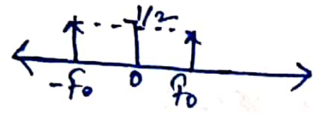
Problem 4: $x(t) = \cos(2\pi f_0 t + \theta) = \frac{1}{2}\left[e^{2\pi f_0 t j} \cdot e^{j\theta} + e^{-j\theta} \cdot e^{-2\pi f_0 t j}\right]$

$T_s = 1\,\mu s$

$x_s(t) = \frac{1}{T_s}\sum_k x(kT_s)\cdot \delta(t - kT_s)$

$\Downarrow$

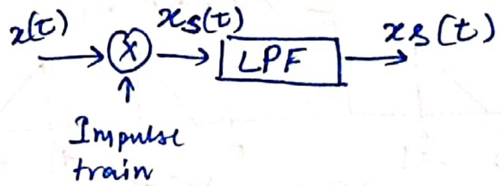
$|X(f)|$ $\quad$ ~~$X(f) = \frac{1}{2}\left[e^{-j\theta} X(f+f_0) + e^{j\theta} X(f-f_0)\right]$~~



$x(t) = \frac{1}{2}\left[e^{(2\pi f_0 t + \theta)j} + e^{-(2\pi f_0 t + \theta)j}\right]$

$X(f) = \frac{1}{2}\left[e^{j\theta}\delta(f - f_0) + e^{-j\theta}\delta(f + f_0)\right]$

$X_s(f) = \frac{1}{T_s}\sum_k X(f - kf_s)$



as $T_s = 1\,\mu s$.

$f_s = 10^6$ Hz.

for (a), (b), (c) parts $f_{0\,max} = 750$ Hz.

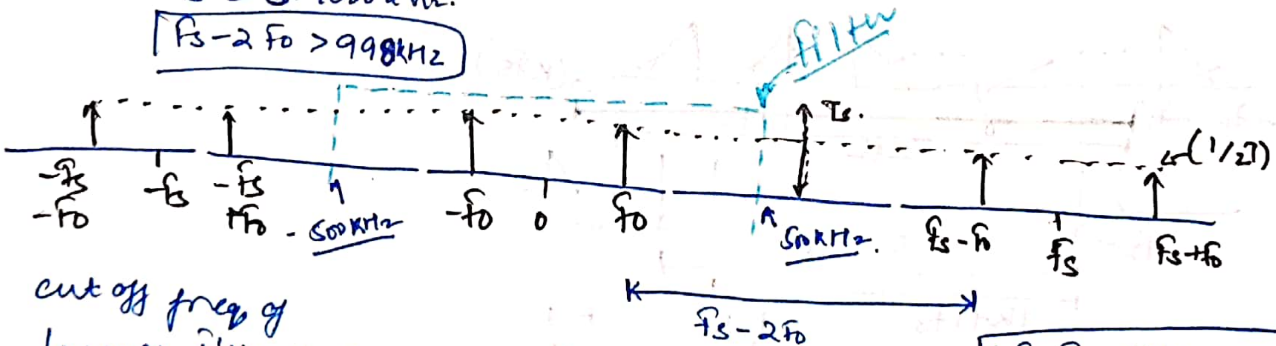
$X_s(f)$ will have copies of $X(f)$ seperated by ~~at least~~ (f_s)

$\Rightarrow$ two pulses has gap of atleast ~~has~~ $f_s - 2f_0$

as $f_0 < 1$ kHz.

$f_s = 1000$ kHz.

$\boxed{f_s - 2f_0 > 998\text{ kHz}}$

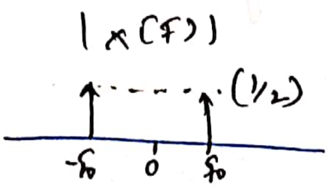


cut off freq of low pass filter: 500 kHz centred at (zero)

$\boxed{f_s - f_0 > 999\text{ kHz}}$

filter will only allow zero component to pass.

$\Rightarrow$



(a) $f_0 = 250$ Hz, $\theta = \pi/4$

$\boxed{\cos(500\pi t + \pi/4)}$

(b) $f_0 = 500$ Hz, $\theta = \pi/2$

$\boxed{\cos(1000\pi t + \pi/2)}$

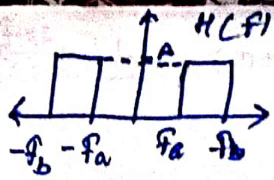
(c) $f_0 = 750$ Hz, $\theta = \pi/2$

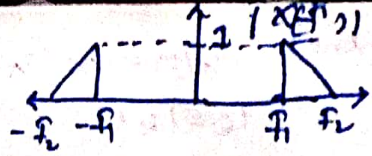
$\boxed{\cos(2\pi(750)t + \pi/2)}$

$X_s(f) = \left(\frac{1}{T_s}\right) \times \frac{T_s}{(2)}\left[e^{j\theta}\cdot\delta(f - f_0) + \delta(f + f_0)e^{-j\theta}\right]$

$x_s(t) = \cos(2\pi f_0 t + \theta)$

Problem 5:





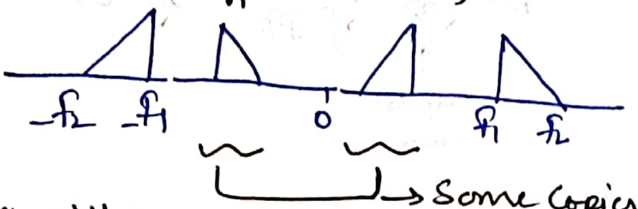
Consider sampling the signal $x(t)$ at the rate of f_s. such that $F_s > 2B$. but $f_s < 2f_2$ (highest freq. component)

Fourier transform of sampled $x(t)$ will have multiple copies of $X(f)$ shifted by (f_s) placed on same axis.

as per our assumption $2B \le f_s \le 2f_2$

$\Rightarrow$ different copies of $X(f)$ will occur in b/w ($-f_1$ to f_1).

$$\sum_n X(f+nf_s)$$

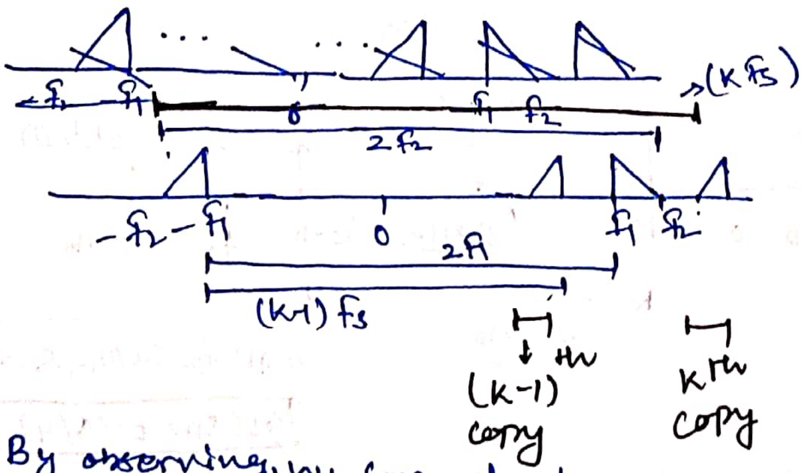


→ Some copies of $X(f)$.

Graph will look like this.

height of signal is not under consideration.

for some k^{th} copy.



By observing, we can clearly state that to prevent aliasing.

$$(k-1)f_s \le 2f_1 \quad — (1)$$

$$2f_2 \le kf_s \quad — (2)$$

$$\Rightarrow \boxed{\frac{2f_2}{k} \le f_s \le \frac{2f_1}{k-1}}$$

where $k \in N$ (Natural No.)

multipling respective terms of eq(1) & eq(2).

$(K-1)\, f_s \cdot 2f_2 \leq 2f_1 \cdot k f_s$ {by property}.

$(K-1)\, f_2 \leq f_1 k$ $\qquad , f_2 - f_1 = B.$

$(K-1) f_2 \leq f_2 K - BK$

$\Rightarrow -f_2 \leq -BK$

$\Rightarrow f_2 \geq BK$

$k \leq \frac{f_2}{B}$

as k can be a Natural No.

$K \leq \left[\frac{f_2}{B}\right]$; greatest Integer. func.

$K_{max} = \left[\frac{f_2}{B}\right].$

$$f_{s_{min}} = \frac{2f_2}{K_{max}} = \frac{2f_2}{[f_2/B]}$$

check whether it will follow nyquist criteria.

$\frac{f_2}{B} = m, \quad \left[\frac{f_2}{B}\right] = n.$

$$f_{s\,min} = \frac{2(mB)}{(n)}$$

$$\Rightarrow \boxed{\frac{f_{s\,min}}{2B} = \frac{m}{n} \geq 1}$$

If $m = n$ (f_2 is multiple of B).

$\Rightarrow \boxed{f_{s\,min} = 2B}.$

It will follow nyquist criteria.

$$\boxed{\begin{array}{l} f_a = f_1 \\ f_b = f_2 \\ \boxed{A = f_{max}} \end{array}}$$

$\underline{A = T_s},$

$f_1 = f_a$

$f_2 = f_b.$

$x_s(t) = h(t) * \frac{1}{T_s} \sum_K x(KT_s) \cdot \delta(t - KT_s)$

$X_s(f) = X(f) = H(f) \cdot \frac{1}{T_s} \sum_K X(f - K f_s)$