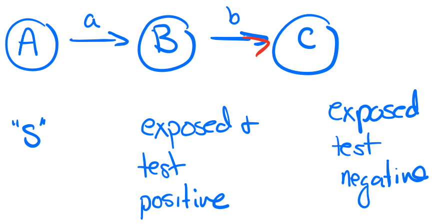
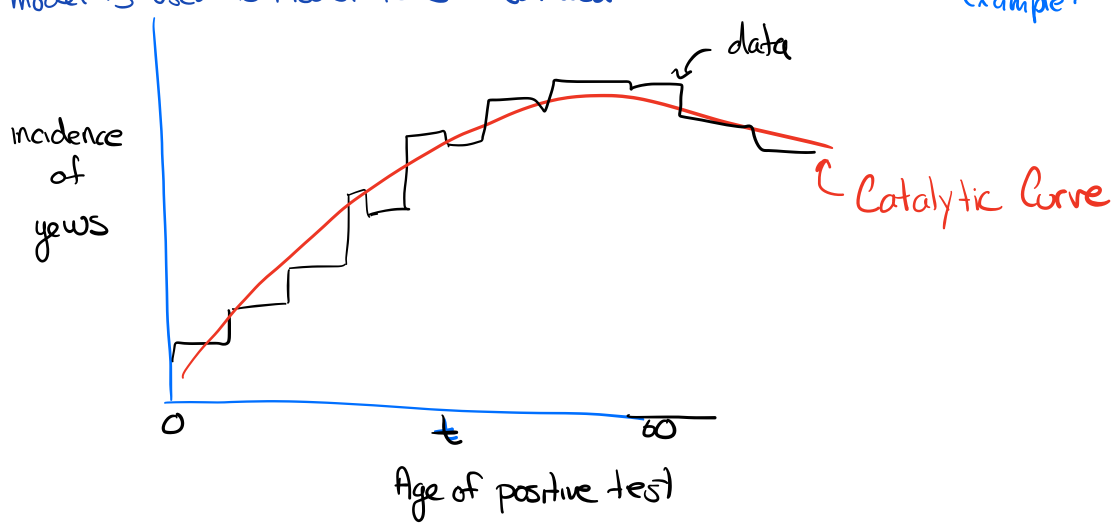
Muench Models

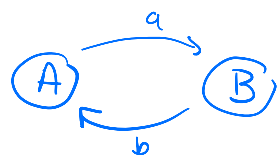
VI Two Stage Example



this model is used to model Yews in Jamaca

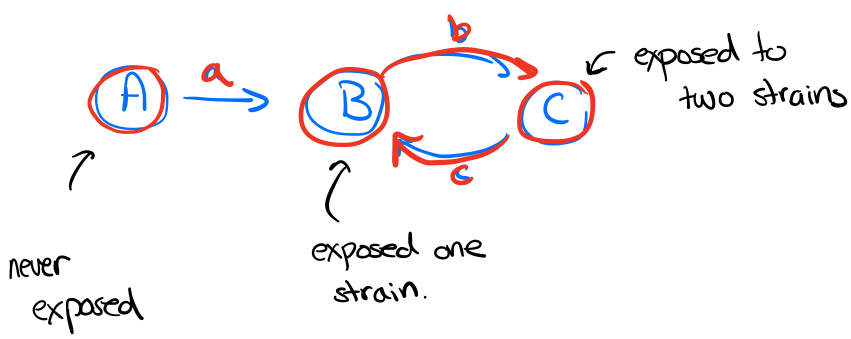


V Two-Way Reaction



"loss of immunity example"

Proposed Cara Model I



(want this!)

lets assign variables:

x – never exposed
y – exposed 1 strain
z – exposed 2 strains.

Let's first solve for 'x'

$$\frac{dx}{dt} = -ax \quad \rightsquigarrow \quad x(t) = 1e^{-at}, \quad x(0) = 1$$

$$\boxed{x + y + z = 1}$$

integrate by 'separation of variables'

$$\frac{1}{x}\,dx = -a\,dt$$

$$\int \frac{1}{x}\,dx = \int -a\,dt$$

$$\ln(x) = -at + C$$

$$x = e^{-at + C_1} = e^{-at} e^{C_1}$$

C_2

$$x = C_2 e^{-at}$$

let $x(0) = 1$, solve for C_2., $C_2 = 1$

$$\boxed{x(t) = e^{-at}} \quad 1$$

Now we can solve for z

$$\frac{dz}{dt} = by - cz$$

, recall $x + y + z = 1 \implies \boxed{y = 1 - z - x}$

$$\frac{dz}{dt} = b(1 - z - x) - cz$$

Note. this is rearranged so that it is in the "integrating factor" form.

$$\frac{dz}{dt} + (b + c)z = b - bx$$

replace x with 1 above

$$\frac{dz}{dt} + (b + c)z = b(1 - e^{-at}) \quad 2$$

notice only two variables remain 'z' and 't'

Integrate via 'integrating factor'

Note. the integrating factor is $\mu(t) = e^{\int (b+c)\,dt}$ coefficient of second term.

$= e^{(b+c)t}$

if we multiply every term in 2 by: $\mu(t) = e^{(b+c)t}$ and simplify, we get

$$\underbrace{\frac{dz}{dt} e^{(b+c)t} + (b+c) z e^{(b+c)t}} = b(1-e^{-at})\, e^{(b+c)t}$$

note - this is the big trick with this technique. the left is the product rule of differentiation, which we can re-write in derivative form.

$$\overbrace{\frac{d}{dt}\left(z\, e^{(b+c)t}\right)} = be^{(b+c)t} - be^{(b+c-a)t}$$

integrate both sides w.r.t. t

$$ze^{(b+c)t} = \frac{b}{b+c} e^{(b+c)t} - \frac{b}{b+c-a} e^{(b+c-a)t} + C_3$$

using $z(0)=0$, solve for C_3 (the integration constant)

$$0 \cancel{e^{0}}^{=1} = \frac{b}{b+c} \cancel{e^{0}}^{=1} - \frac{b}{b+c-a} \cancel{e^{0}}^{=1} + C_3$$

$$C_3 = \frac{b}{b+c-a} - \frac{b}{b+c}$$

$$C_3 = \frac{ab}{(b+c)(b+c-a)}$$

$$z e^{(b+c)t} = \frac{b}{b+c} e^{(b+c)t} - \frac{b}{b+c-a} e^{(b+c-a)t} + \frac{ab}{(b+c)(b+c-a)}$$

$$z(t) = \frac{b}{b+c} \frac{\cancel{e^{(b+c)t}}}{\cancel{e^{(b+c)t}}} - \frac{b}{b+c-a} \frac{e^{(b+c-a)t}}{e^{(b+c)t}} + \frac{ab}{(b+c)(b+c-a)} e^{-(b+c)t}$$

$$\boxed{z(t) = \frac{b}{b+c} - \frac{b}{b+c-a} e^{-at} + \frac{ab}{(b+c)(b+c-a)} e^{-(b+c)t}}$$

perhaps this can be rearanged to look nicer & be more intuitive.

$$\begin{tikzcd} A \arrow[r, "a"] & B \arrow[r, bend left, "b"] & C \arrow[l, bend left, "c"] \end{tikzcd}$$

Question, what is λ

is $a=\lambda$ and $b=\lambda_i$ ← like in Ferguson?