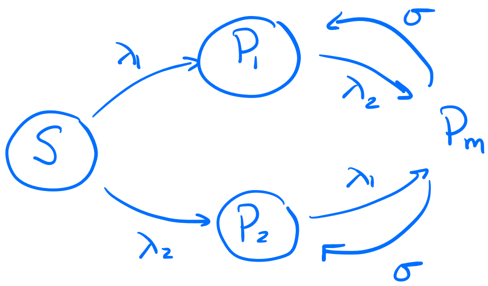

$$\frac{dS}{dt} = -\lambda_1 S - \lambda_2 S$$

$$= -(\lambda_1 + \lambda_2) S$$

$$= -\lambda S$$

$$\frac{1}{S}\, dS = -\lambda\, dt$$

$$\ln(S) + C_1 = -\int \lambda(t)\, dt$$

$$\ln(S) = -\int \lambda(t)\, dt - C_1$$

$$S(t) = e^{-\int \lambda(t) dt - C_1}$$

$$= C_2 e^{-\int \lambda(t) dt}$$

$S(0) = 1 \ldots \quad C_2 = 1 \quad \rightsquigarrow \quad 1 = C_2 \cancel{e^{\int_0^0 \lambda(t)\,dt}}^{=1}$

$$\boxed{S(t) = e^{-\int \lambda(t) dt}} \quad \text{1a}$$

if $\lambda(t) = \lambda$ $\quad \boxed{S(t) = e^{-\lambda t}} \quad$ 1b

Next solve for P_i, infection with any strain 'i'

let P_i represent exposure (sero-positive) to any single strain.

Option 1

$$\frac{dP_i}{dt} = \lambda S - \lambda_K + \sigma P_m \quad \text{, where } K \neq i$$

this is an extension of the Ferguson model particularly the second term that lacks a P_i

$$\frac{dP_i}{dt} = \lambda S - \lambda_K + \sigma(1 - S - P_i)$$

$$= \lambda S - \lambda_K - \sigma - \sigma S - \sigma P_i$$

$$S + P_1 + P_2 + P_m = 1$$

$$P_m = 1 - S - P_1 + P_2$$
$$= 1 - S - P_i$$

$$\frac{dP_i}{dt} + \sigma P_i = (\lambda - \sigma) S - \lambda_K - \sigma$$

Recall from 1a $S = e^{-\int \lambda(t)dt}$

$$\frac{dP_i}{dt} + \sigma P_i = (\lambda - \sigma) e^{-\int \lambda(t)dt} - \lambda_K - \sigma$$

apply integrating factor ....

integrating factor
$$\mu = e^{\int \sigma dt}$$
$$= e^{\sigma t}$$

$$\underbrace{\frac{dP_i}{dt} e^{\sigma t} + \sigma P_i e^{\sigma t}}_{A} = e^{\sigma t}\left((\lambda - \sigma) e^{-\int \lambda(t)dt} - \lambda_K - \sigma\right)$$

$$\underbrace{\frac{d}{dt}\left(e^{\sigma t} P_i\right)}_{B} = e^{\sigma t}\left((\lambda - \sigma) e^{-\int \lambda(t)dt} - \lambda_K - \sigma\right)$$

side note

product rule

$$\frac{d}{dx}\left(f(x)g(x)\right)$$

some function of x — some function of x

$$= \frac{df}{dx} g(x) + f(x)\frac{dg}{dx}$$

integrate both sides.

* Note, we could include an integration constant here if we want... I think. on the LHS.

$$e^{\sigma t} P_i = \int e^{\sigma t}\left((\lambda - \sigma) e^{-\int \lambda(t)dt} - \lambda_K - \sigma\right) dt$$

ie $\underbrace{\frac{d}{dt}\left(\overbrace{P_i(t)}^{f(x)} \overbrace{e^{\sigma t}}^{g(x)}\right)}_{B} = \underbrace{\frac{dP_i}{dt} e^{\sigma t} + P_i \sigma e^{\sigma t}}_{A}$

$$P_i(t) = e^{-\sigma t}\left(\int e^{\sigma t}\left((\lambda-\sigma)e^{-\int \lambda(t)dt} - \lambda_K - \sigma\right) dt\right)$$

what if λ's are constants?

$$P_i(t) = e^{-\sigma t}\left(\int e^{\sigma t}\left((\lambda-\sigma)e^{-\lambda t} - \lambda_K - \sigma\right) dt\right)$$

$$= e^{-\sigma t}\int\left((\lambda-\sigma)e^{(\sigma-\lambda)t} - (\lambda_K+\sigma)e^{\sigma t}\right) dt$$

$$= e^{-\sigma t}\left[\overset{=-1}{\cancel{\frac{(\lambda-\sigma)}{(\sigma-\lambda)}}}e^{(\sigma-\lambda)t} - \frac{\lambda_K+\sigma}{\sigma}e^{\sigma t} + C_1\right]$$

$P_i(0)=0$

$$0 = \overset{1}{\cancel{e^{0}}}\left[-\overset{1}{\cancel{e^{0}}} - \frac{\lambda_K+\sigma}{\sigma}\overset{1}{\cancel{e^{0}}} + C_1\right]$$

$$0 = -1 - \frac{\lambda_K+\sigma}{\sigma} + C_1$$

$$C_1 = 1 + \frac{\lambda_K+\sigma}{\sigma}$$

$$= \frac{\sigma+\lambda_K+\sigma}{\sigma} = \frac{2\sigma+\lambda_K}{\sigma}$$

$$P_i(t) = e^{-\sigma t}\left[-e^{(\sigma-\lambda)t} - \frac{\lambda_K+\sigma}{\sigma}e^{\sigma t} + \frac{2\sigma+\lambda_K}{\sigma}\right]$$

$$\boxed{P_i(t) = -e^{-\lambda t} - \frac{\lambda_K + \sigma}{\sigma} + \frac{2\sigma + \lambda_K}{\sigma} e^{-\sigma t}}$$

... then $P_m = 1 - P_i - S - e^{-S\lambda t}$

we could simplify and say $\lambda_1 = \lambda_2$

so "λ" $= 2\lambda_1$ or "2λ"

and $\lambda_K = \lambda_1$ or "λ"

Option 2.

$$\frac{dP_i}{dt} = \lambda S - \underline{\underline{\lambda_K P_i}} + \sigma P_m$$

$$= \lambda S - \lambda_K P_i + \sigma(1 - S - P_i)$$

all the math would be the same.

I'm inclined to stick with option 1 since the λ's implicitly already contain a 'P' like term built into them.... right?

$$\frac{dP_i}{dt} + \sigma P_i + \lambda_K P_i = \lambda S + \sigma - \sigma S$$

$$\frac{dP_i}{dt} + (\sigma + \lambda_K) P_i = \cdots$$

$$\mu(t) = e^{\int (\sigma + \lambda_K) dt}$$