Assignment 3

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Question 1.

The Euler-Lagrange dynamics of the two-link planar robotic manipulator arm in Fig.1 are defined as

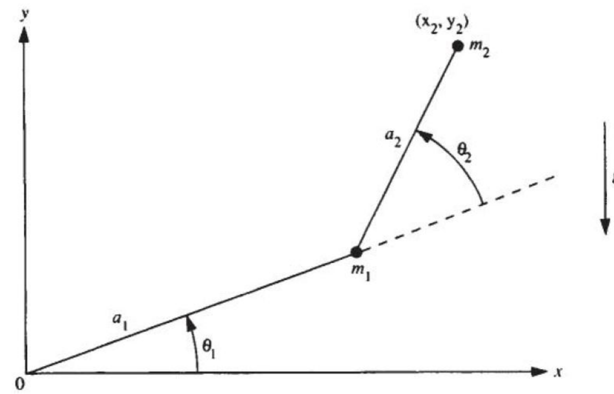

Figure 1: Two-Link Planar Elbow Arm

$$M(q)\ddot{q} + V(q,\dot{q}) + F\dot{q} + G(q) = \tau \tag{1}$$

where $q(t) = [\theta_1(t), \theta_2(t)]^T$ is the position vector (joint angles) and $\tau(t) \in \mathbb{R}^2$ is the external torque input (control input), $M(q) \in \mathbb{R}^{2\times 2}$ denotes the Inertia matrix, $V(q,\dot{q}) \in \mathbb{R}^2$ denotes the Coriolis/centripetal force vector, $F \in \mathbb{R}^{2\times 2}$ denotes the frictional coefficient matrix, and $G(q) \in \mathbb{R}^2$ is the gravity force vector, which are subsequently defined.

$$M(q) = \begin{bmatrix} (m_1+m_2)a_1^2 + m_2a_2^2 + 2m_2a_1a_2cos\theta_2 & m_2a_2^2 + m_2a_1a_2cos\theta_2 \\ m_2a_2^2 + m_2a_1a_2cos\theta_2 & m_2a_2^2 \end{bmatrix}$$

$$V(q,\dot{q}) = \begin{bmatrix} -m_2a_1a_2\left(2\dot{\theta}_1\dot{\theta}_2 + \dot{\theta}_2^2\right)sin\theta_2 \\ m_2a_1a_2\theta_1^2 sin\theta_2 \end{bmatrix}$$

$$F = \begin{bmatrix} f_1 & 0 \\ 0 & f_2 \end{bmatrix}$$

$$G(q) = \begin{bmatrix} (m_1+m_2)ga_1cos\theta_1 + m_2ga_2cos(\theta_1+\theta_2) \\ m_2ga_2cos(\theta_1+\theta_2) \end{bmatrix}$$

Part-1>

Design a model-based nonlinear control input [1] or torque $\tau(t)$ to track the desired trajectory $q_d = [\frac{\pi}{3} + \frac{\pi}{6}sin(t), \frac{\pi}{4} + \frac{\pi}{6}cos(t)]^T$ (oscillatory behaviour around an operating point).

Part-2>

[1] as discussed in class

Consider the following proportional derivative (PD) control input

$$\tau(t) = -K_P e(t) - K_D \dot{e}(t) \tag{2}$$

where $e(t) \triangleq q(t) - q_d(t)$ is tracking error and $k_P > 0$ and $k_D > 0$ are positive definite controller gain matrices. (Choose the gains suitably to get a reasonable performance, however, note that high magnitude of controller gain will lead to high control effort. Hence, a trade off is required between tracking performance and control effort.)

Simulate the dynamics with both of the controllers for the above mentioned desired trajectory with model parameters as: $m_1 = 3.473kg$, $m_2 = 0.196kg$, $a_1 = a_2 = 1m$, $g = 9.81m/sec^2$, $f_1 = 5.3Nm.sec$, $f_2 = 1.1Nm.sec$. (You can use Simulink or Matlab script depending on your preference.)

What you have to submit :

1> Simulink file / matlab code

2> Plot of time evolution of the following variables for both the controllers during the time-span $t \in [0, 10]$ seconds.

a> e-vs-t,

b> $\dot{e}$-vs-t

c> τ-vs-t.

3> A brief comparative remark about the performance of both the controllers in terms of tracking accuracy and control effort etc. (This part should be qualitative based on your observation; No quantitative measure is required. Also note that K_P and K_D should be same for both the controllers for a fair comparison.)

The plots and the remark should be included in a single pdf file.