Self Consistent Calculation of Δ for Pauli Limited Superconductors

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At zero temperature the equation for Δ is:

$$\Delta_{\mathbf{k}} = -\frac{1}{2}\sum_{\mathbf{l}} \frac{\Delta_{\mathbf{l}} V_{\mathbf{kl}}}{E_{\mathbf{l}}} \quad (Tinkham) \tag{1}$$

At finitie temperature the equation in:

$$\Delta_{\mathbf{k}} = -\frac{1}{4}\sum_{\mathbf{l}} \frac{\Delta_{\mathbf{l}} V_{\mathbf{kl}}}{E_{\mathbf{l}}} (\tanh((E_{\mathbf{k}} + \mu_e H)/2T) + \tanh((E_{\mathbf{k}} - \mu_e H)/2T)) \quad (Tinkham) \tag{2}$$

Where we have used $E_{\mathbf{l}} = \sqrt{(\epsilon_{\mathbf{l}} - \mu)^2 + \Delta_{\mathbf{l}}^2}$ and $\epsilon_{\mathbf{l}}$ is the free particle energy. Now we assume some momentum space (relative coordinate) profile for the BCS potential $V_{\mathbf{kl}} = -V\Gamma_{\mathbf{k}}\Gamma_{\mathbf{l}}$ and $\Delta_{\mathbf{k}} = \Delta_{T,H}\Gamma_{\mathbf{k}}$. With this definition we can rewrite equations 1 and 2:

$$1 = \frac{V}{2}\sum_{\mathbf{l}} \frac{\Gamma_{\mathbf{l}}^2}{E_{\mathbf{l}}} \tag{3}$$

$$1 = \frac{V}{4}\sum_{\mathbf{l}} \frac{\Gamma_{\mathbf{l}}^2}{E_{\mathbf{l}}} (\tanh((E_{\mathbf{k}} + \mu_e H)/2T) + \tanh((E_{\mathbf{k}} - \mu_e H)/2T)) \tag{4}$$

To compute the free energy we use the self consistent equation for Δ (equation 2), but write it as the differential of free energy with Δ^*, because the self consistent Δ minimizes the free energy.

$$\frac{dF}{d\Delta^*} = 0 = \Delta_{\mathbf{k}} + \frac{1}{4}\sum_{\mathbf{l}} \frac{\Delta_{\mathbf{l}} V_{\mathbf{kl}}}{E_{\mathbf{l}}} (\tanh((E_{\mathbf{k}} + \mu_e H)/2T) + \tanh((E_{\mathbf{k}} - \mu_e H)/2T)) \tag{5}$$

Since equality must hold for all values of $\mathbf{k}$ we can drop the $\Gamma_{\mathbf{k}}$. Integrating this equation yields the free energy relative to the normal state.

$$F_{SC} - F_N = \int_0^{\Delta_{B,T}} d\Delta \left(\Delta - \frac{V}{4}\sum_{\mathbf{l}} \frac{\Delta\Gamma_{\mathbf{l}}^2}{E_{\mathbf{l}}} (\tanh((E_{\mathbf{k}} + \mu_e H)/2T) + \tanh((E_{\mathbf{k}} - \mu_e H)/2T)) \right) \tag{6}$$

Where $\Delta_{B,T}$ is the self consistent value of Δ calculated from equation 2

S wave

For S wave superconductors the momentum space profile $\Gamma_{\mathbf{k}} = 1$. We now can relate the zero field transition temperature to the value of Δ (Δ_{00}) at zero field and temperature by solving equation 4 for $H = 0$, $\Delta = 0$ and $T = T_c$ along with equation 3.

$$1 = VN_0 \int_0^{\epsilon_m} \frac{d\xi}{\sqrt{\Delta_{00}^2 + \xi^2}} = VN_0 \sinh^{-1} \frac{\epsilon_m}{\Delta_{00}} \tag{7}$$

$$1 = VN_0 \int_0^{\epsilon_m} \frac{d\xi}{\xi} \tanh(\xi/2T) = VN_0 \ln\left[\frac{2e^\gamma \epsilon_m}{\pi T_c}\right] \tag{8}$$

Where $\gamma = 0.57721$ is the EulerMascheroni constant. In the weak coupling limit ($VN_0 >> 1$) we arrive at $\frac{\Delta_{00}}{T_c} = \frac{pi}{e^\gamma} = 1.764$

D wave

D wave superconductors have a $x^2 - y^2$ ($\Gamma_{\mathbf{k}} = \cos(2\theta_k)$)or xy ($\Gamma_{\mathbf{k}} = \sin(2\theta_k)$) symmetry. The solutions to equations three and four are now:

$$1 = \frac{VN_0}{2} \ln\left[\frac{4\epsilon_m}{\Delta_{00} e^{1/2}}\right] \tag{9}$$

$$1 = \frac{VN_0}{2} \ln\left[\frac{2e^\gamma \epsilon_m}{\pi T_c}\right] \tag{10}$$

Where we have taken the limit of $\epsilon_m >> \Delta_{00}$ to do the θ integral in equation 3 we arrive at the solution $\frac{\Delta_{00}}{T_c} = 2.1397$

Sharp D wave

For a "Sharp" D wave order parameter we assume that the order parameter still has $x^2 - y^2$ or xy symmetry, but instead of using sine or cosine we use a polynomial of degree n:

$$\Gamma_{\mathbf{k}} = -\left(\theta_{\mathbf{k}} \frac{4}{\pi} - 1\right)^n + 1 \tag{11}$$

$$\Gamma_{\mathbf{k}} = \left(\theta_{\mathbf{k}} \frac{4}{\pi} - 3\right)^n - 1 \tag{12}$$

$$\Gamma_{\mathbf{k}} = -\left(\theta_{\mathbf{k}} \frac{4}{\pi} - 5\right)^n + 1 \tag{13}$$

$$\Gamma_{\mathbf{k}} = \left(\theta_{\mathbf{k}} \frac{4}{\pi} - 7\right)^n - 1 \tag{14}$$