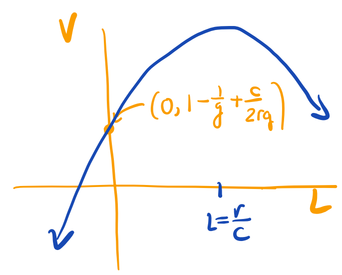
$$V(L) = -\frac{c}{2rg}L^2 + \frac{1}{g}L + \left(1 - \frac{1}{g} + \frac{c}{2rg}\right)$$

what does this parabola look like.

if $L = 0$

$V = 1 - \frac{1}{g} + \frac{c}{2rg}$

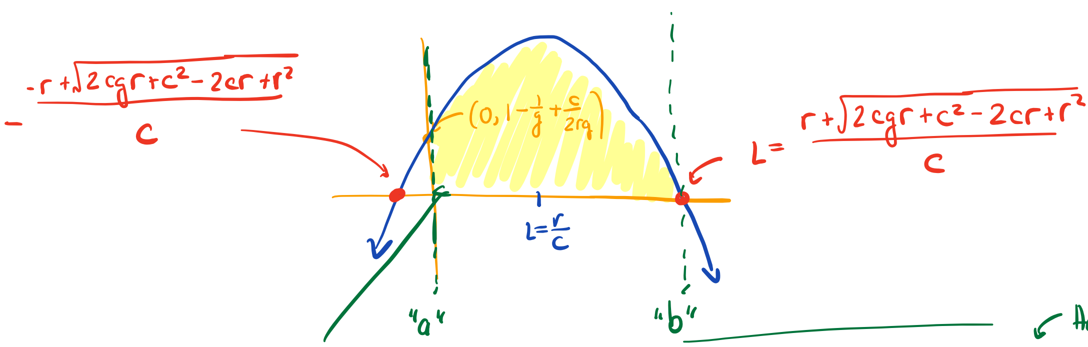


what are the roots of this parabola?

$$V = 0 \quad -\frac{c}{2rg}L^2 + \frac{1}{g}L + \left(1 - \frac{1}{g} + \frac{c}{2rg}\right)$$

quadratic formula, J/k... I used MAPLE

$$L = \frac{r + \sqrt{2cgr + c^2 - 2cr + r^2}}{c} \quad \text{and} \quad L = -\frac{-r + \sqrt{2cgr + c^2 - 2cr + r^2}}{c}$$



goal, find the average. → $\boxed{\frac{1}{b-a}\int_a^b f(x)\,dx}$ How you find the average of a function

$$\frac{1}{\frac{r+\sqrt{2cgr+c^2-2cr+r^2}}{c} - 0} \int_0^{\frac{r+\sqrt{2cgr+c^2-2cr+r^2}}{c}} \left(-\frac{c}{2rg}L^2 + \frac{1}{g}L + 1 - \frac{1}{g} + \frac{c}{2rg}\right) dL$$

again I used MAPLE

$$\boxed{V_{average} = \frac{r\sqrt{c^2 + 2r(g-1)c + r^2} + r^2 + 4r(g-1)c + 2c^2}{6cgr}}$$