The Euler-Lagrangian of the Ein Concept LaTeX

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Abstract

Using the Denavit-Hartenberg Parameters the Lagrangian of Ein was Calculated to be L

1 Question: The Lagrangian

1.1 question about the lagrangian

when i take the derivative of the lagrangian w/ respect to q do i take the derivative of each q? and are the dependent on one another? if i differentiate $q_1 q_2$ with respect to q_1 what do i get?

$$M_i = \begin{bmatrix} m_i & 0 & 0 \\ 0 & m_i & 0 \\ 0 & 0 & m_i \end{bmatrix} \tag{1}$$

$$I_{b,i} = \begin{bmatrix} I_x & 0 & 0 \\ 0 & I_y & 0 \\ 0 & 0 & I_z \end{bmatrix} \tag{2}$$

$$L = K - P \tag{3}$$

$$K = \frac{M_i V_i^2}{2} = \frac{1}{2}\dot{q}^T \left[\sum_{i=1}^{n} M_i J_{v,i}^T J_{v,i} + J_{\omega,i}^T R_i I_{b,i} R_i^T J_{\omega,i} \right] \dot{q} \tag{4}$$

$$J_i = \begin{bmatrix} J_{v,i} \\ J_{\omega,i} \end{bmatrix} = \begin{bmatrix} J_{v_0} & J_{v_1} & \dots & J_{v_n} \\ J_{\omega_0} & J_{\omega_1} & \dots & J_{\omega_n} \end{bmatrix} \tag{5}$$

$$P = M_i gh = \sum_{i=1}^{n} \vec{g}^T M_i \vec{r}_{c,i} \tag{6}$$

2 Results

2.1 Denavit-Hartenberg Parameters

Link_i	a_i	α_i	d_i	θ_i
1	0	$\frac{\pi}{2}$	0	θ_t
2	0	0	L_s	0

2.2 Jacobian

2.2.1 Jacobian of Link One's Center of Mass

$$J_1 = \begin{bmatrix} 0.5l_1 \sin(q_1) & 0 \\ -0.5l_1 \cos(q_1) & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 1 & 0 \end{bmatrix} \tag{7}$$

2.2.2 Jacobian of Link Two's Center of Mass

$$J_2 = \begin{bmatrix} 0.5q_2 \cos(q_1) & 1.0 \sin(q_1) \\ 0.5q_2 \sin(q_1) & -1.0 \cos(q_1) \\ 0 & 6.12323399573677 \cdot 10^{-17} \\ 0 & 0 \\ 0 & 0 \\ 1 & 0 \end{bmatrix} \tag{8}$$

2.3 Lagrangian Results

$$L = 0.5gl_1m_1\sin(q_1) + 0.5gm_2q_2\cos(q_1)$$
$$+ 0.5m_2\dot{q}_2^2 + \dot{q}_1^2\big(0.125l_1^2m_1$$
$$+ 0.166666666666667l_2^2m_2 + 0.523598775598567m_1r_1^2$$
$$+ 0.125m_2q_2^2 + 1.9631809647333\cdot 10^{-33}m_2r_2^2$$

2.4 Torque: Derived from the Lagrangian

$$\tau_k = -0.5gl_1m_1\cos(q_1) + 0.5gm_2q_2\sin(q_1)$$
$$- 0.5gm_2\cos(q_1) - 0.25m_2q_2\dot{q}_1^2$$
$$+ 0.5m_2q_2\dot{q}_1\dot{q}_2 + 1.0m_2\ddot{q}_2$$
$$+ \ddot{q}_1\left(0.25l_1^2m_1 + 0.333333333333333l_2^2m_2 + 1.04719755119713m_1r_1^2\right)$$
$$+ \ddot{q}_1\left(0.25m_2q_2^2 + 3.9263619294666\cdot 10^{-33}m_2r_2^2\right)$$

2.5 State Space

2.5.1 Rotational

$$L_\theta = \frac{d}{dt}\left(\frac{\partial L}{\partial\theta}\right)$$
$$= 1.0gl_1m_1\cos(q_1) - 1.0gm_2q_2\sin(q_1)$$
$$+ 0.5m_2q_2\dot{q}_1\dot{q}_2 + \ddot{q}_1\big(0.25l_1^2m_1$$
$$+ 0.333333333333333l_2^2m_2 + 1.04719755119713m_1r_1^2$$
$$+ 0.25m_2q_2^2 + 3.9263619294666\cdot 10^{-33}m_2r_2^2$$

2.5.2 Radial

$$L_{l_2} = m_2\left(1.0g\cos(q_1) + 0.5q_2\dot{q}_1^2 + 1.0\ddot{q}_2\right) \tag{9}$$

2.5.3 Total

$$
\begin{aligned}
L_{\theta,l_2} = {} & 1.0gl_1m_1\cos(q_1) - 1.0gm_2q_2\sin(q_1) \\
& + 1.0gm_2\cos(q_1) + 0.5m_2q_2\dot{q}_1^2 \\
& + 0.5m_2q_2\dot{q}_1\dot{q}_2 + 1.0m_2\ddot{q}_2 \\
& + \ddot{q}_1\left(0.25l_1^2m_1 + 0.333333333333333l_2^2m_2\right) \\
& + \ddot{q}_1\left(1.04719755119713m_1r_1^2 + 0.25m_2q_2^2\right) \\
& + \ddot{q}_1\left(3.9263619294666\cdot 10^{-33}m_2r_2^2\right)
\end{aligned}
$$

2.5.4 Rotational State Space Representation

$$
\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} x_2 \\ \frac{4.0gl_1m_1\cos(x_1) - 8.0gl_2m_2\sin(x_1) + 4.0gm_2y_1\sin(x_1) + 4.0l_2m_2x_2y_2 - 2.0m_2x_2y_1y_2}{4.0I_1 + 4.0I_2 + l_1^2m_1 + 4.0l_2^2m_2 - 4.0l_2m_2y_1 + m_2y_1^2} \end{bmatrix} \tag{10}
$$

2.5.5 Radial State Space Representation

$$
\begin{bmatrix} \dot{y}_1 \\ \dot{y}_2 \end{bmatrix} = \begin{bmatrix} y_2 \\ -g\cos(x_1) + l_2x_2^2 - 0.5x_2^2y_1 \end{bmatrix} \tag{11}
$$

3 Path Generation

Kinematic Values	Max	Min
$\dot{\theta}$ $\left(\frac{rad}{s}\right)$	0.0	-2.7668
$\ddot{\theta}$ $\left(\frac{rad}{s^2}\right)$	11.1814	-11.1071
T_θ (Nm)	0.1583	-0.1573
$\dot{r}$ $\left(\frac{m}{s}\right)$	0.0	-2.4092
$\ddot{r}$ $\left(\frac{m}{s^2}\right)$	15.3600	-15.0558
T_d (Nm)	1.1588	-1.2503