HW 12: Physics 545

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1a

3a

The diagonalized Hamiltonian is:

$$\mathcal{H} = \sum_{\mathbf{k},s} E_{\mathbf{k}s} b^\dagger_{\mathbf{k}s} b_{\mathbf{k}s} \tag{1}$$

and the b operators are defined through the Bogolioubov transformation:

$$a_{\mathbf{k}s} = u_{\mathbf{k}} b_{\mathbf{k}s} - (i\sigma_y)_{ss'} v^*_{\mathbf{k}} b^\dagger_{-\mathbf{k}s'} \tag{2}$$

which results in the Bogolioubov-de Gennes equations

$$E_{\mathbf{k}s} \begin{pmatrix} u_{\mathbf{k}} \\ v_{\mathbf{k}} \end{pmatrix} = \begin{pmatrix} \epsilon_{\mathbf{k}s} & \Delta \\ \Delta^* & -\epsilon_{\mathbf{k}\bar{s}} \end{pmatrix} \begin{pmatrix} u_{\mathbf{k}} \\ v_{\mathbf{k}} \end{pmatrix} \tag{3}$$

Where $\epsilon_{\mathbf{k}s} = \xi_{\mathbf{k}} - \mu_B H s$ The eigenvalues of such an equation are:

$$E_{\mathbf{k}s} = \sqrt{\xi^2_{\mathbf{k}} + |\Delta|^2} - \mu_B H s \tag{4}$$

3b

The magnetisation is:

$$\begin{aligned}
M &= \mu_B \sum_{\mathbf{k},s} \langle a^\dagger_{\mathbf{k}\uparrow} a_{\mathbf{k}\uparrow} - a^\dagger_{\mathbf{k}\downarrow} a_{\mathbf{k}\downarrow} \rangle && (5)\\
&= \mu_B \sum_{\mathbf{k},s} \langle (u_{\mathbf{k}} b_{\mathbf{k}\uparrow} - v^*_{\mathbf{k}} b^\dagger_{-\mathbf{k}\downarrow})^\dagger (u_{\mathbf{k}} b_{\mathbf{k}\uparrow} - v^*_{\mathbf{k}} b^\dagger_{-\mathbf{k}\downarrow}) - (u_{\mathbf{k}} b_{\mathbf{k}\downarrow} + v^*_{\mathbf{k}} b^\dagger_{-\mathbf{k}\uparrow})^\dagger (u_{\mathbf{k}} b_{\mathbf{k}\downarrow} + v^*_{\mathbf{k}} b^\dagger_{-\mathbf{k}\uparrow}) \rangle && (6)\\
&= \mu_B \sum_{\mathbf{k},s} |u_{\mathbf{k}}|^2 \langle b^\dagger_{\mathbf{k}\uparrow} b_{\mathbf{k}\uparrow} - b^\dagger_{\mathbf{k}\downarrow} b_{\mathbf{k}\downarrow} \rangle + |v_{\mathbf{k}}|^2 \langle b_{-\mathbf{k}\downarrow} b^\dagger_{-\mathbf{k}\downarrow} - b_{-\mathbf{k}\uparrow} b^\dagger_{-\mathbf{k}\uparrow} \rangle && (7)\\
&= \mu_B \sum_{\mathbf{k},s} (f_{\mathbf{k}\uparrow} - f_{\mathbf{k}\downarrow}) && (8)
\end{aligned}$$

3c