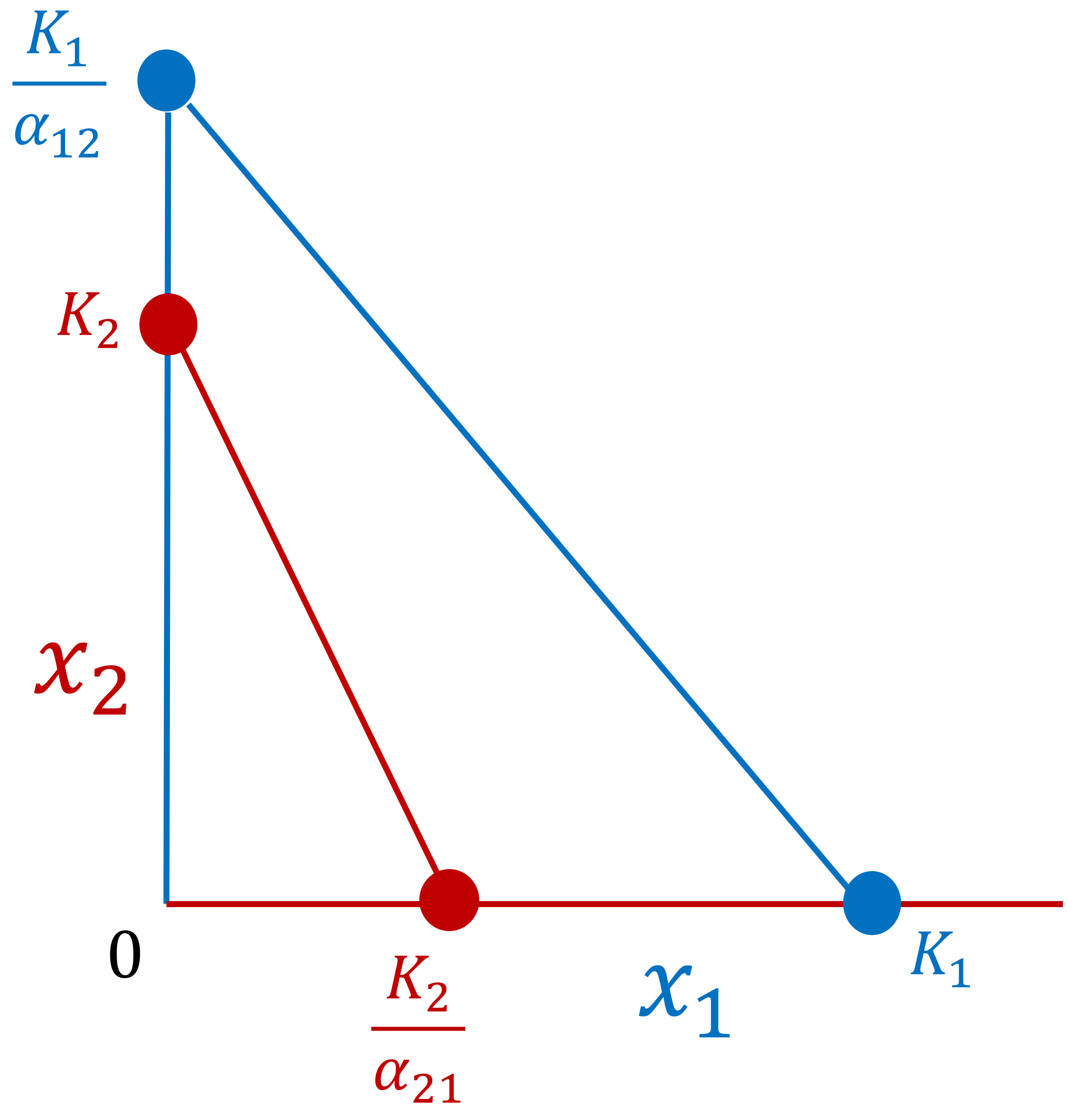


Four possible outcomes for competition

Case 2:

Species 1 outcompetes species 2

$$\frac{K_1}{\alpha_{12}} > K_2 \ \& \ \frac{K_2}{\alpha_{21}} < K_1$$



Four possible outcomes for competition

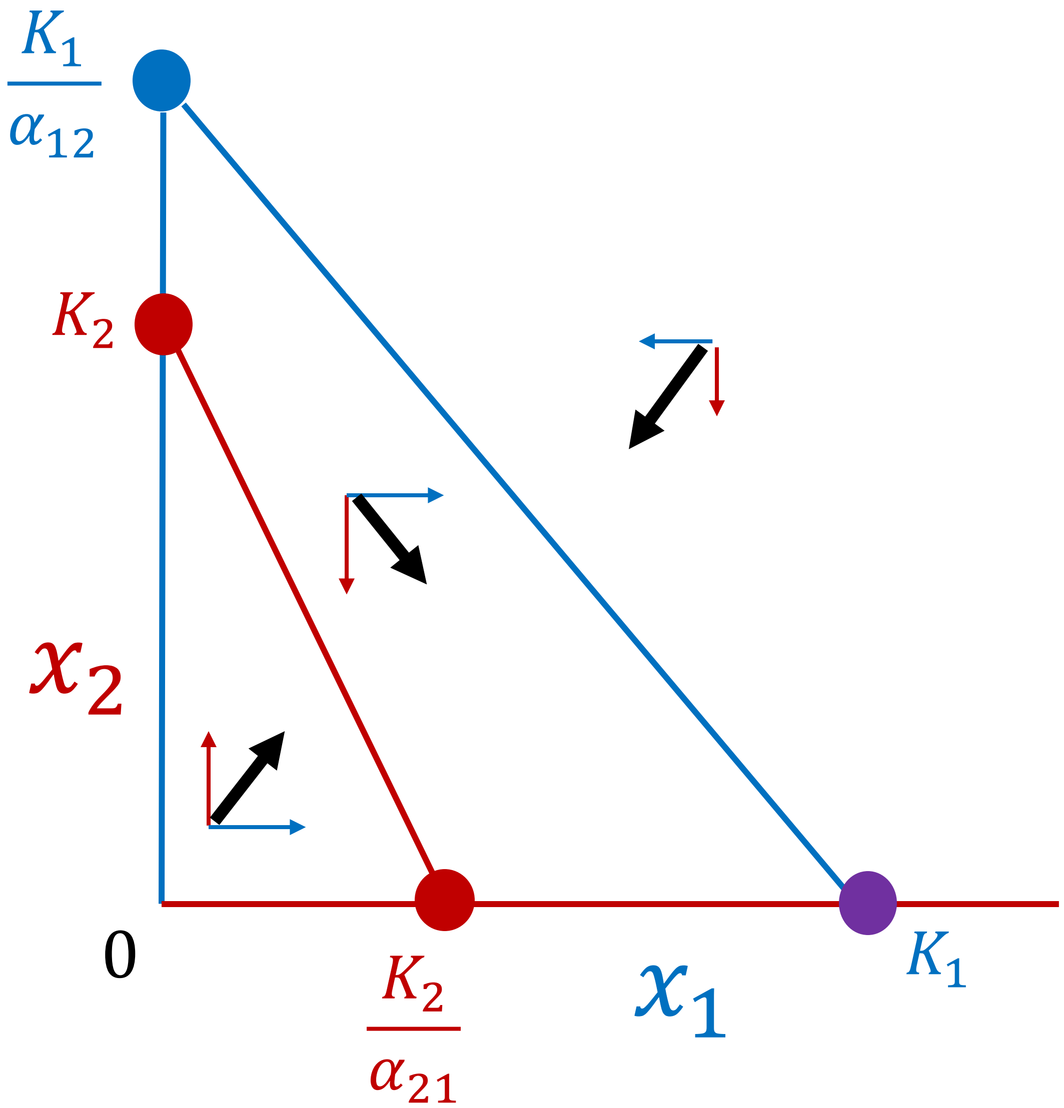
Case 2:

Species 1 outcompetes species 2

$$\frac{K_1}{\alpha_{12}} > K_2 \;\&\; \frac{K_2}{\alpha_{21}} < K_1$$

System collapses to equilibrium:

$$x_1^* = K_1;\; x_2^* = 0$$



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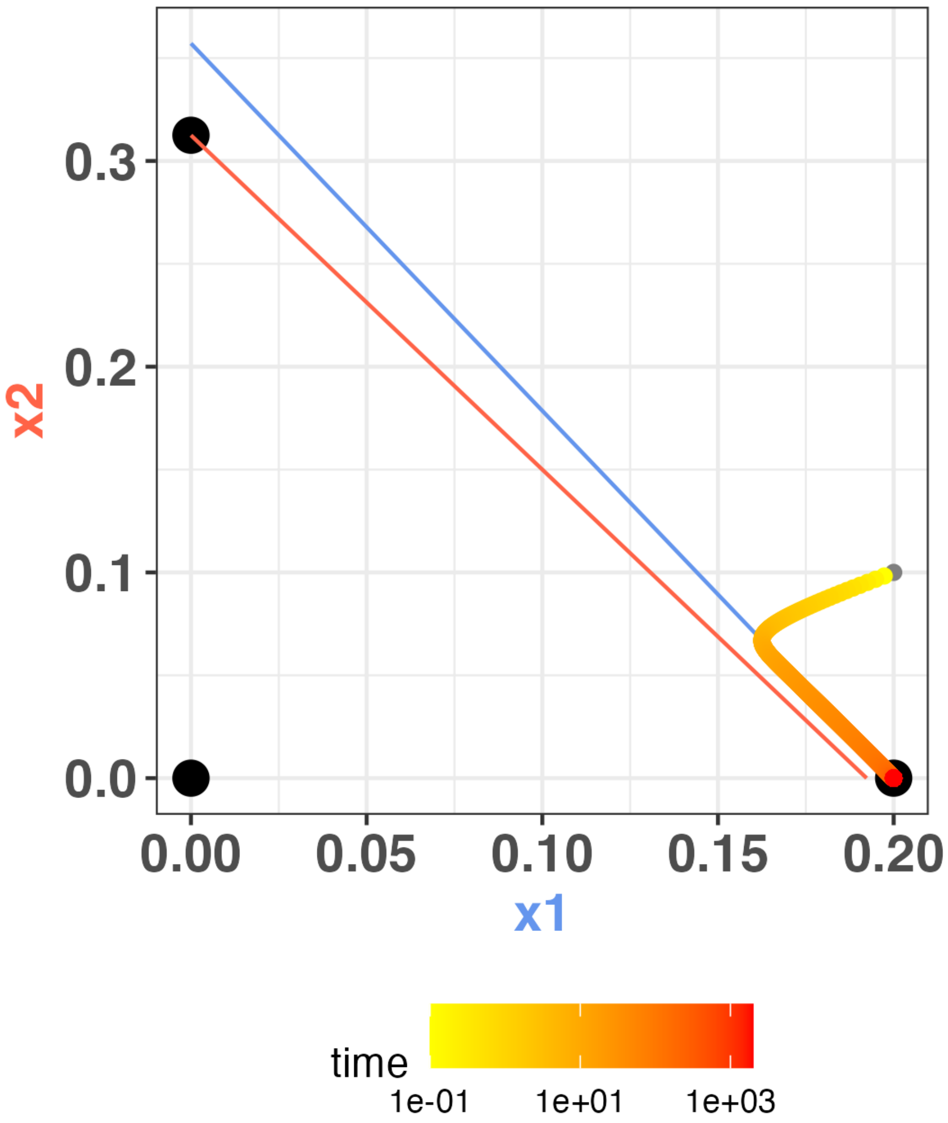
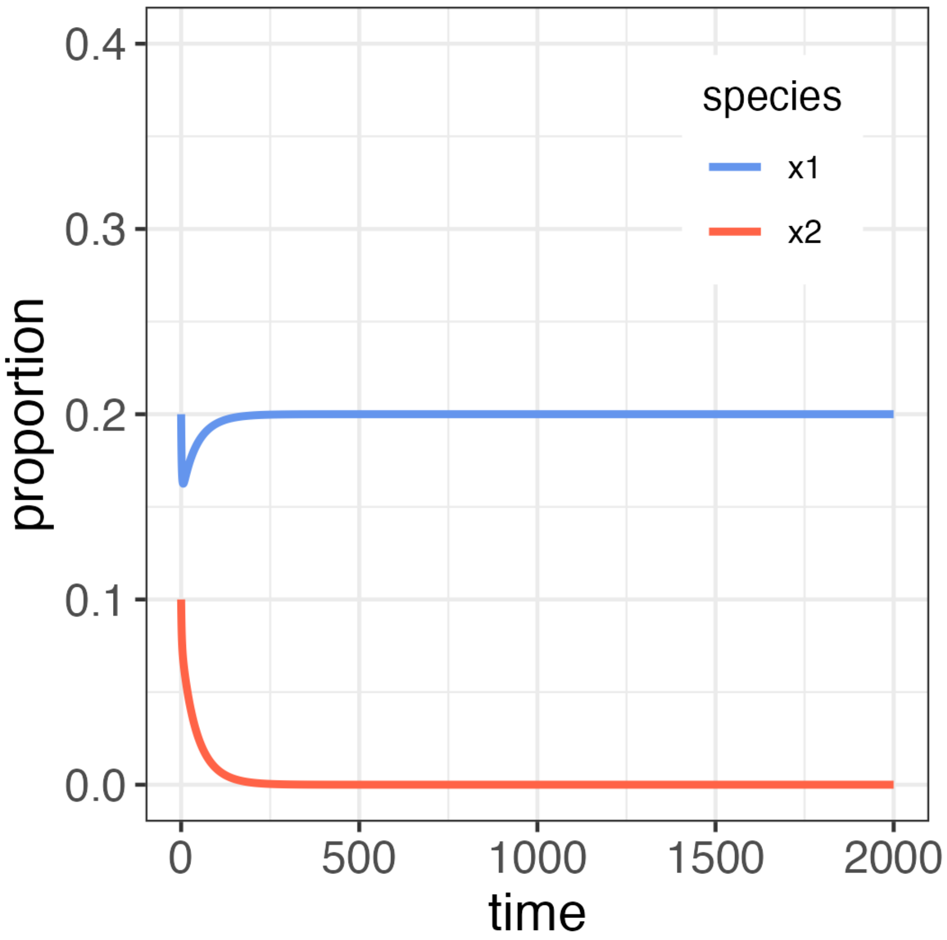


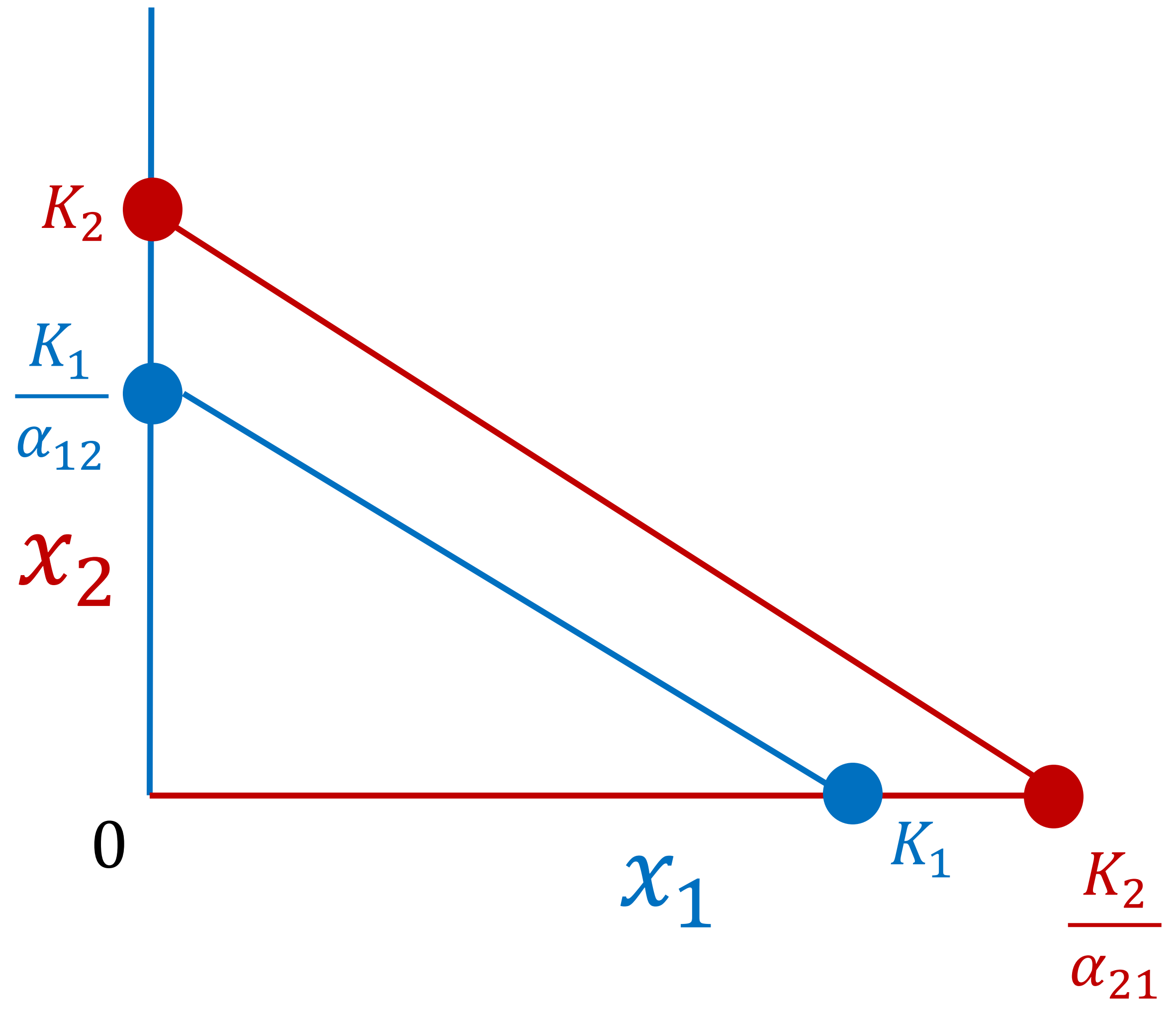


Four possible outcomes for competition

Case 3:

Species 2 outcompetes species 1

$$\frac{K_1}{\alpha_{12}} < K_2 \ \& \ \frac{K_2}{\alpha_{21}} > K_1$$



Four possible outcomes for competition

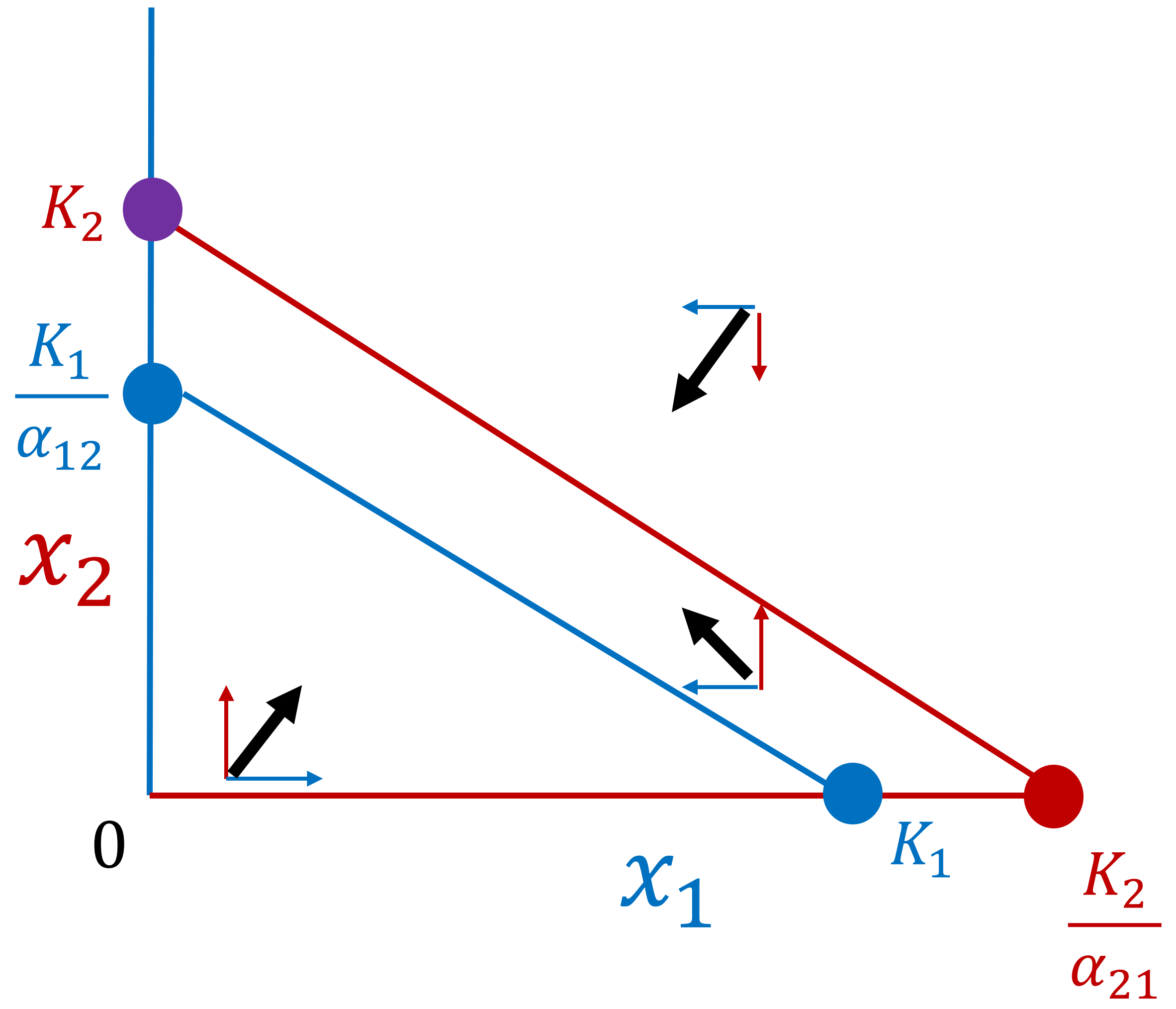
Case 3:

Species 2 outcompetes species 1

$$\frac{K_1}{\alpha_{12}} < K_2 \;\&\; \frac{K_2}{\alpha_{21}} > K_1$$

System collapses to equilibrium:

$$x_1^* = 0;\; x_2^* = K_2$$



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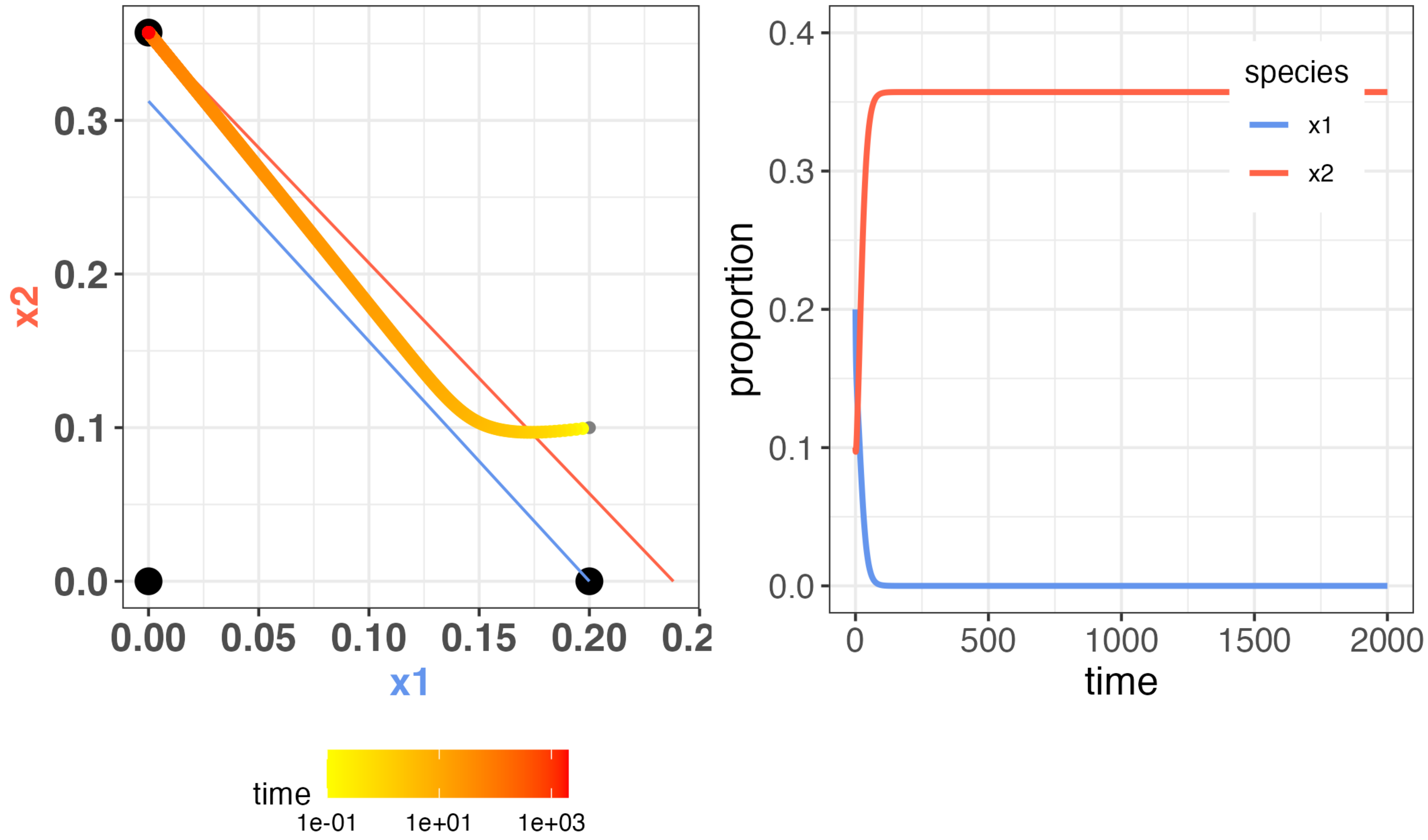
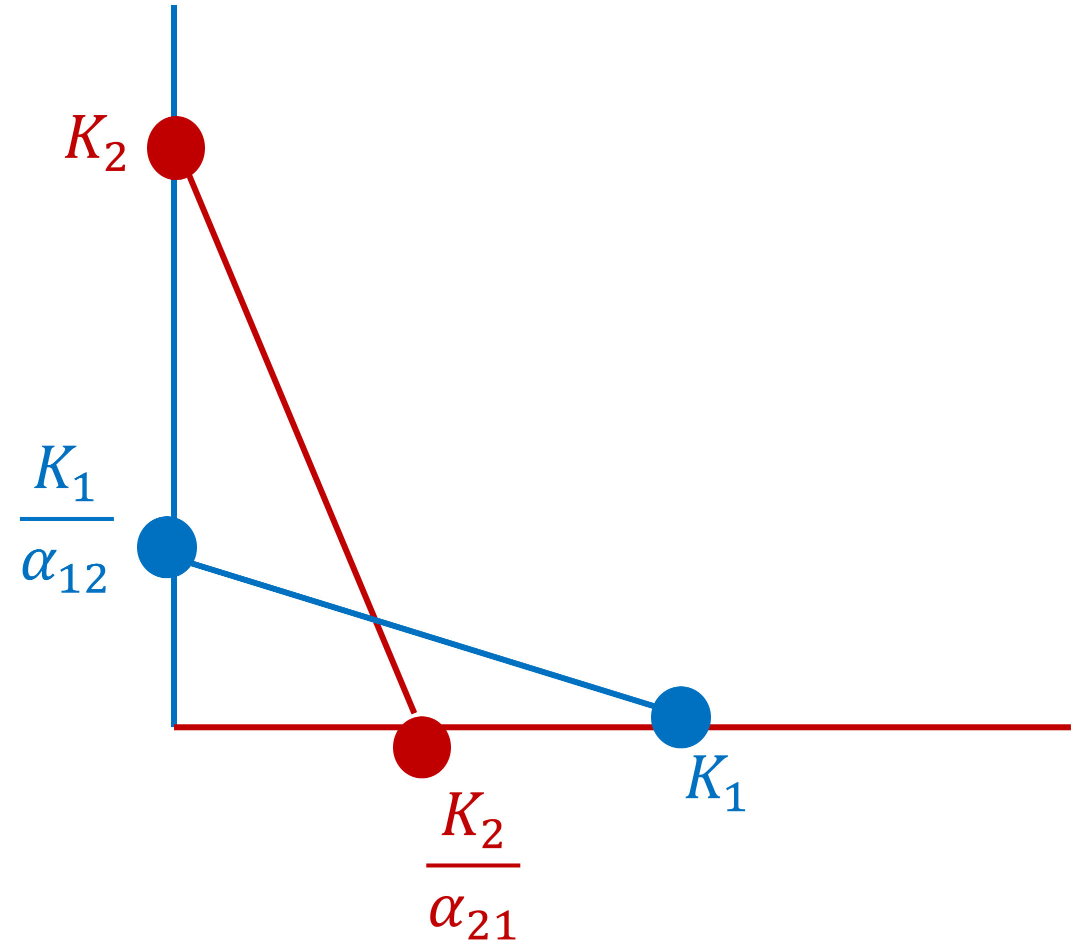


Four possible outcomes for competition

Case 4: Precedence

Aggressive interspecific competition.

Outcome depends on starting conditions.



Four possible outcomes for competition

Case 4: Precedence

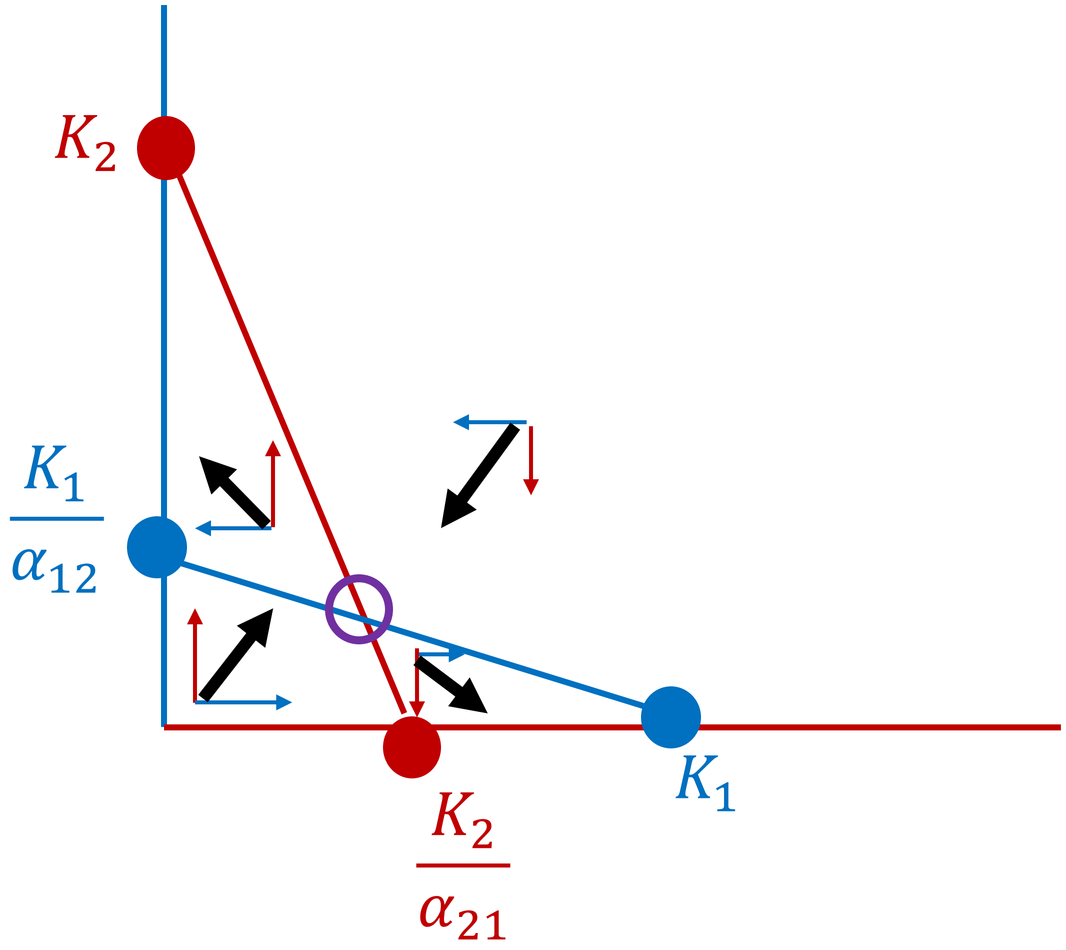
Aggressive interspecific competition.

Outcome depends on starting conditions.

$$\frac{\boldsymbol{K_1}}{\boldsymbol{\alpha_{12}}} < \boldsymbol{K_2} \;\&\; \frac{\boldsymbol{K_2}}{\boldsymbol{\alpha_{21}}} < \boldsymbol{K_1}$$

$$(\alpha_{12} * \alpha_{21} > 1)$$

- **System will move towards either single species equilibria** ($x_1^* = 0\,; x_2^* = K_2$ OR $x_1^* = K_1; x_2^* = 0$), depending on starting conditions.
- Under extremely unrealistic starting conditions, system will sit at an **unstable equilibrium** that will collapse in either direction following slight perturbation.

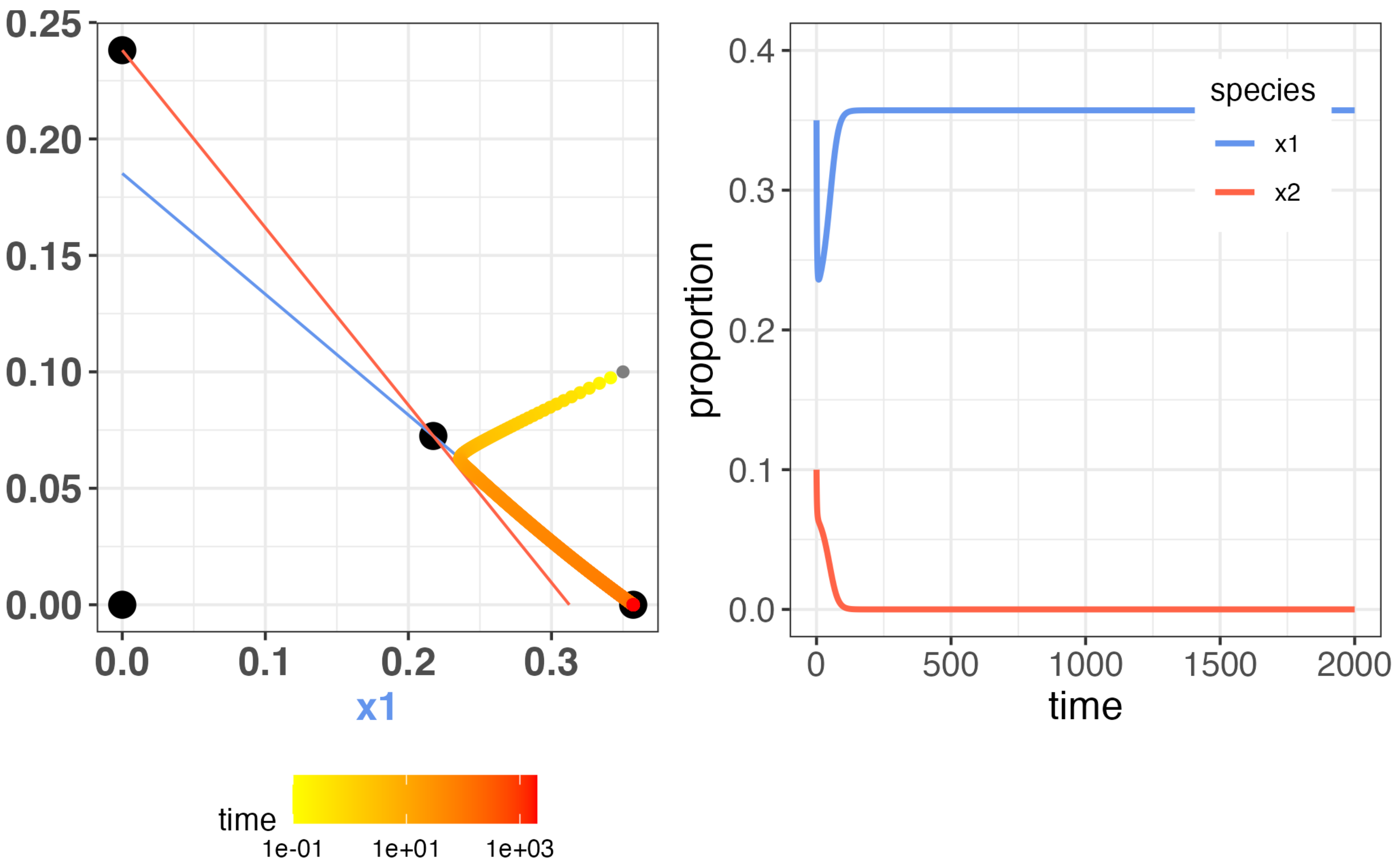


Case 4: Precedence

Aggressive interspecific competition.

Outcome depends on starting conditions.

Here, **x1 outcompetes x2**

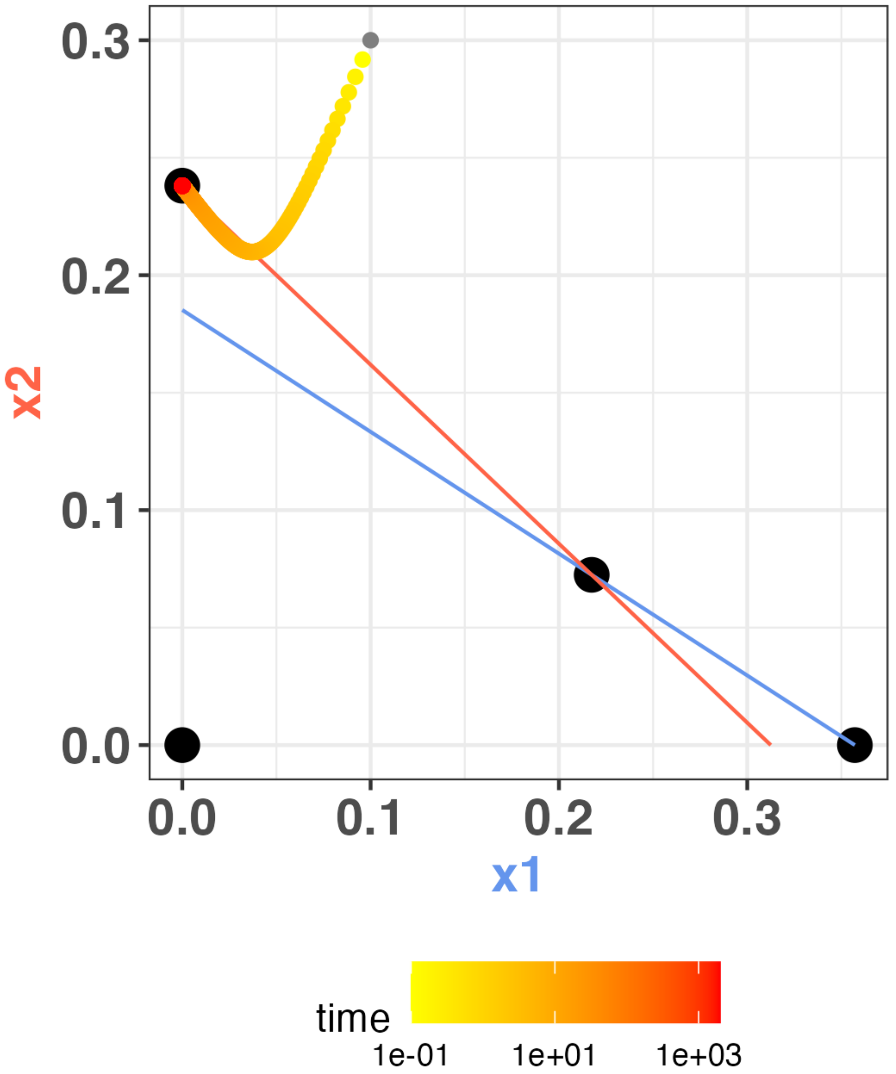


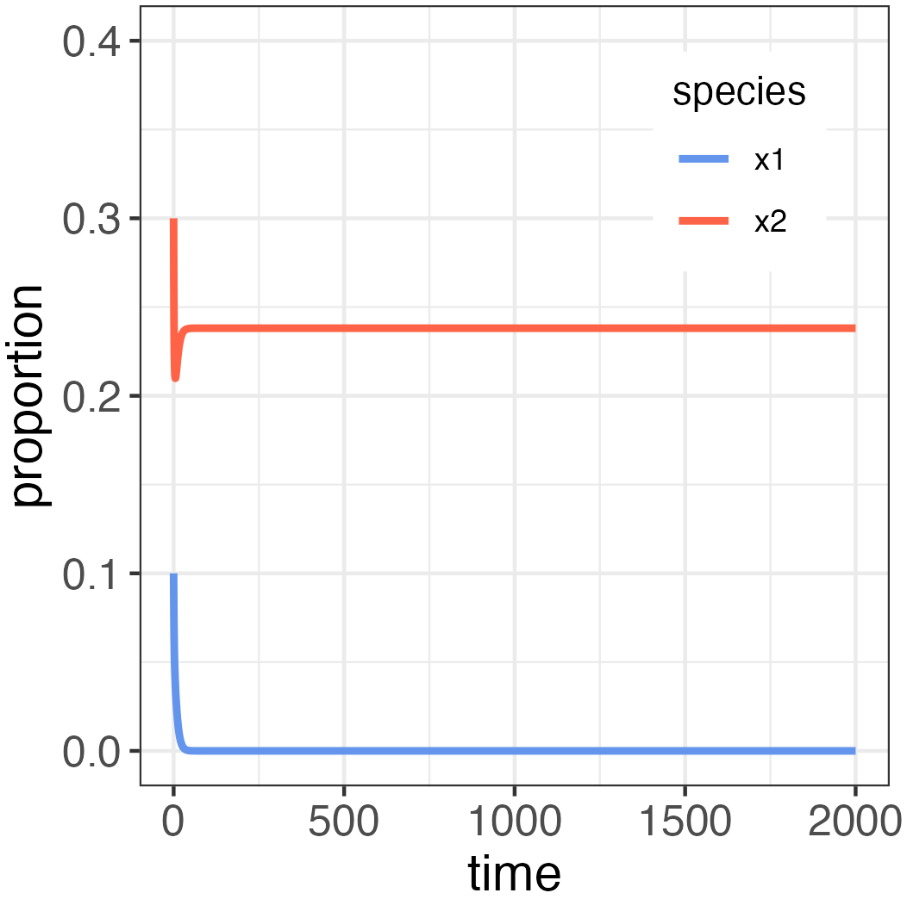
Case 4: Precedence

Aggressive interspecific competition.

Outcome depends on starting conditions.

Here, x2 outcompetes x1



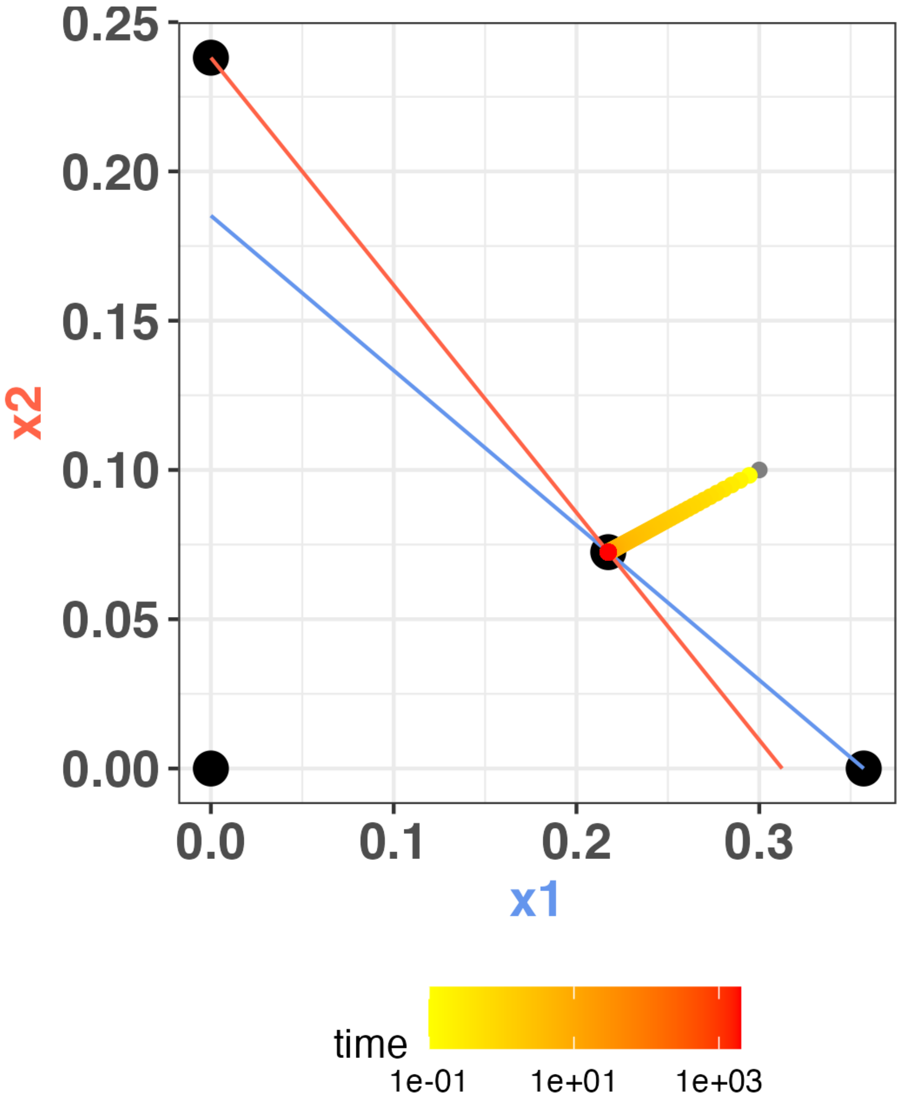


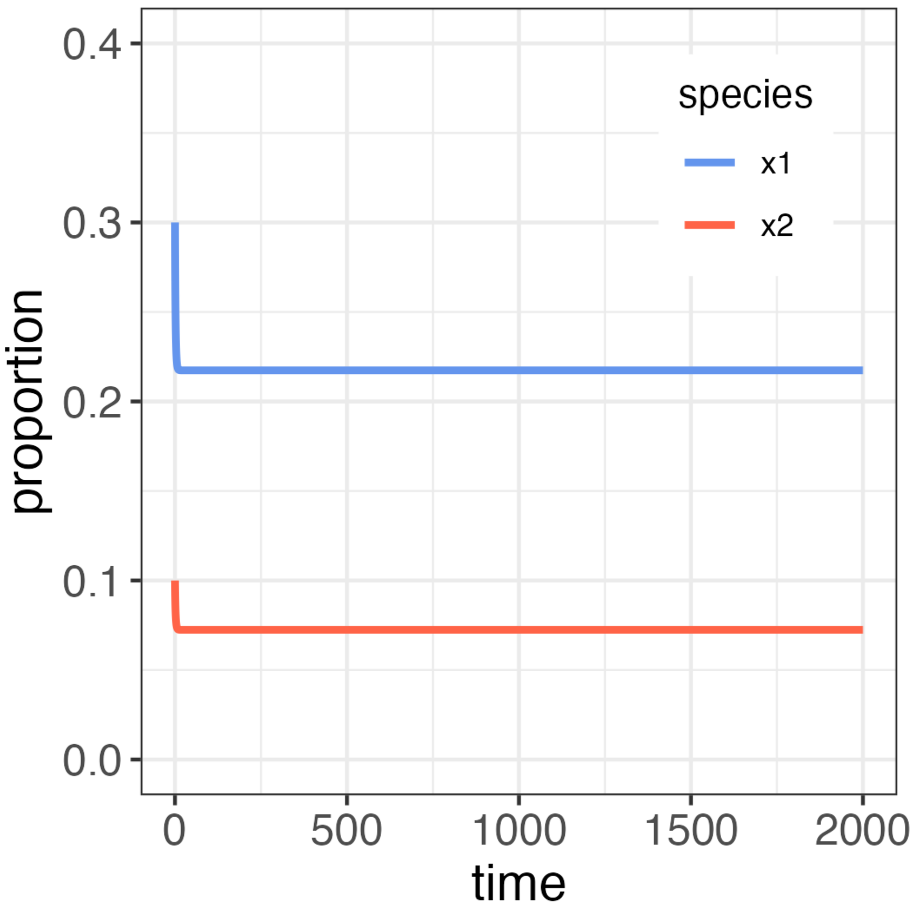
Case 4: Precedence

Aggressive interspecific competition.

Outcome depends on starting conditions.

Here, coexistence



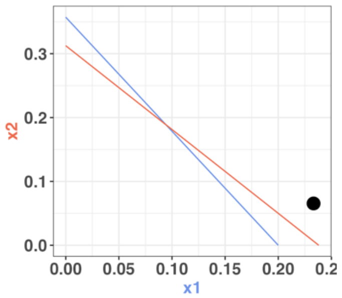


Wk7-Lect5-Feb20

Visual settings

Edit

What will be the outcome of competition for a population with this nullcline structure, starting at the quantities shown by the black dot?

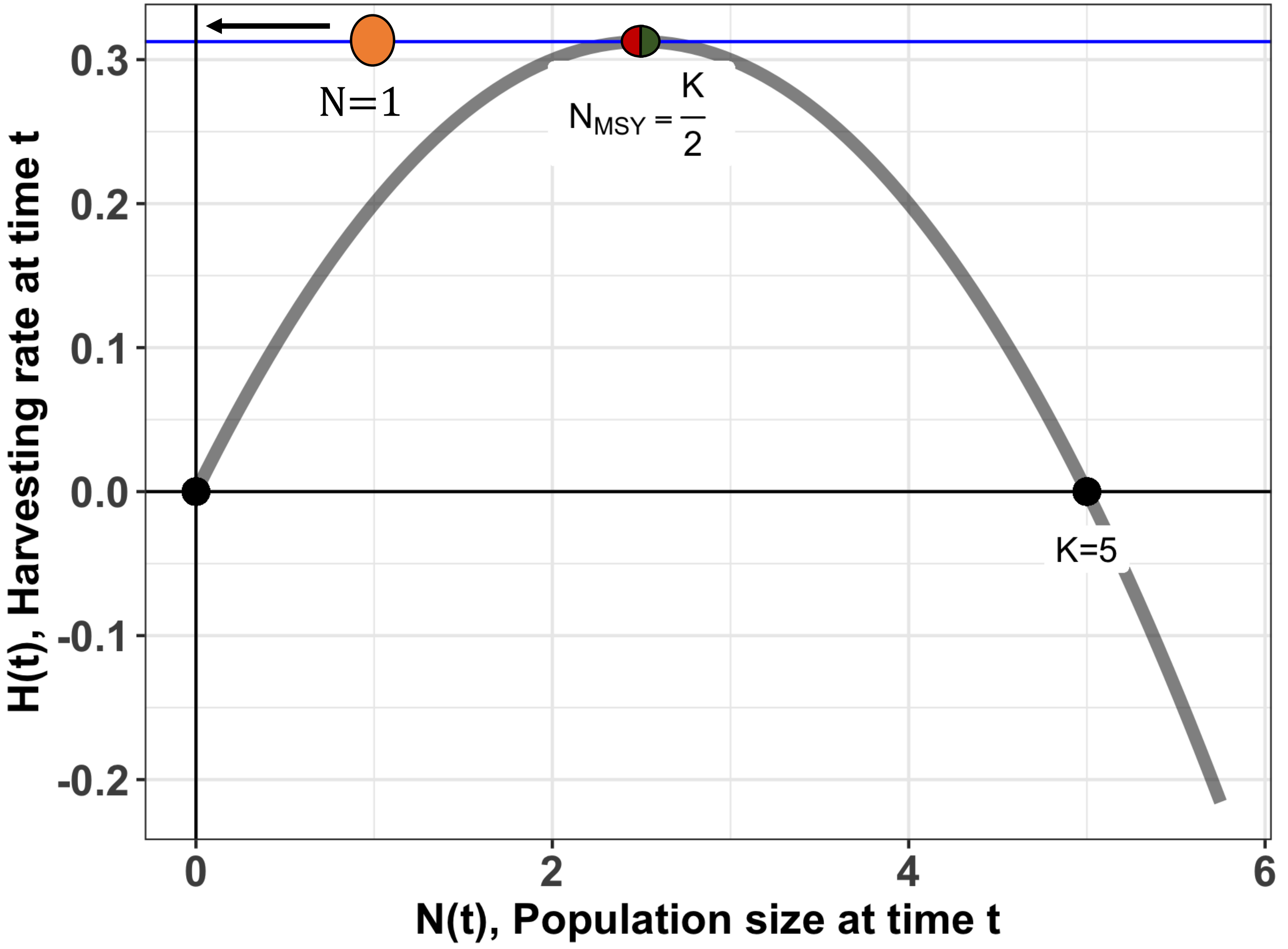


A Species x2 will outcompete x1, so we will e…		21%
B Species x1 will outcompete x2, so we will e…		34%
C Both species will coexist at the sta…		32%
D The outcome depends on the initial conditions.		13%

Instructions Responses Correctness More Clear responses

Remember MSY, the semi-stable equilibrium:

$$\frac{dN}{dt} = rN\left(1 - \frac{N}{K}\right) - H$$



K=5

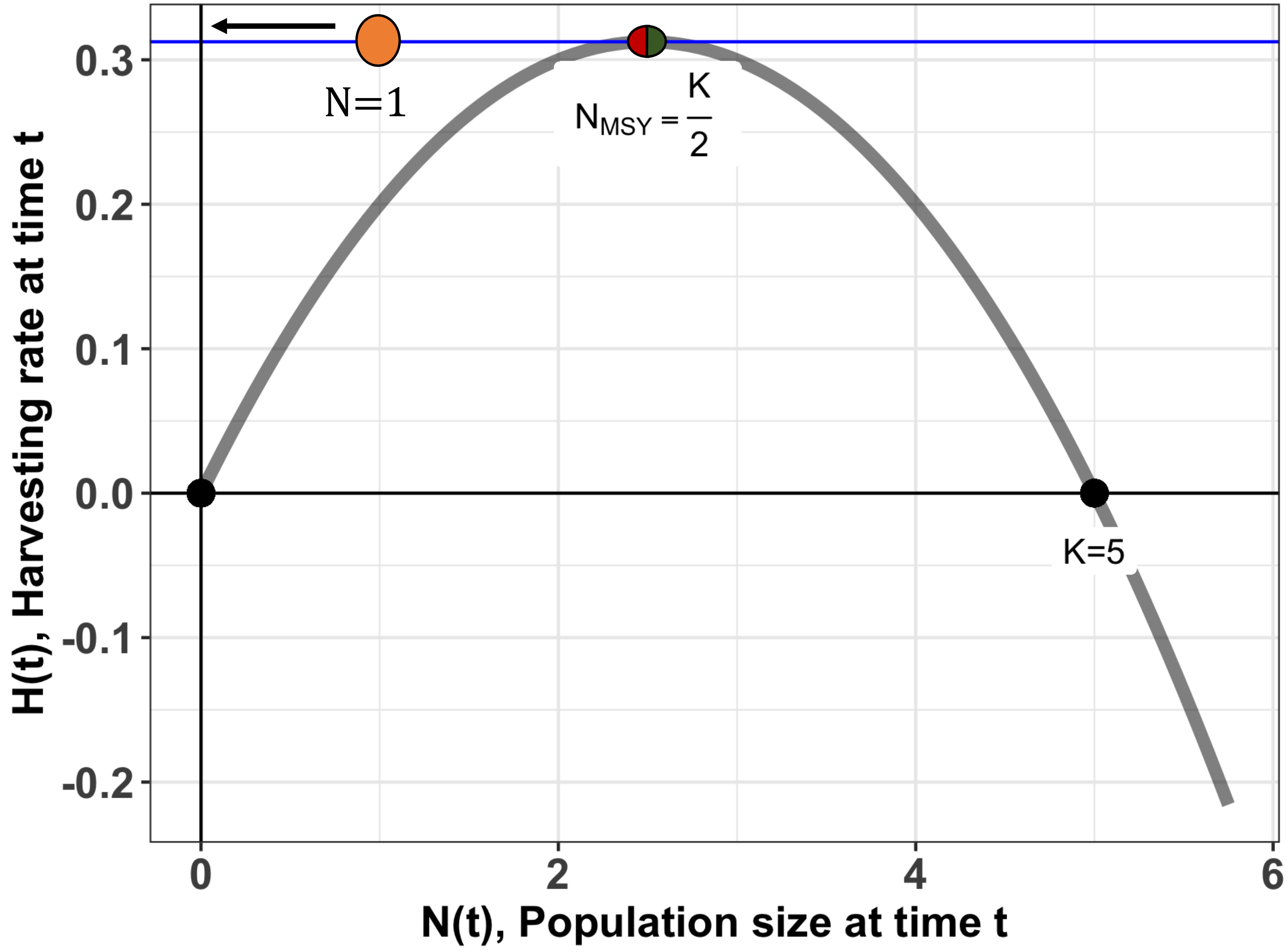
r=0.25

H=.3125

$$\frac{dN}{dt} = 0.25 * 1\left(1 - \frac{1}{5}\right) - .3125$$

$$\frac{dN}{dt} = (-)$$

Remember MSY, the semi-stable equilibrium: $$\frac{dN}{dt} = rN\left(1-\frac{N}{K}\right) - H$$



K=5
r=0.25
H=.3125

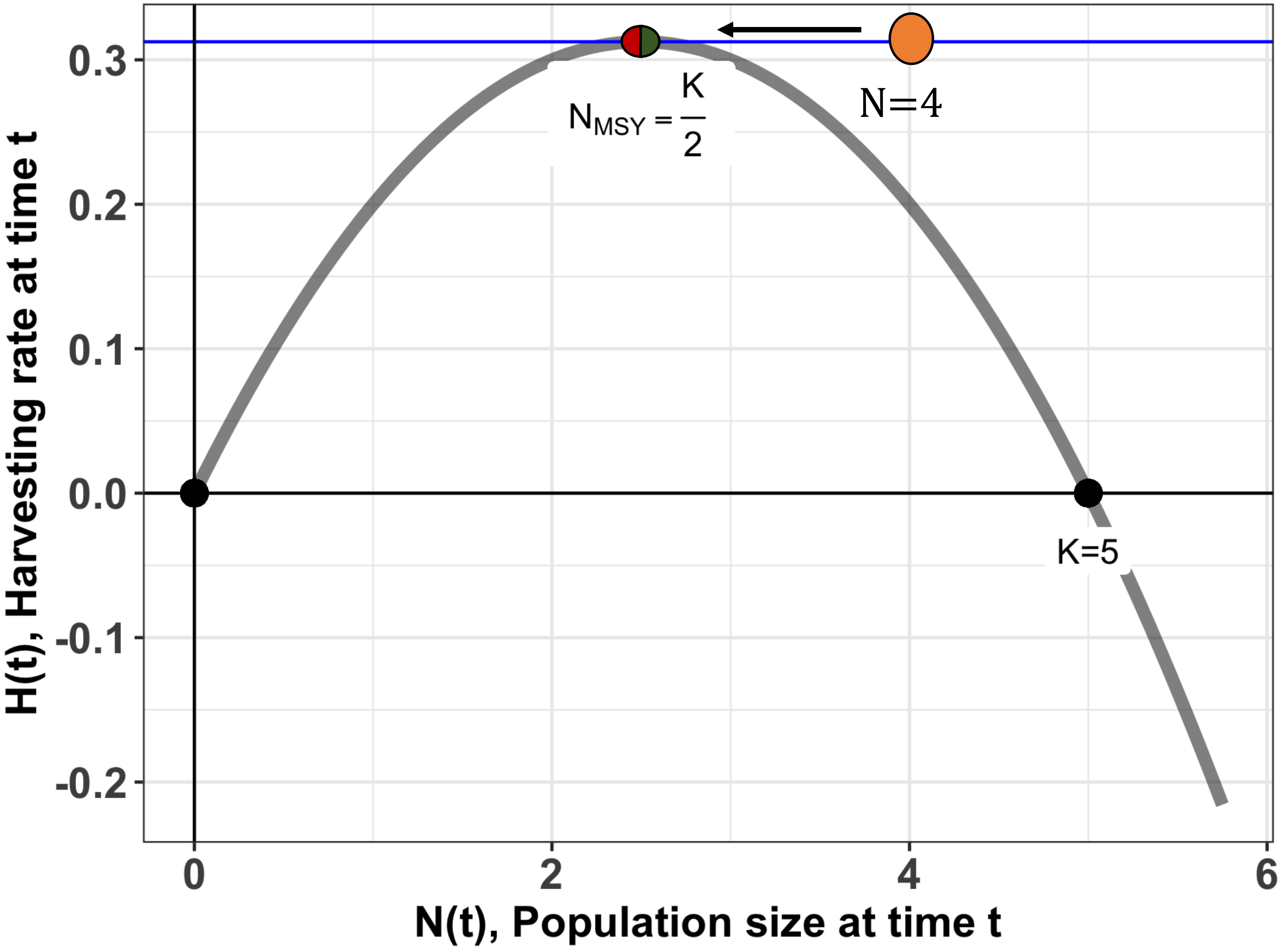
$$\frac{dN}{dt} = 0.25 * 1\left(1-\frac{1}{5}\right) -.3125$$

$$\frac{dN}{dt} = (-)$$

In a **semi-stable equilibrium**, perturbations ***sometimes*** drive the system away from equilibrium.

Remember MSY, the semi-stable equilibrium:

$$\frac{dN}{dt} = rN\left(1 - \frac{N}{K}\right) - H$$



K=5
r=0.25
H=.3125

$$\frac{dN}{dt} = 0.25 * 4\left(1 - \frac{4}{5}\right) - .3125$$

$$\frac{dN}{dt} = (-)$$

Four possible outcomes for competition

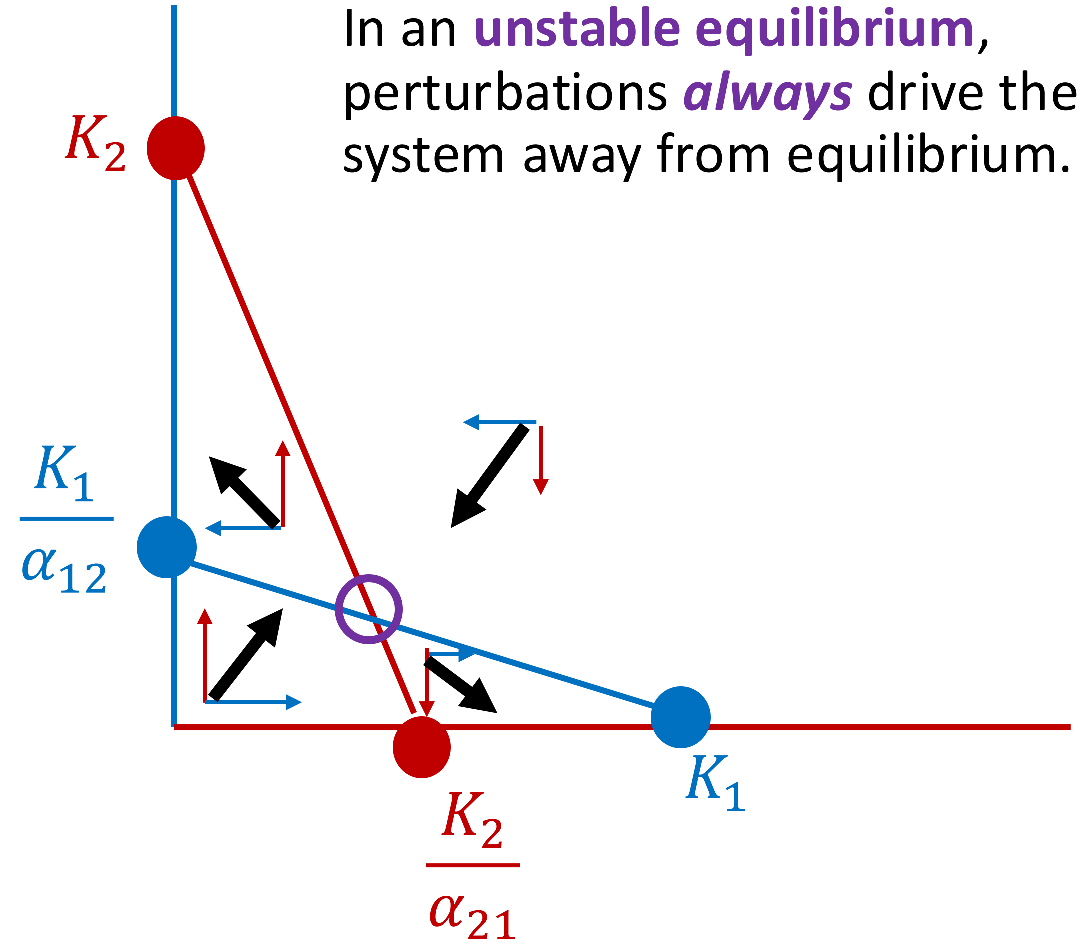
Case 4: Precedence

Aggressive interspecific competition.

Outcome depends on starting conditions.

$$\frac{K_1}{\alpha_{12}} < K_2 \ \& \ \frac{K_2}{\alpha_{21}} < K_1$$

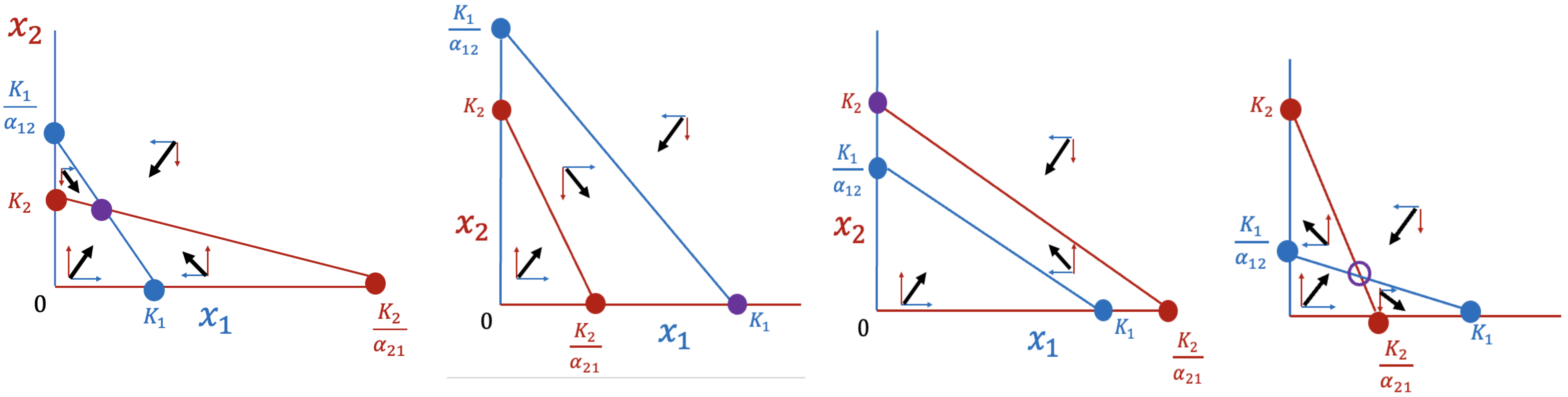
$$(\alpha_{12} * \alpha_{21} > 1)$$



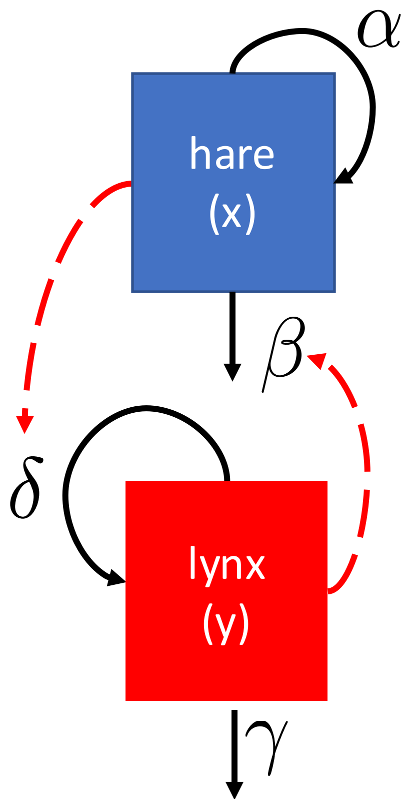
In an **unstable equilibrium**, perturbations ***always*** drive the system away from equilibrium.

Phase plane analysis: graphical determination of the behavior of the state variables in a dynamical system (here, populations of animals)

Case 1: Stable coexistence	Case 2: Spp. 1 wins	Case 3: Spp. 2 wins	Case 4: Precedence



Note the difference with a predator-prey model!

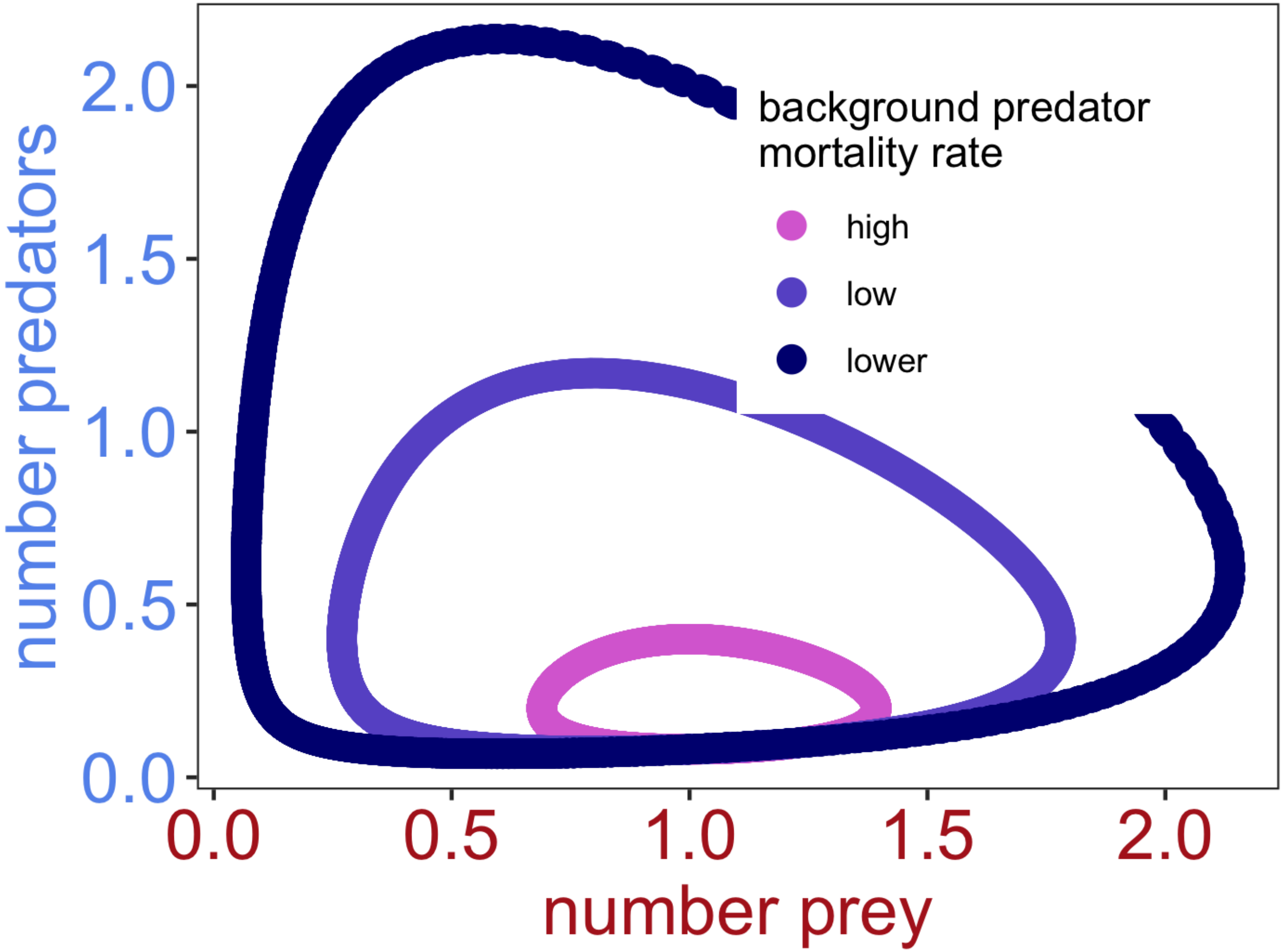


$$\frac{dx}{dt} = \alpha x - \beta xy$$

$$\frac{dy}{dt} = \delta xy - \gamma y$$

*How does **hare** abundance **vary** with **changes** in **lynx** abundance?*

Predator-prey cycles can be visualized as oscillations.



Phase plane analysis: Why do we care?

We can use these tools to make predictions about the coexistence of species!

Principle of **competitive exclusion**

"Two species of approximately the same food habits are not likely to remain long evenly balanced in numbers in the same region. ***One will crowd out the other.****"*

- Joseph Grinnell, 1904:

Museum of Vertebrate Zoology

University of California at Berkeley

Principle of **competitive exclusion**

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Museum of Vertebrate Zoology

University of California at Berkeley

"Neither can live while the other survives."

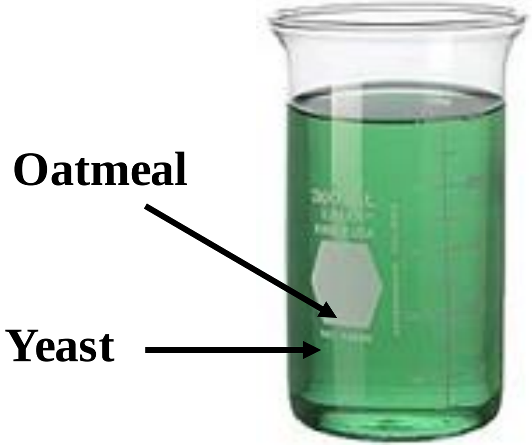
- J.K. Rowling, 2003

Principle of **competitive exclusion**

Paramecium aurelia

Paramecium caudatum

Paramecium bursaria



Gause 1934. *J Experimental Biology.*

Gause 1935. *Science.*

Gause first grew each species in isolation

Paramecium aurelia

Paramecium caudatum

Paramecium bursaria

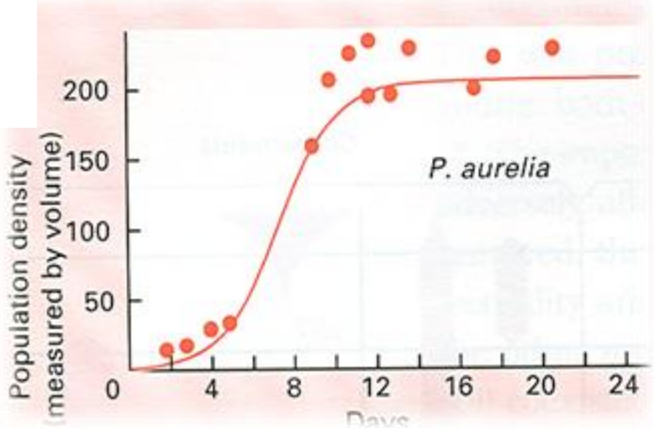
Gause 1934. *J Experimental Biology.*
Gause 1935. *Science.*

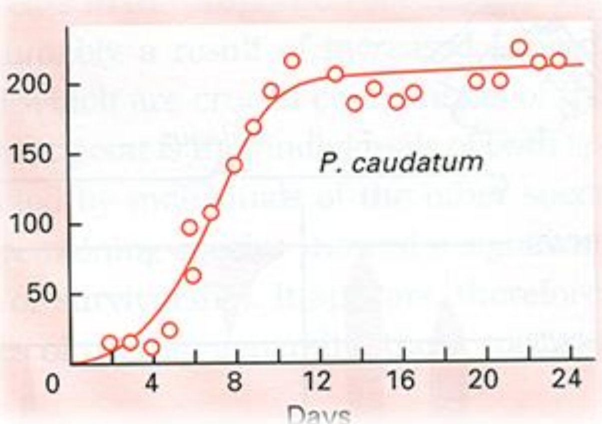
In isolation, each species grew logistically.

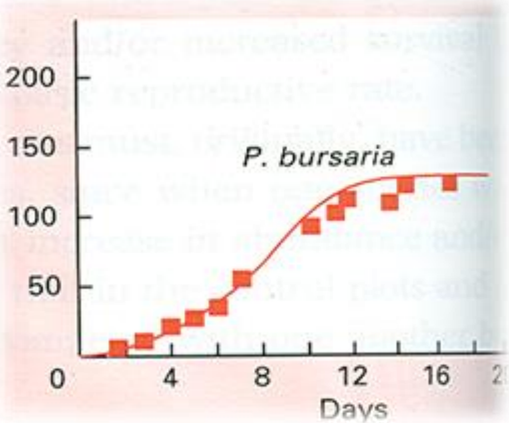
Paramecium aurelia

Paramecium caudatum

Paramecium bursaria







Gause 1934. *J Experimental Biology.*
Gause 1935. *Science.*

Then, pairs of species were placed in the same beaker.

Paramecium aurelia

Paramecium caudatum

Paramecium caudatum

Paramecium bursaria

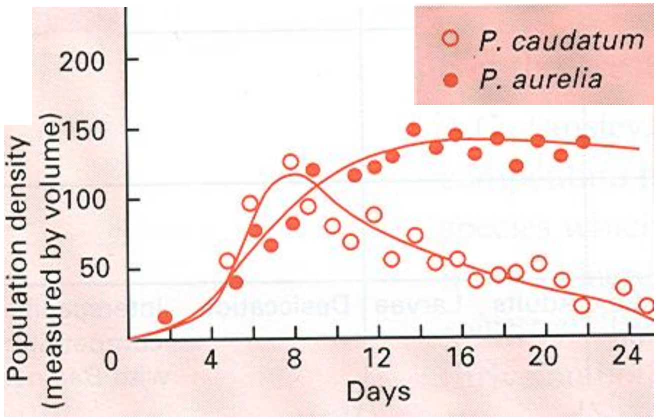
Gause 1934. *J Experimental Biology.*

Gause 1935. *Science.*

Competitive exclusion was observed:

Paramecium aurelia

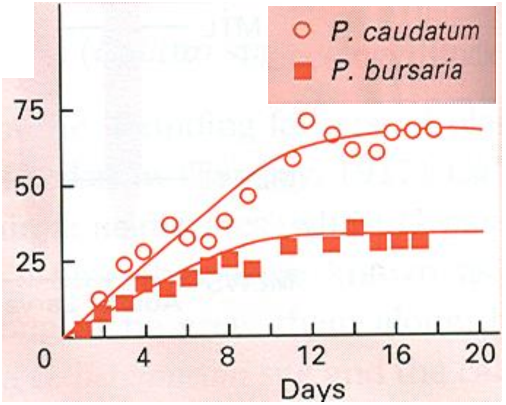
Paramecium caudatum



Coexistence was observed:

Paramecium caudatum

Paramecium bursaria



Gause 1934. *J Experimental Biology.*

Gause 1935. *Science.*

Competitive exclusion was observed:

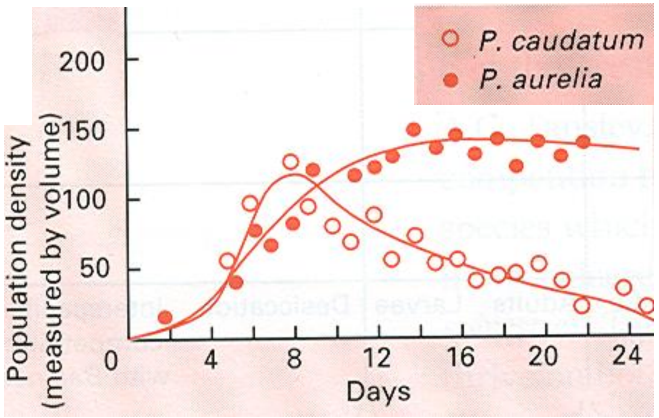
Coexistence was observed:

Paramecium aurelia

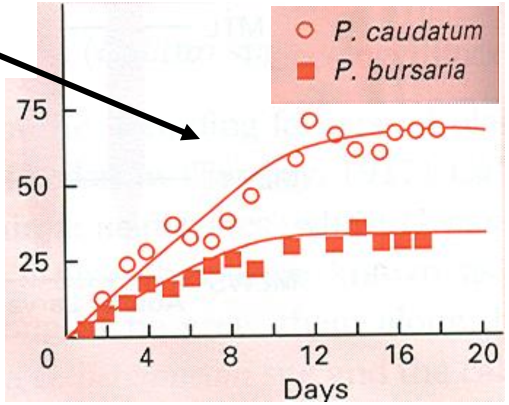
Paramecium caudatum

Paramecium caudatum

Paramecium bursaria



Species coexisted below each species' respective individual carrying capacity.



Gause 1934. *J Experimental Biology.*

Gause 1935. *Science.*

Competitive exclusion was observed:

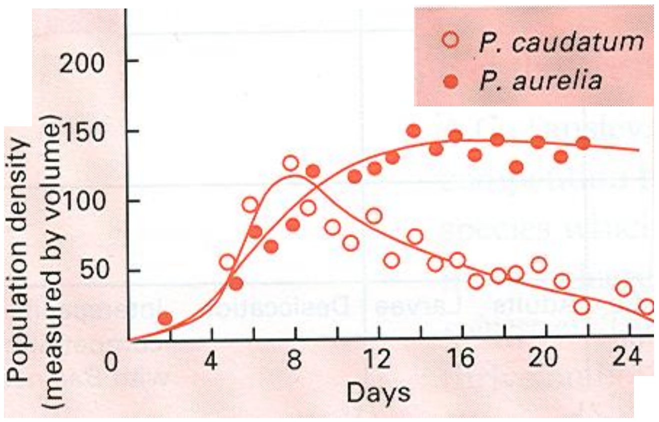
Coexistence was observed:

Paramecium aurelia

Paramecium caudatum

Paramecium caudatum

Paramecium bursaria



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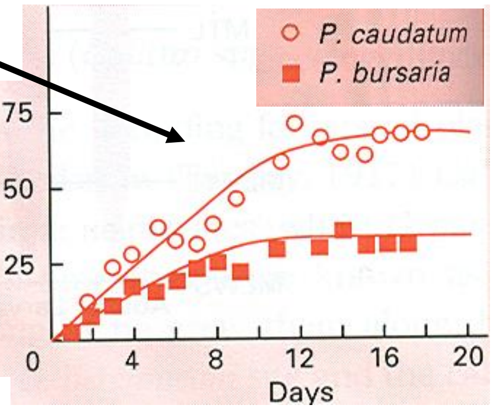
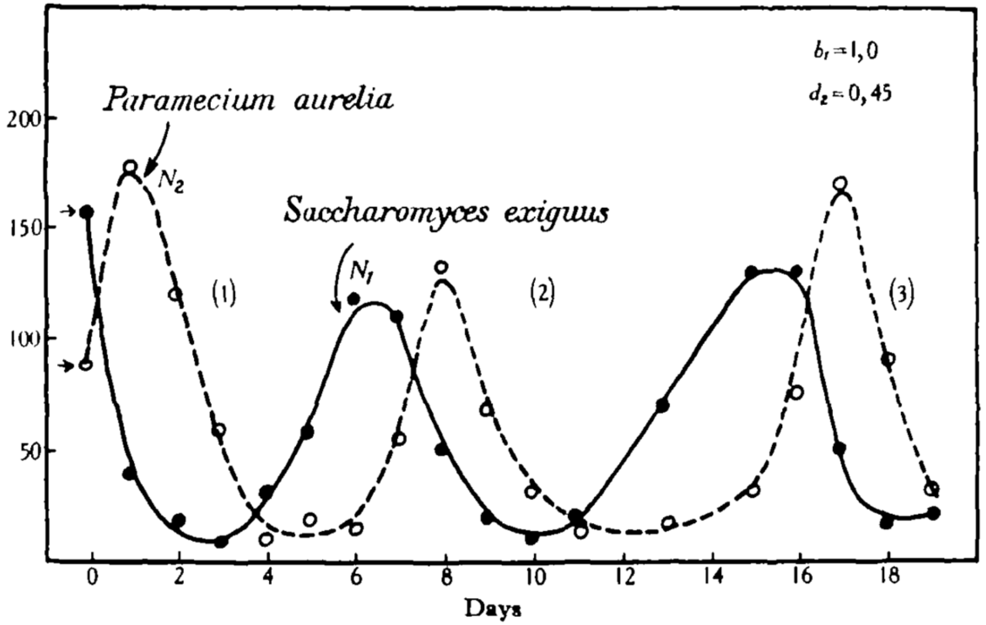


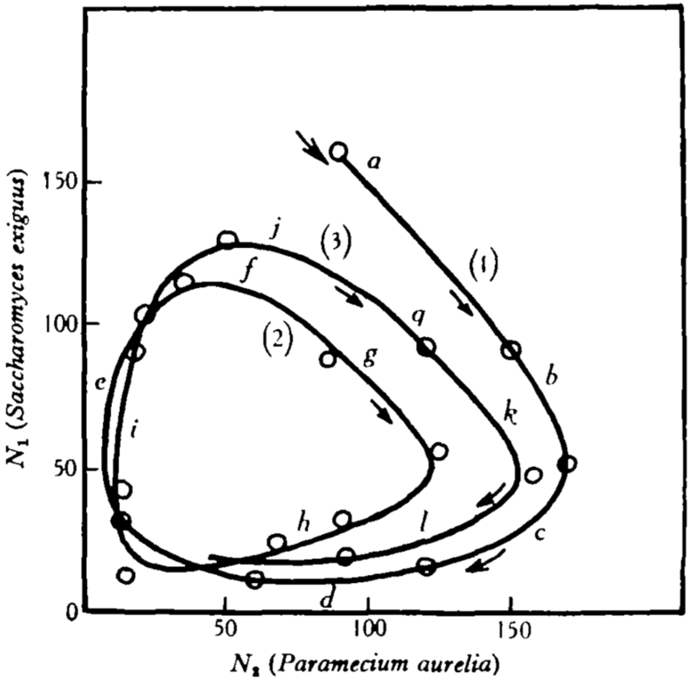
The two coexisting species were largely partitioned in space. *P. bursaria* ate yeast at the bottom and *P. caudatum* consumed bacteria suspended in the medium.

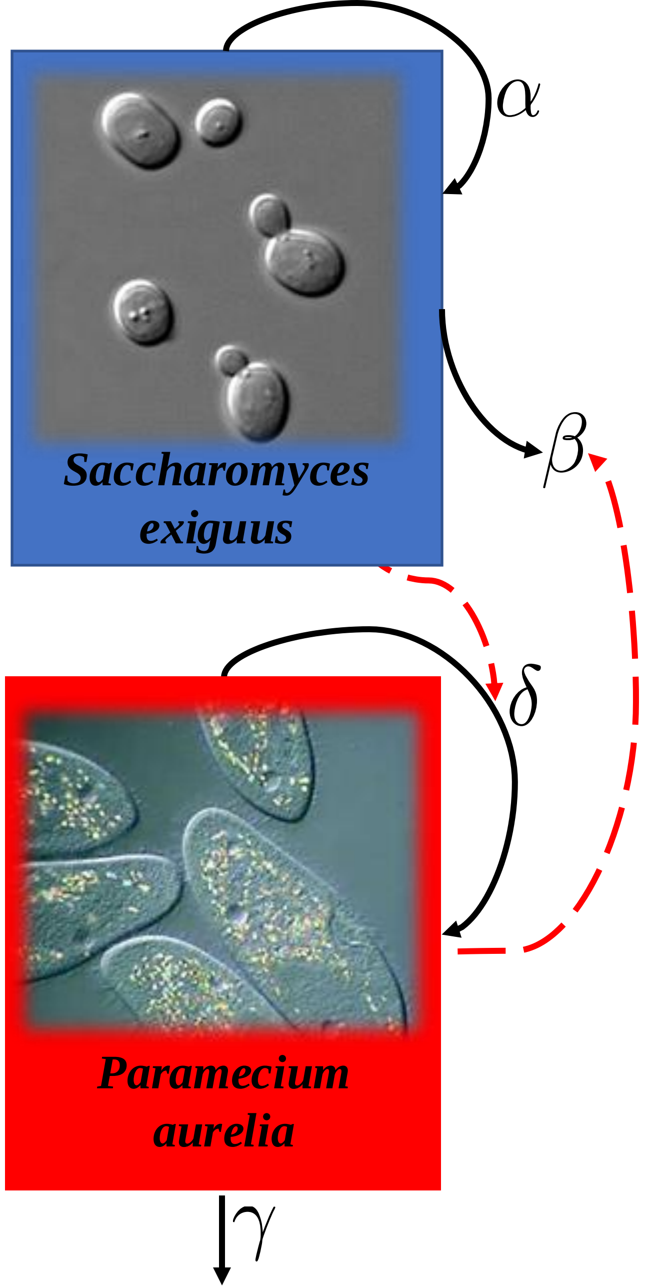
Gause 1934. *J Experimental Biology.*

Gause 1935. *Science.*

Gause also demonstrated real-life predator-prey cycles with his experiments:







Gause 1935. *Science*.

Gause's experiments:

Competitive exclusion was observed:

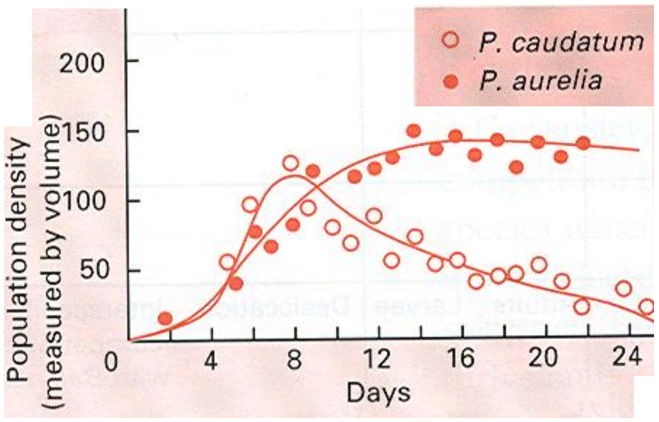
Paramecium aurelia

Paramecium caudatum

Coexistence was observed:

Paramecium caudatum

Paramecium bursaria



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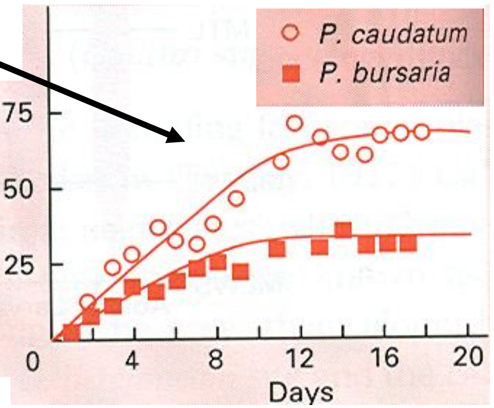

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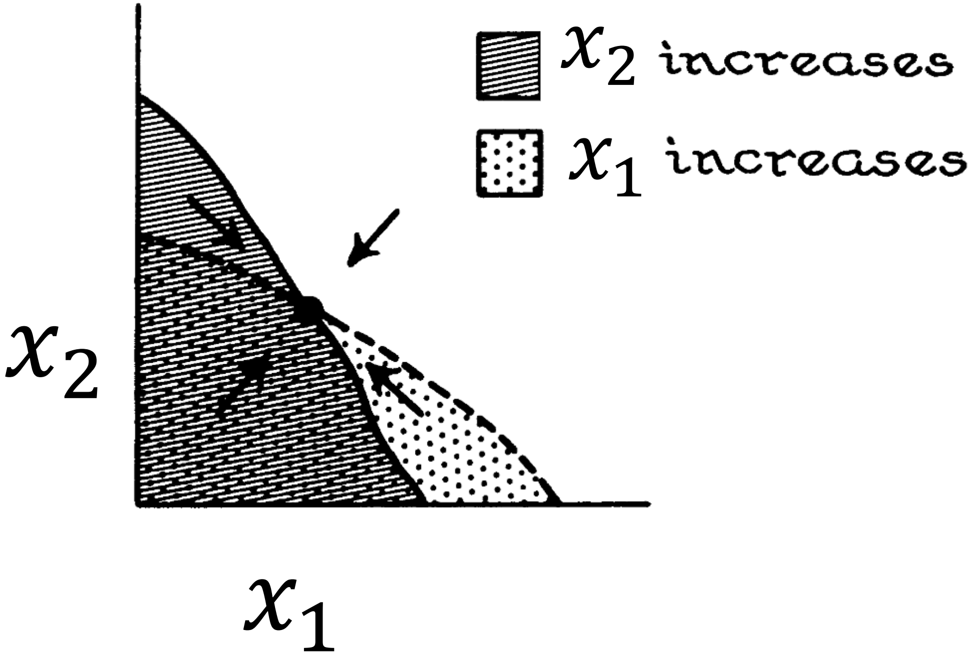
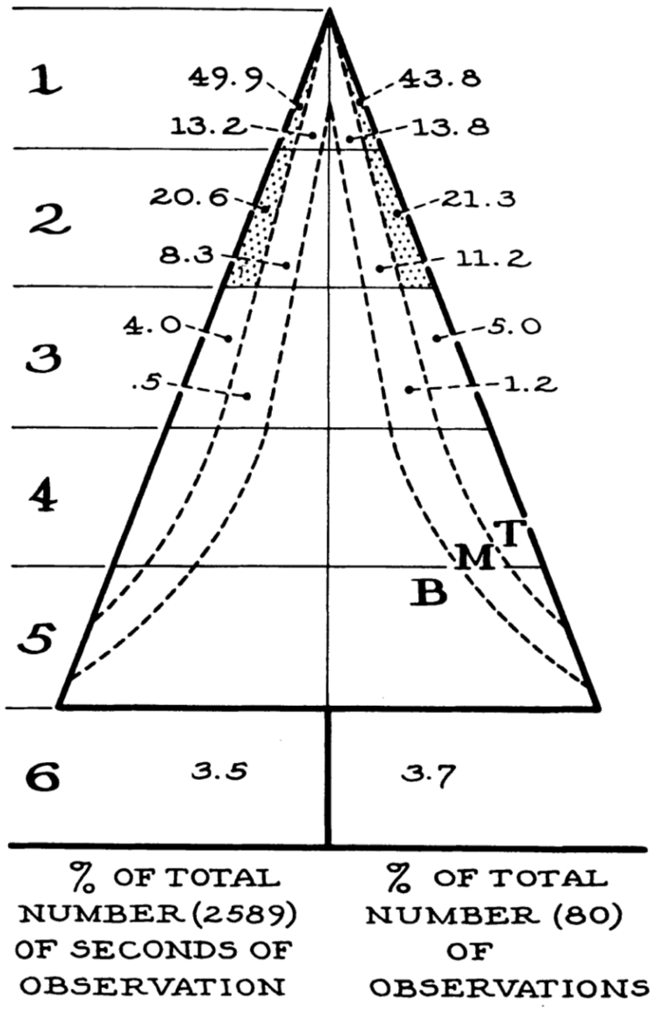
Gause 1935. *Science.*

Niche partitioning enables organisms to avoid competitive exclusion

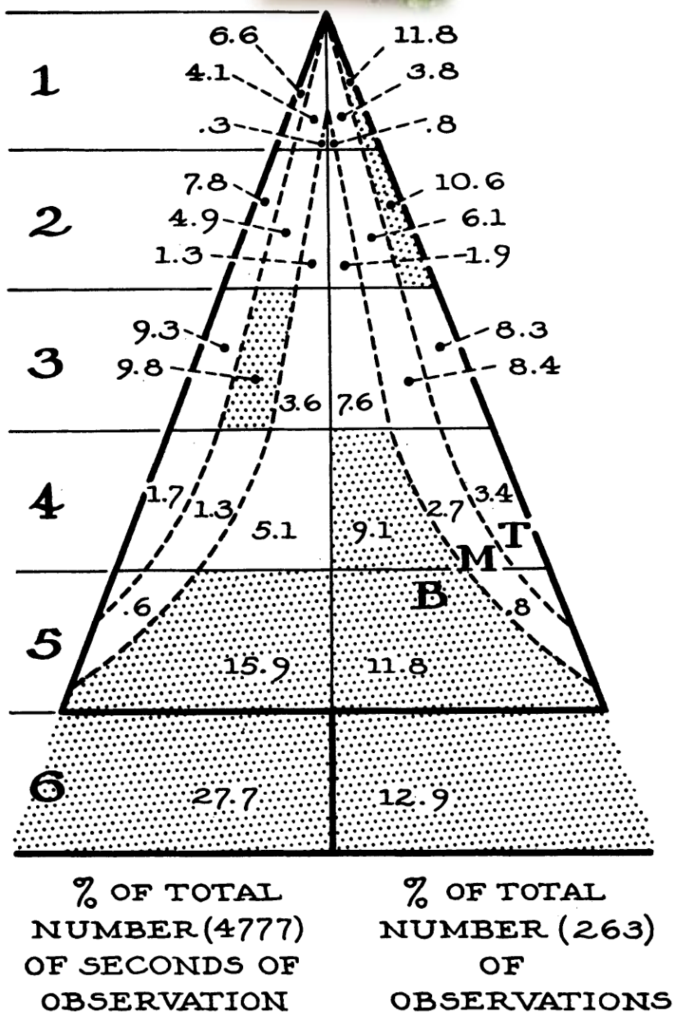
Niche: a match of a species to a specific environmental condition



Cape May Warbler



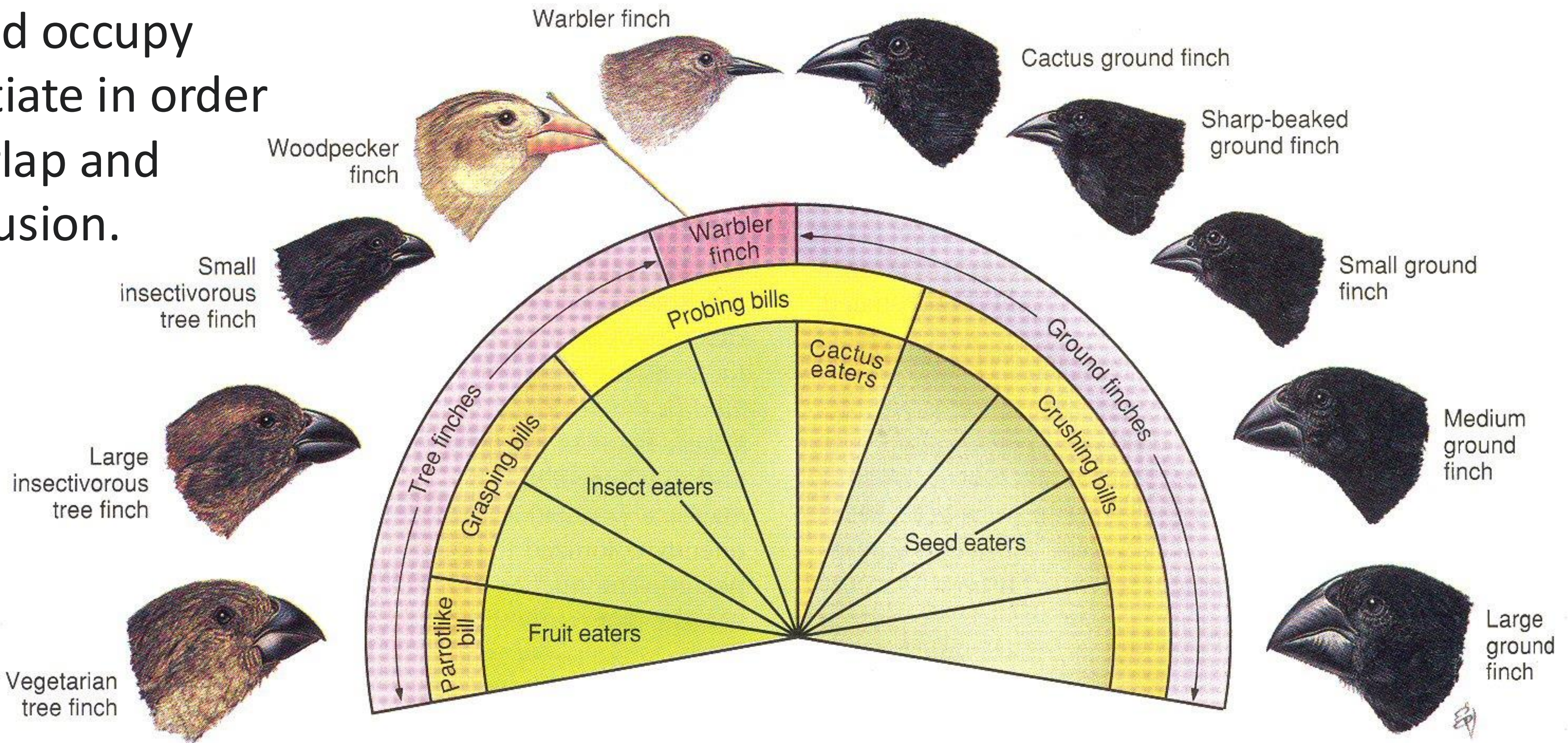
Myrtle Warbler



MacArthur 1968. *Ecology.*

Niche partitioning enables organisms to avoid competitive exclusion

Character displacement: similar species that live in the same geographical region and occupy similar niches differentiate in order to minimize niche overlap and avoid competitive exclusion.



Grant & Grant 2006. *Science.*

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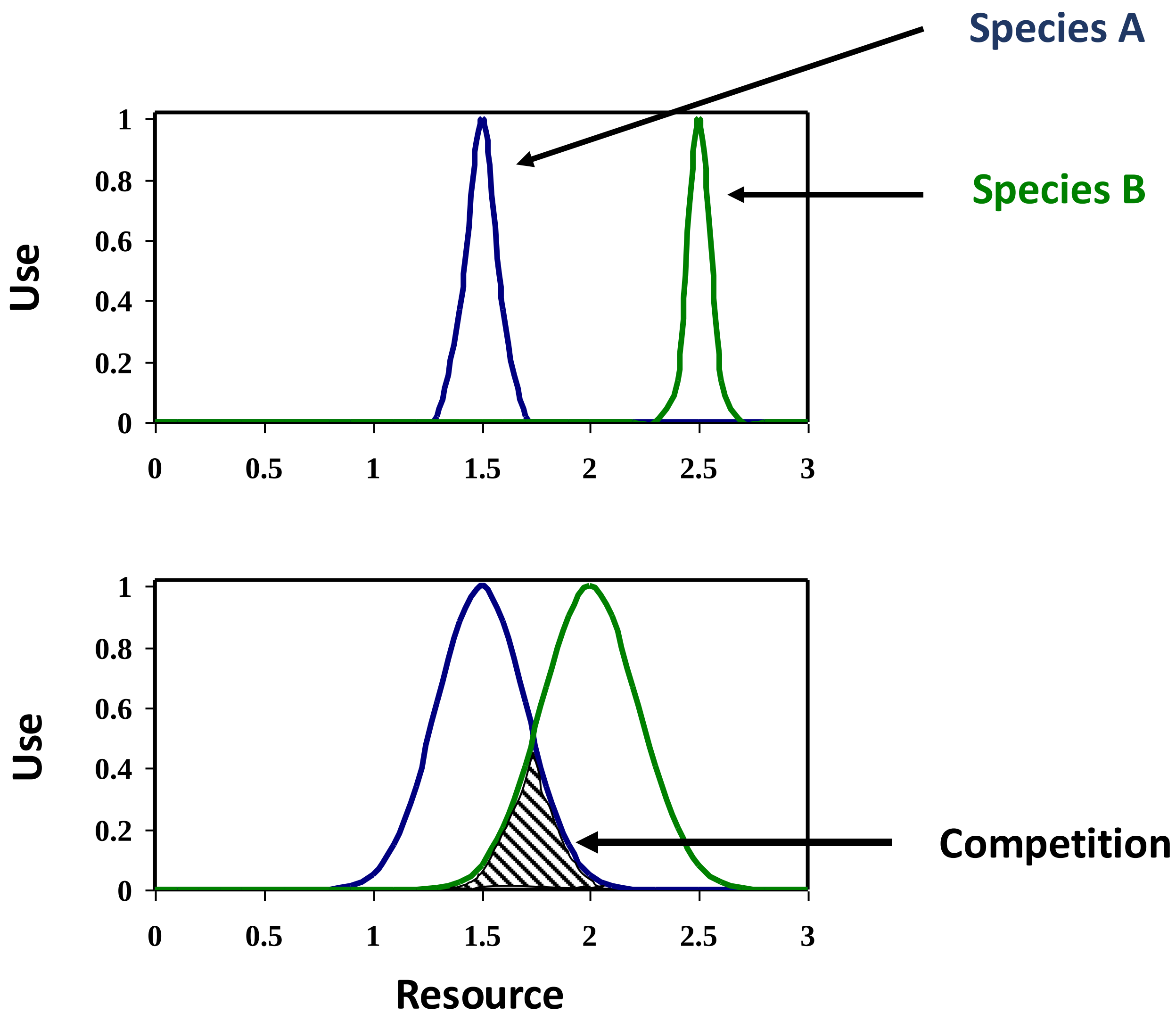
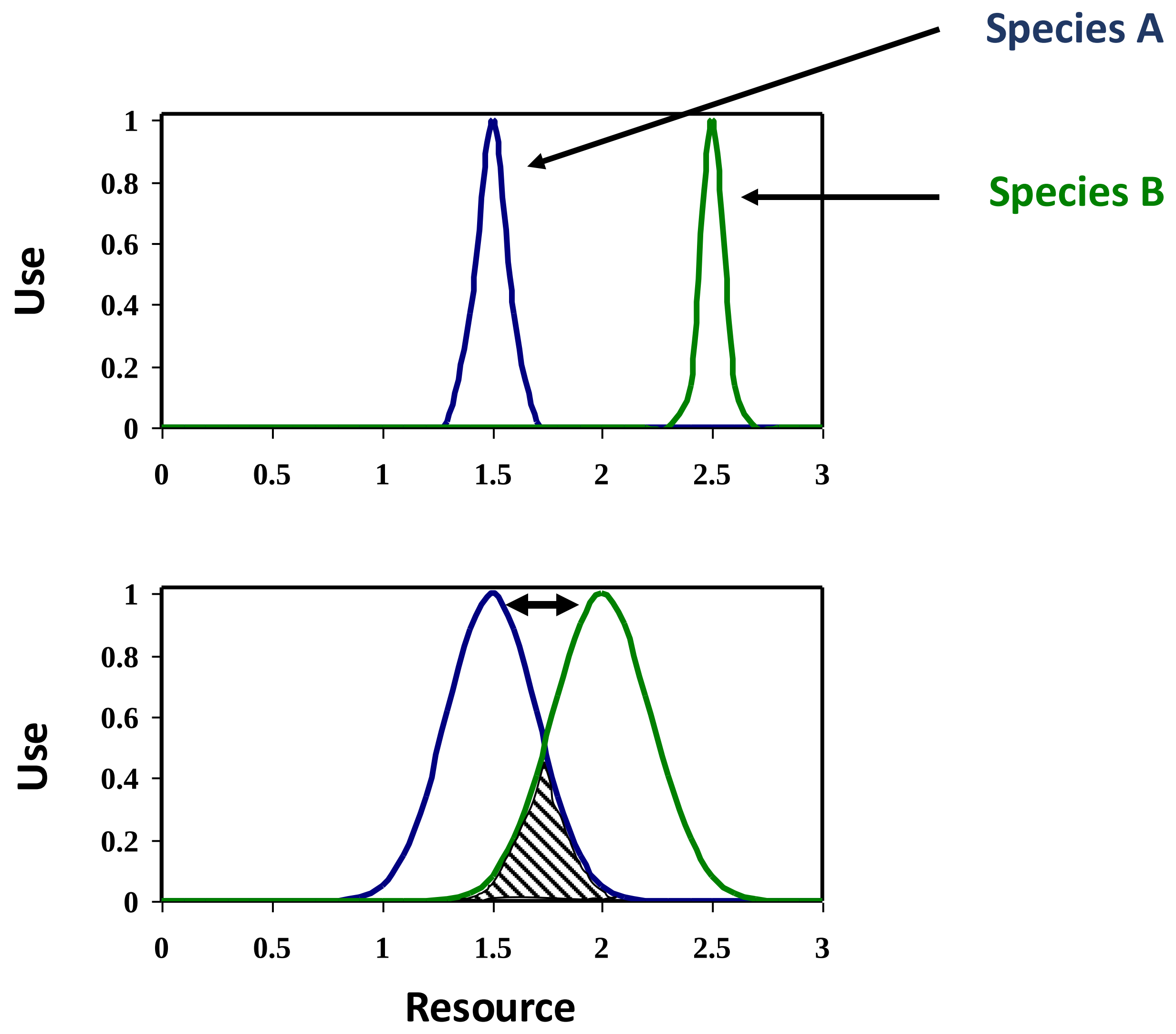


When niches overlap, competition results

Niche partitioning enables organisms to avoid competitive exclusion

Character displacement: similar species that live in the same geographical region and occupy similar niches differentiate in order to minimize niche overlap and avoid competitive exclusion.

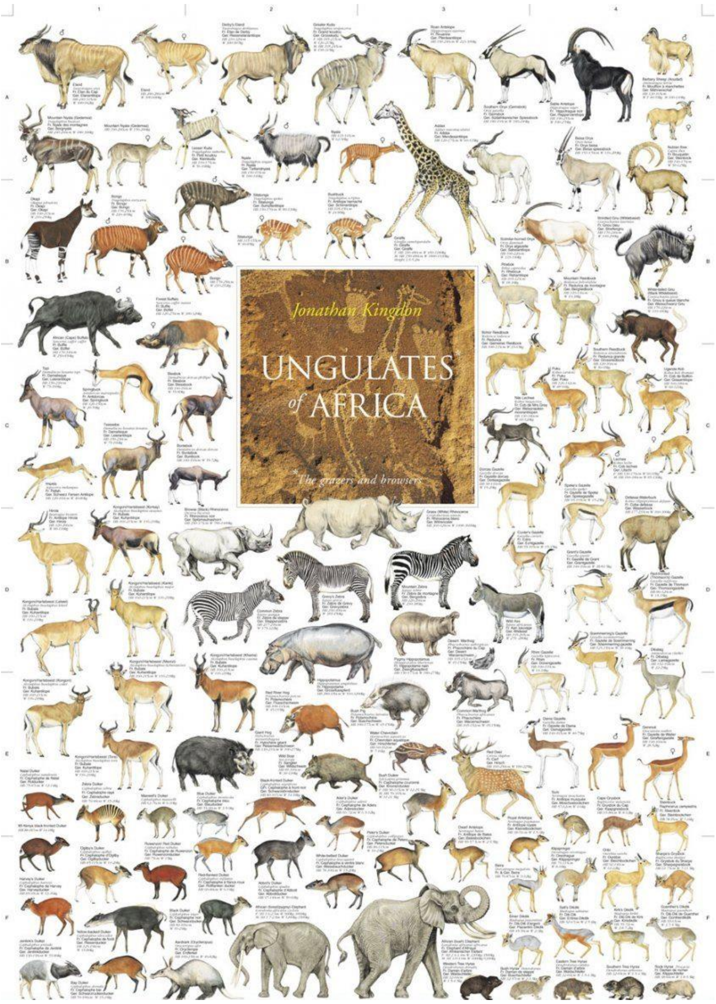


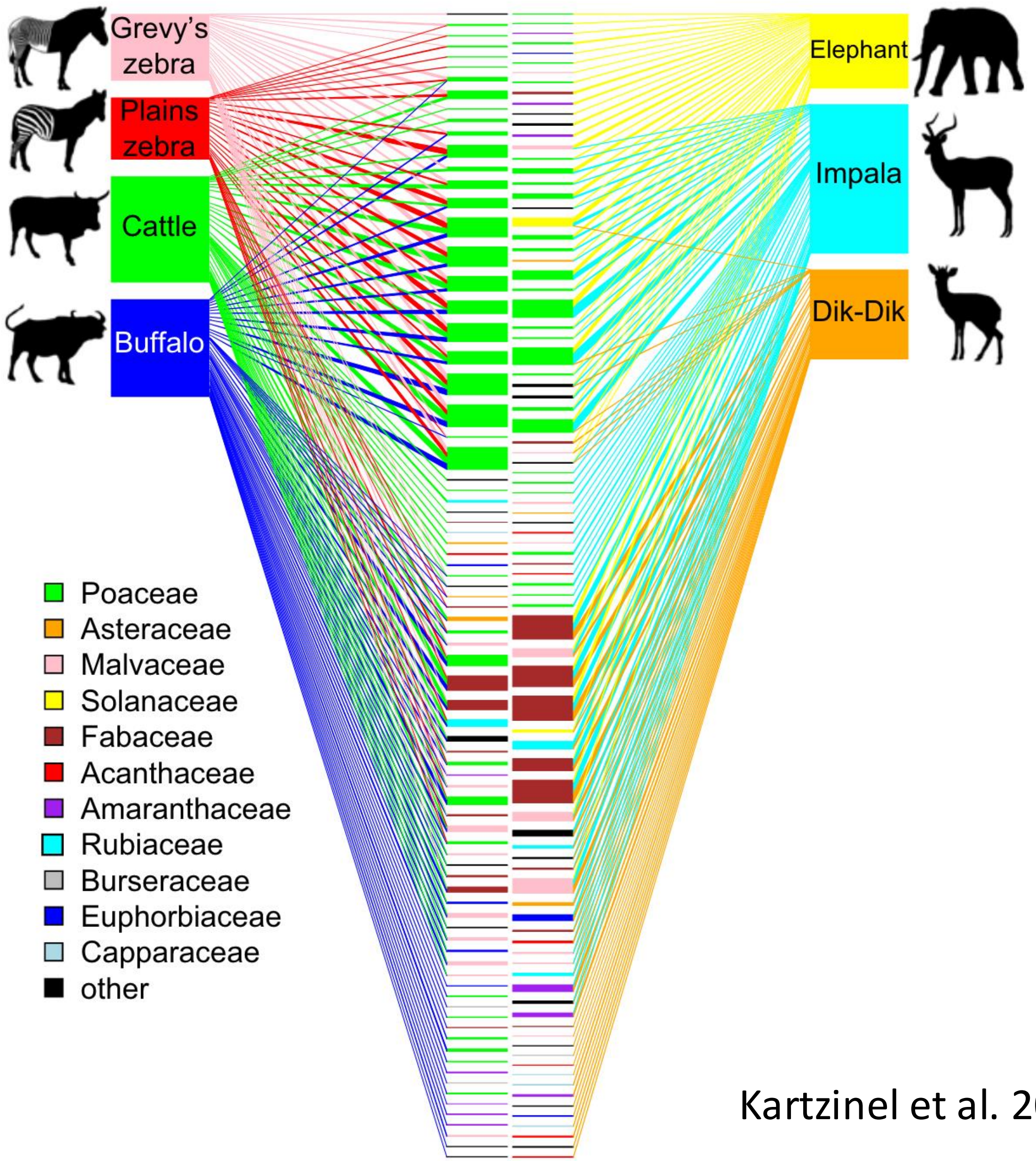
Character displacement forces niches apart.

Grant & Grant 2006. *Science*.

Niche partitioning enables organisms to avoid competitive exclusion

Often 10-25 large mammalian herbivores coexisting in the same African savanna!

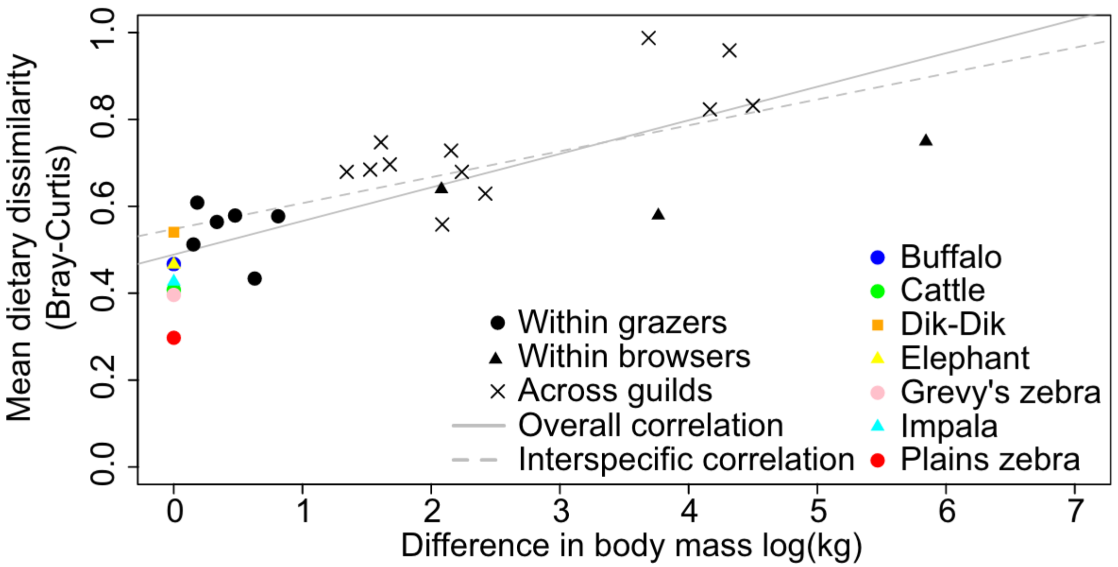


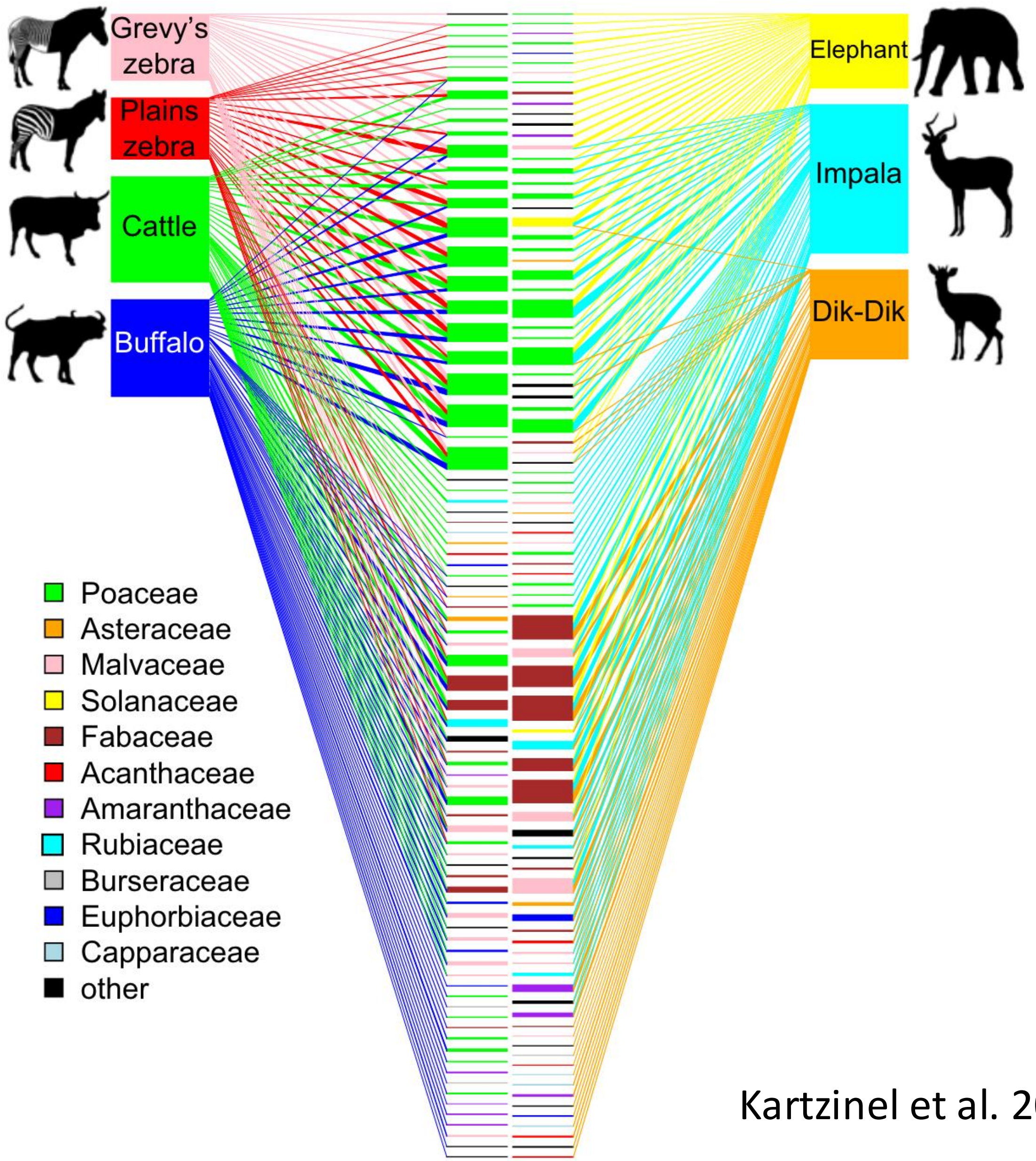


Kartzinel et al. 2015. *PNAS.*

Niche partitioning enables organisms to avoid competitive exclusion

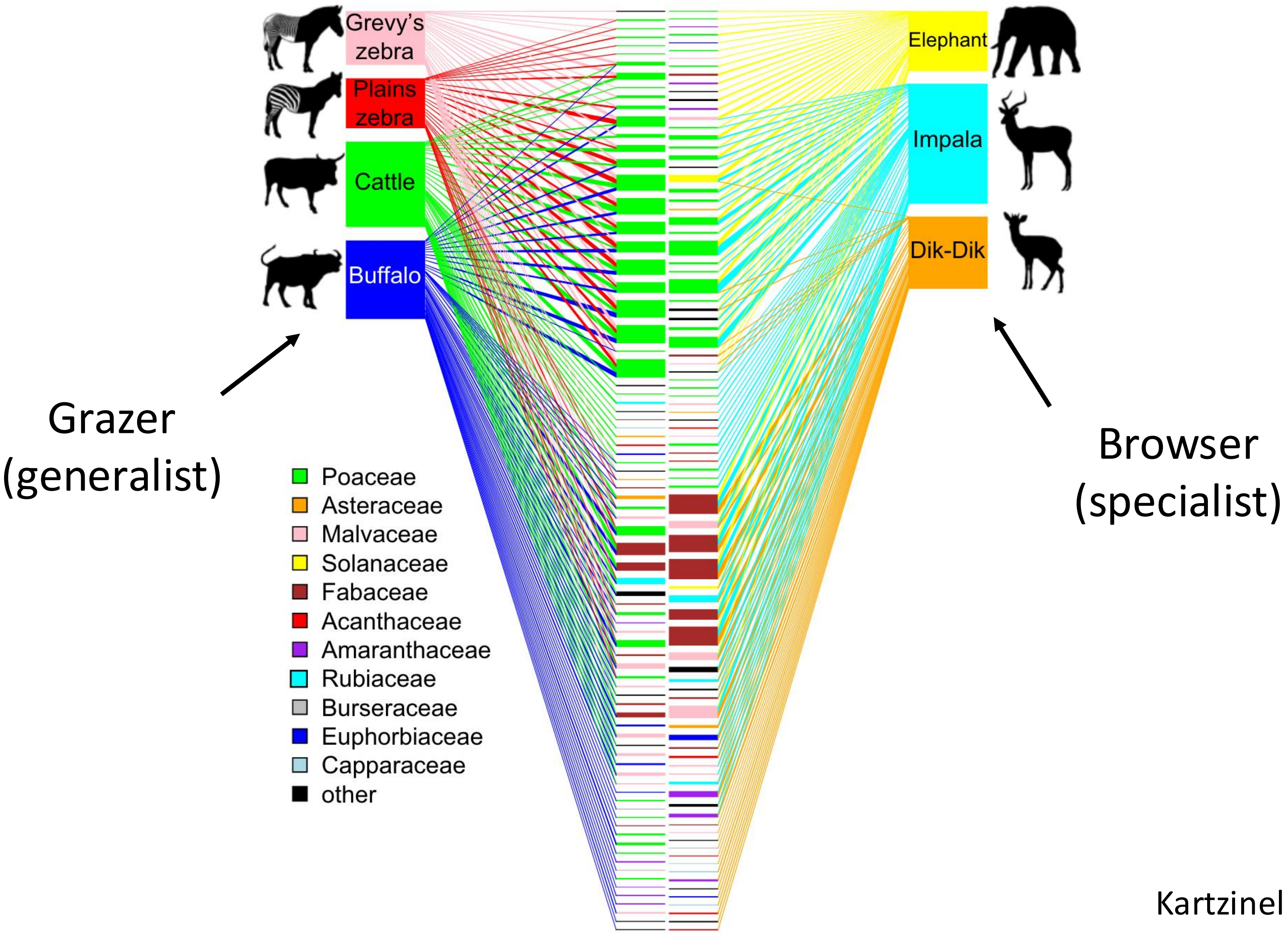
Dietary overlap is higher in similar-sized mammals





Kartzinel et al. 2015. *PNAS.*

Consumer resource partitioning in the African savanna



Kartzinel et al. 2015. *PNAS.*

"Complete competitors cannot coexist" unless:

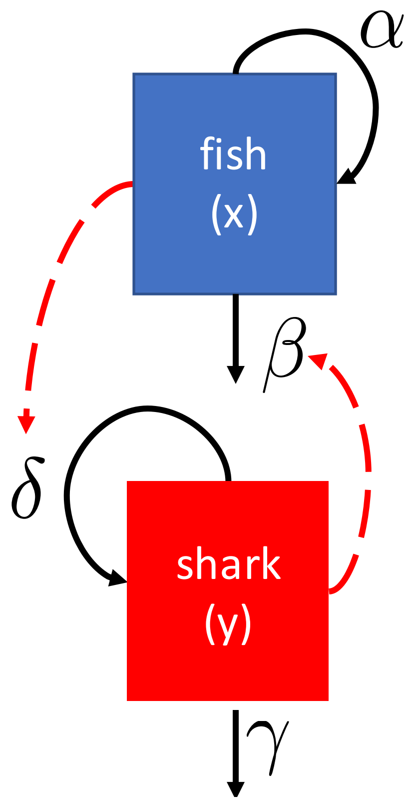
127

a. there is a precedence effect

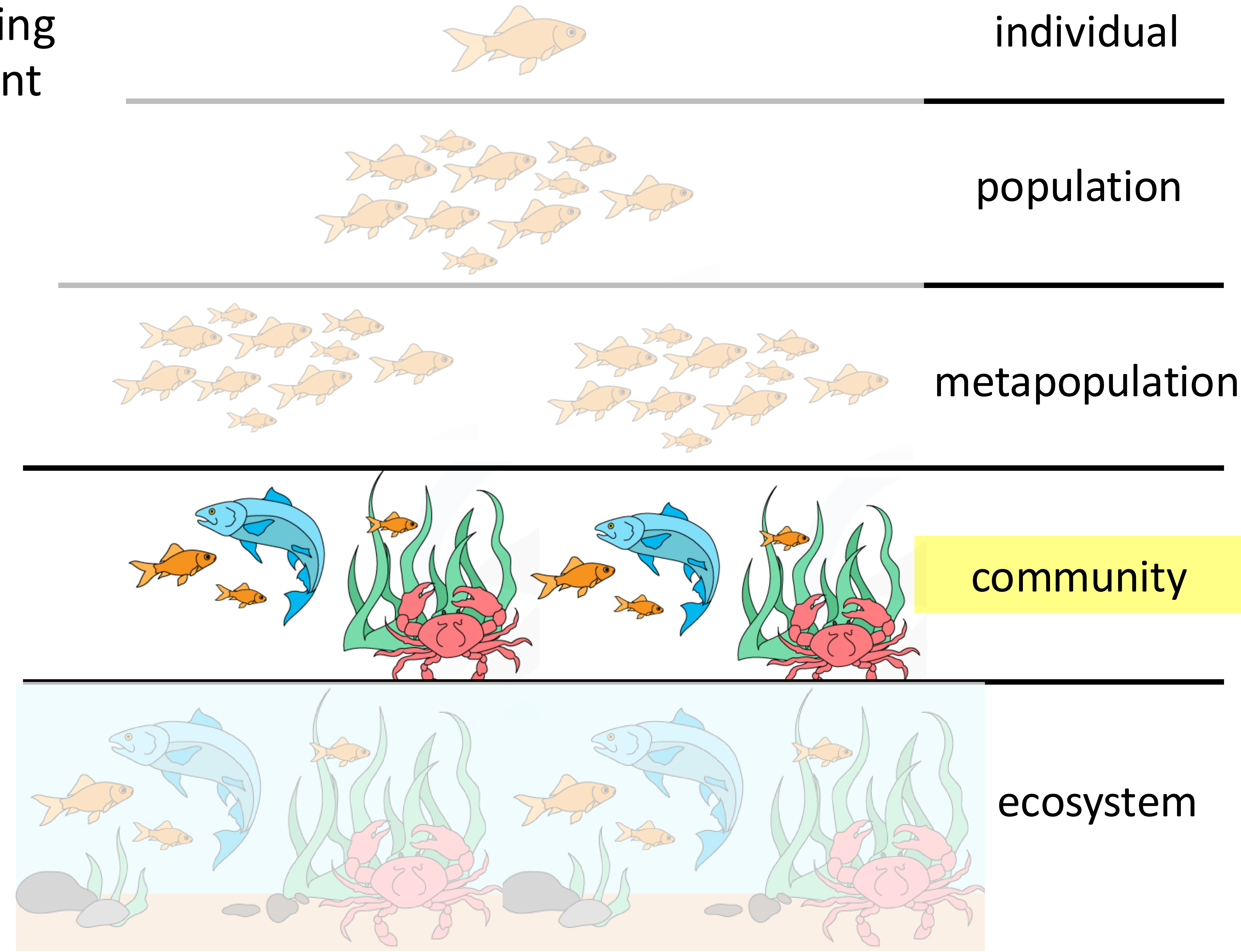
b. space, low dispersal, and stochasticit…

SEE MORE

Community = interacting populations of different species



*How does fish abundance **vary** with **changes** in shark abundance?*



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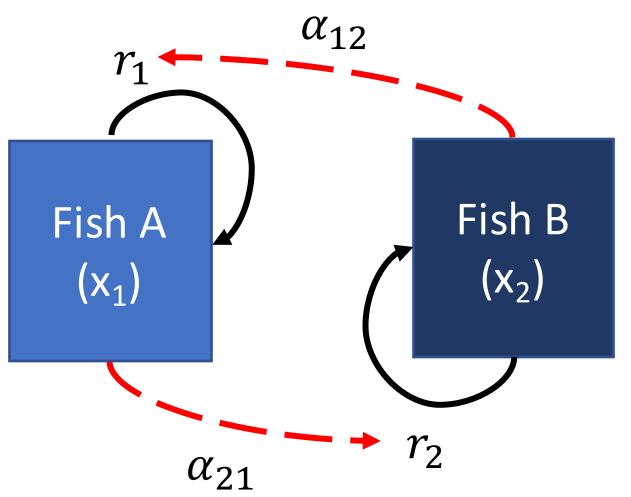
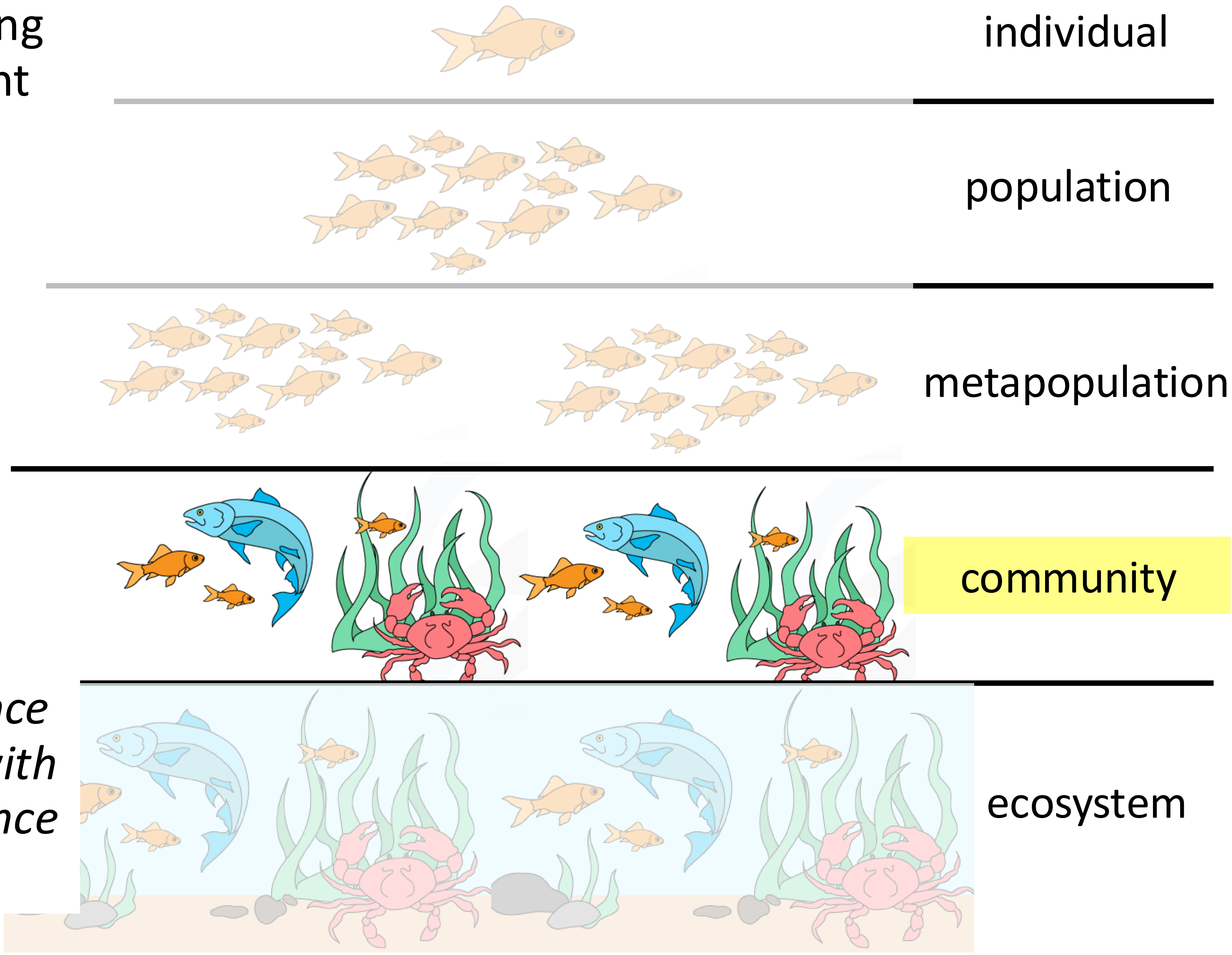


*How does the abundance of **fish species A** vary with changes in the abundance of **fish species B**?*



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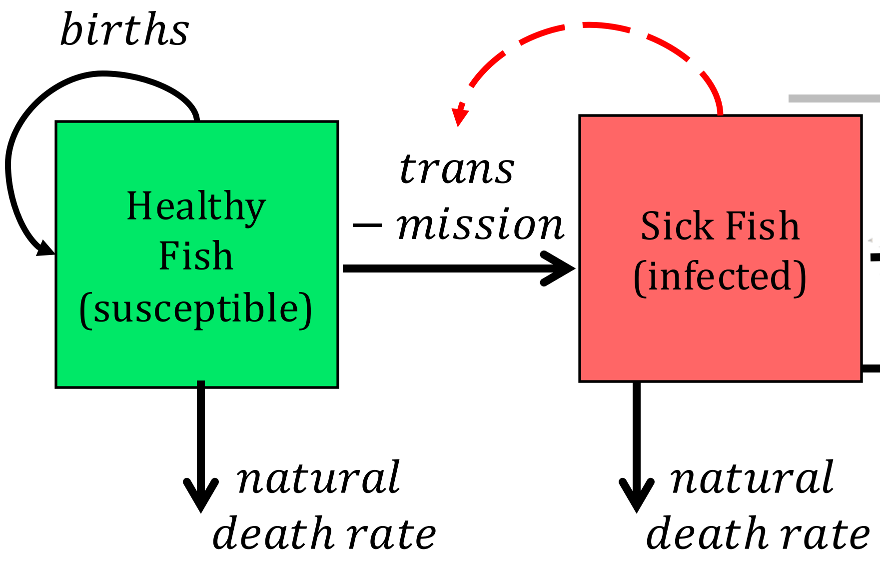

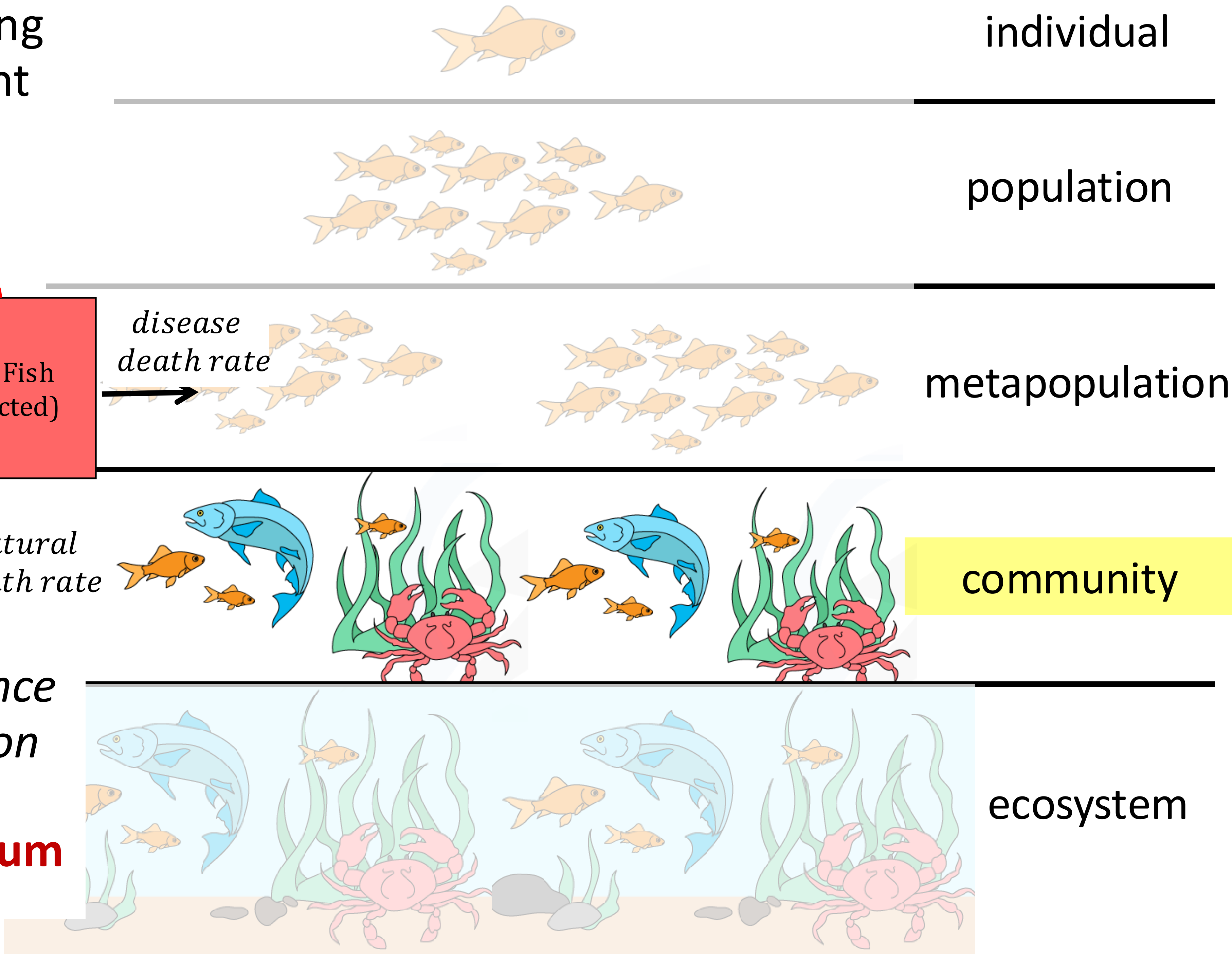

*How does the abundance of **fish** change based on **infection with Mycobacterium marinum** (wasting disease)?*

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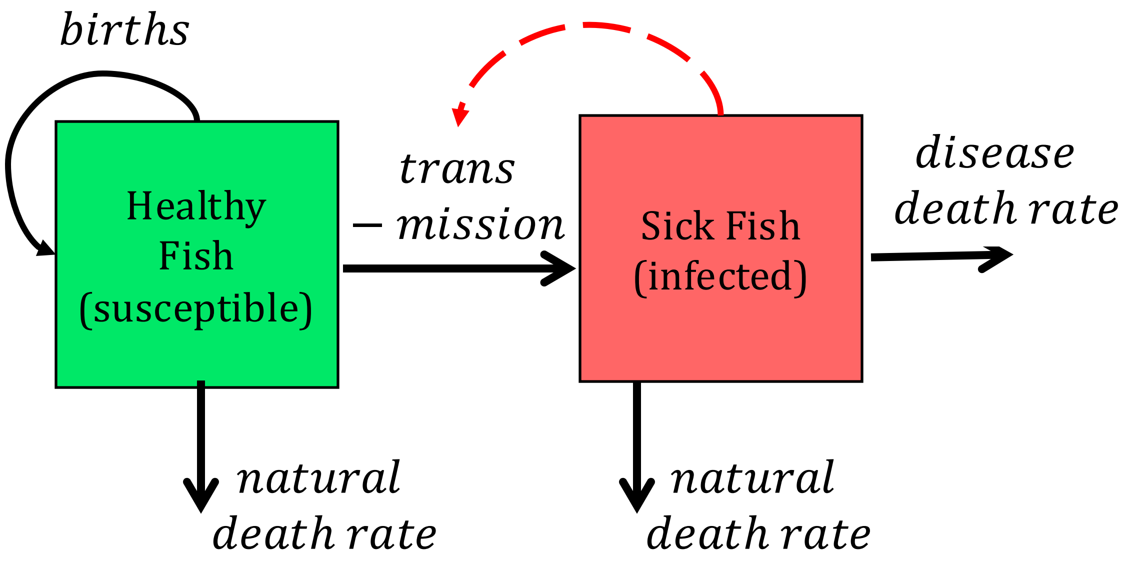

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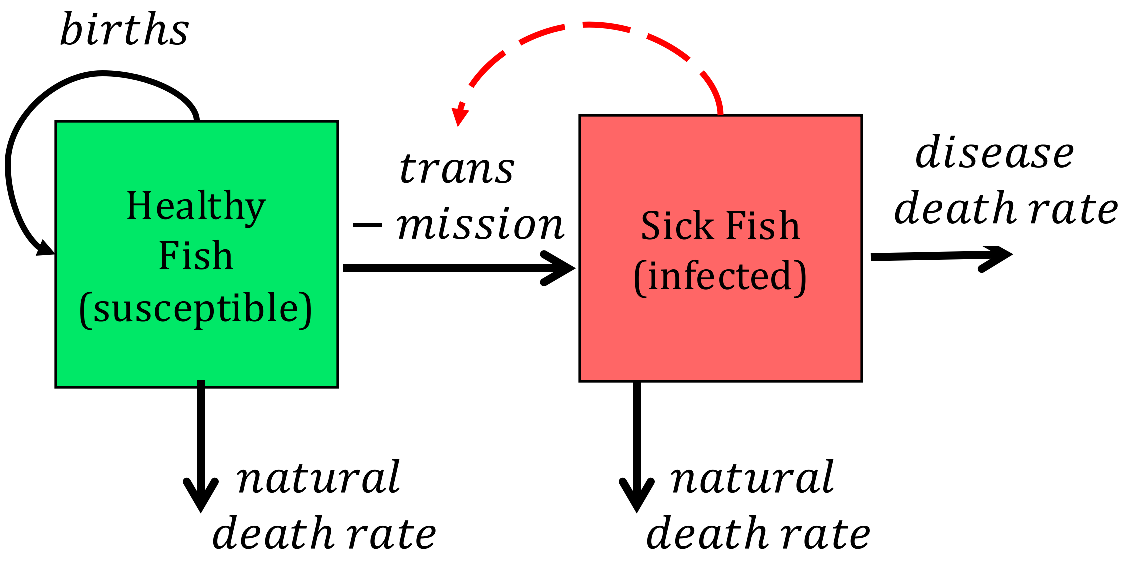


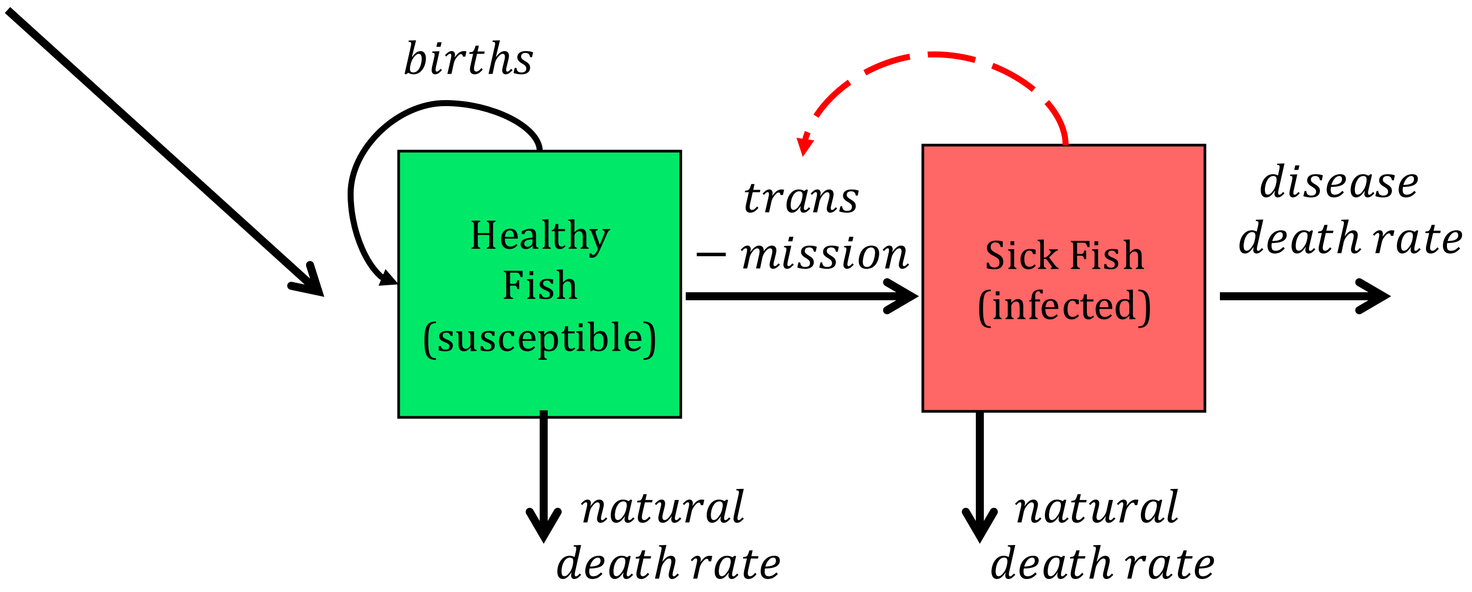
How does the abundance of fish change based on ***infection with*** **Mycobacterium marinum** *(wasting disease)?*

Species can interact in several distinct ways.

- **Mutualism** – both species benefit
- **Commensalism** – one species benefits, the other is unaffected
- **Predation** – one species benefits, the other is harmed (eaten!)
- **Competition** – two species compete for the same limiting resource, both harmed by the interaction
    - Direct = wolves and coyote at a moose carcass
    - Indirect = diurnal cheetah, nocturnal leopard at a giraffe carcass
- **Parasitism** – one species (the parasite) lives *in* or *on* the other species (the host)

Population Biology

Each box is a distinct population!



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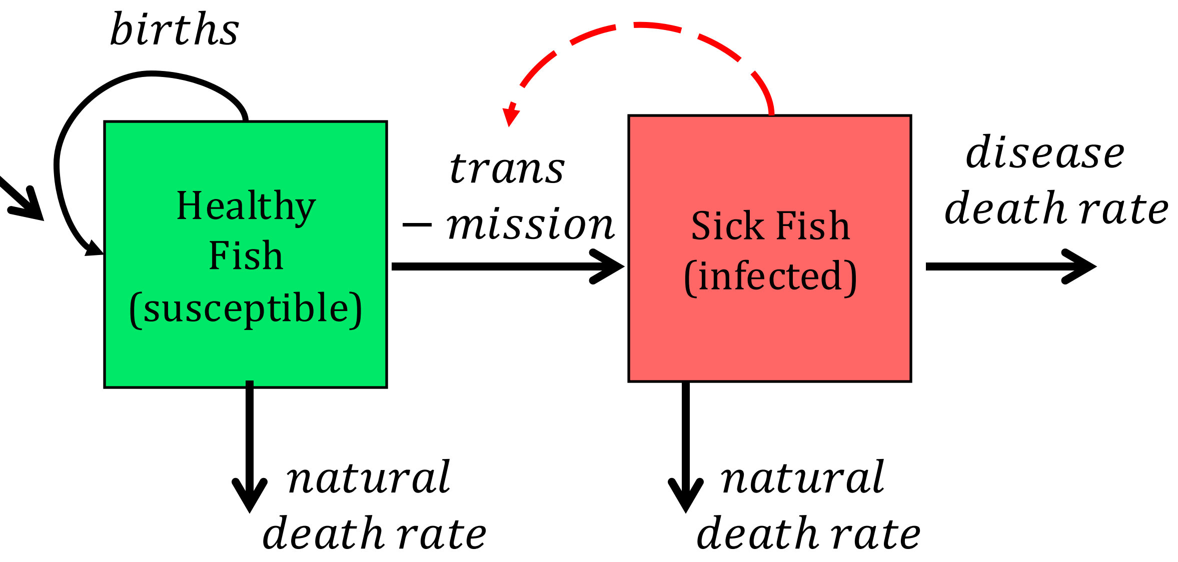
When we model these populations, everyone in the box is considered the same.



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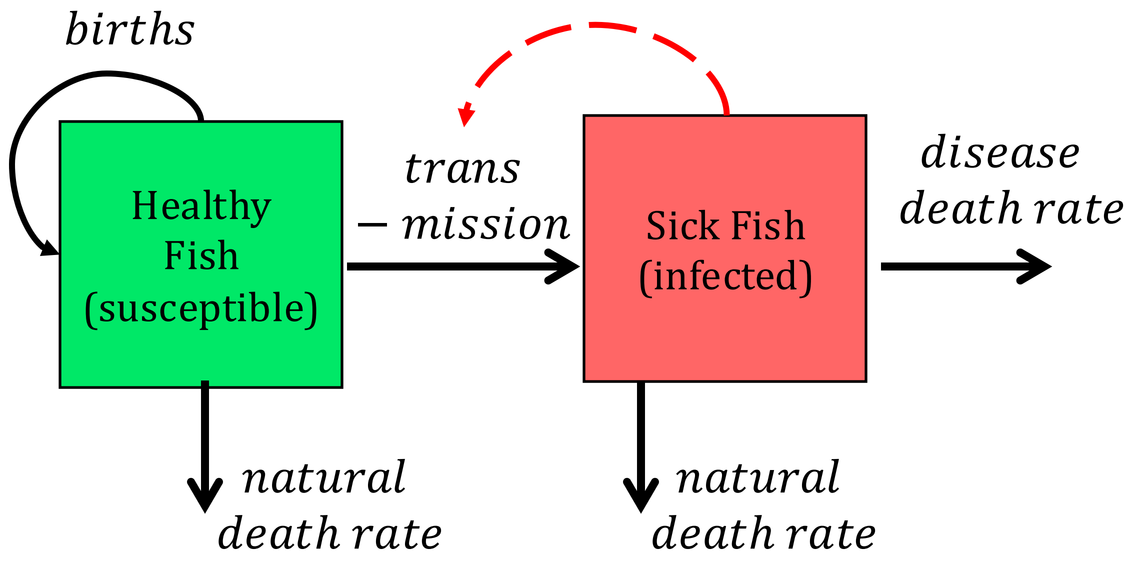

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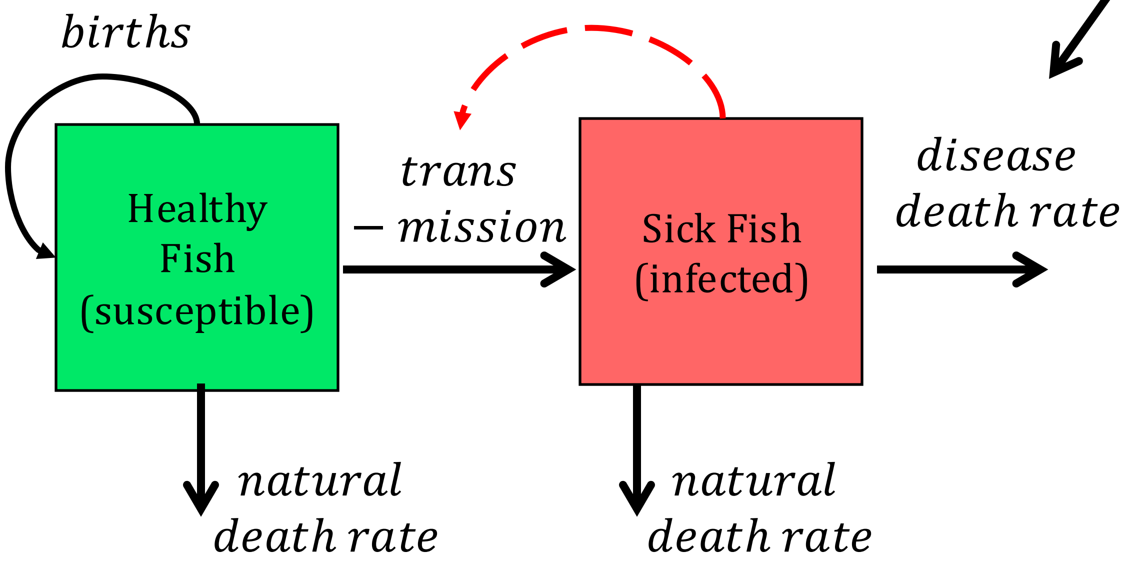

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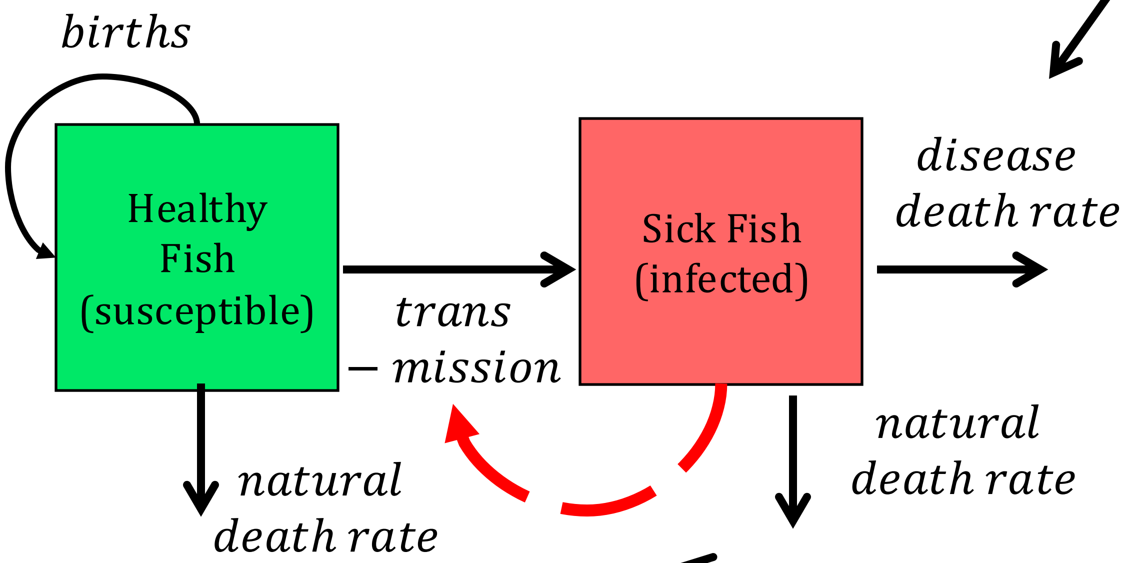
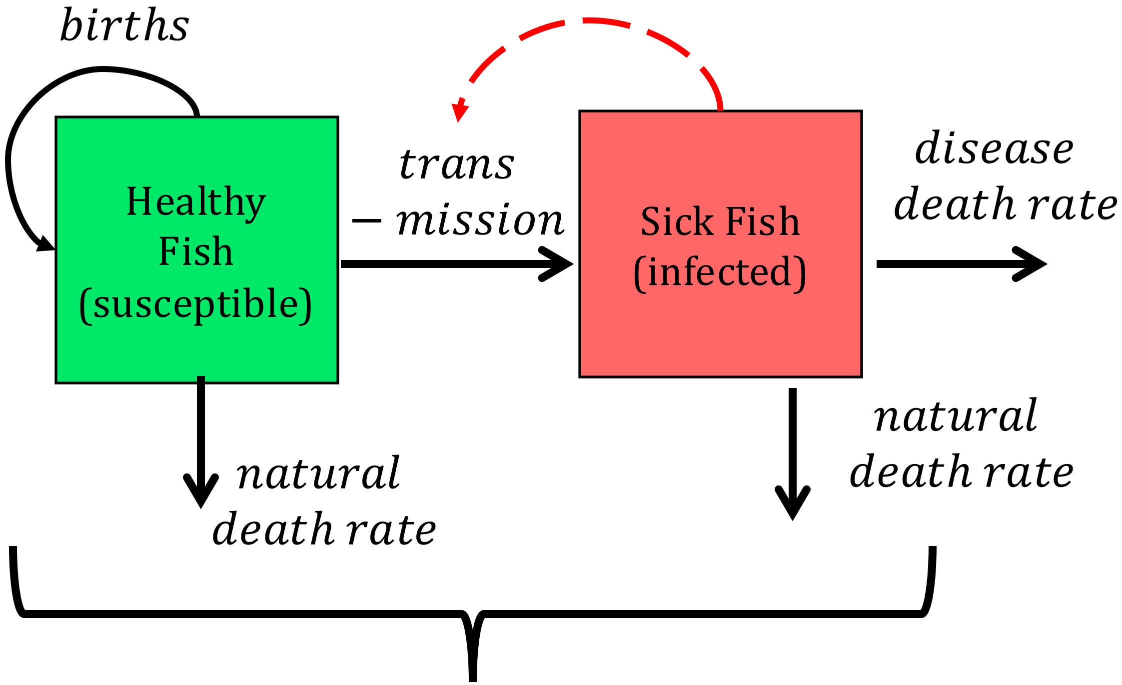


Solid lines are processes by which individuals in each population move between boxes.

Dashed lines are influences of populations on rates (transmission is higher when there are more sick fish)

*How does the abundance of **fish** change based on **infection with Mycobacterium marinum** (wasting disease)?*

Population Biology



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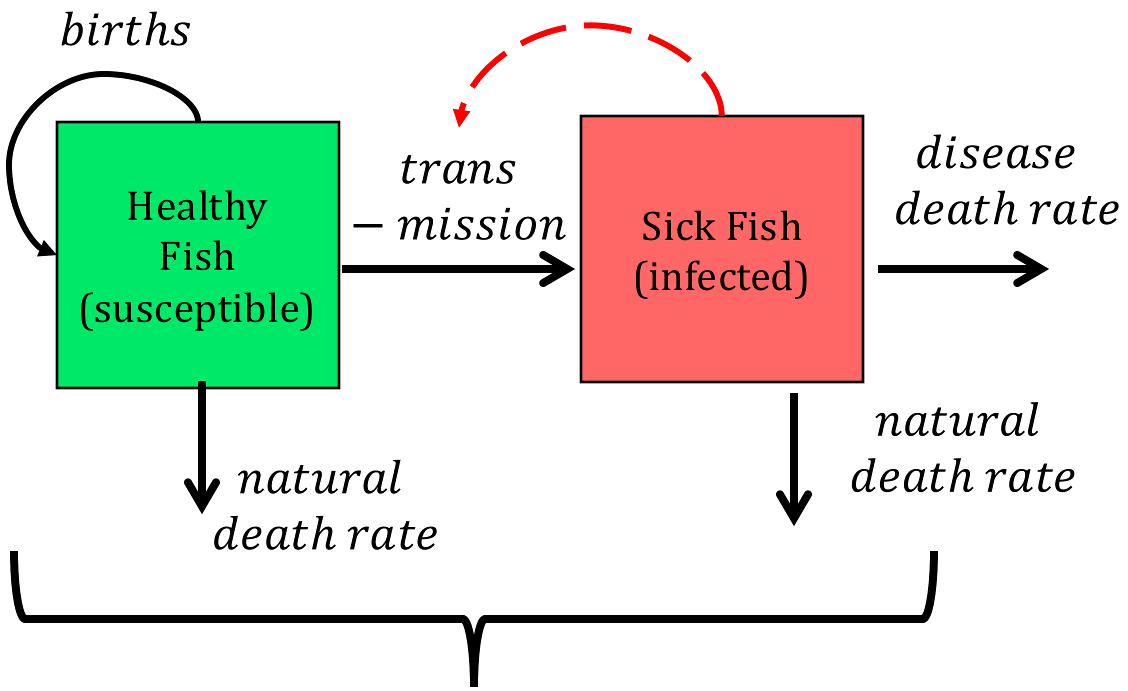
In disease ecology, we model populations based on their **infection status**.

Population Biology



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In disease ecology, we model populations based on their **infection status**.

Previously, we modeled populations of **different species**, or of **distinct life history classes within a species**.

Parasites and Pathogens

- Parasite: an organism that lives in or on another organism and benefits at the expense of others.
  - Ex: helminths (parasitic worms: tapeworms, roundworms, hookworms), ectoparasites (ticks, fleas)

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Pathogen: *Yersinia pestis*
Disease: Plague

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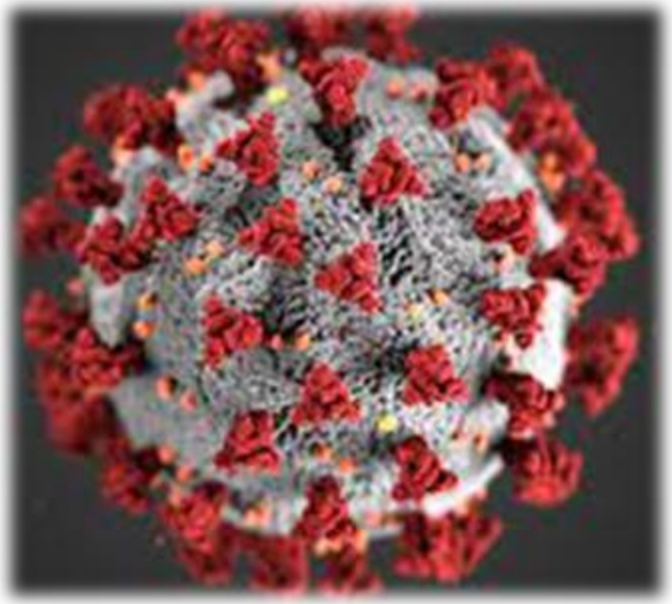
Pathogen: SARS-CoV-2
Disease: COVID-19



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Pathogen: *Yersinia pestis*
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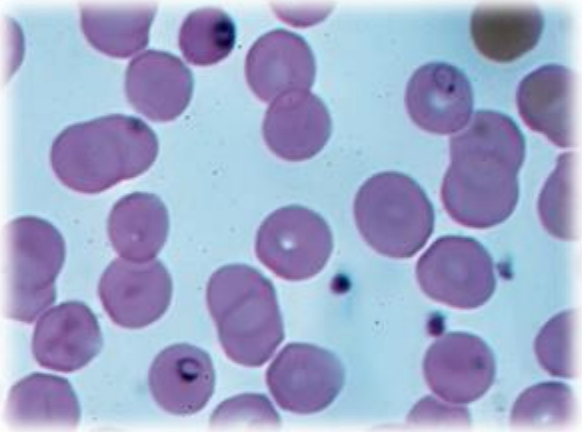
Pathogen: SARS-CoV-2
Disease: COVID-19

Pathogen: *Plasmodium falciparum, P. vivax, P. malariae, P. ovale, P. knowlesi*
Disease: malaria



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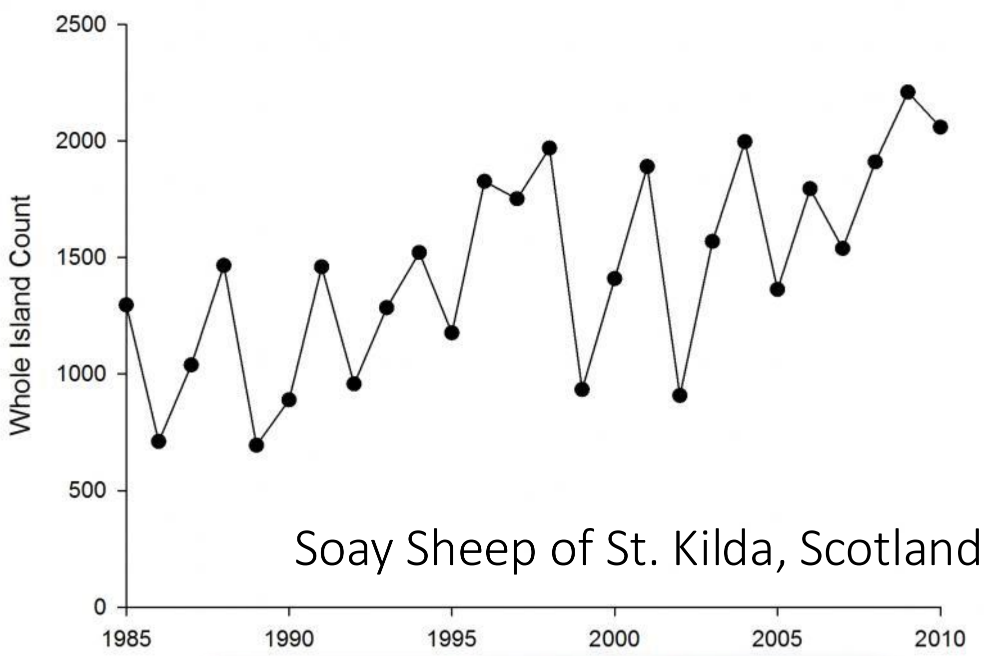
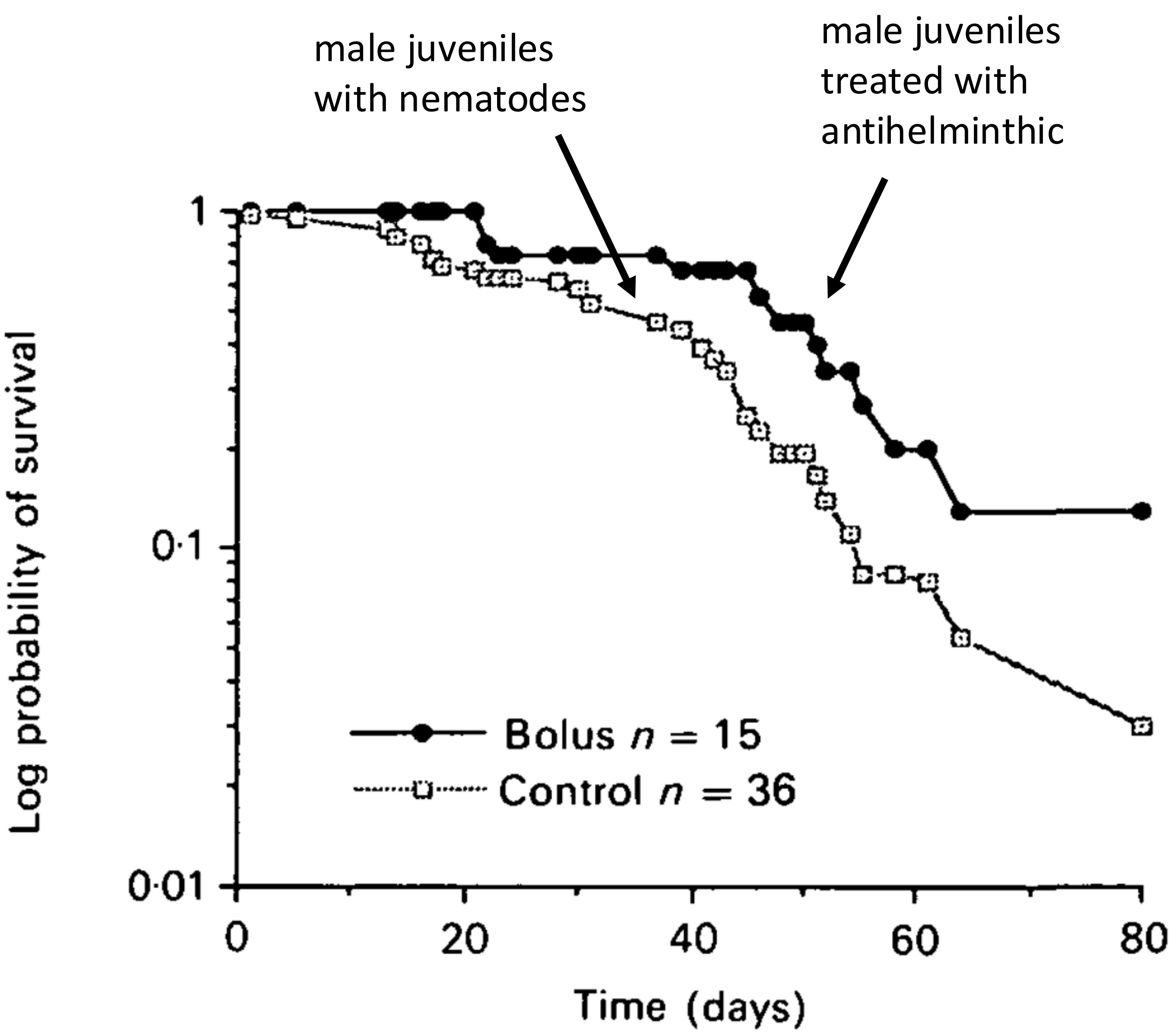
  - Ex: Soay sheep, grapes
- Vector: an arthropod agent that carries and transmits a pathogen or parasite from host to host
  - Ex: mosquitoes, ticks, fleas

We already know that parasites can play a role in regulating populations!





Gulland 1992. *Parasitology.*

Early work in red grouse experimentally demonstrated the power of parasites to regulate populations.

Hudson 1986. *J Animal Ecology.*
Hudson et al 1992. *J Animal Ecology.*
Hudson et al 1998. *Science.*

What factors do we know contribute to annual fluctuations in the population density of the Soay sheep of St Kilda?

A Density-dependent effects …	1%
B Higher mortality in b…	0%
C Higher ewe mortality at hi…	2%
D Lowered survival in…	3%
E Geometric growth	1%
F. A, B	14%
G. A, B, C, D, E	79%

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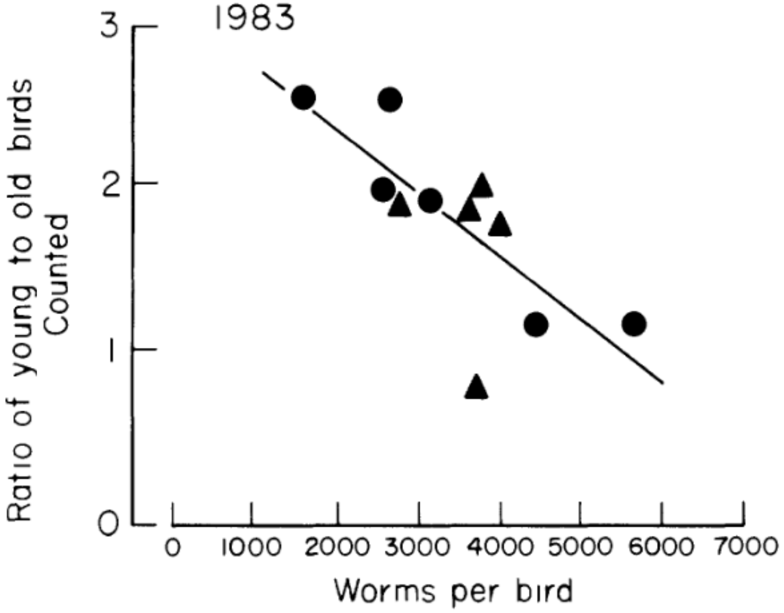
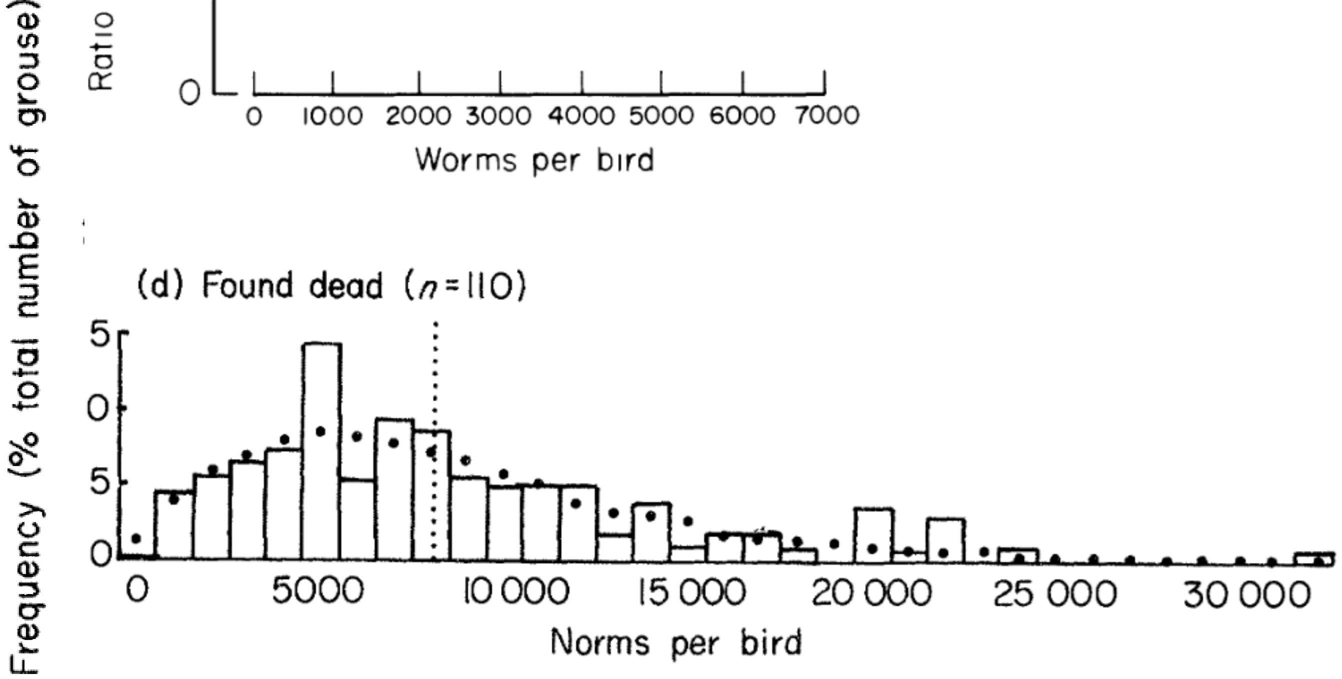
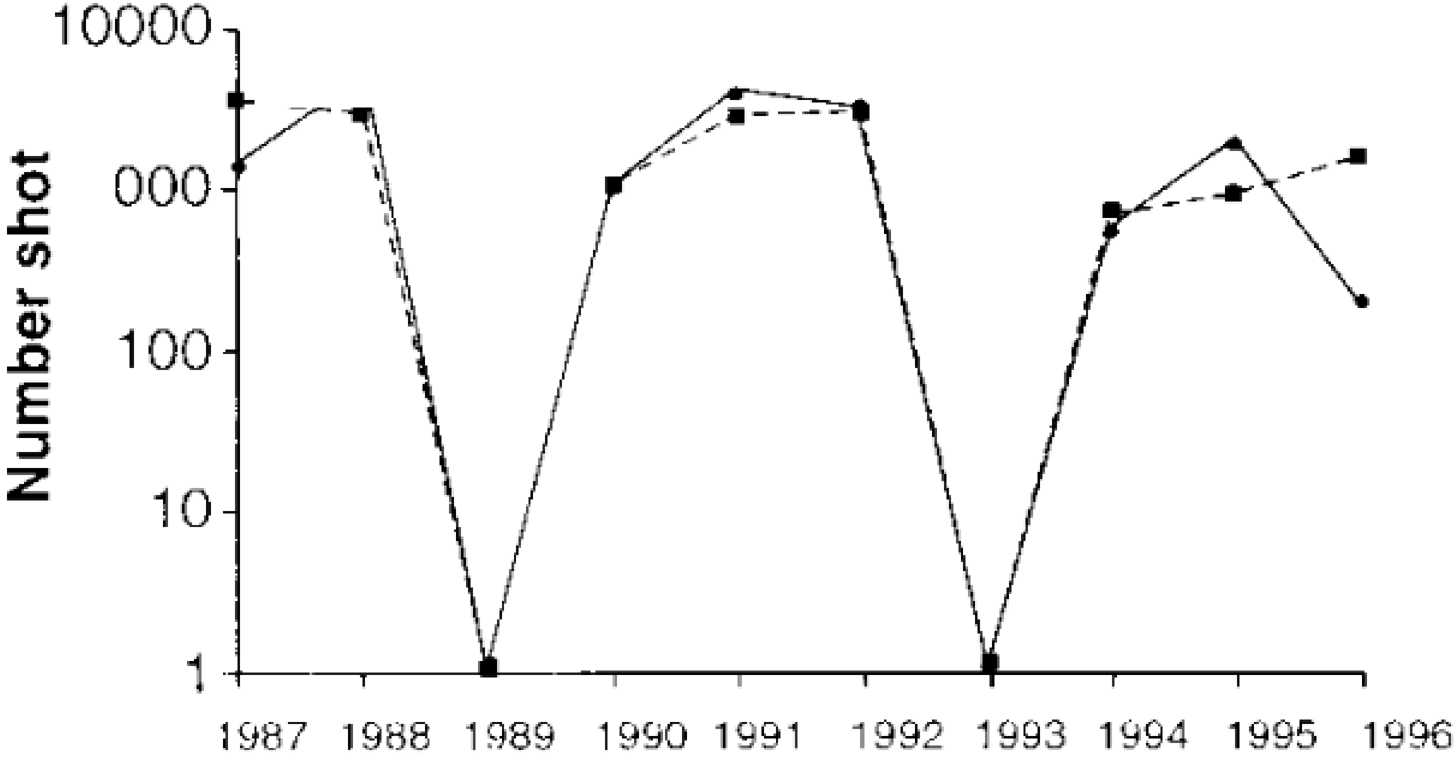




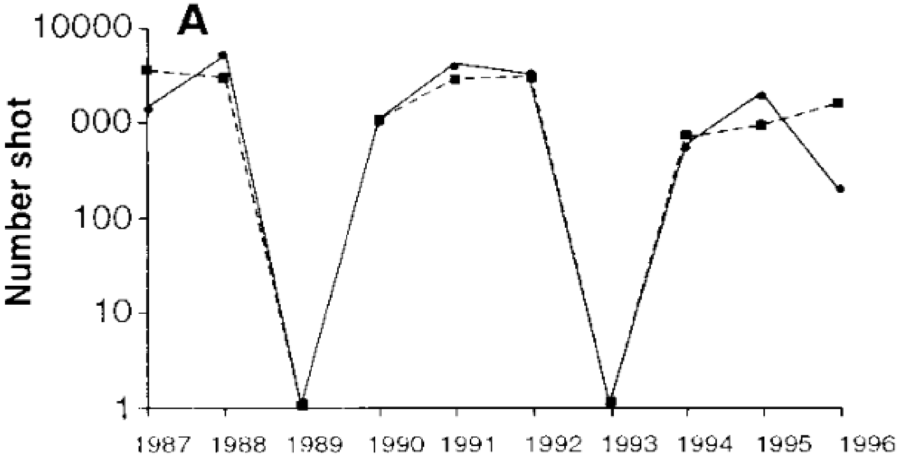
Higher burden of the intestinal strongyle worm, *Trichostrongylus tenuis,* both reduces breeding success and increases mortality in red grouse.

Hudson 1986. *J Animal Ecology.*
Hudson et al 1992. *J Animal Ecology.*

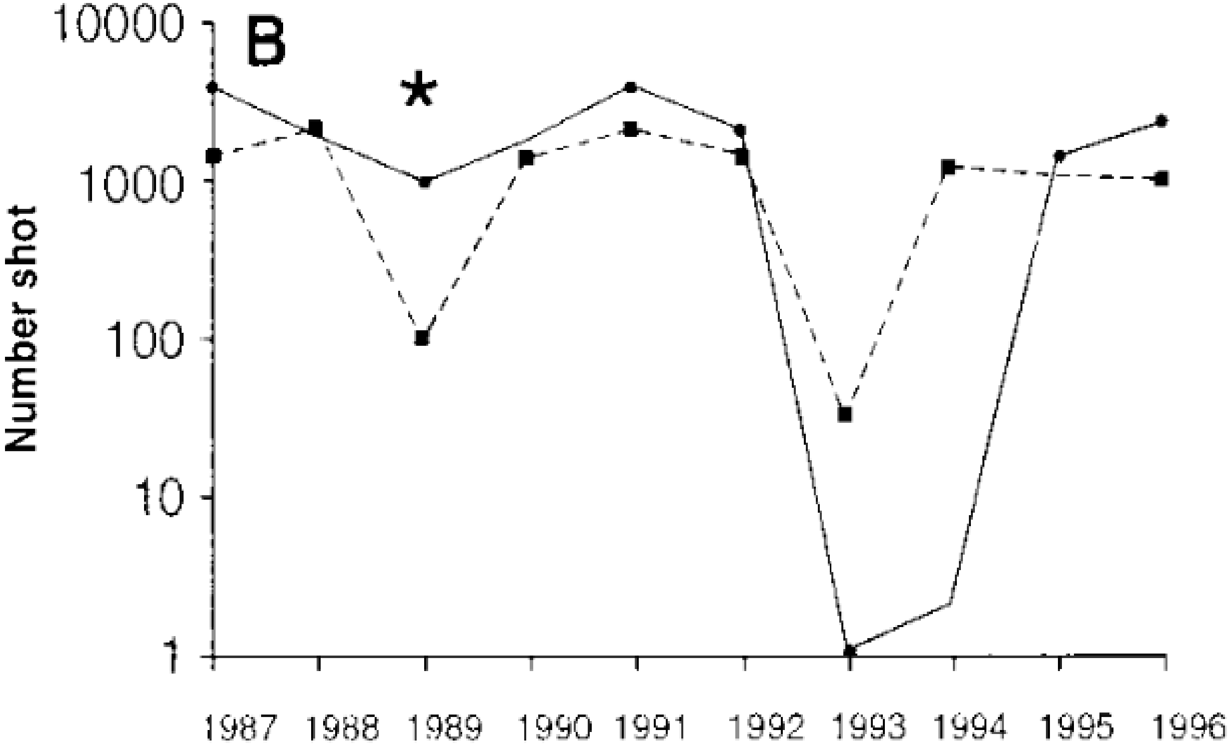
Worms were hypothesized to be responsible for the observed population cycles in ‘bag data’ from northern England.

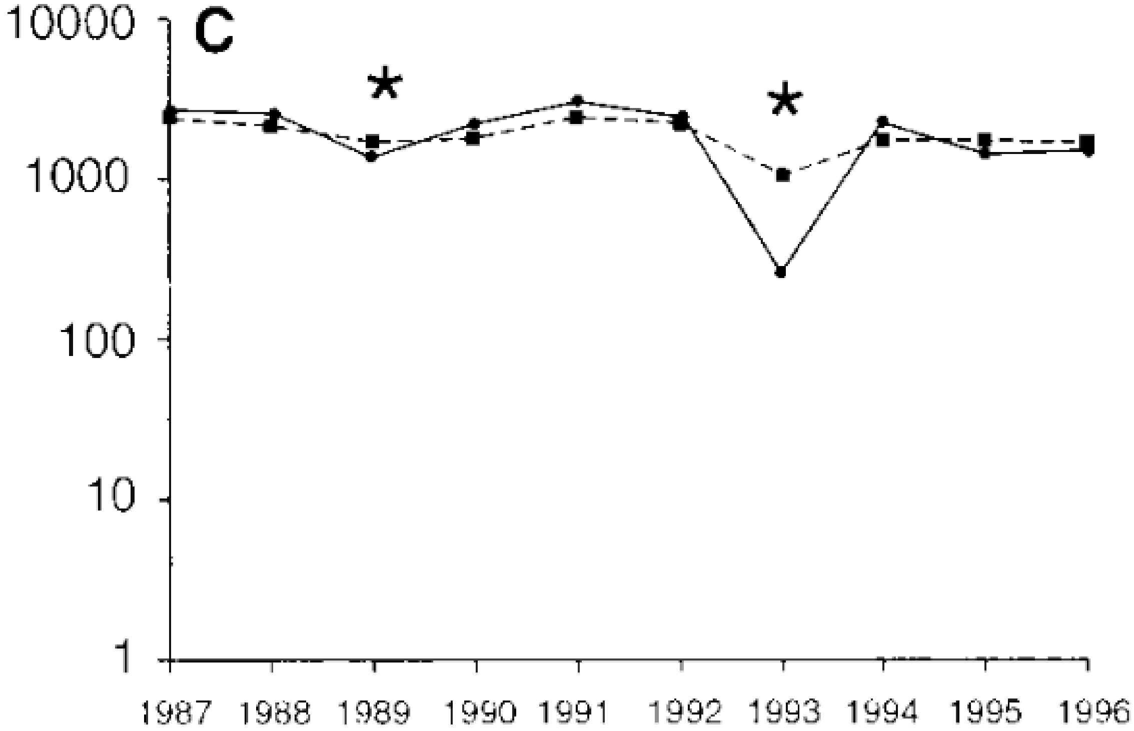


Hudson et al 1998. *Science.*



Deworming eliminated population cycles to prove this effect!





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Parasites and pathogens have also shaped human history.

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But we have not always known they were responsible for disease!

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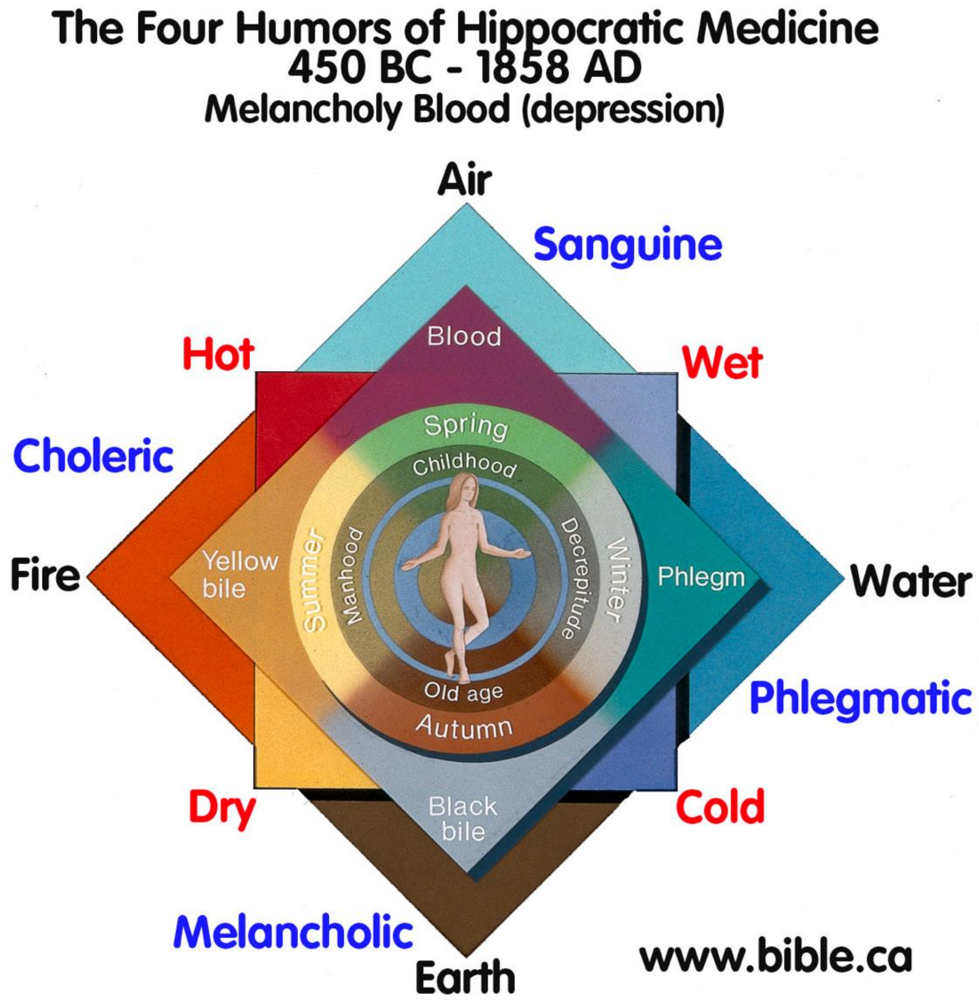
1. **Four Humors:** Hippocrates (c. 400 BC) wrote that disease results from an imbalance of the four humors



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3. **<u>Germ Theory of Disease:</u>** Idea that disease results from germs
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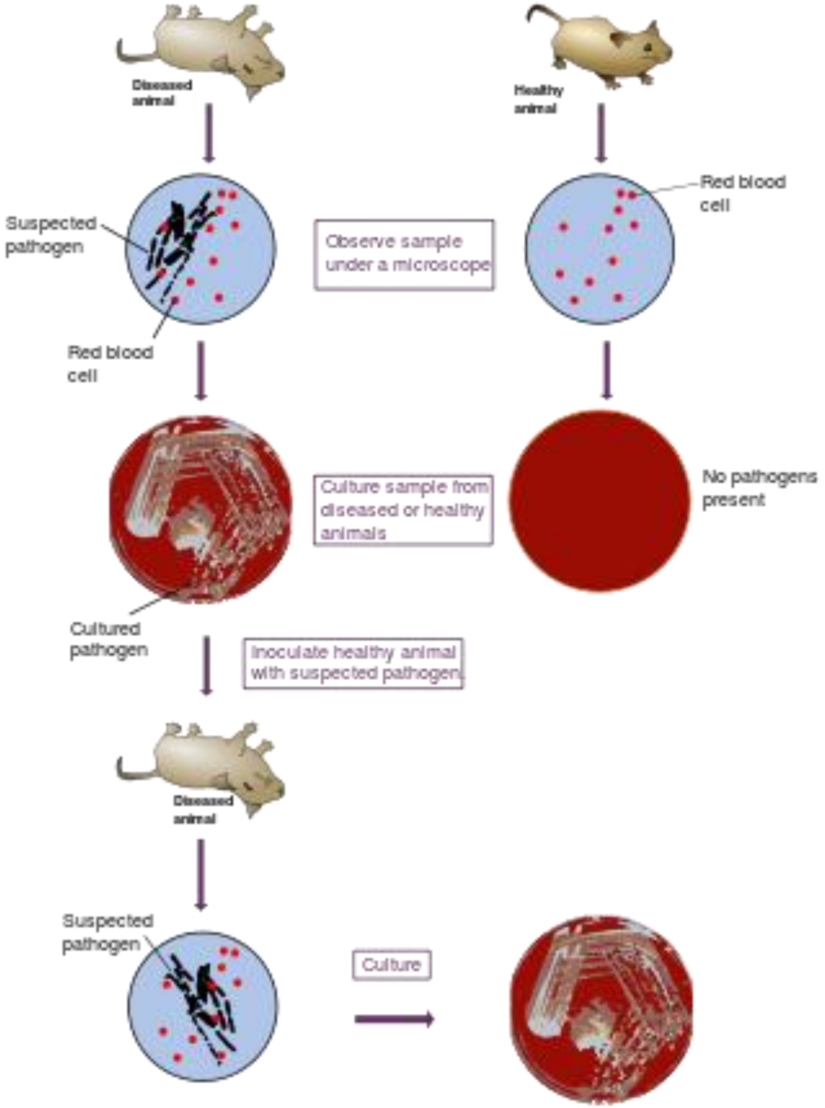

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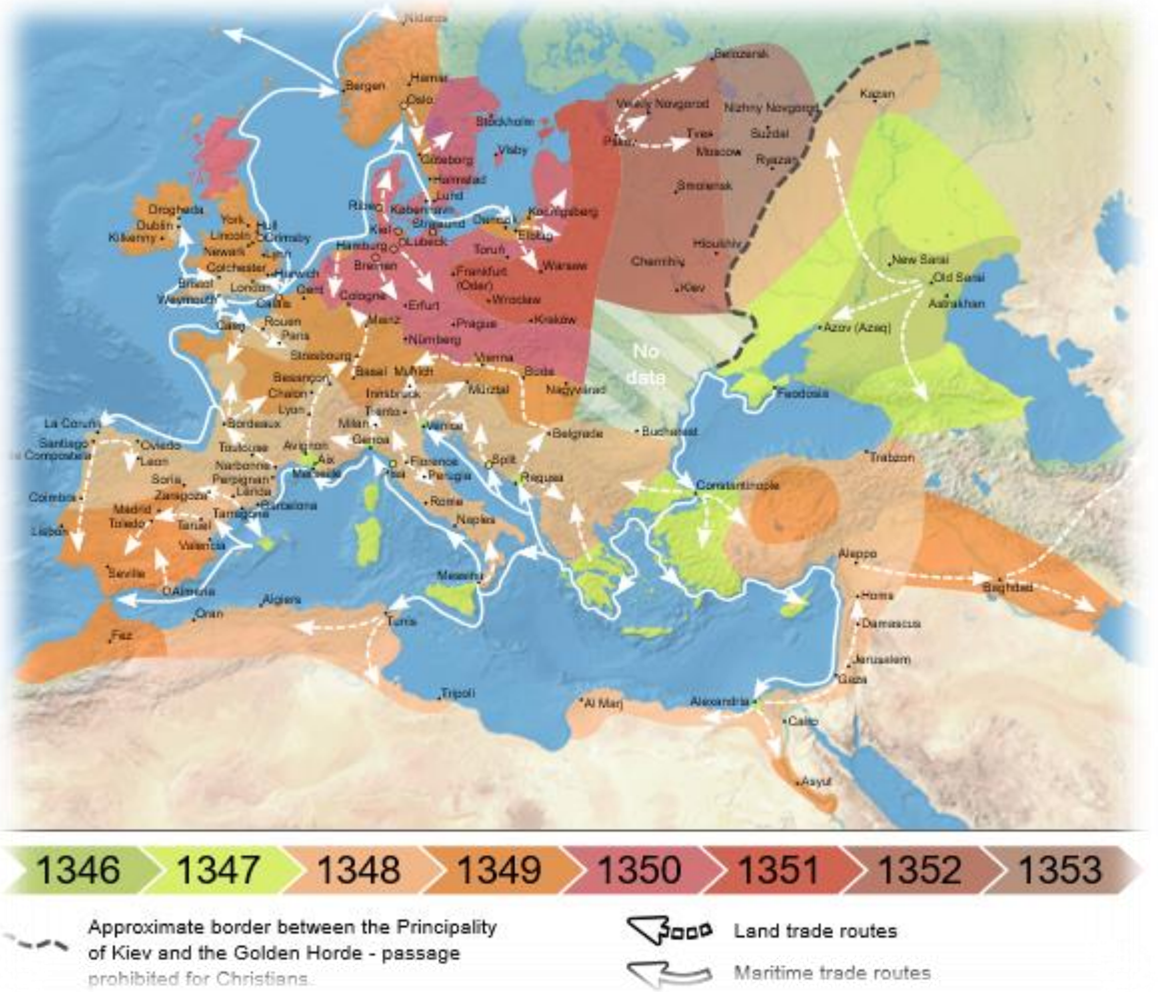
4. **<u>Classic epidemiology</u>**
   - Risk factors for disease = John Snow (1854)

Parasites and pathogens have also shaped human history.

- Plague of Justinian (541-549 AD)
  - First historically recorded pandemic of *Yersinia pestis*
  - Launched the ‘first plague pandemic’ resulting in the deaths of 15-100 million people, 25-60% of Europe’s population at the time

Parasites and pathogens have also shaped human history.

- Plague of Justinian (541-549 AD)
- Black Death (1346-1353 AD)
    - Most fatal pandemic in human history, resulting in deaths of 75-200 million people
    - Killed 30-60% of Europe's population at the time; 17-54% of global population



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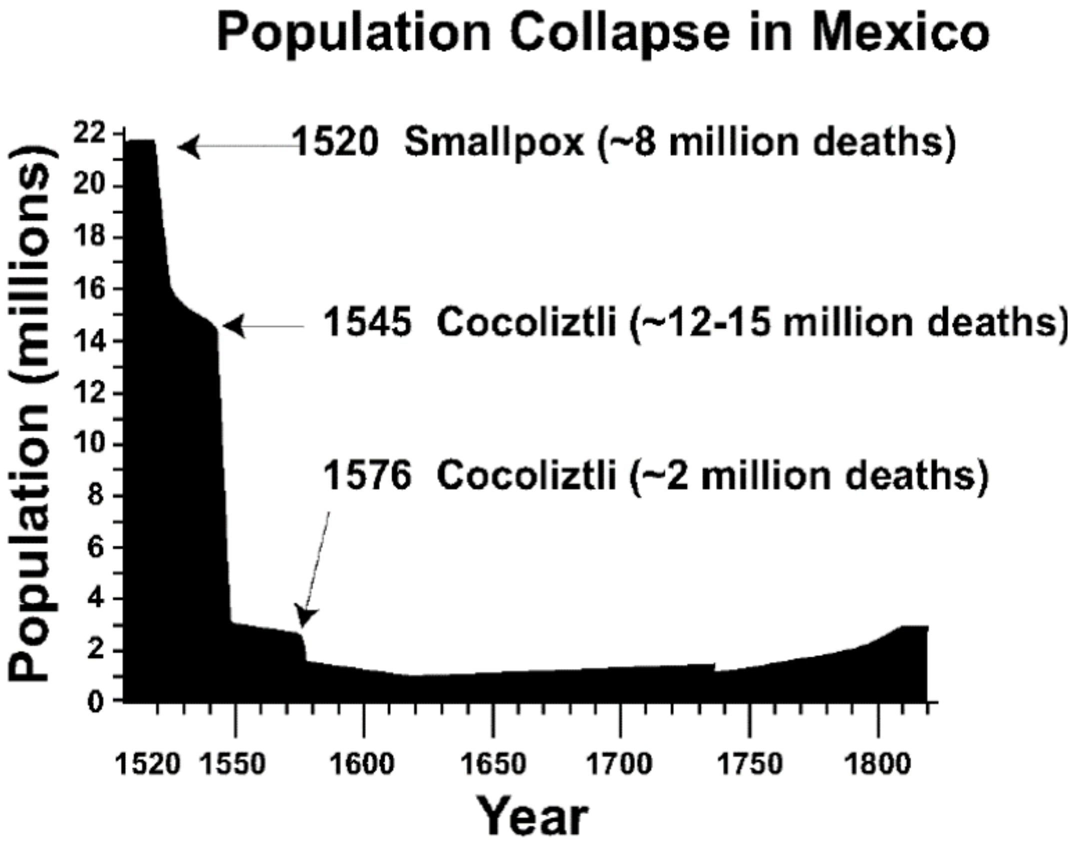
- Plague of Justinian (541-549 AD)
- Black Death (1346-1353 AD)
- Cocoliztli (1545-1548)
  - Pathogen still unknown! Maybe viral hemorrhagic fever, maybe bacterium
  - Killed 80% of the population of Mexico



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- Plague of Justinian (541-549 AD)
- Black Death (1346-1353 AD)
- Cocoliztli (1545-1548)
- Spanish Influenza (1918-1920)
  - 17-100 million deaths worldwide.
  - 1-5% of global population
  - 2nd-most devastating pandemic in history (after Black Death)

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- Plague of Justinian (541-549 AD)
- Black Death (1346-1353 AD)
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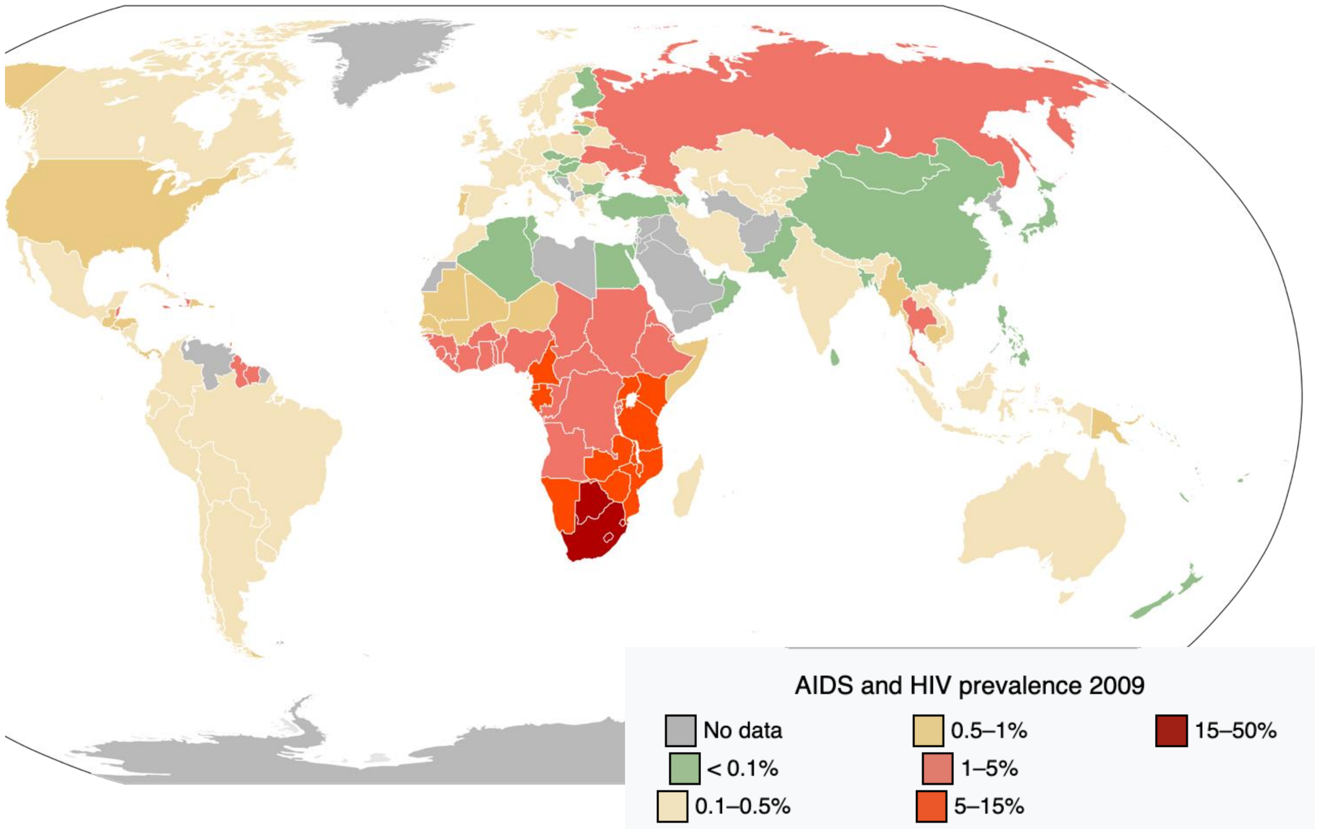
- Spanish Influenza (1918-1920)
- HIV (~1960-now)
  - >40 million deaths and counting
  - Prevalence still >20% in some countries in southern Africa



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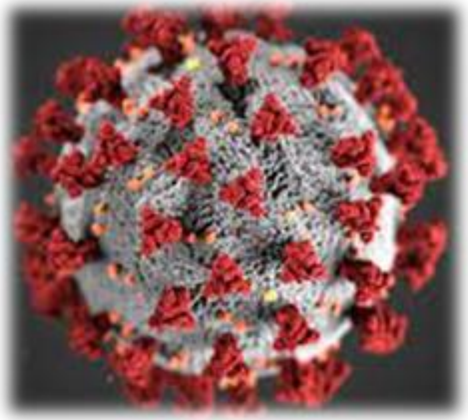

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- Spanish Influenza (1918-1920)
- HIV (~1960-now)
- COVID-19 (2019-now)
  - ~7-29 million deaths worldwide
  - ~0.1-0.4% of population

Parasites and pathogens have also shaped human DNA.

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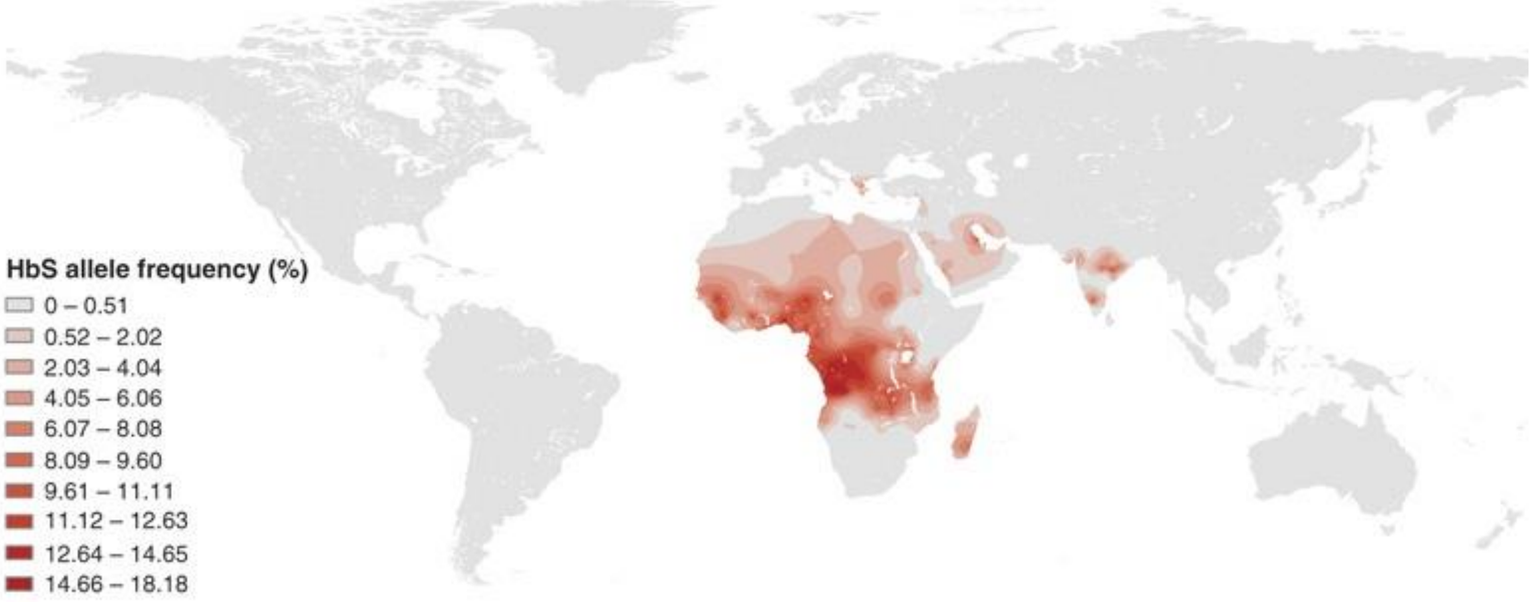
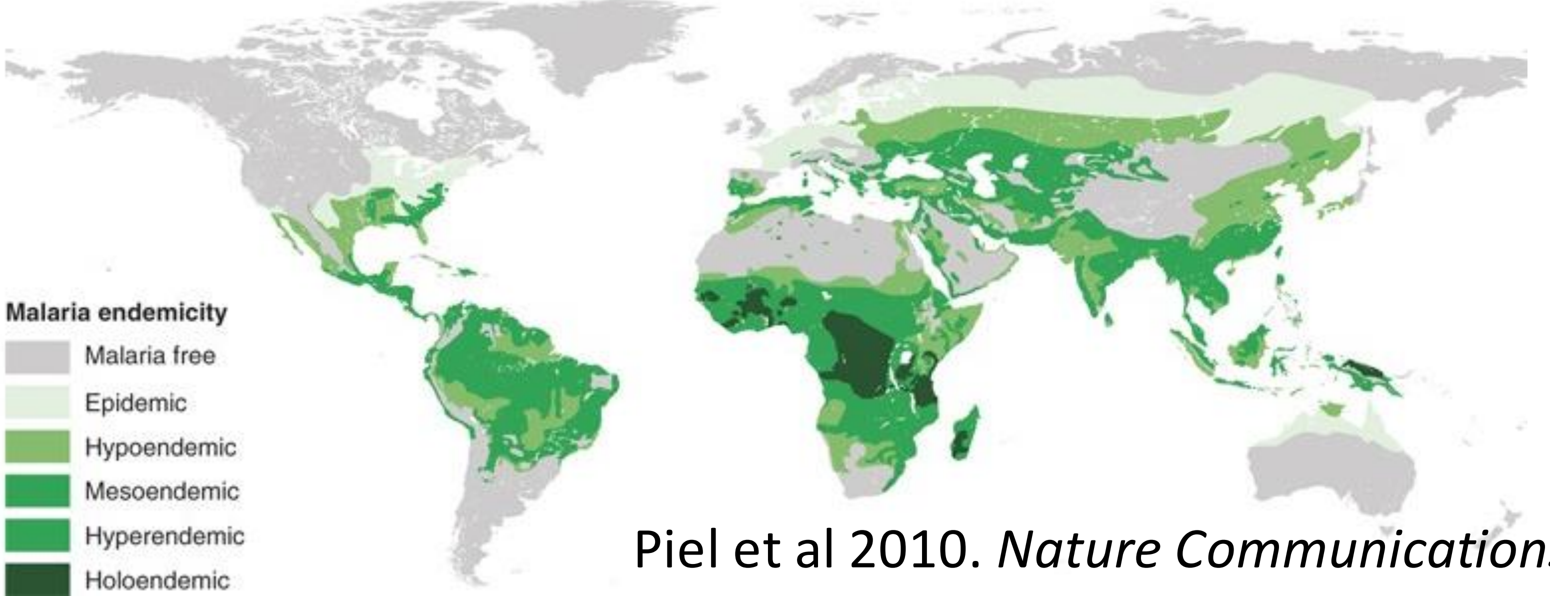
Sickle cell anemia

- The HbS allele confers resistance to malaria but also results in sickle cell anemia when homozygous.
- Natural selection has favored this trait in malaria-endemic regions of the planet.
- As of 2021, WHO estimates 247 million malaria cases worldwide and >600,000 deaths, 95% in Africa.
- Children <5 account for 80% of malaria deaths.

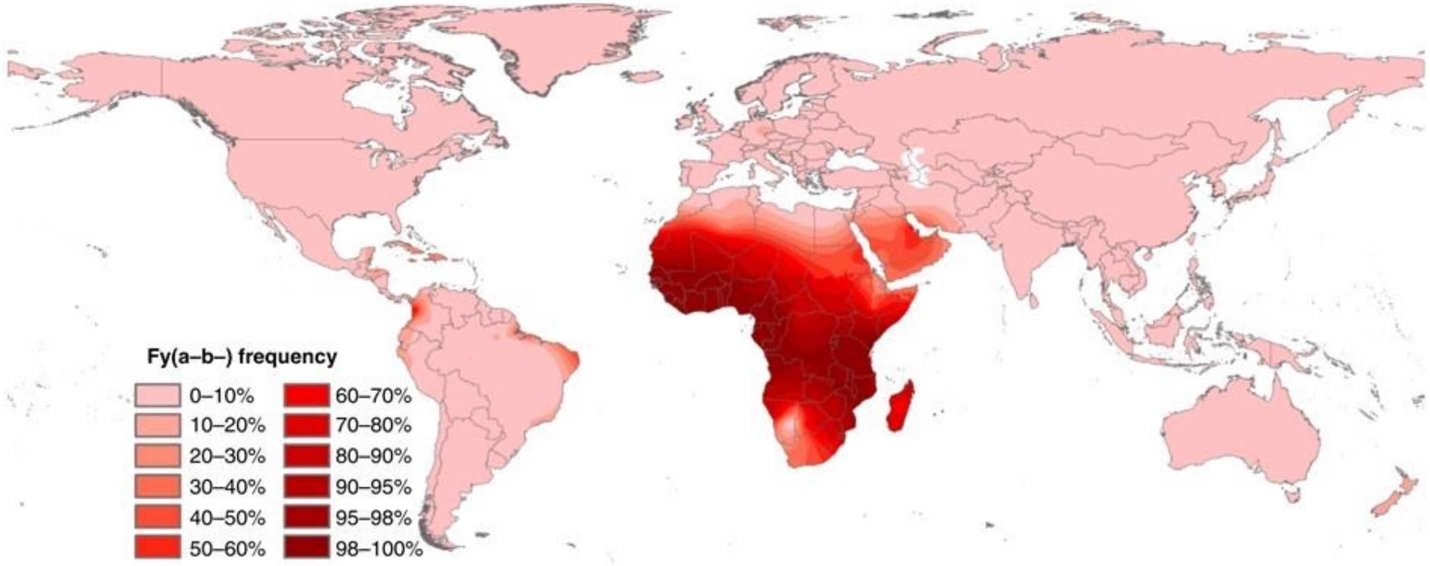




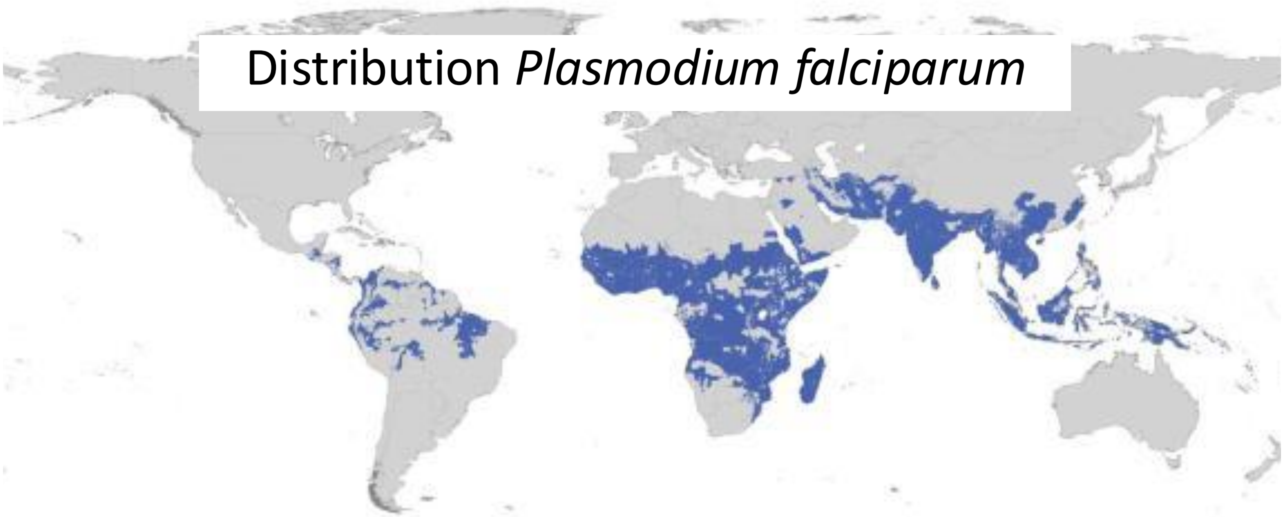
Piel et al 2010. *Nature Communications.*

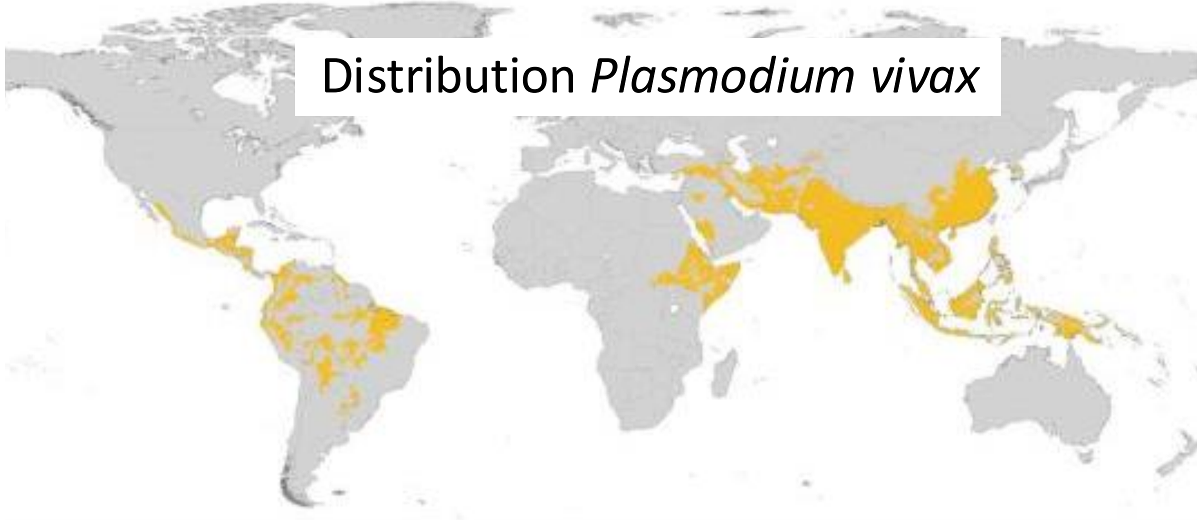
Parasites and pathogens have also shaped human DNA.

Duffy antigen



- Modeled distribution of Duffy-negative human population





Guerra et al. 2006. *Trends in Parasitology*
Howes et al 2011. *Nature Communications.*

Epidemiology vs. Disease Ecology

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- Disease Ecology = the study of **how** a disease spreads
  - Emphasis on the **interactions** of organisms with each other and the environment…when interactions result in disease

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  - Including chronic diseases!
- Often uses **cross-sectional** data to demonstrate associations of variables with outcome (disease)

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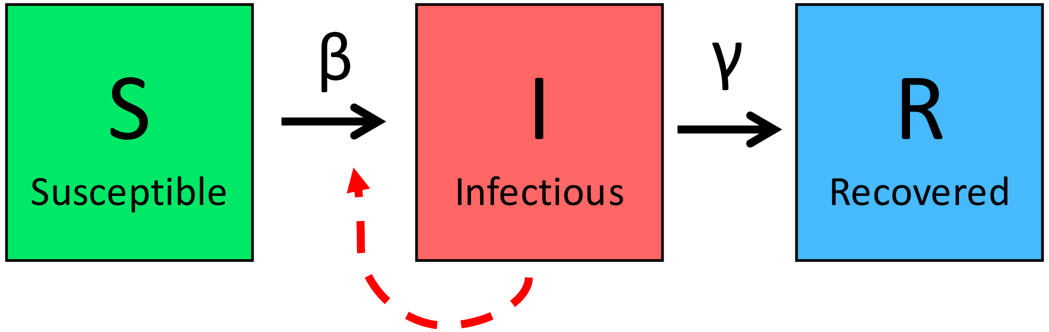
- Emphasis on understanding the **transmission dynamics** of infectious disease
- Typically involves fitting dynamical (population) models to **time series** data

Epidemiology vs. Disease Ecology

- Epidemiology = “the study of **what** is on the people”
  - Coined by Spanish physician Villalba in 1802
- Emphasis on the study and analysis of the **distribution and determinants** of health and disease (“risk factors”)
  - Including chronic diseases!
- Often uses **cross-sectional** data to demonstrate associations of variables with outcome (disease)
- More statistical (y=mx+b)
  - **Pattern**

- Disease Ecology = the study of **how** a disease spreads
  - Emphasis on the **interactions** of organisms with each other and the environment…when interactions result in disease
- Emphasis on understanding the **transmission dynamics** of infectious disease
- Typically involves fitting dynamical (population) models to **time series** data
- More mathematical (dN/dt)
  - **Process**

The SIR Model

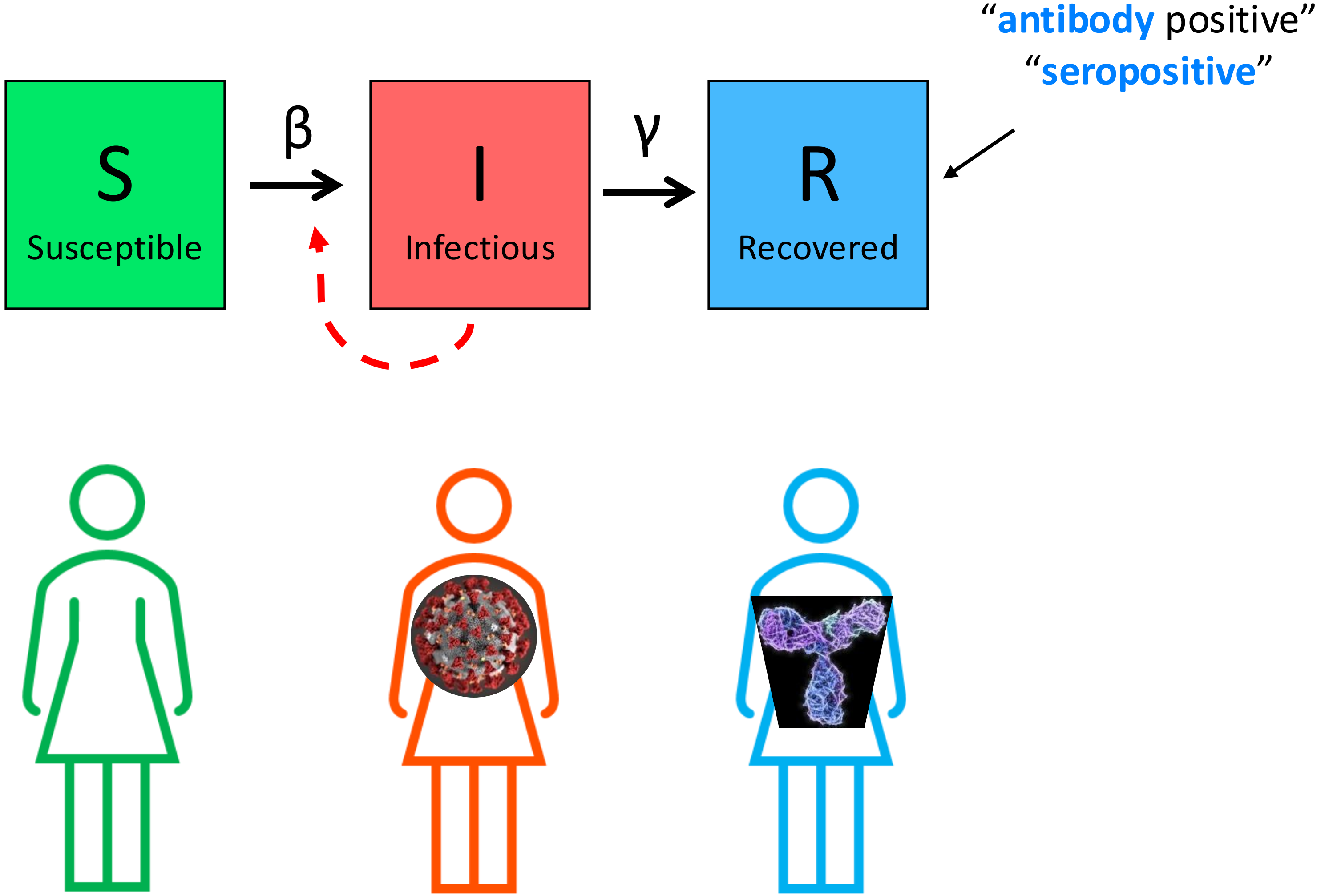


β = transmission rate

γ = recovery rate

Kermack and McKendrick 1927 *Proc Roy Soc A*

We class hosts into categories of **susceptible**, **infectious**, and **recovered** to model **pathogen dynamics**.



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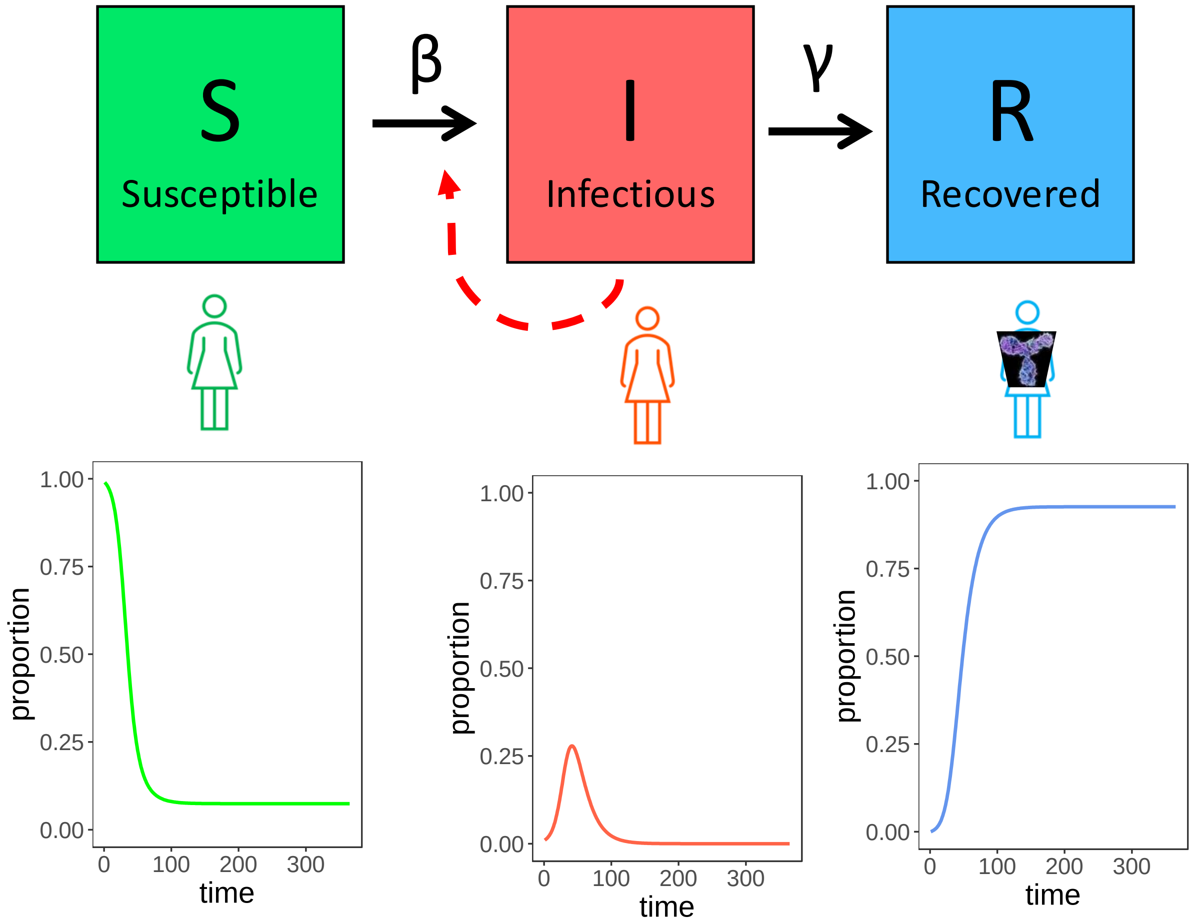
We use computers to simulate systems of equations in the SIR framework.

$$\frac{d\mathbf{S}}{dt} = -\beta \mathbf{S}\mathbf{I}$$

$$\frac{d\mathbf{I}}{dt} = \beta \mathbf{S}\mathbf{I} - \gamma \mathbf{I}$$

$$\frac{d\mathbf{R}}{dt} = \gamma \mathbf{I}$$



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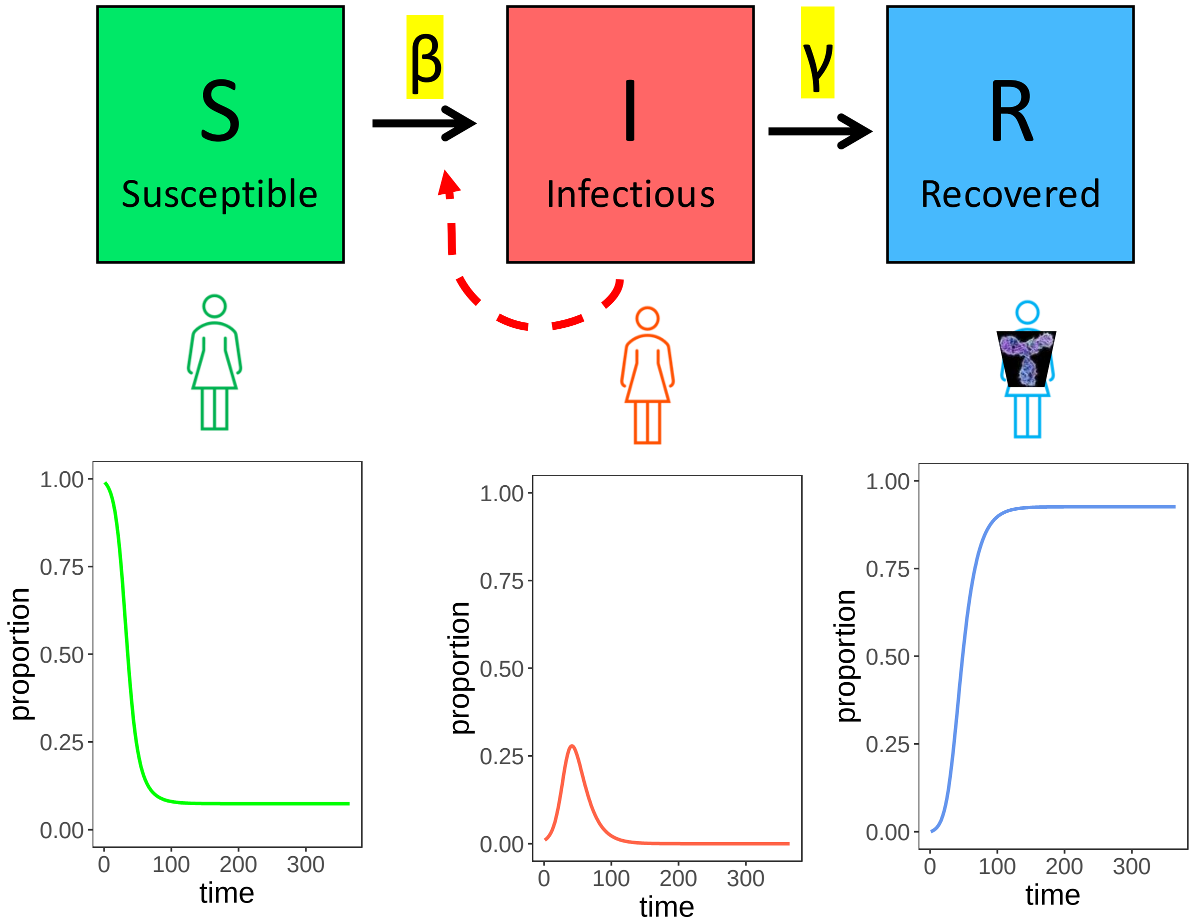

$$\mathbf{R_0} = \frac{\boldsymbol{\beta}}{\boldsymbol{\gamma}}$$

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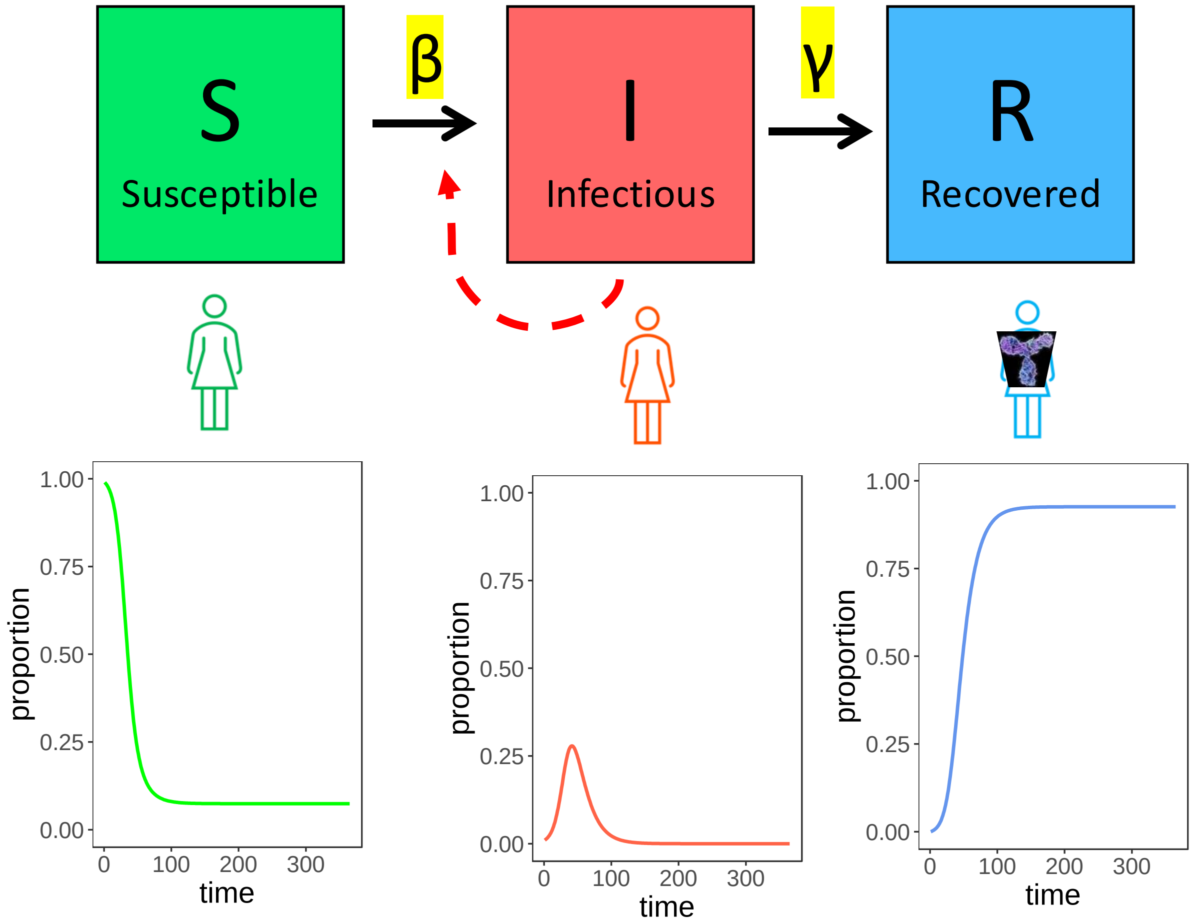

$$\mathbf{R_0} = \frac{\boldsymbol{\beta}}{\boldsymbol{\gamma}}$$

β } infections created

γ } infections lost

R_0 must be >1 for a disease to start spreading!

β = transmission rate

γ = recovery rate

R_o

- The **basic reproduction number** for a pathogen

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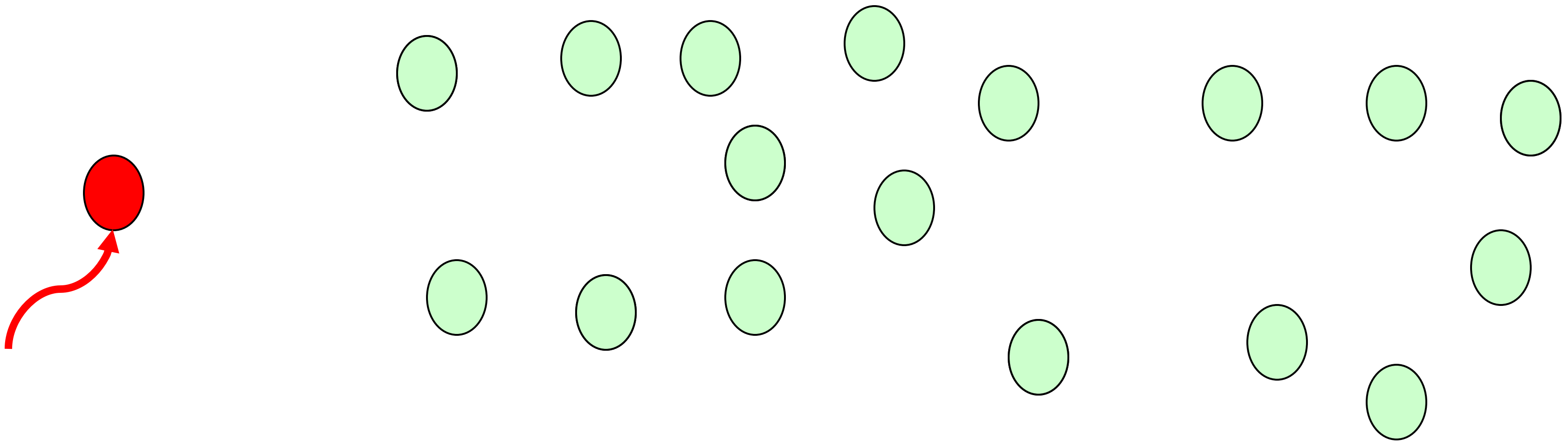

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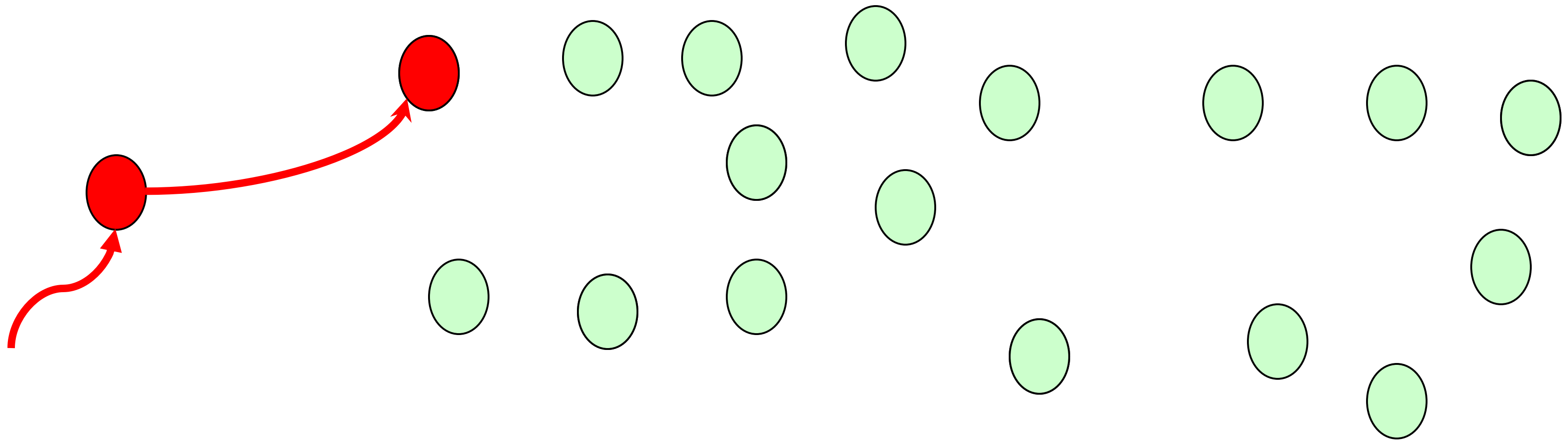

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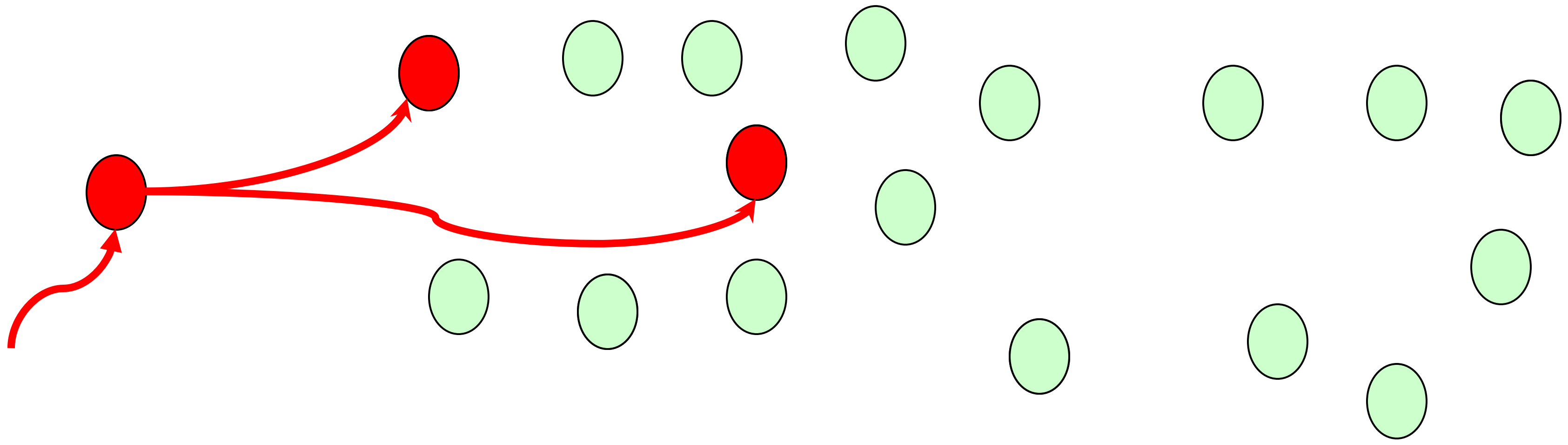

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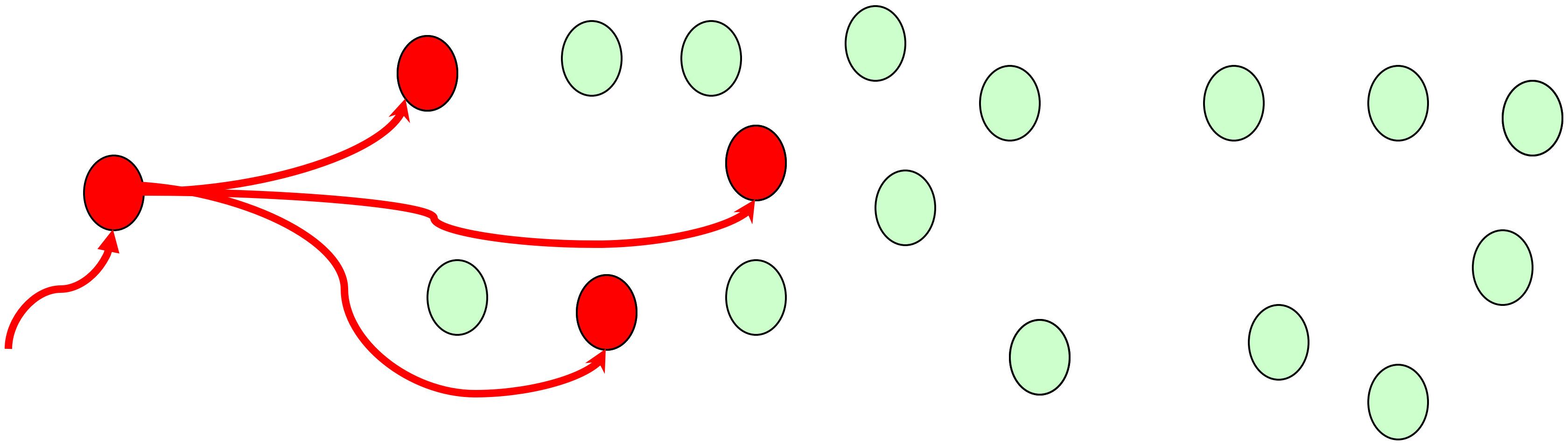

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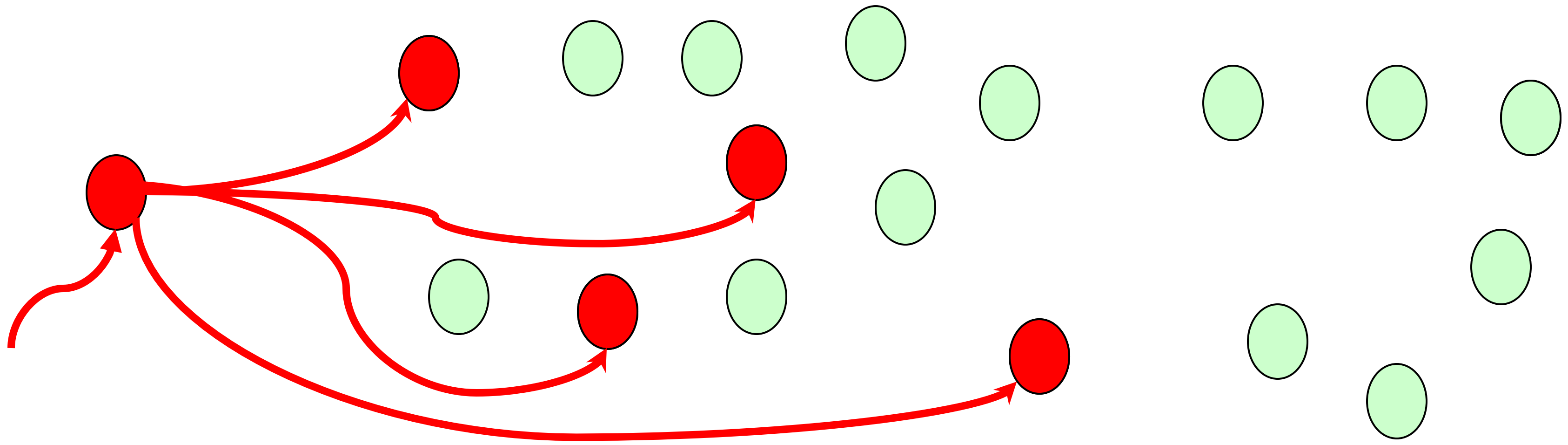

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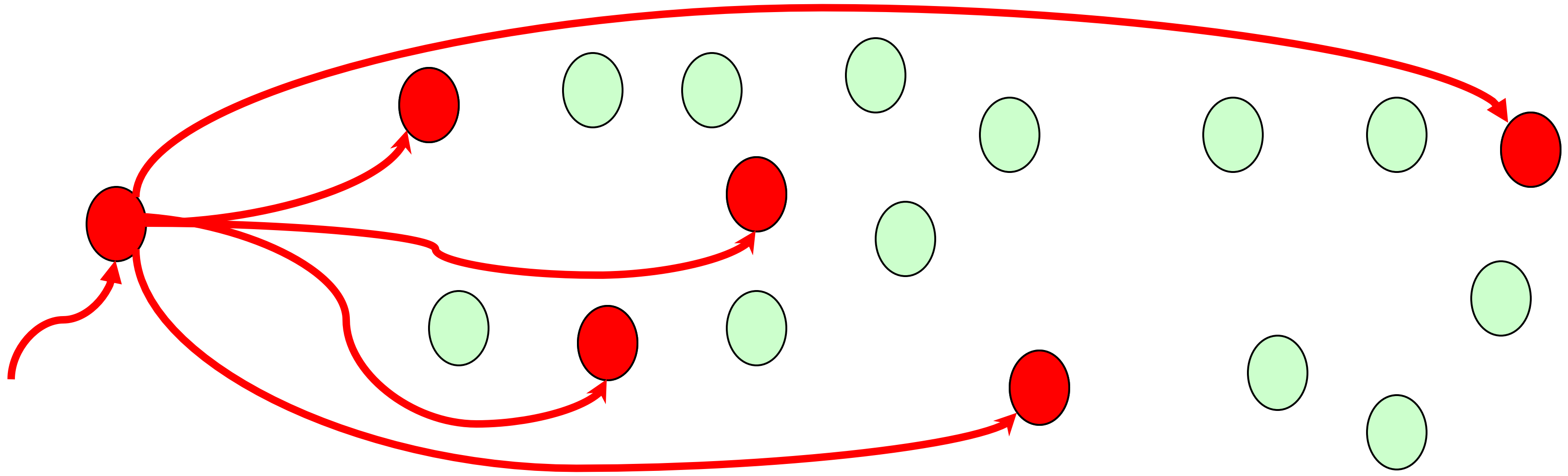

What is R_o?

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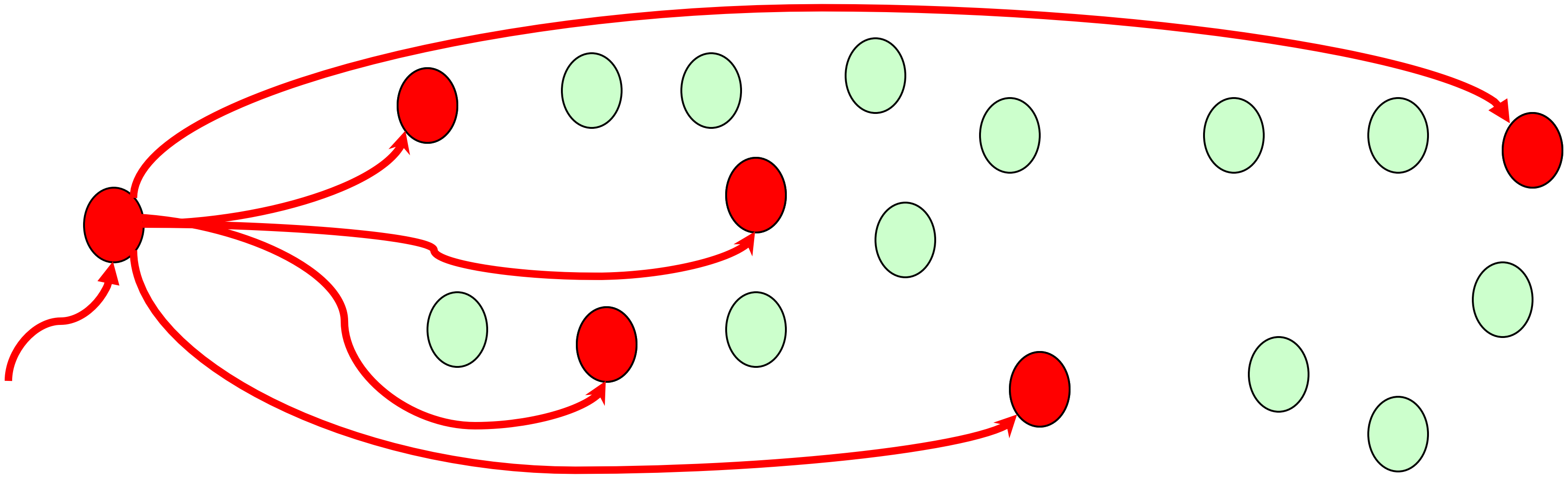


What is R_o?

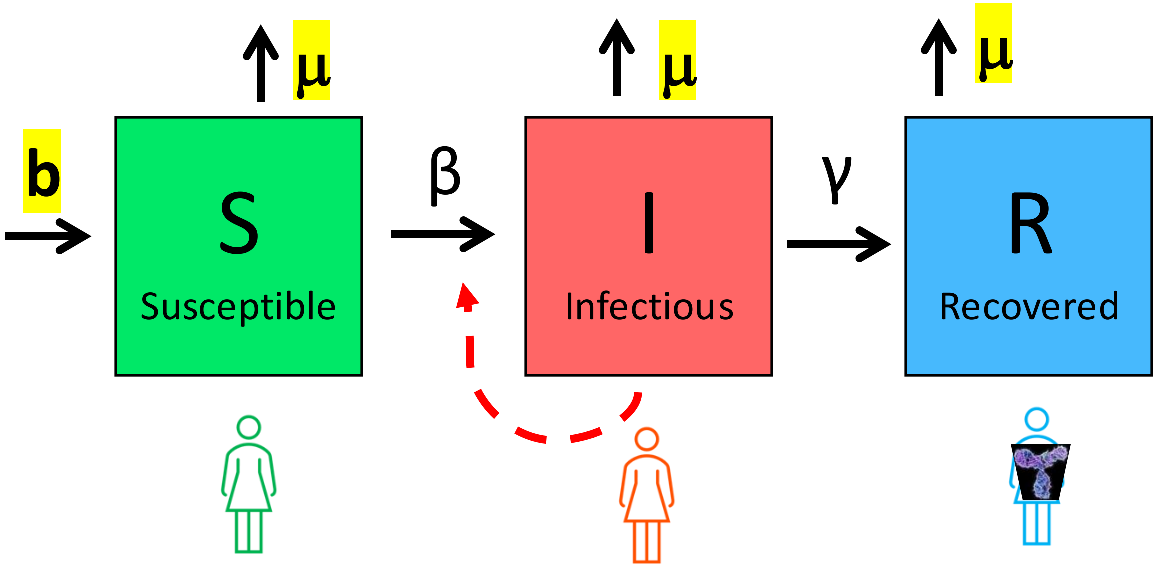
R_o = 5

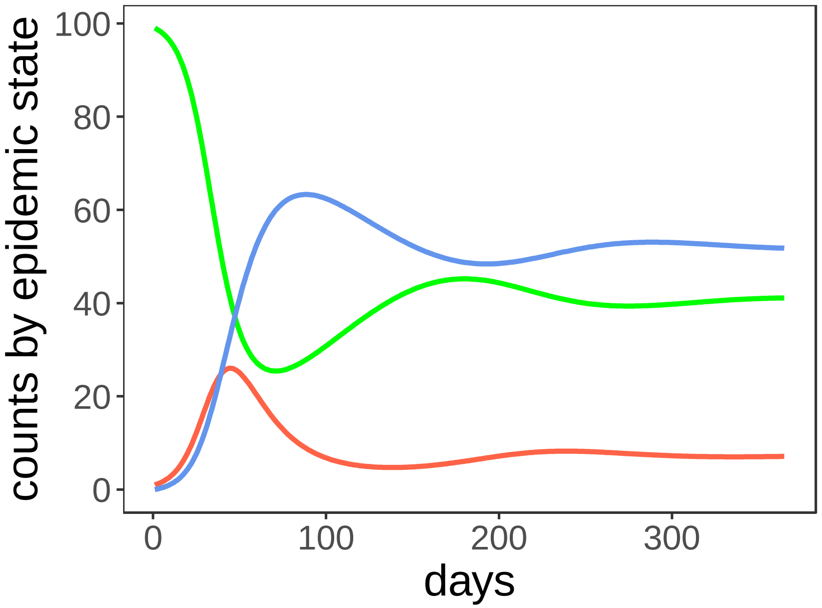
We can add realism to our models with births and deaths to **maintain** endemic pathogens.

$$\frac{dS}{dt} = b(S + I + R) - \beta SI - \mu S$$

$$\frac{dI}{dt} = \beta SI - \gamma I - \mu I$$

$$\frac{dR}{dt} = \gamma I - \mu R$$





b = birth rate

μ = death rate

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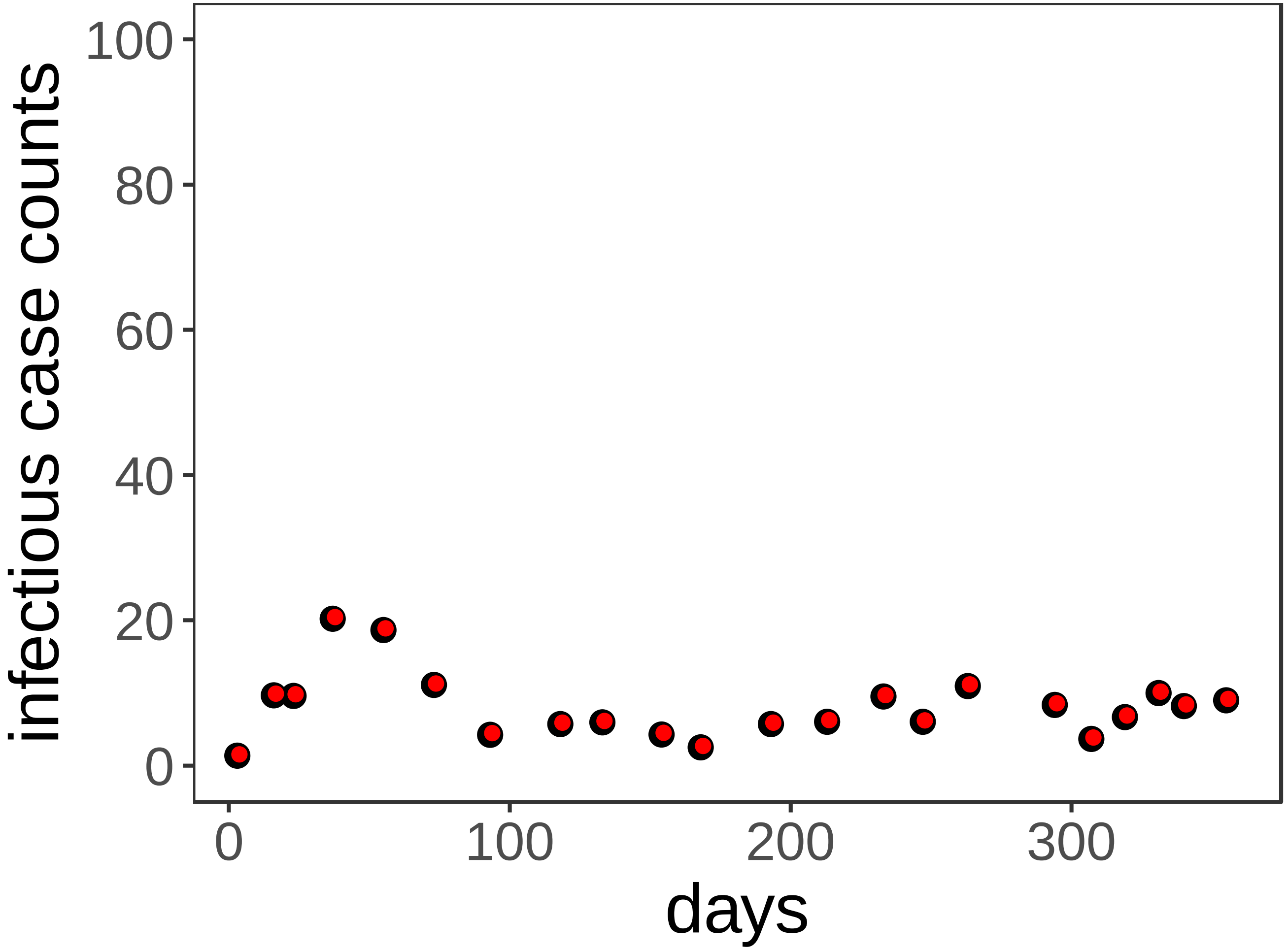
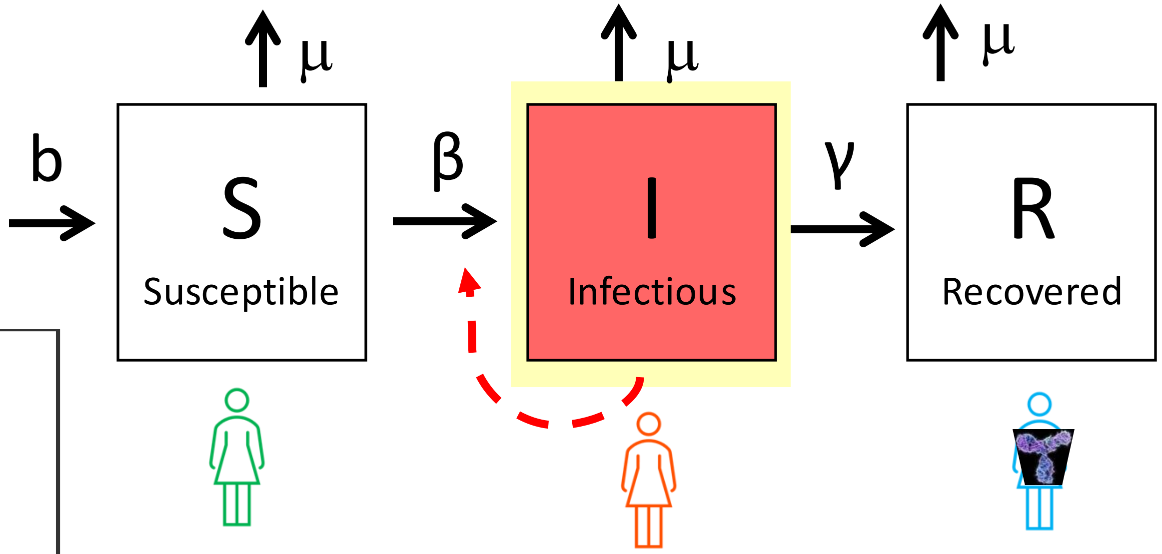
We can **estimate epidemic trajectories** by fitting SIR models to infectious case count data.





Anderson & May 1991. *Infectious Diseases of Humans*

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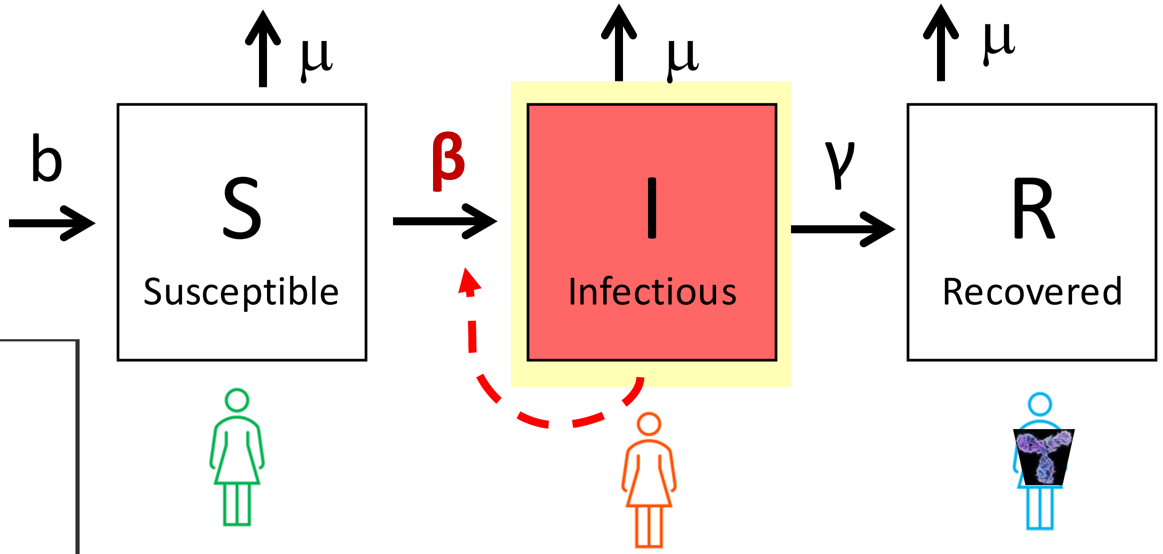

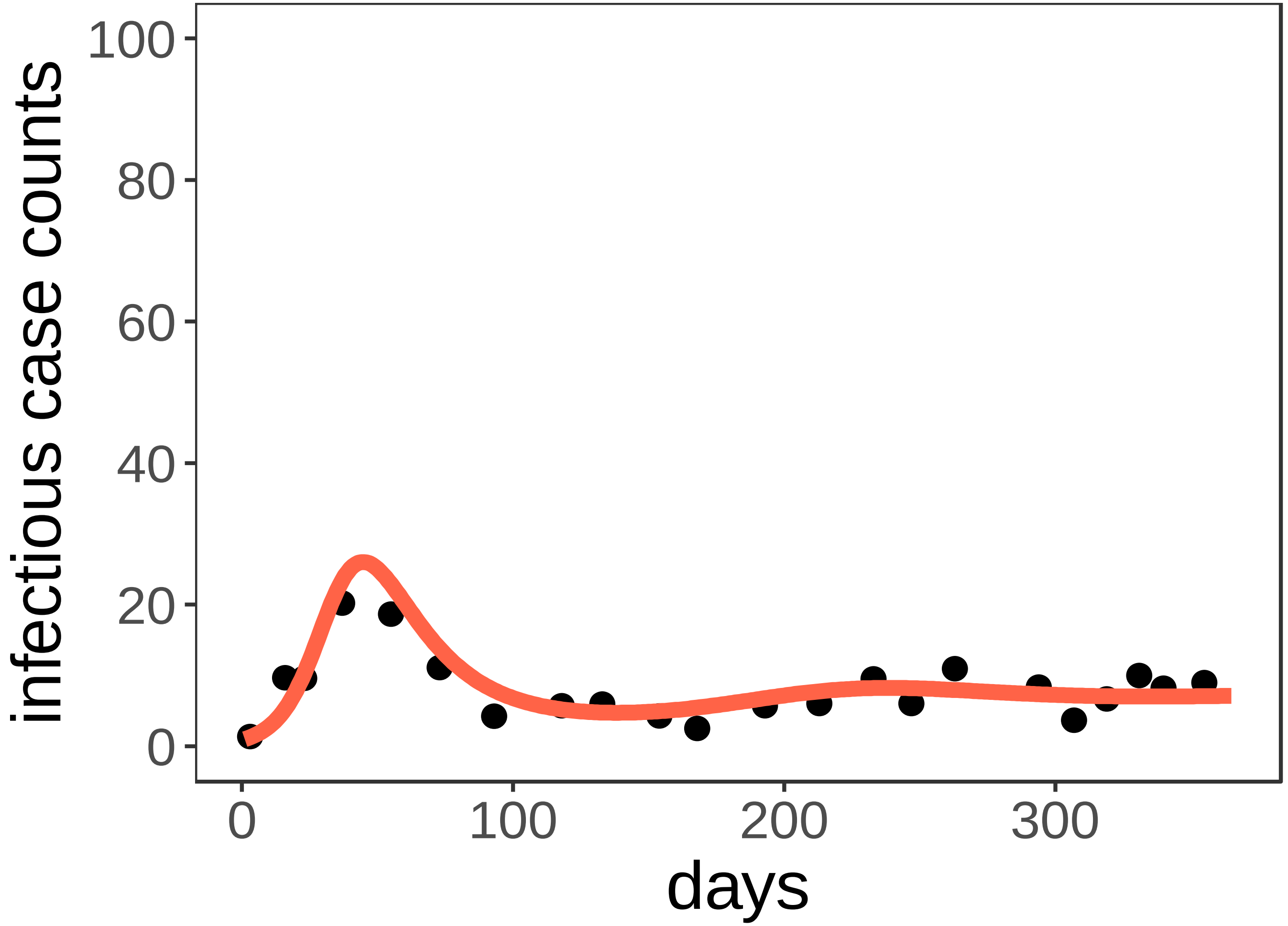

$$R_0 = \frac{\beta}{\gamma + \mu}$$

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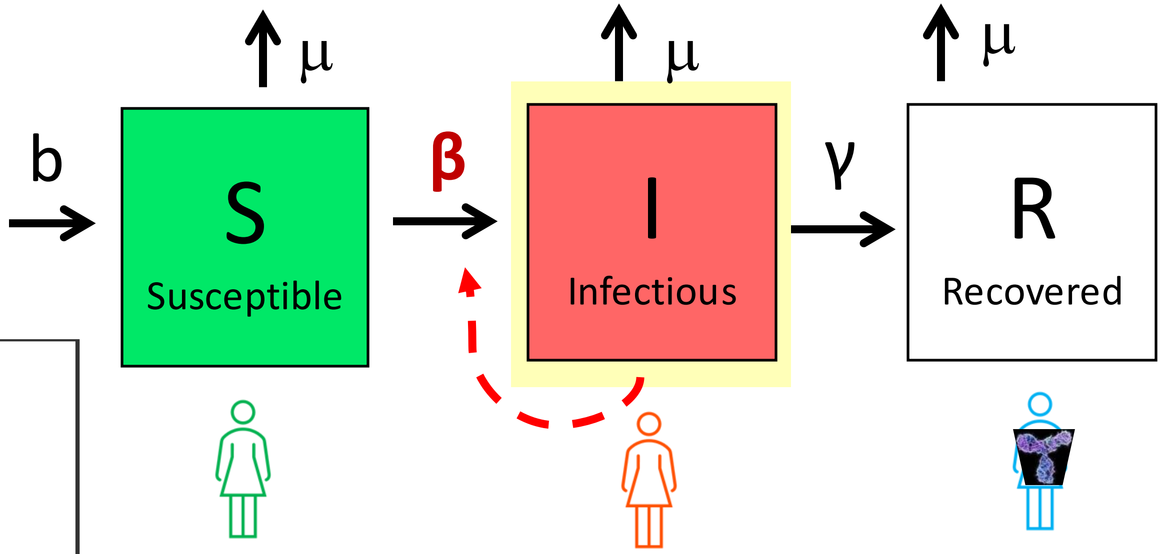

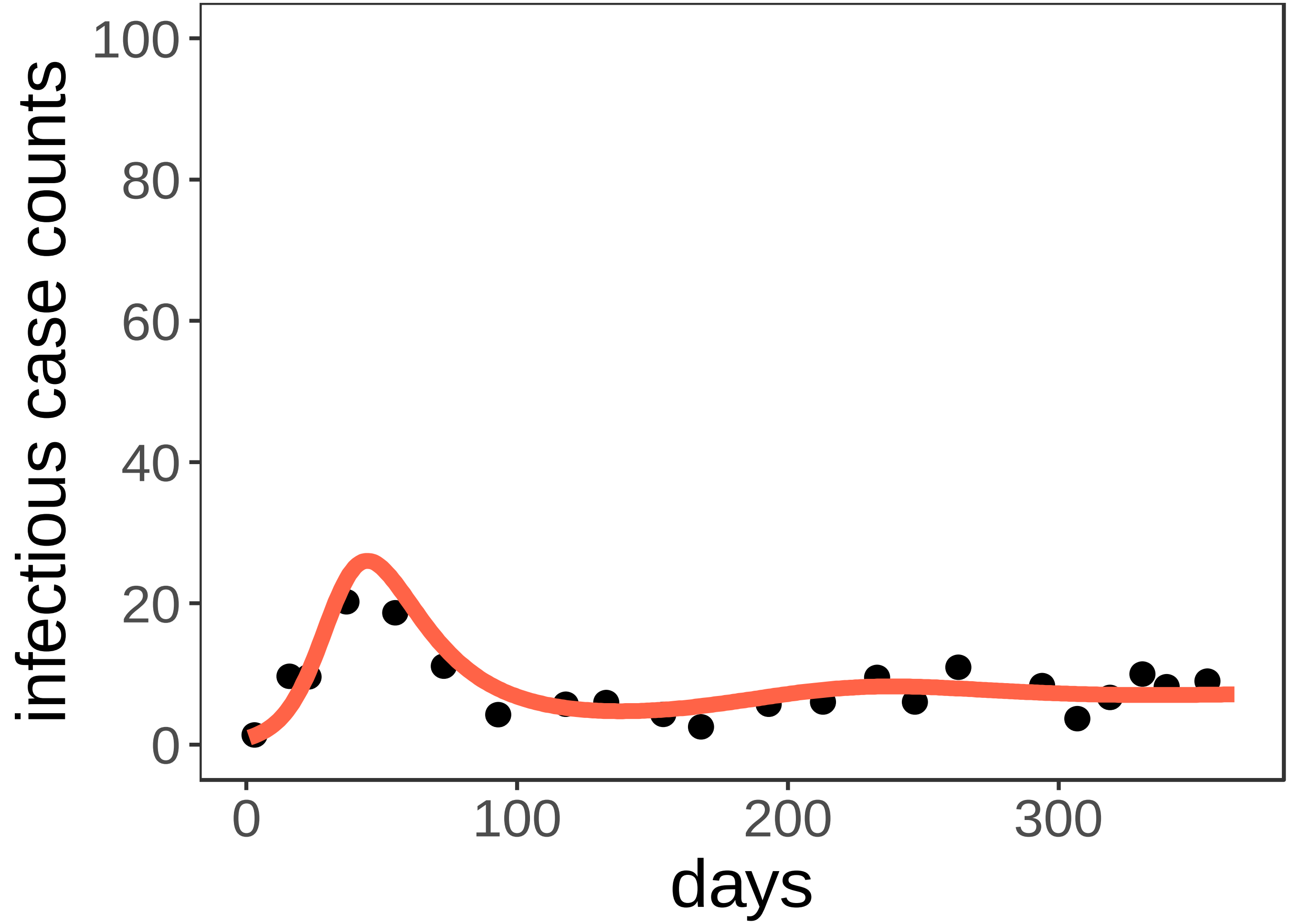

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$$R_E = R_0 \frac{S}{N}$$

$\approx \lambda$ *for a population model*

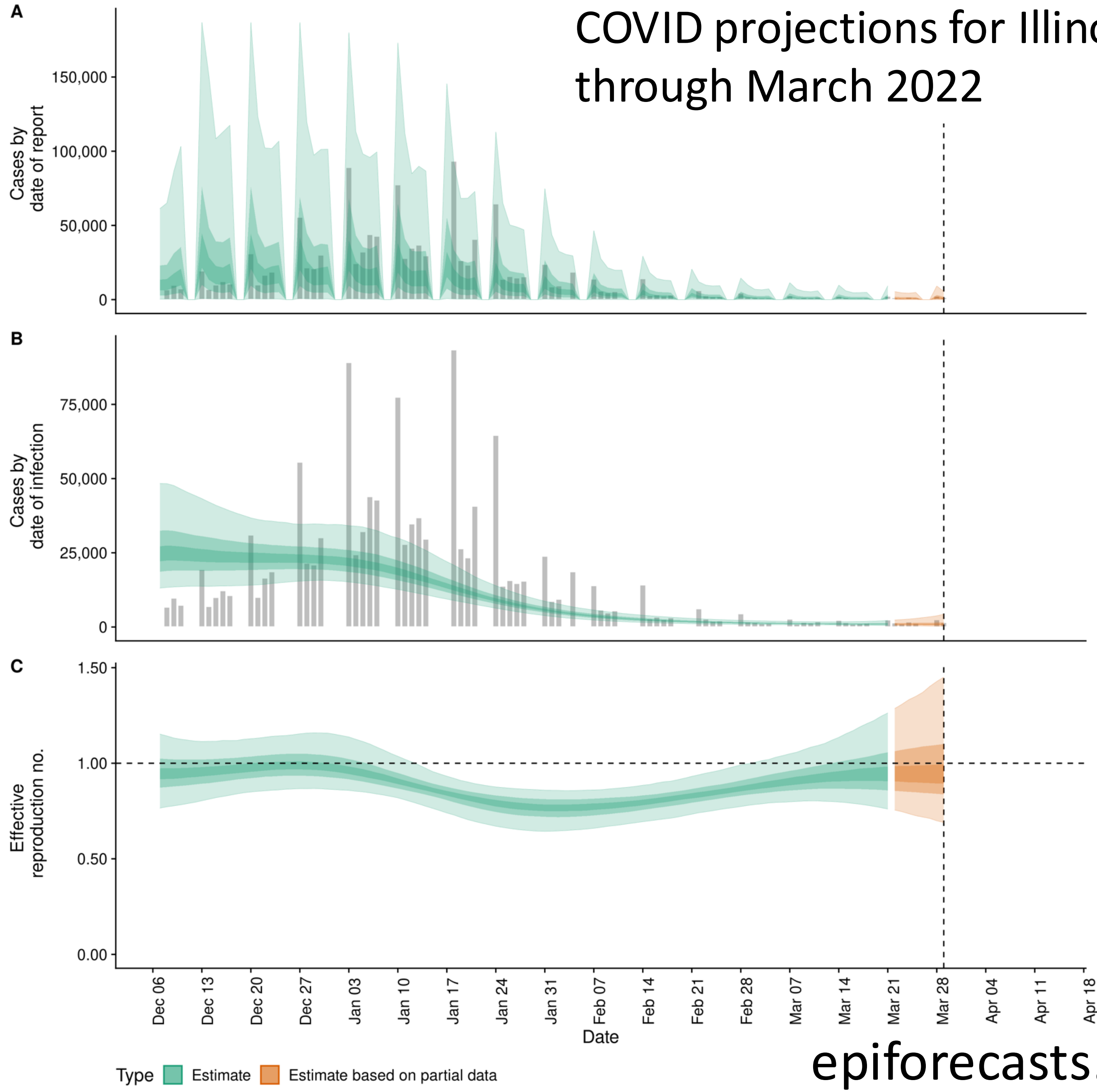
(epidemics spread @R_E>1 and decline @R_E<1)

Anderson & May 1991. *Infectious Diseases of Humans*

R_E OR R_t

- The **effective reproduction number** for a pathogen

COVID projections for Illinois through March 2022



epiforecasts.io

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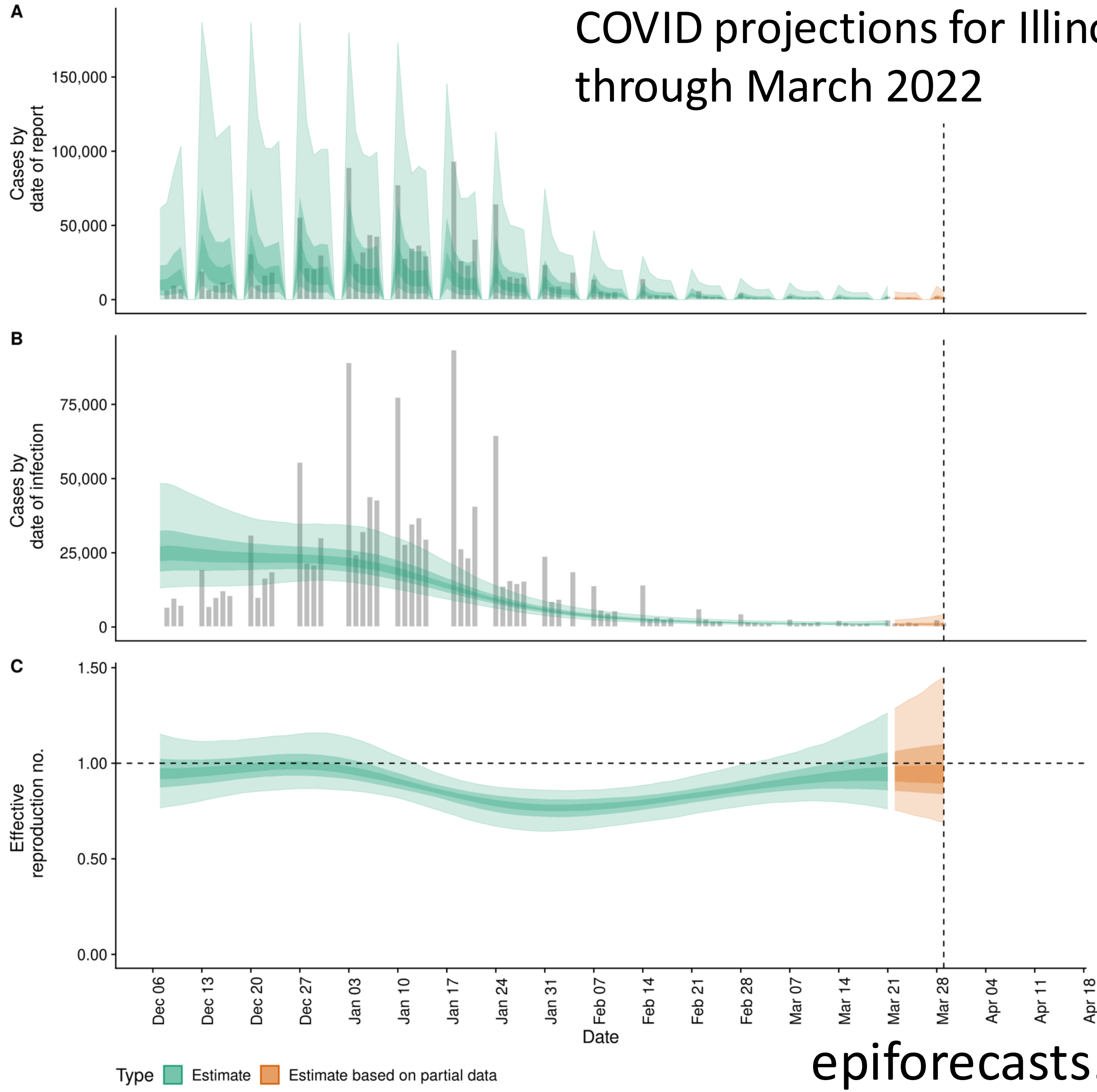

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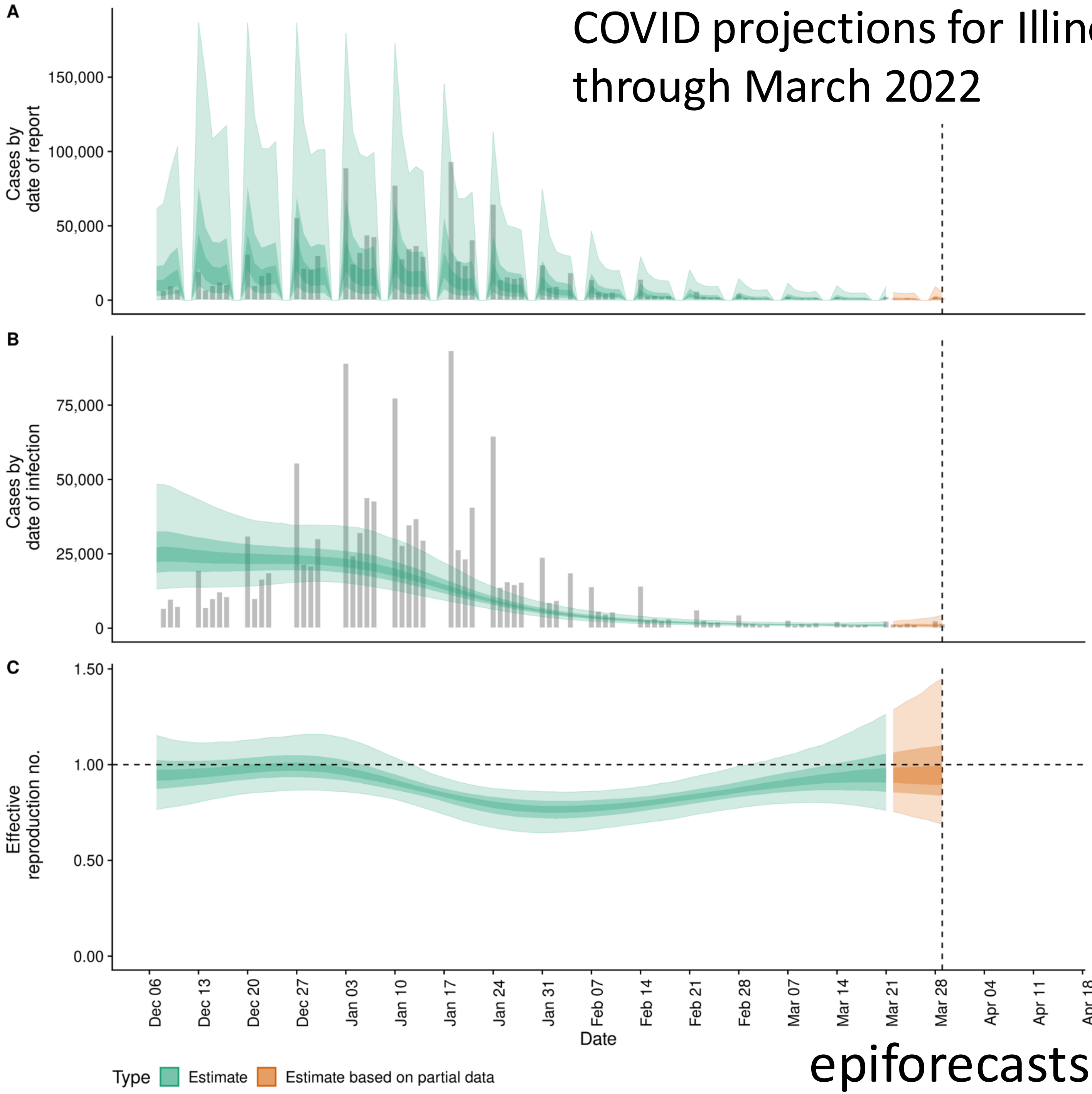

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- Gives a realistic pulse of the current pace of the epidemic!

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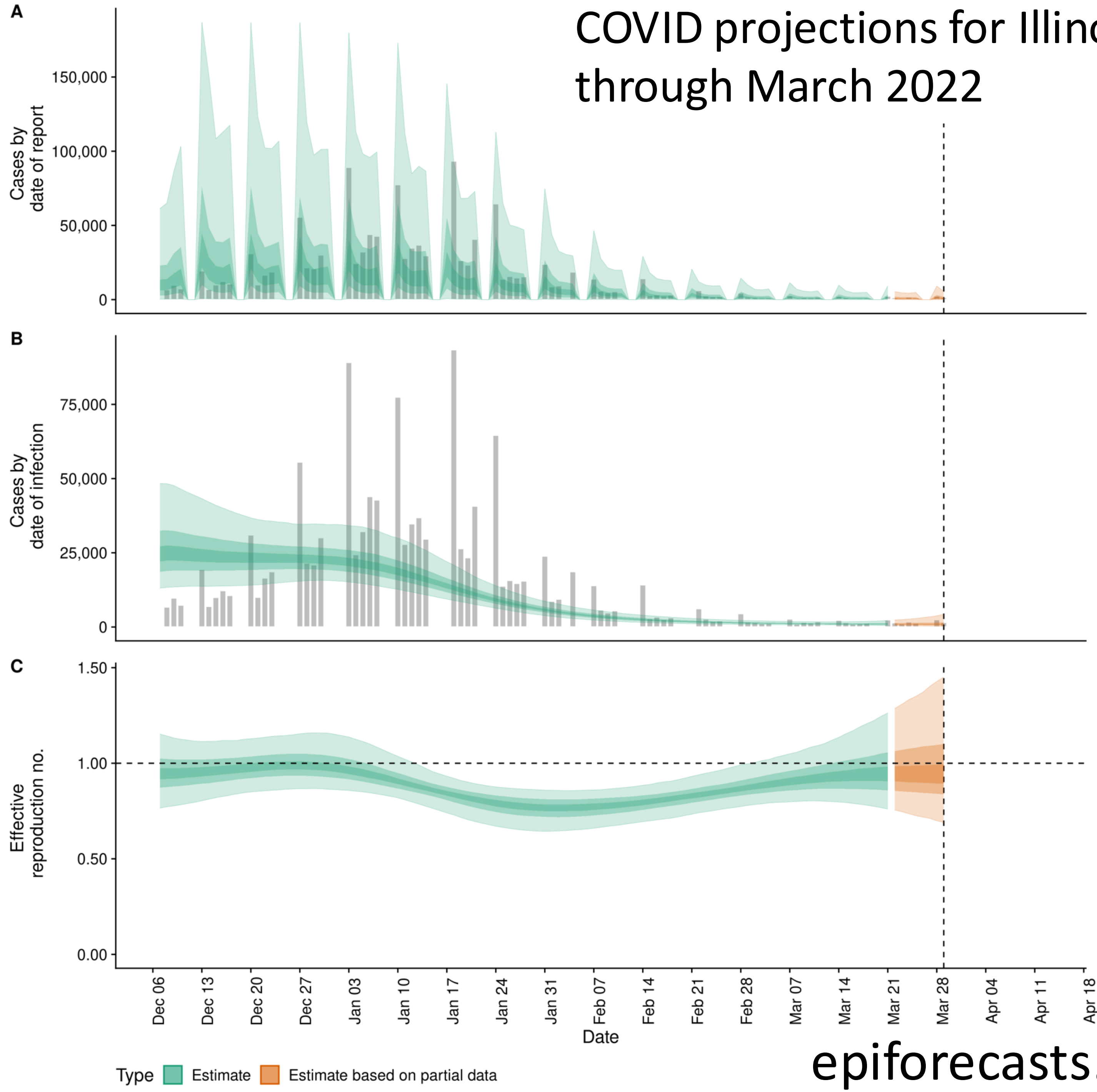

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No matter the dynamics, not everyone gets infected before the epidemic ends!

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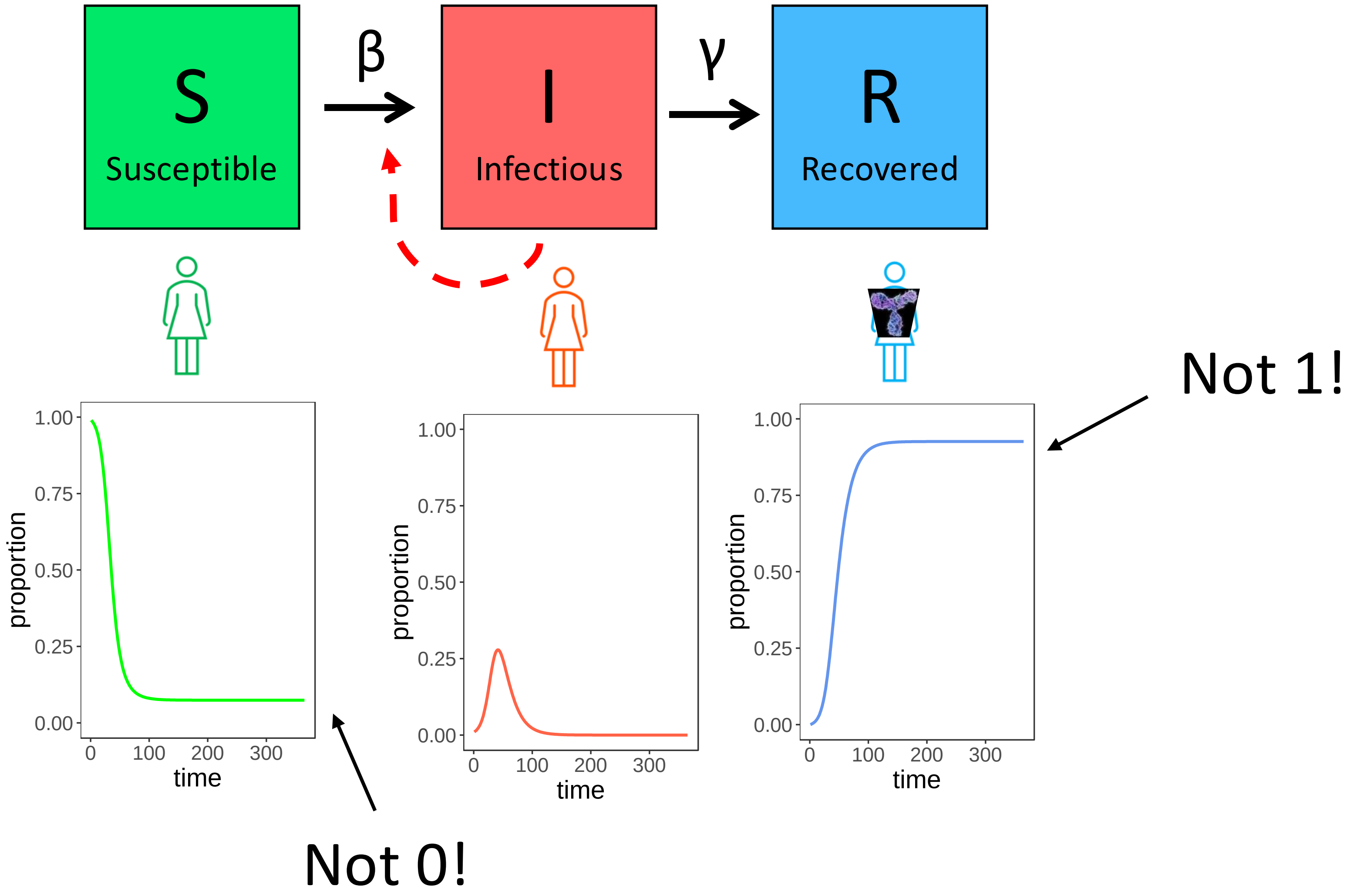

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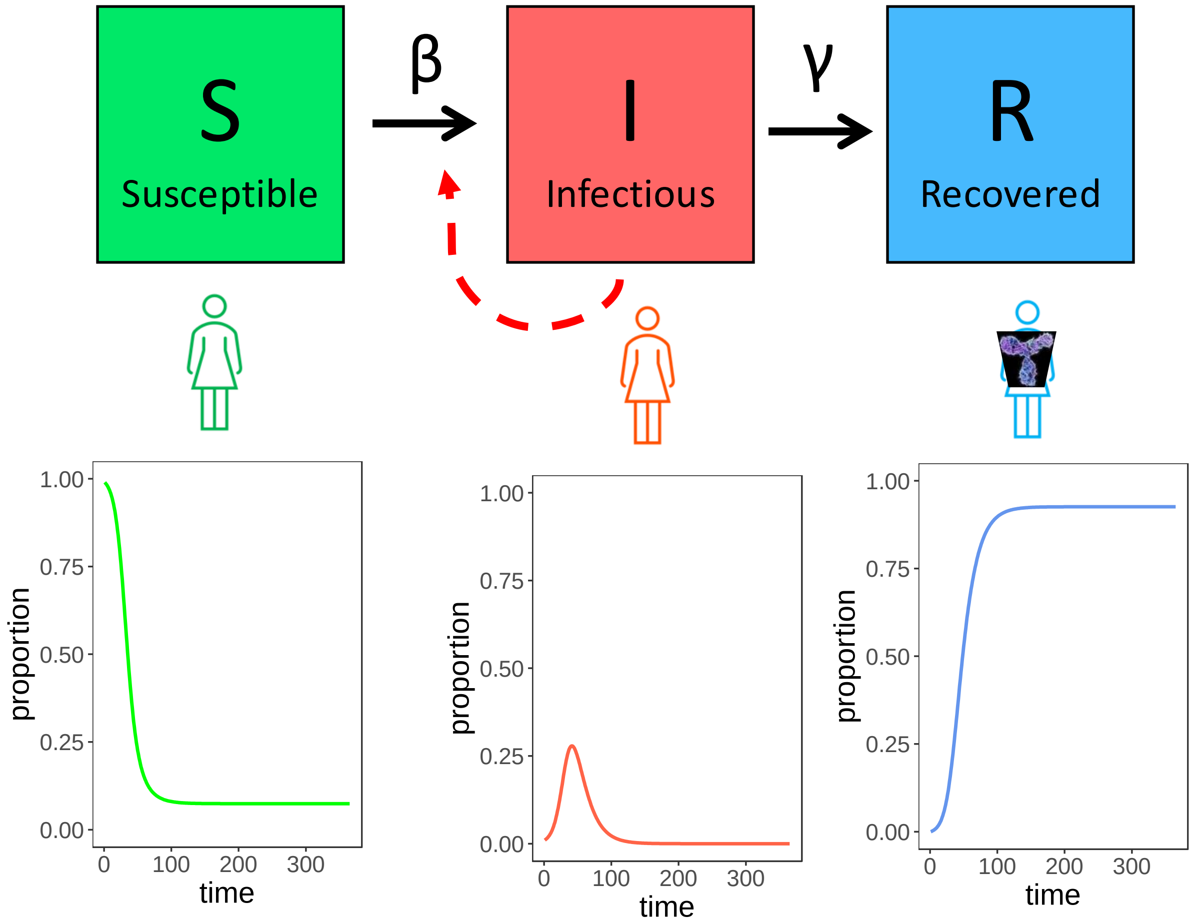

- The epidemic does not end because all individuals have been infected and have either died or recovered.
- Rather, finding new susceptibles becomes more difficult and $R_E < 1$

β = transmission rate

γ = recovery rate

Public health interventions can be employed to reduce both R_0 and R_E

$\boldsymbol{R_0}$ interventions

- Social distancing
- Masking
- Limits to gathering sizes
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- Goal: **Reduce $R_E < 1$** by removing individuals from the susceptible population.

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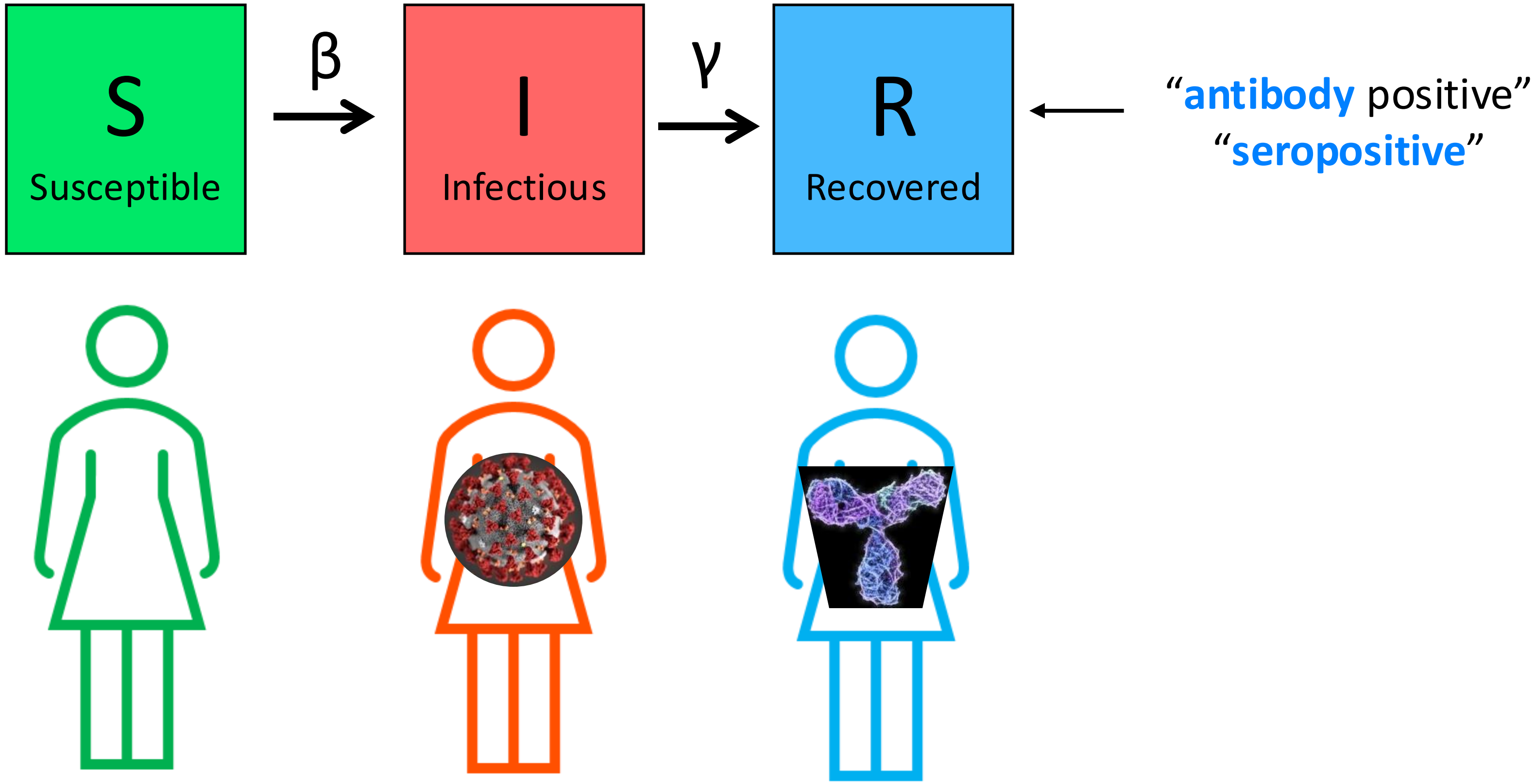

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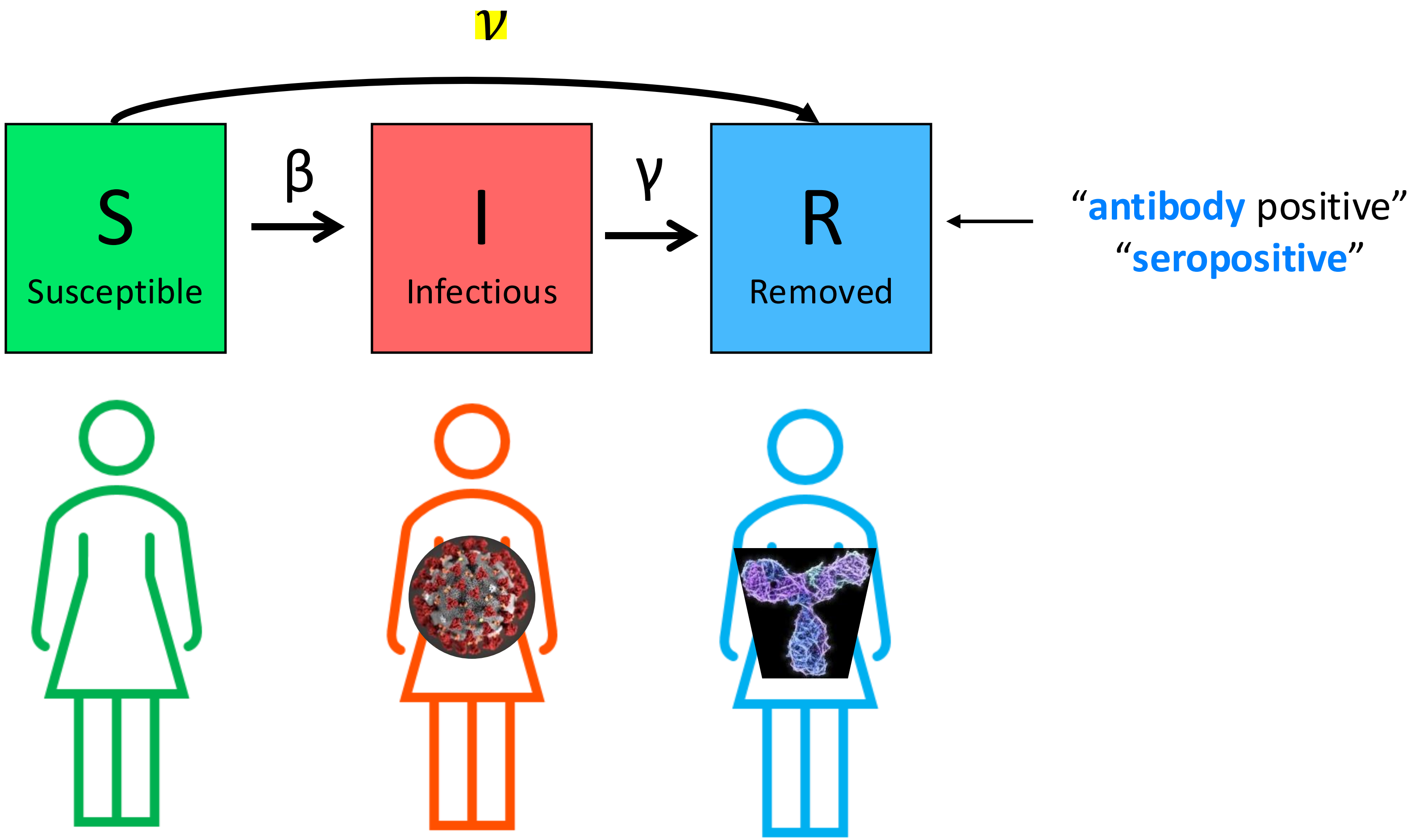

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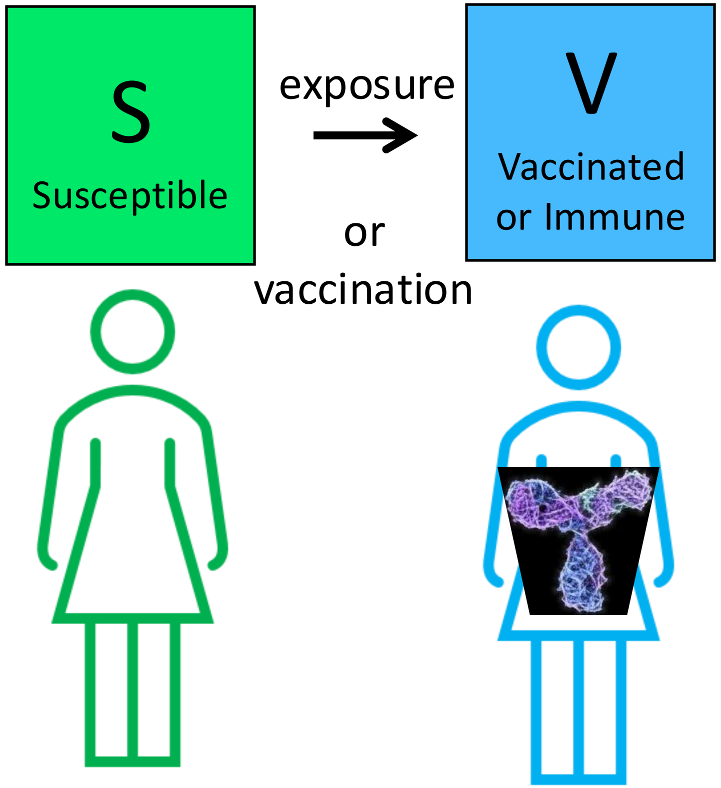

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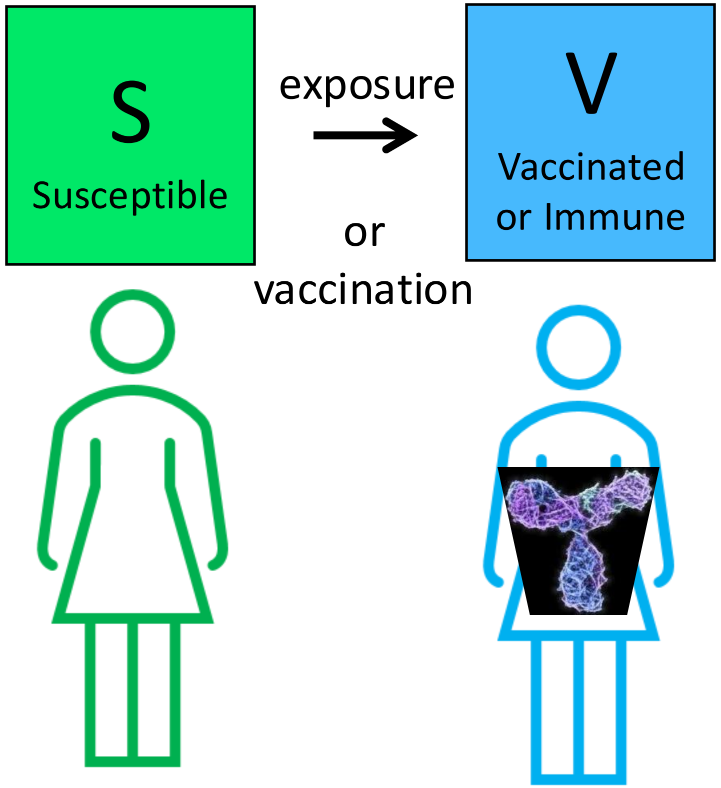

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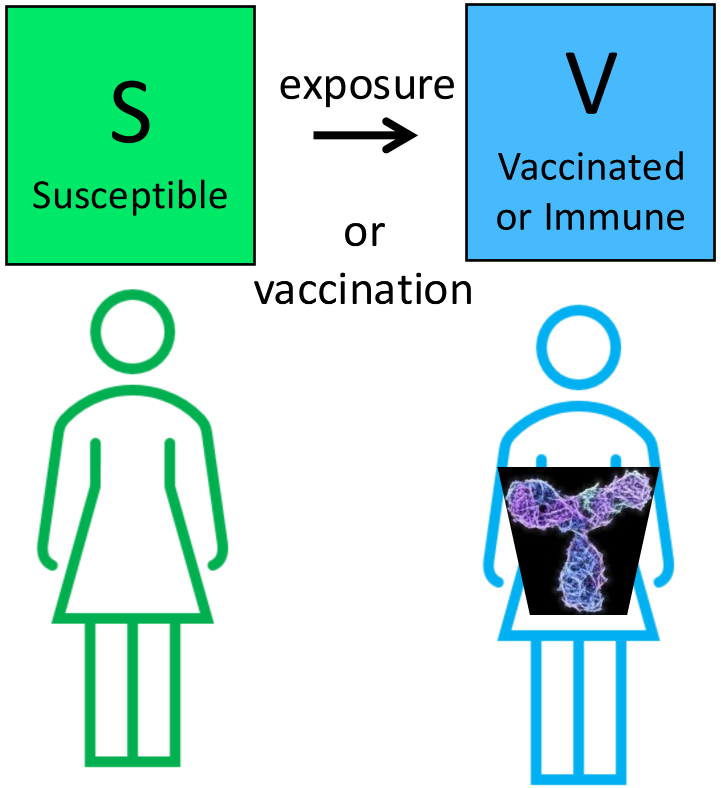

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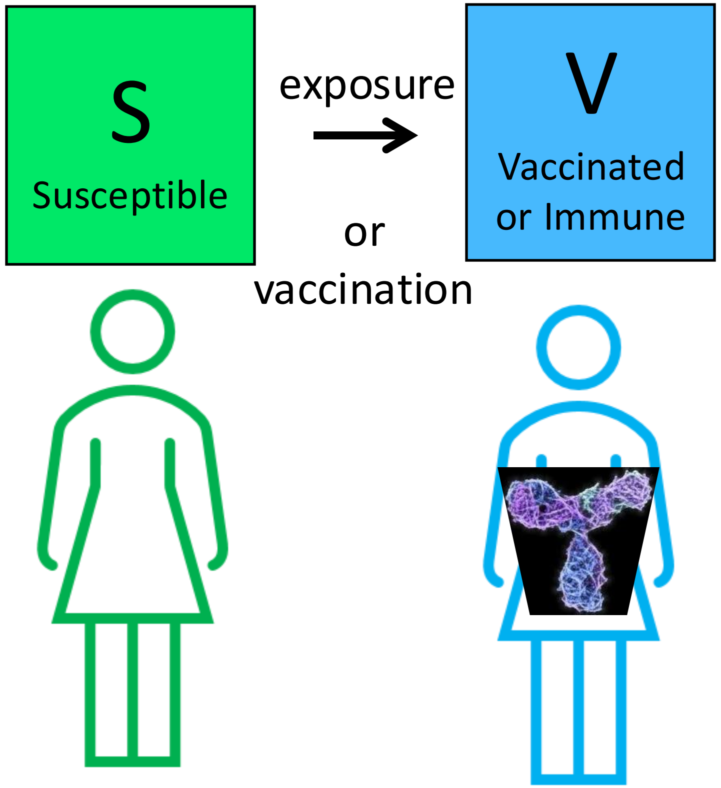

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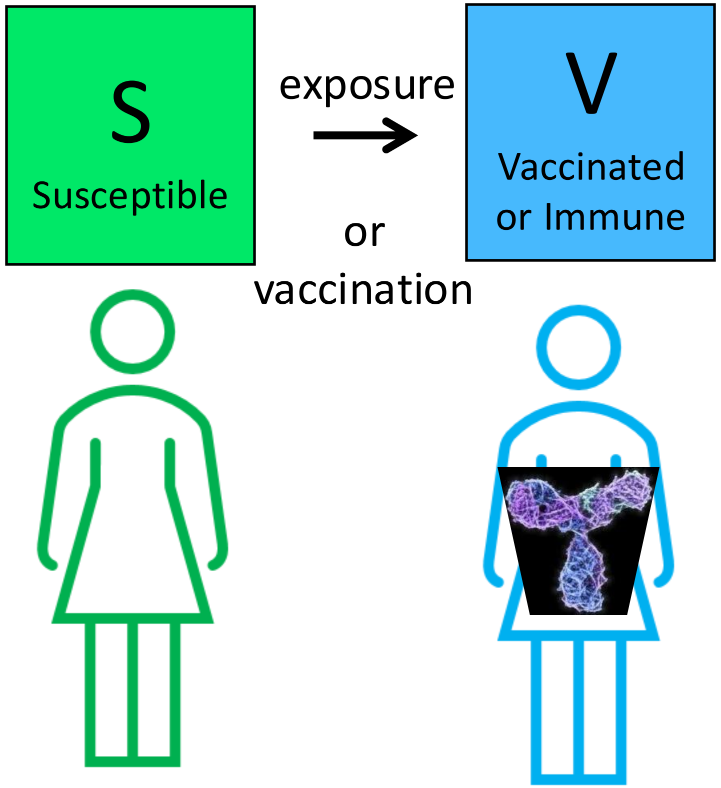

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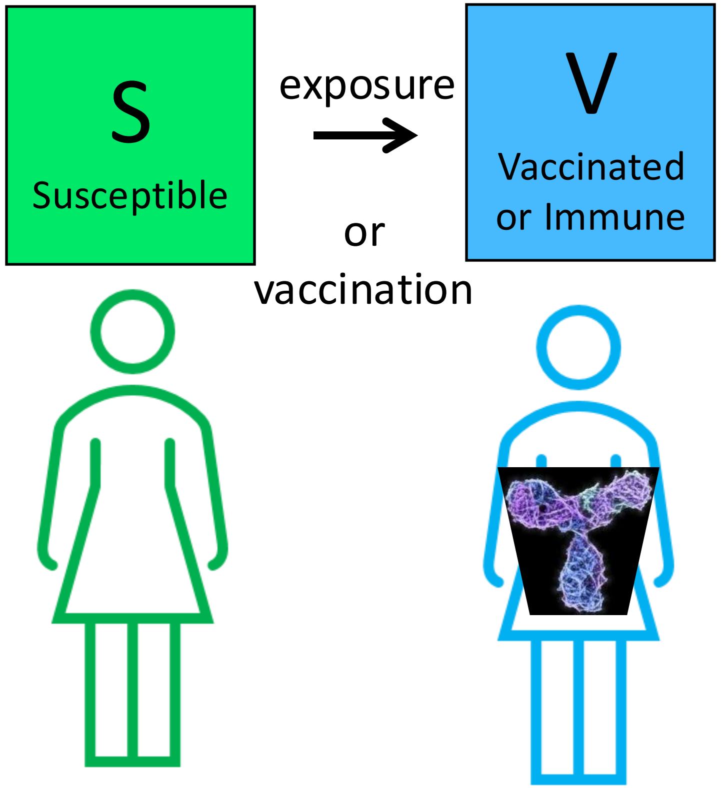

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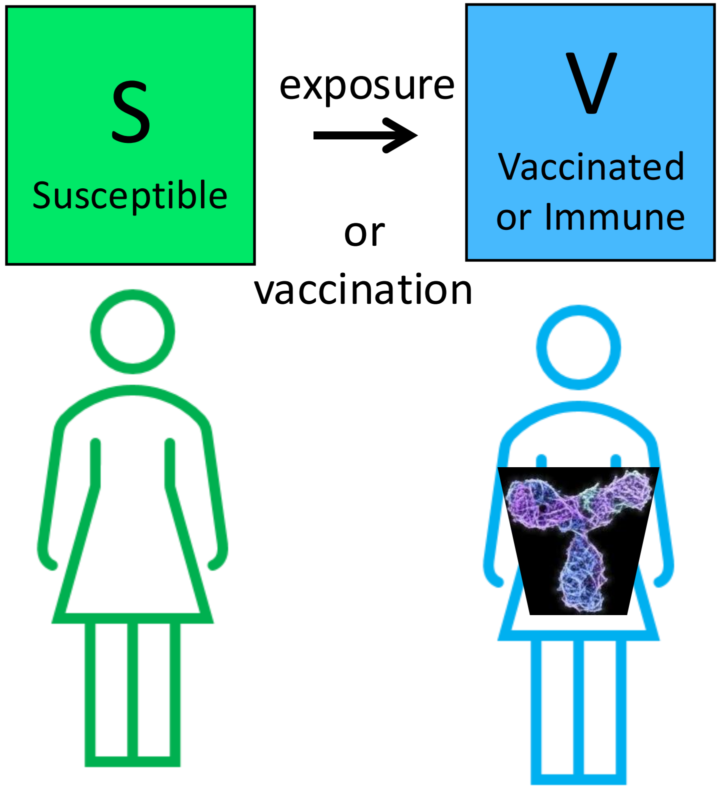
- **Even susceptibles will not become infected because the disease will not spread ($R_E < 1$).**



Extra

Population Biology

Conservation Biology	Disease Ecology

Population Biology

Conservation Biology

- **Goal:**
  - protect **populations** from **extinction**

Disease Ecology

Population Biology

Conservation Biology	Disease Ecology

- **Goal:**
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Population Biology

Conservation Biology	Disease Ecology

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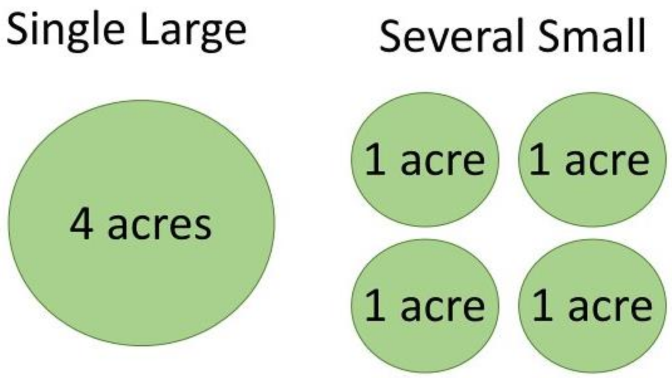

Population Biology

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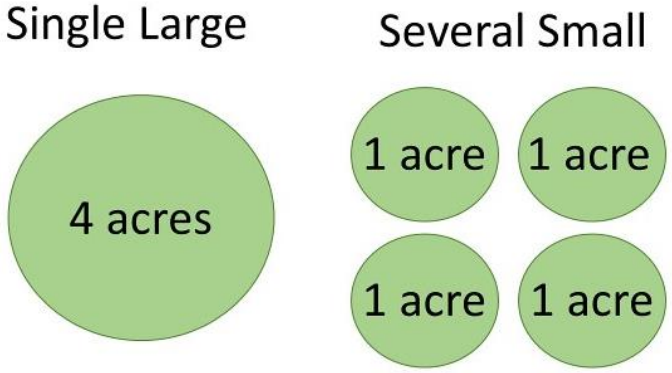

Disease Ecology

- **Goal:**
  - protect **populations** from disease via pathogen **extinction**

Population Biology

Conservation Biology

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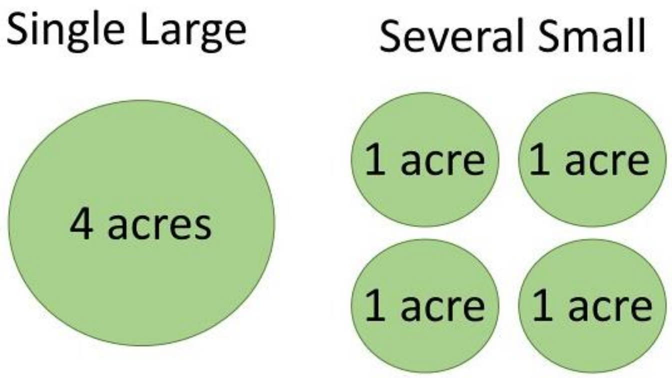

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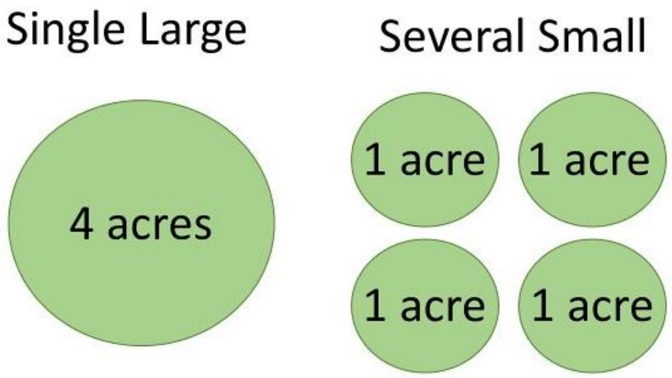

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- **Approach:**
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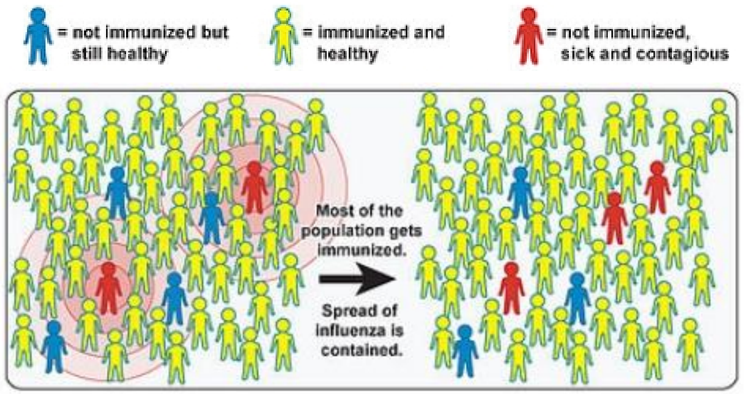
  - **vaccination**

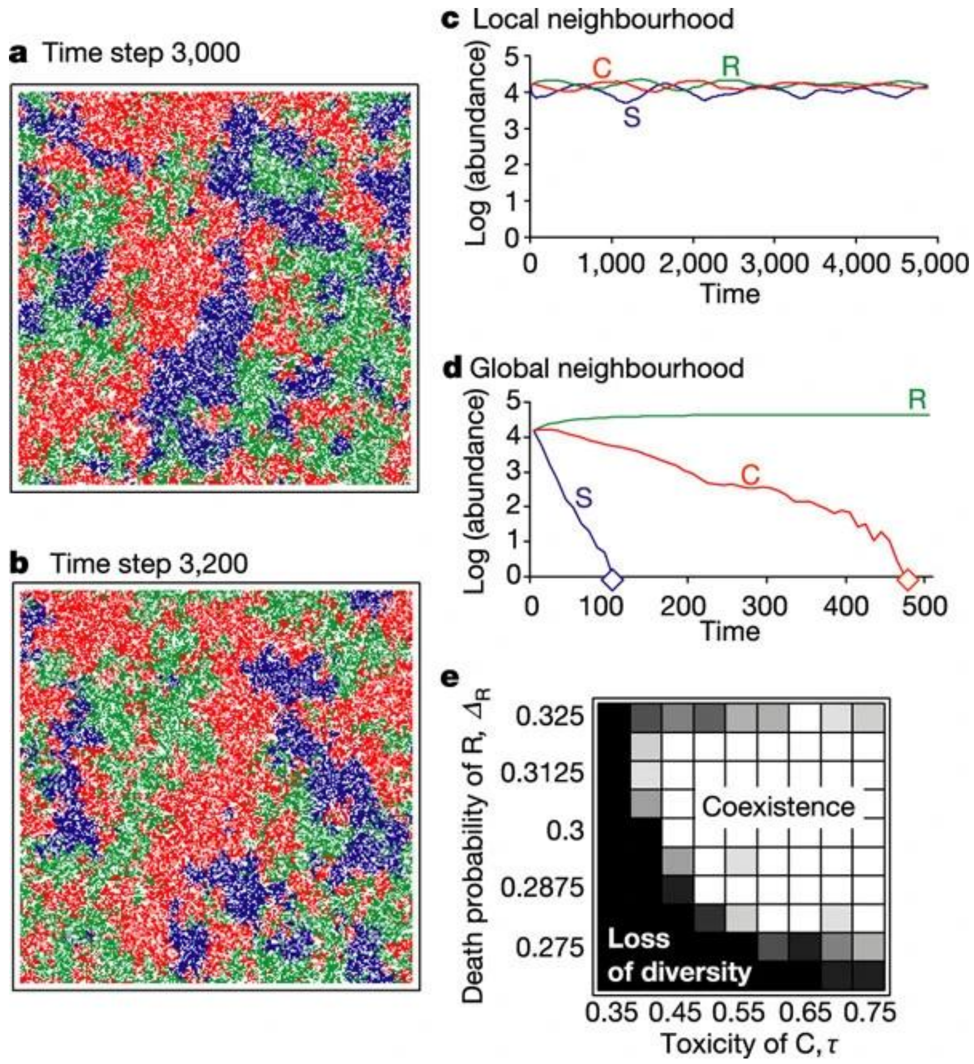


E^2M^2: Ecological and Epidemiological Modeling in Madagascar

December 16, 2022

E2M2.org

Sometimes **stochasticity** and **space** are all you need to ensure **coexistence** in a competitive environment



- Kerr et al. modeled a community of 3 strains of *E. coli* with **overlapping resource requirements** (C,R, and S), occupying distinct spatial patches in a metapopulation.

Kerr et al. 2002. *Nature.*

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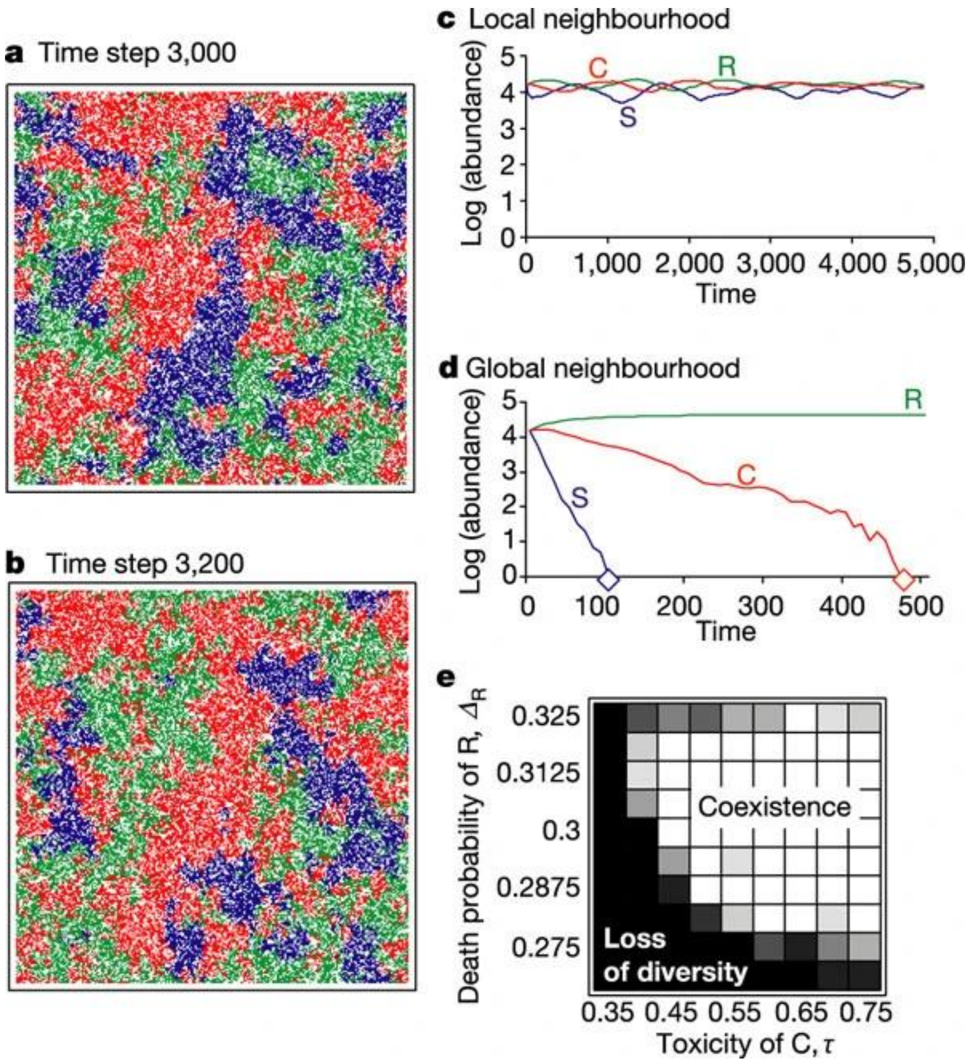

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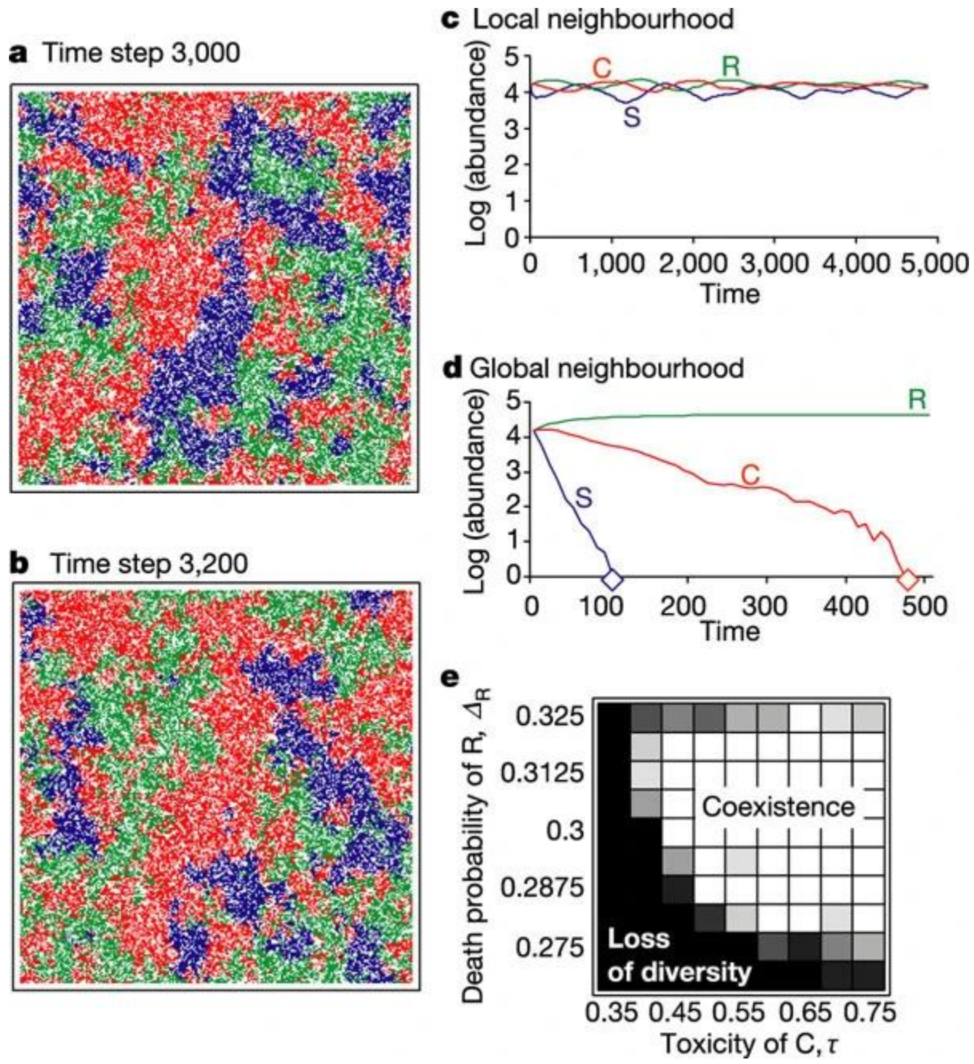

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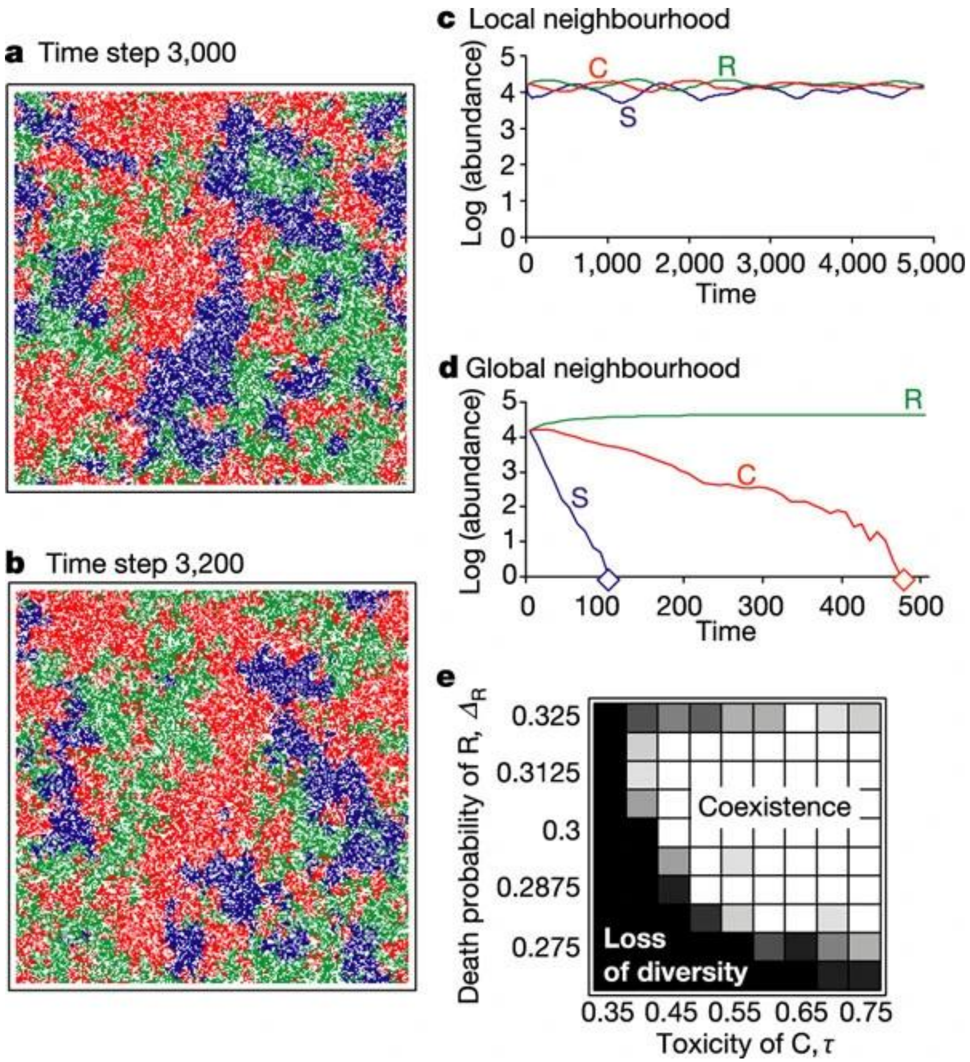
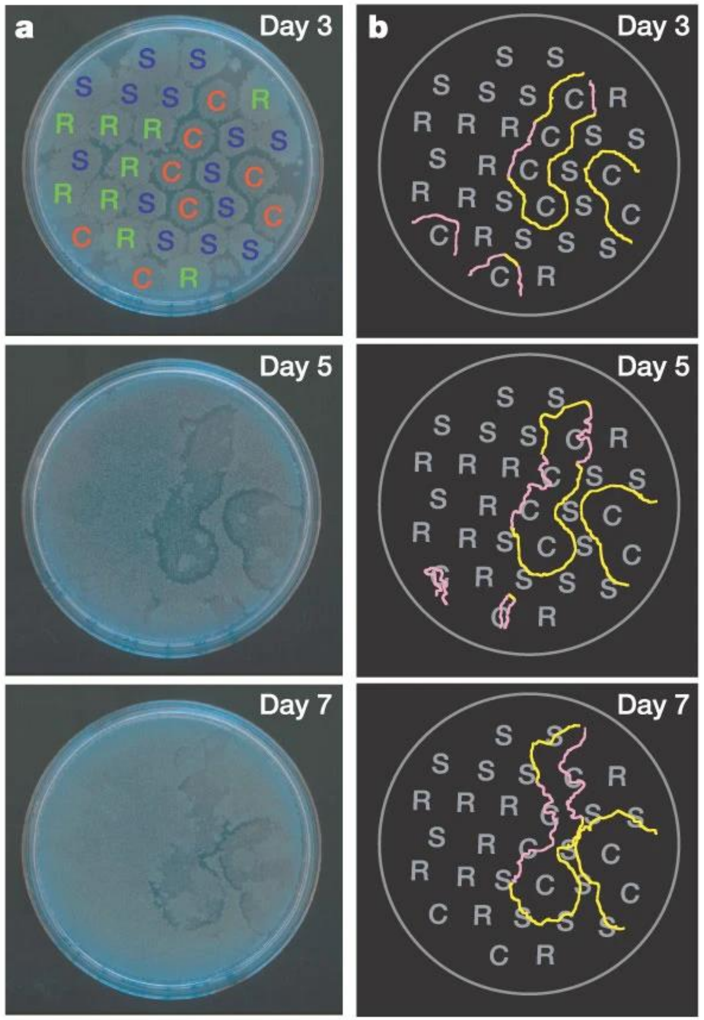
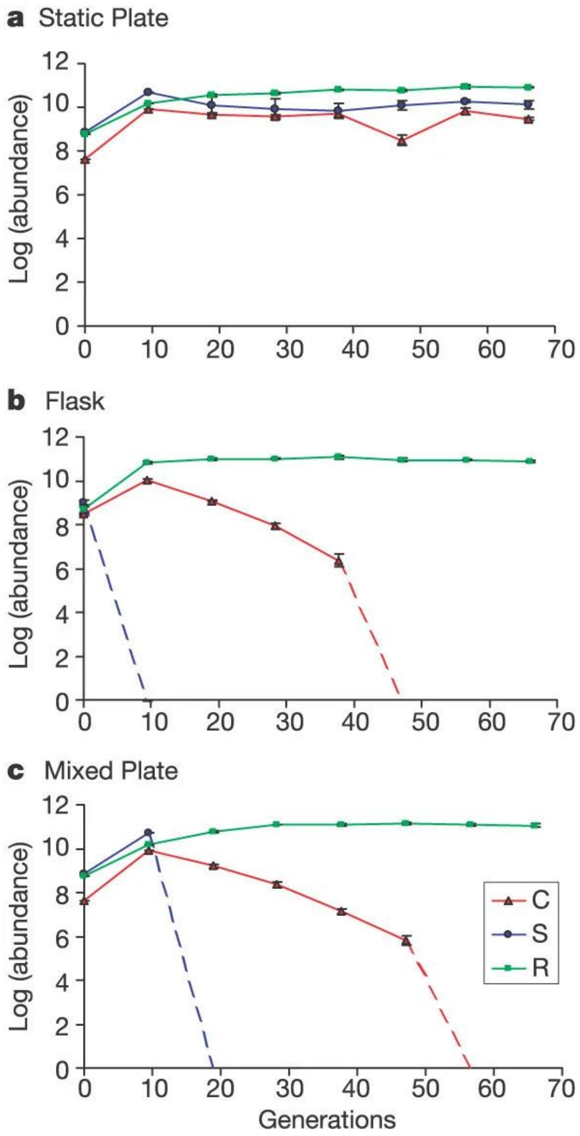
- **Local interactions allowed for the coexistence of all three strains** (in theory). What about experimentally?

Sometimes adding **stochasticity** and **space** (remember metapopulations!) is all you need to ensure **coexistence**!







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