Assignment 1

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Question 1.

Part-1>

Consider Fig.1. Determine the dynamics of the manipulator based on Euler-Lagrange formulation, which is covered in class. The final structure of the dynamics will be as follows

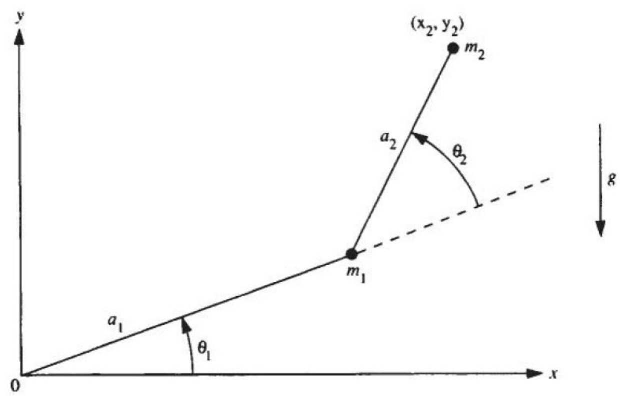

Figure 1: Two-Link Planar Elbow Arm

$$M(q)\ddot{q} + V(q,\dot{q}) + G(q) = \tau \tag{1}$$

where $q(t) = [\theta_1(t), \theta_2(t)]^T$ is the position vector (joint angles) and $\tau(t) \in \mathbb{R}^2$ is the external torque input (control input). Your task is to figure out the exact expressions of the different components $M(q)$, $V(q,\dot{q})$ and $G(q)$ for the given manipulator.

Part-2>

case a) Desired trajectory : $q_d = [\pi/3, \pi/4]^T$, (set-point regulation)

case b) Desired trajectory : $q_d = [sin(t), cos(t)]^T$ (sinusoidal tracking)[1]

Consider the following proportional derivative (PD) control input[2]

$$\tau(t) = -k_1 e(t) - k_2 \dot{e}(t) \tag{2}$$

where $e(t) \triangleq q(t) - q_d(t)$ is tracking error and $k_1 > 0$ and $k_2 > 0$ are controller gains. Consider the initial conditions as $q(0) = [0,0]^T$ and $\dot{q}(0) = [0,0]^T$, assuming $t = 0$ as the initial time and choose $k1 = k2 = 1$.

Simulate the dynamics and the controller for both of the above mentioned cases of desired trajectories. (You can use Simulink or Matlab script depending on your preference. Refer to the recorded discussion for further details.) Also visualize the motion of the manipulator using Peter Corke's Toolbox corresponding to both the cases.

[1]Note that desired trajectory is an outcome of a trajectory planning algorithm depending on the application at hand, however, here some test trajectories are chosen for the assignment.

[2]The rational behind the designed controller will be discusses later in the class.

What you have to submit :

1> Mathematical derivation of the dynamics

2> Simulink file / matlab code (including the visualization part; you may consider a separate code for visualization if required!)

3> Plot of time evolution of the following variables for both the cases during the time-span $t \in [0, 10]$.

a> e-vs-t,

b> $\dot{e}$-vs-t

c> τ-vs-t.