Robotics MIDSEM

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1. Modelling

$$\begin{bmatrix} x_1 \\ y_1 \end{bmatrix} = \begin{bmatrix} a_1\cos(\theta_1) \\ a_1\sin(\theta_1) \end{bmatrix}$$
$$\begin{bmatrix} \dot{x_1} \\ \dot{y_1} \end{bmatrix} = \begin{bmatrix} -a_1\sin(\theta_1) \\ a_1\cos(\theta_1) \end{bmatrix}\dot{\theta_1}$$
$$\begin{bmatrix} x_2 \\ y_2 \end{bmatrix} = \begin{bmatrix} a_1\cos(\theta_1)+a_2\cos(\theta_1+\theta_2) \\ a_1\sin(\theta_1)+a_2\sin(\theta_1+\theta_2) \end{bmatrix}$$
$$\begin{bmatrix} \dot{x_2} \\ \dot{y_2} \end{bmatrix} = \begin{bmatrix} -a_1\sin(\theta_1)-a_2\sin(\theta_1+\theta_2) & -a_2\sin(\theta_1+\theta_2) \\ a_1\cos(\theta_1)+a_2\cos(\theta_1+\theta_2) & a_2\cos(\theta_1+\theta_2) \end{bmatrix}\begin{bmatrix} \dot{\theta_1} \\ \dot{\theta_2} \end{bmatrix}$$

$$\begin{aligned}
K_1 &= \frac{1}{2}m_1((\dot{x_1})^2+(\dot{y_1})^2) \\
&= \frac{1}{2}m_1(a_1)^2(\dot{\theta_1})^2 \\
K_2 &= \frac{1}{2}m_2((\dot{x_2})^2+(\dot{y_2})^2) \\
&= \frac{1}{2}m_2(a_1^2(\dot{\theta_1})^2+a_2^2(\dot{\theta_1}+\dot{\theta_2})^2+2a_1a_2(\dot{\theta_1})(\dot{\theta_1}+\dot{\theta_2})\cos(\theta_2)) \\
&= \frac{1}{2}m_2((a_1^2+a_2^2+2a_1a_2\cos(\theta_2))(\dot{\theta_1})^2+2a_2\dot{\theta_1}\dot{\theta_2}(a_2+a_1\cos(\theta_2))+a_2^2(\dot{\theta_2})^2) \\
P_1 &= m_1ga_1\sin(\theta_1) \\
P_2 &= m_2g(a_1\sin(\theta_1+a_2\sin(\theta_1+\theta_2))) \\
L &= K_1+K_2-(P_1+P_2) \\
\tau &= \frac{d}{dt}\frac{\partial L}{\partial \dot{q}}-\frac{\partial L}{\partial q}
\end{aligned}$$

$$\begin{aligned}
\tau_1 &= (m_1a_1^2+m_2(a_1^2+2a_1a_2\cos(\theta_2)+a_2^2))\ddot{\theta_1}+m_2a_2(a_1\cos(\theta_2)+a_2)\ddot{\theta_2} \\
&\quad -m_2a_1a_2\sin(\theta_2)(2\dot{\theta_2}\dot{\theta_1}+(\dot{\theta_2})^2)+(m_1+m_2)a_1g\cos(\theta_1)+m_2ga_2\cos(\theta_1+\theta_2) \\
\tau_2 &= m_2a_2(a_1\cos(\theta_2)+a_2)\ddot{\theta_1}+m_2a_2^2\ddot{\theta_2}+m_2a_1a_2(\dot{\theta_1})^2\sin(\theta_2)+m_2ga_2\cos(\theta_1+\theta_2) \\
\tau &= M(q,\dot{q})\ddot{q}+V(q,\dot{q})+G(q) \\
M(q,\dot{q}) &= \begin{bmatrix} m_1a_1^2+m_2(a_1^2+2a_1a_2\cos(\theta_2)+a_2^2) & m_2a_2(a_1\cos(\theta_2)+a_2) \\ m_2a_2(a_1\cos(\theta_2)+a_2) & m_2a_2^2 \end{bmatrix} \\
V(q,\dot{q}) &= \begin{bmatrix} -m_2a_1a_2\sin(\theta_2)(2\dot{\theta_2}\dot{\theta_1}+(\dot{\theta_2})^2) \\ m_2a_1a_2(\dot{\theta_1})^2\sin(\theta_2) \end{bmatrix} \\
G(q) &= \begin{bmatrix} (m_1+m_2)a_1g\cos(\theta_1)+m_2ga_2\cos(\theta_1+\theta_2) \\ m_2ga_2\cos(\theta_1+\theta_2) \end{bmatrix}
\end{aligned}$$

With given parameters ($m_1 = m_2 = a_1 = a_2 = 1$(standard unit))

$$M(q, \dot{q}) = \begin{bmatrix} 3 + 2\cos(\theta_2) & 1 + \cos(\theta_2) \\ 1 + \cos(\theta_2) & 1 \end{bmatrix}$$

$$V(q, \dot{q}) = \begin{bmatrix} -\sin(\theta_2)(2\dot{\theta}_2\dot{\theta}_1 + (\dot{\theta}_2)^2) \\ (\dot{\theta}_1)^2 \sin(\theta_2) \end{bmatrix}$$

$$G(q) = \begin{bmatrix} 2g\cos(\theta_1) + g\cos(\theta_1 + \theta_2) \\ g\cos(\theta_1 + \theta_2) \end{bmatrix}$$

Run the parameters.m to load all the parameters in base workspace. Simulate either model sim_1.slx or sim_2.slx . To visual 2DOF motion of two link robot run show_vis.m after simulation.