Electronic Spin Susceptibility in Superconductor-Normal Metal Interfaces

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We calculate the wave-vector dependent electronic spin susceptibility $\chi_{\alpha\beta}(\mathbf{q},\mathbf{H}_0,\mathbf{R})$ around a Superconductor-Normal metal interface with and without a uniform magnetic field $\mathbf{H}_0$ at zero temperature. We consider the 2D cylindrical Fermi surface of free electrons ($\xi_{\mathbf{k}} = \frac{\mathbf{k}^2}{2m^*} - \epsilon_F$) on both sides of the interface. For the superconductor we consider both S and D-wave order parameters. We identify several features such as the tendency for enhanced susceptibility for wave vectors $\mathbf{q}$ which correspond to those defined by the distance $\mathbf{R}$ from the interface and it's steepness.

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INTRODUCTION

EQUATIONS

Our model is a position dependent, mean-field SC Hamiltonian with 2D cylindrical FS, and electrons interacting with uniform magnetic field $\mathbf{H}_0$ through Zeeman term: $\mathcal{H} = \mathcal{H}_0 + V$

$$\mathcal{H}_0(\mathbf{r}) = \sum_{\mathbf{k}\mu} \xi_{\mathbf{k}} c^\dagger_{\mathbf{k}\mu} c_{\mathbf{k}\mu} + \sum_{\mathbf{k}} \left(\Delta_{\mathbf{k}}(\mathbf{r}) c^\dagger_{\mathbf{k}\uparrow} c^\dagger_{-\mathbf{k}\downarrow} + h.c. \right) + \mu_{\rm B} \sum_{\mathbf{k}\mu\nu} c^\dagger_{\mathbf{k}\mu}\, \boldsymbol{\sigma}_{\mu\nu} \mathbf{H}_0\, c_{\mathbf{k}\nu} \tag{1}$$

and for linear response we include a $\mathbf{q}$-dependent perturbation of the magnetic field $\delta\mathbf{H}(\mathbf{R}) = \delta\mathbf{H}_{\mathbf{q}} e^{i\mathbf{q}\cdot\mathbf{R}}$, $V = \mu_{\rm B} \sum_{\mathbf{k}\mu\nu} c^\dagger_{\mathbf{k}+\mathbf{q}\mu}\, \boldsymbol{\sigma}_{\mu\nu} \delta\mathbf{H}_{\mathbf{q}}\, c_{\mathbf{k}\nu}$, where $\mu_{\rm B}$ is the magnetic moment of electron. The electronic dispersion in the normal state is $\xi_{\mathbf{k}} = \frac{\mathbf{k}^2}{2m^*} - \epsilon_F$. The resulting magnetization has uniform part and $\mathbf{q}$-dependent perturbation:

$$M_\alpha(\mathbf{R}) = M_{0\alpha}(\mathbf{R},\mathbf{H}_0) + \chi_{\alpha\beta}(\mathbf{R},\mathbf{q}) \delta H_\beta e^{i\mathbf{q}\cdot\mathbf{R}} \tag{2}$$

with $\mathbf{M}_0(\mathbf{R},t) = \mu_{\rm B}\langle \mathbf{S}(\mathbf{R},t)\rangle_0$, and susceptibility is a two particle correlation function [1]:

$$\chi_{\alpha\beta}(\mathbf{x},\mathbf{x}',t) = \frac{i\mu_{\rm B}^2}{\hbar} \langle [S_\alpha(\mathbf{x},t), S_\beta(\mathbf{x}',0)]\theta(t)\rangle_0 \tag{3}$$

where $\mathbf{S}(\mathbf{x},t) = \sum_{\mu\nu} \psi^\dagger_\mu(\mathbf{x},t)\, \boldsymbol{\sigma}_{\mu\nu}\, \psi_\nu(\mathbf{x},t)$, $\psi_\nu(\mathbf{x},t) = \sum_{\mathbf{k}} c_{\mathbf{k}\nu}(t) \varphi_{\mathbf{k}\nu}(\mathbf{x})$, $c_{\mathbf{k}\nu}(t) = e^{i\mathcal{H}_0 t} c_{\mathbf{k}\mu} e^{-i\mathcal{H}_0 t}$, $\varphi_{\mathbf{k}\nu} = e^{i\mathbf{k}_\nu\cdot\mathbf{x}}$; subscript 0 indicates the average over ensemble (1).

$$\begin{aligned}
\chi(\mathbf{r}_x, q_y, \mathbf{R}=0) = \frac{\mu_B^2}{\hbar} \sum_{\mathbf{k}\mathbf{p}_x\mu} &\Bigg[(u_{\mathbf{k}1}\, v_{\mathbf{k}2}\, v_{\mathbf{p}1}\, u_{\mathbf{p}2} - u_{\mathbf{k}1}\, u_{\mathbf{k}2}\, v_{\mathbf{p}1}\, v_{\mathbf{p}2}) e^{i(-\mathbf{k}1_x\mathbf{r}_x/2 - \mathbf{k}2_x\mathbf{r}_x/2 - \mathbf{p}1_x\mathbf{r}_x/2 - \mathbf{p}2_x\mathbf{r}_x/2)}\Big|_{-\mathbf{k}1_y - \mathbf{p}1_y - q_y = 0} \\
&+ (u_{\mathbf{k}2}\, v_{\mathbf{k}1}\, v_{\mathbf{p}2}\, u_{\mathbf{p}1} - v_{\mathbf{k}1}\, v_{\mathbf{k}2}\, u_{\mathbf{p}1}\, u_{\mathbf{p}2}) e^{i(\mathbf{k}1_x\mathbf{r}_x/2 + \mathbf{k}2_x\mathbf{r}_x/2 + \mathbf{p}1_x\mathbf{r}_x/2 + \mathbf{p}2_x\mathbf{r}_x/2)}\Big|_{\mathbf{k}1_y + \mathbf{p}1_y - q_y = 0} \Bigg] \frac{1 - f_{-\mathbf{p}1} - f_{\mathbf{k}1}}{\omega_{\mathbf{k}1} + \omega_{-\mathbf{p}1}} \\
&+ \Bigg[(u_{\mathbf{k}1}\, u_{\mathbf{k}2}\, u_{\mathbf{p}1}\, u_{\mathbf{p}2} + u_{\mathbf{k}1}\, v_{\mathbf{k}2}\, v_{\mathbf{p}2}\, u_{\mathbf{p}1}) e^{i(-\mathbf{k}1_x\mathbf{r}_x/2 - \mathbf{k}2_x\mathbf{r}_x/2 + \mathbf{p}1_x\mathbf{r}_x/2 + \mathbf{p}2_x\mathbf{r}_x/2)}\Big|_{-\mathbf{k}1_y + \mathbf{p}1_y - q_y = 0} \\
&+ (v_{\mathbf{k}1}\, v_{\mathbf{k}2}\, v_{\mathbf{p}1}\, v_{\mathbf{p}2} + u_{\mathbf{k}2}\, v_{\mathbf{k}1}\, v_{\mathbf{p}1}\, u_{\mathbf{p}2}) e^{i(\mathbf{k}1_x\mathbf{r}_x/2 + \mathbf{k}2_x\mathbf{r}_x/2 - \mathbf{p}1_x\mathbf{r}_x/2 - \mathbf{p}2_x\mathbf{r}_x/2)}\Big|_{\mathbf{k}1_y - \mathbf{p}1_y - q_y = 0} \Bigg] \frac{f_{\mathbf{k}1} - f_{\mathbf{p}1}}{\omega_{\mathbf{k}1} - \omega_{\mathbf{p}1}}
\end{aligned}$$

To find the Fourier Transform wrt the x coordinate we can use a fast fourier transform from the spacial domain $x \in [-L : L]$ to momentum space $q_x \in [-n\pi : n\pi]$ using $2N+1$ points ($N = nL$):

$$\chi(\mathbf{q}) = \sum_{i=1}^{2*N+1} e^{-iq_x\mathbf{r}_x(i)} \chi(\mathbf{r}_x(i), q_y, \mathbf{R}=0) \tag{4}$$

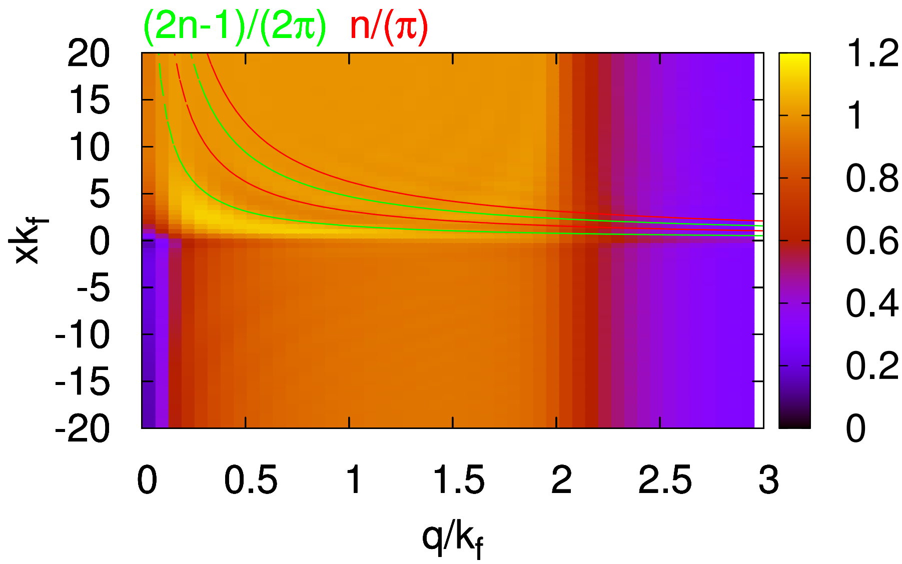

FIG. 1. S wave, No Field

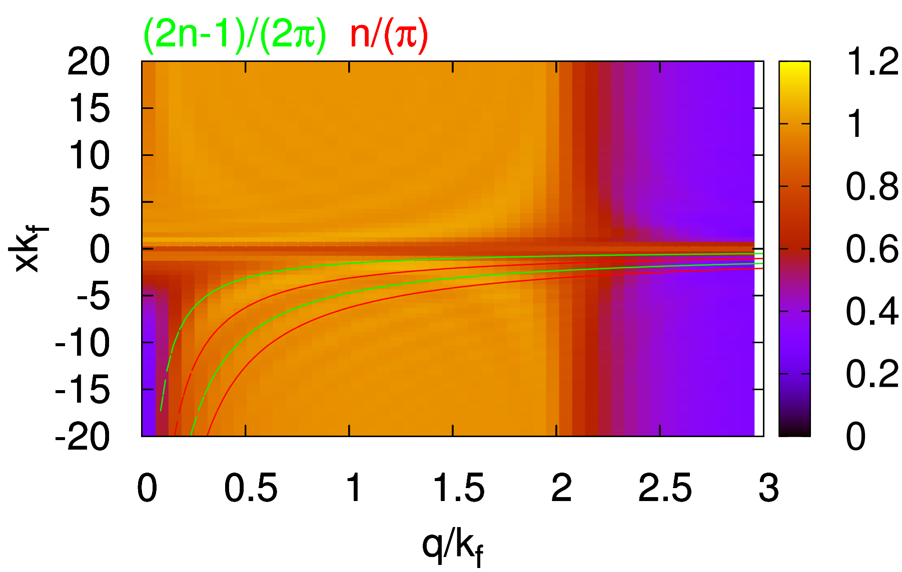

FIG. 2. D wave, No Field

RESULTS

Below are surface plots for the 2D Normalized susceptibility for various conditions.
The q/k_f axis is the in the $\hat{x}$, and the xk_f axis indicates the distance from the domain wall which is at $xk_f = 0$.
For all the plots, $\Delta = 0.05\epsilon_f$, and $T = 0$

[1] G. D. Mahan, *Many-Particle Physics*, 3rd ed. (Plenum Publishers, 2000).

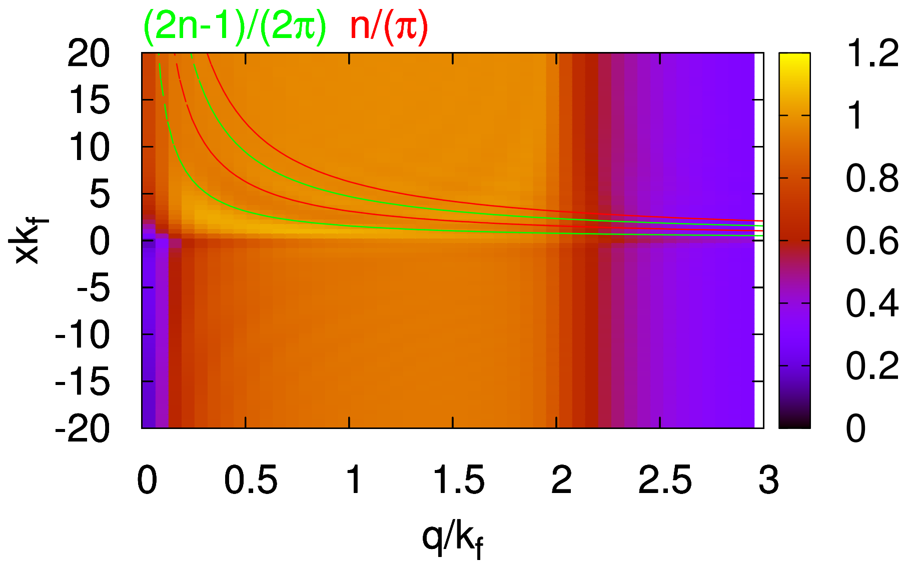

FIG. 3. S wave, $\perp$ Field ($\mu_B H = 0.6\Delta_0$)

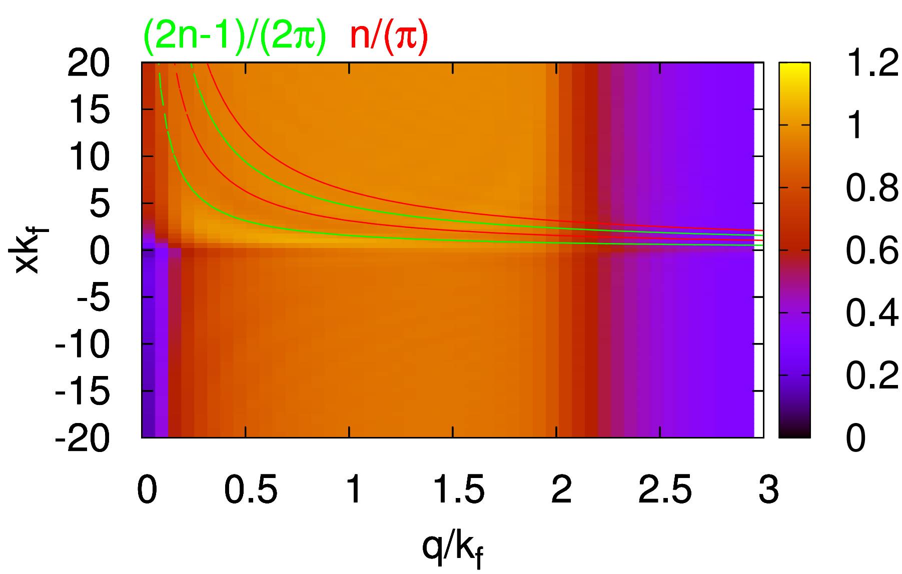

FIG. 4. S wave, $\parallel$ Field ($\mu_B H = 0.6\Delta_0$)

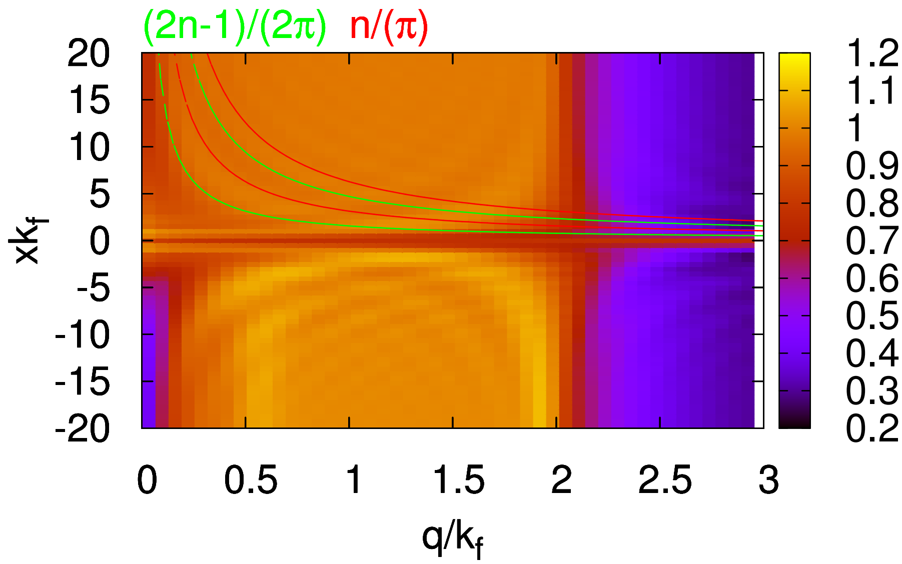

FIG. 5. D wave, $\perp$ Field ($\mu_B H = 0.6\Delta_0$)

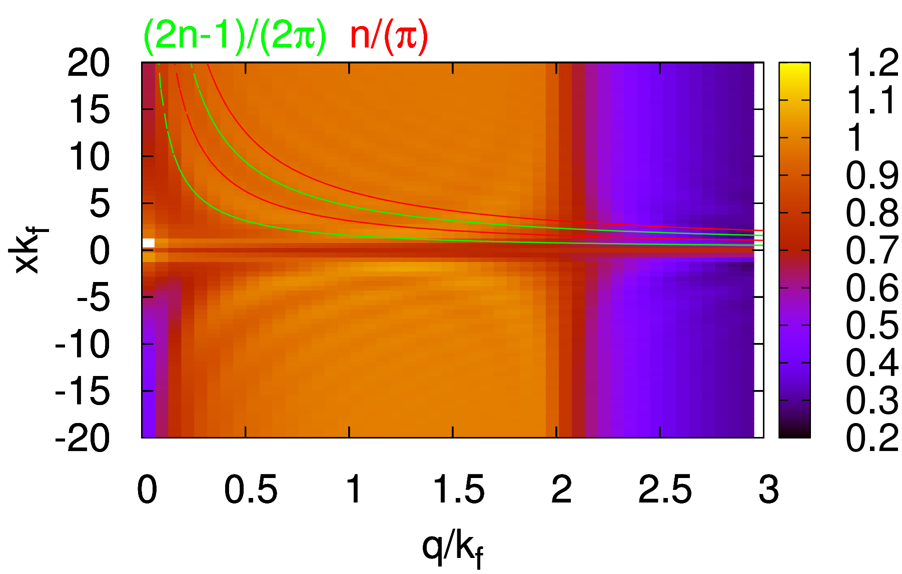

FIG. 6. D wave, $\parallel$ Field ($\mu_B H = 0.6\Delta_0$)

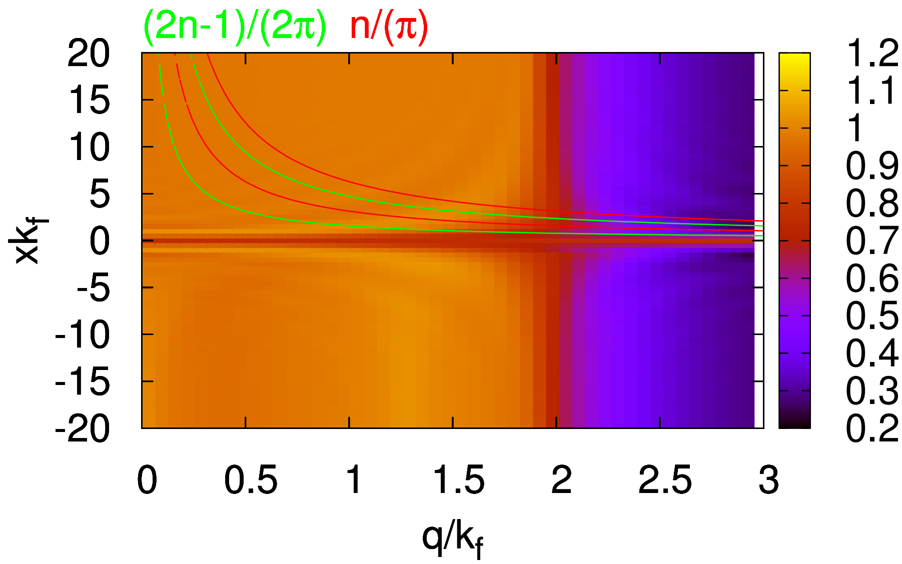

FIG. 7. D wave, $\perp$ Field ($\mu_B H = 0.6\Delta_0$), with $q_y = 0.63$

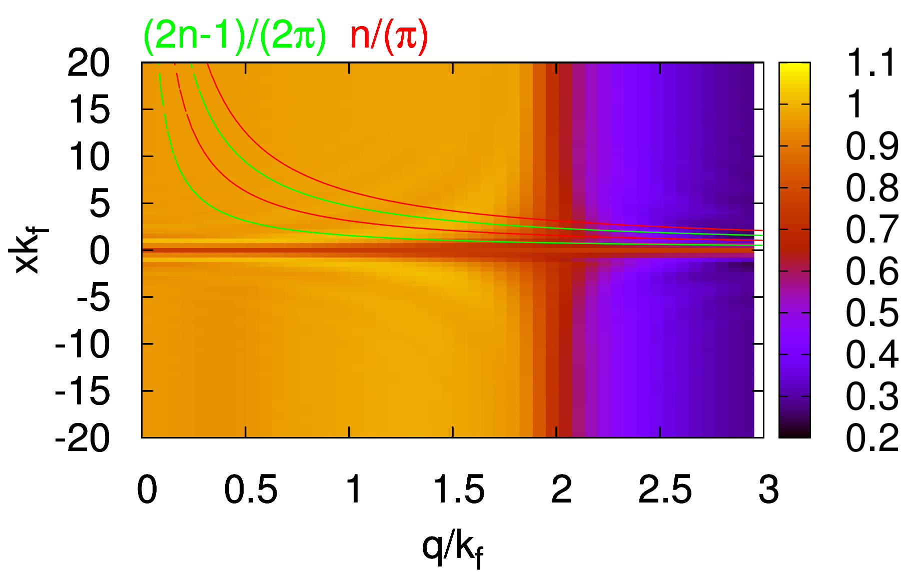

FIG. 8. D wave, $\parallel$ Field ($\mu_B H = 0.6\Delta_0$), with $q_y = 0.63$

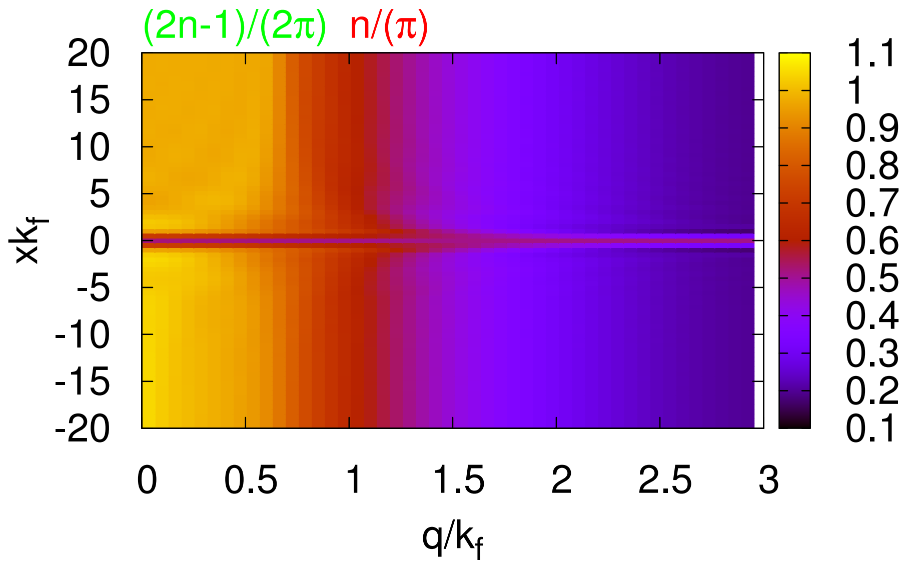

FIG. 9. D wave, $\perp$ Field ($\mu_B H = 0.6\Delta_0$), with $q_y = 1.89$

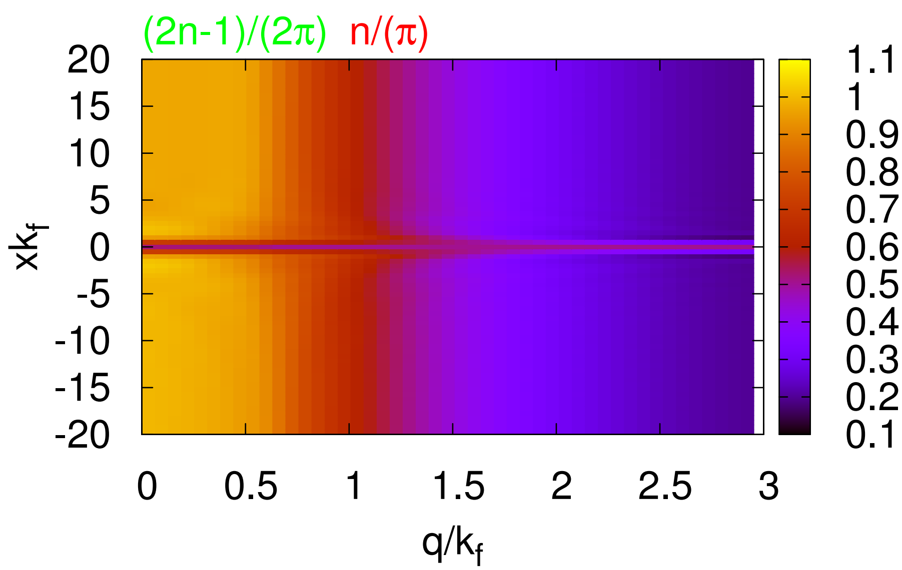

FIG. 10. D wave, $\parallel$ Field ($\mu_B H = 0.6\Delta_0$), with $q_y = 1.89$