ECE-363 Assignment 0

Lavanya Verma (lavanya18155@iiitd.ac.in)

January 31, 2021

Probability Theory

1. Problem 1

Let the A_e, B_e, C_e and D_e be the respective events that the app was made by A, B, C and D respectively. Let E be the app event that app have bug.
Given:

$$\begin{aligned} P(A_e) &= .15 \\ P(B_e) &= .20 \\ P(C_e) &= .30 \\ P(D_e) &= .35 \\ P(E|A_e) &= 0.08 \\ P(E|B_e) &= 0.05 \\ P(E|C_e) &= 0.04 \\ P(E|D_e) &= 0.02 \end{aligned}$$

Part(a):

$$P(A_e|E) = \frac{P(E|A_e)P(A_e)}{P(E|A_e)P(A_e) + P(E|B_e)P(B_e) + P(E|C_e)P(C_e) + P(E|D_e)P(D_e)}$$
$$P(A_e|E) = 0.29268$$

Part(b):

$$P(B_e|E) = \frac{P(E|B_e)P(B_e)}{P(E|A_e)P(A_e) + P(E|B_e)P(B_e) + P(E|C_e)P(C_e) + P(E|D_e)P(D_e)}$$
$$P(B_e|E) = 0.24390$$

Part(c):

$$P(C_e|E) = \frac{P(E|C_e)P(C_e)}{P(E|A_e)P(A_e) + P(E|B_e)P(B_e) + P(E|C_e)P(C_e) + P(E|D_e)P(D_e)}$$
$$P(C_e|E) = 0.29268$$

Part(d):

$$P(D_e|E) = \frac{P(E|D_e)P(D_e)}{P(E|A_e)P(A_e) + P(E|B_e)P(B_e) + P(E|C_e)P(C_e) + P(E|D_e)P(D_e)}$$
$$P(D_e|E) = 0.17073$$

2. Problem 2

Let events be A_w and B_w be that A and B wins respectively.

$$
\begin{aligned}
P(A_w) &= 2 * P(B_w) \\
P(A_w) + P(B_w) &= 1 \\
\Rightarrow P(B_w) &= \frac{1}{3} \\
\Rightarrow P(A_w) &= \frac{2}{3} \;\textit{;(Part a)}
\end{aligned}
$$

Let the X be the random variable denoting the points achieved by A in a single toss.

$$
\begin{aligned}
X &\in \{0, 1\} \\
E[X] &= \sum_{i=1}^{2} P(X_i) * X_i \;\textit{;Part(b)} \\
&= P(X = 1) * 1 + P(X = 0) *) \\
&= \frac{2}{3}
\end{aligned}
$$

$$
\textit{Distribution of X(Part c)}
$$

$$
P(X) = \begin{cases} \frac{2}{3}, X = 1 \\ \frac{1}{3}, X = 0 \end{cases}
$$

$$
\begin{aligned}
E[(X - E[X])^2] &= E[X^2] - (E[X])^2 \;\textit{;Part(d)} \\
&= \frac{2}{3} - \frac{4}{9} \\
&= \frac{2}{9}
\end{aligned}
$$

3. Problem 3

Let X be the random variable denoting the score achieved by A in 10 successive tosses of a biased coin.

X	0	1	2	3	4	5	6	7	8	9	10
P(X)	$\frac{1}{3^{10}}$	$\frac{20}{3^{10}}$	$\frac{180}{3^{10}}$	$\frac{960}{3^{10}}$	$\frac{3360}{3^{10}}$	$\frac{8064}{3^{10}}$	$\frac{13440}{3^{10}}$	$\frac{15360}{3^{10}}$	$\frac{11520}{3^{10}}$	$\frac{5120}{3^{10}}$	$\frac{1024}{3^{10}}$

$$
\begin{aligned}
E[X] &= \sum_{i=0}^{10} \binom{10}{i} \frac{2^i i}{3^{10}} \\
&= 6.6667 \\
E[(X - E[X])^2] &= E[X^2] - E[X]^2 \\
&= 2.2222 \\
P(X \geq 2) &= 1 - P(X = 0) - P(X = 1) \\
&= 0.999644
\end{aligned}
$$

4. Problem 4

Part(a):

$$\begin{aligned} P(B=1) &= P(B=1|A\in\{1,4,6\})P(A\in\{1,4,6\}) + P(B=1|A\in\{2,3,5\})P(A\in\{2,3,5\}) \\ &= \frac{1}{6}*\frac{1}{2}+\frac{1}{2}*\frac{1}{2} \\ &= \frac{1}{3} \end{aligned}$$

Part(b):

$$\begin{aligned} E[A] &= \sum_{i=1}^{6}\frac{i}{6} \\ &= 3.5 \end{aligned}$$

Part(C):

$$\begin{aligned} E[B|A\in\{2,3,5\}] &= P(B=1|A\in\{2,3,5\})*1 + P(B=0|A\in\{2,3,5\})*0 \\ &= \frac{1}{2} \\ E[B|A\in\{1,4,6\}] &= \sum_{i=1}^{6}\frac{i}{6} \\ &= 3.5 \end{aligned}$$

$$P(A=i|B=1) = \frac{P(B=1|A=i)P(A=i)}{P(B=1)} \tag{1}$$

i	P(A = i | B = 1)
1	$\frac{1}{12}$
2	$\frac{1}{4}$
3	$\frac{1}{4}$
4	$\frac{1}{12}$
5	$\frac{1}{4}$
6	$\frac{1}{12}$

$$E[A|B=1] = 3.4166$$

5. Problem 5

Part(a): Average Value: 60
Part(b): Standard deviation: 15
Part(c):

$$P[T>75] = 0.1587$$

Part(d):

$$P[T<30] = 0.0228$$

Part(e):

$$P[45 \leq T \leq 75] = 0.6827$$

Vector Calculus: Gradients

6. Problem 1

Part(a):

$$f(x,y) = -x^4 + 4*(x^2 - y^2) + 20$$
$$\nabla f(x,y) = \begin{bmatrix} -4x^3 + 8x \\ -8y \end{bmatrix}$$

Part(b):

$$f(x,y) = -2x^3 + 5yz + z^4$$
$$\nabla f(x,y) = \begin{bmatrix} -6x^2 \\ 5z \\ 5y + 4z^3 \end{bmatrix}$$

7. Problem 2

$$\begin{aligned} F(x) &= b^T x \\ &= \sum_{i=1}^{n} b_i x_i \\ \nabla_x F(x) &= b \end{aligned}$$

8. Problem 3

Given:

$$x \in \mathbb{R}^n$$
$$A \in \mathbb{R}^{nxn}$$

$$\begin{aligned} Q(x) &= x^T A x \\ &= \begin{bmatrix} x_1 & x_2 & \dots & x_n \end{bmatrix} \begin{bmatrix} a_{11} & \dots & a_{1n} \\ \vdots & \ddots & \vdots \\ a_{n1} & \dots & a_{nn} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix} \\ &= \sum_{i=1}^{n} \sum_{j=1}^{n} x_j x_i a_{ij} \\ \nabla_x Q &= \begin{bmatrix} 2a_{11}x_1 + \sum_{i=2}^{n} (a_{i1} + a_{1i}) x_i \\ \vdots \\ 2a_{nn}x_n + \sum_{i=1}^{n-1} (a_{in} + a_{ni}) x_i \end{bmatrix} \end{aligned}$$

$$j^{th} \ element \ of \ \nabla Q$$

$$\nabla_x Q = 2a_{jj}x_j + \sum_{i=1;i\neq j}^{n} (a_{ij} + a_{ji})x_i$$

Special Case: A is Symetric

$$\nabla_x Q = 2Ax$$

9. Problem 4

$$F(x,y) = \begin{bmatrix} -x + y^2 \\ \sin(x) + 10y^2 \end{bmatrix}$$

$$\nabla_{[xy]^T} F(x,y) = \begin{bmatrix} -1 & 2y \\ \cos(x) & 20y \end{bmatrix}$$