$$\frac{dV}{d\tau} = rV - cVL$$

$$\frac{dL}{d\tau} = rgV \qquad (29)$$

$$\frac{dV}{dL} = \frac{rV - c\cancel{V}L}{rg\cancel{V}} = \frac{r - cL}{rg} = \frac{\cancel{r}}{\cancel{r}g} - \frac{cL}{rg}$$

$$\boxed{\frac{dV}{dL} = \frac{1}{g}\left(1 - \frac{cL}{r}\right)}$$

$$\frac{dV}{dL} = \frac{1}{g}\left(1 - \frac{cL}{r}\right) \qquad (30)$$

— separate the variables —

$$dV = \frac{1}{g}\left(1 - \frac{cL}{r}\right) dL$$

integrate. ——

$$\int_{V(0)}^{V(\tau)} 1\, dV = \int_{L(0)}^{L(\tau)} \frac{1}{g}\left(1 - \frac{cL}{r}\right) dL \quad \rightsquigarrow \quad V \Big|_{V(0)}^{V(\tau)} = \frac{1}{g}\left(L - \frac{c}{2r}L^2\right)\Big|_{L(0)}^{L(\tau)}$$

$$\cancel{V = \frac{1}{g}\int 1 - \frac{c}{r}L \, dL}$$

$$\cancel{V = \frac{1}{g}\left(L - \frac{c}{2r}L^2\right)}$$

oops, guess we need integration bounds. we will integrate from $t=0$ to $t=$"τ" but since time is gone, it will be $V(0), L(0)$ to some other value that we call V and L. (or $V(\tau)$, $V(L)$)

$$V(\tau) - V(0) = \frac{1}{g}\left[L(\tau) - \frac{c}{2r}(L(\tau))^2 - \left(L(0) - \frac{c}{2r}(L(0))^2\right)\right]$$

if $V(0)=1$, $L(0)=1$ and let $V(\tau)=V$, $L(\tau)=L$

$$V - 1 = \frac{1}{g}\left[L - \frac{c}{2r}L^2 - 1 + \frac{c}{2r}\right]$$

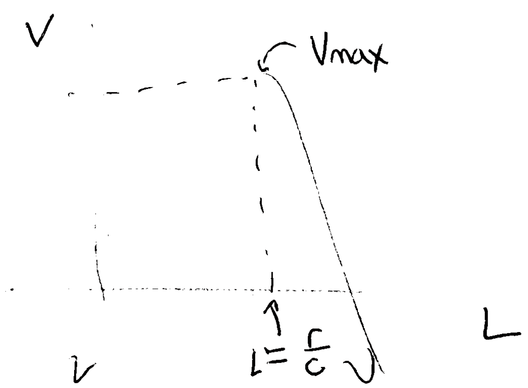
$$\boxed{V = \frac{1}{g}L - \frac{c}{2rg}L^2 - \frac{1}{g} + 1 + \frac{c}{2rg}} \qquad (31)$$

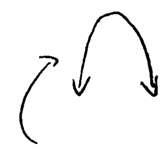
what if we treat V as a function of L

$$V(L) = -\frac{c}{2rg}L^2 + \left(\frac{1}{g}\right)L + \left(1 - \frac{1}{g} + \frac{c}{2rg}\right)$$

its a quadratic

$y = ax^2 + bx + c$





since $a<0$ the parabola has a global max, we can use Calculus to find it

$$\frac{dV}{dL} = -\frac{\cancel{2}c}{\cancel{2}rg}L + \frac{1}{g}$$

set $\frac{dV}{dL} = 0 = -\frac{c}{rg}L + \frac{1}{g}$

$$\frac{c}{rg}L = \frac{1}{g}$$

$$L^* = \frac{r\cancel{g}}{c\cancel{g}} = \frac{r}{c}$$ ← plug this "x" value into the "y" equation

$$V_{max} = V\left(\frac{r}{c}\right) = \frac{-c}{2rg}\left(\frac{r}{c}\right)^2 + \frac{1}{g}\left(\frac{r}{c}\right) + \left(1 - \frac{1}{g} + \frac{c}{2rg}\right)$$

$$= \frac{-r}{2cg} + \frac{r}{cg} + \left(1 - \frac{1}{g} + \frac{c}{2rg}\right)$$

$$\boxed{V_{max} = \frac{r}{gc} - \frac{r}{2gc} + 1 - \frac{1}{g} + \frac{c}{2rg}} \quad (82)$$