Non-uniform Superconductors and Magnetism

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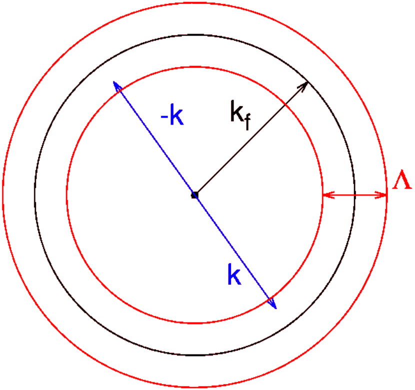
What about a uniform superconductor???

FORMATION OF "COOPER PAIRS" FROM ATTRACTION BETWEEN ELECTRONS WITH OPPOSITE MOMENTUM NEAR FS



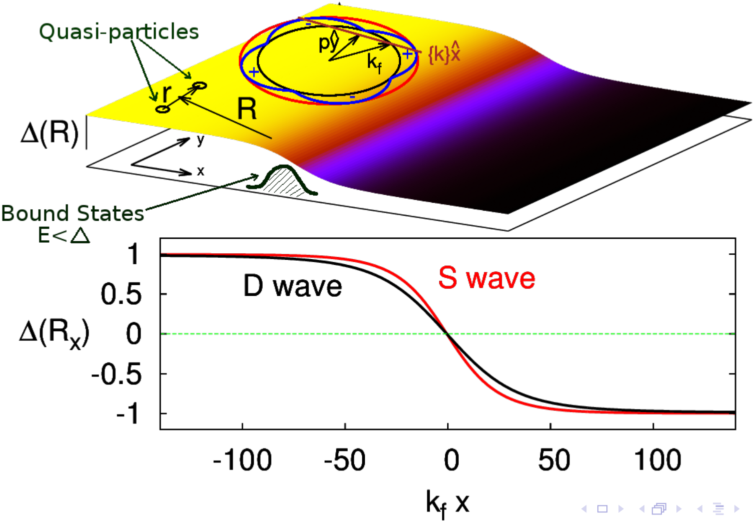
MEAN FIELD HAMILTONIAN $\Rightarrow$ $\mathcal{H}_\Delta = \sum_{\mathbf{k}} \left[\Delta_{\hat{k}} a^\dagger_{\mathbf{k}\uparrow} a^\dagger_{-\mathbf{k}\downarrow} + h.c. \right]$

BdG Transformation $\Rightarrow$ $\boxed{\text{QUASI-PARTICLES} = \text{PARTICLE} + \text{HOLE}}$

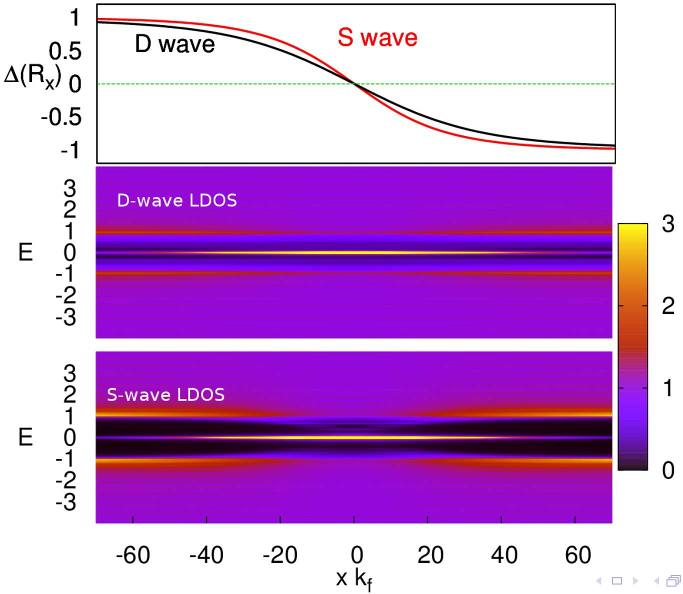
What is a non-uniform superconductor???

$\hat{y}(\hat{x})$ momentum $\Rightarrow$ $\boxed{\text{GOOD(BAD)}}$ quantum number

$$\Rightarrow \Delta(\mathbf{x},\mathbf{x}') = \Delta(R_x)\int d\mathbf{k}\;\Delta_{\hat{k}} e^{-i\mathbf{k}\cdot\mathbf{r}} \qquad \Delta_{\hat{k}} = \{1, sin(2\theta_{\hat{k}})\}$$

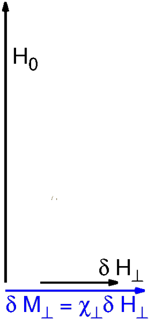


Local Density of States



Question???

How do bound states in a non-uniform superconductor affect TRANSVERSE magnetic properties?



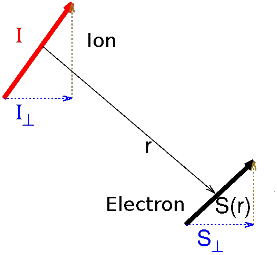
MAGNETIZATION

$\delta M_\perp(\mathbf{q}) = \bar{\chi}_\perp(\mathbf{q})\delta H_\perp(\mathbf{q})$

HYPERFINE INT.

$\Rightarrow$RELAXATION RATE

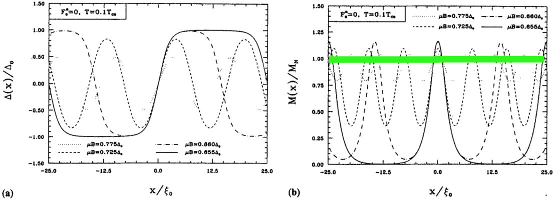
$\Rightarrow\Rightarrow$NMR measurement $T_\perp^{-1}$



Motivation

-FFLO

Burkhardt and Rainer (1994)



-Vortex

Kakuyanagi et al. (2003)

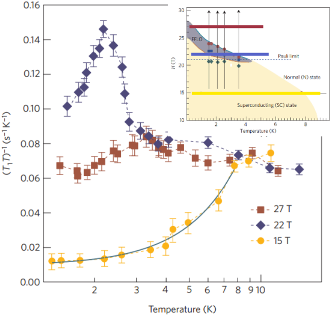
VOLUME 90, NUMBER 19 PHYSICAL REVIEW LETTERS week ending 16 MAY 2003

Antiferromagnetic Vortex Core in $Tl_2Ba_2CuO_{6+\delta}$ Studied by Nuclear Magnetic Resonance

K. Kakuyanagi,[1] K. Kumagai,[1] Y. Matsuda,[2] and M. Hasegawa[3]

-Organic SCs

Mayaffre et al. (2014)



-Topological States

Kashiwaya and Tanaka (2000)

Tunnelling effects on surface bound states in unconventional superconductors

Satoshi Kashiwaya† and Yukio Tanaka‡

† Electrotechnical Laboratory and CREST, Japan Science and Technology Cooperation (JST) Umezono, Tsukuba, Ibaraki 305-8568, Japan
‡ Department of Applied Physics, Nagoya University and CREST, Japan Science and Technology Cooperation (JST) Chikusaku, Nagoya 464-8603, Japan

Received 25 November 1999

SUSCEPTIBILITY

PERTURBATION $\Rightarrow$ $\boxed{\mathcal{V} = \mu_B \int d\mathbf{x} \quad \Psi^\dagger_\mu(\mathbf{x}, t)\boldsymbol{\sigma}_{\mu\nu} \cdot \delta\mathbf{H}(\mathbf{x}, t)\Psi_\nu(\mathbf{x}, t)}$

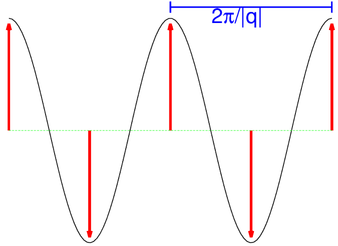
$$\chi_{\alpha\beta}(\mathbf{x}, \mathbf{x}', \omega) = \frac{i\mu_B^2}{\chi_0} \int dt \quad e^{i\omega t}\langle[S_\alpha(\mathbf{x}, t), S_\beta(\mathbf{x}', 0)]\theta(t)\rangle$$

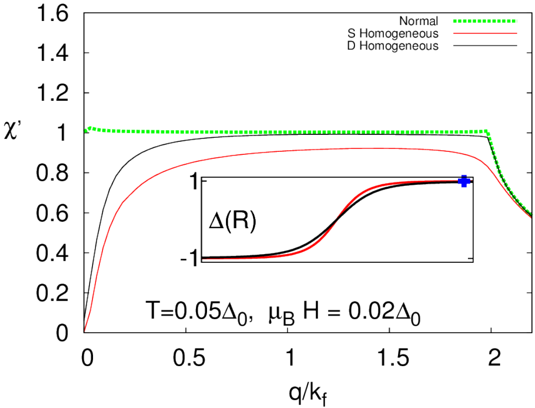
$$S_\alpha(\mathbf{x}, t) = \Psi^\dagger_\mu(\mathbf{x}, t)\sigma^\alpha_{\mu\nu}\Psi^\dagger_\nu(\mathbf{x}, t)$$

$\Rightarrow$Transverse response

$(\alpha, \beta) = (x, x)$ *or* (y, y)

$$\boxed{\chi(\mathbf{R}, \mathbf{q}) = \int d\mathbf{r} e^{-i\mathbf{q}\cdot\mathbf{r}}\chi(\mathbf{R}, \mathbf{r})}$$





$$\frac{\Delta_0}{\epsilon_f} = 0.05, \quad \text{static limit } \omega \to 0$$

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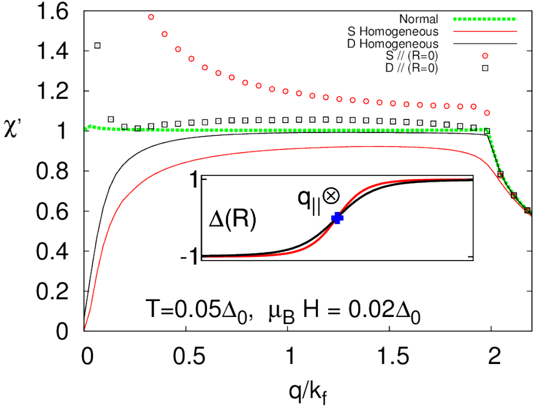

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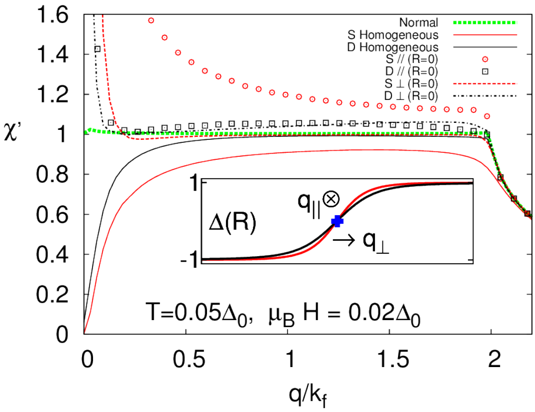

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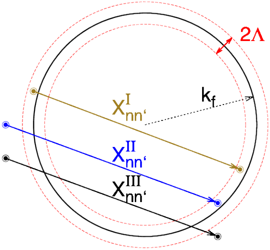
SUSCEPTIBILITY

$$\chi_i(\mathbf{R},\mathbf{q}) = \sum_{\mathbf{nn}'}{}^{i} X^{i}_{\mathbf{nn}'}(\mathbf{R},\mathbf{q}), \quad i = \{I,\ II,\ III\}$$



I: $\epsilon_{\mathbf{n}} < \Lambda,\ \epsilon_{\mathbf{n}'} < \Lambda$

II: $\epsilon_{\mathbf{n}} < \Lambda,\ \epsilon_{\mathbf{n}'} > \Lambda$

$\mathbf{n} \leftrightarrow \mathbf{n}'$

III: $\epsilon_{\mathbf{n}} > \Lambda,\ \epsilon_{\mathbf{n}'} > \Lambda$

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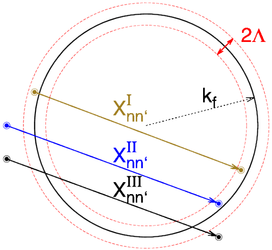
$$\delta\chi_i = \chi_i - \chi_i^N$$



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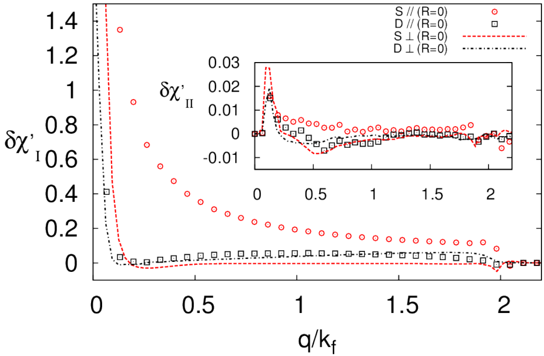
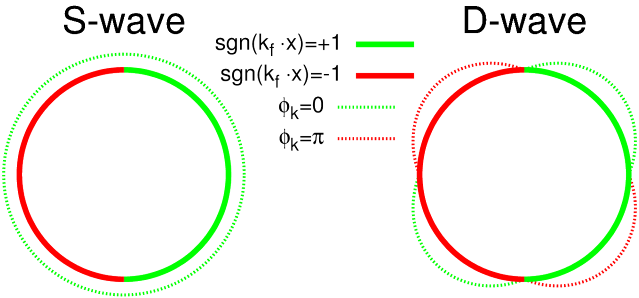


ANDREEV APPROXIMATION

$$\boxed{\mathbf{n} \rightarrow (\hat{k}, n)}$$

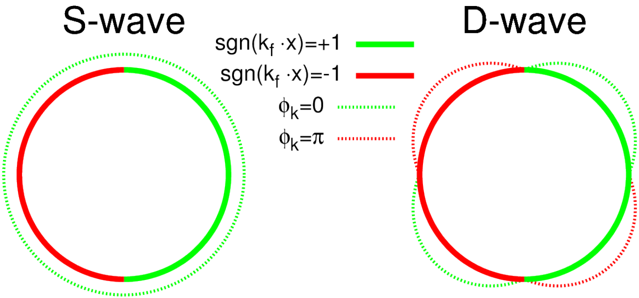


LOWEST ENERGY STATE ALONG PARTICULAR $\hat{k}$, $\rightarrow \epsilon_{\hat{k}0} << |\Delta_{\hat{k}}|$

TRY TO MAXIMIZE SCATTERING AMPLITUDE $X^{l}_{\mathbf{nn}'}(\mathbf{R}, \mathbf{q})$

ANDREEV APPROXIMATION

$$\boxed{\mathbf{n} \to (\hat{k}, n)}$$



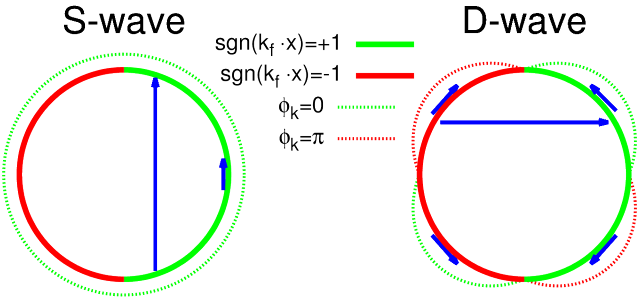
$$\chi_l^{ABS} = \sum_{\hat{k}\hat{k}'\mu} (1 + S_{\hat{k}\hat{k}'})\Pi^{+}_{\hat{k}\hat{k}'\mu} + (1 - S_{\hat{k}\hat{k}'})\Pi^{-}_{\hat{k}\hat{k}'\mu}$$

$$\Pi^{\pm}_{\hat{k}\hat{k}'\mu} \propto \frac{1}{\epsilon_{\hat{k}0} \mp \epsilon_{\hat{k}'0} + 2\sigma^z_{\mu\mu}\mu_B H}$$

$$S_{\hat{k}\hat{k}'} \approx e^{-i\zeta}, \quad \zeta = \frac{\pi}{2}(sgn(\hat{k}\cdot\hat{x}) - sgn(\Delta_{\hat{k}}) - sgn(\hat{k}'\cdot\hat{x}) + sgn(\Delta_{\hat{k}'}))$$

ANDREEV APPROXIMATION

$$\boxed{\mathbf{n} \rightarrow (\hat{k}, n)}$$



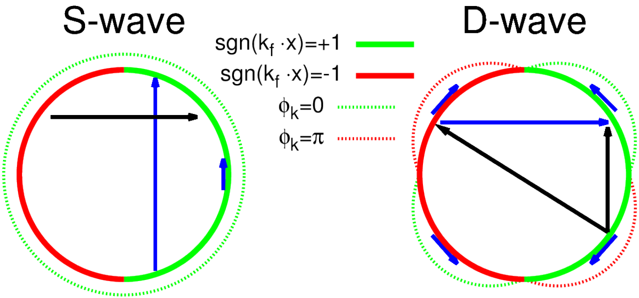
$$\chi_I^{ABS} = \sum_{\hat{k}\hat{k}'\mu} (1 + S_{\hat{k}\hat{k}'})\Pi^{+}_{\hat{k}\hat{k}'\mu} + (1 - S_{\hat{k}\hat{k}'})\Pi^{-}_{\hat{k}\hat{k}'\mu}$$

$$S_{\hat{k}\hat{k}'} = +1,\ \zeta = 0, \pm 4$$

$$S_{\hat{k}\hat{k}'} \approx e^{-i\zeta}, \quad \zeta = \frac{\pi}{2}(sgn(\hat{k}\cdot\hat{x}) - sgn(\Delta_{\hat{k}}) - sgn(\hat{k}'\cdot\hat{x}) + sgn(\Delta_{\hat{k}'}))$$

ANDREEV APPROXIMATION

$$\boxed{\mathbf{n} \rightarrow (\hat{k}, n)}$$



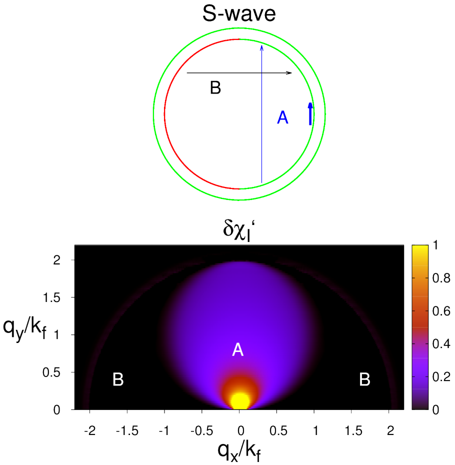
$$\boxed{\chi_l^{ABS} = \sum_{\hat{k}\hat{k}'\mu} (1 + S_{\hat{k}\hat{k}'})\Pi^{+}_{\hat{k}\hat{k}'\mu} + (1 - S_{\hat{k}\hat{k}'})\Pi^{-}_{\hat{k}\hat{k}'\mu}}$$

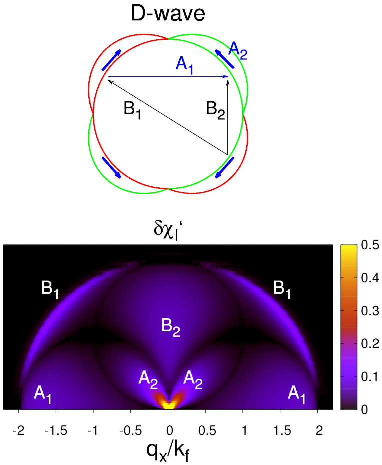
$$\boxed{S_{\hat{k}\hat{k}'} = -1,\ \zeta = \pm 2}$$

$$\boxed{S_{\hat{k}\hat{k}'} \approx e^{-i\zeta}, \quad \zeta = \frac{\pi}{2}(sgn(\hat{k}\cdot\hat{x}) - sgn(\Delta_{\hat{k}}) - sgn(\hat{k}'\cdot\hat{x}) + sgn(\Delta_{\hat{k}'}))}$$

SUSCEPTIBILITY (ANDREEV APPROXIMATION)

REAL PART $T = 0.05\Delta_0,\ \mu_B H = 0.02\Delta_0,\ R = 0$

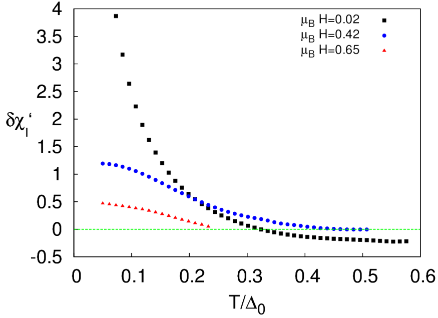




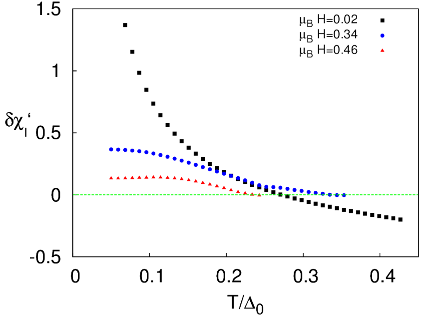
FERROMAGNETIC RESPONSE

$\rightarrow$ decreasing with temperature and field*

S WAVE (R=0)

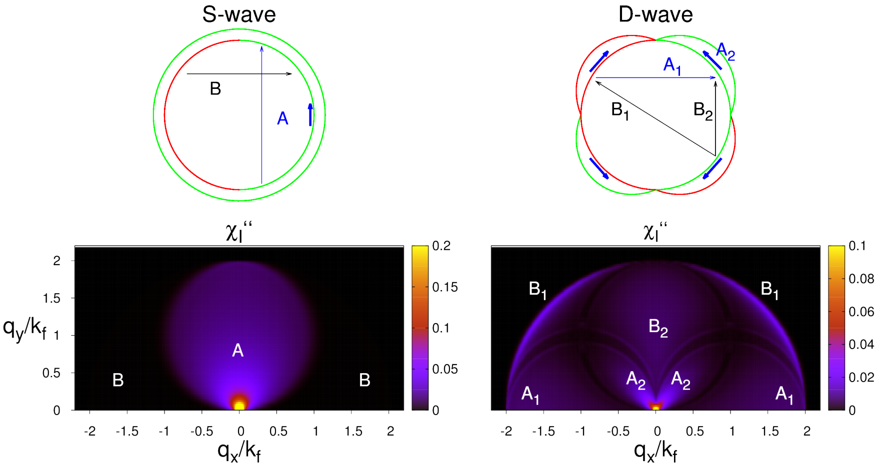


D WAVE (R=0)



* Excluding narrow region near $\mu_B H = 0$

IMAGINARY PART $T = 0.05\Delta_0,\ \mu_B H = 0.02\Delta_0,\ R = 0$

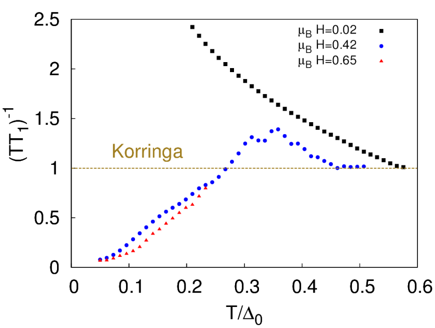


RELAXATION RATE

$$(TT_1)^{-1}(\mathbf{R}) = \frac{1}{\omega} \sum_{\mathbf{q}} \mathcal{I}m(\chi_{\perp I}(\mathbf{R}, \mathbf{q}))$$

$$\boxed{\text{Korringa (normal) limit} \quad \Rightarrow \quad (TT_1)^{-1} = constant}$$

S WAVE (R=0)



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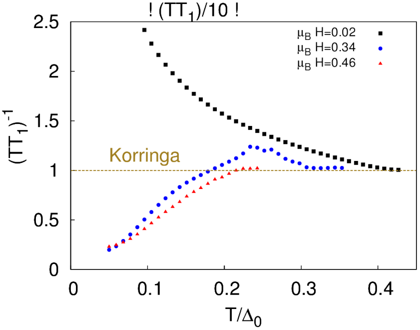

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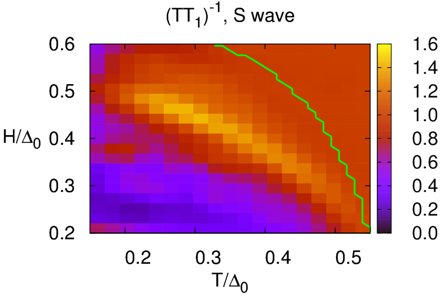
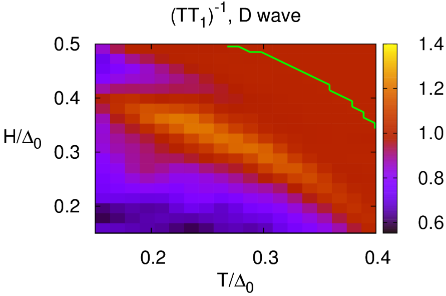
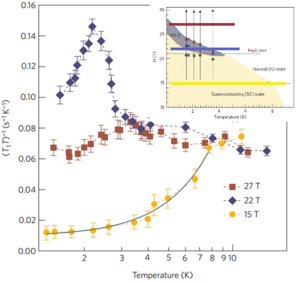






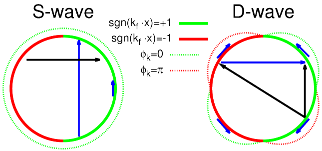
?? Will this enhancement change in H-T if...

a) we move DWs closer together?

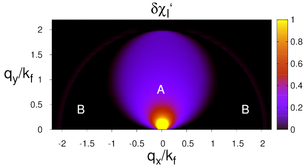
b) we allow Δ to change phase?

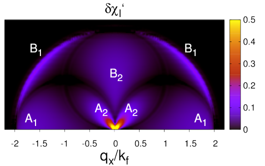
Conclusions

Conditions for magnetic coherence of bound states near a domain wall



Increases $\chi_\perp(\mathbf{q})$ $\Rightarrow$





Increases $(TT_1)^{-1}_\perp$ $\Rightarrow$