HW 4: Physics 545

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February 17, 2015

1a

Using the relaxation time approximation, we can write the Boltzmann equation as:

$$\dot{n_{\mathbf{p}}} + (\nabla_{\mathbf{p}}\epsilon_{\mathbf{p}}) \cdot (\nabla_{\mathbf{r}} n_{\mathbf{p}}) - (\nabla_{\mathbf{r}}\epsilon_{\mathbf{p}}) \cdot (\nabla_{\mathbf{p}} n_{\mathbf{p}}) = -\delta\bar{n}_{\mathbf{p}}/\tau_{\mathbf{p}} \tag{1}$$

Where the energies on the LHS are assumed to be that of the non-interacting system and do not depend on position. We are also interested in the static case only. Thus, the first and third terms above become 0 and we are left with

$$(v_p\hat{p}) \cdot (\nabla_{\mathbf{r}} n_{\mathbf{p}}) = -\delta\bar{n}_{\mathbf{p}}/\tau_{\mathbf{p}} \tag{2}$$

$$= -v_p\frac{\xi_{\mathbf{p}}}{T^2}\hat{p} \cdot \nabla T\frac{df}{dx} \tag{3}$$

Where f(x) is the Fermi function, and $\frac{df}{dx} = \frac{-1}{4\cosh(x/2)}$, $x = \xi/T$. Solving for $\delta\bar{n}_{\mathbf{p}}$:

$$\delta\bar{n}_{\mathbf{p}} = -\frac{\tau_{\mathbf{p}} v_{\mathbf{p}} \xi_{\mathbf{p}}}{4T^2\cosh(\xi_{\mathbf{p}}/2T)}\hat{p} \cdot \nabla T \tag{4}$$

1b

using the definition of $A(p)$ in the problem statement we get

$$A(p) = -\frac{\tau_{\mathbf{p}} v_{\mathbf{p}} \xi_{\mathbf{p}}}{4T^2\cosh(\xi_{\mathbf{p}}/2T)} \tag{5}$$

which we assume to be isotropic and only depends on the magnitude of $\mathbf{p}$. Plugging this into the RHS of the self-consistent collision integral:

$$\frac{2\pi}{\hbar}\int d^3p' \quad |V(\mathbf{p}-\mathbf{p}')|^2\delta(\epsilon_{\mathbf{p}}-\epsilon'_{\mathbf{p}})(\delta\bar{n}_{\mathbf{p}}-\delta\bar{n}_{\mathbf{p}'}) \tag{6}$$

$$=\frac{2\pi N_0}{\hbar}\int\frac{d\Omega_{p'}}{4\pi}d\xi_p\sqrt{\frac{\xi_p+\mu}{\epsilon_f}} \quad |V(\mathbf{p}-\mathbf{p}')|^2\delta(\xi_{\mathbf{p}}-\xi'_{\mathbf{p}})(\delta\bar{n}_{\mathbf{p}}-\delta\bar{n}_{\mathbf{p}'}) \tag{7}$$

$$=\frac{2\pi N_0 A(p)}{\hbar}\sqrt{\frac{\xi_p+\mu}{\epsilon_f}}\int\frac{d\Omega_{p'}}{4\pi} \quad |V(p(\hat{p}-\hat{p}'))|^2(\hat{p}\cdot\nabla T-\hat{p}'\cdot\nabla T) \tag{8}$$

$$=N_0\delta\bar{n}_{\mathbf{p}}\sqrt{\frac{\xi_p+\mu}{\epsilon_f}}V_{av}(p) \tag{9}$$

Where we have written the solid angle average of the interaction as $V_{av}(p)=\frac{2\pi}{\hbar}\int\frac{d\Omega_{p'}}{4\pi} \quad |V(p(1-\hat{p}\cdot\hat{p'}))|^2(1-\hat{p'}\cdot\hat{p})$ (assuming that the perpendicular bit vanishes), and used the result of 1a to get $\delta\bar{n}_{\mathbf{p}}$. We can also sub in the density of states in equilibrium $N(\epsilon_p)=N_0\sqrt{\frac{\xi_p+\mu}{\epsilon_f}}=N_0\sqrt{\frac{\epsilon_p}{\epsilon_f}}$ and finally solve for the relaxation time:

$$\tau_p=(N(\epsilon_p)V_{av}(p))^{-1} \tag{10}$$

1c

The energy current is:

$$\mathbf{q}=v_f\int d\xi_p\frac{d\Omega_p}{4\pi}N(\epsilon_p)\hat{p}\xi_p A(p)\hat{p}\cdot\nabla T \tag{11}$$

$$=-\frac{v_f}{4T^2}\int d\xi_p\frac{d\Omega_p}{4\pi}\hat{p}\frac{\xi_p^2 v_p\hat{p}\cdot\nabla T}{V_{av}(p)\cosh(\xi_p/2T)} \tag{12}$$

To do the integral above we must note that the $1/cosh$ bit is strongly peaked at the fermi level and that $v_p\approx v_f$ and $V_{av}(p)\approx V_{av}(p_f)=V_{fs}$ are both nearly constant in this range. We will also use the integral result $\int dx\frac{x^2}{\cosh(x)}=\frac{\pi^2}{6}$. With all this, the jth component of $\mathbf{q}$ is:

$$\mathbf{q}_j=-\frac{v_f^2}{4V_{fs}T^2}\left[\int\frac{d\Omega_p}{4\pi}\hat{p}_j\hat{p}_i\right](\delta_i T)\int 2T dx\frac{4T^2x^2}{\cosh(x)}=-\frac{v_f^2 T\pi^2}{9V_{fs}}(\delta_j T) \tag{13}$$

According to the definition:

$$\mathbf{q}_j=-\kappa\nabla T \tag{14}$$

$$\kappa=\frac{v_f^2 T\pi^2}{9V_{fs}} \tag{15}$$