ECE570: Control Theory
Monsoon 2020

Group Project Report

Project#2: Radar control in 3-dimensional space

Lavanya Verma 2018155 | **Jayant Chhillar** 2018151 | **Raghav Avasthi** PhD14803

Abstract

The project asks to design a target tracking system for a radar in 3-dimensional space. The radar antenna can change its azimuth and elevation angles using two servo motors.

Methodology

The project is dependent on formulating the control equations for two servo motors, one for azimuth control and the other for controlling the elevation angle of the radar. The first step required was to finalize on a model of the mounting orientation of the two motors.
The orientation chosen was alt-azimuth mount (elevation motor over the azimuth motor.)
Then we needed to derive the length parameters from the given moments of inertias. To be able to get an assumption of the length parameter we decided to assume the mass parameter to be an average of real world satellite dishes which came out to be 60kg.
We used this data to design a simple linear model modified from the standard single servo motor linear control equations.
The inputs to our system are both the positions to track and the current angles of the motors.

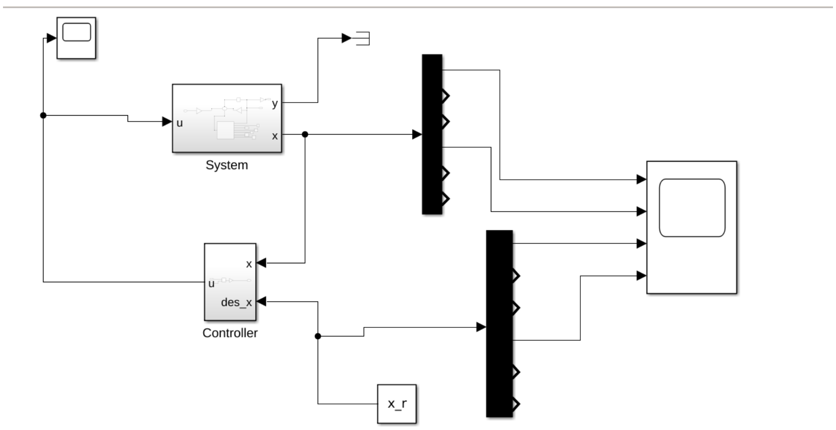

Figure 1: Simulink Model

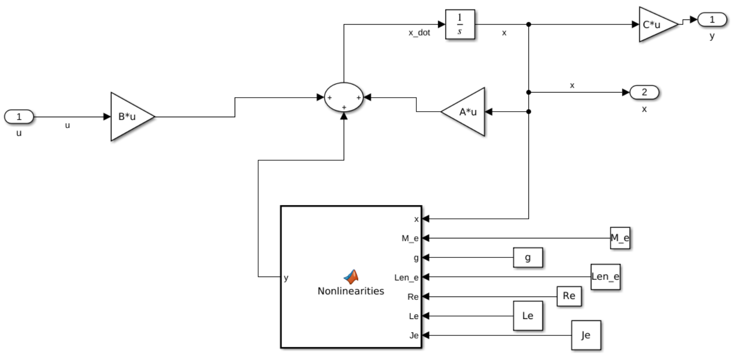

Figure 2: Control System Model

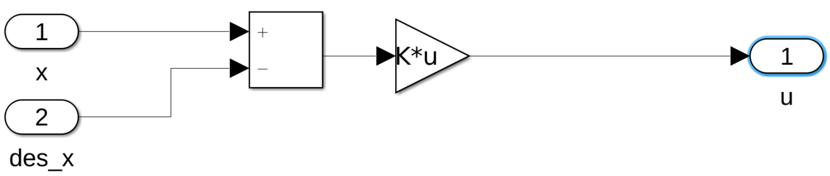

Figure 3: Controller Model

Constants

Constants	**Values**
K_{2a}	0.01 Nm/A
K_{3a}	0.01 V/rad/s
R_a	1 Ω
L_a	0.5 H
J_a	500 kgm^2
b_a	0.1 Nms
K_{2e}	0.01 Nm/A
K_{3e}	0.01 V/rad/s
R_e	1 Ω
L_e	0.5 H
J_e	600 kgm^2
b_e	0.1 Nms
m	60 kg
l	$\sqrt{300}m$

Table 1: Constants

System Equations

- ## Linear

$$
\begin{aligned}
u_1 &= R_a I_a + L_a \dot{I}_a + K_{3a}\dot{\theta} \\
I_a &= \frac{J_a\ddot{\theta} + b_a\dot{\theta}}{K_{2a}} \\
u_1 &= \dddot{\theta}_a \left[\frac{L_a J_a}{K_{2a}}\right] + \ddot{\theta}_a \left[\frac{R_a J_a + L_a b_a}{K_{2a}}\right] + \dot{\theta}_a \left[\frac{R_a b_a}{K_{2a}} + K_{3a}\right]
\end{aligned}
$$

- ## Non Linear

Abstraction of radar wave reception system is taken to be a rod of length(l).

$$
\begin{aligned}
u_2 &= R_e I_e + L_e \dot{I}_e + K_{3e}\dot{\theta}_e \\
I_a &= \frac{1}{K_{2e}}\left[J_e\ddot{\theta}_e + b_e\dot{\theta}_e + \frac{mgl\cos\theta_e}{2}\right] ;\ (J_e = \frac{ml^2}{3}) \\
u_2 &= \dddot{\theta}_e \left[\frac{L_e J_e}{K_{2e}}\right] + \ddot{\theta}_e \left[\frac{R_e J_e + L_e b_e}{K_{2e}}\right] + \dot{\theta}_e \left[\frac{R_e b_e}{K_{2e}} + K_{3e} - \frac{mglR_e\sin\theta_e}{2K_{2e}}\right] + \frac{R_e mgl\cos\theta_e}{2K_{2e}}
\end{aligned}
$$

- ## State Vector

$$
X = \begin{bmatrix}\theta_a & \dot{\theta}_a & \ddot{\theta}_a & \theta_e & \dot{\theta}_e & \ddot{\theta}_e\end{bmatrix}'
$$

- **Dynamics**

$$\dot{X} = \begin{bmatrix} \dot{\theta}_a \\ \ddot{\theta}_a \\ \dddot{\theta}_a \\ \dot{\theta}_e \\ \ddot{\theta}_e \\ \dddot{\theta}_e \end{bmatrix} = \begin{bmatrix} \dot{\theta}_a \\ \ddot{\theta}_a \\ \frac{u_1 K_{2a}}{L_a J_a} - \ddot{\theta}_a \left(\frac{R_a}{L_a} + \frac{b_a}{J_a} \right) - \dot{\theta}_a \left(\frac{R_a b_a}{L_a J_a} + \frac{K_{3a} K_{2a}}{L_a J_a} \right) \\ \dot{\theta}_e \\ \ddot{\theta}_e \\ \frac{u_2 K_{2e}}{L_e J_e} - \ddot{\theta}_e \left(\frac{R_e}{L_e} + \frac{b_e}{J_e} \right) - \dot{\theta}_e \left(\frac{R_e b_e}{L_e J_e} + \frac{K_{3e} K_{2e}}{L_e J_e} - \frac{mglR_e \sin(\theta_e)}{L_e b_e} \right) - \frac{R_e mgl \cos\theta_e}{L_e J_e} \end{bmatrix}$$

$$\dot{X} = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & -\left(\frac{R_a b_a}{L_a J_a} + \frac{K_{3a} K_{2a}}{L_a J_a} \right) & -\left(\frac{R_a}{L_a} + \frac{b_a}{J_a} \right) & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & -\left(\frac{R_e b_e}{L_e J_e} + \frac{K_{3e} K_{2e}}{L_e J_e} \right) & -\left(\frac{R_e}{L_e} + \frac{b_e}{J_e} \right) \end{bmatrix} X$$

$$+ \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ \frac{K_{2a}}{L_a J_a} & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & \frac{K_{2e}}{L_e J_e} \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ \frac{\dot{\theta}_e mglR_e \sin\theta_e}{L_e b_e} - \frac{R_e mgl \cos\theta_e}{L_e J_e} \end{bmatrix}$$

$\dot{X} = AX + BU + N$; *(N accounts for nonlinearities, feedback linearisation)*

- **Output**

$$C = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \end{bmatrix}; D = 0_{2x2}$$

$$Y = CX + DU$$

Simulation Results

System is completely controllable and observable.
To check the behaviour of system in simulation, gain block is added with gain matrix computed from eigenvalues.
Since, this part was not focused on controller, state is taken as output from plant. State can be estimated from y itself as system is completely observable.

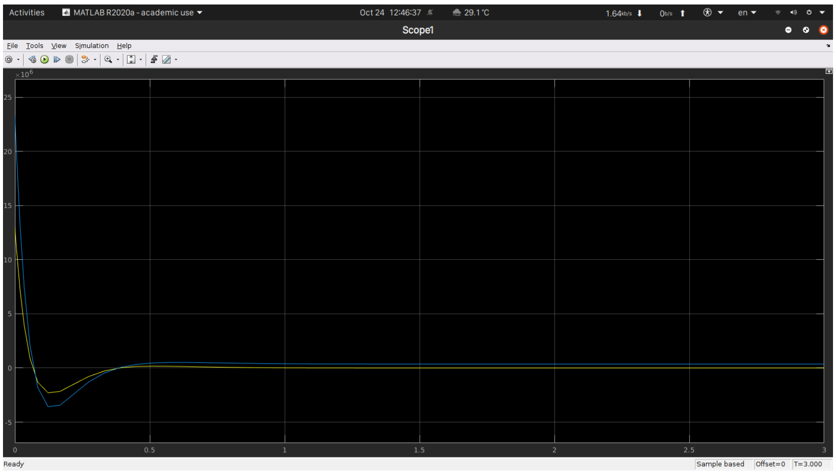

Figure 4: Control Actuation [INPUT]

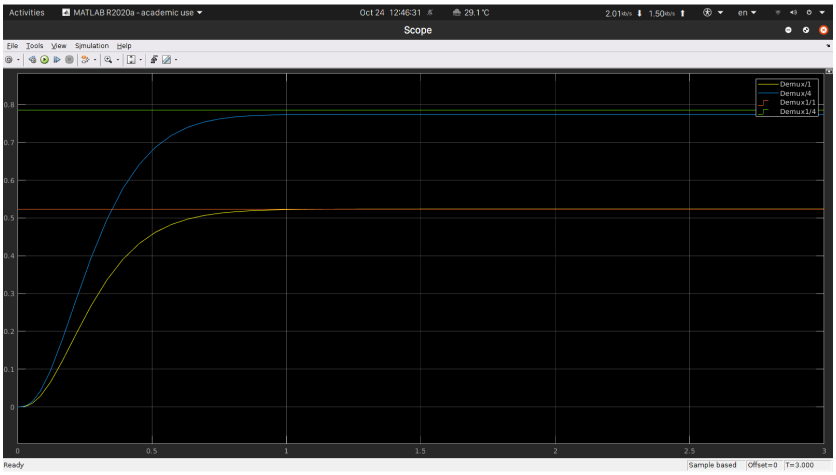

Figure 5: Reference vs Actual State [OUTPUT]