ECE570: Control Theory
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Group Project Report

Project#2: Radar control in 3-dimensional space

Lavanya Verma 2018155
Jayant Chhillar 2018151

Abstract

The project asks to design a target tracking system for a radar in 3-dimensional space. The radar antenna can change its azimuth and elevation angles using two servo motors.

PART 2: Controller design and demonstration
Assume the target is taking a horizontal flight at 10km altitude in a straight line at an arbitrary direction with a constant velocity of 50m/s. Also assume that the tracking system can estimate the position of the target (x, y, z coordinates) accurately. Design a controller which can stair the antenna such that the antenna maintains the Line of Sight with the target.

Methodology

The model developed in Part I of the project was to be updated.

- Controller model was updated to include an integrator.
- Luenberger observer added to the closed loop.
- A dynamic object model added, that simulates an airplane moving in the 3D space.
- Manual calculations to be done for the new integrator gains, for the estimations for realistic conversation between desired points to desired angles.
- A VR sink added to animate the system's working in a 3D environment.

Simulink Models

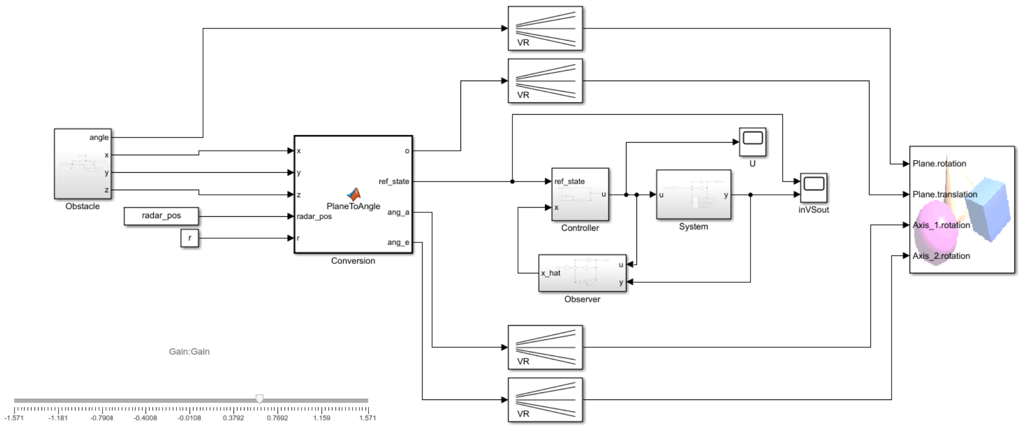


Figure 1: Simulink Model

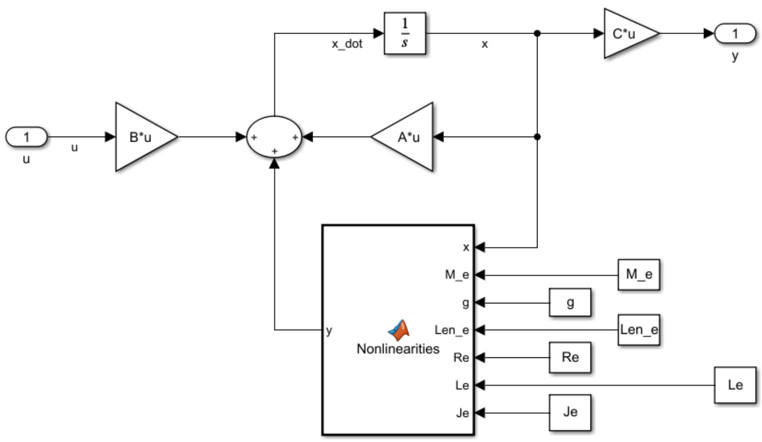


Figure 2: Dynamic System Model

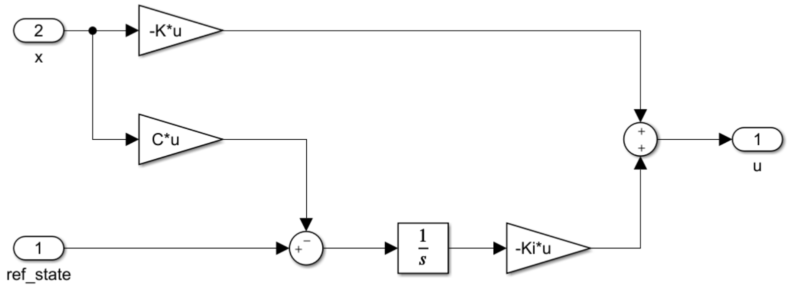


Figure 3: Controller Model

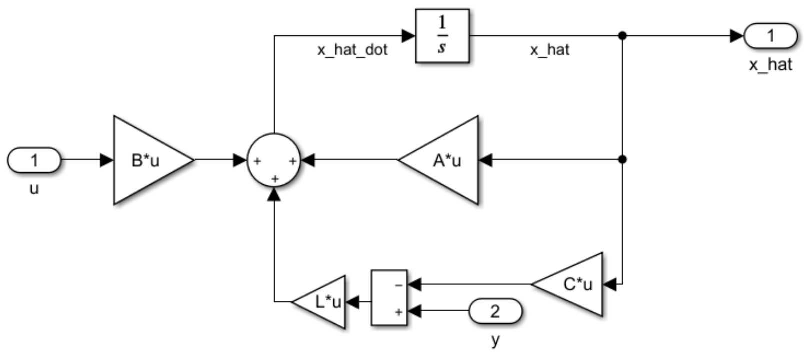


Figure 4: Observer Model

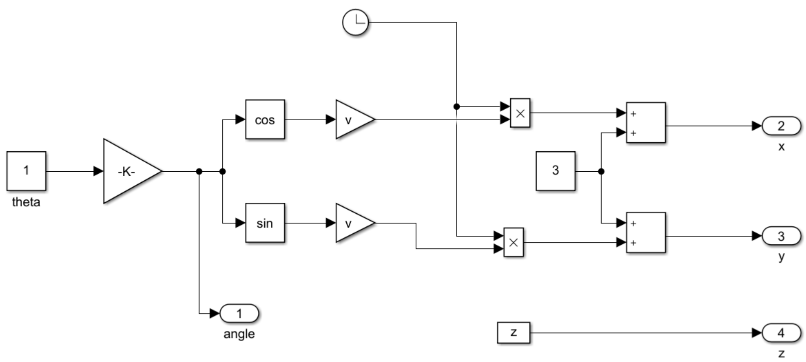


Figure 5: Object Simulation Model

Constants

Constants	**Values**
K_{2a}	0.01 Nm/A
K_{3a}	0.01 V/rad/s
R_a	1 Ω
L_a	0.5 H
J_a	500 kgm^2
b_a	0.1 Nms
K_{2e}	0.01 Nm/A
K_{3e}	0.01 V/rad/s
R_e	1 Ω
L_e	0.5 H
J_e	600 kgm^2
b_e	0.1 Nms
m	60 kg
l	$\sqrt{300}m$

Table 1: Constants

System Equations

- ## Controller

Now that we have an integrator in the system, we don't need two equation for each $u_1,\ u_2$ the controller is defined by the equation:

$$U = -K \begin{bmatrix} x \\ x_I \end{bmatrix} = -\begin{bmatrix} K_r & k_3 \end{bmatrix} \begin{bmatrix} x \\ x_I \end{bmatrix}$$
$$u(t) = -K_r x - k_3 x_I = -K_r x(t) - k_3 \int_0^t (r(\tau) - y_r(\tau)\, d\tau$$

- ## State Vector

$$X = \begin{bmatrix} \theta_a & \dot{\theta}_a & \ddot{\theta}_a & \theta_e & \dot{\theta}_e & \ddot{\theta}_e \end{bmatrix}'$$

- ## Dynamics

$$\dot{X} = \begin{bmatrix} \dot{\theta}_a \\ \ddot{\theta}_a \\ \dddot{\theta}_a \\ \dot{\theta}_e \\ \ddot{\theta}_e \\ \dddot{\theta}_e \end{bmatrix} = \begin{bmatrix} \dot{\theta}_a \\ \ddot{\theta}_a \\ \frac{u_1 K_{2a}}{L_a J_a} - \ddot{\theta}_a \left(\frac{R_a}{L_a} + \frac{b_a}{J_a} \right) - \dot{\theta}_a \left(\frac{R_a b_a}{L_a J_a} + \frac{K_{3a} K_{2a}}{L_a J_a} \right) \\ \dot{\theta}_e \\ \ddot{\theta}_e \\ \frac{u_2 K_{2e}}{L_e J_e} - \ddot{\theta}_e \left(\frac{R_e}{L_e} + \frac{b_e}{J_e} \right) - \dot{\theta}_e \left(\frac{R_e b_e}{L_e J_e} + \frac{K_{3e} K_{2e}}{L_e J_e} - \frac{mgl R_e \sin(\theta_e)}{L_e b_e} \right) - \frac{R_e mgl \cos\theta_e}{L_e J_e} \end{bmatrix}$$

$$\dot{X} = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & -\left(\frac{R_a b_a}{L_a J_a} + \frac{K_{3a} K_{2a}}{L_a J_a}\right) & -\left(\frac{R_a}{L_a} + \frac{b_a}{J_a}\right) & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & -\left(\frac{R_e b_e}{L_e J_e} + \frac{K_{3e} K_{2e}}{L_e J_e}\right) & -\left(\frac{R_e}{L_e} + \frac{b_e}{J_e}\right) \end{bmatrix} X + \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ \frac{K_{2a}}{L_a J_a} & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & \frac{K_{2e}}{L_e J_e} \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ \frac{\dot{\theta_e} mgl R_e \sin\theta_e}{L_e b_e} - \frac{R_e mgl \cos\theta_e}{L_e J_e} \end{bmatrix}$$

$\dot{X} = AX + BU + N$; *(N accounts for nonlinearities, feedback linearisation)*

- ## System Equation

$$C = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \end{bmatrix}; D = 0_{2x2}$$

$$Y = CX + DU$$

Simulation Results

System is completely controllable and observable.
To check the behaviour of system in simulation, gain block is added with gain matrix computed from eigenvalues.
Since, this part was not focused on controller, state is taken as output from plant. State can be estimated from y itself as system is completely observable.
The results bellow are from a scaled down value of 'z' attribute of the object.
(Changes in the angle will be immune to uniform scaling.)

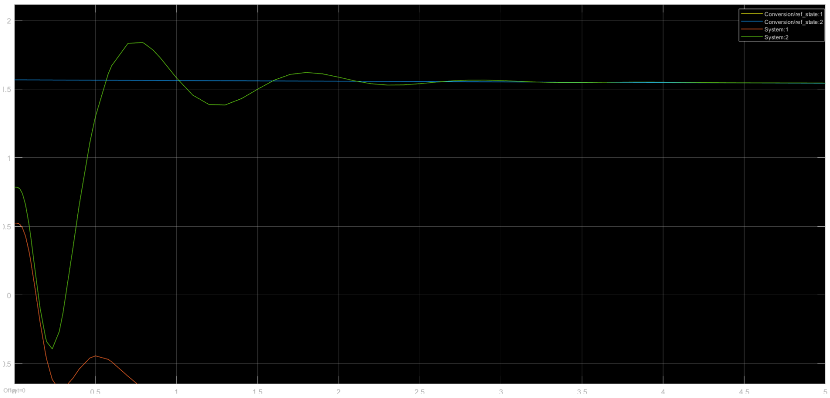
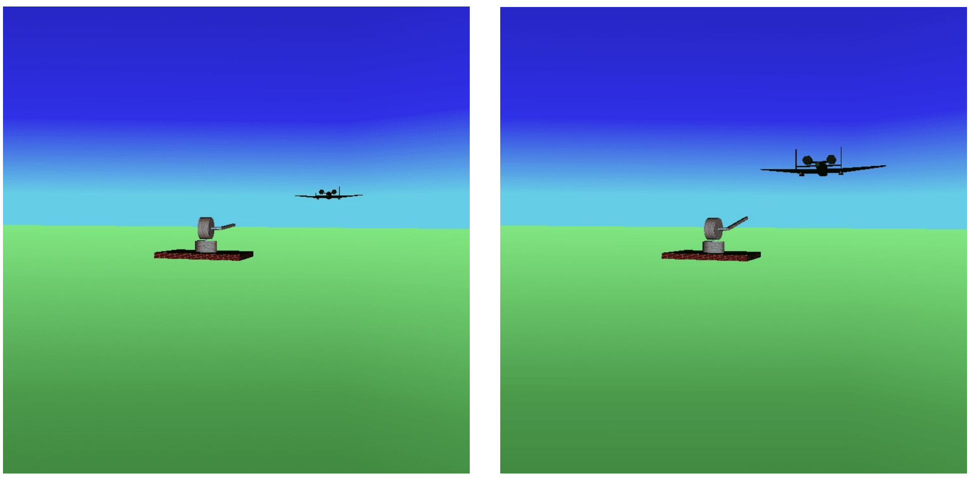


Figure 6: An input vs Output graph (of the first 5 seconds)



(a) Position 1 (b) Position 2

Figure 7: Simulation Screenshots