(2) $z_i(a,t) = x(a,t)\left[e^{\int_0^a \lambda_i(a-\tau,t-\tau)\,d\tau} - 1\right]$ ← How do we get here?

going from (10) → (11) in Ferguson 1999

first let's consider (9)

$$x(a,t) = e^{-\int_0^a \sum_k \lambda_k(a-\tau,t-\tau)\,d\tau}$$

suppose this is a two strain model.

$$x(a,t) = e^{-\int_0^a \sum_{k=1}^{2} \lambda_k(a-\tau,\, t-\tau)\,d\tau}$$

$$x(a,t) = e^{-\int_0^a \left(\lambda_1(a-\tau,t-\tau) + \lambda_2(a-\tau,\,t-\tau)\right)d\tau}$$

$$x(a,t) = e^{-\int_0^a \lambda_1(a-\tau,t-\tau)\,d\tau}\, e^{-\int_0^a \lambda_2(a-\tau,t-\tau)\,d\tau}$$

Boot this over

$$\boxed{x(a,t)e^{\int_0^a \lambda_1(a-\tau,t-\tau)\,d\tau} = e^{-\int_0^a \lambda_2(a-\tau,t-\tau)\,d\tau}} \quad (A)$$

Now lets get back to (10)

$$z_i(a,t) = \left[e^{-\int_0^a \sum_{k\neq i} \lambda_k(a-\tau,t-\tau)\,dt}\right]\left[1 - e^{-\int_0^\infty \lambda_i(a-\tau,t-\tau)\,d\tau}\right]$$

suppose again we have two strains. Lets also solve for a particular i. let $z_i = z_1$

$$z_1(a,t) = \underbrace{\left[e^{-\int_0^a \lambda_2(a-\tau,\, t-\tau)\, d\tau} \right]}_{\text{this is the RHS of (A). "sub" in the LHS of (A)}} \left[1 - e^{-\int_0^\infty \lambda_1(a-\tau,\, t-\tau)\, d\tau} \right]$$

$$z_1(a,t) = \left[x(a,t)\, e^{\int_0^a \lambda_1(a-\tau,\, t-\tau)\, d\tau} \right] \left[1 - e^{-\int_0^\infty \lambda_1(a-\tau,\, t-\tau)\, d\tau} \right]$$

factor

$$z_1(a,t) = x(a,t) \left[e^{\int_0^a \lambda_1(a-\tau,\, t-\tau)\, d\tau} - e^{\int_0^a \lambda_1(a-\tau,\, t-\tau)\, d\tau}\, e^{-\int_0^\infty \lambda_1(a-\tau,\, t-\tau)\, d\tau} \right]$$

notice both integrating λ_1 (or λ_i in the generic model)

IF

$$\int_0^a \lambda_1(a-\tau,\, t-\tau)\, d\tau = \int_0^\infty \lambda_1(a-\tau,\, t-\tau)\, d\tau$$

THEN

$$z_1(a,t) = x(a,t)\left[e^{\int_0^a \lambda_1(a-\tau,\, t-\tau)\,d\tau} - 1\right]$$

which is the Ferguson 11